Ch-8 Introduction to Trigonometry

1. Prove that $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + (\sec\theta.\csc\theta) = 1 + \tan\theta + \cot\theta$.

2. If $\tan A = n(\tan B)$ and $\sin A = m(\sin B)$, prove that $\cos^2 A = \frac{m^2-1}{n^2-1}$.

3. Prove that $\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \csc A + \cot A$, using the identity $\csc^2 A = 1 + \cot^2 A$.

4. If $x(\sin^3\theta) + y(\cos^3\theta) = \sin\theta.\cos\theta$ and $x(\sin\theta) = y(\cos\theta)$, prove that, $x^2 + y^2 = 1$.

5. Find the value of $(\sin 45^\circ - \cos 45^\circ)$.

6. If $\tan\theta + \sin\theta = m$ and $\tan\theta - \sin\theta = n$, show that $(m^2 - n^2) = 4\sqrt{mn}$.

7. If $\sin A = \frac{1}{2}$, then find the value of $\cos A$.

8. If $\csc\theta = 3x$ and $\cot\theta = \frac{3}{x}$, then find the value of $\left(x^2 - \frac{1}{x^2}\right)$.

9. What is the value of $\sin^2\theta + \frac{1}{1+\tan^2\theta}$?

10. Given, $\csc\theta = \frac{4}{3}$, calculate all other trigonometric ratios.

11. Evaluate : $\frac{\cos 60^\circ - \cot 45^\circ + \csc 30^\circ}{\sec 60^\circ + \tan 45^\circ - \sin 30^\circ}$.

12. Prove that $\frac{\cot\theta}{1+\tan\theta} = \frac{\cot\theta-1}{2-\sec^2\theta}$.

13. If $\sin 3\theta = \cos(\theta - 6^\circ)$, find value of θ.

14. If $\tan A = \frac{4}{3}$, then find value of $\frac{\sin A + \cos A}{\sin A - \cos A}$.

15. If $\tan A = \frac{b}{a}$, where a and b are real numbers, find value of $\sin^2 A$.

16. Prove that $\frac{\cot A - \cos A}{\cot A + \cos A} = \frac{\csc A - 1}{\csc A + 1}$.

17. If $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$, show that $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$.

18. Prove that $\sin\theta(1+\tan\theta) + \cos\theta(1+\cot\theta) = \sec\theta + \csc\theta$.

19. Prove that $(\sin\theta + \csc\theta)^2 + (\cos\theta + \sec\theta)^2 = 7 + \tan^2\theta + \cot^2\theta$.

20. If $\sec 4A = \csc(A - 20^\circ)$, find the value of A.

21. Find : $\frac{2\tan 30^\circ}{1+\tan^2 30^\circ}$.

22. If $\sin B = \frac{12}{13}$, then find the value of $\cot B$.

23. If $\csc^2\theta(1+\cos\theta)(1-\cos\theta) = x$, then find the value of x.

24. In a $\triangle ABC$, if $\angle C = 90^\circ$, then prove that $\csc^2 A - \tan^2 B = 1$.

25. Prove that $(\sin^4\theta - \cos^4\theta + 1)\csc^2\theta = 2$.

26. If $a\cos\theta - b\sin\theta = c$, then prove that $a\sin\theta + b\cos\theta = \pm\sqrt{a^2 + b^2 - c^2}$.

27. Prove that $\frac{1}{\sec A - 1} + \frac{1}{\sec A + 1} = 2.\csc A.\cot A$.

28. Prove that $(\csc A - \sin A).(\sec A - \cos A) = \frac{1}{\tan A + \cot A}$.

29. Find value of $\sqrt{3}\sin x = \cos x$.

30. Find x, if $\tan x = \sin 45^\circ.\cos 45^\circ + \sin 30^\circ$.