Answer Key 6

1.

(1) Note that $f_1(x) = \dfrac{(6x^2+3)}{22}$ $\quad for\ 0 \le x \le 2$ and

$f_2(y) = \dfrac{(8+6y)}{11}$ $\quad for\ 0 \le y \le 1$.(from Problem Set 3, # 1)

And also, for $0 \le x \le 2$,

$$g_2(y \mid x) = \frac{f(x,y)}{f_2(x)} = \frac{2x^2+2y}{2x^2+1} \quad for\ 0 \le y \le 1.$$

Therefore, $E(Y \mid X) = \int_{-\infty}^{\infty} y g_2(y \mid x) dy = \dfrac{3x^2+2}{6x^2+3}$ $\quad for\ 0 \le x \le 2$.

(2)

$$E(X) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xf(x,y)dxdy = \int_0^2 xf(x)dx = 15/11.$$

Similarly, $E(Y) = 6/11, \quad E(X^2) = 116/55, \quad E(Y^2) = 25/66$.

$$E(XY) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xyf(x,y)dxdy = 8/11, \quad V(X) = E(X^2) - [E(X)]^2 = 151/605,$$

$$V(Y) = 59/726, \quad C(X,Y) = E(XY) - E(X)E(Y) = -2/121.$$

(3)

$$E^*(Y \mid X) = \alpha + \beta X,$$

$$where\ \beta = C(X,Y)/V(X) = -10/151,\ \alpha = E(Y) - \beta E(X) = \frac{6}{11} - \left(-\frac{10}{151}\right)\frac{5}{11} = 96/151.$$

So, $E^*(Y \mid X) = \dfrac{96}{151} - \dfrac{10}{151}X$.

2.

(1) $E(Y \mid x=1) = \sum_{y=0}^{1} y g_2(y \mid x=1) = \dfrac{f(1,1)}{f_1(1)} = 0.5,$

$$E(Y \mid x=2) = \sum_{y=0}^{1} y g_2(y \mid x=2) = \frac{f(2,1)}{f_1(2)} = 0.75,$$

$$E(Y \mid x=3) = \sum_{y=0}^{1} y g_2(y \mid x=3) = \frac{f(3,1)}{f_1(1)} = 0.5.$$

(2) Since $E(X) = 2,\ E(Y) = 0.6,\ E(X^2) = 4.6,\ V(X) = 0.6,\ C(X,Y) = 0$,

(2) From $E^*(Y|X) = \alpha + \beta X$, $\alpha = 0.6$, $\beta = 0$, $E^*(Y|X) = 0.6$.

(3)

	$E(Y \mid x)$	$E^*(Y \mid x)$
$x = 1$	0.5	0.6
$x = 2$	0.75	0.6
$x = 3$	0.5	0.6

3.

(1)
$$\begin{aligned} &E(X) = E(Z) + E(W) = 42 + 0 = 42, \\ &C(Z, X) = C(Z, Z + W) = V(Z) = 2500, \\ &C(W, X) = C(W, Z + W) = V(W) = 500, \\ &V(X) = V(Z) + V(W) + 2C(Z, W) = 3000. \end{aligned}$$

(2) $E^*(X \mid Z) = \alpha + \beta Z = Z$

since $\beta = C(X, Z)/V(Z) = 2500/2500 = 1, \alpha = E(X) - \beta E(Z) = 0$.

(3) $E^*(X \mid z = 54) = 54.$

(4) $E^*(Z \mid X) = 7 + 5/6X$

since $\gamma = C(X, Z)/V(X) = 2500/3000 = 5/6, \delta = E(Z) - \gamma E(X) = 42 - 5/6 \times 42 = 7$.

(5) $E^*(Z \mid x = 54) = 7 + 5/6 \times 54 = 52.$

4. Since X_1, X_2, X_3 are mutually independent, X_1, X_2, X_3^2 are also independent.

Therefore,

$$E(Z) = E(X_3^2)E\left[(X_2 - 2X_1)^2\right] = E(X_3^2)\left[E(X_2^2) - 4E(X_2)E(X_1) + 4E(X_1^2)\right] = 1 \times 5 = 5.$$