



# Balance and persistence

Reconstructing signed relations and social groups from interaction data

**Georges Andres**  
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# The persistence of social groups

[...] at every moment **destructive forces** attack the life both from within and from without, and, if these alone operated, the unity would soon be resolved into its elements or transformed into other combinations. But opposed to these destructive forces there are **preservative influences**[...]"

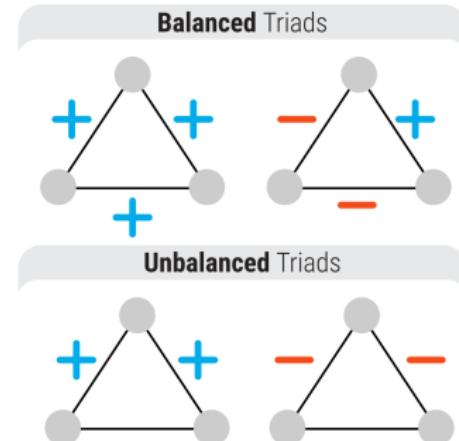
(Georg Simmel, 1898)

## The theory of structural balance (SB) (Heider, 1946)

- ▶ Balanced vs unbalanced (Cartwright and Harary, 1956)
- ▶ **Degree of balance** ⇒ destructive vs preservative

## Structural balance and persistence

- ▶ Resolving unbalance by **leaving the group**
- ▶ Balance ~ **low maintenance cost** (Morrisette et al., 1967)
- ▶ (+++) triad ~ **cohesion** (Davis, 1963)



## Hypothesis

The **higher** the **degree of balance** within a group is, the **more likely** the group is **to persist**.

# From interaction data to signed relations and social groups

## The data we require is rare

- ▶ (i) **temporal**, (ii) **signed relations** (SR), (iii) **groups**
- ▶ Surveys or online likes/follows?

## Interaction data more abundant

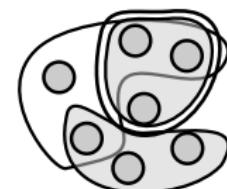
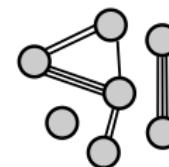
- ▶ E.g., email exchanges, phone-calls, FtF contacts etc.
- ▶ Short-lived and repeated

**Problem:** missing signed relations or groups

## Dyadic vs polyadic interactions

- ▶ **Networks:** edges between node pairs
- ▶ **Hypergraphs:** hyperedges between node groups

| Interaction ID | Individual ID 1 | Individual ID 2 | timestamp |
|----------------|-----------------|-----------------|-----------|
| I <sub>1</sub> | A               | B               | 1         |
| I <sub>2</sub> | C               | B               | 2         |
| I <sub>3</sub> | A               | B               | 3         |
| I <sub>3</sub> | A               | C               | 3         |
| I <sub>3</sub> | A               | D               | 3         |
| I <sub>4</sub> | A               | B               | 4         |

