Ch-2 Polynomials

1. If $x + y = 12$ and $xy = 32$, find the value of $x^2 + y^2$.
2. If $3x + 2y = 12$ and $xy = 6$, find the value of $9x^2 + 4y^2$.
3. Write the following cubes in the expanded form –
   a. $(3a + 4b)^3$ b. $(5p – 3q)^3$
4. If $x^2 + \frac{1}{x^2} = 27$, find the values of each of the following –
   a. $x + \frac{1}{x}$ b. $x - \frac{1}{x}$
5. If $a + b + c = 15$ and $a^2 + b^2 + c^2 = 83$, find the value of $a^3 + b^3 + c^3 – 3abc$.
6. Factorize –
   a. $6ab – b^2 + 12ac – 2bc$ b. $9(2a – b)^2 – 4(2a – b) – 13$
7. If $x^3 + ax^2 – bx + 10$ is divisible by $x^2 – 3x + 2$, find the values of a and b.
8. Using factor theorem, factorize each of the following polynomials –
   a. $x^3 – 6x^2 + 3x + 10$ b. $2y^3 – 5y^2 – 19y + 42$
9. The number of zeros of $x^2 + 4x + 2$ are?
10. Find the value of k, if $(x – 1)$ is a factor of $4x^3 + 3x^2 – 4x + k$.
11. If $x^2 + \frac{1}{x^2} = 18$, then find the value of $x - \frac{1}{x}$.
12. Factorize : $(a – b)^3 + (b – c)^3 + (c – a)^3$.
13. Factorize : $14x^6 – 45x^3y^3 – 14y^6$.
14. Find the product : $(x – 3y)(x + 3y)(x^2 + 9y^2)$.
15. If $x^2 – 3x + 2$ divides $x^3 – 6x^2 + ax + b$ exactly, then find the value of ‘a’ and ‘b’.
16. The polynomials $P(t) = 4t^3 – st^2 + 7$ and $Q(t) = t^2 + st + 8$ leave the same remainder when divided by $(t – 1)$. Find the value of s.
17. Find the value of k for which the cubic polynomial $3y^3 – \frac{3}{2}y^2 + ky + 5$ is exactly divisible by $\left(y - \frac{1}{2}\right)$.
18. Verify whether the indicated numbers are zeroes of their corresponding polynomials –
   a. $Q(s) = –4s^3 + 7s^2 – 24$ $s = –4$ and 1
   b. $P(t) = 8t^2 + 4t – 4$ $t = \frac{1}{2}$ and -1
19. If $x = –2$ is a root of the polynomial $P(x) = –2x^4 – 7x^3 – 3x^2 – tx – 10$, then find the value of t.
20. State whether the following statements are true or false. Give reasons to justify your answers –
   a. The degree of polynomial $–5x^5 – 6x^4 – 8x^2$ is 4.
   b. The algebraic expression $x^3 + \frac{1}{x^3} – 2x + 1$ is a polynomial.
   c. The polynomial $\sqrt{x^4} + 4x + 1$ is a quadratic trinomial.
21. Using the long division method, determine the remainder when the polynomial $4x^5 + 2x^4 – x^3 + 4x^2 – 7$ is divided by $(x – 1)$.
22. Evaluate the following products using algebraic identities – a. 993^3 b. 1002^3
23. Factorize – a. $2y^3 – 4y^2 – 2y + 4$ b. $x^3 + 13x^2 + 32x + 20$