ECO 4004: Mathematical Statistical Economics
Problem Set 12: Maximum Likelihood Estimation

1. The random variable Y has pdf:

$$f(y;\theta)=\begin{cases}\frac{1}{\theta}e^{-y/\theta} & for \quad y>0\\ 0 & otherwise\end{cases}$$

where the unknown parameter θ is positive.

(1) Write down the joint probability function for a random sample of size n from that population.

(2) Find the maximum, likelihood estimator of θ.

(3) Obtain the asymptotic distribution of the estimator.

(4) Is your estimator unbiased? Explain briefly.

(5) Verify the second order condition.

(6) What is Cramer-Rao lower bound?

2. Let $(X_1,\cdots,X_n)$ be an i.i.d. sample of Bernoulli random variables; that is, each X_i has density $f(x;\theta)=\theta^x(1-\theta)^{1-x}$.

(1) Find the MLE of θ.

(2) Sketch the asymptotic distribution of MLE.

(3) Suppose that you are interested in $\gamma=1/\theta$. What is the MLE for γ?

(4) Deduce the asymptotic distribution of the MLE for γ.