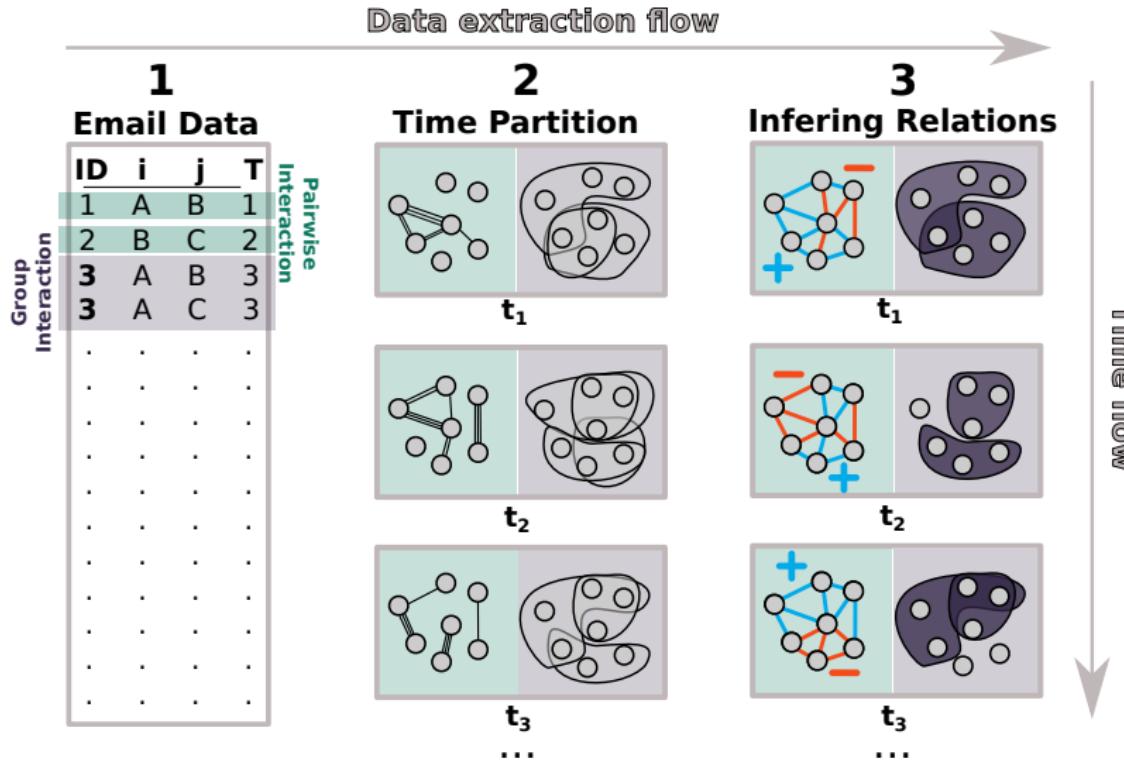


## Data challenge

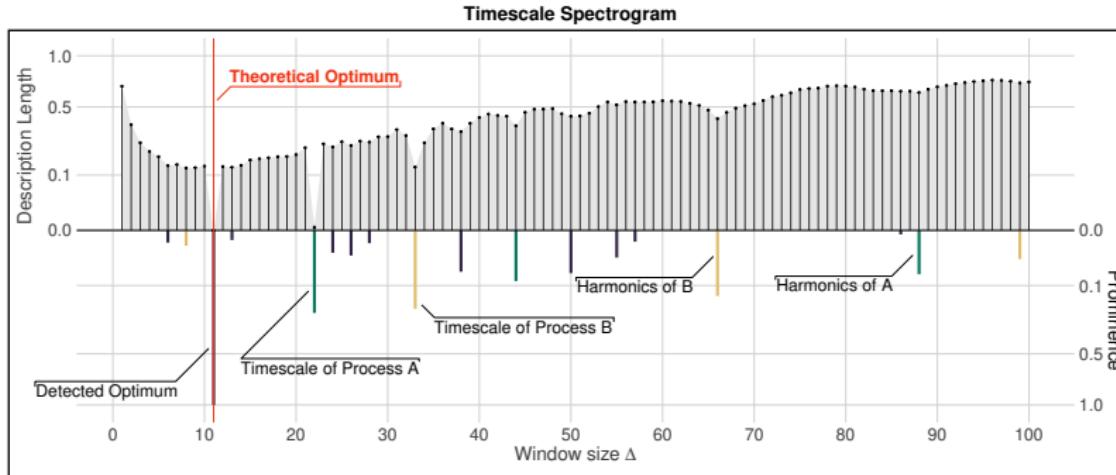
**Inferring** (i) time-decomposition, (ii) signed relations and (iii) groups from interaction data

# Overview of the data challenge

- Data requirement
- ▶ time dependence
  - ▶ signed relations
  - ▶ groups



# Disentangling the spectrum of timescales



Timescale detection\*

Bayesian framework for timescale detection in temporal network models

- Employing the **description length** to infer the **spectrum of timescales** in the network

Optimal timescales reveal different system reactions to major shocks

- Tracking the **slowdown** before and **speedup** after shocks in OSS projects

\*Casiraghi and Andres, *Disentangling the Timescales of a Complex System*, arXiv (2024)

