Planetary Astrophysics: Homework 3

Samuel Factor

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Code for this assignment can be found at `https://github.com/sfxfactor/PlanetaryHW3`

1. Part 1

The method `calcF` in `diskModel` returns the flux density, F_ν, of a star with temperature T, radius R_s, at a distance of D_p. A plot of the flux density of Fomalhaut at orbital radii of 10 and 130 AU is shown in Figure 1. Stellar parameters are from Mamajek (2012) ($T_{\rm eff} = 8590$, $R_* = 1.842 R_\odot$).

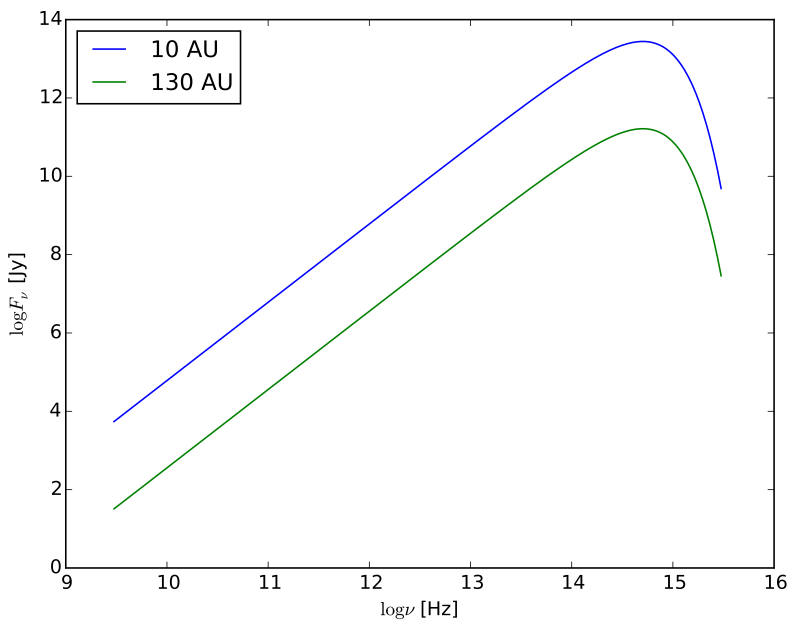

Fig. 1.— Flux density of Fomalhaut

2. Part 2

The method `Pin` in `diskModel` returns the total power absorbed by a grain of radius r_g [μm] and in a radiation field with flux density F_ν. It first imports the Q data obtained from the "Smoothed UV Astronomical Silicate" models found at `http://www.astro.princeton.edu/~draine/dust/dust.diel.html` and extrapolates $Q_{\rm abs}$ to longer wavelengths with a $1/\lambda^2$ power-law. It then numerically integrates, using the trapezoid rule, Equation 1 from $\lambda = 1$ cm – 1 nm, where r_g is now in cgs units. $P_{\rm in}$ for different grain sizes, r_g for different orbital radii D_p are given in Table 1.

$$P_{\rm in} = \pi r_g^2 \int Q_{{\rm abs},\nu} F_\nu d\nu \tag{1}$$

r_g	D_p [AU]	$P_{\rm in}$ [erg s^{-1}]
0.1 μm	10	1.46×10^{-5}
1 μm	10	6.25×10^{-3}
10 μm	10	6.61×10^{-1}
1 mm	10	7.13×10^{3}
0.1 μm	130	8.66×10^{-8}
1 μm	130	3.70×10^{-5}
10 μm	130	3.91×10^{-3}
1 mm	130	4.22×10^{1}

Table 1: $P_{\rm in}$ for Fomalhaut

3. Part 3

The method `Teq` in `diskModel` returns the equilibrium temperature of a grain of radius r_g [μm] and incident power $P_{\rm in}$. Similar to `Pin`, it first imports the Q data and extrapolates $Q_{\rm abs}$ to longer wavelengths with a $1/\lambda^2$ power-law. It then finds T such that Equation 2, where B_ν is the Planck function, is satisfied, using `scipy.optimize.root`. Again the integration uses the trapezoid rule. The luminosity of each grain is then simply equal to the input power given in Table 1.

$$P_{\rm in} - 4\pi^2 r_g^2 \int_{\lambda=1\ \rm cm}^{\lambda=1\ \rm nm} Q_{{\rm abs},\nu} B_\nu(T) d\nu = 0 \tag{2}$$

The method `calcFQ` in `diskModel` returns the flux density, F_ν of a grain with temperature T, radius r_g, at a distance (from the observer) of D. Figures 2 and 3 show the flux density of a given set of grains at orbital radii of 10 and 130 AU, respectively, around Fomalhaut at a distance of 7.7 pc (Mamajek 2012). The general trends of the flux densities make sense: smaller grains are fainter, as they have less surface area, but are hotter, as they are less like black-bodies.

