Chapter 12 Autocorrelation

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1. Sources, Nature of Autocorrelation
2. Problems of Least Squares Estimator under Autocorrelation
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7. Other forms of Autocorrelation

(Example)

1. Sources, Nature of Autocorrelation

- $C(\varepsilon_t, \varepsilon_s) = C(y_t, y_s) \neq 0 \text{ for } t \neq s$: serial correlation, autocorrelation.

► Frequently encountered in time series data.

(1) Inertia

- Time series(ex: GDP, Export, Unemployment etc.) data exhibit cycle.

► When recovering from recession, the series continue increasing for some period:

$Y_{t+1} \uparrow, Y_{t+2} \uparrow, \cdots$

► Time series are sluggish to change.

Any shock at t $\rightarrow$ $\Delta Y_t, \Delta Y_{t+1}, \Delta Y_{t+2}, \cdots$

(Example)

► Effect of labor strike at t to output at t, t+1, …

► Effect of temperature at t to consumption of electricity at t, t+1, …

► Spatial Correlation

(2) Specification bias: excluded variables

• True model: $Y_t = \beta_1 + \beta_2 X_t + \beta_3 Z_t + \varepsilon_t, \quad \varepsilon_t \sim \text{uncorrelated}$.

Estimated model: $Y_t = \beta_1 + \beta_2 X_t + v_t, \quad v_t = \beta_3 Z_t + \varepsilon_t$,

► If Z_t is correlated with Z_{t-1}, then v_t is correlated with v_{t-1}.

► If we add some variables, sometimes autocorrelation is removed.

(3) <u>Specification bias: incorrect functional form</u>

• True model: $Y_t = \beta_1 + \beta_2 X_t + \beta_3 X_t^{\,2} + \varepsilon_t, \quad \varepsilon_t \sim \text{uncorrelated}$.

Estimated model: $Y_t = \beta_1 + \beta_2 X_t + v_t$,

► v_t would have a pattern of autocorrelation.

(4) Lagged variable model

- $Y_t = \beta_1 + \beta_2 X_t + \beta_3 Y_{t-1} + \varepsilon_t$: autoregressive model,

► $Y_{t-1} \rightarrow Y_t$: habit persistence, due to psychological, technological or institutional factor.

► If we neglect Y_{t-1}, then $v_t = \beta_3 Y_{t-1} + \varepsilon_t$ will show systematic pattern.

(5) Data transformation

- $Y_t = \beta_1 + \beta_2 X_t + \varepsilon_t$,

  $Y_{t-1} = \beta_1 + \beta_2 X_{t-1} + \varepsilon_{t-1}$: level form.

$\Rightarrow$ $\Delta Y_t = \beta_2 \Delta X_t + v_t, \quad v_t = \varepsilon_t - \varepsilon_{t-1}, \quad v_t \sim \text{correlated}$.

(6) Nonstationarity

- If $Y_t, X_t \sim$ nonstationary, then $\varepsilon_t = Y_t - \beta_1 - \beta_2 X_t$ would be correlated.

► Unit root problem, Cointegration.

2. Problems of Least Squares Estimator under Autocorrelation

- $b_2 = \sum_{t=1}^{T} w_t y_t$, where $w_t = \dfrac{(X_t - \bar{X})}{\sum_{t=1}^{T}(X_t - \bar{X})^2}$.

  ► $E(b_2) = \beta_2$.

  ►
$$\begin{aligned} V(b_2)\Big|_{\text{autocorrelation}} &= V\left(\sum_{t=1}^{T} w_t y_t\right) \\ &= \sigma^2 \sum_t w_t^{\,2} + 2\sum_{t>s}\sum_{s=1} w_t w_s \sigma_{ts} \qquad \text{where } \sigma_{ts} = C(y_t, y_s) \\ &\neq \sigma^2 \sum_t w_t^{\,2} = \frac{\sigma^2}{\sum_{t=1}^{T}(X_t - \bar{X})^2} = V(b_2)\Big|_{\text{no autocorrelation}} \end{aligned}$$

  ► OLS estimator $s_{b_2}{}^2 = \dfrac{s^2}{\sum_{i=1}^{n}(X_i - \bar{X})^2}$, $E(s_{b_2}{}^2) \neq V(b_2)\Big|_{\text{autocorrelation}}$.

- So, inferences based on OLS is invalid.

- OLS estimators still unbiased.
- However, standard errors are wrong.
- OLS is inefficient: There are other estimators that have lower variances.

$\Rightarrow$

(a) t-test, F-test are invalid.

(b) Classical assumptions do not satisfy.

- Gauss-Markov theorem does not hold.
- There is more efficient estimator than OLS.

3. <u>Typical Type of Autocorrelation</u>: AR(1)

- Model: $Y_t = \beta_1 + \beta_2 X_t + \varepsilon_t$, $\varepsilon_t = \rho\varepsilon_{t-1} + u_t$, $|\rho| < 1$,

  with $E(u_t) = 0$, $V(u_t) = \sigma_u{}^2$, $C(u_t, u_s) = 0$ for $t \neq s$. $\Rightarrow$ $u_t \sim$ white noise.

► AR(1)

$$\begin{aligned}
\varepsilon_t &= \rho\varepsilon_{t-1} + u_t \\
&= \rho\left(\rho\varepsilon_{t-2} + u_{t-1}\right) + u_t = \cdots \qquad \text{(if let } \varepsilon_0 = 0\text{)}. \\
&= \sum_{s=0}^{t} \rho^s u_{t-s}
\end{aligned}$$

① $E(\varepsilon_t) = 0$.

