Chapter 6 Extensions

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1. Unit of Measurement
2. Functional Form
3. Typical Nonlinear Functional Form

1. Unit of Measurement

• Motivation:

► Suppose that the inflation rate is 5%, in inputting data, which one will you use, $X_i = 0.05$ or $X_i = 5$?

► For GDP=2,000조원, $X_i = 2,000$ or $X_i = 2,000\cdots000$?

► If annual income=1,200만원, $X_i = 1,200$(만원/연) or $X_i = 100$(만원/월)?

• Answer: Freely choose your own but be careful of interpretation.

◎ If we change the units of data, <u>what's change</u>?

► $$b_2 = \frac{\sum_{i=1}^{n}\left(X_i - \overline{X}\right)\left(y_i - \overline{y}\right)}{\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^2} = \frac{\text{'unit of } y\text{'}}{\text{'unit of } X\text{'}},\qquad b_1 = \overline{y} - b_2\overline{X} = \text{'unit of } y\text{'}.$$

► $$V(b_2) = \frac{\sigma^2}{\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^2} = \left(\frac{\text{'unit of } y\text{'}}{\text{'unit of } X\text{'}}\right)^2,\qquad \sigma_{b_2} = \frac{\text{'unit of } y\text{'}}{\text{'unit of } X\text{'}}.$$

► t-ratio=$\dfrac{b_2}{s_{b_2}}$: unit-free measure

► Any other unit-free statistics in Econometrics?

► What about in Economic Statistics course?

◎ If we change the units of data, <u>any change in understanding the value of statistics</u>?

(Example) Income file

연령: AGE(세), AGE_M(개월).

소득: INCOME(백만원), INCOME_M(천원).

◎ Descriptive Statistics

Variable	Mean	Median	Minimum	Maximum	Std. Dev.	C.V.	Skewness
AGE	34.682	35.000	23.000	45.000	5.4864	0.15819	-0.17337
AGE_M	416.19	420.00	276.00	540.00	65.836	0.15819	-0.17337
INCOME	1.5598	1.4290	0.22000	3.9500	0.81954	0.52541	0.89810
INCOME_M	1559.8	1429.0	220.00	3950.0	819.54	0.52541	0.89810

① LS of INCOME on AGE

Model 1: Dependent variable: INCOME

	Coefficient	*Std. Error*	*t-ratio*	*p-value*	
const	−0.0360046	0.547663	−0.0574	0.9477	
AGE	0.0460126	0.0155991	2.950	0.0041	***

② LS of INCOME on AGE_M

Model 4: Dependent variable: INCOME

	Coefficient	*Std. Error*	*t-ratio*	*p-value*	
const	−0.0360046	0.547663	−0.06574	0.9477	
AGE_M	0.00383438	0.00129993	2.950	0.0041	***

③ LS of INCOME_M on AGE

Model 5: Dependent variable: INCOME_M

	Coefficient	*Std. Error*	*t-ratio*	*p-value*	
const	−36.0046	547.663	−0.06574	0.9477	
AGE	46.0126	15.5991	2.950	0.0041	***

④ LS of INCOME_M on AGE_M

Model 7: Dependent variable: INCOME_M

	Coefficient	*Std. Error*	*t-ratio*	*p-value*	
const	−%s	547.663	−%#.4g	0.9477	
AGE_M	3.83438	1.29993	2.950	0.0041	***

◎ What's relation between the estimates in ①-④?

(a) $y_i = b_1 + b_2 X_i + e_i$

(b) $y_i^* = b_1^* + b_2^* X_i^* + e_i^*$

where (c) $y_i^* = w_1 y_i, \qquad X_i^* = w_2 X_i$

► Putting (c) into (b),

$$w_1 y_i = b_1^* + b_2^* w_2 X_i + e_i^*$$

$$\Rightarrow \; y_i = \frac{1}{w_1} b_1^* + b_2^* \frac{w_2}{w_1} X_i + \frac{1}{w_1} e_i^*$$

So, $b_1 = \dfrac{b_1^*}{w_1}, \qquad b_2 = b_2^* \dfrac{w_2}{w_1}$.

2. Functional Form

(1) 모든 변수에 대해 선형인 기본 모형

$$Y_i = \alpha_0 + \alpha_1 A_i + \varepsilon_i$$

► Marginal effect of Age: $\dfrac{\Delta E(y_i)}{\Delta A_i} = \alpha_1$.