# Inferring SR from dyadic interactions

## Working Assumption

**more** interactions  $\Rightarrow +$  signed relation/links

**less** interactions  $\Rightarrow -$  signed relation/links

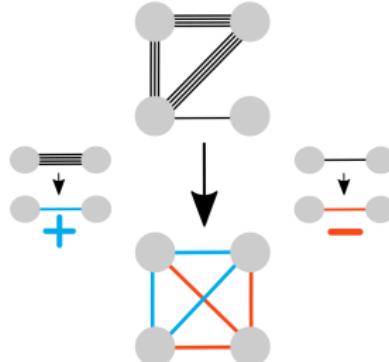
## Comparing to random expectation

- ▶ Require **null-model for interactions**
- ▶  $a_{ij} \in \mathbb{N}$ , *observed* interactions between  $i$  and  $j$
- ▶  $A_{ij} \in \mathbb{N}$ , *random* variable for the interactions

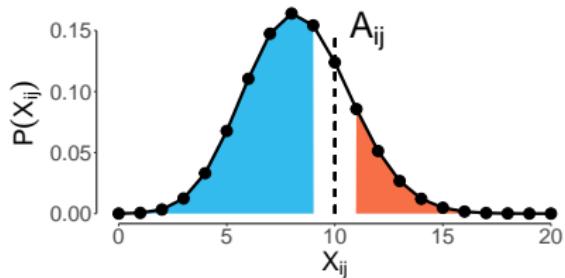
## Over- and under-representation of interactions

- ▶  $P(X_{ij} < A_{ij})$  large  $\Rightarrow \geq A_{ij}$  interactions is **unlikely**  
Interpret: over-representation and **positive SR**
- ▶  $P(X_{ij} > A_{ij})$  large  $\Rightarrow \leq A_{ij}$  interactions is **unlikely**  
Interpret: under-representation and **negative SR**

## Interaction Network



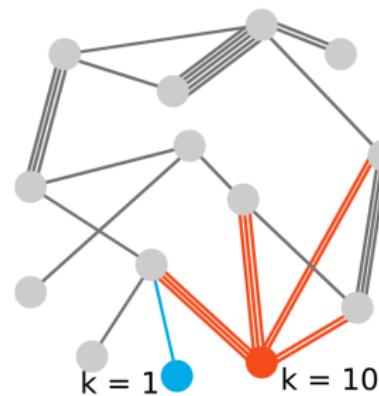
## Signed Network



# Hypergeometric configuration model

## Capturing individuals' heterogeneities

- ▶ Individuals exhibit different levels of **fitness** & **activity**
- ▶ in-degree  $\hat{k}_i^{in} \rightarrow$  fitness; out-degree  $\hat{k}_i^{out} \rightarrow$  activity  
⇒ **Use a network null-model**

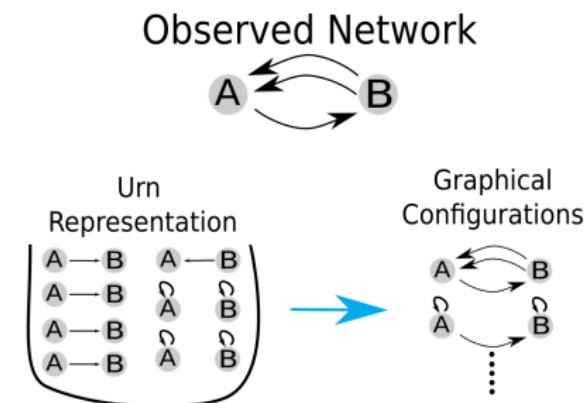


## Defining the null-model (Casiraghi and Nanumyan, 2021)

- ▶ Possible links from  $i$  to  $j$ :  $\Xi_{ij} \equiv \hat{k}_i^{out} \hat{k}_j^{in}$
- ▶ Unif. sampling w/o replacement of  $m$  links from urn  
⇒ **Multivariate hypergeometric distribution**
- ▶ **Degrees preserved in expectation**:  $\mathbb{E}[k_i^{out}] = \hat{k}_i^{out}$

