Answer Key 2

1. $F(x)=\sum_{t\le x} f(t)=\sum_{t\le x}\frac{t}{15}=\frac{1}{15}\sum_{t\le x} t=\frac{1}{15}\frac{x(x+1)}{2}$.

That is,

$$F(x)=\begin{cases}0 & for\ x<1\\ 1/15 & for\ 1\le x<2\\ 3/15 & for\ 2\le x<3\\ 6/15 & for\ 3\le x<4\\ 10/15 & for\ 4\le x<5\\ 1 & for\ 5\le x\end{cases}$$

For discrete distributions, $F(x)$ has a shape of step function.

2.

(1) $P(X>0.6)=\int_{0.6}^{1} g(x)dx=\int_{0.6}^{1} 6x(1-x)dx=\left[3x^2-2x^3\right]_{0.6}^{1}=0.352.$

(2) $G(x)=\int_{0-\infty}^{x} g(x)dx=3x^2-2x^3$.

Therefore, $G(x)=\begin{cases}0 & for\ x<0\\ 3x^2-2x^3 & for\ 0\le x<1\\ 1 & for\ 1\le x\end{cases}$

3.

(1) Since, for a proper pdf, $\int_{-\infty}^{\infty} f(x)dx=1$,

from $\int_{-\infty}^{\infty} f(x)dx=\int_0^1 xdx+\int_1^c (2-x)dx=1$, $c=2$.

(2) For $0<x<1$, $F(x)=\int_0^t xdx=0.5x^2$.

For $1\le x<2$, $F(x)=F(1)+\int_1^x (2-x)dx=0.5+2x-0.5x^2-1.5=2x-0.5x^2-1$.

Therefore, $F(x) = \begin{cases} 0 & for\ x \le 0 \\ 0.5x^2 & for\ 0 < x < 1 \\ -0.5x^2 + 2x - 1 & for\ 1 \le x < 2 \\ 1 & for\ 2 \le x \end{cases}$

(3) From $F(m) = 0.5$, $m = 1$.

(4) $P(0.8 < X < 0.6\,c) = P(0.8 < X < 1.2) = F(1.2) - F(0.8) = 0.36$.

4. 생략.

5.

Since $F(x) = \begin{cases} 0 & for\ x \le 0 \\ 1 - e^{-x/30} & for\ 0 < x \end{cases}$

(1) $P(X \le 19) = F(19) = 1 - e^{-19/30}$.

(2) $P(29 \le X \le 38) = F(38) - F(29) = -e^{-38/30} + e^{-29/30}$.

(3) $P(48 \le X) = 1 - F(48) = e^{-48/30}$.