② $V(\varepsilon_t) = \sigma_\varepsilon{}^2 = \sigma_u{}^2(1 + \rho^2 + \rho^4 + \cdots + \rho^{2t}) = \dfrac{\sigma_u{}^2}{1-\rho^2}$.

③ $Cov(\varepsilon_t, \varepsilon_{t-1}) = \rho\sigma_\varepsilon{}^2$.

$Cov(\varepsilon_t, \varepsilon_{t-2}) = \rho^2\sigma_\varepsilon{}^2$.

$\cdots$

$Cov(\varepsilon_t, \varepsilon_{t-s}) = \rho^s\sigma_\varepsilon{}^2$; autocovariance function.

④ $Corr(\varepsilon_t, \varepsilon_{t-1}) = \rho$.

$Cov(\varepsilon_t, \varepsilon_{t-2}) = \rho^2$.

$\cdots$

$Cov(\varepsilon_t, \varepsilon_{t-s}) = \rho^s$; autocorrelation function.

► As $s \to \infty$, $\rho^s \to 0$ (fading memory), but $\rho^s \neq 0$ (memory size= ∞).

• The case of $|\rho| = 1$; unit root $\Rightarrow$ Non-stationary.

► Random walk

• AR(2): $\varepsilon_t = \rho_1 \varepsilon_{t-1} + \rho_2 \varepsilon_{t-2} + u_t$.

• AR(p): $\varepsilon_t = \rho_1 \varepsilon_{t-1} + \rho_2 \varepsilon_{t-2} + \cdots + \rho_p \varepsilon_{t-p} + u_t$.

► Gaussian

4. Detecting Autocorrelation

(1) Graphical Method

Run OLS and get e_t. Plot e_t on e_{t-1} or plot e_t on t.

(2) Durbin-Watson Test

Model: $Y_t = \beta_1 + \beta_2 X_t + \varepsilon_t, \quad \varepsilon_t = \rho \varepsilon_{t-1} + u_t$.

$H_0 : \rho = 0, \qquad H_1 : \rho \neq 0$.

◎ Mechanics of Durbin-Watson test:

① Run OLS and calculate $d = \dfrac{\sum_{t=2}^{T} \left(e_t - e_{t-1}\right)^2}{\sum_{t=1}^{t} e_t^{\,2}}$.

② Find critical value d_L, d_U.

③ Decision rule:

If $d < d_L$, reject $H_0: \rho = 0$ $\Rightarrow$ positive autocorrelation.

If $d_L < d < d_U$, no decision.

If $d_U < d < 4 - d_U$, do not reject $H_0: \rho = 0$ $\Rightarrow$ no autocorrelation.

If $4 - d_U < d < 4 - d_L$, no decision.

If $4 - d_L < d$, reject $H_0: \rho = 0$ $\Rightarrow$ negative autocorrelation.

◎ <u>Rationale</u>:

①

$$d=\frac{\sum_{t=2}^{T}\left(e_t-e_{t-1}\right)^2}{\sum_{t=1}^{t}e_t^{\,2}}=\frac{\sum_{t=2}^{T}e_t^{\,2}+\sum_{t=2}^{T}e_{t-1}^{\,2}-2\sum_{t=2}^{T}e_te_{t-1}}{\sum_{t=1}^{t}e_t^{\,2}}.$$

$$\approx 2\left(1-\frac{\sum_{t=2}^{T}e_te_{t-1}}{\sum_{t=1}^{t}e_t^{\,2}}\right)=2\left(1-\hat{\rho}\right)$$

Since $-1\le\rho\le 1,\quad 0\le d\le 4$.

► If $\hat{\rho}\to 0, d\to 2.$ If $\hat{\rho}\to 1, d\to 0.$ If $\hat{\rho}\to -1, d\to 4.$

② <u>Inconclusive region</u>:

ⓐ There exists an exact critical value d^* which satisfies $P(d < d^* | \rho = 0) = 0.05$.

However, d^* depends on the particular entries of the matrix X.

ⓑ Durbin and Watson shows, for any X with constant, $d_L \le d^* \le d_U$.

Therefore,

(if $d < d^*$, reject $H_0 : \rho = 0$) $\Rightarrow$ (if $d < d_L$, reject $H_0 : \rho = 0$).

(if $d > d^*$, do not reject $H_0 : \rho = 0$) $\Rightarrow$ (if $d > d_U$, do not reject $H_0 : \rho = 0$).

(if $d_L < d < d_U$) $\Rightarrow$ (don't know whether $d < d^*$ or $d > d^*$)

$\Rightarrow$ (inconclusive region).

• <u>Practical Decision Rule</u>($H_1 : \rho > 0$):

► In most economic data, $\rho < 0$ is not common.

If $d < d_L$, reject $H_0 : \rho = 0$ $\Rightarrow$ positive autocorrelation.

If $d_L < d < d_U$, no decision. $\Rightarrow$ positive autocorrelation.

If $d_U < d$, do not reject $H_0 : \rho = 0$ $\Rightarrow$ no autocorrelation.

◎ <u>Underlying assumptions for Durbin-Watson test</u>.

① The regression model includes the intercept term.

② Effective for AR(1).

③ For quarterly data, if assume $\varepsilon_t = \rho\varepsilon_{t-4} + u_t$, use other method.

④ No lagged dependent variable. ⇒ Lagged dependent variable: Other test.

(4) Lagrange Multiplier Test(Breusch-Godfrey Test)

- Cover the weakness of Durbin-Watson test (No lagged dependent variable, Inconclusive region).

