FRE6123 MIDTERM PROJECT

Team Daredevils:

- Bhaskar Gudimetla (bpg5084)
- Harsha Gangasani (sg6423)
- Navil Zaman (nz837)
- Jason Wang (jw6542)

Team Members: Contributions:

We all discussed and collaborated with each other for all tasks and worked on our assigned tasks. However, each team member's major contribution is tabulated below:

Team Member	Contribution
Bhaskar Gudimetla	Part 1: Univariate and Multivariate Stylized facts analysis and results. Part 3: 1-day Linear Approximation and Joint Multivariate Distribution assumption computation of 95% VaR and ES. Part 3: Historical Simulation computation of 1-day 95% VaR and ES.
Harsha Gangasani	Part 1: Multivariate Stylized facts analysis and results modifications. Part 2: Computed Option greeks (Delta and Vega). Part 3: Historical Simulation computation of 1-day 95% VaR and ES.
Navil Zaman	Part 2: Drift test and Portfolio Optimization to choose minimum variance portfolio and Portfolio Value computations. Part 3: Scenario creation and Scenario Analysis for portfolio loss.
Jason Wang	Part 2: Choosing Option Strategy and Options. Part 3: 1-day Linear Approximation and Joint Multivariate Distribution assumption computation of 95% VaR and ES.

Selected Stocks: Information

We have selected the following stocks for performing the Midterm Project Analysis. The analysis has been done for prices and returns for the stocks for the past three years (12^{th} March 2018 – 12^{th} March 2021) (Source: Yahoo Finance).

Activision Blizzard, Inc.

Ticker: ATVI

Market Cap: $70.107B

Company Description: Activision Blizzard, Inc. publishes, develops, and distributes interactive entertainment software and peripheral products. The Company's products cover diverse game categories.

Take Two Interactive Software, Inc.

Ticker: TTWO

Market Cap: $19.55B

Company Description: Take-Two Interactive Software, Inc. develops, markets, distributes, and publishes interactive entertainment software games and accessories. The Company's products are for console systems, handheld gaming systems and personal computers and are delivered through physical retail, digital download, online, and cloud streaming services.

GameStop Corp.

Ticker: GME

Market Cap: $13.968B

Company Description: GameStop Corporation operates specialty electronic game and PC entertainment software stores. The Company stores sell new and used video game hardware and software, as well as accessories. GameStop markets its products worldwide.

Zynga, Inc.

Ticker: ZNGA

Market Cap: $10.808B

Company Description: Zynga Inc. designs and develops video game software. The Company offers wide range of online social games. Zynga serves customers worldwide.

We have also used return data for US Equity Indices like the S&P 500, NASDAQ, DJIA, Russell 2000 (Source: Yahoo Finance) and return data for major FX rates like the USDGBP, USDEUR, USDJPY, USDCHF (Source: Bloomberg) for the same dates as above to analyze certain stylized facts.

PART 1: Stylized facts on financial returns

The Stylized Facts of Financial return series for both Univariate and Multivariate Series are presented below:

Univariate Series:

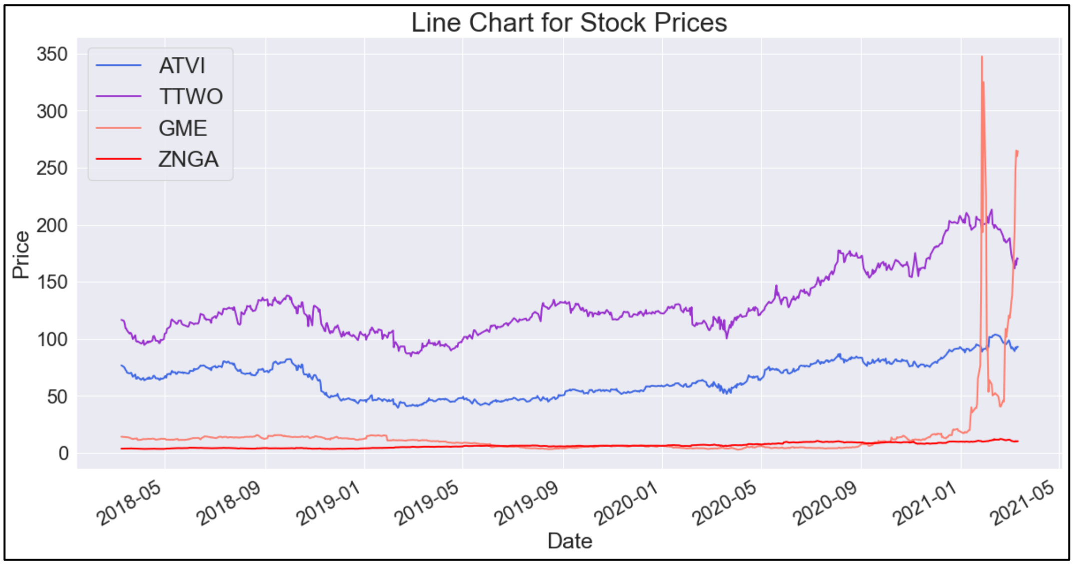
- Return series are not independent and identically distributed (i.i.d), although they show little serial correlation: $Corr(R_t, R_{t-\tau}) \approx 0$, , $\tau = 1, ...100$.
- Series of absolute or squared returns show profound serial correlation; for small τ, $Corr(R^2_t, R^2_{t-\tau}) > 0$.
- The standard deviation of returns completely dominates the mean of returns at short horizons: it is impossible to reject statistically zero mean.
- Volatility appears to vary in time.
- Extreme returns appear in clusters (volatility clustering).
- Return series are leptokurtic or heavily tailed.
- Equity and equity indices display negative correlation between variance and returns.
- The stock market exhibits occasional large drops but not equally large up moves: the returns distribution is negatively skewed. Other markets as FX tend to show less evidence for skewness.

