ECO 4004: Mathematical Statistical Economics
Problem Set 10: Parameter Estimation

1. In one population, $E(Y_1) = \mu$ and $V(Y_1) = \sigma_1^2$; in a second population, $E(Y_2) = \mu$ and $V(Y_2) = \sigma_2^2$. The population variances are known, but their common expectation μ is unknown. Random samples of size n_1 and n_2 respectively are drawn from the two populations. The two samples are independent. It is proposed to combine the sample means $\overline{Y_1}$ and $\overline{Y_2}$ linearly into a single estimator of the common mean μ.

(1) Consider all possible linear combinations $\overline{Y_1}$ and $\overline{Y_2}$. Determine the one that is minimum variance unbiased as an estimator of μ.

(2) Verify that the variance of that estimator is less than the variance of each of the two sample means.

2. We are interested in estimating the proportion of the population whose incomes are below the poverty line, a pre-specified level of income. Let $Y =$ income and $c =$ poverty line, so the parameter of interest is $\theta = \Pr(Y \leq c) = G(c)$, where $G(.)$ is the unknown cdf of income. For random sampling, sample size n, from the population, the analogy principle suggests that we estimate θ by $T =$ proportion of sample observations having $Y \leq c$.

(Hint): T can be rewritten to be $T = \dfrac{\# \text{ of } Y_i \leq c}{n}$. Define $X_i = \begin{cases} 1 & \text{if } Y_i \leq c \\ 0 & \text{otherwise} \end{cases}$, then

$T = \dfrac{\sum_{i=1}^{n} X_i}{n}$. What is distribution, mean and variance of X_i?

(1) Find $E(T)$. Is T unbiased?

(2) Find $V(T)$.

(3) Is T consistent? Explain.

(4) Find the limiting/ asymptotic distribution of T.

(5) From (4), suggest a consistent estimator of asymptotic variance of T. Check the consistency.

(6) Find 95% asymptotic confidence interval of θ.