Chapter 10 Dummy Variable Model

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(1) Motivation

(2) Intercept Dummy Variable

(3) Slope Dummy Variable

(4) Intercept and Slope Dummy Variable

(5) Application: Seasonal Adjustment

(1) Motivation

► Independent variables(quantitative variable, qualitative variable) $\Rightarrow$ dependent variable

• Dummy variable: quantifying the qualitative variables.

(Example) Wage discrimination across gender.

• To investigate wage equation for each gender,

Male: $y_i = \alpha_1 + \beta_1 X_i + \varepsilon_i$

Female: $y_i = \alpha_2 + \beta_2 X_i + \varepsilon_i$ where y: wage rate, X: tenure,

► α: starting salary, β: wage promotion

• To compare (conduct hypothesis testing) α's and β's,

we need to merge two separate regression equations into one.

► Need for quantify gender variable.

(2) <u>Intercept Dummy Variable</u>($\alpha_1 \neq \alpha_2,\ \beta_1 = \beta_2$)

Male: $y_i = \alpha_1 + \beta X_i + \varepsilon_i$

Female: $y_i = \alpha_2 + \beta X_i + \varepsilon_i$

► Let $D_i = \begin{cases} 0 & \text{for female} \\ 1 & \text{for male} \end{cases}$.

Then,

$$\begin{aligned} y_i &= \alpha_1 \cdot D_i + \alpha_2 \cdot (1 - D_i) + \beta X_i + \varepsilon_i \\ &\equiv \alpha + \gamma D_i + \beta X_i + \varepsilon_i \end{aligned},$$

γ : wage discrimination against female worker.

► LS of y on (1, D, X).

① Testing $H_0: \gamma = 0$ or $H_0: \gamma \geq 0$ by t-test.

(Example) Wage discrimination across gender:

$$Male_i = \begin{cases} 0 & \text{for female} \\ 1 & \text{for male} \end{cases}$$

Model: $Income_i = \alpha + \gamma \cdot Male_i + \beta \cdot Age_i + \varepsilon_i$

Output:

$$Income_i = \underset{(3.133)}{17.564} + \underset{(2.412)}{6.776} \cdot Male_i + \underset{(3.181)}{0.494} \cdot Age_i + e_i \qquad R^2 = 0.260$$

► Male: $Income_i = 24.340 + 0.494 \cdot Age_i + e_i$

Female: $Income_i = 17.564 + 0.494 \cdot Age_i + e_i$

② Let $F_i = \begin{cases} 0 & \text{for male} \\ 1 & \text{for female} \end{cases}$. $y_i = \alpha + \delta \cdot F_i + \beta X_i + \varepsilon_i$

Then, $\delta = -\gamma$.

③ In time series data, the dummy variable can be used to examine policy effectiveness or structural change.

$$D_t = \begin{cases} 0 & \text{before policy} \\ 1 & \text{after policy} \end{cases}$$

$$y_t = \alpha + \gamma \cdot D_t + \beta X_t + \varepsilon_t$$

④ Suppose there are M categories, use only M-1 dummy variables.

(Example) Investigating the effect of education on income

Suppose there are four groups: (중졸이하, 고졸, 대졸, 대학원졸).

$$H_i = \begin{cases} 1 & \text{if 고졸} \\ 0 & \text{otherwise} \end{cases}, \qquad C_i = \begin{cases} 1 & \text{if 대졸} \\ 0 & \text{otherwise} \end{cases}, \qquad G_i = \begin{cases} 1 & \text{if 대학원졸} \\ 0 & \text{otherwise} \end{cases}$$

► $y_i = \alpha + \delta_1 H_i + \delta_2 C_i + \delta_3 G_i + \beta X_i + \varepsilon_i$

(Example) Effect of education on income

- Education group: (중졸 이하, 고졸, 대졸 이상)

$$H_i = \begin{cases} 1 & \text{if 고졸} \\ 0 & \text{otherwise} \end{cases}, \qquad C_i = \begin{cases} 1 & \text{if 대졸} \\ 0 & \text{otherwise} \end{cases}$$

Model: $Income_i = \alpha + \delta_1 H_i + \delta_2 C_i + \beta Age_i + \varepsilon_i$

Output:

$$Income_i = \underset{(0.551)}{4.217} + \underset{(0.904)}{3.691} H_i + \underset{(2.020)}{8.478} C_i + \underset{(5.680)}{0.816} Age_i + e_i \qquad R^2 = 0.272$$

► 중졸이하: $Income_i = 4.217 + 0.816 Age_i + e_i$

► 고졸: $Income_i = 4.217 + 0.816 Age_i + e_i$

► 대졸이상: $Income_i = 12.695 + 0.816 Age_i + e_i$

⑤ More than 2 qualitative variables

► gender(D) and education(H, C, G)

► $y_i = \alpha + \gamma D_i + \delta_1 H_i + \delta_2 C_i + \delta_3 G_i + \beta X_i + \varepsilon_i$

⑥ What if using M dummy variables for M categories?