- $Y_t = \beta_1 + \beta_2 X_t + \alpha Y_{t-1} + \varepsilon_t, \quad \varepsilon_t = \rho\varepsilon_{t-1} + u_t$

$H_0 : \rho = 0$.

① Estimate by O.L.S. and get e_t.

② Regress e_t on $(1, X_t, e_{t-1})$ and get R^2.

③ Since $LM = n \cdot R^2 \sim \chi^2(1)$, reject H_0 if $LM \geq \chi^2(1;\alpha)$ $\Rightarrow$ autocorrelation.

5. Remedy for Autocorrelation

(1) When ρ is known(GLS)

- $Y_t = \beta_1 + \beta_2 X_t + \varepsilon_t, \quad \varepsilon_t = \rho\varepsilon_{t-1} + u_t$; autocorrelation.

$$\Rightarrow \begin{array}{l} Y_t - \rho Y_{t-1} = \beta_1(1-\rho) + \beta_2\left(X_t - \rho X_{t-1}\right) + u_t, \quad u_t \sim \text{white noise} \\ \quad \mathrm{Y_t}* \quad = \beta_1 * \qquad + \beta_2 X_t * \qquad\qquad + u_t \end{array}.$$

$\Rightarrow$ Classical assumptions are satisfied.

► O.L.S. of $Y_t *$ on (1, $X_t *$), $t = 2,\cdots,T$.

- If want to include (Y_1, X_1),

$Y_1* = \sqrt{1-\rho^2}\,Y_1, \quad X_1* = \sqrt{1-\rho^2}\,X_1$: Prais-Winsten tranformation.

(Note)

- Original model: $Y_t = \beta_1 + \beta_2 X_t + \varepsilon_t, \quad \varepsilon_t = \rho\varepsilon_{t-1} + u_t$.
- Transformed model: $Y_t = \beta_1 * + \beta_2 X_t + \beta_2 \rho X_{t-1} + \rho Y_{t-1} + u_t = f(X_t, X_{t-1}, Y_{t-1}) + u_t$.

So, adding (Y_{t-1}, X_{t-1}) into original model, sometimes can remove autocorrelation.

But, $Cov\left(Y_{t-1}, \varepsilon_t\right) \neq 0$.

(2) FGLS: When ρ is unknown.

$Y_t = \beta_1 + \beta_2 X_t + \varepsilon_t, \quad \varepsilon_t = \rho\varepsilon_{t-1} + u_t$

① Do O.L.S. of Y_t on (1, X_t) and get e_t.

② Regress e_t on e_{t-1} and get $\hat{\rho}$.

Construct $Y_t^* = Y_t - \hat{\rho}Y_{t-1}, \quad X_t^* = X_t - \hat{\rho}X_{t-1}$.

③ Do O.L.S. of Y_t^* on (1, X_t^*).

(3) Cochrane-Orcutt Method

- Rewrite autocorrelated model,

① If ρ is known,

$Y_t - \rho Y_{t-1} = \beta_1(1-\rho) + \beta_2\left(X_t - \rho X_{t-1}\right) + u_t \;\Rightarrow\; (\beta_1, \beta_2)$ can be estimated.

② If (β_1, β_2) is known,

$\left(Y_t - \beta_1 - \beta_2 X_t\right) = \rho\left(Y_{t-1} - \beta_1 - \beta_2 X_{t-1}\right) + u_t \;\Rightarrow\; \rho$ can be estimated.

► Start with any value of ρ(starting value), from ①,

get (β_1, β_2). $\Rightarrow$ From ②, get ρ.

$\Rightarrow$ From ①, get (β_1, β_2). $\Rightarrow$ From ②, get ρ. $\Rightarrow$ ⋯⋯

Do this process until ρ differ only by small change(ex: 0.001).

; iterative Zellner estimator.

6. <u>Prediction with AR(1) Model</u>

Data: $\{(X_t, y_t)\}_{t=1,2,\cdots,T}$

- Suppose the value of X at time t+1 is given, X_{t+1}, what is best predictor of y_{t+1}?

① With no autocorrelated model, $\hat{y}_{T+1} = b_1 + b_2 X_{T+1}$

► Therefore, $\hat{y}_{T+1}$ is nothing to do with y_T.

② With autocorrelated model,

$$
\begin{aligned}
y_{T+1} &= \beta_1 + \beta_2 X_{T+1} + \varepsilon_{T+1} \\
&= \beta_1 + \beta_2 X_{T+1} + \rho\varepsilon_T + u_{T+1}
\end{aligned}
$$

Therefore,

$\hat{y}_{T+1} = \hat{\beta}_1 + \hat{\beta}_2 X_{T+1} + \hat{\rho} e_T$, where $e_T = y_T - (\hat{\beta}_1 + \hat{\beta}_2 X_T)$.

$(\hat{\beta}_1, \hat{\beta}_2)$: FGLS estimator.