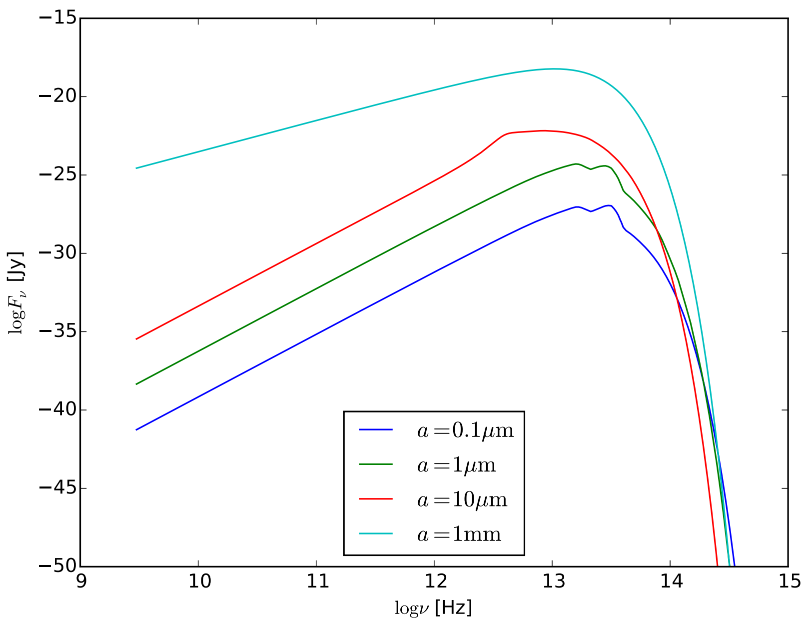

Fig. 2.— Flux density of grains at 10 AU around Fomalhaut.

4. Part 4

An SED of Fomalhaut from Su et al. (2013) is shown in Figure 4. By scaling the peak flux density of the SED of a single grain up to the full SED, we can estimate

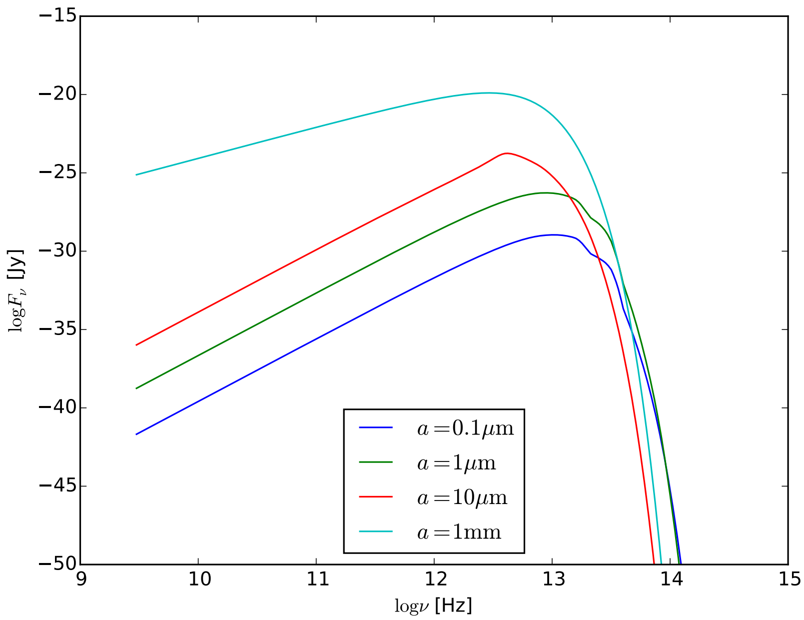

Fig. 3.— Flux density of grains at 130 AU around Fomalhaut.

the number of grains in the disk. If we then assume a density of 2 g cm^{-3} we can also calculate a rough disk mass. These values are given in Table 2. Based on the peak of the spectrum and slope of the long wavelength emission, the qualitative best fit grain size for the 10 and 130 AU belt is 1 mm and 10 μm, respectively.

