ECO 4004: Math. Econ. Statistics

Problem Set 3: Change of Variables

1. Suppose that the pdf of a random variable X is as follows:

$$f_X(x) = \begin{cases} 2x & for\ 0 \le x \le 1, \\ 0 & elsewhere. \end{cases}$$

We were interested in a new random variable given by $Y = 3X - 1$. Find the probability density function for Y by the transformation method.

2. Suppose that $X \sim N(\mu, \sigma^2)$. Prove that $Y = aX + b \sim N\left(a\mu + b, (a\sigma)^2\right)$ by transformation method.

3. For each of the following, obtain the pdf of Y.

(1) X distributed exponential with $\lambda = 2$, $Y = 2X$.

(2) X distributed uniform(or rectangular) on $(0, 1)$, $Y = -\ln(X)$.

(3) X distributed standard normal, $Y = X^2$.

* If you know the pdf type of chi-squares distribution, compare it with the answer of (3)

4. Suppose that the pmf of X is

$$f(x) = \begin{cases} 1/8 & for\ x = -1, \\ 2/8 & for\ x = 0, \\ 5/8 & for\ x = 1, \\ 0 & elsewhere. \end{cases}$$

Suppose that $Y = X^2$. Find pmf of Y.