Similarly,

$$\hat{y}_{T+s} = \hat{\beta}_1 + \hat{\beta}_2 X_{T+s} + \hat{\rho}^s e_T$$

7. Other forms of Autocorrelation

- Moving Average series
- ARMA

(Example) Data file: SNA2012(annual)

표본기간을 1980-2012로 설정

(1) log(cgv) c log(gdp)에 대해 회귀분석

Model 4: OLS, using observations 1980-2012 (T = 33)
Dependent variable: logcgv

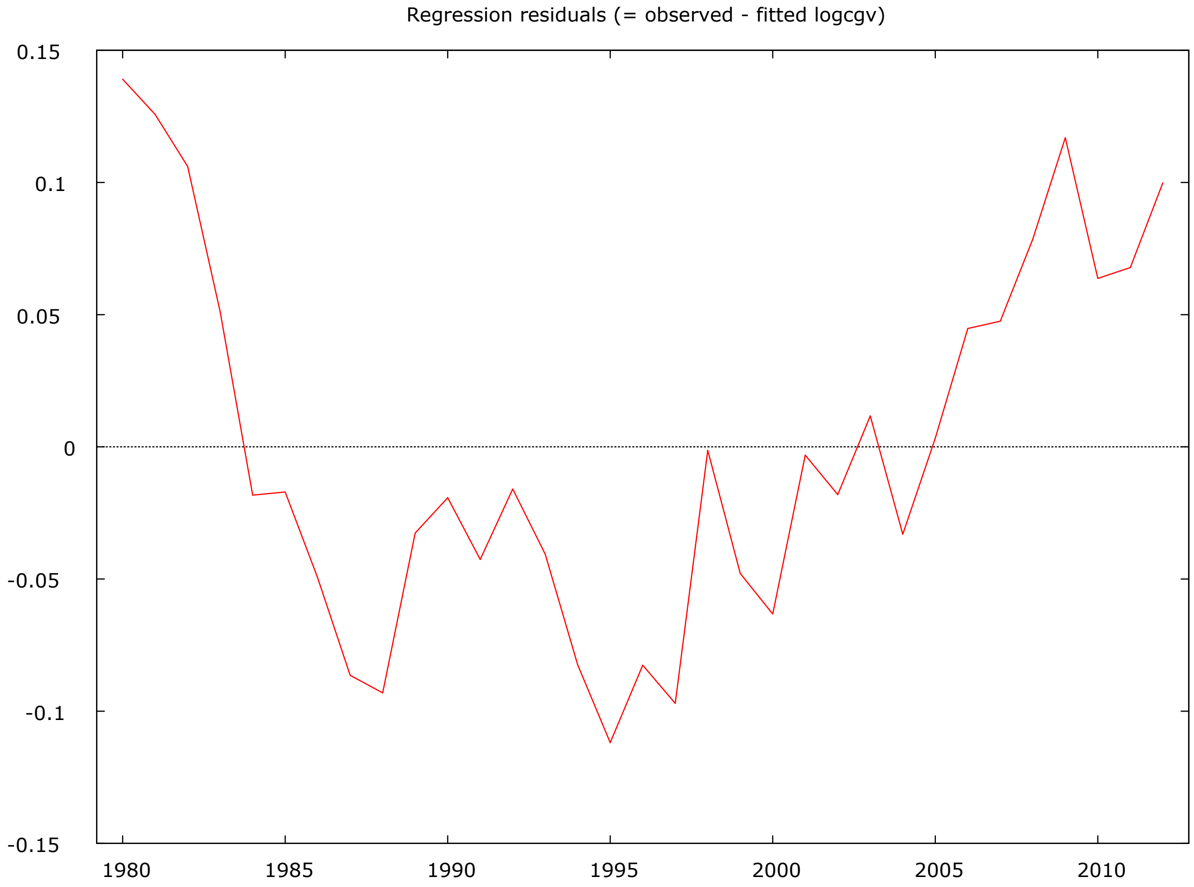
	Coefficient	*Std. Error*	*t-ratio*	*p-value*	
const	−%s	0.174784	−%#.4g	<0.0001	***
loggdp	1.05166	0.0116037	90.63	<0.0001	***

Mean dependent var	10.63029	S.D. dependent var	1.151465
Sum squared resid	0.159522	S.E. of regression	0.071735
R-squared	0.996240	Adjusted R-squared	0.996119
F(1, 31)	8214.037	P-value(F)	3.70e-39
Log-likelihood	41.15438	Akaike criterion	−78.30876
Schwarz criterion	−75.31575	Hannan-Quinn	−77.30170
rho	0.808596	Durbin-Watson	0.300159

