

Polarization Metrics and Opinions Inference in Multipolar Systems

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Based on the idea that a population is perfectly polarized when divided in two groups of the same size and contrasting opinions, we propose a general methodology to study and measure the emergence of polarization from social interactions. We will present polarization metrics to opinion distributions. In particular, we propose the *polarization index* [1] for bipolar systems. This metric is very restrictive since the maximum value is only achieved when the two conditions (groups of the same size and with contrasting opinions), are fulfilled. It was applied to measure the political polarization in several dichotomous Twitter conversations finding a good agreement between our results and offline data. In particular, we analyzed a Venezuelan case [1], the second round of the 2017 Chilean elections [2], and the Catalan Independence issue [3]. For the case of multipolar opinion distributions, we present the trace of the covariance matrix (the total variation) as a global polarization metric [4]. This metric achieves the maximum value when there are only extreme opinions and they are uniformly distributed among all poles. Accordingly, in a similar way to the polarization index, it not only measures how extreme the opinions are but also how evenly is the population divided into the considered factions. In order to further characterize the polarization of multipolar systems, we carry out a principal components analysis to identify the directions of maximum polarization. We will present several case studies corresponding to tetra polar and pent polar electoral campaigns showing that this framework unveils the natural ideological axes of the system.

We also present a method to infer opinions in multidimensional contexts through networks of interactions based on an extension of the DeGroot learning process [1, 3]. To do this, we first introduce a generalization to the ideological space of multipolar systems [4] with n opinion poles modeled by placing each pole (for example, the parties in a multi-party democracy) at the vertex of a regular simplex of dimension $n-1$ (a multidimensional generalization of an equilateral triangle, which would correspond to a tripolar system). This way every pole is at the same distance of the others, avoiding the introduction of a priori biases. By applying this methodology to empirical Twitter data from multi-party Spanish general elections, we find that the main axis of polarization is the left-wing / right-wing split [4]. However, our most striking finding comes from the secondary axes of polarization, as they reveal non-trivial tensions specific to each system. These tensions can be understood in terms of the underlying socio-political context.

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