Eco 4004: Math. Econ. Statistics

Problem Set 6

Mathematical Expectation for Multivariate Distribution and Independence

1. Consider the following bivariate curved-roof distribution, considered at Problem Set 2:

$$f(x,y) = 3\left(x^2 + y\right)/11 \qquad for\ 0 \le x \le 2, \quad 0 \le y \le 1,$$

with $f(x,y) = 0 \qquad elsewhere$.

(1) For $0 \le x \le 2$, find the conditional expectation function(CEF) of Y given X.
(2) Calculate $E(X), E(Y), E(X^2), E(Y^2), E(XY), V(X), V(Y), C(X,Y)$.
(3) Find the best linear predictor(BLP) of Y given X.

2. For the joint pmf in the table below:

	$x=1$	$x=2$	$x=3$
$y=0$	0.15	0.10	0.15
$y=1$	0.15	0,30	0.15

(1) Find the conditional expectation function $E(Y|X)$.

(2) Find the best linear predictor of Y given X, $E^*(Y|X)$.

(3) Prepare a table that gives $E(Y|X)$ and $E^*(Y|X)$ for $x = 1,2,3$.

3. Suppose that the random variables Z (=permanent income) and W (=transitory income) have zero covariance, with $E(Z) = 42, \quad E(W) = 0, \quad V(Z) = 2500, \quad V(W) = 500$. Further, X (=current income) is determined as $X = Z + W$.
(1) Calculate $E(X), C(Z,X), C(W,X)$ and $V(X)$.
(2) Find the BLP of current income given permanent income.
(3) Predict as best you can the current income of a person whose permanent income is $z = 54$.
(4) Find the BLP of permanent income given current income.

(5) Predict as best you can the permanent income of a person whose current income is $x = 54$.

4. Suppose X_1, X_2, X_3 are stochastically independent, have zero expectations, and have unit variances, and $Z = X_3^2 (X_2 - 2X_1)^2$. Find $E(Z)$.