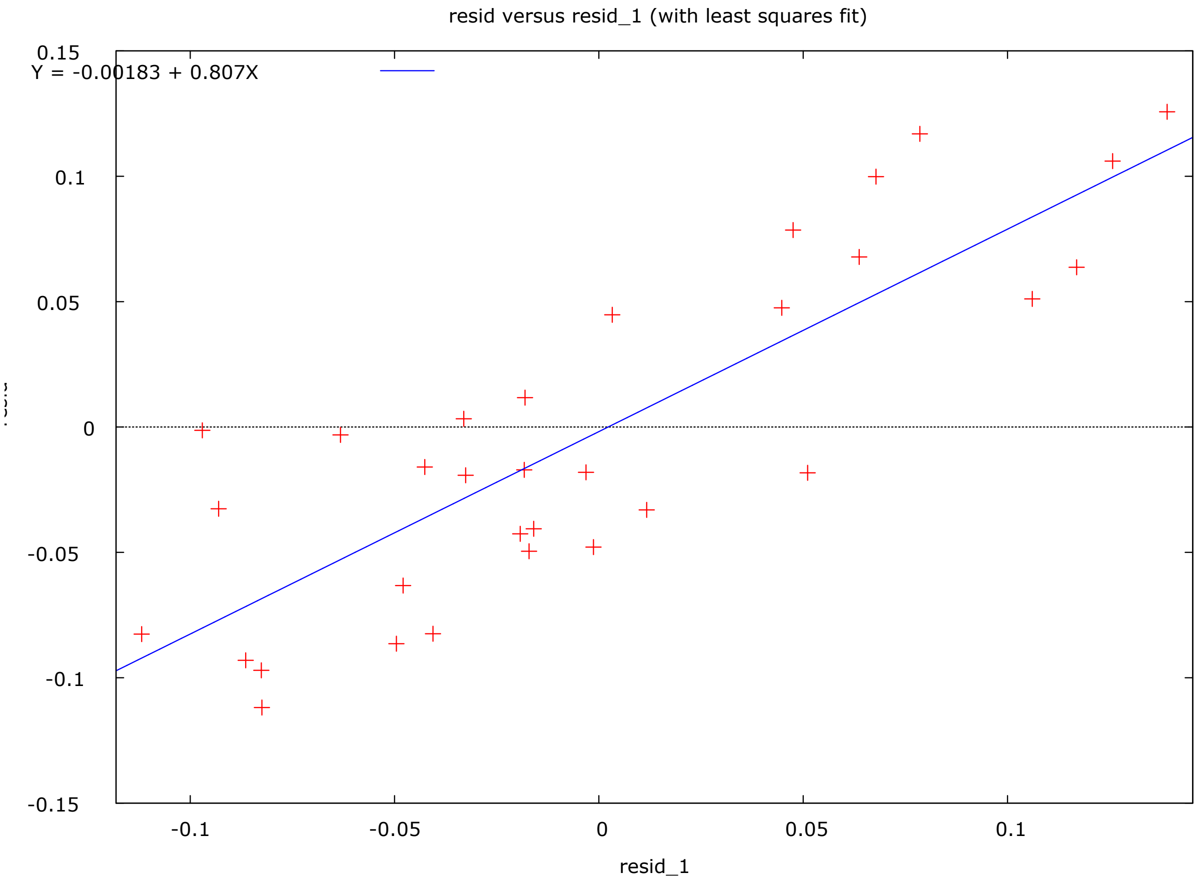
(2) Graphical Analysis

Save / Residuals

Graphs / Residual plot / Against time



Add / Lags of selected variables
View / Graph specified variables / XY scatter / mark show lagged variables



(3) Durbin-Watson Test

D-W=0.300159,

n=33, k'=1 이므로 dL=1.383, dU=1.508

0<D-W=0.300159<dl=1.383이므로 $H_0: \rho = 0$을 기각함.

(4) LM 검정방법

Tests / Autocorrelation

```
Breusch-Godfrey test for first-order autocorrelation
OLS, using observations 1980-2012 (T = 33)
Dependent variable: uhat

             coefficient   std. error   t-ratio   p-value
  --------------------------------------------------------
  const      -0.0447925    0.110208     -0.4064   0.6873
  loggdp      0.00314515   0.00731810    0.4298   0.6704
  uhat_1      0.813347     0.117105      6.945    1.03e-07 ***

  Unadjusted R-squared = 0.616562

Test statistic: LMF = 48.239439,
with p-value = P(F(1,30) > 48.2394) = 1.03e-007

Alternative statistic: TR^2 = 20.346535,
with p-value = P(Chi-square(1) > 20.3465) = 6.46e-006

Ljung-Box Q' = 20.7412,
with p-value = P(Chi-square(1) > 20.7412) = 5.26e-006
```

(4) Cochrane-Orcutt Estimation

$\boxed{\text{Model}}$ / $\boxed{\text{Time series}}$ / $\boxed{\text{AR(1)}}$ / $\boxed{\text{Cochrane-Orcutt}}$

Model 4: Cochrane-Orcutt, using observations 1981-2012 (T = 32)
Dependent variable: logcgv
rho = 0.77576

	Coefficient	*Std. Error*	*t-ratio*	*p-value*	
const	−%s	0.495245	−%#.4g	<0.0001	***
loggdp	1.10120	0.0319476	34.47	<0.0001	***

Statistics based on the rho-differenced data:

Mean dependent var	10.69652	S.D. dependent var	1.104177
Sum squared resid	0.039297	S.E. of regression	0.036192
R-squared	0.998968	Adjusted R-squared	0.998934
F(1, 30)	1188.097	P-value(F)	1.09e-25
rho	−0.140348	Durbin-Watson	2.199813

<표 B-5> Durbin-Watson (5% 유의수준)

