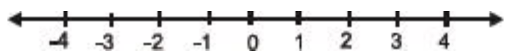
Ch-1 Number System

1. **Natural Numbers –** Numbers from 1 onward are known as Natural numbers, denoted by ‘N’. N = {1, 2, 3, 4,......}.
2. **Whole Numbers –** Numbers from 0 onward are known as Whole numbers, denoted by ‘W’. W = {0, 1, 2, 3, 4, ...}.
3. **Integers –** The collection of all whole numbers and negative of natural numbers are called Integers, denoted by ‘Z’. Z = {...... –3, –2, –1, 0, 1, 2, 3 ......}.
4. **Rational Number –** A number which can be expressed as $\frac{p}{q}$, where q ≠ 0 and p, q ∈ Z, is known as rational number, denoted by ‘Q’.
5. **Irrational Number –** A number which can’t be expressed in the form of $\frac{p}{q}$, and its decimal representation is non-terminating and non-repeating is known as irrational number. E.g., $\sqrt{2}$, $\sqrt{3}$, π, 1.1010010001…, etc.
6. **Number Line –** 

7. Method to find two or more rational numbers between two numbers p and q – If p < q, then one of the number be p < $\frac{p+q}{2}$ < q and other will be in continuation as p < $\frac{p+\frac{(p+q)}{2}}{2}$ < $\frac{p+q}{2}$ and so on.
8. The sum of a rational number and an irrational number is always an irrational number.
9. The product of a non-zero rational number and an irrational number is always an irrational number. E.g., $\frac{2}{3}$ x $\sqrt{5}$ is an irrational number.
10. The sum of two irrational numbers is not always an irrational number. (2 + $\sqrt{2}$) + (2 – $\sqrt{2}$) = 4 (a rational number).
11. The product of two irrational numbers is not always an irrational number. (2 + $\sqrt{2}$).(2 – $\sqrt{2}$) = 4 – 2 = 2.
12. If a is a rational number and n is a positive integer such that the n^{th} root of a is an irrational number, then $\sqrt[n]{a}$ is called a surd. E.g., $\sqrt{5}$, $\sqrt{2}$, $\sqrt{3}$, etc.
13. If $\sqrt[n]{a}$ is a surd, then ‘n’ is known as the order of the surd, and ‘a’ is known as the radicand.
14. Every surd is an irrational number, but every irrational number is not a surd.
15. Laws of radicals –
    a. $\left(\sqrt[n]{a}\right)^n$ = a.
    b. $\sqrt[n]{a}$ x $\sqrt[n]{b}$ = $\sqrt[n]{ab}$. (at least 1 of either a or b should be a non-negative integer.)
    c. $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$.
    d. $\sqrt[n]{\sqrt[m]{a}} = \sqrt[mn]{a} = \sqrt[m]{\sqrt[n]{a}}$.
    e. $\frac{\sqrt[p]{a^n}}{\sqrt[p]{a^m}} = \sqrt[p]{a^{n-m}}$.
    f. $\sqrt[p]{a^n \times a^m} = \sqrt[p]{a^{n+m}}$.
    g. $\sqrt[p]{(a^n)^m} = \sqrt[p]{a^{n.m}}$.
16. A surd which has unity only as rational factor is called a pure surd.
17. A surd which has a rational factor other than unity is called a mixed surd.

18. Order of a given surd can be changed by using following steps –
    a. Let the surd be $\sqrt[n]{a}$ and m be the order of the surd to which it has to be converted.
    b. Compute $\frac{m}{n}$ and let $\frac{m}{n}$ = p.
    c. Write $\sqrt[n]{a} = \sqrt[m]{a^p}$, which is the required result.
19. Surds having same irrational factors are called similar or like surds.
20. Only similar surds can be added or subtracted by adding or subtracting their rational parts.
21. Surds of same order can be multiplied or divided.
22. If the surds to be multiplied or to be divided are not of the same order, we first convert them to the same order and then multiply or divide.
23. If the product of two surds is a rational number, then each one of them is called the rationalizing factor of the other. E.g., $\sqrt[3]{2} \times \sqrt[3]{4} = 2$, then $\sqrt[3]{2}$ and $\sqrt[3]{4}$ are rationalizing factors of one another.
24. A surd consisting of one term only is called a monomial surd.
25. An expression consisting of the sum or difference of two monomial surds or the sum or difference of a monomial surd and a rational number is called binomial surd. E.g., $\sqrt{2} + \sqrt{5}$, $\sqrt{3} + 2$, $\sqrt{2} - \sqrt{3}$, etc., are binomial surds.
26. The binomial surds which differ only in sign (+ or –) between the terms connecting them, are called conjugate surds. E.g., $\sqrt{3} + \sqrt{2}$ and $\sqrt{3} - \sqrt{2}$ or $2 + \sqrt{5}$ and $2 - \sqrt{5}$ are conjugate surds.
27. Rational exponents –
    a. If x, y be any rational numbers different from zero and m be any integer, then $x^m \times y^m = (x \times y)^m$.
    b. If x be any rational number different from zero and m, n be any integers, then $x^m \times x^n = x^{m+n}$ and $(x^m)^n = x^{m \times n}$.
28. Reciprocals of positive integers as exponents – If q be any positive integer other than 1, and x and y be rational numbers, such that $x^q = y$, then $y^{\frac{1}{q}} = x$. We write $y^{\frac{1}{q}}$ as $\sqrt[q]{y}$ and read it as q^{th} root of y. $\sqrt[q]{y}$ is called a radical and q is called the index of the radical.
29. Positive rational numbers as exponents – If $\frac{p}{q}$ be any positive rational number (where p and q are positive integers prime to each other), and let x be any rational number. We have already given a meaning to $x^{\frac{p}{q}}$. This can be done very easily, i.e., $x^{\frac{p}{q}}$ is the q^{th} root of x^p. Thus, $(4)^{\frac{3}{2}} = (4^3)^{\frac{1}{2}} = (64)^{\frac{1}{2}} = 8$.
30. If $\frac{p}{q}$ is a negative rational number, then $x^{-\frac{p}{q}}$ ($x \neq 0$) is equal to $\frac{1}{x^{\frac{p}{q}}}$.
31. If x be any rational number different from zero, and a and b be any rational numbers, then, $x^a \div x^b = x^{a-b}$.