Till now

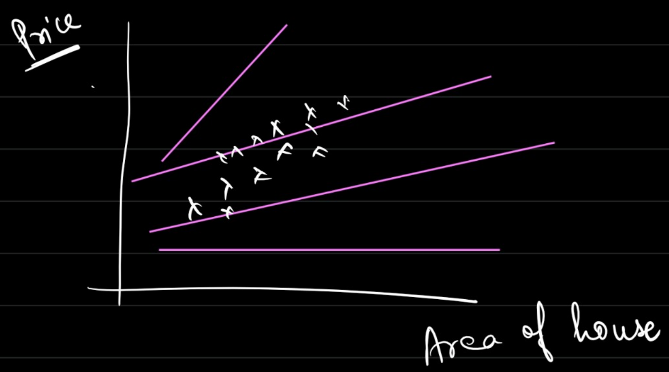
→ intuition of simple linear regression.

→ Error was least.

Agenda

↳ P.S in various forms

↳ implementation.



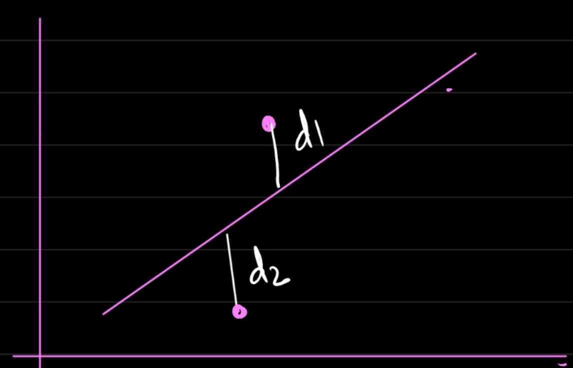
Best fit line

↓

minimise the error.

⇓

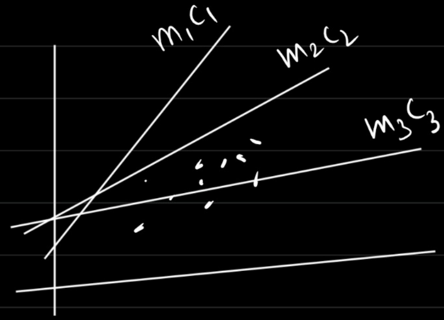
best representative of all the data points.



$$E = d_1 + d_2$$

(should be minimum)

$$T.E = \sum_{i=1}^{n} E_i$$



$$= \sum_{i=1}^{n} (y_{act} - y_{pred})^2$$

↓

for least error

$$= \sum_{i=1}^{n} (y_i - mx_i - c)^2$$

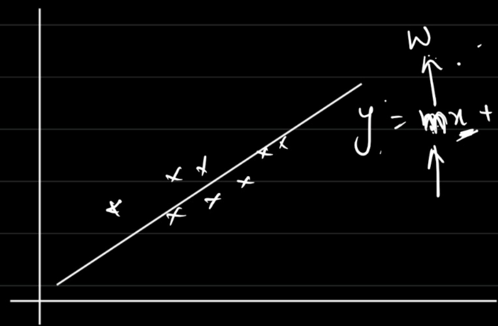
or

$(\beta_0 , \beta_1$

$w \to$ weights , b - bias.

$\Downarrow$ w, b
θ_0, θ_1

deep learning.



$$y = mx + c$$

with w above m and b above c.

for least Error

$$E = \sum_{i=1}^{n} (y_i - mx_i - c)^2$$

$J(m, c)$

$J(\beta_0, \beta_1)$

$J(\theta_0, \theta_1)$

$J(w, b)$

$$E = \sum_{i=1}^{n} (y_i - mx_i - c)^2$$

Closed form soln.

$\to$ formulate a problem statement into mathematical equation.