(2) (변수에 대해) <u>비선형 모형</u>(Quadratic, Cubic)

- $Y_i = \gamma_0 + \gamma_1 A_i + \gamma_2 A_i^2 + \varepsilon_i$: Quadratic model

► Marginal effect of Age: $\dfrac{\Delta E(y_i)}{\Delta A_i} = \gamma_1 + 2\gamma_2 A_i$; depends on A_i.

(예)

$$\widehat{Income_i} = -62.147 + 2.817 Ed_i + 4.297 Age_i - 0.044 Age_i^2,$$
$$(-1.287)\ (3.151)\quad (2.491)\quad (-2.048)\qquad R^2 = 0.328$$

► 연령의 소득에 대한 한계 효과: 4.297-0.088× Age_i로 개인의 연령수준에 의존.

- $Y_i = \gamma_0 + \gamma_1 A_i + \gamma_2 A_i^2 + \gamma_3 A_i^3 + \varepsilon_i$: Cubic model

(3) <u>Interaction Term</u>

If marginal effect of X on y depends on other variable(Z),

$$y_i = \beta_1 + \beta_2 X_i \cdot Z_i + \varepsilon_i$$

(예) $\frac{\Delta E(y_i)}{\Delta A_i}\Big|_{\text{대졸}} > \frac{\Delta E(y_i)}{\Delta A_i}\Big|_{\text{고졸}}$

$\frac{\Delta E(y_i)}{\Delta A_i} = \text{Constant} \cdot \text{Education}_i$; proportional to education.

Then, $y_i = \beta_1 + \beta_2 X_i + \beta_3 A_i \cdot \text{Edu}_i + \varepsilon_i$

4. Typical Nonlinear Functional Form

① Double-log Model (log-log model)

$$\log y_i = \beta_1 + \beta_2 \log X_i + \varepsilon_i$$

► $\beta_2 = \dfrac{\Delta \log y_i}{\Delta \log X_i} = \dfrac{\Delta y_i / y_i}{\Delta X_i / X_i}$

(예)

$$\hat{\log}(sales)_i = 9.809 + 0.413 \log(Adv_i)$$

$$(26.822)\,(8.273) \qquad\qquad R^2 = 0.487$$

► 광고비가 1% 증가할 때 평균 매출액은 약 0.4% 증가하는 것을 의미

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② <u>Log-linear Model</u>

$$\log y_i = \beta_1 + \beta_2 X_i + \varepsilon_i$$

► $\beta_2 = \dfrac{\Delta \log y_i}{\Delta X_i} = \dfrac{\Delta y_i / y_i}{\Delta X_i}$

(예)

$$\hat{\log}(Income_i) = 3.087 + 0.0159 Age_i$$

$$(24.842)\,(5.276) \qquad\qquad R^2 = 0.236$$

► 연령이 1세 증가함에 따라 소득은 평균 1.59% 증가함을 의미한다.

- In time-series data, if $X = time$, $\beta_2 = \dfrac{\Delta \log y_t}{\Delta t} = \dfrac{\Delta y_t / y_t}{\Delta t}$: average growth rate of y_t.

(Example)

(a) $\widehat{\ln GDP_t} = 6.93 + 0.0269t$

(b) $\widehat{GDP_t} = 1{,}040 + 34.99t$

③ Linear-log Model

$$y_i = \beta_1 + \beta_2 \log X_i + \varepsilon_i$$

► $$\beta_2 = \frac{\Delta y_i}{\Delta X_i / X_i}$$

$$\hat{Sales}_i = -1565325 + 404440 \log(Adv_i)$$
$$(-2.323) \qquad (4.399) \qquad\qquad R^2 = 0.212$$

► 광고비가 1% 증가할 때 평균 매출액은 4,044.40(백만원) 즉, 약 40.4억원 증가.

④ <u>Reciprocal Model (Inverse Function)</u>

$$y_i = \beta_1 + \beta_2 \frac{1}{X_i} + \varepsilon_i$$

► $y_t = \pi_t$: inflation rate, $X_t = u_t$: unemployment rate.

⇒ (short-run) Phillips curve estimation.

- In many cases,

► monetary unit $\Rightarrow$ log

► rate $\Rightarrow$ level

►

- (Real) Nonlinear Model

(Example) $y_i = \beta_1 + \dfrac{\beta_2}{(X_i - \beta_3)} + \varepsilon_i$

► Nonlinear Least Squares Estimation.