

# Stellar Structure and Evolution Numerical Project

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Code for this project can be found at  
<https://github.com/sfxfactor/StellarNumericalProj>.

## 1 ZAMS and Starting Model

Figure 1 is a reproduction of Figure 1 of Bromm et al. (2001). As I used the assumptions in the paper, and not real models, the curves are straight and not curved as in Bromm et al. (2001). I used two prescriptions for the effective temperature to see if it would bring the curves into better agreement with the curves in Bromm et al. (2001). One prescription was from Equation 6 of Bromm et al. (2001) and the other was from the Eddington luminosity,

$$T_{eff} = \left( \frac{L_{edd}}{4\pi R^2 \sigma_{SB}} \right)^{1/4}. \quad (1)$$

Using my code from Problem Set 2, I generated an  $n = 3$  polytropic model of a  $100M_{\odot}$  Pop III star. The pressure and density structure is shown in Figures 2 and 3 respectively. To calculate a temperature I assumed  $\beta = 0.58$ . This value comes from Equation 19.56 in KW<sup>2</sup>

$$\frac{1 - \beta}{\mu^4 \beta^4} = 3.02 \times 10^{-3} \left( \frac{M}{M_{\odot}} \right)^2. \quad (2)$$

The temperature is then

$$T = \left( (1 - \beta) P \frac{3}{a} \right)^{1/4} \quad (3)$$

This lead to a temperature structure shown in Figure 4.

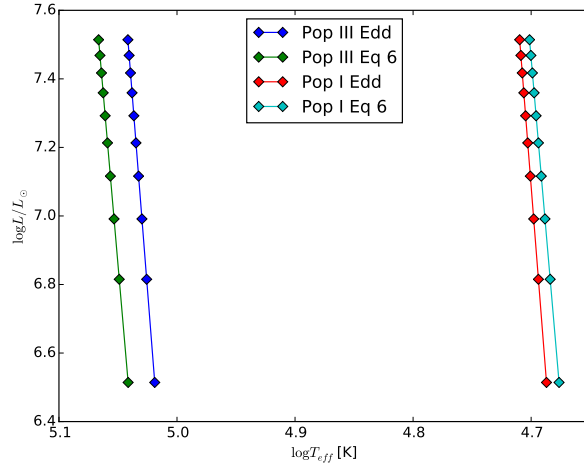


Fig. 1: HR Diagram for high mass ( $M = 100 - 1000 M_{\odot}$ ) Pop III (left) and Pop I stars. Different points are temperatures given by Equation 6 of Bromm et al. (2001) and the Eddington luminosity, Equation 1.

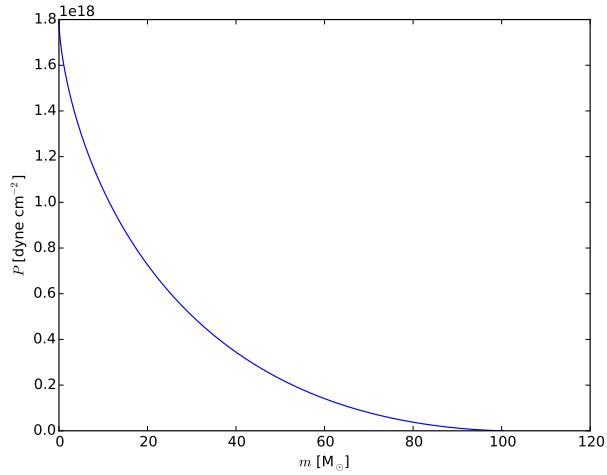


Fig. 2: Pressure as a function of enclosed mass for an  $n = 3$  polytropic model of a  $100 M_{\odot}$  Pop III star.

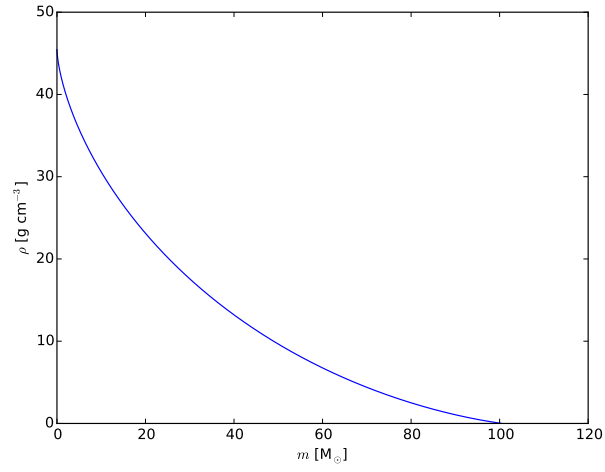


Fig. 3: Density as a function of enclosed mass for an  $n = 3$  polytropic model of a  $100 M_{\odot}$  Pop III star.