$$ax^2 + bx + c = 0$$

$$\alpha, \beta = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

iterative form solution

$\Downarrow$

Gradient Descent

$$TE = \sum_{i=1}^{n}(y_i - mx_i - c)^2$$

$\to m$

$\to c$

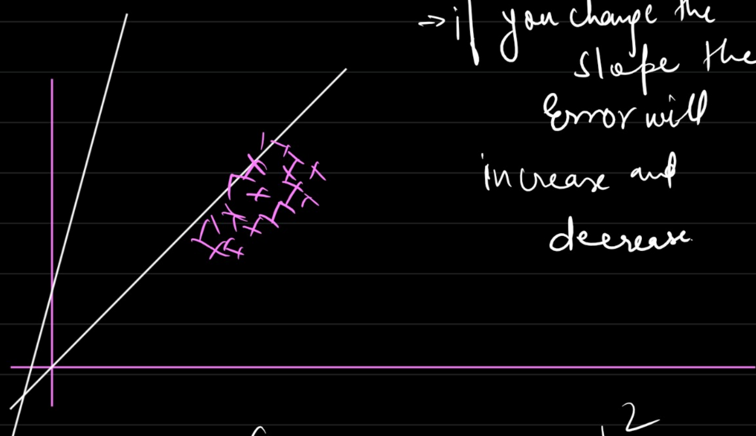
Scen 1

$y = mx + c$

$c = 0$

$y_{pred.} = mx$

→ if you change the slope the Error will increase and decrease

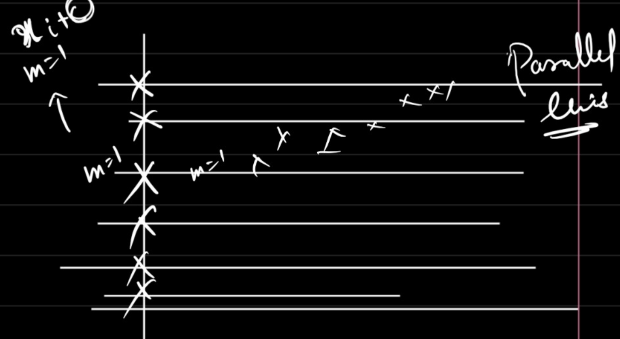


Scen 2

→ $m = 1$

$y = x + c$

$$E(c) = \sum_{i=1}^{n}(y_i - x_i - c)^2$$



→ if $c \uparrow$ Error $\uparrow$

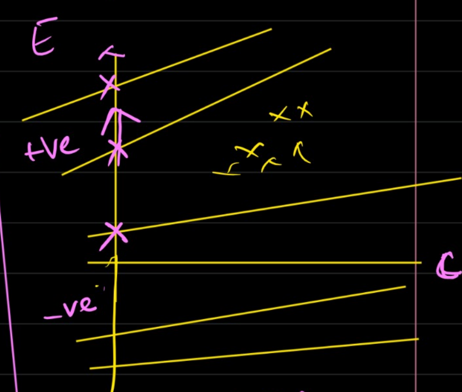
$$E = \sum(y_i - mx_i - c)^2$$

$c \to 0$

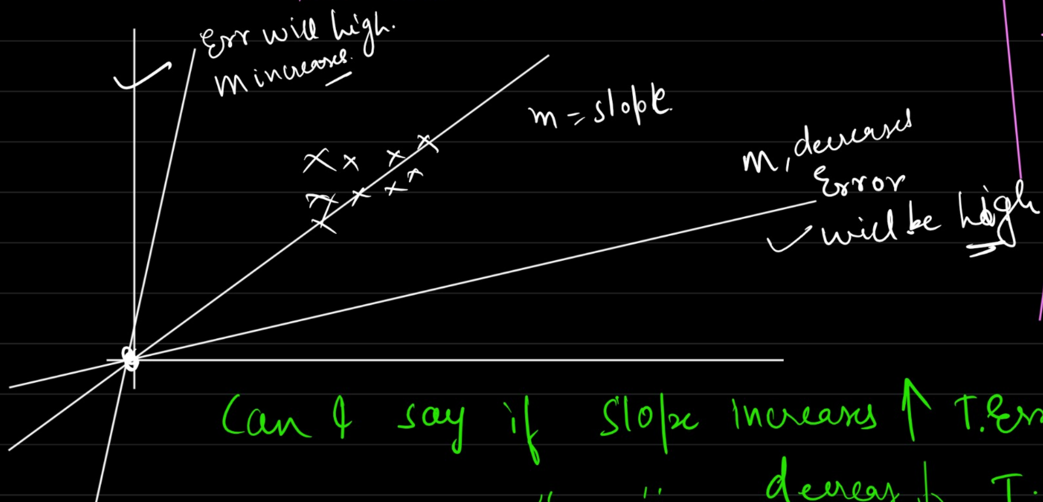
$$E(m) = \sum(y_i - mx_i)^2$$

$y = x^2$ → Quadratic

∪



* Observation → if you increase m or decrease m, the error will change.

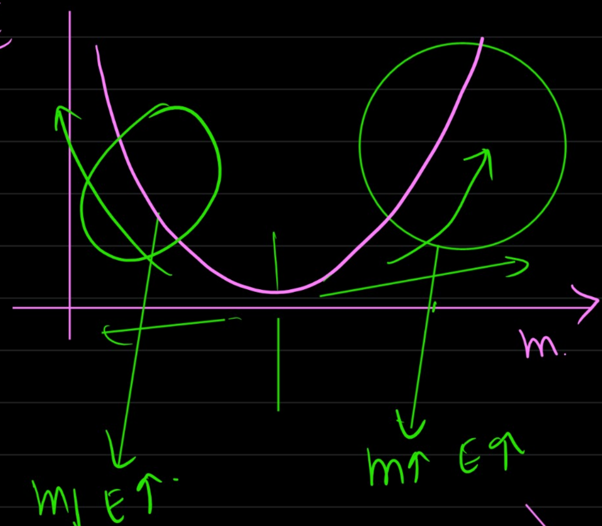


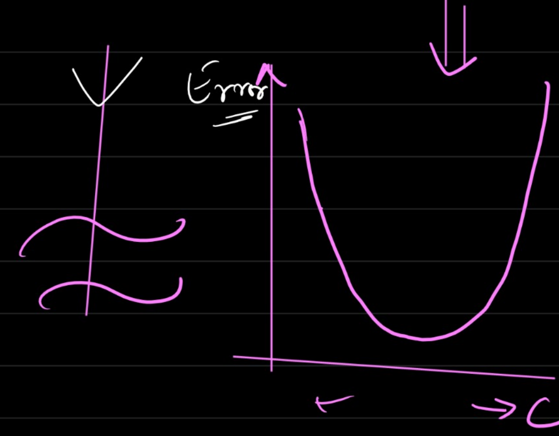
$C \uparrow E \uparrow$

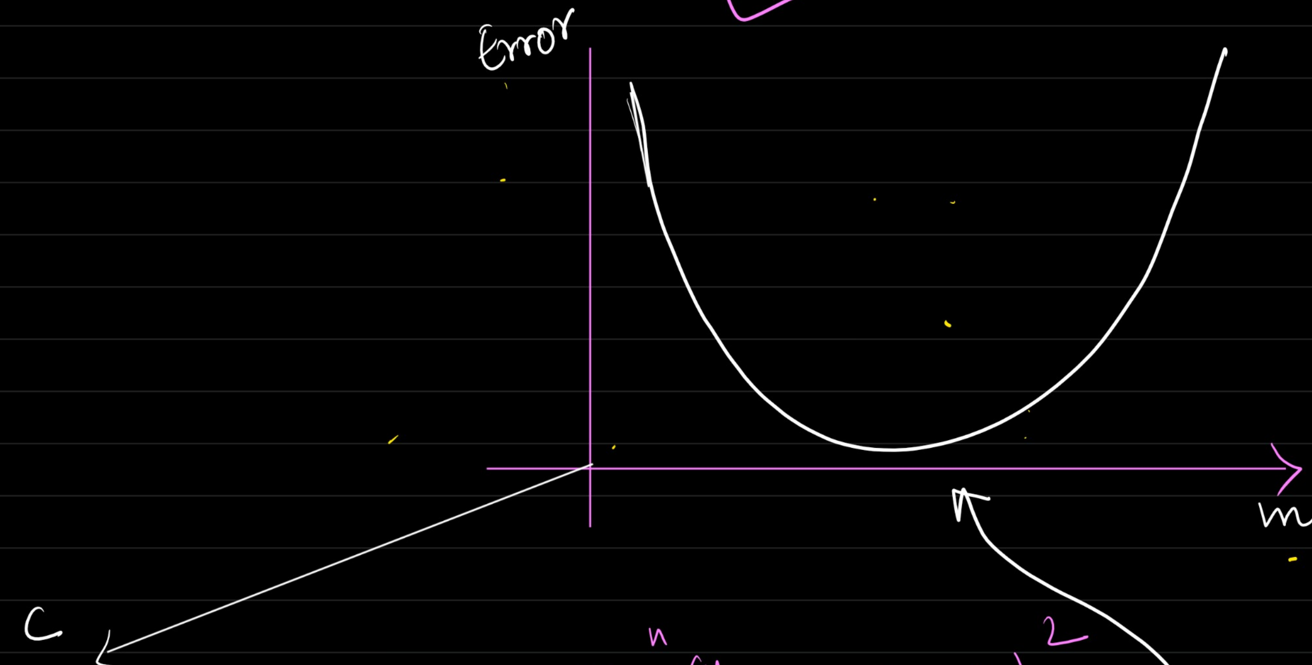
$C \downarrow E \uparrow$

Can I say if slope increases ↑ T.Error ↑

" " decreas ↓ T.Erro ↑.







$$E = \sum_{i=1}^{n} (y_i - m x_i - c)^2$$

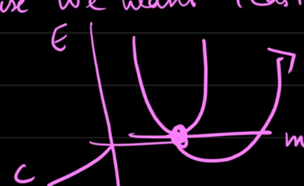
$$y = x^2$$

m, c

$$y = x^2 \longrightarrow \text{Parabola}$$

$$E = \sum_{i=1}^{n} (y_i - m x_i - c)^2$$

because we want least error → $\dfrac{\partial E}{\partial m} = 0$



→ slope = 0 wrt m & c $\dfrac{\partial E}{\partial c} = 0$

differentiation

$\frac{\partial E}{\partial c} = 0$ Why equating 0 ?

