Math. Econ. Stat.

Answer Key 4

1.

Since $f(x) \geq 0$ for $x \geq 1$ and $\int_1^\infty \frac{1}{x^2} dx = \left[-\frac{1}{x} \right]_1^\infty = 1$, $f(x)$ is a valid pdf.

However, $E(X) = \int_1^\infty x \frac{1}{x^2} dx = [\ln x]_1^\infty = \infty$

2. $E\left(\sum_{i=1}^n a_i X_i \right) = \sum_{i=1}^n a_i E(X_i) = \mu \sum_{i=1}^n a_i$

It is required that $\sum_{i=1}^n a_i = 1$

3.

(1)

$$E(X) = \int_0^\infty x e^{-x} dx = \left[-xe^{-x} \right]_0^\infty + \int_0^\infty e^{-x} dx = \left[-e^{-x} \right]_0^\infty = 1$$

$$E(X^2) = \int_0^\infty x^2 e^{-x} dx = \left[-x^2 e^{-x} \right]_0^\infty + 2\int_0^\infty e^{-x} dx = 2$$

$$Var(X) = E(X^2) - [E(X)]^2 = 2 - 1 = 1 \qquad \therefore \mu = 1, \ \sigma = 1$$

$$P(|X-1| \geq 2) = 1 - P(0 < X \leq 3) = 1 - \int_0^3 e^{-x} dx = 1 - \left[-e^{-x} \right]_0^3 = e^{-3} = 0.05$$

(2)

$$P(|X - \mu| \geq 2\sigma) \leq \frac{\sigma^2}{(2\sigma)^2} = \frac{1}{4} = 0.25$$

4. $E(Y) = \int_{-\infty}^\infty y f(y) dy = \int_{-\infty}^0 y f(y) dy + \int_0^\infty y f(y) dy$

$= \int_{-\infty}^0 (-y) f(-y)(-dy) + \int_0^\infty y f(y) dy$ since $f(y) = f(-y)$

$$= -\int_0^{\infty} wf(w)dw + \int_0^{\infty} yf(y)dy = 0 .$$

(Note) As indicated in the class,

$$E(\mathrm{Y}^r) = 0, \text{ for } r = \text{odd numbers, if } f(\mathrm{y}) \text{ is symmetric.}$$