Ch-2 Polynomials

1. An expression $p(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + ... a_n$ is a polynomial Where a_0, a_1, ... a_n are real numbers and n is non-negative integer.
2. Degree of a polynomial is the greatest exponent of the variable in the polynomial.
3. Constant polynomial is a polynomial of degree 0. The **constant polynomial** $f(x) = 0$ is called **zero polynomial.**
4. Degree of zero polynomial is not defined.
5. A polynomial of degree 1 is called a **linear polynomial** e.g. $ax + b$, where $a \neq 0$.
6. A polynomial of degree 2 is called a **quadratic polynomial** e.g. $ax^2 + bx + c$ where $a \neq 0$.
7. A polynomial of degree 3 is called a **cubic polynomial** e.g. $px^3 + qx^2 + rx + s$, $p \neq 0$.
8. A polynomial of degree 4 is called a **biquadratic polynomial** e.g. $px^4 + qx^3 + rx^2 + sx + t$, $p \neq 0$.
9. Value of a polynomial $p(x)$ at $x - a$ is $p(a)$.
10. Zero of a polynomial $p(x)$ is a number 'a' such that $p(a) = 0$.

Factor Theorem

1. If $p(x)$ is a polynomial of degree $n \geq 1$, and a is any real number then.
    a. $x - a$ is a factor of $p(x)$, if $p(a) = 0$.
    b. $p(a) = 0$, if $(x - a)$ is a factor of $p(x)$.

Algebraic Identities

1. $(x + y)^2 = x^2 + 2xy + y^2$
2. $(x - y)^2 = x^2 - 2xy + y^2$
3. $x^2 - y^2 = (x + y)(x - y)$
4. $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$
5. $(x + a)(x + b) = x^2 + (a + b)x + ab$
6. $(x + y)^3 = x^3 + y^3 + {}^3xy(x + y)$
7. $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$
8. $x^3 + y^3 + z^3 - 3xyz = [(x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)]$
    a. If $x + y + z = 0$, then $x^3 + y^3 + z^3 = 3xyz$.
9. $x^3 - y^3 = (x + y)(x^2 + xy + y^2)$
10. $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

Degree of a Polynomial

1. The exponent of the term with the highest power in a polynomial is known as its **degree**. $f(x) = 8x^3 - 2x^2 + 8x - 21$ and $g(x) = 9x^2 - 3x + 12$ are polynomials of degree 3 and 2 respectively.

Zeroes of a Polynomial

1. **Value of polynomial –** The value of a polynomial $f(x)$ at $x = c$ is obtained by substituting $x = c$ in the given polynomial and is denoted by $f(c)$.
2. **Zero or root –** A real number c is a zero of the polynomial $f(x) = a_0x^n + a_1x^{n-1} + ... + a_n$, if $f(c) = 0$. $\Rightarrow a_0c^n + a_1c^{n-1} + a_2c^{n-2} + ... + a_n = 0$.