Error should be least

Slope = 0

$$n x^{n-1} \leftarrow x^n$$

$$\Rightarrow \frac{\partial}{\partial c} \sum_{i=1}^{n} (y_i - m x_i - c)^2$$

$$\cancel{\frac{\partial y_i}{\partial c}} - \cancel{\frac{\partial m x_i}{\partial c}} - \frac{\partial c}{\partial c} \to -1$$

$$\Rightarrow \sum_{i=1}^{n} 2 (y_i - m x_i - c)^{2-1} (-1)$$

$$\Rightarrow \sum_{i=1}^{n} -2 (y_i - m x_i - c)$$

$$\frac{\partial E}{\partial c} = 0$$

$$\Rightarrow \sum -2 (y_i - m x_i - c) = 0$$

dividing by -2

$$\sum_{i=1}^{n} (y_i - m x_i - c) = 0$$

$$\sum_{i=1}^{n} y_i - \sum_{i=1}^{n} m x_i - \sum_{i=1}^{n} c = 0$$

divide by n (no. of DP's)

$$\frac{\sum_{i=1}^{n} y_i}{n} - \frac{\sum_{i=1}^{n} m x_i}{n} - \frac{\sum_{i=1}^{n} c}{n}$$

$c + c + c + c + c \cdots n$

$$\bar{y} - m\bar{x} - \frac{\cancel{n} c}{\cancel{n}} = 0$$

$$\bar{y} - m\bar{x} - c = 0$$

$$\boxed{c = \bar{y} - m\bar{x}}$$

what is m ?

for m

$$\frac{\partial E}{\partial m} = 0 \qquad E = \sum_{i=1}^{n} (y_i - m x_i - c)^2$$

$$c \rightarrow \bar{y} - m\bar{x}$$

$$E = \sum (y_i - m x_i - \bar{y} + m\bar{x})^2$$

$$\frac{\partial E}{\partial m} = \sum \frac{\partial}{\partial m} (y_i - m x_i - \bar{y} + m\bar{x})^2$$

$$= \sum 2 (y_i - m x_i - \bar{y} + m\bar{x})(-x_i + \bar{x}) = 0$$

$$\Rightarrow$$

$$\Rightarrow -2 (y_i - m x_i - \bar{y} + m\bar{x})(x_i - \bar{x}) = 0$$

divide by -2 on both sides

$$\sum_{i=1}^{n} (y_i - m x_i - \bar{y} + m\bar{x})(x_i - \bar{x}) = 0$$

$$\sum_{i=1}^{n} \left((y_i - \bar{y}) - m(x_i - \bar{x})\right)(x_i - \bar{x}) = 0$$

$$\sum_{i=1}^{n} \left((y_i - \bar{y})(x_i - \bar{x}) - m(x_i - \bar{x})^2\right) = 0$$

$$\Rightarrow \sum (y_i - \bar{y})(x_i - \bar{x}) = m \sum (x_i - \bar{x})^2$$

$$m = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n} (x_i - \bar{x})^2}$$

if let's say multiple variable

x_1, x_2, x_3, x_4

$\Rightarrow \frac{\partial E}{\partial x_1}, \frac{\partial E}{\partial x_2}, \frac{\partial E}{\partial x_3}, \frac{\partial E}{\partial x_4}, \frac{\partial E}{\partial x_5} \Rightarrow$ Soln would be more complex