Multivariate Series:

- Multivariate series show little evidence of cross correlations, except for contemporaneous returns.
- Multivariate series of absolute returns show profound evidence of cross correlations.
- Correlations between series (contemporaneous returns) vary over time.
- Extreme returns in one series often coincide with extreme returns in several other series.

Analysis:

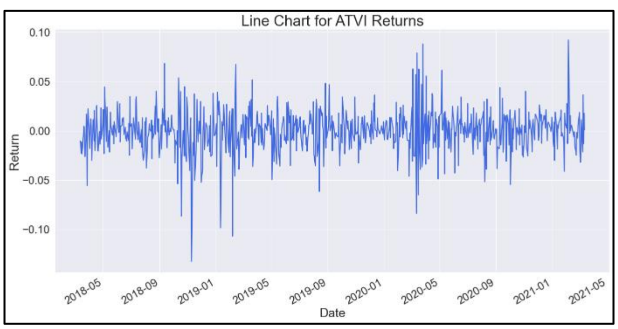
The Line Plot for the price data for the four stocks: ATVI, TTWO, GME and ZNGA are initially plotted:

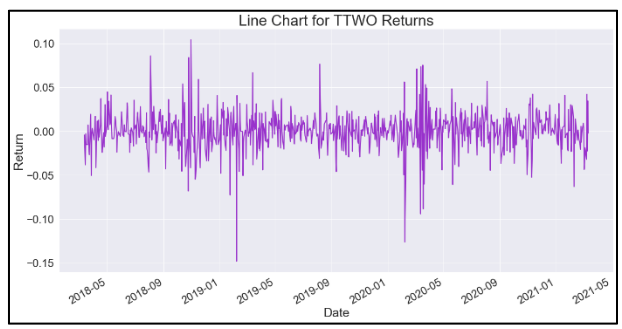


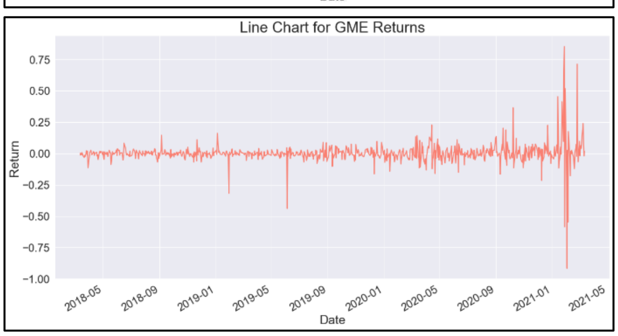
The daily returns for the stocks are calculated from the prices as follows:

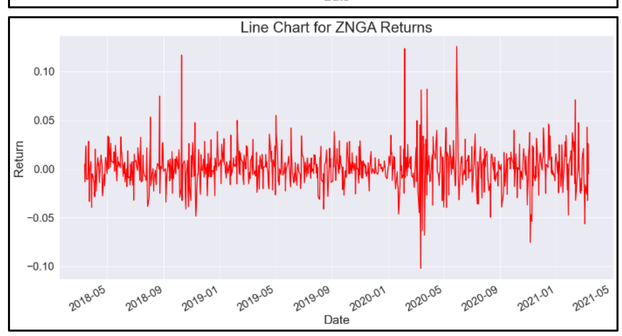
$$R_t = \frac{\ln\left(\frac{P_t}{P_{t-1}}\right)}{\sqrt{(t) - (t-1)}}$$

Where R_t is the return for date 't' and P_t and P_{t-1} are the prices of the stocks on dates 't' and 't-1' respectively. The Line Plots for the return data for the four stocks are also plotted:









An initial look at the return plots for the stock suggests that large returns (positive or negative) are generally followed by large returns (positive or negative) which is a sign of volatility clustering. The volatility also appears to be changing with time. We perform further tests to test these hypotheses.