n	$k'=1$		$k'=2$		$k'=3$		$k'=4$		$k'=5$	
	d_L	d_V	d_L	d_V	d_L	d_V	d_L	d_V	d_L	d_V
15	1.077	1.361	0.946	1.543	0.814	1.75	0.685	1.977	0.562	2.22
16	1.106	1.371	0.982	1.539	0.857	1.728	0.734	1.935	0.615	2.157
17	1.133	1.381	1.015	1.536	0.897	1.71	0.779	1.9	0.664	2.104
18	1.158	1.391	1.046	1.535	0.933	1.696	0.82	1.872	0.71	2.06
19	1.18	1.401	1.074	1.536	0.967	1.685	0.859	1.848	0.752	2.023
20	1.201	1.411	1.1	1.537	0.998	1.676	0.894	1.828	0.792	1.991
21	1.221	1.42	1.125	1.538	1.026	1.669	0.927	1.812	0.829	1.964
22	1.239	1.429	1.147	1.541	1.053	1.664	0.958	1.797	0.863	1.94
23	1.257	1.437	1.168	1.543	1.078	1.66	0.986	1.785	0.895	1.92
24	1.273	1.446	1.188	1.546	1.101	1.656	1.013	1.775	0.925	1.902
25	1.288	1.454	1.206	1.55	1.123	1.654	1.038	1.767	0.953	1.886
26	1.302	1.461	1.224	1.553	1.143	1.652	1.062	1.759	0.979	1.873
27	1.316	1.469	1.24	1.556	1.162	1.651	1.084	1.753	1.004	1.861
28	1.328	1.476	1.255	1.56	1.181	1.65	1.104	1.747	1.028	1.85
29	1.341	1.483	1.27	1.563	1.198	1.65	1.124	1.743	1.05	1.841
30	1.352	1.489	1.284	1.567	1.214	1.65	1.143	1.739	1.071	1.833
31	1.363	1.496	1.297	1.57	1.229	1.65	1.16	1.735	1.09	1.825
32	1.373	1.502	1.309	1.574	1.244	1.65	1.177	1.732	1.109	1.819
33	1.383	1.508	1.321	1.577	1.258	1.651	1.193	1.73	1.127	1.813
34	1.393	1.514	1.333	1.58	1.271	1.652	1.208	1.728	1.144	1.808
35	1.402	1.519	1.343	1.584	1.283	1.653	1.222	1.726	1.16	1.803
36	1.411	1.525	1.354	1.587	1.295	1.654	1.236	1.724	1.175	1.799
37	1.419	1.53	1.364	1.59	1.307	1.655	1.249	1.723	1.19	1.795
38	1.427	1.535	1.373	1.594	1.318	1.656	1.261	1.722	1.204	1.792
39	1.435	1.54	1.382	1.597	1.328	1.658	1.273	1.722	1.218	1.789
40	1.442	1.544	1.391	1.6	1.338	1.659	1.285	1.721	1.23	1.786

45	1.475	1.566	1.43	1.615	1.383	1.666	1.336	1.72	1.287	1.776
50	1.503	1.585	1.462	1.628	1.421	1.674	1.378	1.721	1.335	1.771
55	1.528	1.601	1.49	1.641	1.452	1.681	1.414	1.724	1.374	1.768
60	1.549	1.616	1.514	1.652	1.48	1.689	1.444	1.727	1.408	1.767
65	1.567	1.629	1.536	1.662	1.503	1.696	1.471	1.731	1.438	1.767
70	1.583	1.641	1.554	1.672	1.525	1.703	1.494	1.735	1.464	1.768
75	1.598	1.652	1.571	1.68	1.543	1.709	1.515	1.739	1.487	1.77
80	1.611	1.662	1.586	1.688	1.56	1.715	1.534	1.743	1.507	1.772
85	1.624	1.671	1.6	1.696	1.575	1.721	1.55	1.747	1.525	1.774
90	1.636	1.679	1.612	1.703	1.589	1.726	1.566	1.751	1.542	1.776
95	1.645	1.687	1.623	1.709	1.602	1.732	1.579	1.755	1.557	1.778
100	1.654	1.694	1.634	1.715	1.613	1.736	1.592	1.758	1.571	1.78
150	1.72	1.746	1.706	1.76	1.693	1.774	1.679	1.788	1.665	1.802
200	1.758	1.778	1.748	1.789	1.738	1.799	1.728	1.81	1.718	1.82

k' = 상수항을 제외한 설명변수의 수

n	$k'=6$		$k'=7$		$k'=8$		$k'=9$		$k'=10$	
	d_l	d_v	d_l	d_v	d_l	d_v	d_l	d_v	d_l	d_v
15	0.447	2.472	0.343	2.727	0.251	2.979	0.175	3.216	0.111	3.438
16	0.502	2.388	0.398	2.624	0.304	2.86	0.222	3.09	0.155	3.304
17	0.554	2.318	0.451	2.537	0.356	2.757	0.272	2.975	0.198	3.184
18	0.603	2.257	0.502	2.461	0.407	2.667	0.321	2.873	0.244	3.073
19	0.649	2.206	0.459	2.396	0.456	2.589	0.369	2.783	0.29	2.974
20	0.692	2.162	0.595	2.339	0.502	2.521	0.416	2.704	0.336	2.885
21	0.732	2.124	0.637	2.29	0.547	2.46	0.461	2.633	0.38	2.806
22	0.769	2.09	0.677	2.246	0.588	2.407	0.504	2.571	0.424	2.734
23	0.804	2.061	0.715	2.208	0.628	2.36	0.545	2.514	0.465	2.67
24	0.837	2.035	0.751	2.174	0.666	2.318	0.584	2.464	0.506	2.613
25	0.868	2.012	0.784	2.144	0.702	2.28	0.621	2.419	0.544	2.56
26	0.897	1.992	0.816	2.117	0.735	2.246	0.657	2.379	0.581	2.513
27	0.925	1.974	0.845	2.093	0.767	2.216	0.691	2.342	0.616	2.47
28	0.951	1.958	0.874	2.071	0.798	2.188	0.723	2.309	0.65	2.431
29	0.975	1.944	0.9	2.052	0.826	2.164	0.753	2.278	0.682	2.396
30	0.998	1.931	0.926	2.034	0.854	2.141	0.782	2.251	0.712	2.363
31	1.02	1.92	0.95	2.018	0.879	2.12	0.81	2.226	0.741	2.333
32	1.041	1.909	0.972	2.004	0.904	2.102	0.836	2.203	0.769	2.306
33	1.061	1.9	0.994	1.991	0.927	2.085	0.861	2.181	0.795	2.281
34	1.08	1.891	1.015	1.979	0.95	2.069	0.885	2.162	0.821	2.257
35	1.097	1.884	1.034	1.967	0.971	2.054	0.908	2.144	0.845	2.236
36	1.114	1.877	1.053	1.957	0.991	2.041	0.93	2.127	0.868	2.216
37	1.131	1.87	1.071	1.948	1.011	2.029	0.951	2.112	0.891	2.198
38	1.146	1.864	1.088	1.939	1.029	2.017	0.97	2.098	0.912	2.18
39	1.161	1.859	1.104	1.932	1.047	2.007	0.99	2.085	0.932	2.164
40	1.175	1.854	1.12	1.924	1.064	1.997	1.008	2.072	0.945	2.149
45	1.238	1.835	1.189	1.895	1.139	1.958	1.089	2.002	1.038	2.088
50	1.291	1.822	1.246	1.875	1.201	1.93	1.156	1.986	1.11	2.044
55	1.334	1.814	1.294	1.861	1.253	1.909	1.212	1.959	1.17	2.01
60	1.372	1.808	1.335	1.85	1.298	1.894	1.26	1.939	1.222	1.984
65	1.404	1.805	1.37	1.843	1.336	1.882	1.301	1.923	1.266	1.964