## Over/under-representation: cumul. marginals

$$P(A_{ij} < a_{ij}) = \sum_{A_{ij}=0}^{a_{ij}-1} \frac{\binom{\Xi_{ij}}{A_{ij}} \binom{m^2 - \Xi_{ij}}{m - A_{ij}}}{\binom{m^2}{m}}$$



# Inferring signed relations: combine the ingredients!

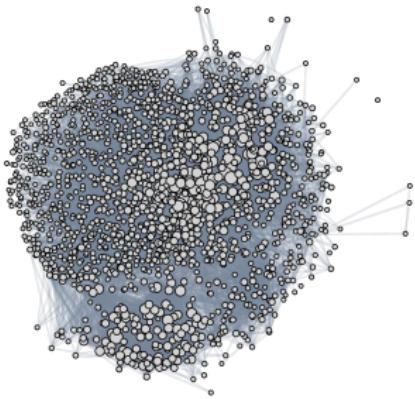
## Introducing the $\Phi$ -measure

- ▶ Combine  $P(A_{ij} < a_{ij})$  and  $P(A_{ij} > a_{ij})$  in **one index**:

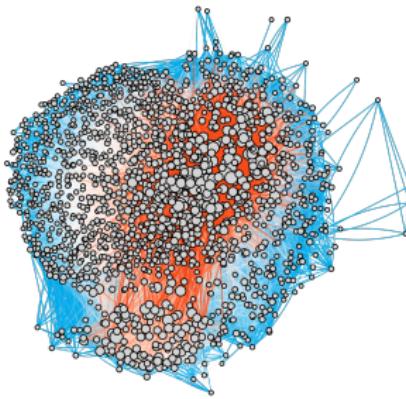
$$\Phi_{ij} = aP(A_{ij} < a_{ij}) + bP(A_{ij} > a_{ij})$$

- ▶  $\text{sign}(\Phi_{ij})$  is the **sign** of the relation
- ▶  $|\Phi_{ij}|$  is the **weight** on the inferred relation
- ▶  $a$  and  $b$  can be **learned from known relations**; usually  $a > 0$  and  $b < 0$
- ▶ In absence of additional information, we propose:  $1 = a = -b$

# Testing the method on real-world data



Interaction network



Inferred signed network

| Inferred Relations |          |          |
|--------------------|----------|----------|
|                    | positive | negative |
| Declared Relations |          |          |
| trust              | 81%      | 19%      |
| distrust           | 26%      | 74%      |

Validation

Inference of signed relations\*

Providing stochastic network method for inferring signed relations

- ▶ High accuracy in predicting known relations tested across 5 datasets

Unlocking interaction data for the study of signed relations

- ▶ Allows for quantifying structural balance for a wide variety of systems

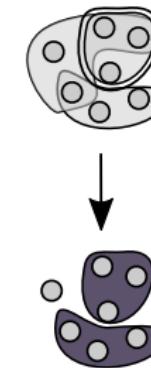
\*Andres et al., Reconstructing signed relations from interaction data, Scientific Reports (2023)

# Inferring groups from polyadic interactions

Working assumption

many polyadic interactions  $\Rightarrow$  individuals part of social group

Interaction Hypergraph



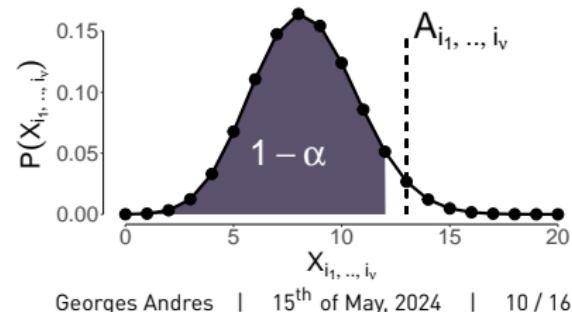
## Comparing to random expectation

- ▶ Which groups interacted **more than expected?**
- ▶ Employ **hypergraph null-model** for interactions
- ▶ Perform a significance test ( $\alpha = 0.05$ ):