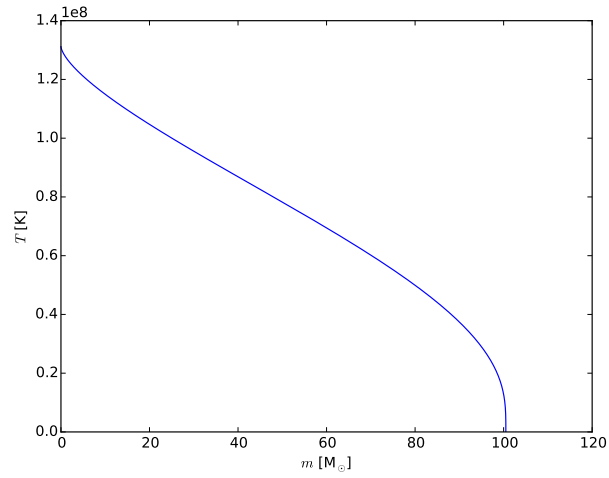


Fig. 4: Temperature as a function of enclosed mass for an  $n = 3$  polytropic model of a  $100 M_{\odot}$  Pop III star.

## 2 Evolving off the MS

I tried very hard to get the `STELLAR` code to work with a  $100 M_{\odot}$  Pop III (metal free) star though was unsuccessful. You can see my commit history on github to look at the specific changes I tried but I will outline the important ones below.

First, the initial conditions, polytropic index, and polytrope constants (in `polytr.F`) must be changed. Then I changed the pressure prescription in `invstate.F` to fix  $\beta$  at the value mentioned above (see Equation 2. Second I replaced all calls to `opacity` in `gi.F` with Thompson scattering opacity  $0.2(1 + X)$ . In `atmos.F` I did the same thing but added  $H^-$  opacity,  $(1 - X - Y)\rho^{1/2}T^9$ . This was of course added in inverse according to  $1/\kappa = 1/\kappa_T + 1/\kappa_{H^-}$ . Finally I changed the luminosity prescription in `polytr.F` to Equation 3 of Bromm et al. (2001).

When looking at the output of the failed stellar run I noticed that the energy generation from the CNO cycle was much too small ( $\sim 10^7$  erg/s when the luminosity is  $\sim 10^{40}$ ). If this could be brought up the model may have converged.

If the  $100 M_{\odot}$  Pop III star would have evolved past the MS, the model would have not been able to model the pulsations caused by pair instability. This causes the rapid burning of oxygen and silicon, stopping the collapse with an enormous explosion. For stars  $\sim 140 - 260 M_{\odot}$  (Heger et al., 2003) this completely disrupts the star leaving behind no remnant. For masses less than this but greater than  $\sim 100 M_{\odot}$  the pulsations through off large amounts of mass while the core collapses into a black hole.

Shown in Figure 5 are evolution tracks for 1, 3, 4, 5, and  $10 M_{\odot}$  solar metallicity stars. You will notice that the 5 and  $10 M_{\odot}$  models fail when they reach the MS. The `STELLAR` code seems to be very sensitive to the initial  $R$  and  $T_{eff}$  of the polytrope model. For example, the initial  $R$  and  $T_{eff}$  of the 3 and  $4 M_{\odot}$  (which evolved past the MS) were almost identical. If I changed the  $4 M_{\odot}$  initial conditions a small amount, the star would evolve to the MS and the model would fail, much like the 5 and  $10 M_{\odot}$  models.

Figures 6, 7, and 8 show the initial and final pressure, density, and temperature structures for the  $4 M_{\odot}$  model, the highest mass model that evolved off the MS.

## References

- Bromm, V., Kudritzki, R. P., & Loeb, A. 2001, *ApJ*, 552, 464  
Heger, A., Woosley, S. E., Fryer, C. L., & Langer, N. 2003, in From Twilight to Highlight: The Physics of Supernovae, ed. W. Hillebrandt & B. Leibundgut, 3–540

