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GRUPO DE SISTEMAS COMPLEJOS
GSC
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Polarization Metrics and Opinions Inference in Multipolar Systems

Rosa M. Benito

Grupo de Sistemas Complejos
Universidad Politécnica de Madrid
rosamaria.benito@upm.es

Outline

- Model of Opinion Inference on Networks
 - Bipolar systems
 - Multipolar systems
- Polarization Metrics
 - Bipolar Case: The Polarization Index
 - Multipolar Cases. Covariance matrix
- Applications. Empirical Studies:
 1. Bipolar systems:
 - Second Round 2017 Chilean Presidential elections, 2017
 - Catalonian Independence issue, 2017
 - Venezuela conversation about Hugo Chavez, 2015
 2. Multipolar systems:
 - Spanish elections (2015). Tetrapolar case.
 - Spanish elections (2019). Pentapolar case.

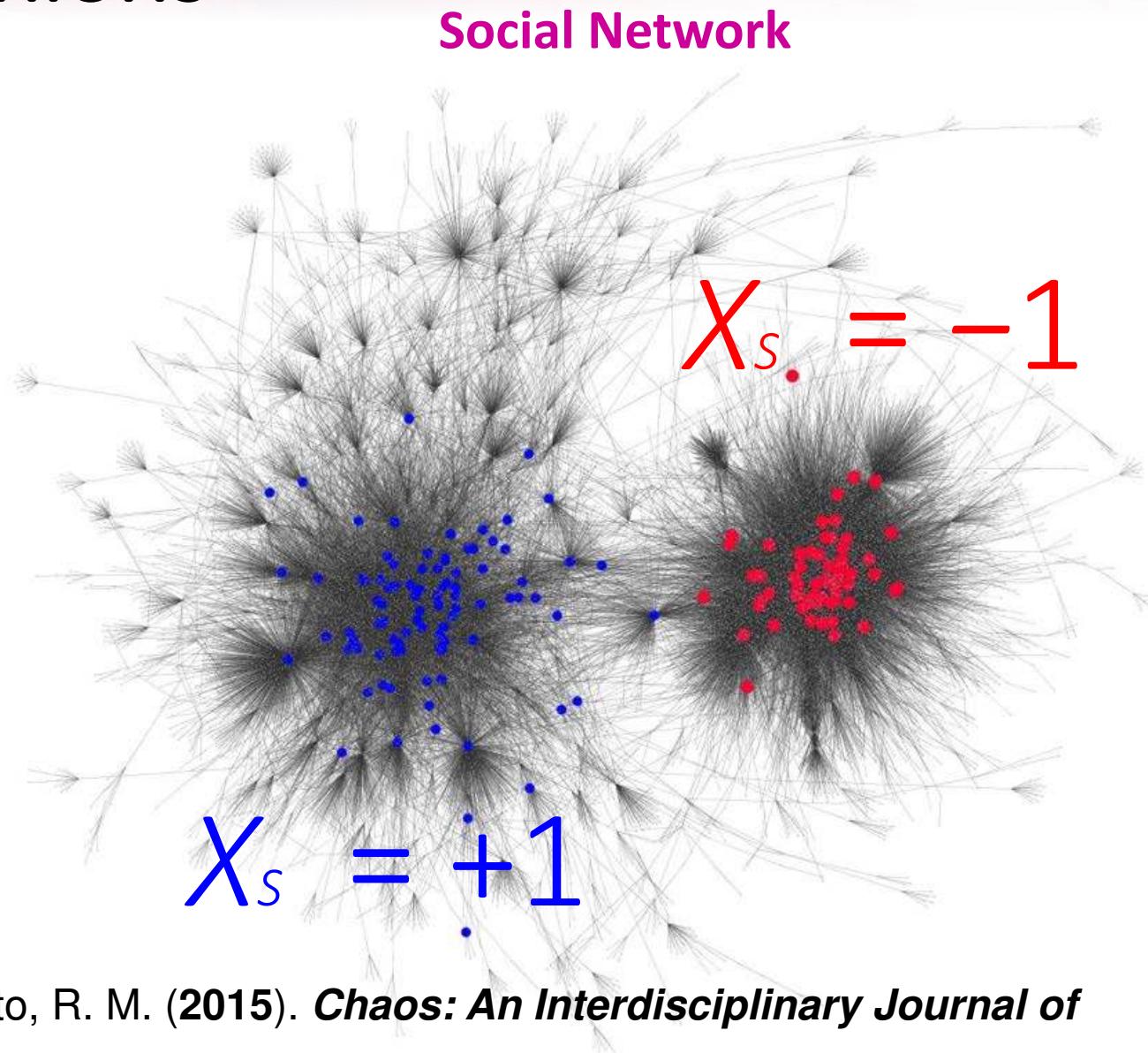
Model to Estimate Opinions

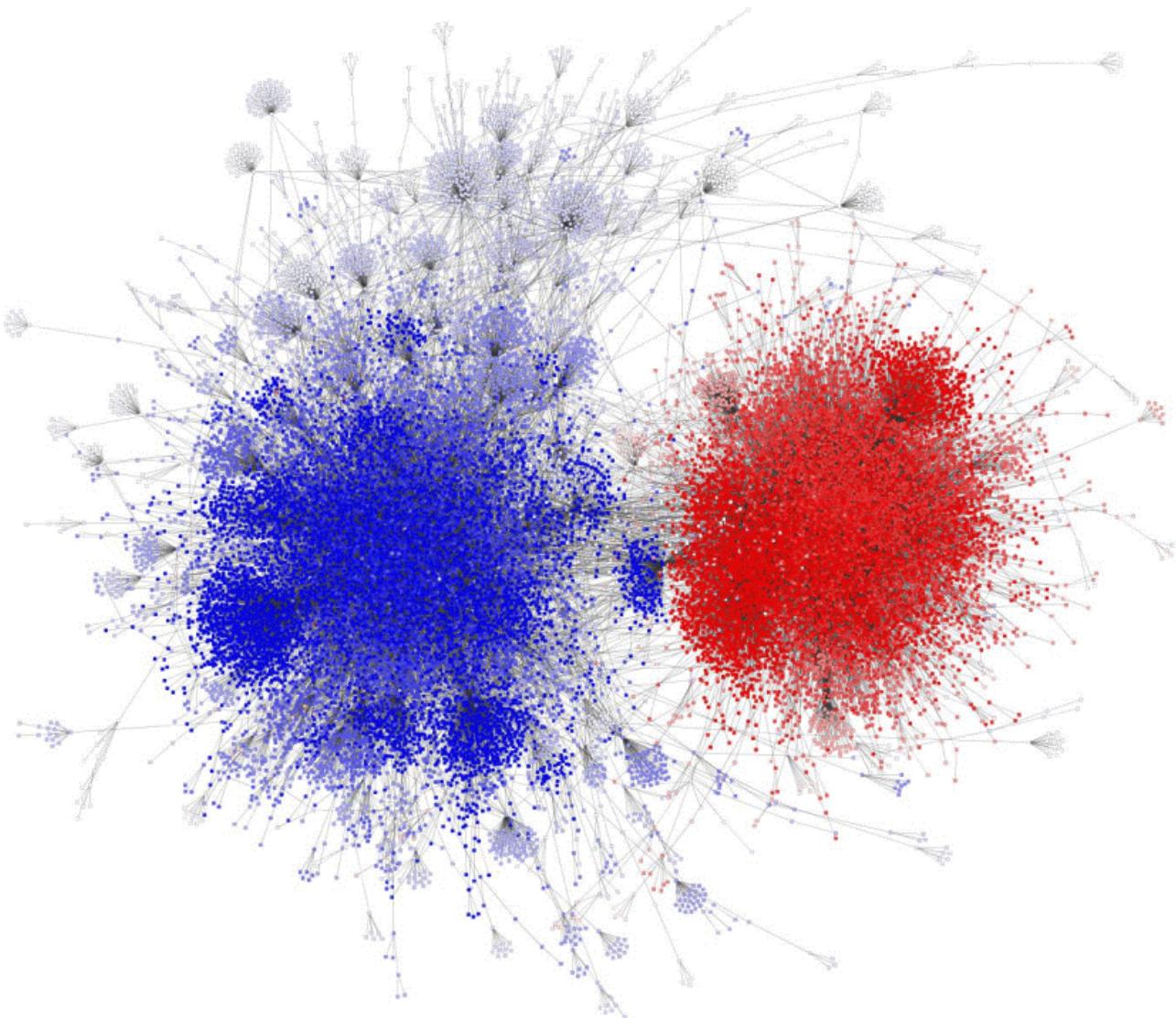
Two type of users:

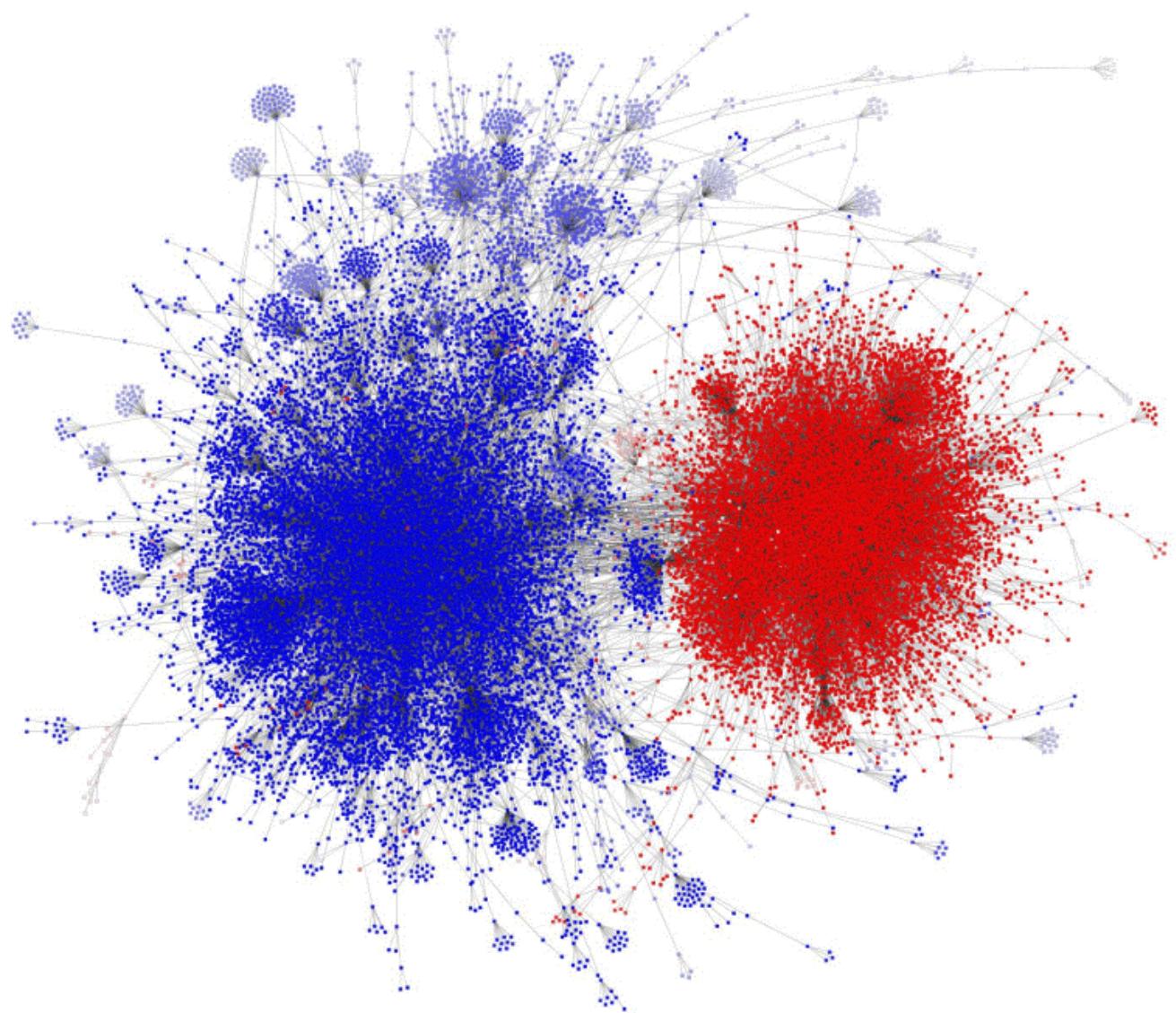
- **Elite:** fixed and antagonist opinions X_s
- **Listeners:** iteratively update their opinions:

$$X_i(t) = \frac{\sum_j A_{ij} X_j(t-1)}{k_i}$$

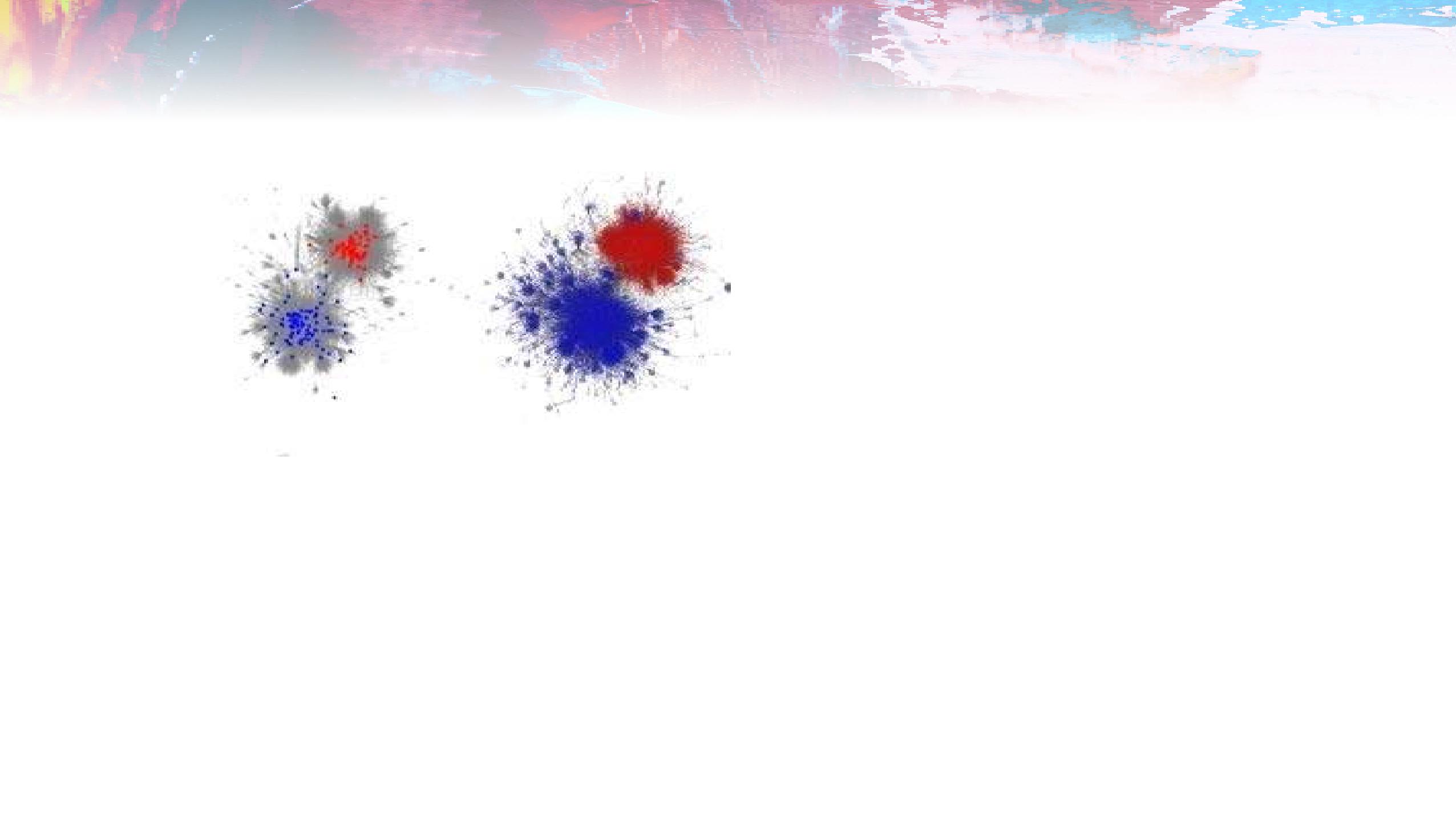
$$X_i(t=0) = 0$$





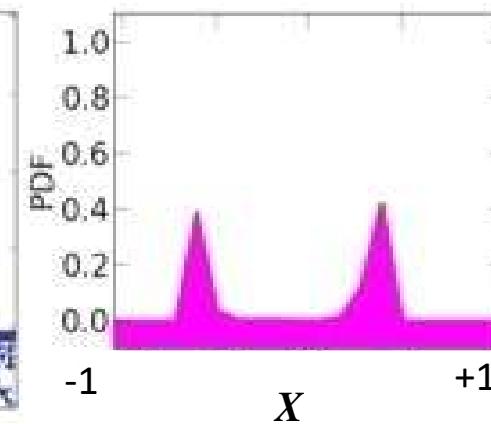
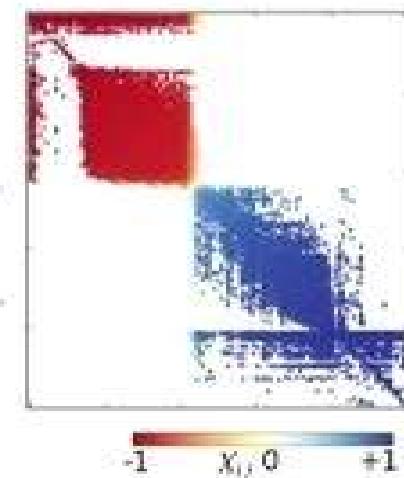






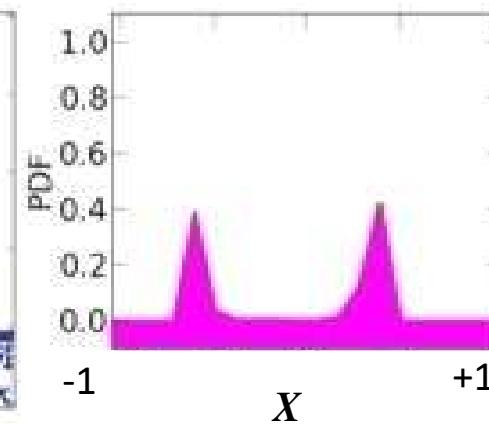
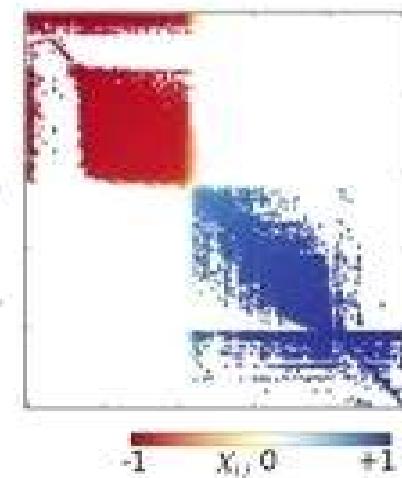
Distribution of opinions

Color Adjacency Matrix



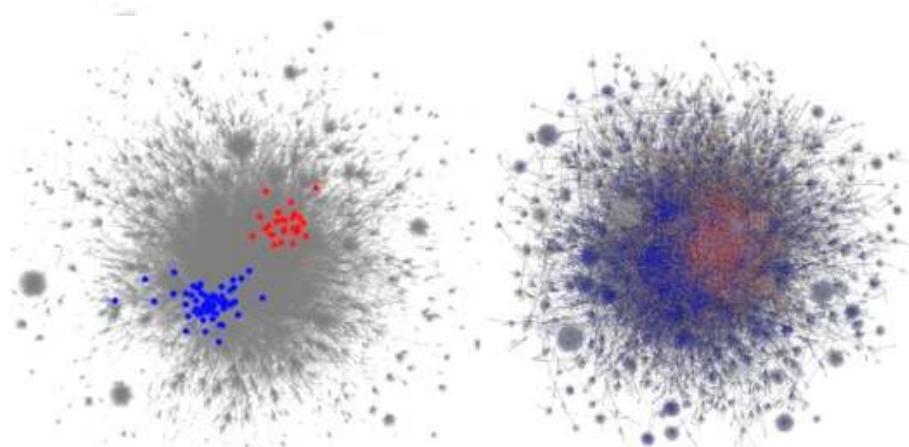
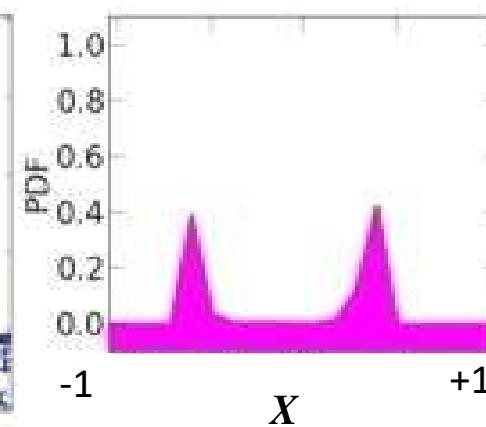
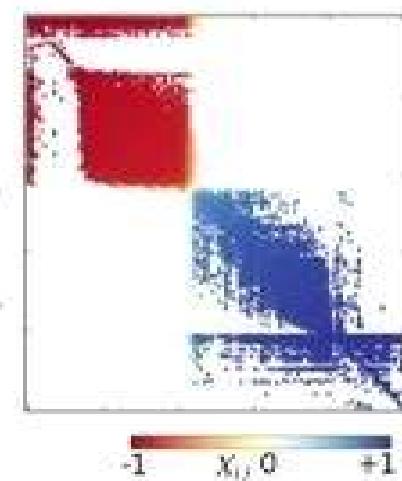
Distribution of opinions

Color Adjacency Matrix



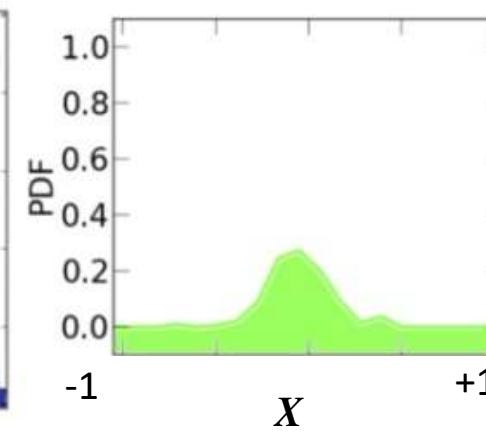
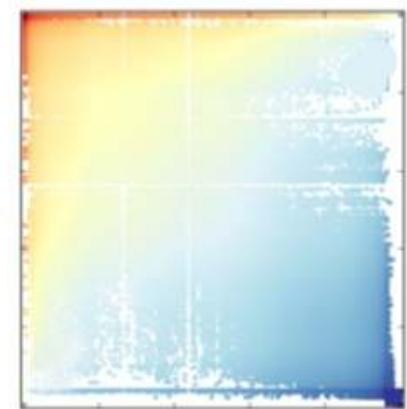
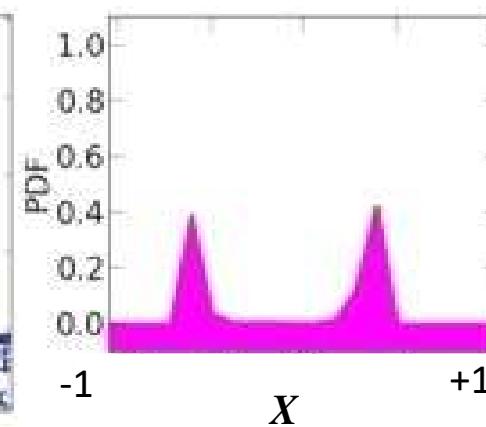
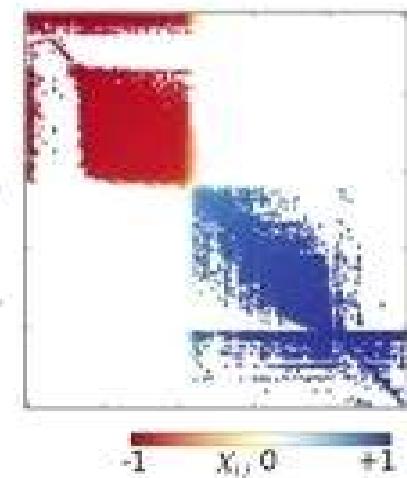
Distribution of opinions

Color Adyacency Matrix



Distribution of opinions

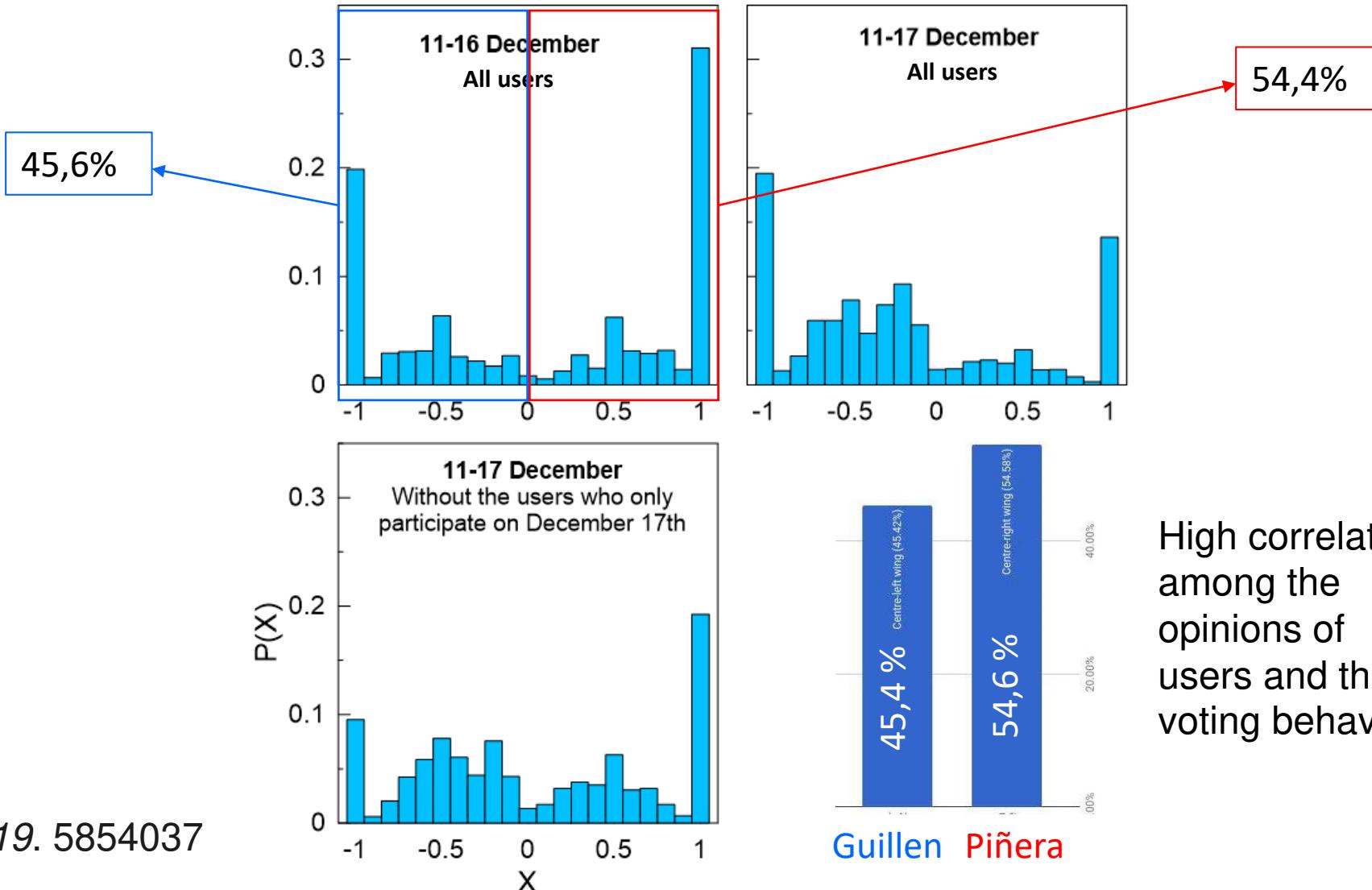
Color Adyacency Matrix



Opinion Polarization during a Dichotomous Electoral Process.