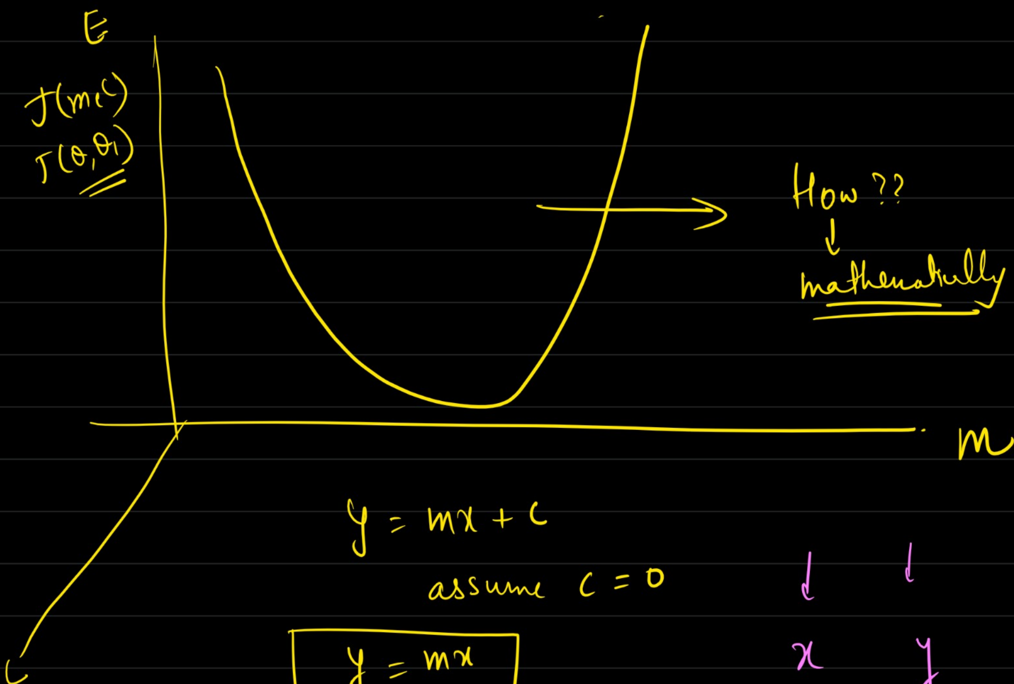
$\Downarrow$

iterative Soln

$\Downarrow$

Gradient descent

$$y = \sum_{i=1}^{n} (y_i - mx_i - c)^2$$

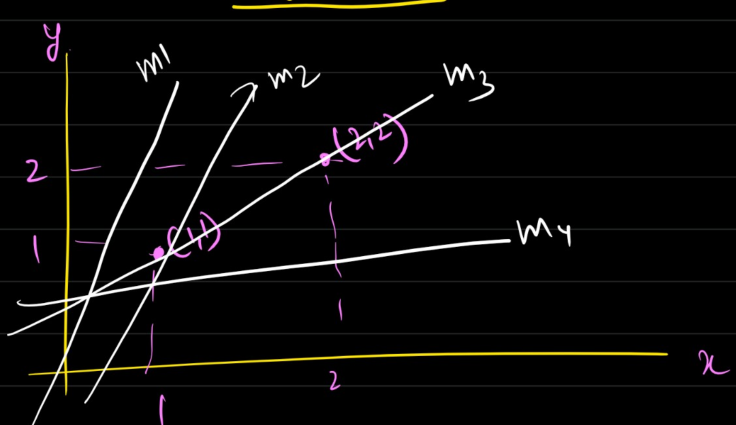


How ??
↓
mathematically

$y = mx + c$

assume $c = 0$

$$\boxed{y = mx}$$



x	y
1	1
2	2
3	3

$\boxed{y = mx}$

x	y
1	1
2	2
3	3

Scen-1

$m = 1$

$x = 1,\ y_{pred} = 1 \times 1 + 0$

$y_{pred} = 1$

$x = 2,\ y_{pred} = 1 \times 2 + 0 = 2$

$x = 3,\ y_{pred} = 1 \times 3 + 0 = 3$

x	y	y_{pred}
1	1	1
2	2	2
3	3	3

$$T.E = \frac{1}{n}\sum_{i=1}^{n}(y_i - y_{pred})^2$$

$$T.E = \frac{1}{3}\left((1-1)^2 + (2-2)^2 + (3-3)^2\right)$$

$$\Downarrow$$

$$\boxed{J(m) = 0}$$

$$\frac{\text{Total Error}}{n} = MSE = J(m,c)$$
$$\Downarrow$$
$$J(m)$$

Scen2

$m = 0.5$

$x = 1 \Rightarrow y_{pred} = 0.5 \times 1 = 0.5$

$x = 2 \Rightarrow y_{pred} = 0.5 \times 2$

$y_{pred} = 1$

$x = 3,\ y_{pred} = 0.5 \times 3 = 1.5$

x	y	y_{pred}
1	1	0.5
2	2	1
3	3	1.5

$m = 0.5$
$c = 0$

$$J(m) = \frac{1}{n}\sum_{i=1}^{n}(y_i - y_{pred})^2$$
$$= \frac{1}{3}\left[(1-0.5)^2 + (2-1)^2 + (3-1.5)^2\right]$$
$$= \frac{1}{3}(0.25 + 1 + 2.25)$$
$$= 1.16$$