► What if we use two dummies $F_i = \begin{cases} 0 & \text{for male} \\ 1 & \text{for female} \end{cases}$ and $D_i = \begin{cases} 0 & \text{for female} \\ 1 & \text{for male} \end{cases}$?

(3) Slope Dummy Variable($\alpha_1 = \alpha_2,\ \beta_1 \neq \beta_2$)

Male: $y_i = \alpha + \beta_1 X_i + \varepsilon_i$

Female: $y_i = \alpha + \beta_2 X_i + \varepsilon_i$

► Let $D_i = \begin{cases} 0 & \text{for female} \\ 1 & \text{for male} \end{cases}$.

Then,

$$\begin{aligned} y_i &= \alpha + \beta_1 D_i \cdot X_i + \beta_2 (1 - D_i) \cdot X_i + \varepsilon_i \\ &\equiv \alpha + \beta X_i + \gamma D_i \cdot X_i + \varepsilon_i \end{aligned},$$

γ: discrimination on wage promotion against female worker.

► LS of y on (1, X, D·X).

(Example) Discrimination on wage promotion against female worker:

$$Male_i = \begin{cases} 0 & \text{for} \quad \text{female} \\ 1 & \text{for} \quad \text{male} \end{cases}$$

Model: $Income_i = \alpha + \ \beta \cdot Age_i + \gamma \cdot Male_i \cdot Age_i + + \varepsilon_i$

Output:

$$Income_i = 22.902 + 0.334\,Age_i + \ 0.193\,Male_i \cdot Age_i + e_i$$

$$(3.626) \quad (1.711) \qquad\qquad (2.442) \qquad\qquad\qquad R^2 = 0.261$$

► Male: $Income_i = 22.902 + \ 0.527 \cdot Age_i + e_i$

Female: $Income_i = 22.902 + \ 0.334 \cdot Age_i + e_i$

(4) Intercept and Slope Dummy Variable($\alpha_1 \neq \alpha_2,\ \beta_1 \neq \beta_2$)

Male: $y_i = \alpha_1 + \beta_1 X_i + \varepsilon_i$

Female: $y_i = \alpha_2 + \beta_2 X_i + \varepsilon_i$

► Let $D_i = \begin{cases} 0 & \text{for female} \\ 1 & \text{for male} \end{cases}$.

Then,

$$y_i = \alpha + \gamma_1 \cdot D_i + \beta X_i + \gamma_2 D_i \cdot X_i + \varepsilon_i,$$

γ_1: wage discrimination against female worker,

γ_2: discrimination of wage promotion against female worker.

► LS of y on (1, D, X, D·X).

(a) $H_0: \gamma_1 = 0$ or $H_0: \gamma_1 \geq 0$ by t-test.

(b) $H_0: \gamma_2 = 0$ or $H_0: \gamma_2 \geq 0$ by t-test.

(c) $H_0: \gamma_1 = \gamma_2 = 0$ by F-test.