2nd round 2017 Chilean Presidential Elections

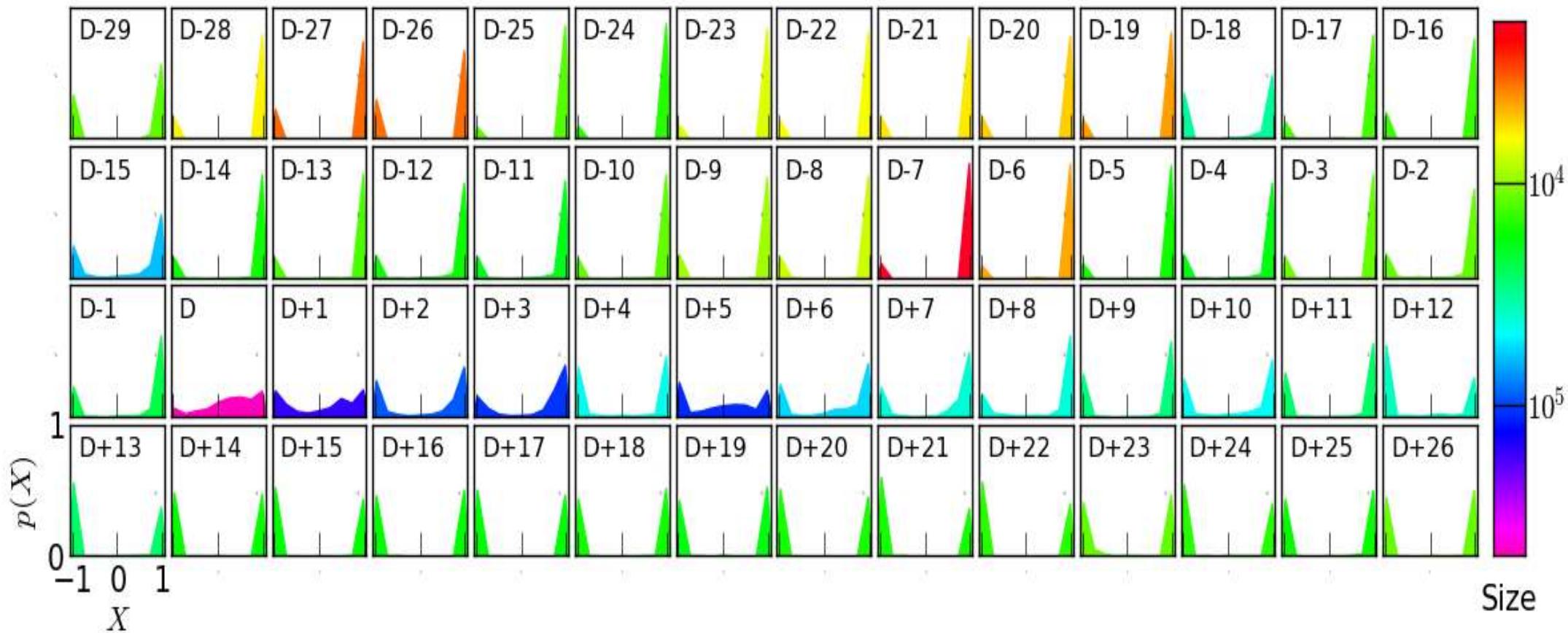
Opinion distribution



Results: Opinion distributions

Twitter conversation on the late Venezuelan president, Hugo Chavez

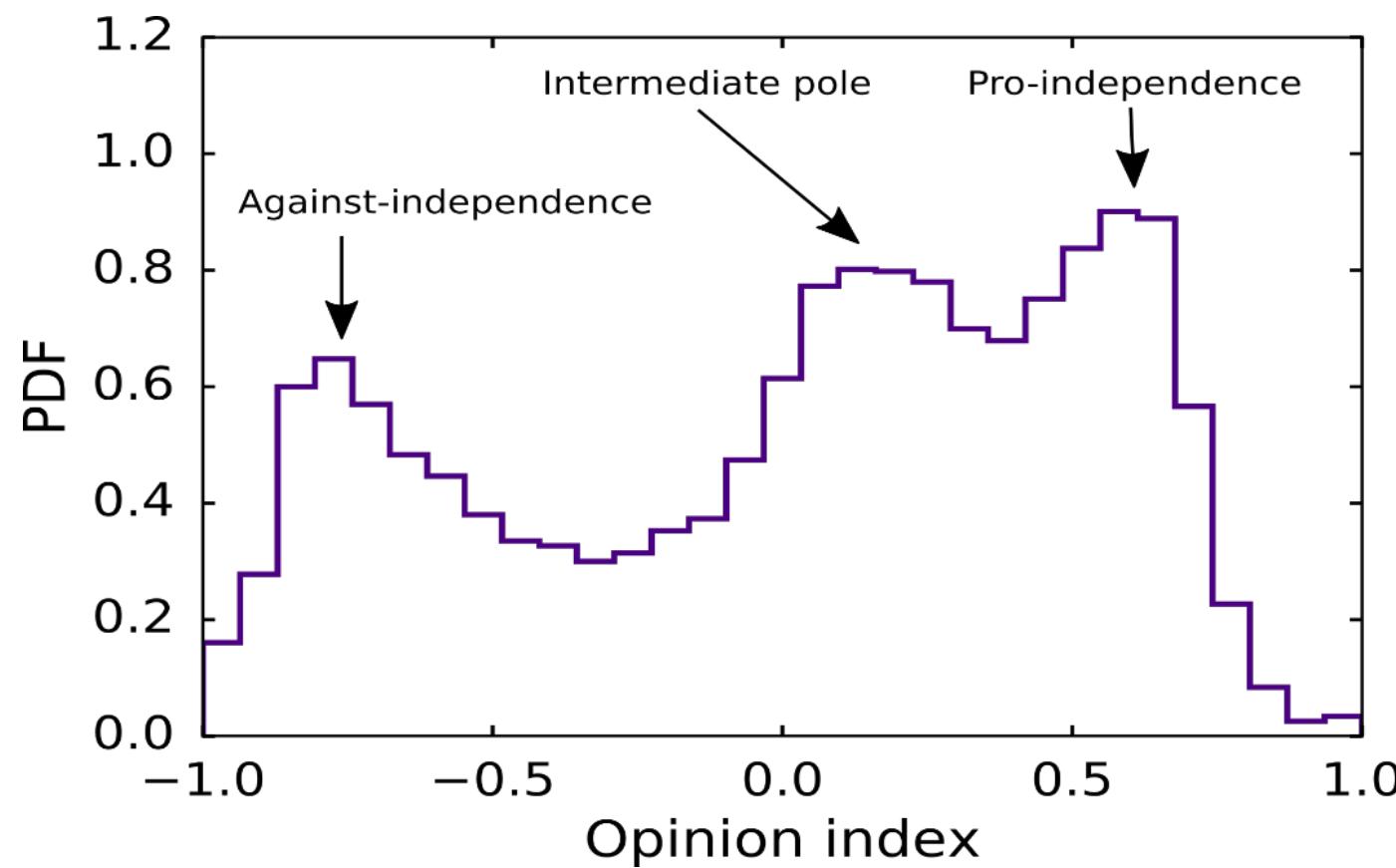
Time evolution of ideological value Probability Density Functions



Measuring political polarization: Twitter shows the two sides of Venezuela

A. J. Morales, J. Borondo, J. C. Losada, and R. M. Benito, **Chaos 25, 033114 (2015)**

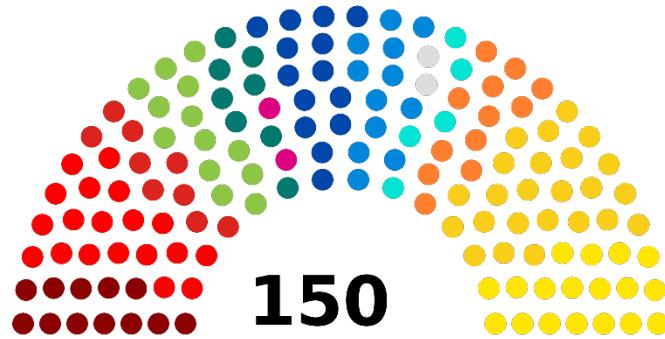
Twitter conversation about the Catalan independence issue around the 1-O referendum



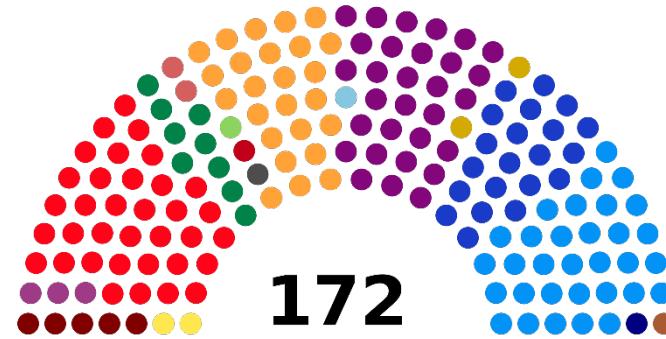
Atienza-Barthelemy, J., Martin-Guitierrez, S., Losada, J. C., Benito, R.M.
Relationship between ideology and language in the Catalan Independence
context. *Scientific Reports*, 2019. DOI: 10.1038-s41598-019-53404-x

