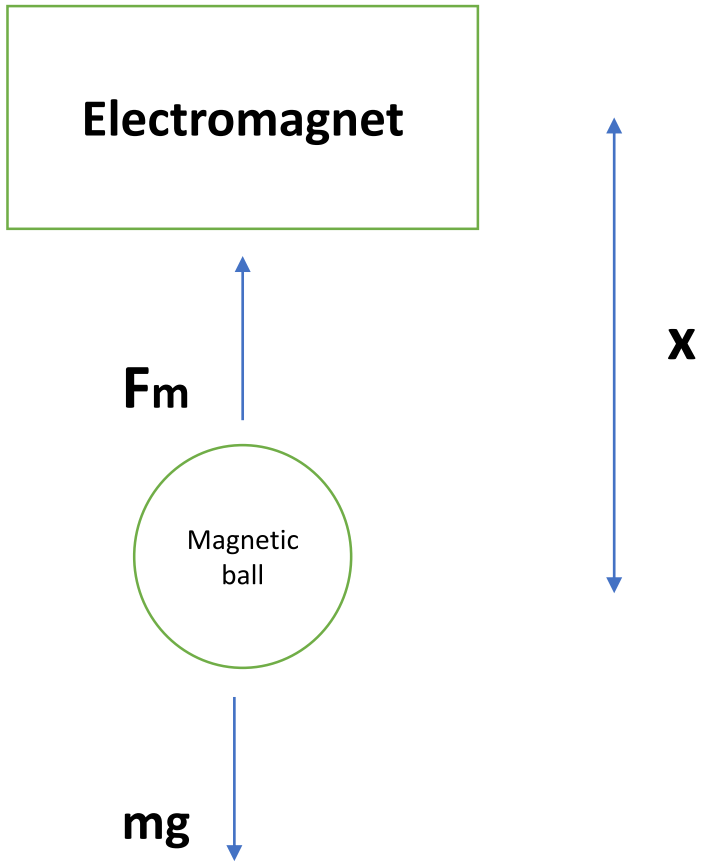
Objective:

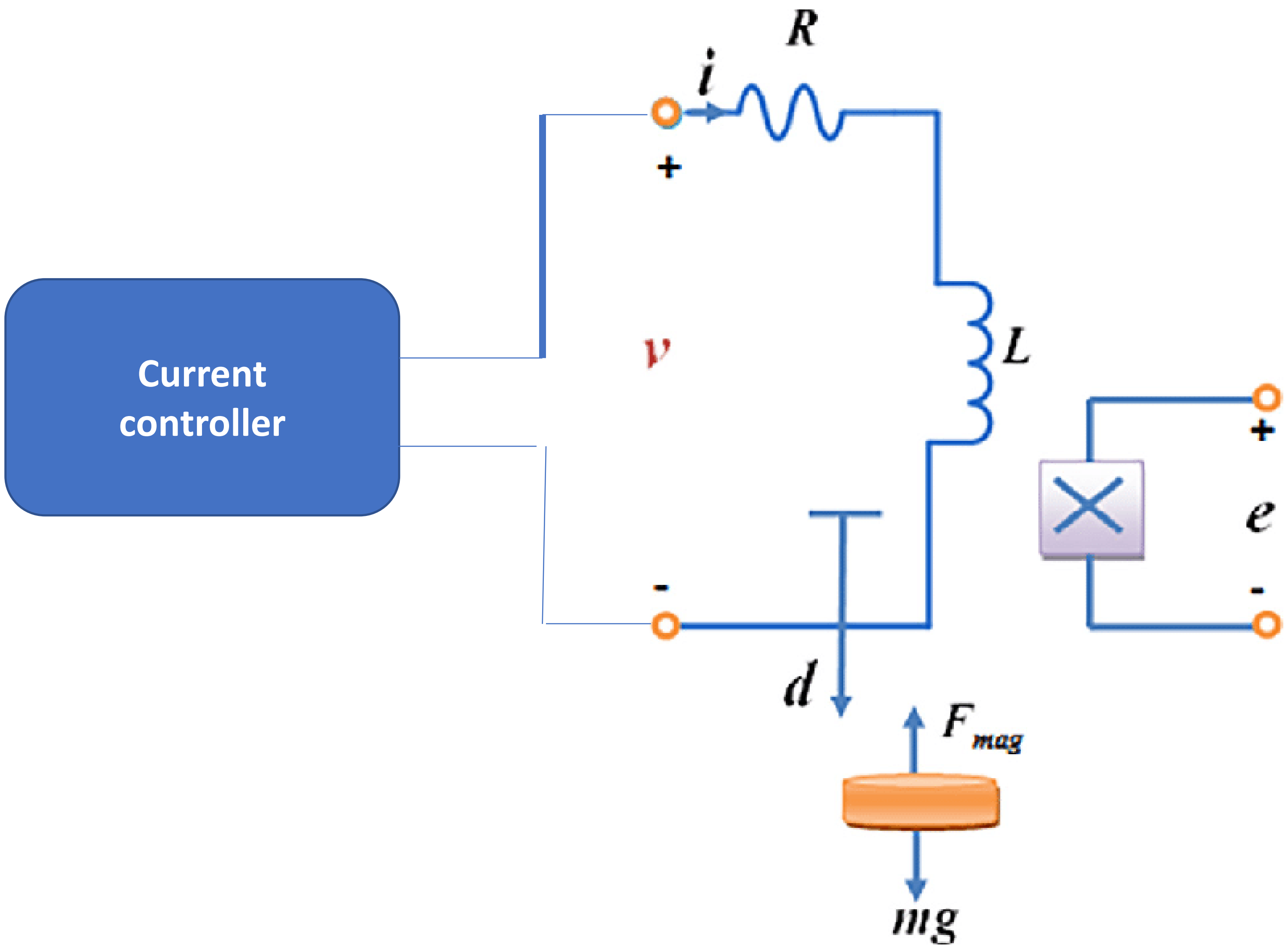
- The concept behind magnetic suspension is to use the magnetic force generated by electromagnet to counteract the effect of gravity on the ball hence when these two forces are balanced, there is a point of equilibrium.
- A way to solve this control problem is to linearize the system around these equilibrium, or operating points and use conventional linear control techniques.
- Taylor series expansion is used and also to approximate the behaviour of the system over a limited range around the operating points.