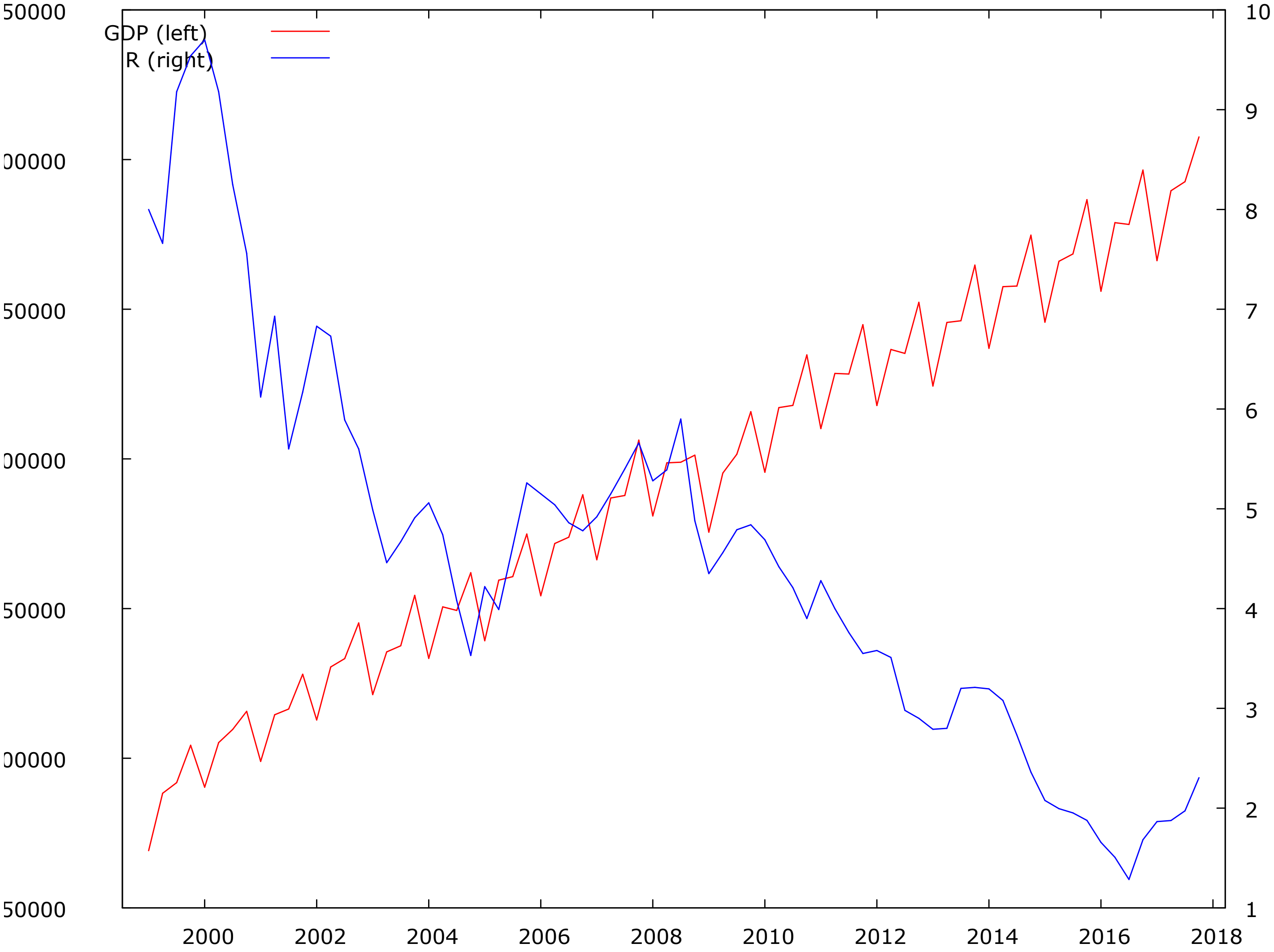
(5) Application: Seasonal Adjustment

- Many economic (monthly or quarterly) time series data based on exhibit seasonal patterns.

(Example) sales on Christmas season, sales of ice-cream, unemployment rate on winter season and etc.

► Often, it is desirable to remove the seasonal factor from a time series, so that one may concentrate on the trend.

► Process of removing seasonal component: deseasonalization, seasonal adjustment.


