Problem Set 1

계량경제학 남 준우 교수

1. Verify the following statement:

$$\sum_{i=1}^{n}\left(X_i-\bar{X}\right)\left(Y_i-\bar{Y}\right)=\sum_{i=1}^{n}\left(X_i-\bar{X}\right)Y_i \;=\sum_{i=1}^{n}X_i\left(Y_i-\bar{Y}\right)$$
$$=\sum_{i=1}^{n}X_iY_i-n\bar{X}\bar{Y}$$

(Hint) Easy way to prove the above equality: derive the first three equations in terms of the last equation, $\sum_{i=1}^{n}X_iY_i-n\bar{X}\bar{Y}$

(Note) Substituting $Y_i = X_i$ into the above equation makes the following equality:

$$\sum_{i=1}^{n}\left(X_i-\bar{X}\right)^2=\sum_{i=1}^{n}\left(X_i-\bar{X}\right)X_i=\sum_{i=1}^{n}X_i^2-n\bar{X}^2$$

2. Prove that the following statement: The sum of squared deviations around the sample mean is as small as the squared deviations around <u>any other</u> number,

$\sum_{i=1}^{n}(X_i-\bar{X})^2 \le \sum_{i=1}^{n}(X_i-a)^2$ for any number a.

This implies that $\sum_{i=1}^{n}(X_i-a)^2$ is minimized by choosing $a=\bar{X}$.

3. Suppose that $V(Y_i) = \sigma_i^2$, that is, $V(Y_1) = \sigma_1^2$, $V(Y_2) = \sigma_2^2$, $\cdots$, $V(Y_n) = \sigma_n^2$. And also, $Cov(Y_i, Y_j) = \sigma_{ij}$, that is, $Cov(Y_1, Y_2) = \sigma_{12}, Cov(Y_1, Y_3) = \sigma_{13}, \cdots, Cov(Y_{n-1}, Y_n) = \sigma_{n-1\,n}$, where $Cov(X, Y)$ denotes covariance of X and Y.

(1) Find $V\left(\sum_{i=1}^{n} w_i Y_i\right)$ where w_i's are constants.

(2) Now suppose $\sigma_{ij} = 0$ for all $i \neq j$. Find $V\left(\sum_{i=1}^{n} w_i Y_i\right)$.

(3) In addition to (2), suppose $\sigma_i^2 = \sigma^2$ for all $i = 1, 2, \cdots, n$. Find $V\left(\sum_{i=1}^{n} w_i Y_i\right)$.