Multipolar systems

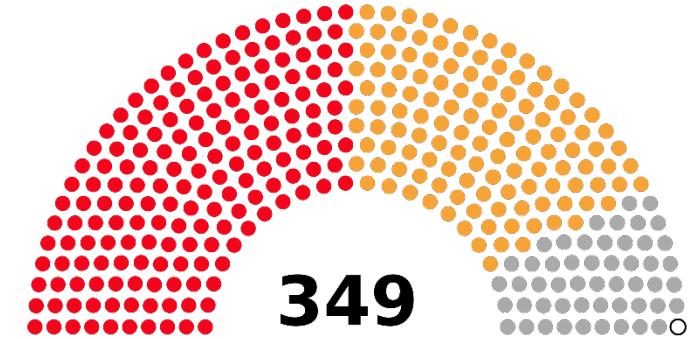
Belgium



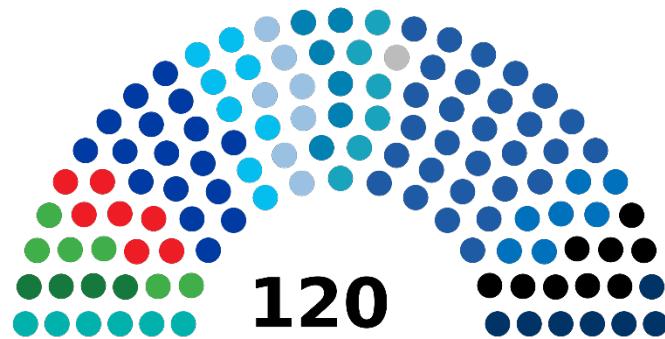
Colombia



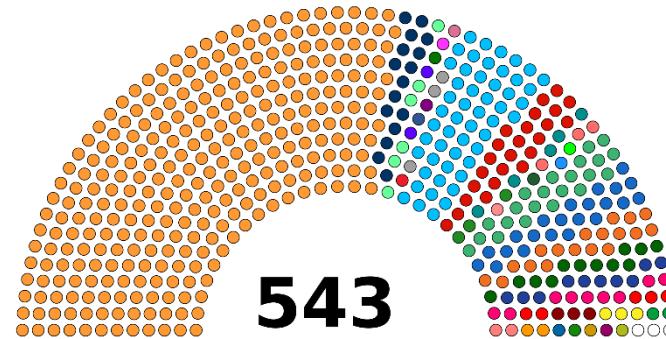
Kenya



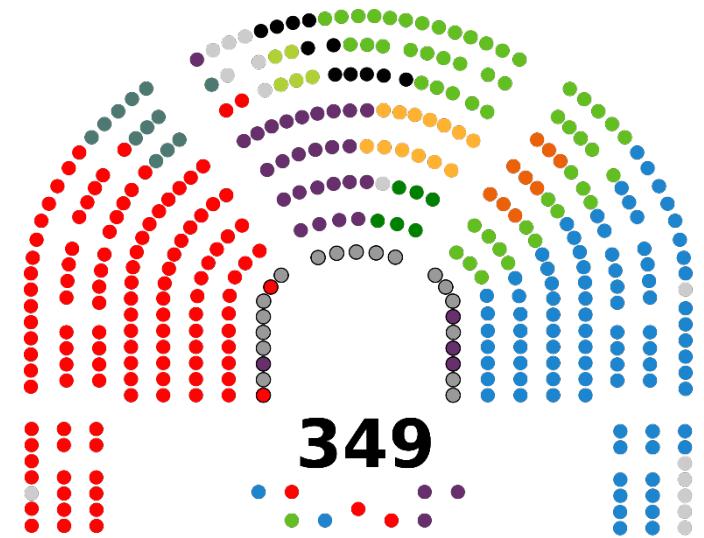
Israel

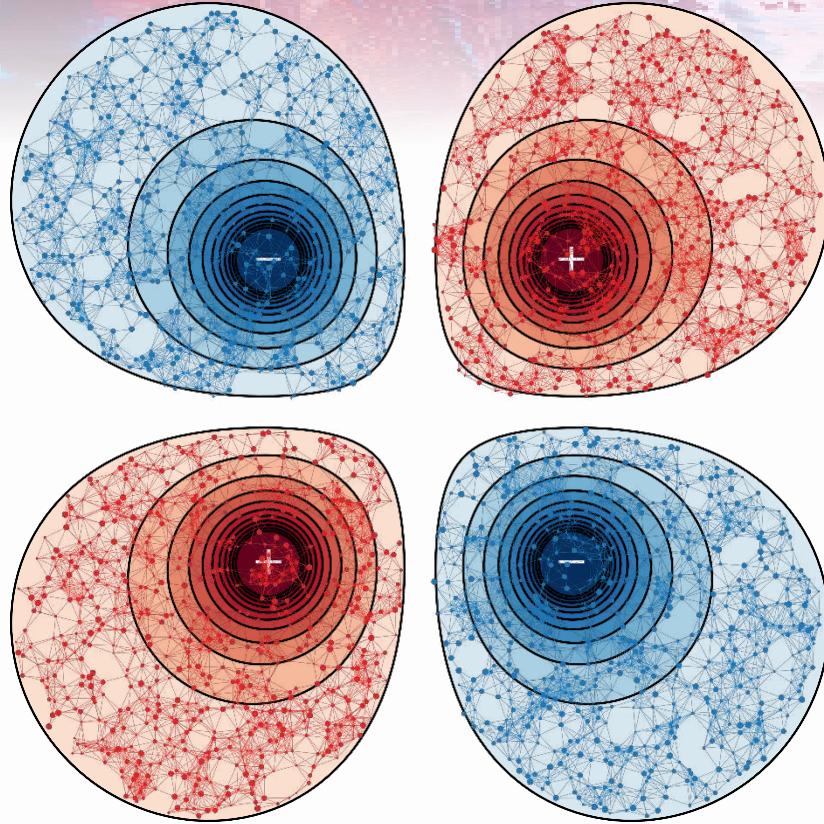


India



Spain





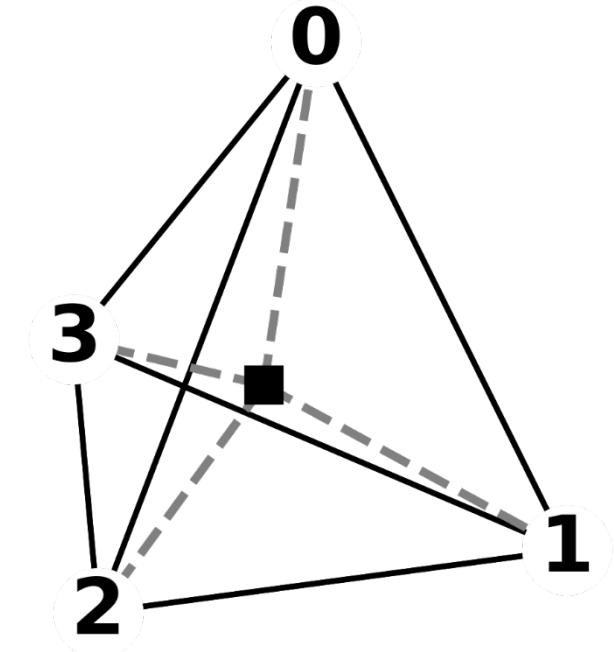
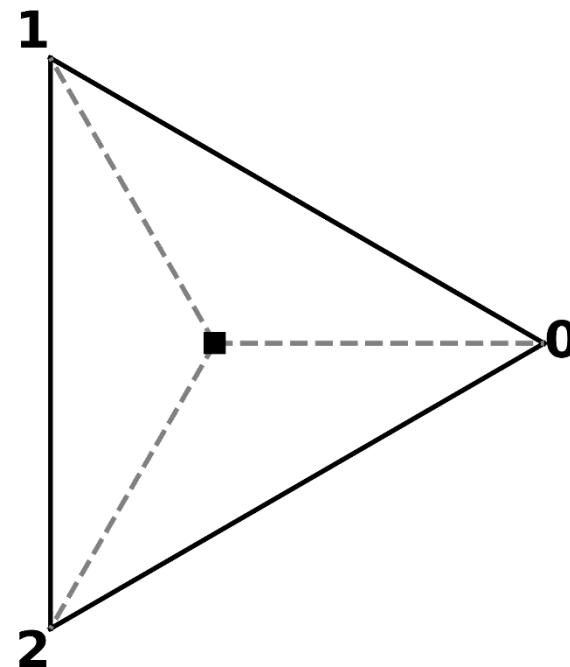
Modeling a multipolar opinion space

Pole vectors: The opinion simplex

Bipolar System
space-1D

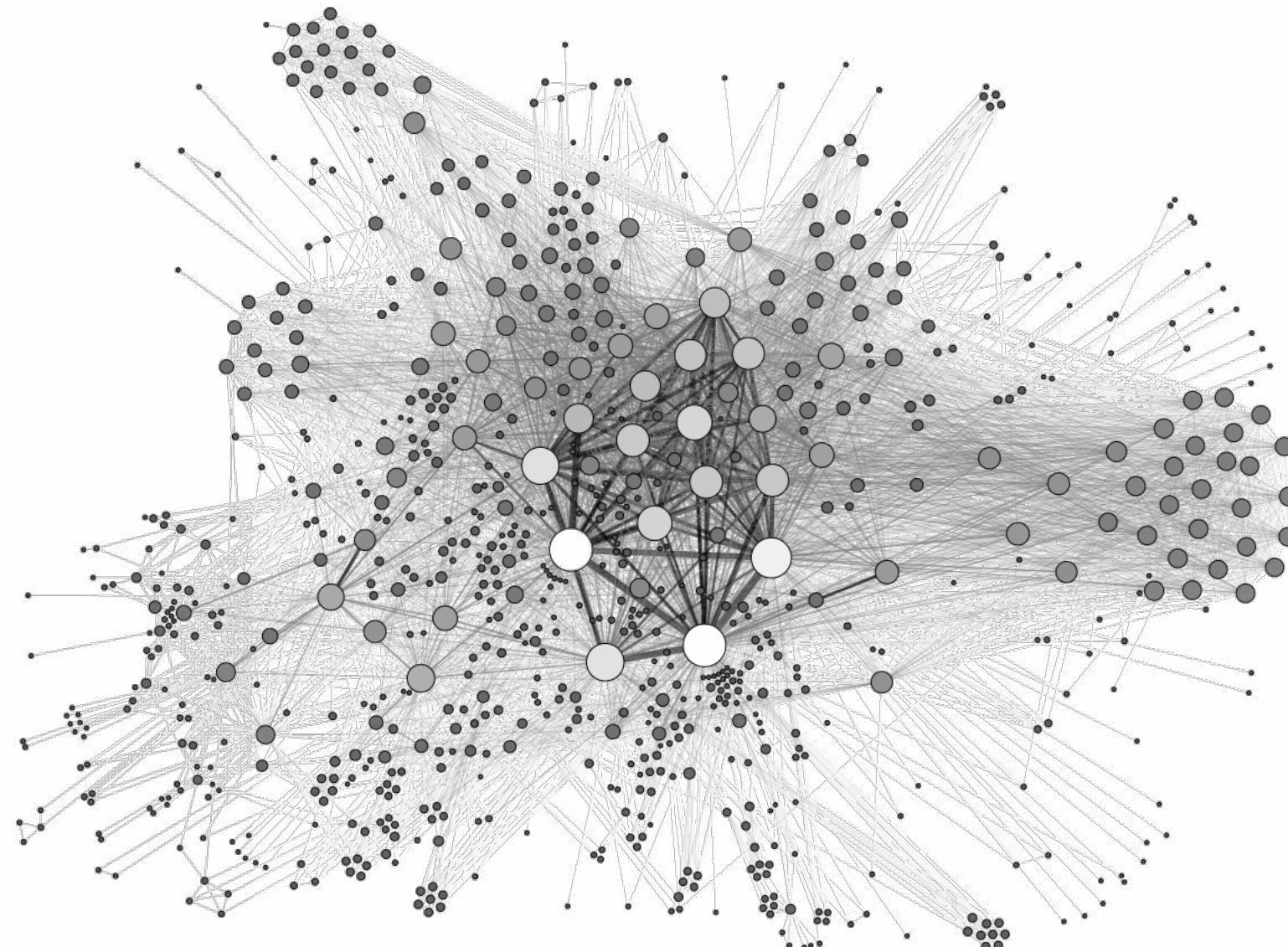
Tripolar System
space-2D

Quadripolar System
space-3D



Interaction networks: elite and listeners

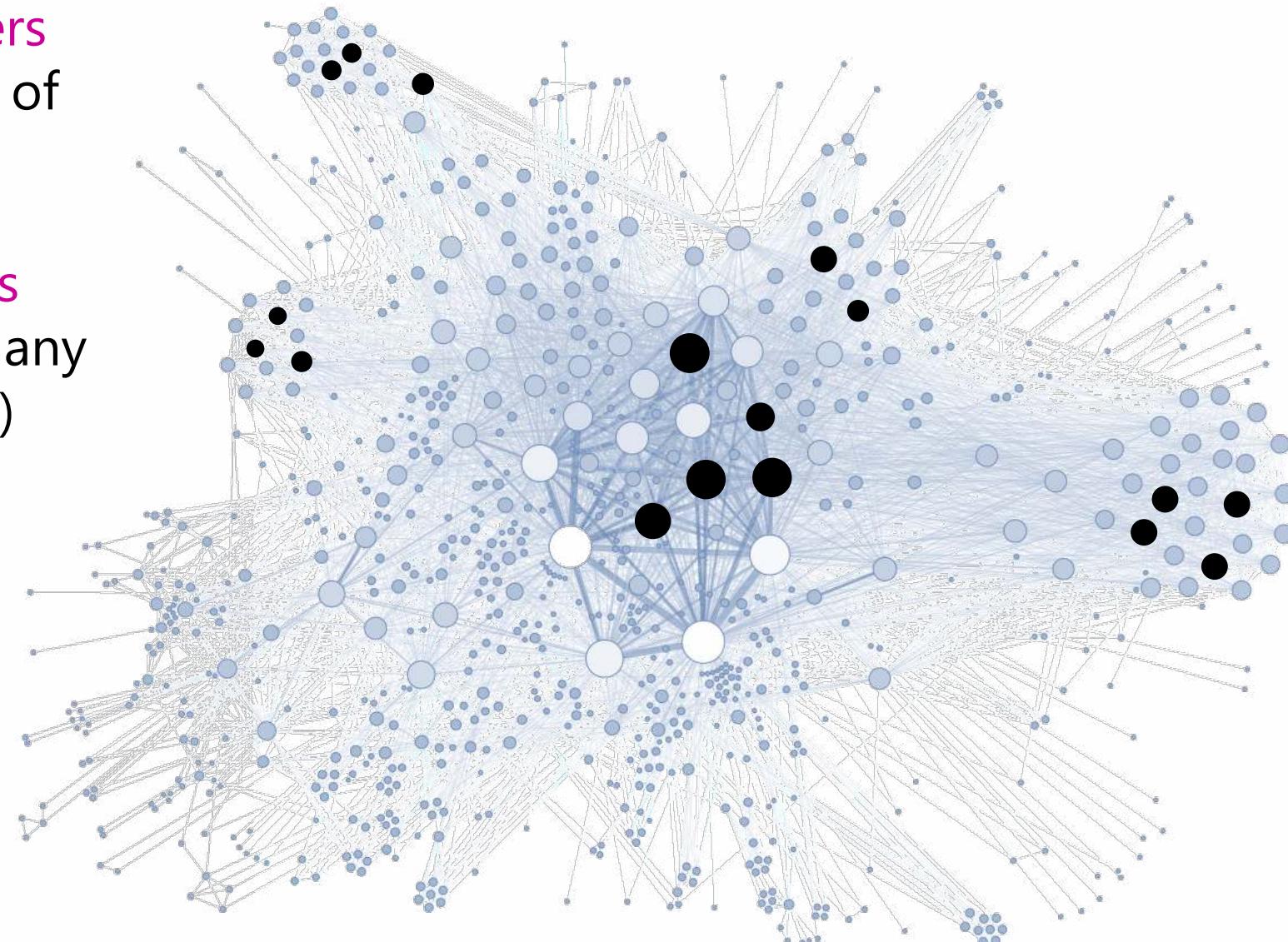
Retweet
network



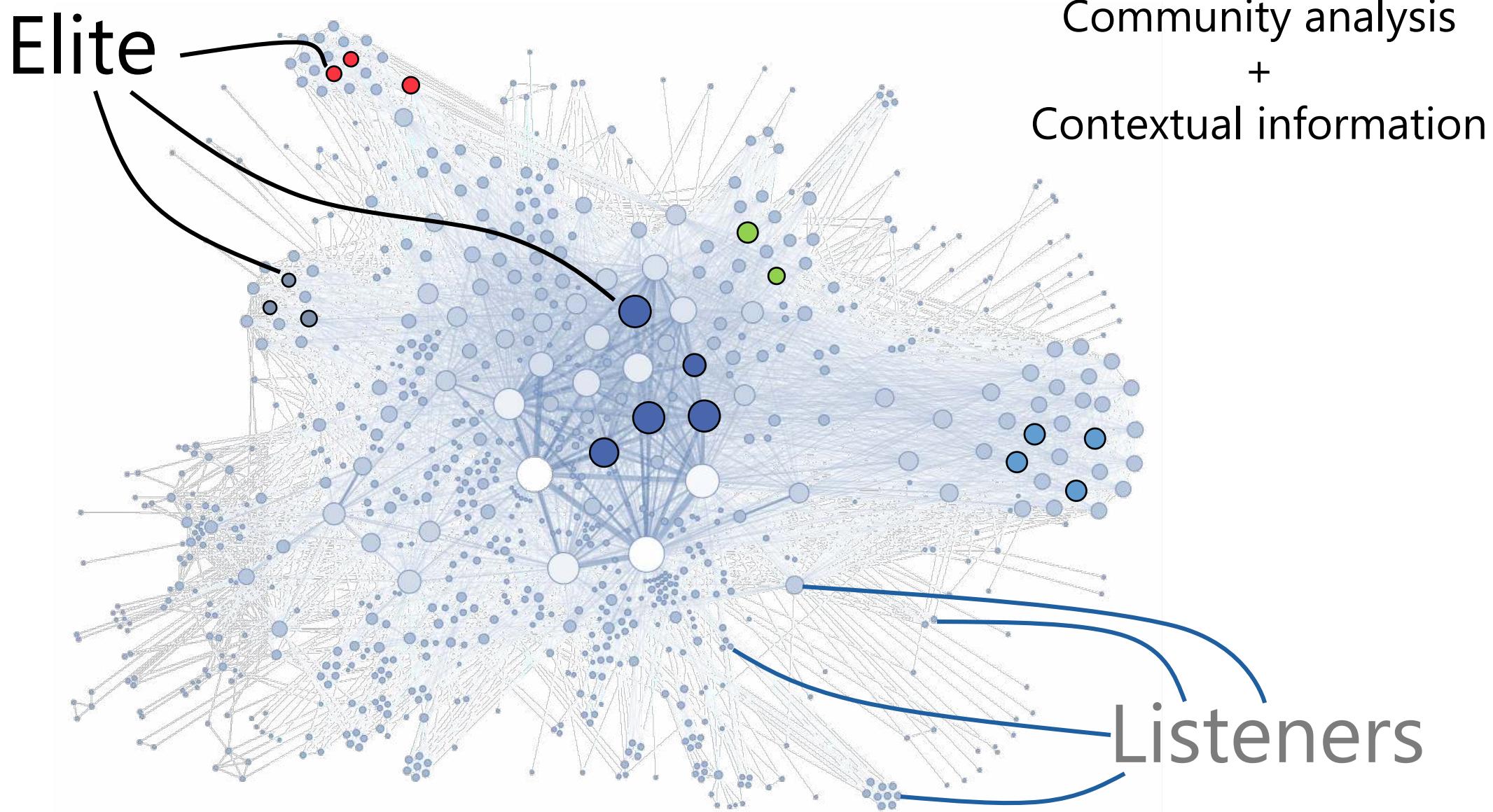
Interaction networks: elite and listeners

