Eco. 4004: Math. Econ. Statistics

**Problem Set 7: Multivariate Normal Distribution**

1. The pair of random variables $X$ and $Y$ is bivariate normally distributed with parameters $\mu_X = 3, \mu_Y = 4, \sigma_X{}^2 = 9, \sigma_Y{}^2 = 20$ and $\sigma_{XY} = 6$. Calculate each of the following:

(1) $E(Y \mid x=3)$ (2) $E(Y \mid x=6)$ (3) $V(Y \mid x=3)$
(4) $V(Y \mid x=6)$ (5) $P(Y \le 8 \mid x=3)$ (6) $P(Y \le 8 \mid x=6)$
(7) $P(Y \le 8)$.

2. Let the random variables $X$ and $Y$ is bivariate normally distributed with parameters $\mu_X, \mu_Y, \sigma_X{}^2, \sigma_Y{}^2$ and $\rho$. Set $U = X+Y, V = X - Y$. Then,

(1) What is the joint distribution of $U$ and $V$? How do you know it?
Find the parameters of mean, variance and correlation coefficient of $U$ and $V$.
(2) What is the marginal distribution of $U$? And what is the marginal distribution of $V$?

3. Suppose that $Z \sim N(42, 2500)$, $W \sim N(0, 500)$ that $Z$ and $W$ are independent, and that $X = Z + W$. Calculate the conditional expectation function $E(Z \mid X)$. How do you know that the CEF is linear?

4. Suppose that $\underline{y} \sim N(\underline{\mu}, \Sigma)$ with $\underline{\mu} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $\Sigma = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 5 & 1 \\ 1 & 1 & 3 \end{pmatrix}$.

(1) Suppose $\underline{g}_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $H_1 = \begin{pmatrix} 1 & 1 & 2 \\ -1 & 2 & 0 \end{pmatrix}$, and assume $\underline{z} = \underline{g}_1 + \underline{H}_1 \text{y}$.
Find $E(\underline{z}), V(\underline{z})$.
(2) Suppose $\underline{g}_2 = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$ and $H_2 = \begin{pmatrix} 2 & 1 & -1 \\ 1 & -1 & 2 \end{pmatrix}$, and assume $\underline{w} = \underline{g}_2 + \underline{H}_2\, y$.
Is $\underline{w}$ multinormally distributed? How do you decide?
Find $E(\underline{w}), V(\underline{w})$.
(3) Find $Cov(\underline{z}, \underline{w})$.
(4) Calculate $E(y_3 \mid y_1, y_2)$ and $V(y_3 \mid y_1, y_2)$.
(5) Find the best prediction of $y_3$ given $y_1 = y_2 = 1$.