Answer Key 1

2.

(1)
$$P(R) = 0.67, P(R \cup S) = 0.55.$$

$$P(R \cup S) = P(R) + P(S) - P(R \cap S) \ge P(R)$$
 should hold.

(2)
$$P(E) = 0.82, P(E \cap F) = 0.86.$$

 $P(E) \ge P(E \cap F)$ should hold.

(3)
$$P(G) = 0.72, P(B) = 0.84, P(G \cap B) = 0.52.$$

$$P(G \cup B) = P(G) + P(B) - P(G \cap B) = 1.04 > 1$$
 violates the axiom of probability.

3.
$$P(A) = 0.5, P(B) = 0.7,$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Since $P(A \cup B) \le 1$ and $P(A \cap B) \le \min\{P(A), P(B)\}, 0.2 \le P(A \cap B) \le 0.5$

4.
$$P(A) = \frac{18}{36} = \frac{1}{2}$$
, $P(B) = \frac{18}{36} = \frac{1}{2}$, $P(C) = \frac{6}{36} = \frac{1}{6}$

$$P(A \cap B) = \frac{9}{36} = \frac{1}{4}, P(B \cap C) = \frac{3}{36} = \frac{1}{12}, P(B \cap C) = \frac{3}{36} = \frac{1}{12}$$

$$\Rightarrow$$
 P(A \cap B) = $\frac{1}{4}$ = P(A)P(B); A and B are independent.

$$P(B \cap C) = \frac{1}{4} = P(B)P(C)$$
; B and C are independent.

$$P(A \cap C) = \frac{1}{A} = P(A)P(C)$$
; A and C are independent.

5.
$$P(A) = 0.7, P(B) = 0.5, P(\overline{A \cup B}) = 0.1$$

$$\Rightarrow$$
 P(A \cup B) = 1 - P($\overline{A \cup B}$) = 0.3

(1)
$$P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.3$$
.

(2)
$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{3}{5}$$

(3)
$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{3}{7}$$