GDP (left)
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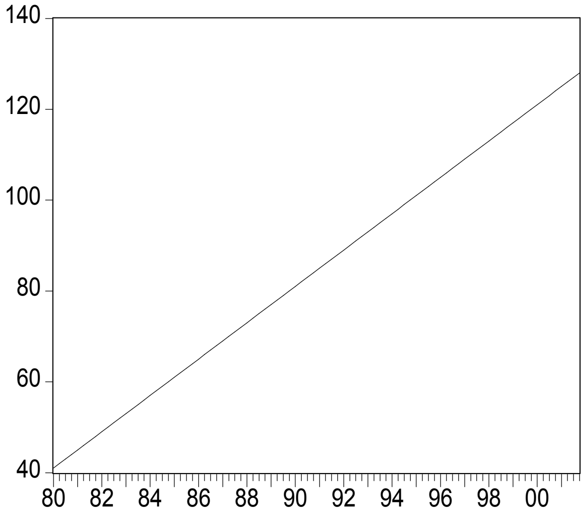
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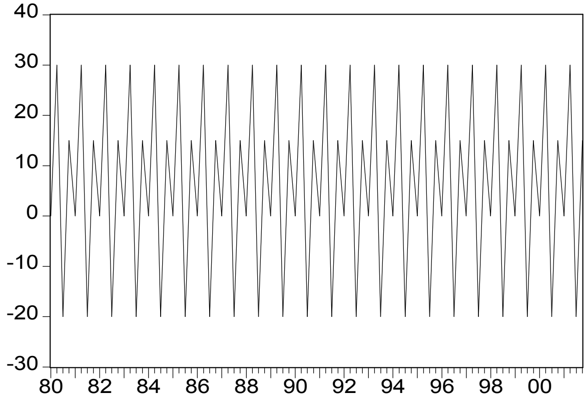
(a)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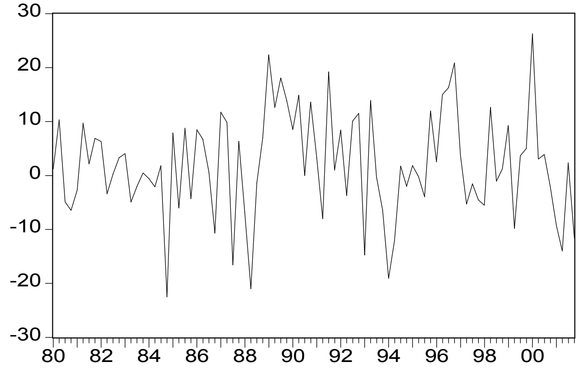
(b)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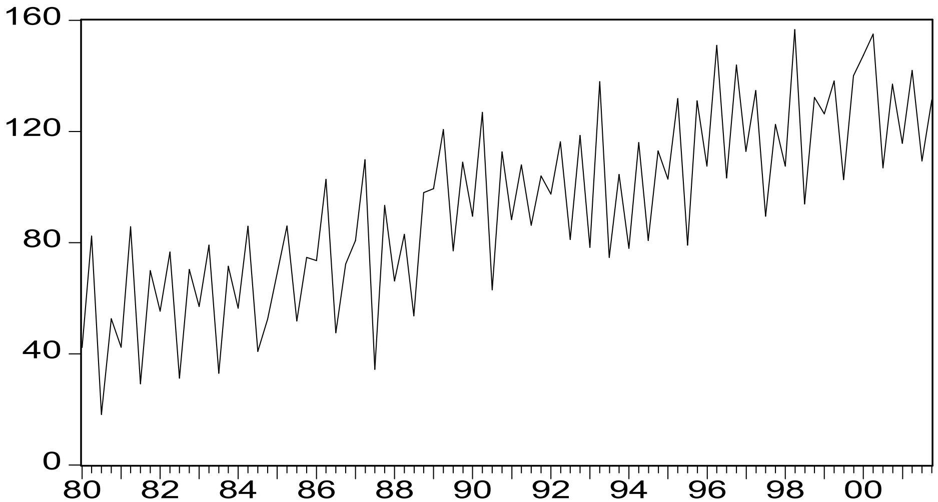
(c)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(d)

- Quarterly Dummy Variables:

► $D_{2t} = \begin{cases} 1 & \text{if 2nd quarter} \\ 0 & \text{otherwise} \end{cases}$, $D_{3t} = \begin{cases} 1 & \text{if 3rd quarter} \\ 0 & \text{otherwise} \end{cases}$, $D_{4t} = \begin{cases} 1 & \text{if 4th quarter} \\ 0 & \text{otherwise} \end{cases}$

- $y_t = \alpha + \alpha_2 D_{2t} + \alpha_3 D_{3t} + \alpha_4 D_{4t} + \beta X_t + \varepsilon_t$

1st quarter: $y_t = \alpha + \beta X_t + \varepsilon_t$,

2nd quarter: $y_t = \alpha + \alpha_2 + \beta X_t + \varepsilon_t$,

3rd quarter: $y_t = \alpha + \alpha_3 + \beta X_t + \varepsilon_t$,

4th quarter: $y_t = \alpha + \alpha_4 + \beta X_t + \varepsilon_t$.