Scen-3

$m = 0$

$x_1 = 1,\ y_{pred} \Rightarrow 0 \times 1 = 0$

$x_1 = 2\ y_{pred} = 0$

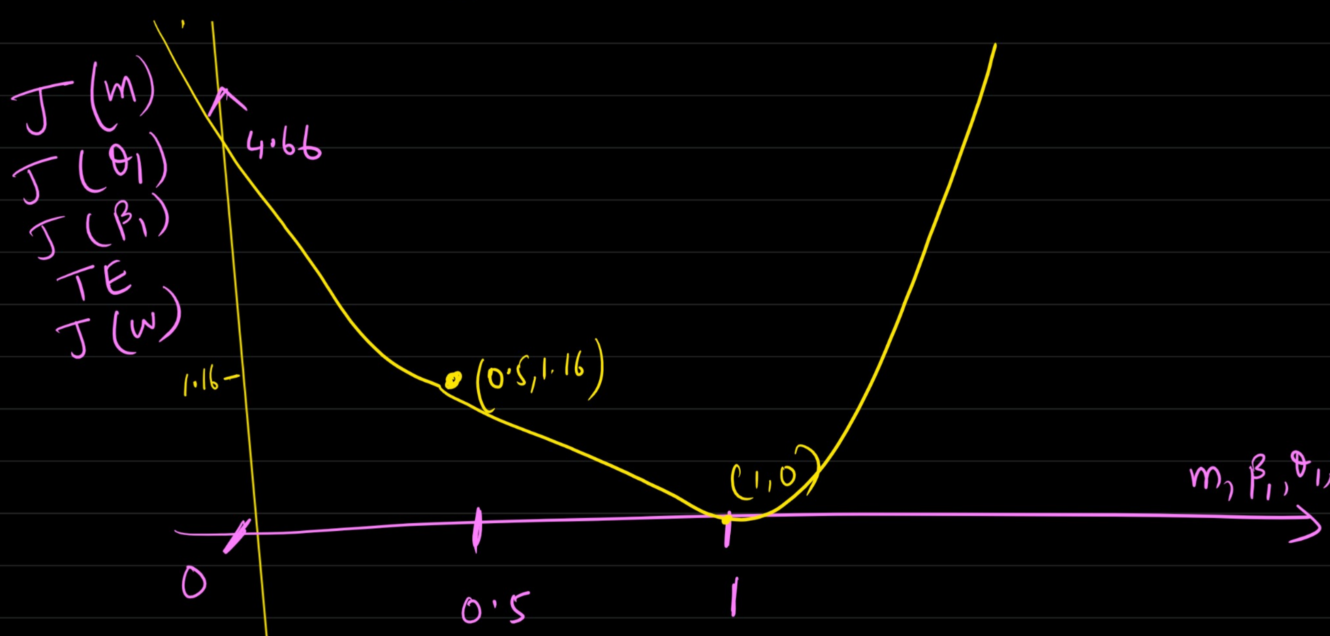
$x_1 = 3\ y_{pred} = 0$

$m = c$
$c = 0$

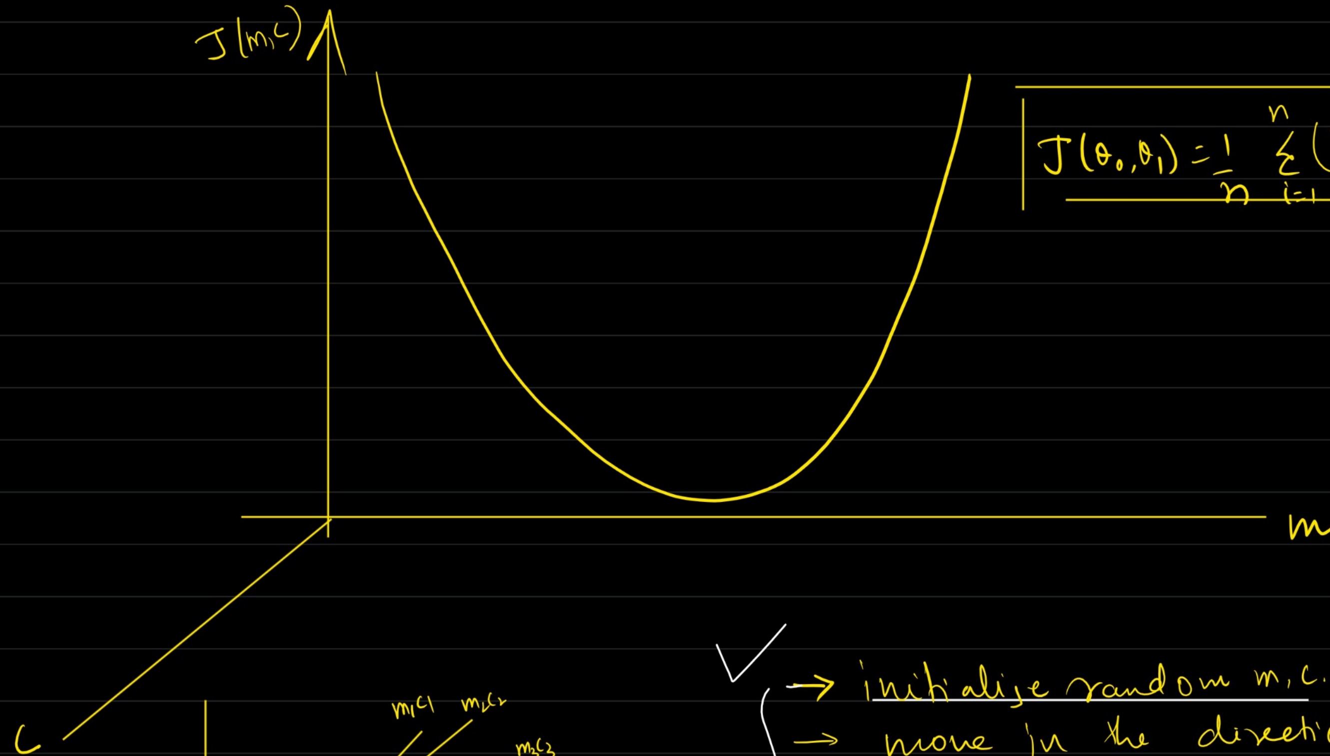
x	y	y_{pred}
1	1	0
2	2	0
3	3	0

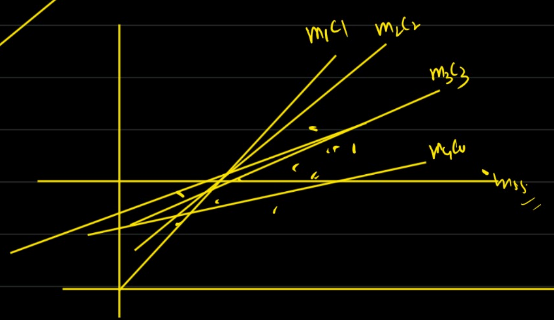
$$= \frac{1}{3}\left((1-0)^2 + (2-0)^2 + (3-0)^2\right)$$
$$= \frac{1}{3}(1+4+9) = 4.66$$

$J(m) = 4.66$



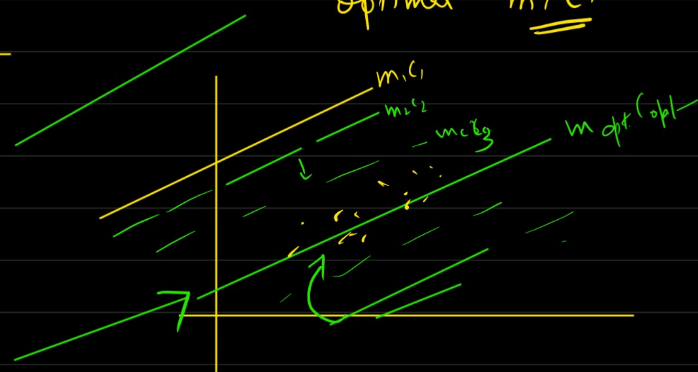
$$J(\theta_0,\theta_1) = \frac{1}{n}\sum_{i=1}^{n}(y_i - y_{pred})^2$$





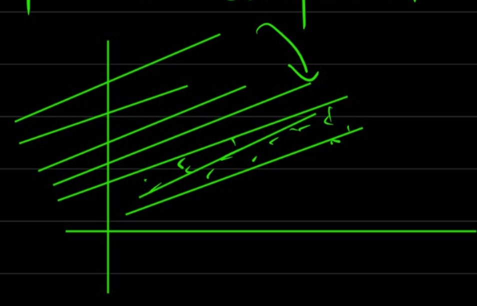
✓

- → initialise random m, c.
- → move in the direction where error decreases
- → least error will be optimal m, c.



* Convergence Algorithm

→ keep making new best fit line in the direction of dp until Error (CF) is reduced as compared to previous line.

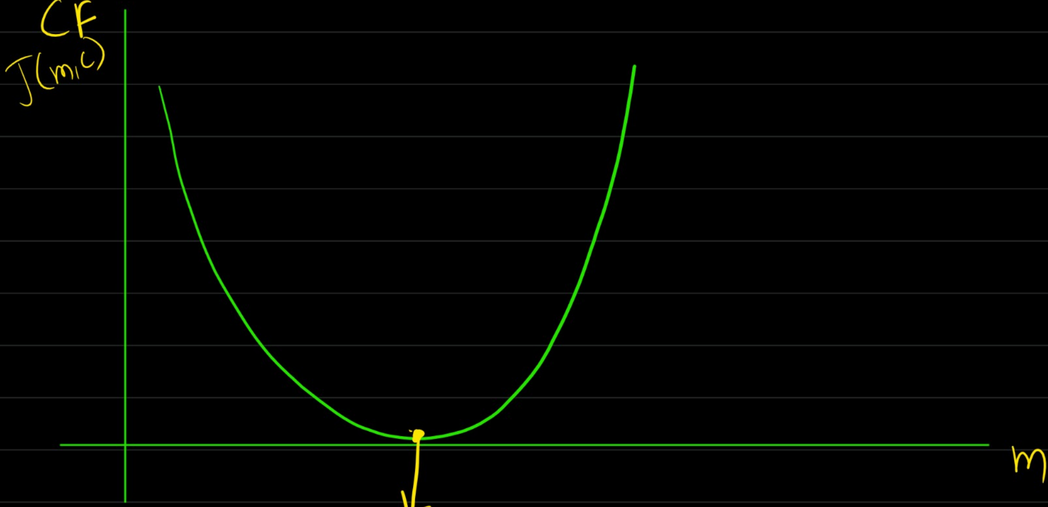


⇓

Till when ??

⇓

(where m, c is optimal



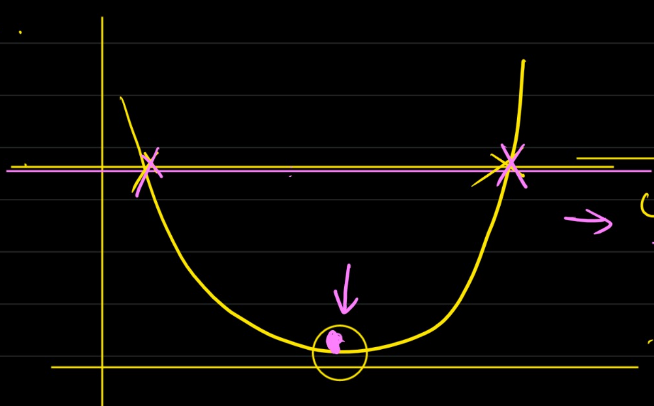
{ Error or is least
or
Cf is least

Cf is least

↓

Point of minima ⟶ global minima

function.



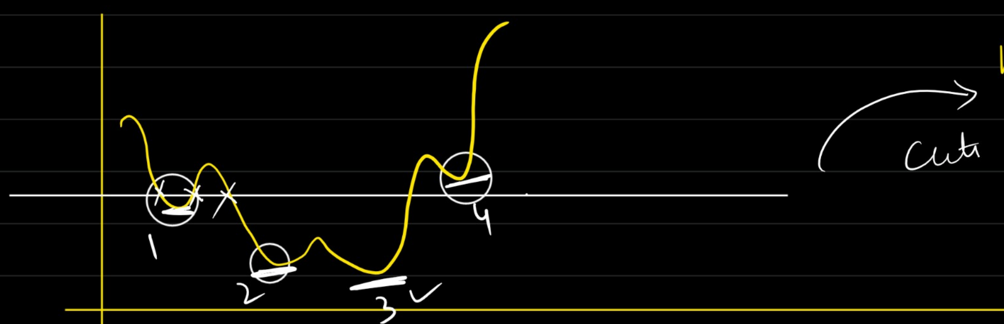
⟶ Convex function.