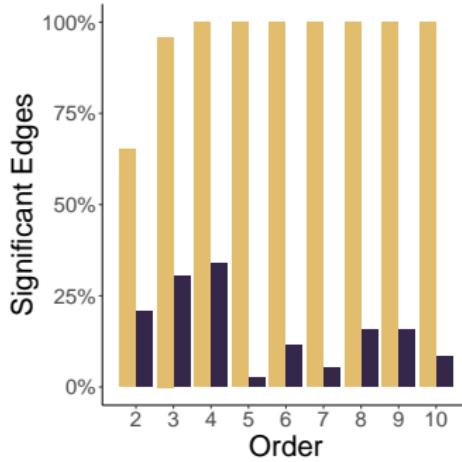
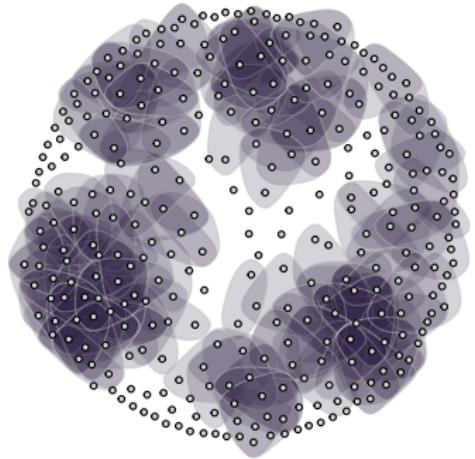
$$\Pr(X_{i_1, \dots, i_v} \leq A_{i_1, \dots, i_v}) > 1 - \alpha$$

## Modeling sparsity

- ▶ Real hypergraphs are **extremely sparse**  
 $\Rightarrow$  **Zero-inflating** the hypergraph model



# Inference of groups despite sparsity



## Inference of groups\*

### Constructing a sparsity-adapted stochastic hypergraph model

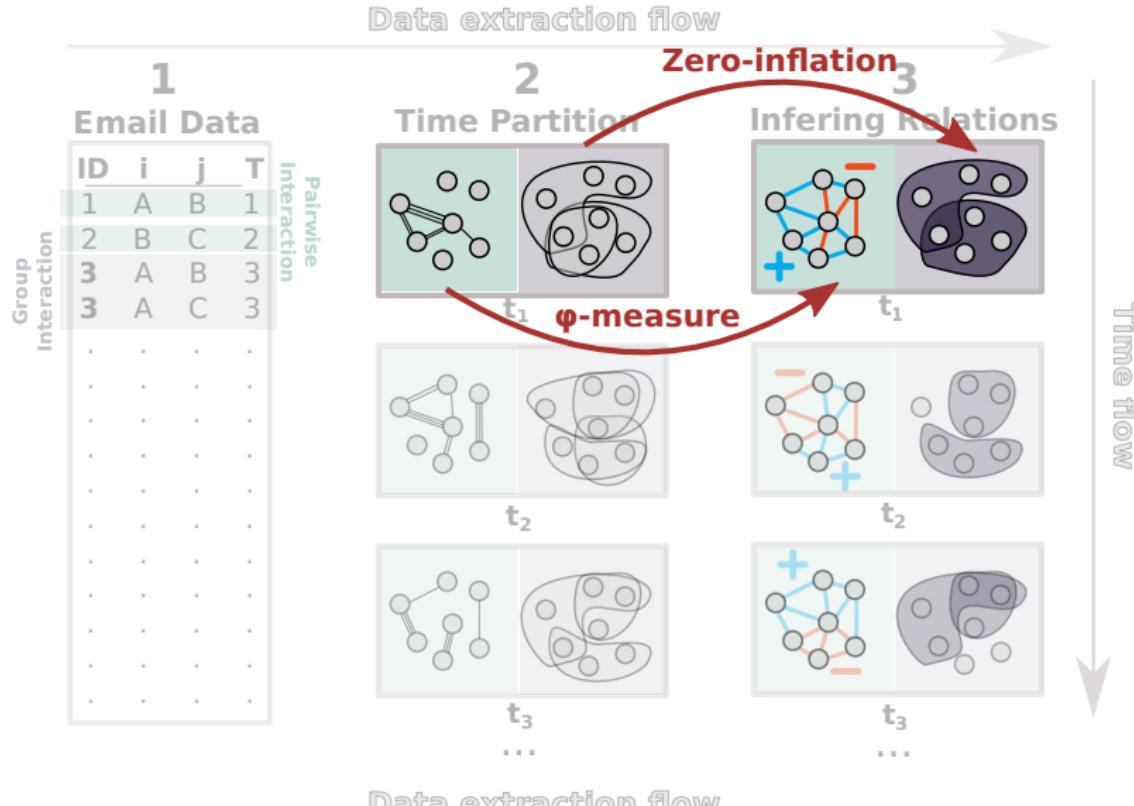
- Derive **analytical formulation** and provide a method for **fitting to data**