(Example) 분기별 자료를 이용한 소비함수의 추정

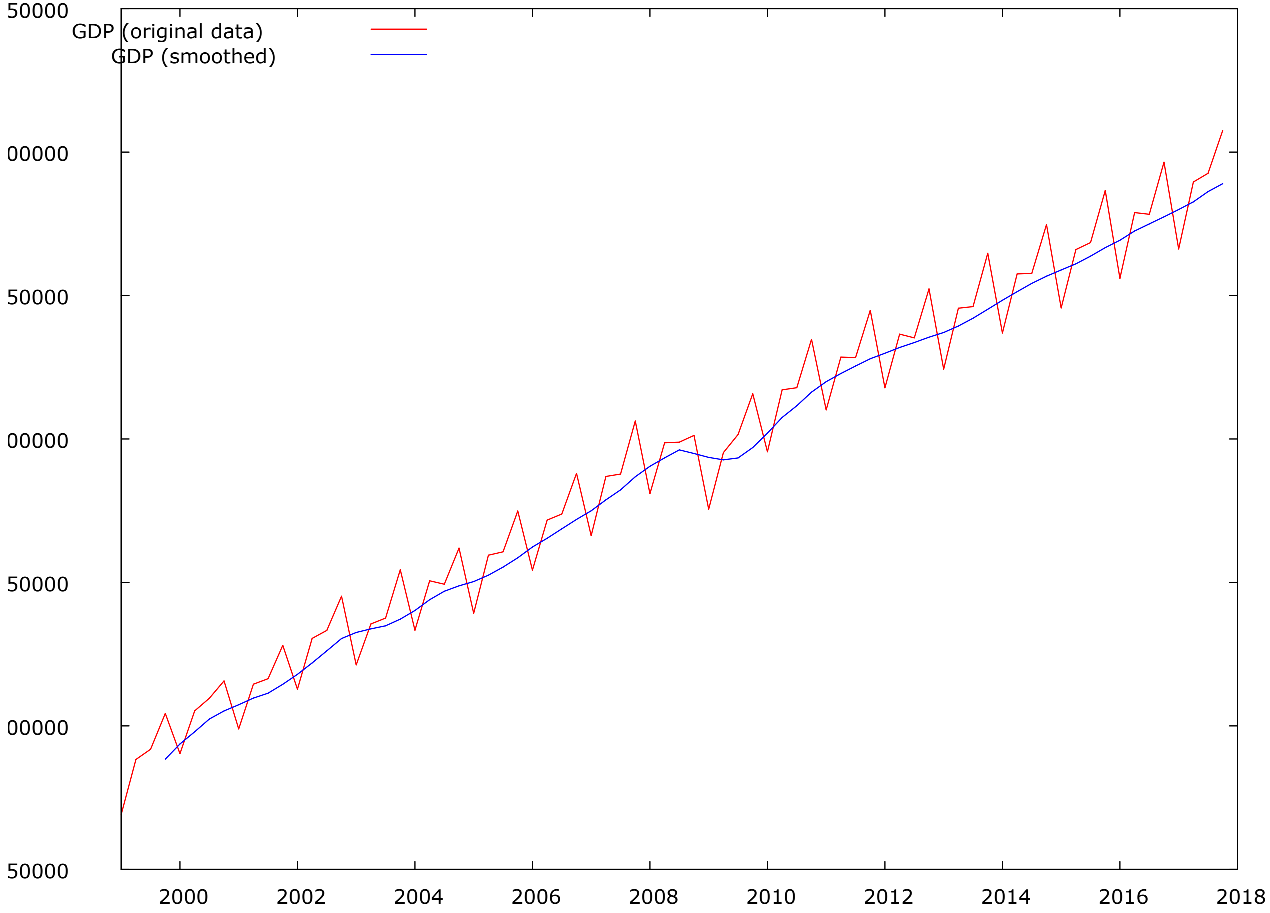
민간소비(CP)를 (상수항, 국민소득(GNI), 이자율(R), 계절더미)에 대해 회귀분석:

$$CPt = 39207.98 + 0.37GNI_t - 560.52R_t + 9070.33D1_t - 684.11D2_t + 1228.48D3_t + e_t$$

$$(4.40) \quad (18.34) \quad (-0.93) \quad (5.78) \quad (-0.46) \quad (0.83)$$

$$R^2 = 0.97$$

• Removing Seasonality(Seasonal Adjustment)



►In gretl, 변수를 선택 후 [variable]/[simple moving average]