Autocorrelation Tests:

Stylized Facts Addressed:

- Return series are not independent and identically distributed (i.i.d), although they show little serial correlation: Corr(R_t , $R_{t-\tau}$) ≈ 0, , τ = 1, ...100.
- Series of absolute or squared returns show profound serial correlation; for small τ, Corr(R^2_t , $R^2_{t-\tau}$) > 0.
- Extreme returns appear in clusters (volatility clustering).

Autocorrelation is the correlation of a signal with a delayed copy of itself.

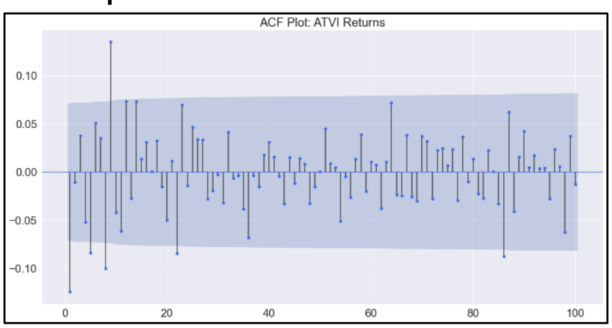
Autocorrelation: Return Timeseries

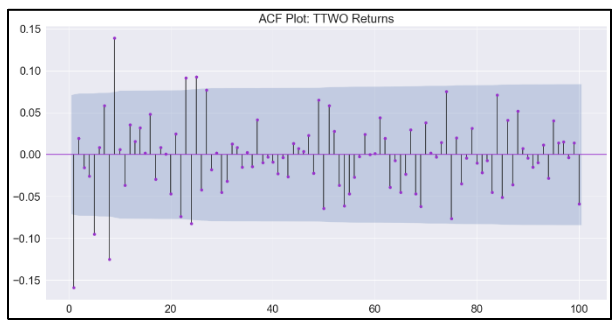
The Autocorrelation for stock returns is given by:

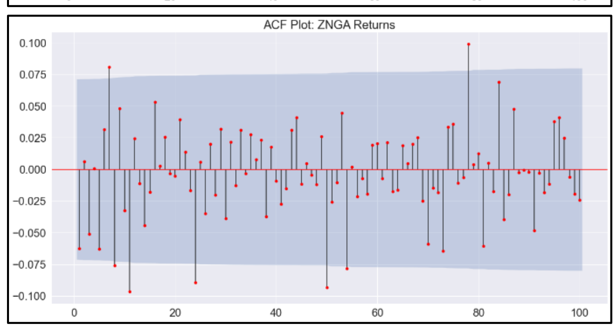
$$\rho_\tau = Corr(R_t, R_{t-\tau})$$

For this analysis, we considered lags(τ) up to 100 days for Stock Returns. ACF plots (Autocorrelation function plots, also called as "correlogram") are used to visualize the autocorrelation values. We analyzed the values of autocorrelation and checked for a presence of a pattern. The ACF plots here show the value for autocorrelation [y-axis] for various lags τ, τ = 0, 1, 2, …, 100 [x-axis] and a shaded region that represents the 95% Confidence Interval around Autocorrelation 0.

The **ACF plots for Return timeseries** for the chosen stocks are as under:





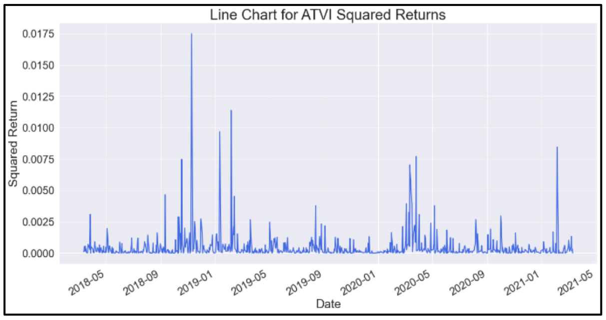


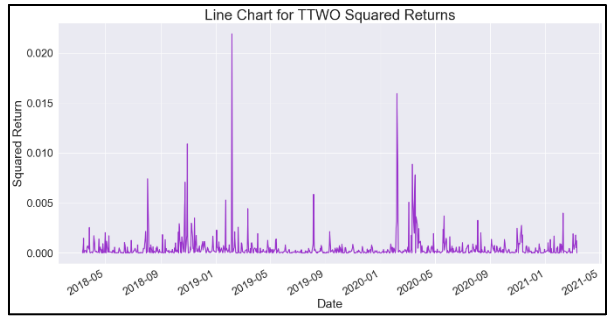
Observations:

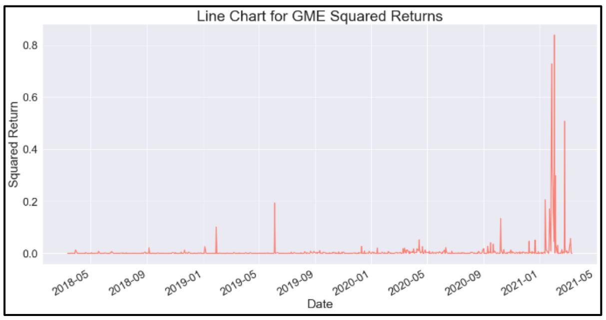
From the ACF plots for Stock Returns, we can observe that for most of the lags, the autocorrelation value lies in the shaded 95% Confidence Interval which means that the autocorrelations are not statistically significant from 0. There is no pattern for the autocorrelations of Stock Return series.

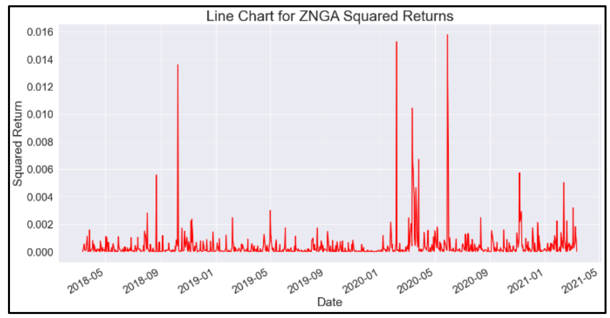
Autocorrelation: Squared Return Timeseries

We plotted line plots for the Squared Returns for all the stocks:









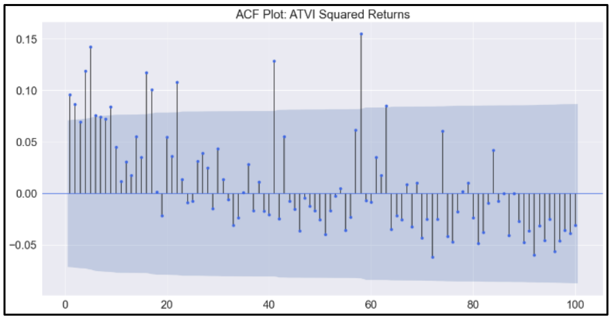
Observations:
The Line Plots for Squared Returns show visual evidence of volatility clustering. The phenomenon is particularly evident during March, 2020 where large values of Squared Returns occur consecutively.

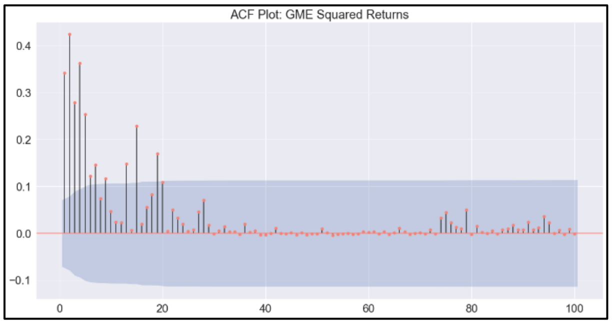
The presence of Volatility clustering is further established by analyzing the autocorrelations for Squared Returns. The Autocorrelation for Squared Stock Returns is given by:

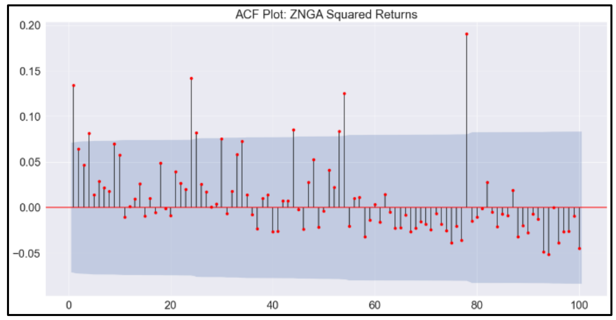
$$\rho_{\tau}^{squared\ returns} = Corr(R_t^2, R_{t-\tau}^2)$$

Similar to ACF Plots for Stock Returns, ACF Plots for the Squared Stock Returns are also plotted for lags up to 100 days.

The **ACF plots for Squared Return timeseries** for the chosen stocks are as under:







Observations:
From the ACF plots for Squared Stock Returns, we can observe that there is a statistically significant positive correlation for a 1-day lag. The autocorrelation however decreases and falls into the 95% Confidence Interval around 0 autocorrelation when the lag increases. This suggests a decaying autocorrelation for Squared Stock Returns.