### Enabling the inference of groups through ensemble perspective

- Social groups can be differentiated** from spurious interactions

\*Andres et al. *Stochastic Modeling of Hypergraphs: Zero-inflation to counteract the curse of dimensionality* (in preparation)

## Final data representation



# The persistence of social groups

## Hypothesis

The higher the **degree of balance** within a group is, the **more likely** the group is **to persist**.

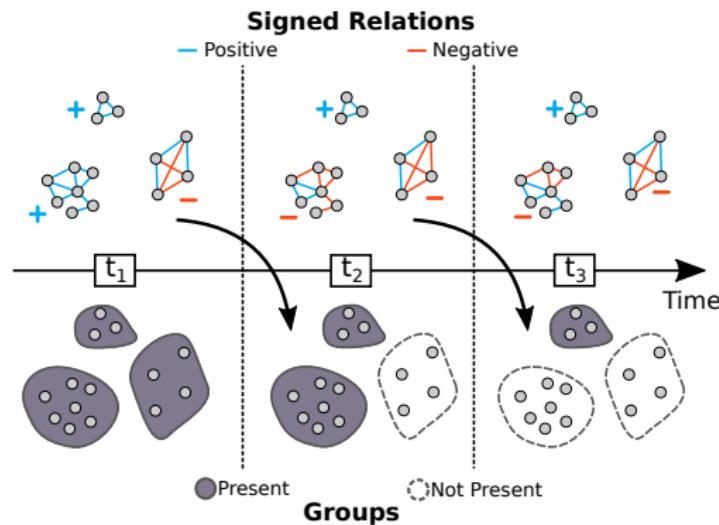
## Performing a logistic regression

- Dependent variable: individual  $i$  being in a group  $g_j$  that persists from  $t \rightarrow t + 1$
- Independent variables: importance of triads  $n(g_j^t)$

## Quantifying the degree of balance

- Calculated from **inferred signed network**
- Importance** associated with each **type of triad**:

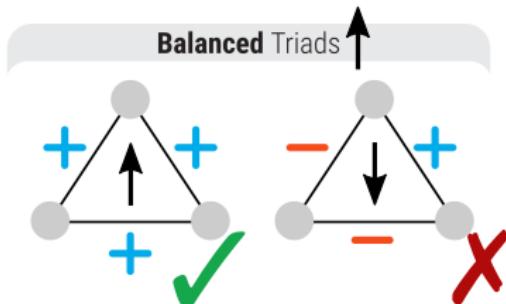
$$n(g_j^t) = (n_{+++}, n_{++-}, n_{+-+}, n_{---})$$



# Structural balance and group persistence

## How different triads affect the persistence of groups

- ▶ **(+++)** consistently has a **positive effect** on persistence  
⇒ **cohesion**
- ▶ **(+--)** has a **negative effect** when significant  
⇒ **polarization**