→ cuts at only two points

↳ convex

↓

one minima.



non-convex fn

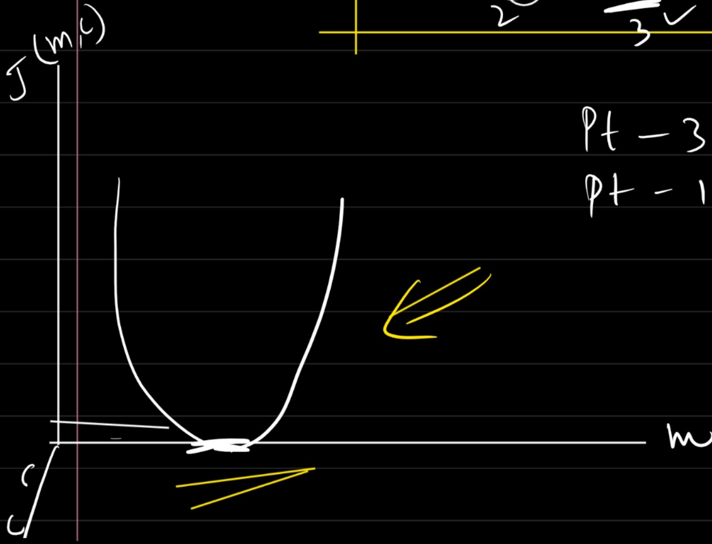
cuts more than two points

⇓

Local, global minima.

Pt − 3 Global minima

Pt − 1, 2, 4 ⟶ local minima



* Convergence Algorithm

Repetat unil convergence

$$m_{new} = m_{old} - \eta \frac{\partial(CF)}{\partial m_{old}}$$

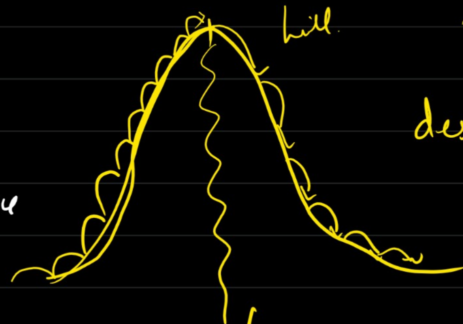
$$C_{new} = C_{old} - \eta \frac{\partial(CF)}{\partial(C_{old})}$$

gradient descent

↓ slope

↓ Coming down.

⇒ slope guides for convergence



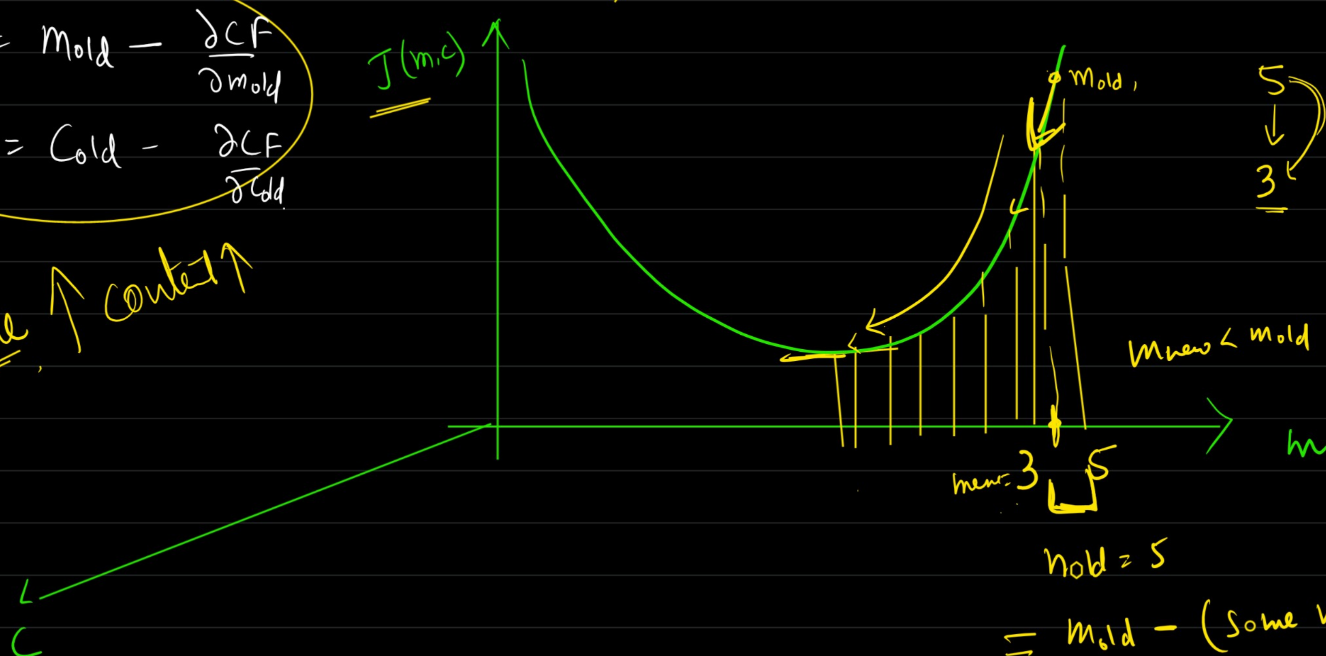
trekking

descend the slope

$$m_{new} = m_{old} - \frac{\partial CF}{\partial m_{old}}$$

$$C_{new} = C_{old} - \frac{\partial CF}{\partial C_{old}}$$

⇒ Time ↑ content ↑



5 ↓ 3

$n_{old} = 5$

$= m_{old} - (\text{some val})$

$5 - 2$

$m_{new} = 3$

$\frac{dy}{dx} = \text{slope}$

$m_{new} = m_{old} - (\text{Some value})$

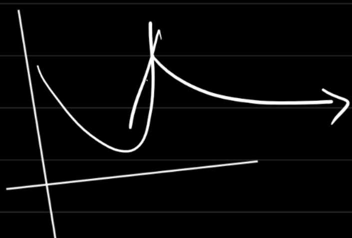
$\Rightarrow \left\{ \frac{\partial CF}{\partial m_{old}} \right\} \Rightarrow \text{slope}$

$\to +\text{ve value}$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$\downarrow$

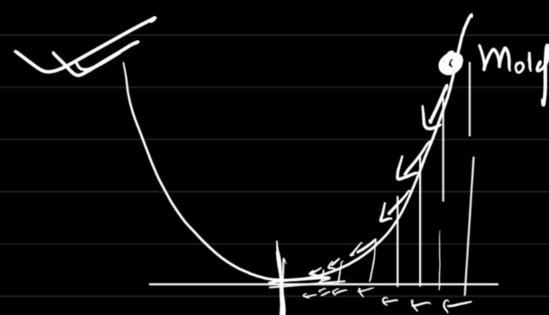
$\frac{\text{Change in } y}{\text{Change in } x}$