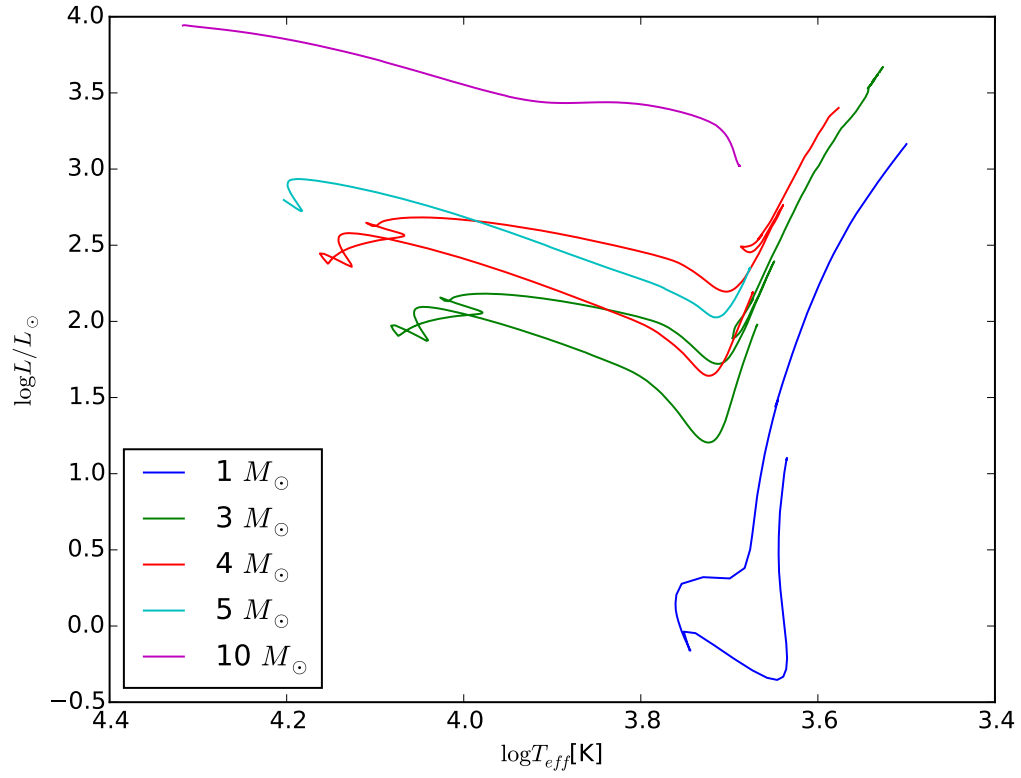


Fig. 5: Evolutionary tracks for solar metallicity stars. The high mass models fail when they reach the MS.

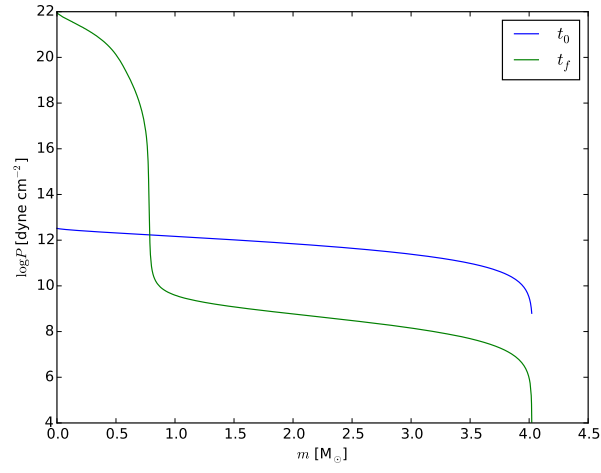


Fig. 6: Pressure as a function of enclosed mass for a  $4 M_{\odot}$  Pop I star at the beginning and end of its life.

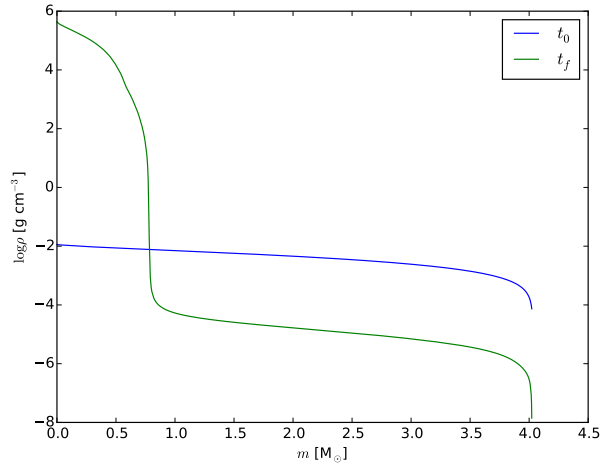


Fig. 7: Density as a function of enclosed mass for a  $4 M_{\odot}$  Pop I star at the beginning and end of its life.

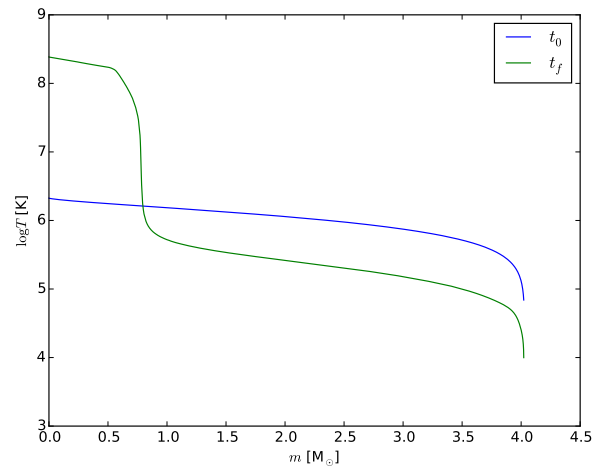


Fig. 8: Temperature as a function of enclosed mass for a  $4 M_{\odot}$  Pop I star at the beginning and end of its life.