70	1.433	1.802	1.401	1.837	1.369	1.873	1.337	1.91	1.305	1.948
75	1.458	1.801	1.428	1.834	1.399	1.867	1.369	1.901	1.339	1.935
80	1.48	1.801	1.453	1.831	1.425	1.861	1.397	1.893	1.369	1.925
85	1.5	1.801	1.474	1.829	1.448	1.857	1.422	1.886	1.396	1.916
90	1.518	1.801	1.494	1.827	1.469	1.854	1.445	1.881	1.42	1.909
95	1.535	1.802	1.512	1.827	1.489	1.852	1.465	1.877	1.442	1.903
100	1.55	1.803	1.528	1.826	1.506	1.85	1.484	1.874	1.462	1.898
150	1.651	1.817	1.637	1.832	1.622	1.847	1.608	1.862	1.594	1.877
200	1.707	1.831	1.697	1.841	1.686	1.852	1.675	1.863	1.665	1.874

<표 B-5(계속)> Durbin-Watson (1% 유의수준)

n	$k'=1$		$k'=2$		$k'=3$		$k'=4$		$k'=5$	
	d_L	d_V	d_L	d_V	d_L	d_V	d_L	d_V	d_L	d_V
15	0.811	1.07	0.7	1.252	0.591	1.464	0.488	1.704	0.391	1.967
16	0.844	1.086	0.737	1.252	0.633	1.446	0.532	1.663	0.437	1.9
17	0.874	1.102	0.772	1.255	0.672	1.432	0.574	1.63	0.48	1.847
18	0.902	1.118	0.805	1.259	0.708	1.422	0.613	1.604	0.522	1.803
19	0.928	1.132	0.835	1.265	0.742	1.415	0.65	1.584	0.561	1.767
20	0.952	1.147	0.863	1.271	0.773	1.411	0.685	1.567	0.598	1.737
21	0.975	1.161	0.89	1.277	0.803	1.408	0.718	1.554	0.633	1.712
22	0.997	1.174	0.914	1.284	0.831	1.407	0.748	1.543	0.667	1.691
23	1.018	1.187	0.938	1.291	0.858	1.407	0.777	1.534	0.698	1.673
24	1.037	1.199	0.96	1.298	0.882	1.407	0.805	1.528	0.728	1.658
25	1.055	1.211	0.981	1.305	0.906	1.409	0.831	1.523	0.756	1.645
26	1.072	1.222	1.001	1.312	0.928	1.411	0.855	1.518	0.783	1.635
27	1.089	1.233	1.019	1.319	0.949	1.413	0.878	1.515	0.808	1.626
28	1.104	1.244	1.037	1.325	0.969	1.415	0.9	1.513	0.832	1.618
29	1.119	1.254	1.054	1.332	0.988	1.418	0.921	1.512	0.855	1.611
30	1.133	1.263	1.07	1.339	1.006	1.421	0.941	1.511	0.877	1.606
31	1.147	1.273	1.085	1.345	1.023	1.425	0.96	1.51	0.897	1.601
32	1.16	1.282	1.1	1.352	1.04	1.428	0.979	1.51	0.917	1.597
33	1.172	1.291	1.114	1.358	1.055	1.432	0.996	1.51	0.936	1.594
34	1.184	1.299	1.128	1.364	1.07	1.435	1.012	1.511	0.954	1.591
35	1.195	1.307	1.14	1.37	1.085	1.439	1.028	1.512	0.971	1.589
36	1.206	1.315	1.153	1.376	1.098	1.442	1.043	1.513	0.988	1.588
37	1.217	1.323	1.165	1.382	1.112	1.446	1.058	1.514	1.004	1.586
38	1.227	1.33	1.176	1.388	1.124	1.449	1.072	1.515	1.019	1.585
39	1.237	1.337	1.187	1.393	1.137	1.453	1.085	1.517	1.034	1.584
40	1.246	1.344	1.198	1.398	1.148	1.457	1.098	1.518	1.048	1.584
45	1.288	1.276	1.245	1.423	1.201	1.474	1.156	1.528	1.111	1.584
50	1.324	1.403	1.285	1.446	1.245	1.491	1.205	1.538	1.164	1.587
55	1.356	1.427	1.32	1.466	1.284	1.506	1.247	1.548	1.209	1.592