Linear Regression Test: 1-Day Lag

Since, statistically significant positive 1-day lag autocorrelation was observed for Squared Stock Returns, we performed an additional Linear Regression test for Squared Stock Return and it's one day lagged value. This is also a test for volatility clustering.

If the daily stock returns (R) are assumed to be normally distributed with mean (μ) = 0 (Addressed in the **Sample Mean Test**) and standard deviation σ. Then σ is given by:

$$\sigma^2 = E[R^2] - \mu^2$$

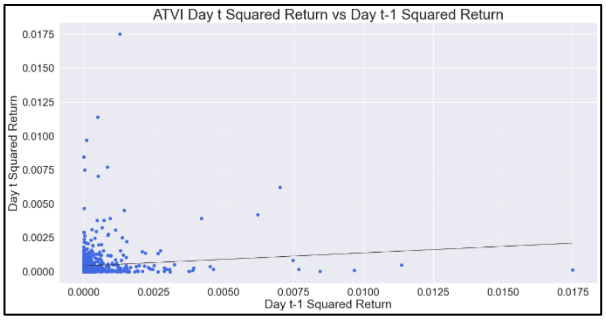
Since μ = 0,

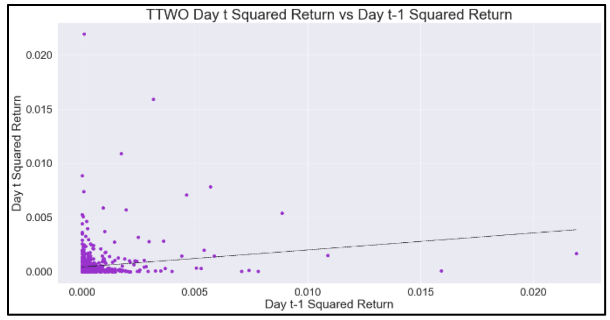
$$\sigma^2 = E[R^2]$$

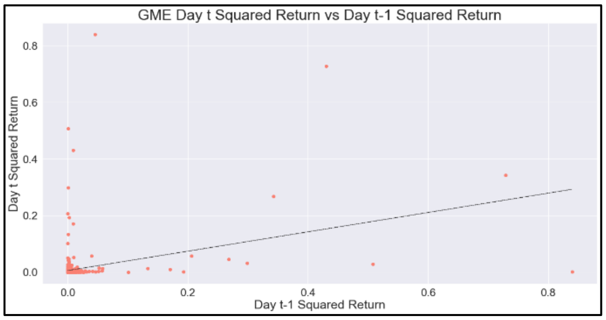
This means the Squared Stock Return is a measure of volatility. We therefore, test the presence of volatility clustering by running a linear regression on R_t^2 and R_{t-1}^2 and test the significance of the slope parameter. The linear regression equation is given by:

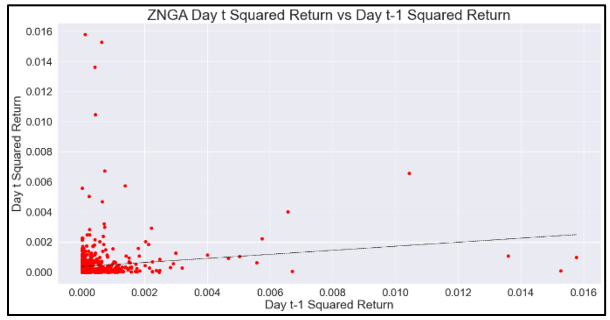
$$R_t^2 = aR_{t-1}^2 + b + \epsilon$$

Test Results:









The Linear Regression results are tabulated below:

Parameter/Stock	ATVI	TTWO	GME	ZNGA
Slope	0.0958	0.1570	0.3421	0.1342
Intercept	0.0004	0.0004	0.0052	0.0004
R squared	0.0092	0.0247	0.1170	0.0180
p-value (slope)	0.0084	1.445e-05	3.489e-22	0.00021

Observations:
For all 4 stocks, the slope is positive and the p-value for the slope in the linear regression is less than 0.01. This means that the slope parameter lies outside the 99% Confidence Interval around 0. This means that the slope parameters are statistically significant.
There is a relationship between 't' Squared Return and 't-1' Squared Return, and hence it can be said that a large squared return on day 't' can occur when a large squared return occurs on day 't-1'. (Volatility clustering)

Ljung-Box Test

We performed the Ljung-Box Test which tests whether any of a group of autocorrelations of a time series are different from zero.

The Ljung-Box test is defined as:

- **H_0:** The data are independently distributed (i.e., the correlations in the population from which the sample is taken are 0, so that any observed correlations in the data result from randomness of the sampling process)
- **H_a:** The data are not independently distributed; they exhibit serial correlation.

