

CAPM

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- Let's recall the generalized case: N risky assets and one riskfree asset.
- The optimization problem was

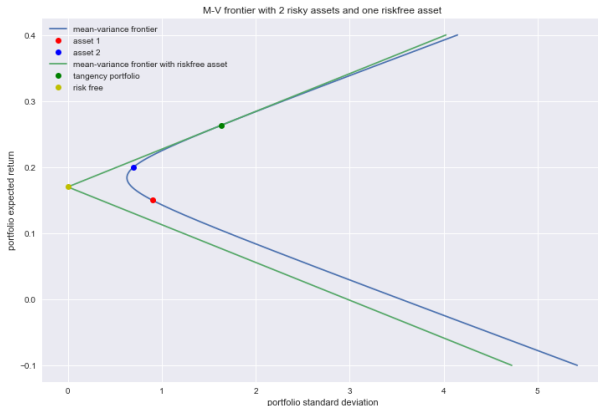
$$\min_w w' \sum w$$

s.t

$$w' (E - r_f \mathbb{I}) = \mu - r_f$$

(Case 3: Example)

$$\mu_1 = 0.15, \sigma_1 = 0.9, \sigma_{12} = 0.2, \mu_2 = 0.2, \sigma_2 = 0.7, r_f = 0.17$$



- Are there more implications?
- Let's make things a bit easier.
- N risky assets and one riskfree asset \rightarrow 2 risky assets and one riskfree asset.

$$\min_w \text{Var}(r_p) = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1 w_2 \sigma_{12}$$

s.t

$$w_0 r_f + w_1 \mu_1 + w_2 \mu_2 = \mu_p,$$

$$w_0 + w_1 + w_2 = 1$$

Interpreting F.O.C

- F.O.Cs are

$$(w_1) : 2w_1\sigma_1^2 + 2w_2\sigma_{12} = \lambda(\mu_1 - r_f)$$

$$(w_2) : 2w_2\sigma_1^2 + 2w_1\sigma_{12} = \lambda(\mu_2 - r_f)$$

- By rearranging them, we obtain

$$\frac{\mu_1 - r_f}{\text{Cov}(r_1, r_p)} = \frac{\mu_2 - r_f}{\text{Cov}(r_2, r_p)} = \frac{\mu_p - r_f}{\text{Var}(r_p)}$$

- At equilibrium, excess return relative to risk are equalized for all assets in the mean variance frontier.
- Risk that matters comes from the covariance between assets. Thus, the risk of adding an asset to the portfolio are generated from the asset's covariance with the portfolio.
- $\mu_i - r_f$ and $\text{Cov}(r_i, r_p)$ can be understood as the marginal benefit and cost of adding asset i to the portfolio, respectively.

From Markowitz to CAPM

- Lets get a step closer to CAPM.
- We restart from

$$\frac{\mu_1 - r_f}{\text{Cov}(r_1, r_p)} = \frac{\mu_2 - r_f}{\text{Cov}(r_2, r_p)} = \frac{\mu_p - r_f}{\text{Var}(r_p)}.$$

- By rearranging this, we get

$$E(R_i) = R_f + \frac{\text{Cov}(r_i, r_p)}{\text{Var}(r_p)} [E(R_p) - R_F]$$

From Markowitz to CAPM

- Let $\beta_{i,p} = \frac{\text{Cov}(r_i, r_p)}{\text{Var}(r_p)}$, then

$$E(R_i) = R_f + \beta_{i,p} [E(R_p) - R_f].$$

- This relation holds for every optimal portfolio. In other words, it holds for any portfolio p on the mean-variance efficient frontier. Thus, we can replace p with the tangency portfolio.

$$E(R_i) = R_f + \beta_{i,T} [E(R_T) - R_f].$$

- How can we interpret $\beta_{i,T}$?

From Markowitz to CAPM

- Even though it looks like an asset pricing model, it still has two problems.
 - There is no concept of market equilibrium.
 - How can we identify the tangency portfolio?
- CAPM solve these problems.

Market Equilibrium

- The supply and demand of investors adjust the prices of all traded assets so that they will be included in the tangency portfolio.
 - Lets say that CJ CGV is not included.
 - Then the demand (in turn, price) for CJ CGV will decrease until it is attractive.
 - If the price for CJ CGV is attractive, then it will be included.
- When all the price adjustment stops, the market will be at equilibrium.
 - The market price of each asset will be at the level where demand equals supply.
 - The riskfree rate will be at the level where the total amount of money borrowed equals the total amount of money lent.
 - Net supply of riskfree rate is zero.

- CAPM: "Market portfolio is tangency portfolio"

$$\begin{aligned} E(r_i) &= r_f + \beta_{i,M} [E(r_M) - r_f] \\ &= \text{riskfree rate} + \text{risk premium} \\ &= \text{riskfree rate} + \text{beta} \times \text{market price of risk} \end{aligned}$$

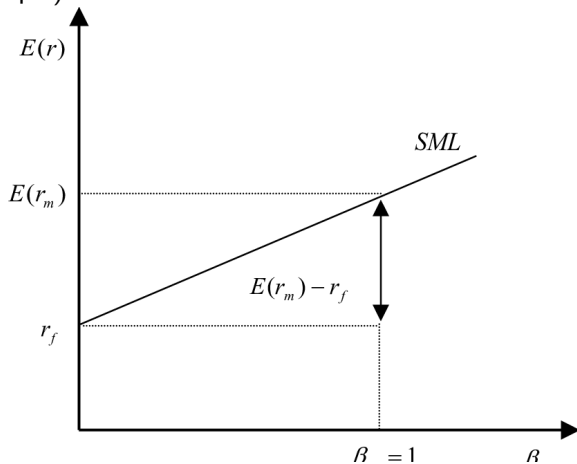
- Asset price is a function of how much the asset is exposed to systematic risk.

Security Market Line

- Security market line (SML): The relationship between expected return and beta.
- Given the risk of an asset (which is measured by its beta), the SML gives the required return necessary to compensate investors for risk.
- The slope of SML is the market price of risk: $E(r_M) - r_f$

Security Market Line

(Case 3: Example)



- Lets assume that we are fund managers. How can we evaluate our performance?
- Denote our portfolio as p , we got the following results.

$$R_p - R_f = 0.07 + 0.95^{**} (R_M - R_f)$$

- How can we interpret this result?

- CAPM alpha (α): actual return - fair expected return (what does 'fair' mean?)
- Securities lying above (below) the SML are under(over)priced.
- Now, let's get back to our example.

$$R_p - R_f = 0.07 + 0.95^{**} (R_M - R_f)$$

- In the sense that portfolio p was built by us, CAPM alpha can also show how much a fund manager outperformed (or, underperformed) compared to benchmark.

- CAPM holds for portfolios as well as for individual assets.
- SML holds for all possible portfolios whereas CML holds only for efficient portfolios.
- 'All portfolios, whether efficient or not, must lie on the SML. This is not true for the CML.' True or false?

Contributions and Limitations

- Market participants are using CAPM for remuneration or world portfolio composition or consulting for new industry.
- Identify systematic and non-system risk with a very simple model.
- However, empirical results supporting CAPM is limited or very scarce.
- Roll's critique: What is market portfolio?
 - One can never fully diversify a portfolio.
 - Even a 'market portfolio' is only a proxy for a fully-diversified portfolio.