Influential users
(high number of
retweets)

Engaged users
(participate many
different days)



Interaction networks: elite and listeners



Opinion inference: DeGroot model

$\vec{x}_i \rightarrow$ Opinion vector of user i

$A_{ij} \rightarrow$ Adjacency matrix of the network

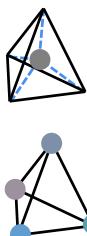
Replace elite's
outgoing links by
single self-loop

$$A_{ij}^* = \begin{cases} A_{ij} & \text{if } i \text{ is Listener} \\ \delta_{ii} & \text{if } i \text{ is Elite} \end{cases}$$

Iteratively average the
opinion of the neighbors

$$\vec{x}_i(t=0) = \begin{cases} \vec{0} & \text{if } i \text{ is Listener} \\ \vec{x}_p & \text{if } i \text{ is Elite} \end{cases}$$

Vertex of the
opinion simplex

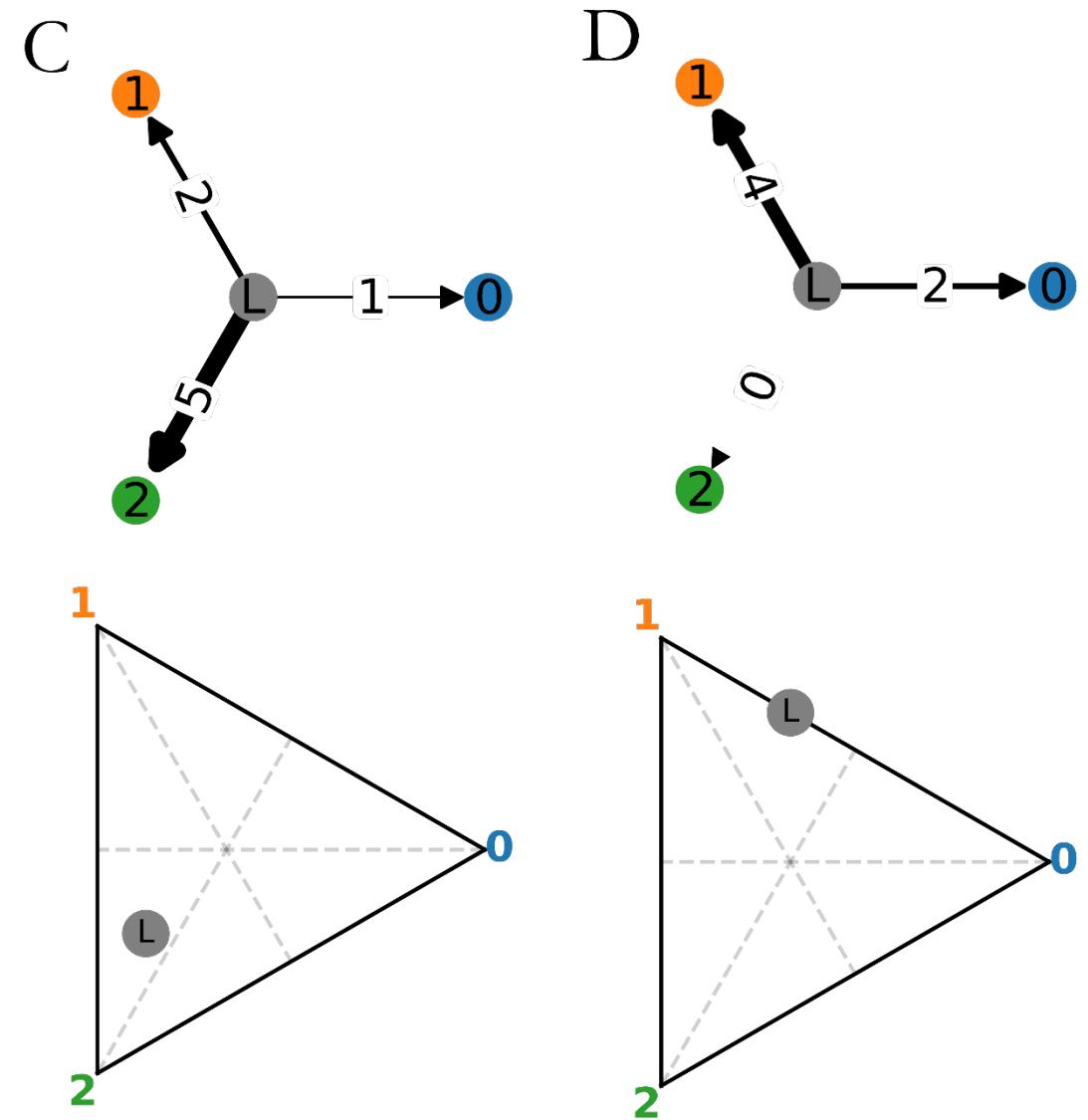


$$\vec{x}_i(t) = \frac{\sum_j A_{ij}^* \vec{x}_j(t-1)}{\sum_j A_{ij}^*}$$



Example of a tripolar system

Network→



Example of a quadripolar system





MEASURE OF POLARIZATION IN OPINION DISTRIBUTIONS:

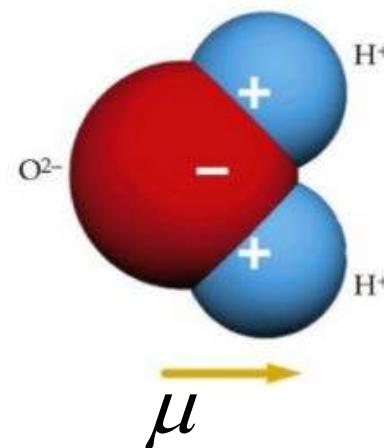
The Polarization Index

The Polarization Index

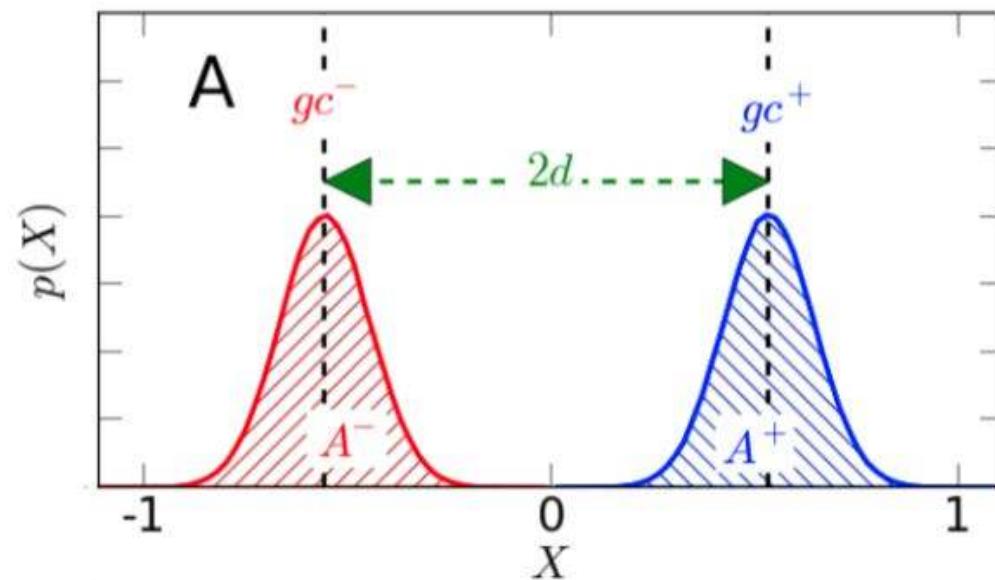
Based on:

- A Population is Perfectly Polarized:
Divided in two groups of the same size and opposite opinions
- Dipolar Moment

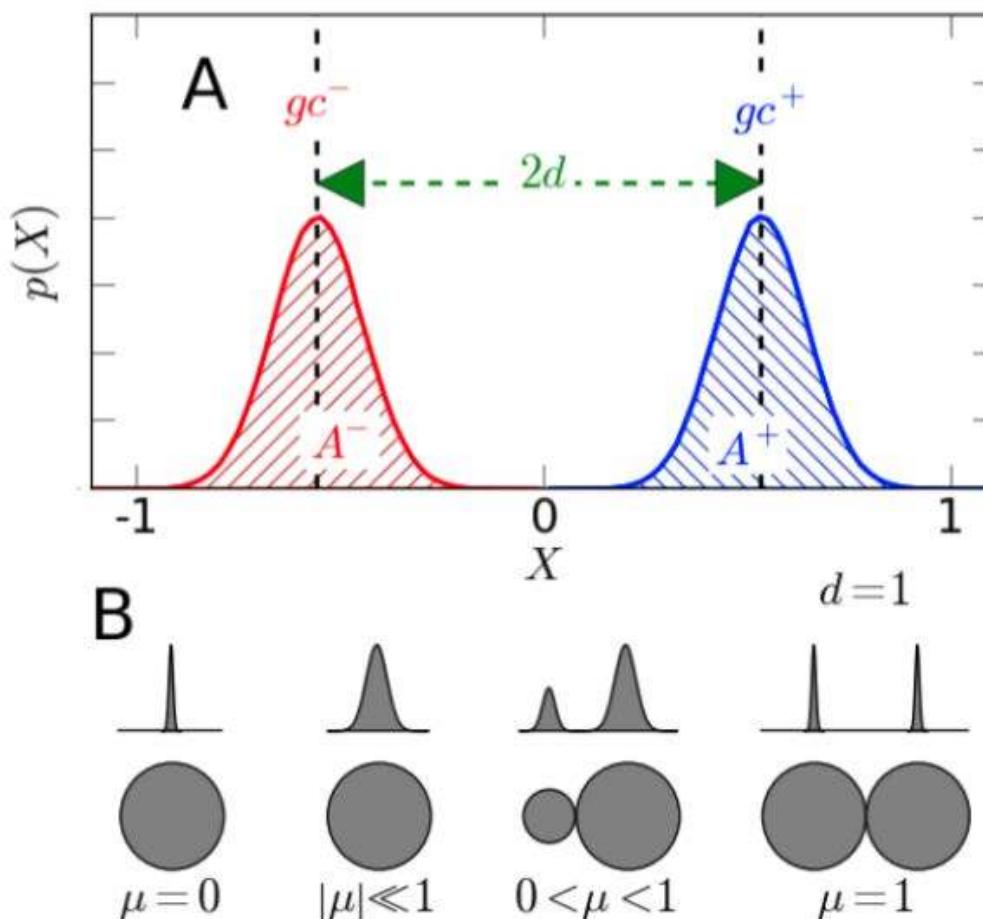
$$\begin{array}{c} -q \quad +q \\ \text{---} \quad \text{d} \\ - \quad + \end{array}$$
$$\mu = q \cdot d$$



A Population is Perfectly Polarized: Divided in two groups of the same size and opposite opinions



A Population is **Perfectly Polarized**: Divided in two groups of the **same size** and **opposite opinions**