The test statistic is given by:

$$Q = n(n+1)\sum_{k=1}^{h}\frac{\hat{p}_k^2}{n-k}$$

Where ‘n’ is the sample size, $\hat{p}_k$ is the sample autocorrelation at lag k, and h is the number of lags being tested.

We performed the Ljung-Box test and the Box-Pierce test for lags 1 day to 10 days. The test results are presented below:

Test Results:

ATVI				
Lag	Ljung-Box Statistic	Ljung-Box p-value	Box-Pierce Statistic	Box-Pierce p-value
1	11.7242	6.17e-04	11.6779	6.32e-04
2	11.8143	2.72e-03	11.7675	2.78e-03
3	12.8892	4.88e-03	12.8353	5.01e-03
4	14.9388	4.83e-03	14.8687	4.98e-03
5	20.3626	1.07e-03	20.2425	1.13e-03
6	22.3156	1.06e-03	22.1749	1.13e-03
7	23.2436	1.55e-03	23.0919	1.64e-03
8	30.9273	1.45e-04	30.6743	1.61e-04
9	44.8976	9.64e-07	44.4422	1.17e-06
10	46.2591	1.29e-06	45.7822	1.57e-06

TTWO				
Lag	Ljung-Box Statistic	Ljung-Box p-value	Box-Pierce Statistic	Box-Pierce p-value
1	19.1158	1.23e-05	19.0402	1.28e-05
2	19.3980	6.13e-05	19.3210	6.38e-05
3	19.5871	2.07e-04	19.5088	2.15e-04
4	20.0869	4.80e-04	20.0046	4.98e-04
5	26.9661	5.79e-05	26.8204	6.18e-05
6	27.0215	1.43e-04	26.8753	1.53e-04
7	29.6461	1.10e-04	29.4687	1.19e-04
8	41.6455	1.58e-06	41.3100	1.82e-06
9	56.4246	6.51e-09	55.8749	8.29e-09
10	56.4506	1.69e-08	55.9006	2.14e-08

GME				
Lag	Ljung-Box Statistic	Ljung-Box p-value	Box-Pierce Statistic	Box-Pierce p-value
1	0.0025	0.96	0.0025	0.96
2	9.6425	8.06e-03	9.5917	8.26e-03
3	41.9117	4.19e-09	41.6483	4.76e-09
4	93.0582	2.95e-19	92.3905	4.09e-19
5	94.2399	8.63e-19	93.5613	1.20e-18
6	100.9777	1.57e-19	100.2281	2.25e-19
7	110.1228	8.66e-21	109.2647	1.30e-20
8	120.8513	2.21e-22	119.8519	3.56e-22
9	121.3411	7.10e-22	120.3345	1.14e-21
10	122.4106	1.64e-21	121.3872	2.65e-21

ZNGA				
Lag	Ljung-Box Statistic	Ljung-Box p-value	Box-Pierce Statistic	Box-Pierce p-value
1	2.9876	0.083903	2.9758	0.084518
2	3.0159	0.221362	3.0040	0.222689
3	5.0042	0.171487	4.9792	0.173328
4	5.0047	0.286820	4.9796	0.289398
5	8.0310	0.154534	7.9781	0.157449
6	8.7812	0.186261	8.7203	0.189927
7	13.7899	0.055048	13.6696	0.057380
8	18.1899	0.019847	18.0117	0.021139
9	19.9646	0.018132	19.7607	0.019448
10	20.7825	0.022662	20.5657	0.024334

The Box-Pierce test is also performed. It is an identical test but with a different test statistic. The Box-Pierce test statistic is given by:

$$Q = n \sum_{k=1}^{h} \hat{p}_k^2$$

Observations:
For ATVI and TTWO, in both Ljung-Box Test and Box-Pierce Test for up to 10 lags, the p value is always below 0.05. Therefore, we can reject the null hypothesis that the returns are independently distributed and exhibit no serial correlation at 95% Confidence Interval.

For GME, the p-value for 1-day lag in both Ljung-Box and Box-Pierce is 0.96, thus we can't reject the null hypothesis. The 1-day autocorrelation is not statistically significant from zero. But, when the lag increases, the p-value tend to be less than 0.05.

For ZNGA, the p-values for lags up to 7 days are greater than 0.05, thus we can't reject the null hypothesis for these lags. This can also be seen in the ACF plot for ZNGA, where the autocorrelations lie within the band that represents the 95% Confidence Interval around 0 autocorrelation.

Sample Mean Test:

Stylized Facts Addressed:

- The standard deviation of returns completely dominates the mean of returns at short horizons: it is impossible to reject statistically zero mean.

We tested for the sample mean for non-overlapping returns for different horizons: 1 day, 5 days, 10 days, 20 days, and 25 days. The returns are scaled down to 1 day returns for ease of comparability by dividing them with the square root of the difference between dates. The T-statistic of the sample mean estimate is used to check if the mean is statistically significant from zero.

