ECO 4004: Math. Econ. Statistics

Problem Set 8: Sampling Distributions

1. If the random variable $X$ has the exponential distribution with parameter $\theta$, then we know that $E(X^j) = \frac{\theta \times j!}{\theta^{j+1}} = \frac{j!}{\theta^j}$, using the following definite integral:

$$\int_0^\infty t^n e^{-at} dt = \frac{n!}{a^{n+1}} \text{ if } a > 0 \text{ and } n \text{ is positive integer.}$$

Suppose $\theta = 2$.

(1) Show that $E(X) = 1/2, \quad E(X^2) = 1/2, \quad E(X^3) = 3/4, \quad E(X^4) = 3/2$.

Consider random sampling, sample size 20, from that population. Let $\overline{W} = \frac{1}{20}\sum_{i=1}^{20} X_i^{\ 2}$.

(2) Calculate $E(\overline{W})$.

(3) Calculate $V(\overline{W})$.

2. Let $\overline{X}$ and $S^2$ denote the sample mean and sample variance in random sampling, sample size 20, from a $N(10, 80)$ population. Calculate the probabilities of each of the following events:

A: $\overline{X} \le 14$, B: $10 \le \overline{X} \le 12$,

C: $S^2 \le 108.8$, D: $B \cap C$

E: $\frac{\sqrt{20}(\overline{X} - 10)}{S} \le 1.066$, F: $\overline{X} \le 10 + 0.3047S$.

For Exercise #2, to get values of $F(a), G(a)$ and $H(a)$, where $F(.), G(.)$ and $H(.)$ denote the cdf of $N(0,1), \chi^2(19)$ and $t(19)$ and a is a real number, you can refer to either of Wackerly et al. (2008)'s tables or you can calculate those values by GAUSS.

3. Suppose that $X_i$ is normally distributed, calculate $V(S^2)$.