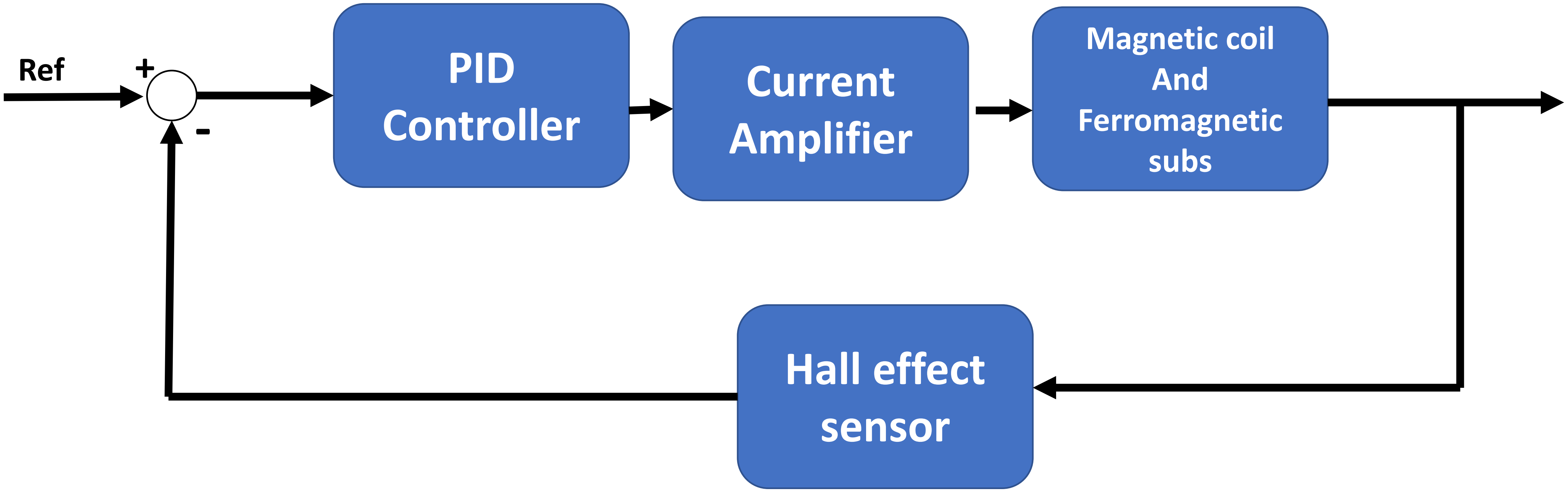
List of variables:

- **m** mass of ball (100 g)
- **x** equilibrium position ball (7cm below from coil)
- **i** equilibrium position
- **L** inductance of coil
- **L1** mutual inductance due to permanent magnet



Schematic Diagram




Ref
+
-
PID
Controller
Current
Amplifier
Magnetic coil
And
Ferromagnetic
subs
Hall effect
sensor

Equations:

The magnetic ball suspension system can be categorized into two systems: a mechanical system and an electrical system

Sum of the forces in vertical direction yields

$$m\ddot{x} = mg - F_m \qquad \text{...........(1)}$$

$$V = iR + L_1\frac{di}{dt} \qquad \text{..........(2)}$$

The inductance of the coil changes with the change in position of the ball
So the Total inductance will be,

$$L = L_1 + 2C/x \qquad \text{..........(3)}$$

The electromagnetic levitation force can be determined using the theorems of the generalized forces:

$$F_m = C\left(\frac{i}{x}\right)^2 \quad .....(4)$$

Substituting for magnetic force from equation (4) , we can write

$$m\ddot{x} = mg - C\left(\frac{i}{x}\right)^2 \quad ......(5)$$

From equation (5) it can be seen that a nonlinear term is present which depends on current and air gap. Thus it is needed to linearize the system about these two variables by taking a first-order Taylor series expansion about points that describe a desired operating point and small deviations from:

$$i = \bar{\imath} + \tilde{\imath}$$

$$x = \bar{x} + \tilde{x} \quad ....(6)$$

In Equation (6), the bar denotes the equilibrium operating point and tilde represents small deviations from the operating point. Applying a first order approximation of the Taylor series expansion of the magnetic force gives

$$F_m = C\left(\frac{i}{x}\right)^2 + \frac{\partial F}{\partial x}\Big|_{\bar{\imath},\bar{x}}\ \tilde{x} + \frac{\partial F}{\partial i}\Big|_{\bar{\imath},\bar{x}}\ \tilde{\imath} \quad \ldots\ldots.(7)$$

Solving the partial differential equation we get,

$$\frac{\partial F}{\partial x}\Big|_{\bar{\imath},\bar{x}}\ \tilde{x} = -2C\,\frac{\bar{\imath}^2}{\bar{x}^3}\ \tilde{x}$$

$$\frac{\partial F}{\partial x}\Big|_{\bar{\imath},\bar{x}}\ \tilde{\imath} = 2C\,\frac{\bar{\imath}}{\bar{x}^2}\ \tilde{\imath} \quad \ldots\ldots.(8)$$

For Simplicity, Let $k_1 = -2C\,\frac{\bar{\imath}^2}{\bar{x}^3}$

And $k_2 = 2C\,\frac{\bar{\imath}}{\bar{x}^2}$ $\quad \ldots\ldots.(9)$

Combining Equations 5-9, we yield the final differential equation representing the linearized system:

$$m\ddot{x} = k_1\,\tilde{x} - k_2\tilde{\imath}$$

Now, the transfer function of the system can be find that describes the air gap as a result of the current input by taking the Laplace transform of the linearized differential equation. So the Laplace transfer of the above equation is

$$mS^2X(s) = k_1X(s) - k_2I(s)$$

$$\text{Therefore,}\quad \frac{X(s)}{I(s)} = \frac{-k_2}{mS^2 - k_1} \quad \ldots\ldots.(10)$$

The current through the coil is produced with the help of a current amplifier. Both the sensor and the current amplifier are linear elements which can be described by the proportional gain transfer functions *Ksens*, *Kamp*. By considering these additional elements the following transfer function of the process is found:

$$\frac{X(s)}{I(s)} = \frac{-k_2}{mS^2 - k_1} K_{sens}\, K_{amp}$$

$$= \frac{-k_3}{mS^2 - k_1} \quad \ldots\ldots..(11)$$

$$k_3 = k_2 K_{sens}\, K_{amp}$$