Polarization indices

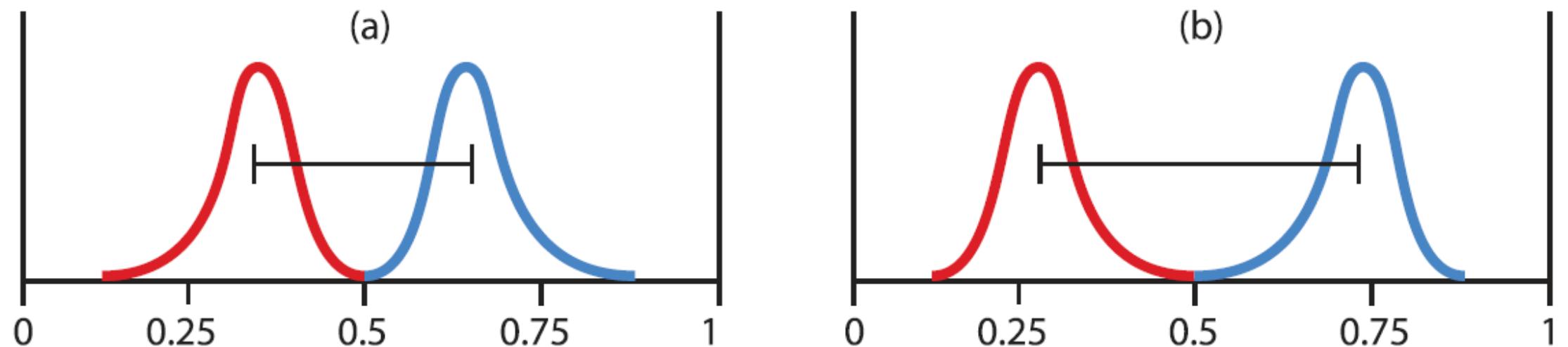
$$\mu = (1 - \Delta A)d$$

$$d = \frac{|gc^+ - gc^-|}{|X_{max} - X_{min}|}$$

$$\Delta A = |A^+ - A^-|$$

Measures of Polarization in multipolar systems: The covariance and PCA

$$\text{Var}(X) = E[(X - E[X])^2]$$

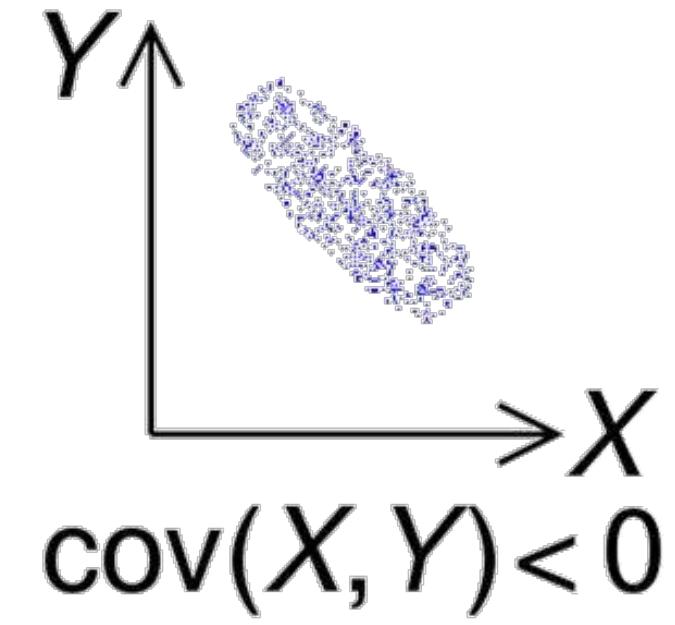
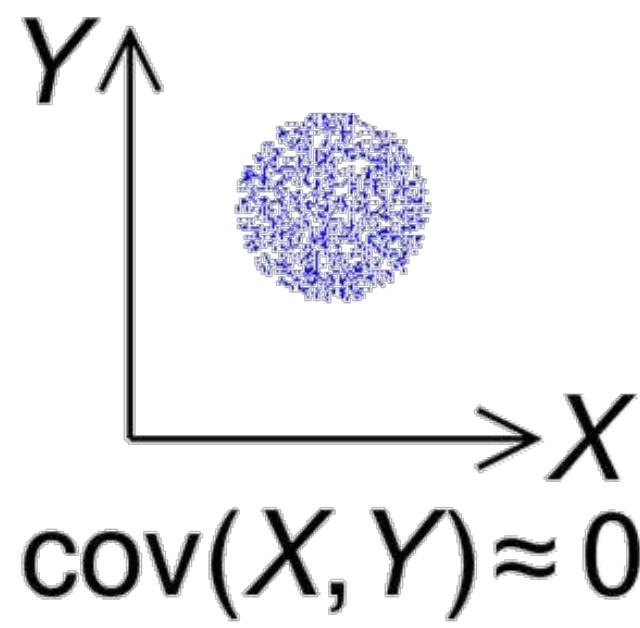
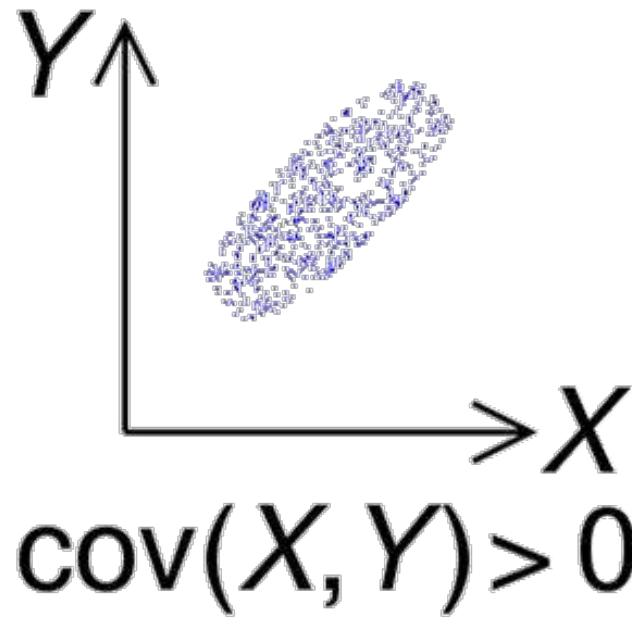


Aaron Bramson, Patrick Grim, Daniel J. Singer, Steven Fisher, William Berger, Graham Sack & Carissa Flocken (2016) Disambiguation of social polarization concepts and measures, The Journal of Mathematical Sociology, 40:2, 80-111, DOI: [10.1080/0022250X.2016.1147443](https://doi.org/10.1080/0022250X.2016.1147443)

Measures of Polarization in multipolar systems: The covariance and PCA

Multidimensional generalization of the variance: **the covariance**

$$Cov(X_i, X_j) = E[(X_i - E[X_i])(X_j - E[X_j])]$$



Measures of Polarization in multipolar systems: The covariance and PCA

$$Cov(X_i, X_j) = E[(X_i - E[X_i])(X_j - E[X_j])]$$

$$Cov[\vec{X}, \vec{X}] = \begin{bmatrix} E[(X_1 - E[X_1])(X_1 - E[X_1])] & E[(X_1 - E[X_1])(X_2 - E[X_2])] & \cdots & E[(X_1 - E[X_1])(X_n - E[X_n])] \\ E[(X_2 - E[X_2])(X_1 - E[X_1])] & E[(X_2 - E[X_2])(X_2 - E[X_2])] & \cdots & E[(X_2 - E[X_2])(X_n - E[X_n])] \\ \vdots & \vdots & \ddots & \vdots \\ E[(X_n - E[X_n])(X_1 - E[X_1])] & E[(X_n - E[X_n])(X_2 - E[X_2])] & \cdots & E[(X_n - E[X_n])(X_n - E[X_n])] \end{bmatrix}$$

Measures of Polarization in multipolar systems: The covariance and PCA

The **trace of the covariance matrix** :

as a measure of ***multidimensional variance***, usually called ***total variation (TV)***.

- the **maximum** attainable TV is 1,

so this metric **is normalized** by design when the distance between the barycenter of the opinion simplex and the poles is $\mu = 1$,

- . The **maximum TV** is achieved **when**:

- there are only **extreme opinions**
- they are **uniformly distributed** among all the poles.

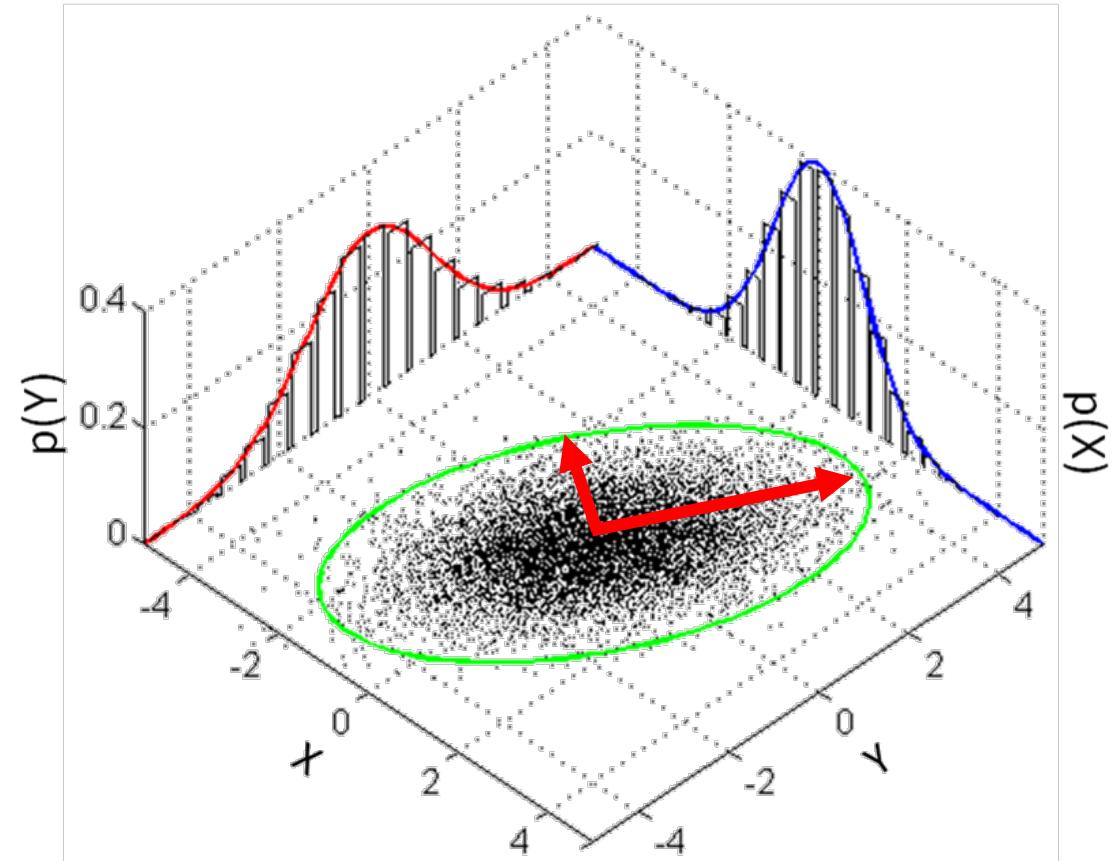
- Therefore, the TV can be used as a measure of **global polarization** combining the aspects:

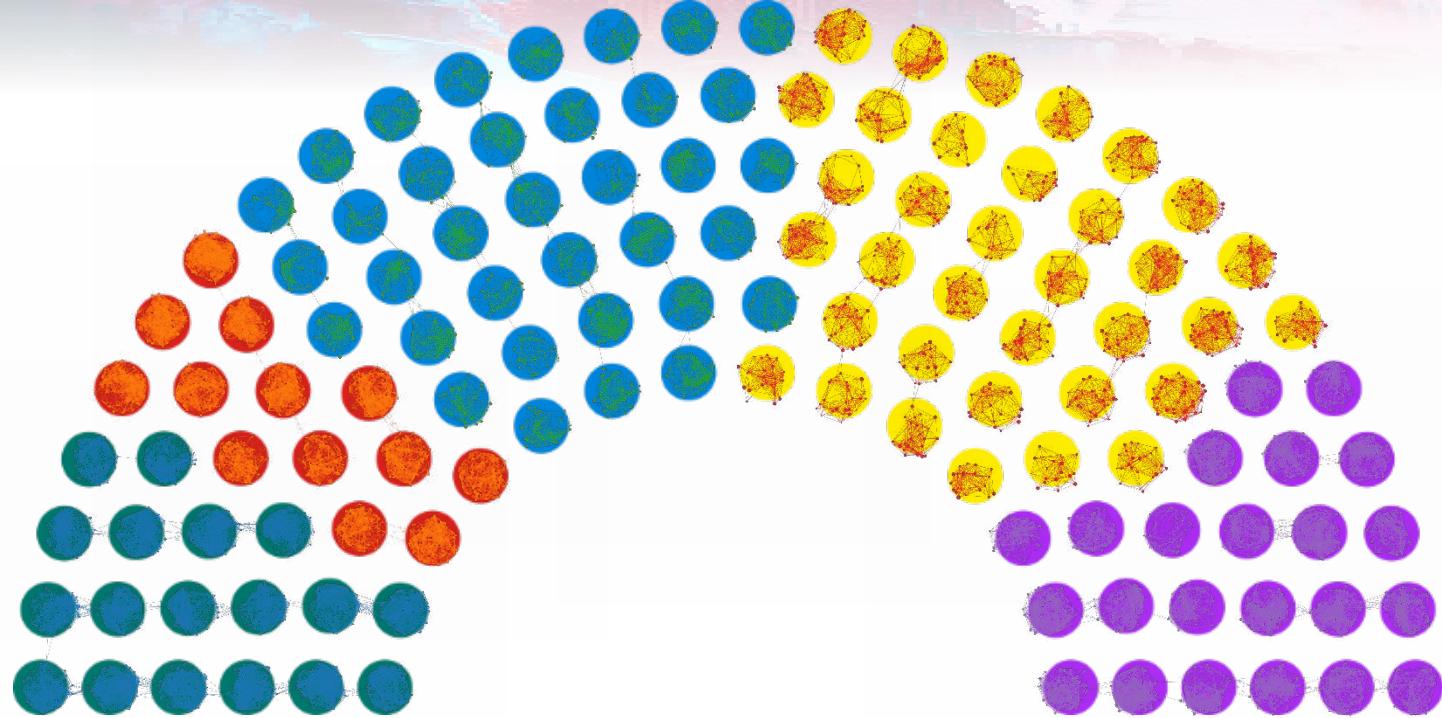
of ***opinion extremeness*** and ***community fragmentation***.

