Eco 4004: Math. Econ. Statistics

Problem Set 5: Bivariate Distribution

(Moment Generating Function)

1. Consider the following pdf.

$$f(x)=\begin{cases} e^{-x} & \text{for } x>0 \\ 0 & \text{elsewhere} \end{cases}$$

(1) Find the mgf of the distribution.

(2) Find the $\mu_1' = M'_X(0)$ and $\mu_2' = M''_X(0)$.

Check the above pdf is exponential with $\lambda = 1$ and mean and variance in the class.

(Bivariate Distribution)

2. Consider the continuous bivariate probability distribution whose joint probability density function is

$$f(x,y) = 3\left(x^2 + y\right)/11 \qquad for\ 0 \le x \le 2, \quad 0 \le y \le 1,$$

with $f(x,y) = 0 \qquad elsewhere$.

(1) Find the marginal pdf of X.

(2) Find the marginal pdf of Y.

(3) For $0 \le x \le 2$, derive $g_2(y|x)$, the conditional pdf of Y given X .

(4) Let $A = \{0 \le Y \le 1/2\}$. Calculate $\Pr(A)$, and calculate $\Pr(A|x)$ for $x = 0, 1, 2$.

3. Suppose $f(x,y) = \frac{3}{2}(x^2 + y^2) \qquad for\ 0 < x < 1, 0 < y < 1$, with $f(x,y) = 0$ elsewhere.

(1) Find the marginal pdf of Y.

(2) Find the conditional density $f(x|y)$. Note that this is a function of y as well as a function of x .

(3) Find the conditional density $f(x|y = 0.5)$. Note that this is not a function of y but a

function of x.

4. Let X and Y be continuous random variables with joint pdf

$$f(x,y)=\begin{cases}\frac{1}{8}(6-x-y) & 0<x<2,\ 2<y<4\\ 0 & \text{elsewhere}\end{cases}.$$

(1) Find marginal density of X.

(2) Find conditional density of Y on X.

(3) Find $P(2<Y<3|x=1)$.