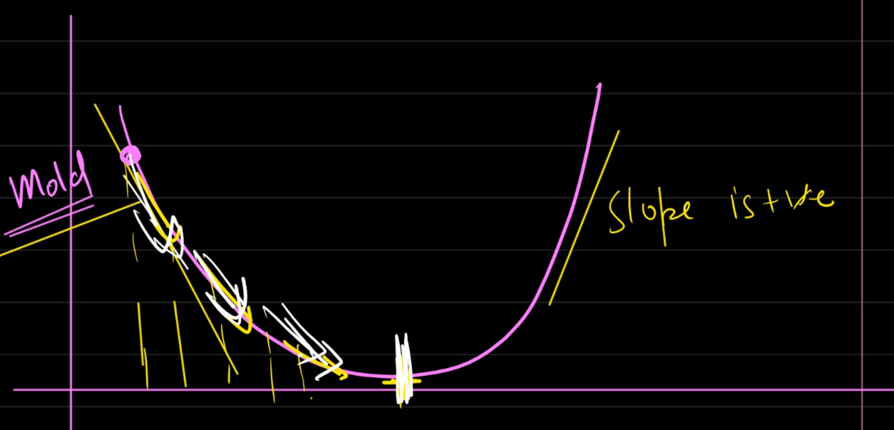


$\frac{dy}{dx} = \text{slope} = \frac{\partial y}{\partial m_{old}}$

$m_{new} = m_{old} - (+\text{ve val})$

$m_{new} <<< m_{old}$





$\Rightarrow m_{new} = m_{old} - \frac{\partial CF}{\partial m_{old}}$

$m_{new} >> m_{old}$

$\to \; = m_{old} - (-\text{ve slope})$

$\to m_{new} = m_{old} + \text{ve value}$

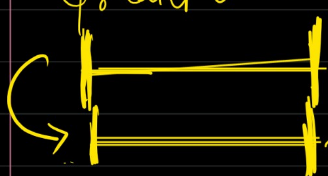
Slope is negative

When slope is positive

$$\begin{cases} m_{new} = m_{old} - \eta \dfrac{\partial CF}{\partial m_{old}} \\ C_{new} = C_{old} - \eta \dfrac{\partial CF}{\partial C_{old}} \end{cases}$$

multithreading	multiprocessing
True multith	True multi.
	Parallel Executio

Gradient descent

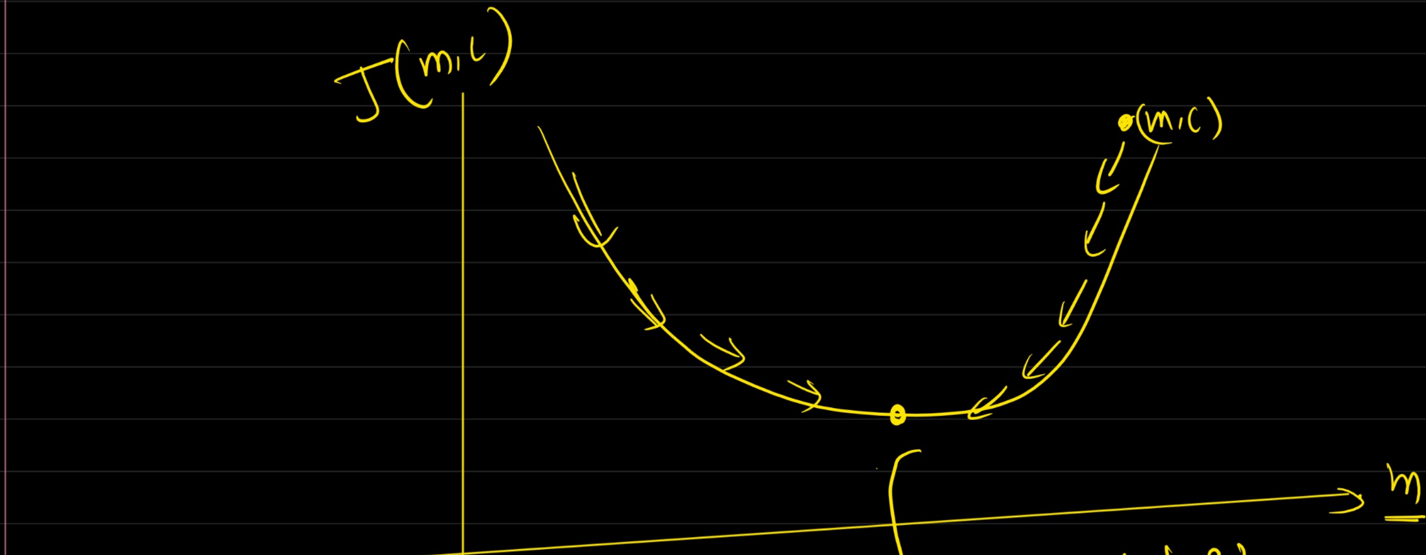


→ practical demons

Gradien descent → PyH

equ — ||
↓
m, c → ||
↓
Optimus ⟷ ||
Opt. → ||
← eq ||

→ multiprocessing



When to stop??

$$m_{new} = m_{old} - \eta \frac{\partial CF}{\partial m_{old}} \rightarrow 0$$

$m_{new} \approx m_{old}$

$C_{new} \approx C_{old}$

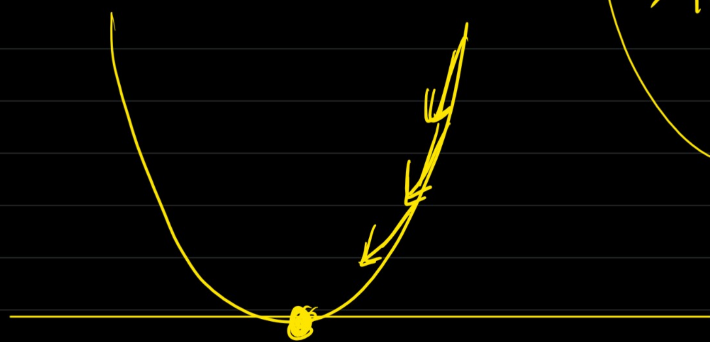
Gradient descent visualisation :-

blog.skz.dev gradient-descent

$$m_{new} = m_{old} - \eta \frac{\partial CF}{\partial m_{old}}$$

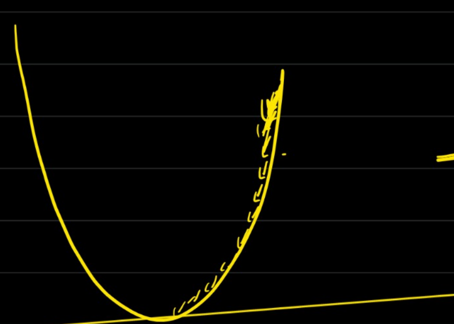
→ learning rate, α.