It not only measures how extreme the opinions are but also how evenly is the population divided into the considered factions

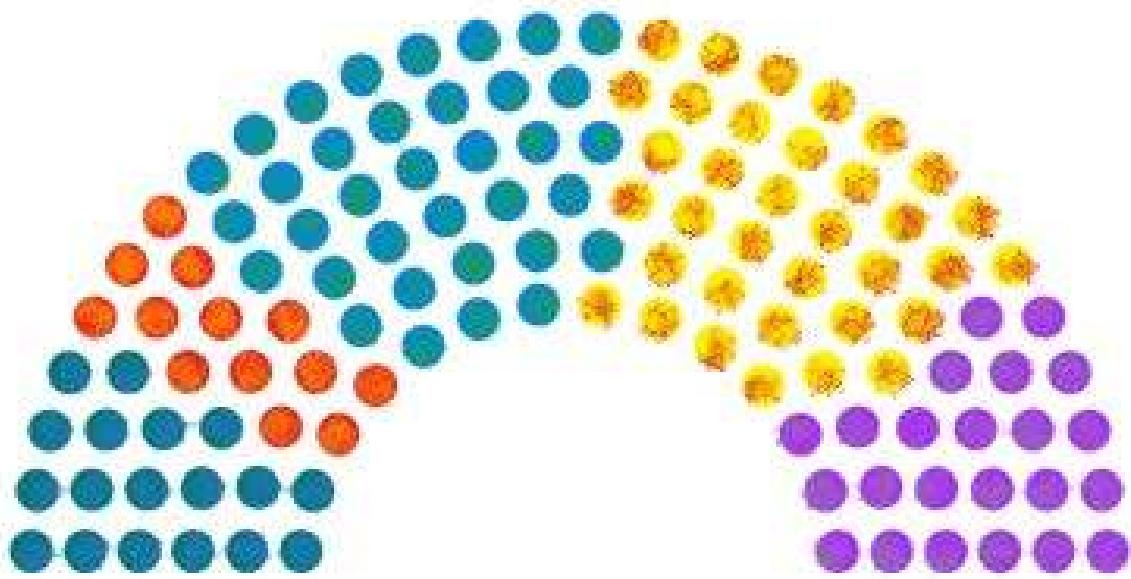
Measures of Polarization in multipolar systems: The covariance and PCA

- To characterize and **quantify pole alignment** we compute the eigendecomposition of the covariance matrix, (*Principal Component Analysis (PCA)*).
 - The **eigenvectors** (or Principal Components — PCs) form an orthogonal basis of the opinion space
 - Each **eigenvalue** corresponds to the projected variance along the direction defined by the eigenvector.
- The eigenvector with the largest eigenvalue (**first PC**): corresponds to the **direction of maximum variance**: direction of **maximum polarization**.





Empirical study of multipolar systems



Empirical study of multipolar systems

Quadripolar system: Spanish elections of December 2015



0 - Christian conservatism



1 - Social democratic

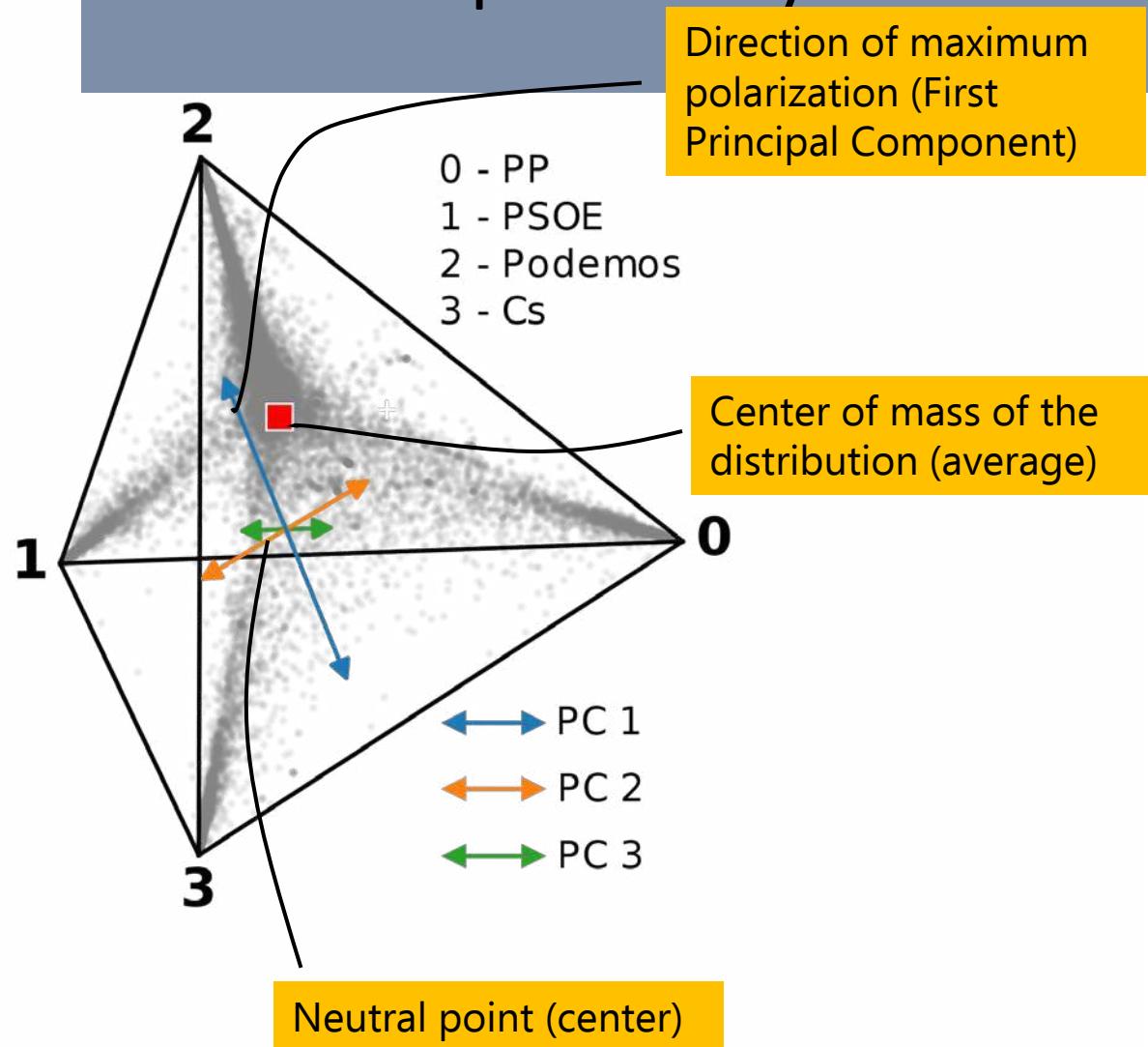


PODEMOS
**2 - Left-wing
populism**



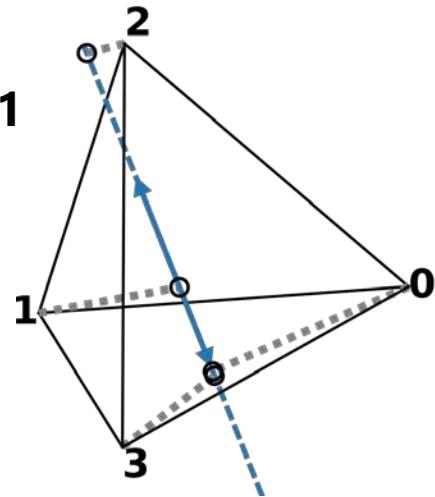
3 - Liberalism

Quadripolar system: Spanish elections 2015

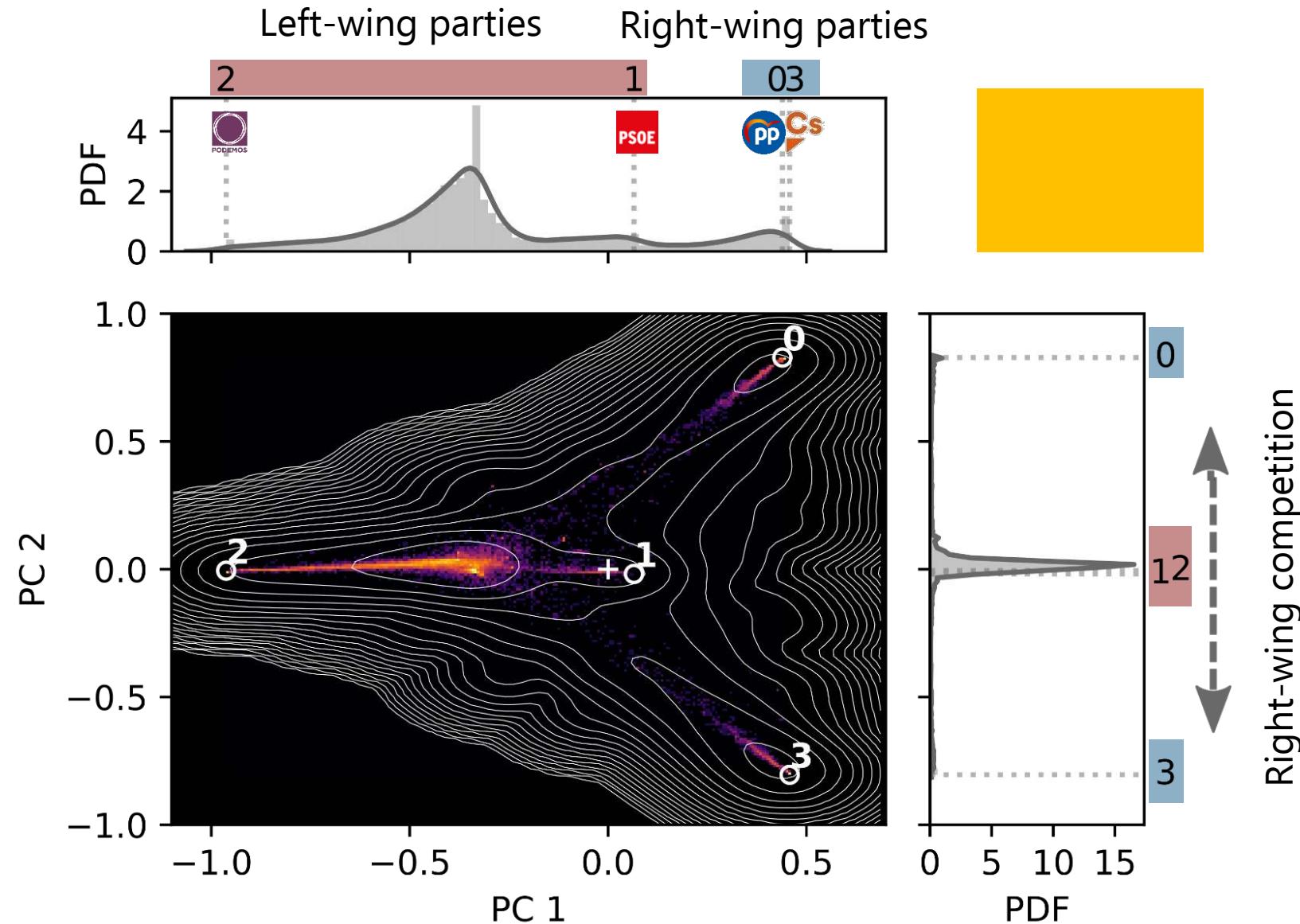
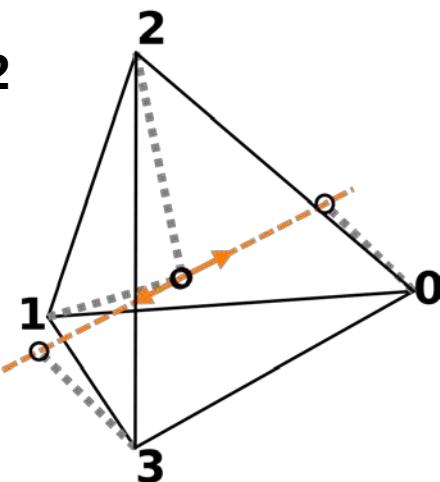


