ECO 4004: Math. Econ. Statistics

Problem Set 2: Univariate Distributions

1. Consider the following pmf of a random variable X:

$$f(x)=\begin{cases}\dfrac{x}{15} & \text{for} \quad x=1,2,3,4,5\\ 0 & \text{elsewhere}\end{cases}$$

Find the (cumulative) distribution function $F(x)$ and sketch its graph.

Check whether it is a step function and right continuous.

2. If the density function of the random variable X is given by

$$g(x)=\begin{cases}6x(1-x) & \text{for } 0<x<1\\ 0 & \text{elsewhere}\end{cases}$$

find

(1) $P(X>0.6)$;

(2) the cumulative distribution function of this random variable

3. If the density function of the random variable X is given by

$$f(x)=\begin{cases}x & \text{for } 0<x<1\\ 2-x & \text{for } 1\le x<c\\ 0 & \text{elsewhere}\end{cases},$$

find

(1) the value of c;

(2) the distribution function of X;

(3) the value of m such that $F(m)=0.5$, namely, such that m is the median of the distribution of X.

(4) $P(0.8<X<0.6\,c)$.

4. Find the distribution function of the random variable X whose density function is given by

$$f(x)=\begin{cases}\dfrac{x}{2} & \text{for } 0<x\le 1\\ \dfrac{1}{2} & \text{for } 1<x\le 2\\ \dfrac{3-x}{2} & \text{for } 2<x<3\\ 0 & \text{elsewhere}\end{cases}$$

Also sketch the graph of the density and distribution function.

5. The tread wear (in thousands of kilometers) which car owners get with a certain kind of tire is a random variable whose probability density function is given by

$$f(x)=\begin{cases}\dfrac{1}{30}e^{-\frac{x}{30}} & \text{for } x>0\\ 0 & \text{for } x\le 0\end{cases}$$

Find the probabilities that one of these tires will last

(1) at most 19,000 kilometers;

(2) anywhere from 29,000 to 38,000 kilometers;

(3) at least 48,000 kilometers.