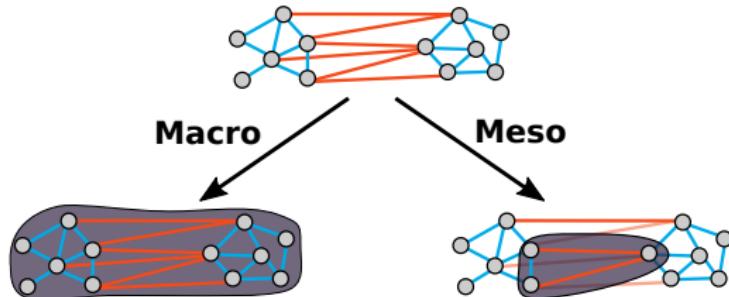
### Structural balance vs cohesion

For small groups, only the **cohesive triad (+++)** positively affects persistence

# Structural balance and group persistence

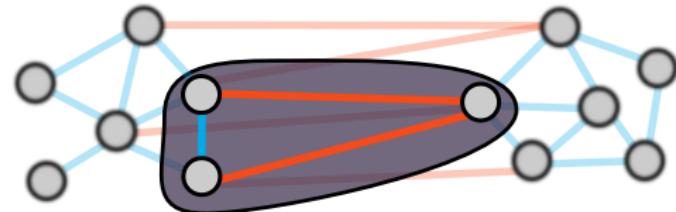
Interpretation: scales matter!

- ▶ **Macro**: polarization can be **very persistent**
- ▶ **Meso**: polarization is **not persistent**



Uncovering the effects of balance

- ▶ Signed network  $\Rightarrow$  groups
- vs
- ▶ **Signed network  $\neq$  groups**



Structural balance: beneficial or not?

To answer this, we must consider the **interplay** between **group and signed structures**

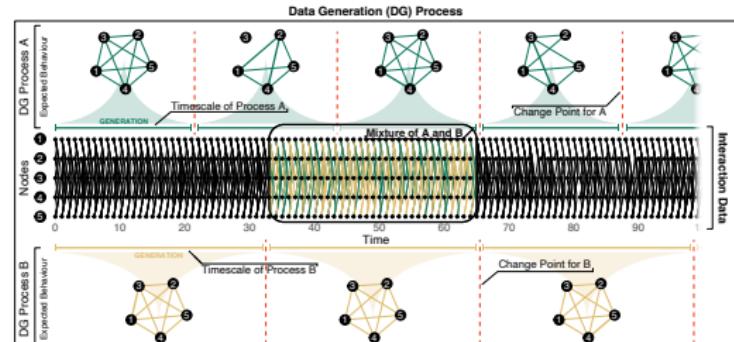
## Conclusion

### Structural balance and group dynamics

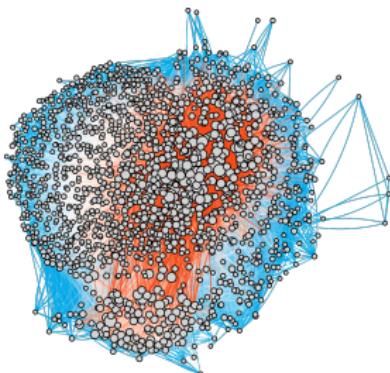
- Small groups: **cohesion vs balance**

### Unlocking the potential of interaction data

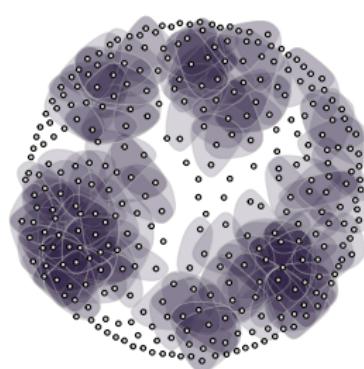
- Disentangle** a network's **timescales**
- Infer signed **relations** with  $\Phi$ -method
- Infer social **groups** via zero-inflation



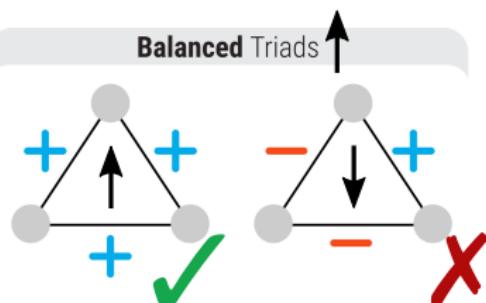
Timescale inference



Signed inference



Group inference



Balance & persistence