Quadripolar system: Spanish elections 2015

Principal Component 1



Principal Component 2



Pentapol system: Spanish elections of April 2019



0 - Christian conservatism



1 - Social democratic



PODEMOS
2 - Left-wing
populism

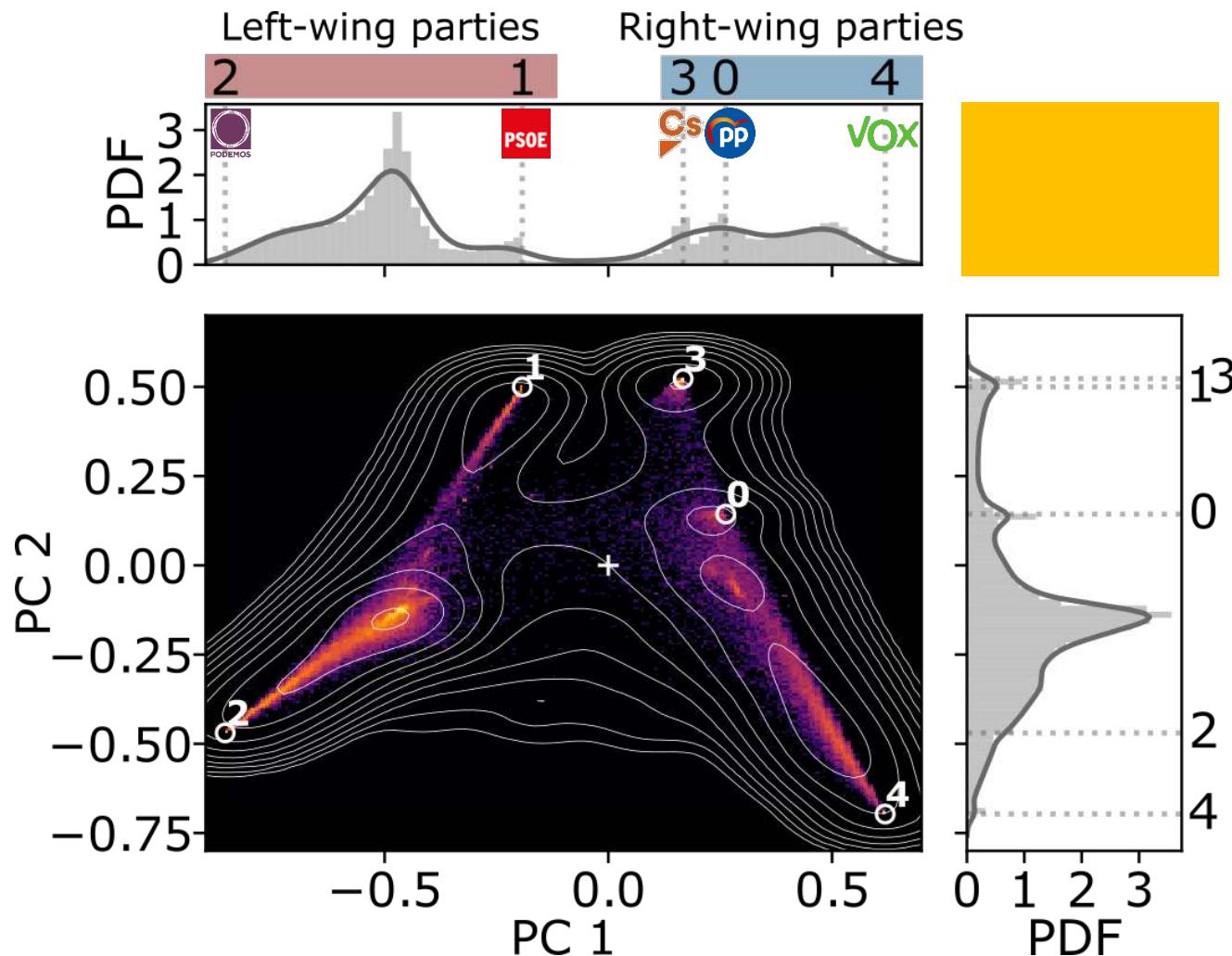


3 - Liberalism



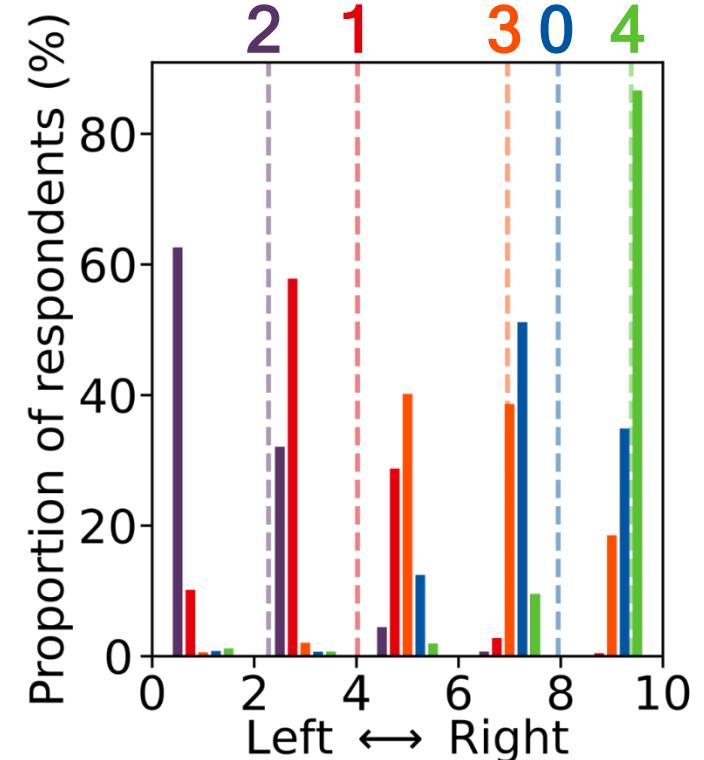
4 - Right-wing populism

Pentapolular system: Spanish elections 2019



CIS Centro de
Investigaciones
Sociológicas

**National survey on
parties' ideology**



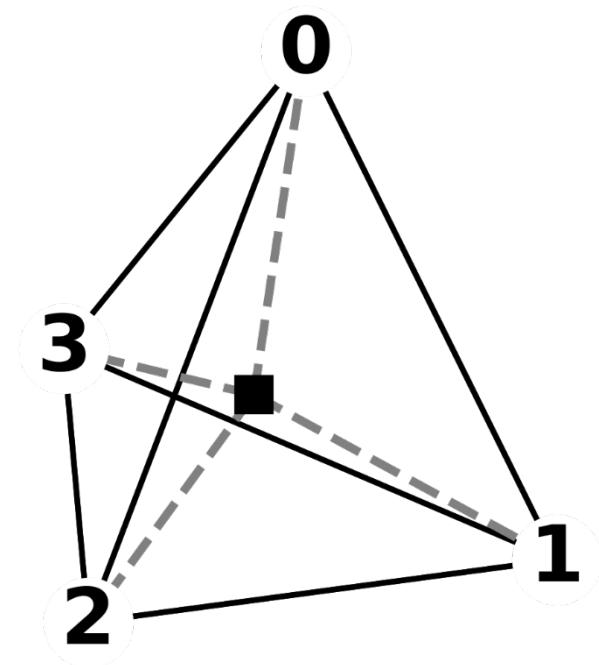
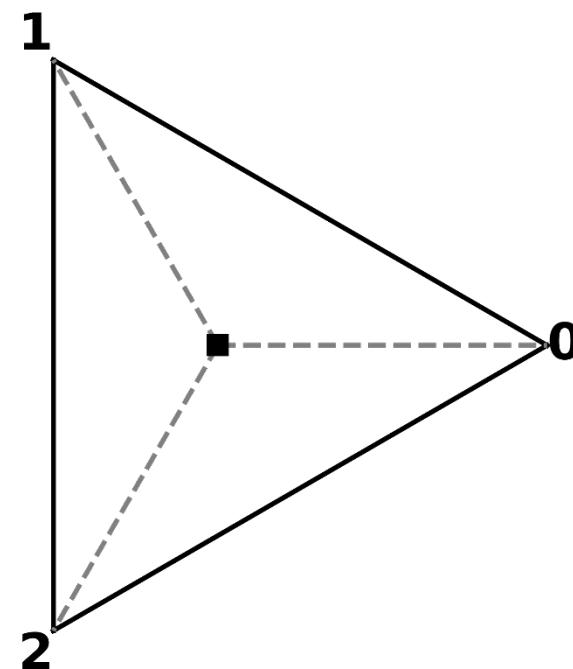
Parties' perceived extremism was computed
from national-wide surveys

Conclusions

Bipolar Opinion space-1D

Tripolar Opinion space-2D

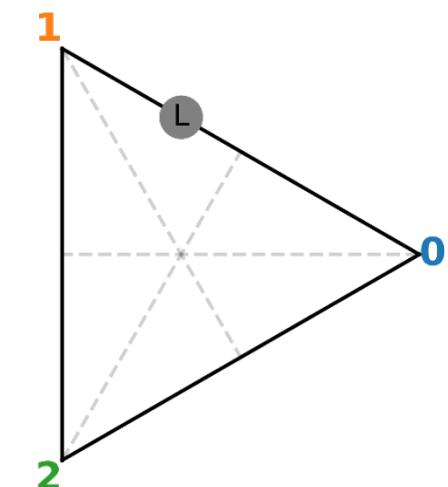
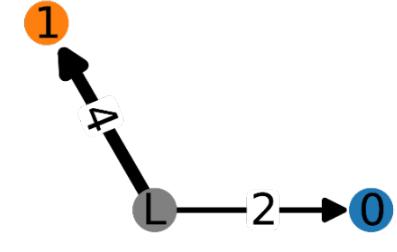
Quadripolar Opinion space-3D



We have proposed an **unbiased model** for the ideological space of **multipolar social systems** based on the multidimensional **regular simplex**.

Conclusions

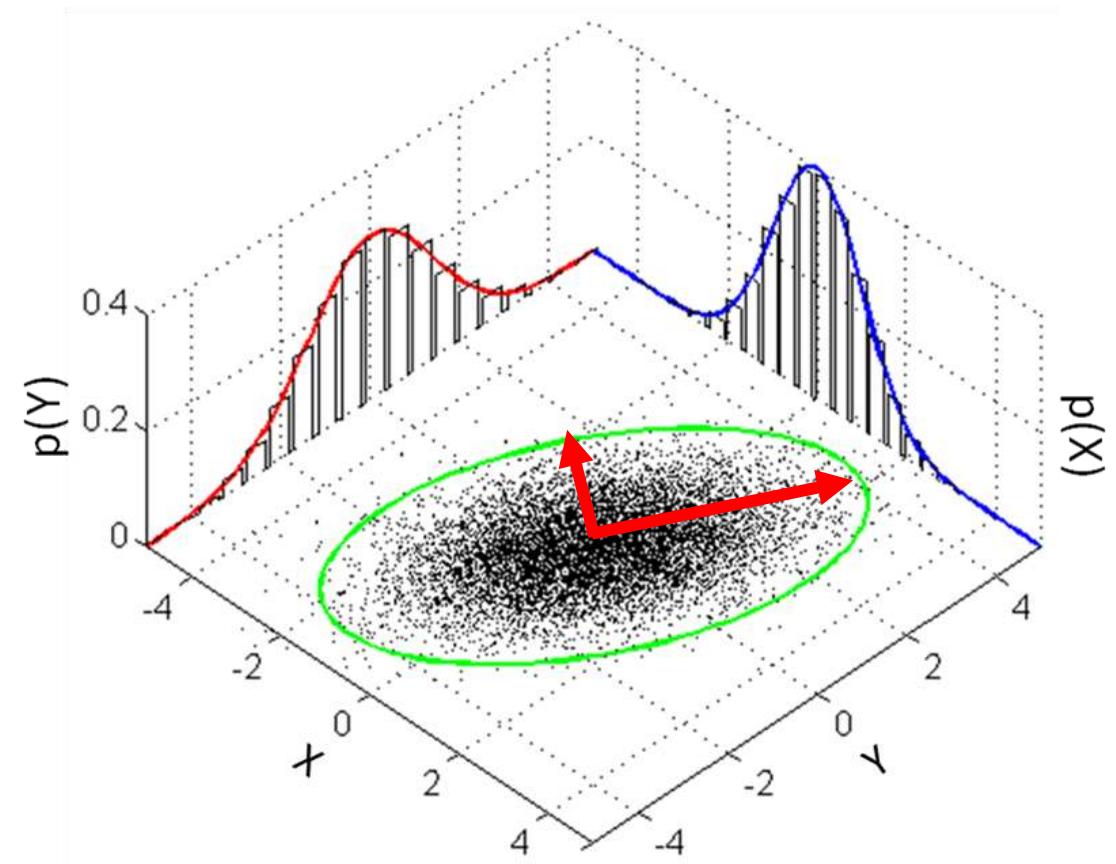
We have developed a methodology to infer
multidimensional opinion distributions
using online user interaction networks.



Conclusions

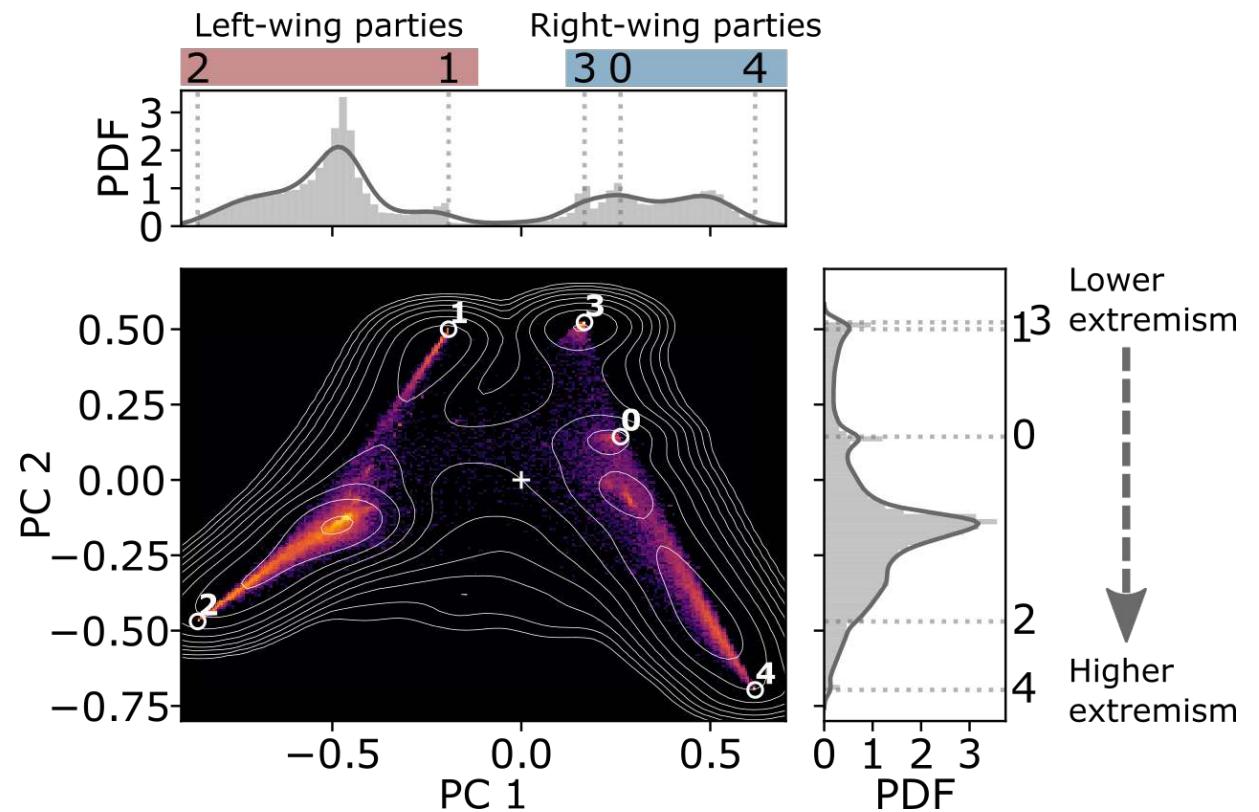
We have derived **multi-polarization** metrics from the **covariance matrix**:

- A measure of **global polarization**.
- A way to obtain the **directions of maximum polarization**.



Conclusions

We have **quantitatively validated** the traditional left / right scheme of party systems and revealed new **non-trivial axes of polarization** in real-world multipolar contexts.



Collaborators



Julia Atienza-Barthelemy



Samuel Martin-Gutierrez



Juan Carlos Losada



Juan Pablo Cárdenas



Alfredo J Morales



Javier Borondo



Gastón Olivares Fernández



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Thanks for your attention!

rosamaria.benito@upm.es

www.gsc.upm.es/gsc3

Samuel Martin-Gutierrez, Juan C. Losada, Rosa M. Benito,
Multipolar social systems: Measuring polarization beyond dichotomous contexts
Chaos, Solitons and Fractals 169 (2023) 113244