→ Eta (0.1 – 0.001)



η – learning rate

It decides the convergence speed.



$$- \boxed{\eta \left(\frac{\partial CF}{\partial m_{old}}\right)}$$

η → too small

very slow for convergence.

η – too high.

→ Exploding gradient problem.

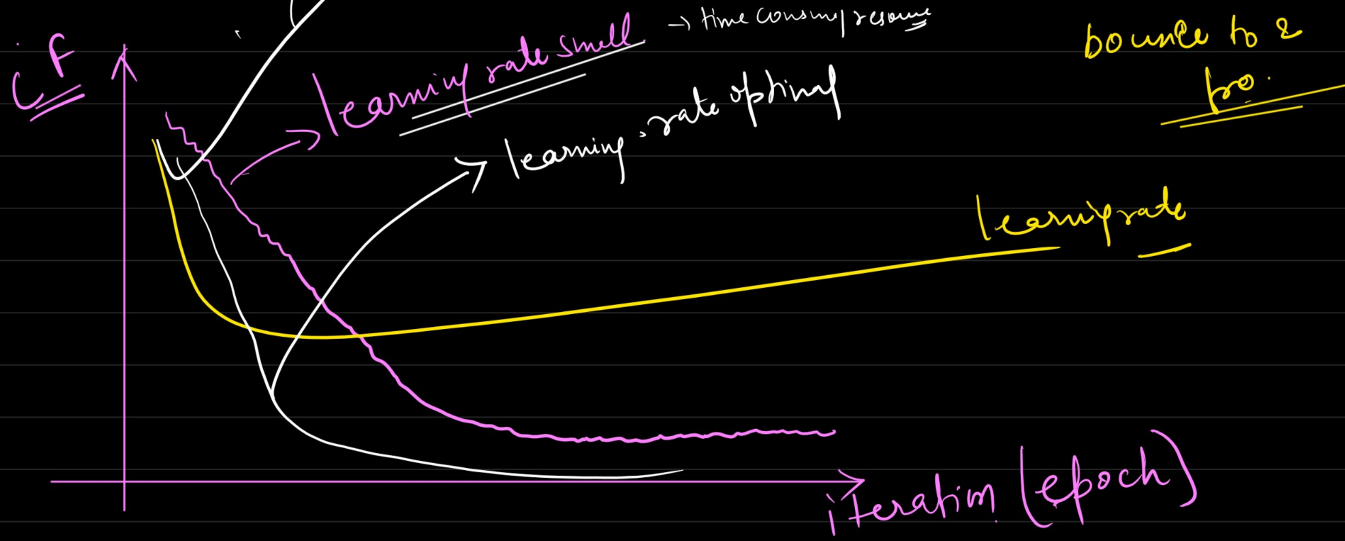
⇓

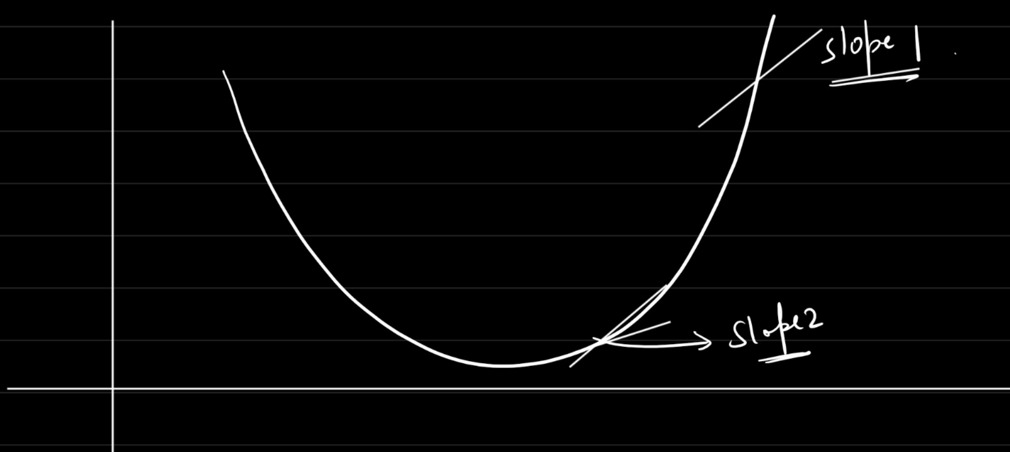
will never converge,

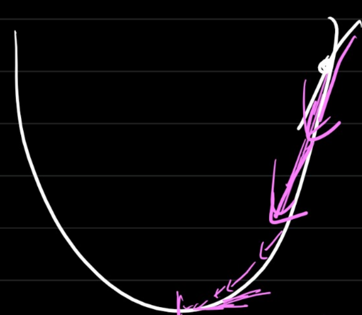
high learning

0.1 to 0.01

→ very high learning rate.

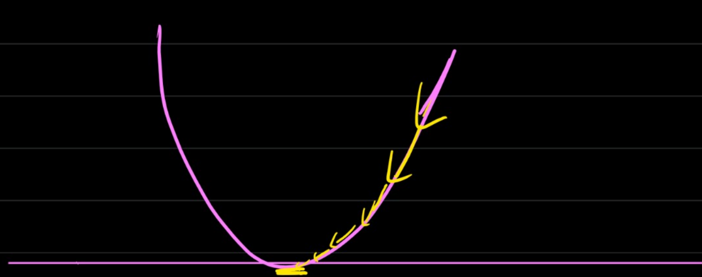




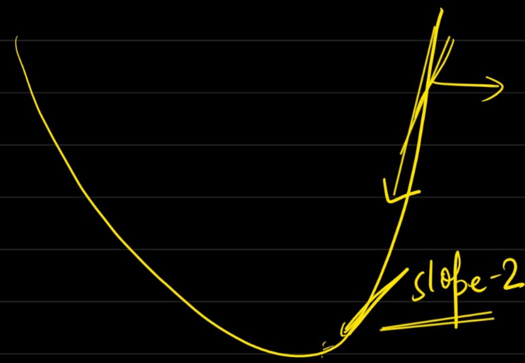


$$m_{new} = m_{old} - \eta \frac{\partial cf}{\partial m_{old}}$$

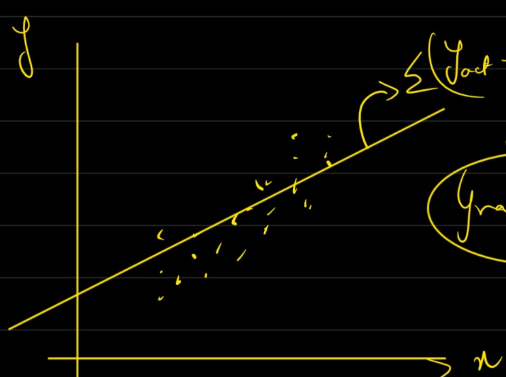
initially → slope will be higher, it will take big jump and while close to minima, slope decreases and steps become smaller.



developers.google.com/machine-learning/crash-course/fitter/graph



$-\eta\,(\text{slope})$



$\sum (y_{act} - y_{pr})^2$

Gradient Desc

.fit (X_{train}, Y_{train})

$\Rightarrow$ β_0, β_1

To predict on test

r squar