60	1.383	1.449	1.35	1.484	1.317	1.52	1.283	1.558	1.249	1.598
65	1.407	1.468	1.377	1.5	1.346	1.534	1.315	1.568	1.283	1.604
70	1.429	1.485	1.4	1.515	1.372	1.546	1.343	1.578	1.313	1.611
75	1.448	1.501	1.422	1.529	1.395	1.557	1.368	1.587	1.34	1.617
80	1.466	1.515	1.441	1.541	1.416	1.568	1.39	1.595	1.364	1.624
85	1.482	1.528	1.458	1.553	1.435	1.578	1.411	1.603	1.386	1.63
90	1.496	1.54	1.474	1.563	1.452	1.587	1.429	1.611	1.406	1.636
95	1.51	1.552	1.489	1.573	1.468	1.596	1.446	1.618	1.425	1.642
100	1.522	1.562	1.503	1.583	1.482	1.604	1.462	1.625	1.441	1.647
150	1.611	1.637	1.598	1.651	1.584	1.665	1.571	1.679	1.557	1.693
200	1.664	1.684	1.653	1.693	1.643	1.704	1.633	1.715	1.623	1.725

k' = 상수항을 제외한 설명변수의 수

n	$k'=6$		$k'=7$		$k'=8$		$k'=9$		$k'=10$	
	d_L	d_V	d_L	d_V	d_L	d_V	d_L	d_V	d_L	d_V
15	0.303	2.244	0.226	2.53	0.161	2.817	0.107	3.101	0.068	3.374
16	0.349	2.153	0.269	2.416	0.2	2.681	0.142	2.944	0.094	3.201
17	0.393	2.078	0.313	2.319	0.241	2.566	0.179	2.811	0.127	3.053
18	0.435	2.015	0.355	2.238	0.282	2.467	0.216	2.679	0.16	2.925
19	0.476	1.963	0.396	2.169	0.322	2.381	0.255	2.597	0.196	2.813
20	0.515	1.918	0.436	2.11	0.362	2.308	0.294	2.51	0.232	2.714
21	0.552	1.881	0.474	2.059	0.4	2.244	0.331	2.434	0.268	2.625
22	0.587	1.849	0.51	2.015	0.437	2.188	0.368	2.367	0.304	2.548
23	0.62	1.821	0.545	1.977	0.473	2.14	0.404	2.308	0.34	2.479
24	0.652	1.797	0.578	1.944	0.507	2.097	0.439	2.255	0.375	2.417
25	0.682	1.766	0.61	1.915	0.54	2.059	0.473	2.209	0.409	2.362
26	0.711	1.759	0.64	1.889	0.572	2.026	0.505	2.168	0.441	2.313
27	0.738	1.743	0.669	1.867	0.602	1.997	0.536	2.131	0.473	2.269
28	0.764	1.729	0.696	1.847	0.63	1.97	0.566	2.098	0.504	2.229
29	0.788	1.718	0.723	1.83	0.658	1.947	0.595	2.068	0.533	2.193
30	0.812	1.707	0.748	1.814	0.684	1.925	0.622	2.041	0.562	2.16
31	0.834	1.698	0.772	1.8	0.71	1.906	0.649	2.017	0.589	2.131
32	0.856	1.69	0.794	1.788	0.734	1.889	0.674	1.995	0.615	2.104
33	0.876	1.683	0.816	1.776	0.757	1.874	0.698	1.975	0.641	2.08
34	0.896	1.677	0.837	1.766	0.779	1.86	0.722	1.957	0.665	2.057
35	0.914	1.671	0.857	1.757	0.8	1.847	0.744	1.94	0.689	2.037
36	0.932	1.666	0.877	1.749	0.821	1.836	0.766	1.925	0.711	2.018
37	0.95	1.662	0.895	1.742	0.841	1.825	0.787	1.911	0.733	2.001
38	0.966	1.658	0.913	1.735	0.86	1.816	0.807	1.899	0.754	1.985
39	0.982	1.655	0.93	1.729	0.878	1.807	0.826	1.887	0.774	1.97
40	0.997	1.652	0.946	1.724	0.895	1.799	0.844	1.876	0.789	1.956
45	1.065	1.643	1.019	1.704	0.974	1.768	0.927	1.834	0.881	1.902
50	1.123	1.639	1.081	1.692	1.039	1.748	0.997	1.805	0.955	1.864
55	1.172	1.638	1.134	1.685	1.095	1.734	1.057	1.785	1.018	1.837
60	1.214	1.639	1.179	1.682	1.144	1.726	1.108	1.771	1.072	1.817

65	1.251	1.642	1.218	1.68	1.186	1.72	1.153	1.761	1.12	1.802
70	1.283	1.645	1.253	1.68	1.223	1.716	1.192	1.754	1.162	1.792
75	1.313	1.646	1.284	1.682	1.256	1.716	1.227	1.746	1.199	1.785
80	1.338	1.653	1.312	1.683	1.285	1.714	1.259	1.745	1.232	1.777
85	1.362	1.657	1.337	1.685	1.312	1.714	1.287	1.743	1.262	1.773
90	1.383	1.661	1.36	1.687	1.336	1.714	1.312	1.741	1.228	1.769
95	1.403	1.666	1.381	1.69	1.358	1.715	1.336	1.741	1.313	1.767
100	1.421	1.67	1.4	1.693	1.378	1.717	1.357	1.741	1.335	1.765
150	1.543	1.708	1.53	1.722	1.515	1.737	1.501	1.752	1.486	1.767
200	1.613	1.735	1.603	1.746	1.592	1.757	1.582	1.768	1.571	1.779