The T-statistic is given by:

$$t = \frac{\overline{R} - \mu_0}{\frac{s}{\sqrt{n}}}$$

Where '$\overline{R}$' is the sample mean of stock returns, 'μ_0' is the hypothesized mean of 0, 's' is the sample standard deviation and 'n' is the number of samples.

Parameter	1 day	5 days	10 days	15 days	20 days	25 days
Number of Sample points (N)	757	151	75	50	37	30
Degrees of freedom for T-test (N-1)	756	150	74	49	36	29
Critical Values of two-tailed T-test (5% Significance Level)	±1.9631	±1.9759	±1.9925	±2.0096	±2.0281	±2.0452
Critical Values of two-tailed T-test (1% Significance Level)	±2.5823	±2.609	±2.6439	±2.68	±2.7195	±2.7564

The skew and kurtosis for each horizon is also computed to check if changing the horizon decreases the stock returns' deviation from normality.

Test Results:

The T-test rows are filled with green if the T-statistic lies in the range of the critical values for its corresponding confidence interval and horizon or else, it is filled with red.

ATVI Sample Mean Test Results						
Parameter	1 day	5 days	10 days	15 days	20 days	25 days
Mean	0.0484%	0.0459%	0.0677%	0.0916%	0.1432%	0.1147%
Standard Deviation	2.1464%	1.7110%	1.8551%	1.9091%	1.8821%	1.7613%
T statistic	0.6202	0.3293	0.3162	0.3394	0.4629	0.3567
T test (SL: 5%)						
T test (SL: 1%)						
Skew	(0.5012)	(0.3731)	(0.4291)	(1.0701)	(1.1773)	(1.1047)
Excess Kurtosis	4.5796	1.0353	0.3862	2.7046	2.8564	2.5118

TTWO Sample Mean Test Results						
Parameter	1 day	5 days	10 days	15 days	20 days	25 days
Mean	0.0727%	0.0935%	0.1541%	0.1988%	0.2677%	0.2441%
Standard Deviation	2.2240%	1.6174%	1.6108%	1.7241%	1.6439%	1.6259%
T statistic	0.9000	0.7101	0.8287	0.8153	0.9906	0.8224
T test (SL: 5%)						
T test (SL: 1%)						
Skew	-0.5639	-0.1468	-0.3628	-0.7368	-0.0651	-0.2598
Excess Kurtosis	5.9415	0.2570	0.0304	0.7227	0.1150	-0.4631

GME Sample Mean Test Results						
Parameter	1 day	5 days	10 days	15 days	20 days	25 days
Mean	0.2960%	0.7254%	0.7496%	0.9179%	0.5713%	1.1613%
Standard Deviation	8.8775%	7.4373%	6.7507%	5.2925%	5.6136%	7.5098%
T statistic	0.9174	1.1986	0.9616	1.2264	0.6190	0.8470
T test (SL: 5%)						
T test (SL: 1%)						
Skew	0.7024	2.4407	1.3505	1.0088	1.1957	3.7781
Excess Kurtosis	42.5855	15.0028	4.2832	1.9170	1.5583	17.4219

ZNGA Sample Mean Test Results						
Parameter	1 day	5 days	10 days	15 days	20 days	25 days

Mean	0.1570%	0.2369%	0.3697%	0.4644%	0.5577%	0.5985%
Standard Deviation	2.0577%	1.8846%	1.9116%	1.6729%	1.8233%	1.6699%
T statistic	2.0995	1.5448	1.6748	1.9631	1.8604	1.9630
T test (SL: 5%)						
T test (SL: 1%)						
Skew	0.6386	-0.7618	-0.2375	-0.2573	0.5172	-0.3137
Excess Kurtosis	5.9431	2.8607	0.9745	-0.5358	-0.1016	-0.3510

Observations:
The mean return is not statistically significant from 0 in all cases for all considered horizons and for both 5% and 1% Significance level except for ZNGA returns with 1 day horizon.
The Skew and Kurtosis of the Stock returns tends to decrease as horizon increases and then increase as horizon increases. This suggests stock returns with a moderately high horizon show lesser deviation from normality.

EWMA Volatility:

Stylized Facts Addressed:

- Volatility appears to vary in time.