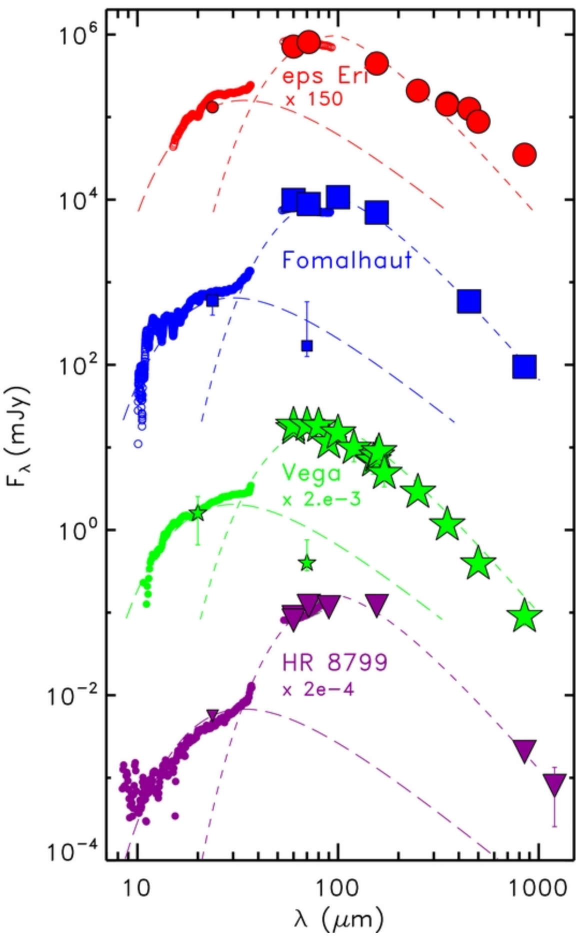

Fig. 4.— SED of the two component disk around Fomalhaut from Su et al. (2013).

r_g	D_p [AU]	N	$M_{\rm dust}$ [g]
0.1 μm	10	5×10^{26}	4×10^{12}
1 μm	10	1×10^{24}	1×10^{13}
10 μm	10	9×10^{21}	7×10^{13}
1 mm	10	1×10^{18}	8×10^{15}
0.1 μm	130	9×10^{29}	7×10^{15}
1 μm	130	2×10^{27}	2×10^{16}
10 μm	130	6×10^{24}	5×10^{16}
1 mm	130	8×10^{20}	7×10^{18}

Table 2: Number of Grains and Dust Mass in Fomalhaut

5. Part 5

Since we have already calculated the power absorbed by each grain it is simple to calculate the force due to radiation pressure ($F_{\rm rad} = P_{\rm in}/c$) and Poynting-Robertson drag ($F_{\rm pr} = P_{\rm in}\sqrt{GM_*/D}/c^2$, where D is the orbital radius and M_* is the mass of the central star). If we define $\beta = F_{\rm rad}/F_{\rm grav}$, we can calculate the characteristic timescale, τ in years, for dust grains to be removed from the system from Equation 3, where r is the initial orbital radius in AU and M_* is the mass of the central star in $M_\odot$ (Klačka & Kocifaj 2008). These values are given in Table 3.

$$\tau = 400\frac{r^2}{\beta M_*} \tag{3}$$

REFERENCES

Klačka, J., & Kocifaj, M. 2008, Monthly Notices of the Royal Astronomical Society, 390, 1491

Mamajek, E. E. 2012, ApJ, 754, L20

Su, K. Y. L., et al. 2013, ApJ, 763, 118

r_g	D_p [AU]	F_{rad} [dyn]	F_{pr} [dyn]	β	τ [yr]
0.1 μm	10	4.9×10^{-16}	2.1×10^{-20}	5.1	4.1×10^{3}
1 μm	10	2.1×10^{-13}	9.1×10^{-18}	2.2	9.5×10^{3}
10 μm	10	2.2×10^{-11}	9.6×10^{-16}	0.23	9.0×10^{4}
1 mm	10	2.4×10^{-7}	1.0×10^{-11}	2.5×10^{-3}	8.4×10^{6}
0.1 μm	130	2.9×10^{-18}	3.5×10^{-23}	5.1	6.9×10^{5}
1 μm	130	1.2×10^{-15}	1.5×10^{-20}	2.2	1.6×10^{6}
10 μm	130	1.3×10^{-13}	1.6×10^{-18}	0.23	1.5×10^{7}
1 mm	130	1.4×10^{-9}	1.7×10^{-14}	2.5×10^{-3}	1.4×10^{9}

Table 3: Number of Grains and Dust Mass in Fomalhaut

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