Math. Econ. Statistics

2021년 1학기 학기말 고사 남 준 우

학번: ____________ 성명: ____________

1. (15 points)

Suppose that $\underline{y} \sim \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \sim N(\underline{\mu}, \Sigma)$ with $\underline{\mu} = \begin{bmatrix} 3 \\ -1 \\ 0 \end{bmatrix}$, $\Sigma = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 4 & 1 \\ 0 & 1 & 1 \end{bmatrix}$.

What is the distribution of $W = \dfrac{(y_1 - 3)^2}{2} + y_3{}^2$. Provide **concretely** your argument.

<다음 문제의 답안은 A4 용지 새 종이에 답안을 기록하세요.>

2. (15 points) Suppose that $Y_1 = X + U_1$ and $Y_2 = X + U_2$, where $X =$ permanent income, $U_1 =$ current income in year 1, and $U_2 =$ current income in year 2. It is known that U_1 and U_2 have zero expectations and are uncorrelated with X. It is also known that $V(X) = 400,\ V(U_1) = 200,\ V(U_2) = 100$, and that $C(U_1, U_2) = 300$. A random sample of size 10 is drawn from the joint probability distribution of Y_1 and Y_2. The objective is to estimate $\mu = E(X)$, which is unknown. The sample means are $\overline{Y}_1$ and $\overline{Y}_2$. Consider all the linear combinations of $\overline{Y}_1$ and $\overline{Y}_2$ which are unbiased estimators of μ, and find the one that has minimum variance.

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3. (15 점*2=30 점) Suppose that $X_1, \cdots, X_n$ form a random sample from $N(\mu, \sigma^2)$.

Consider the following random variable: $\dfrac{n(\bar{X}_n - \mu)^2}{\sigma^2}$.

(1) Derive the distribution of the random variable and state your logic as concrete as possible. What is mean of the above statistics?

(2) Suppose that the above parental distribution is NOT $N(\mu, \sigma^2)$. Find the limiting distribution of $\left(\bar{X}_n\right)^2$.

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4. (각 10점씩 40점) Let $(X_1, \cdots, X_n)$ be an i.i.d. sample of Bernoulli random variables; that is, each X_i has density $f(x;\theta) = \theta^x (1-\theta)^{1-x}$.

(1) Find the MLE of θ.

(2) Sketch the asymptotic distribution of MLE.

(3) Suppose that you are interested in $\gamma = 1/\theta$. What is the MLE for γ?

(4) Deduce the asymptotic distribution of the MLE for γ.