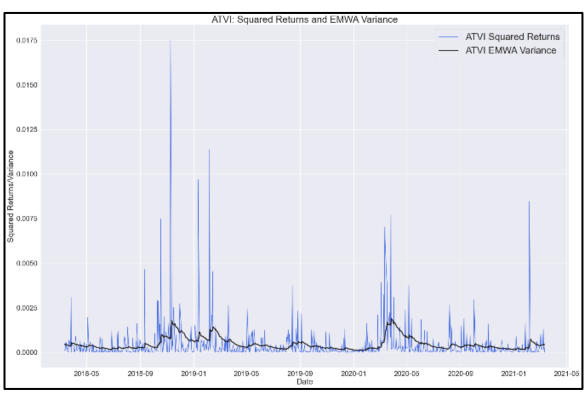
From the Squared Returns plot, we can observe that the volatility changes with time. We can also conclude from the tests performed previously that the mean is not statistically different from 0 and hence the returns are governed by the standard deviation.

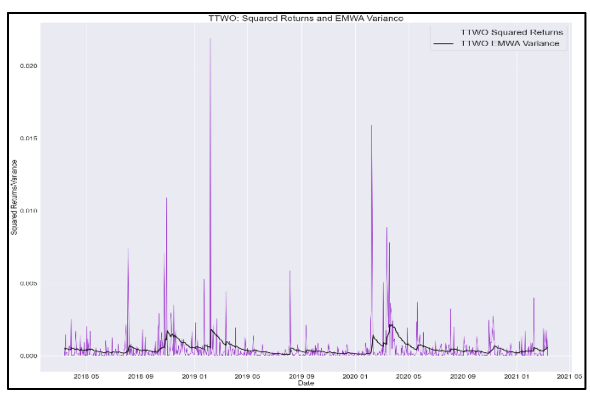
The finding that squared returns are correlated and returns are governed by the standard deviation can be used to model the volatility using the EWMA (Exponentially Weighted Moving Average) model.

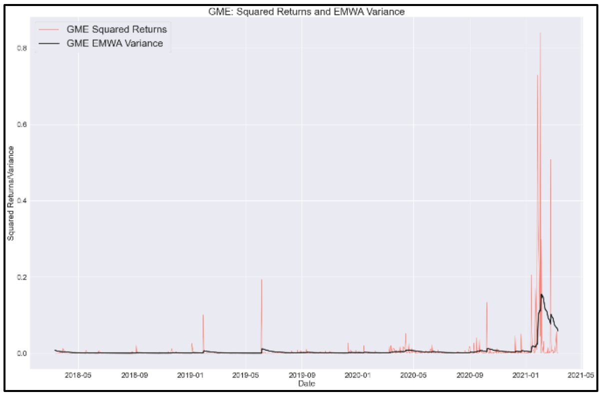
$$\sigma_{t+1}^2 = \lambda\sigma_t^2 + (1 - \lambda)R_t^2$$

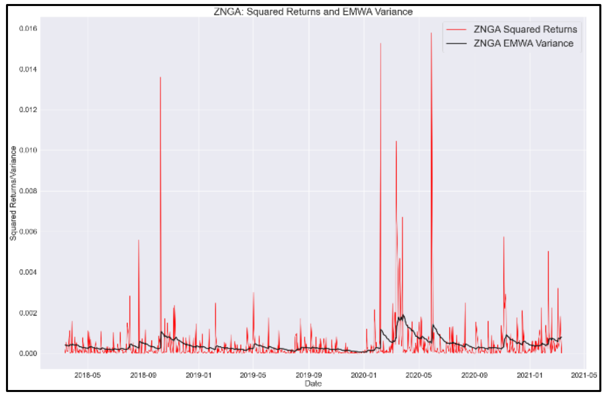
$\lambda = 0.94$ is generally used and σ_0 is set to the standard deviation of the entire return series.

The EWMA variance plots for the four stocks are plotted below:









Observations:
There are significant spikes in the smoothed out EMWA variance at certain dates for all the four stocks. This further establishes that the volatility changes with time.

The Leverage Effect:

Stylized Facts Addressed:

- Equity and equity indices display negative correlation between variance and returns

The stylized fact mentioned in the box above is also called as the 'leverage effect', If the above stylized fact is true, then the variance of returns increases with a decrease in prices.

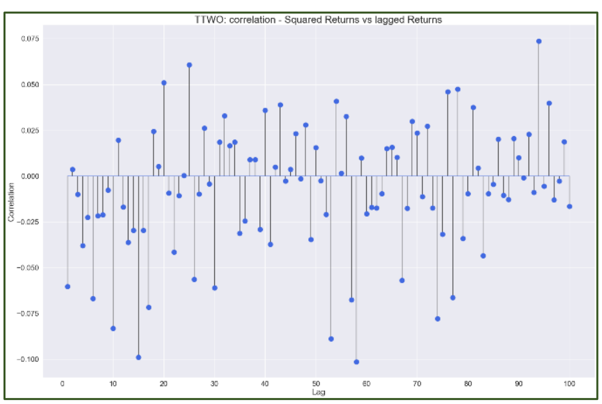
We observe this by computing the correlations 'ρ' between time 't' squared stock returns and time 't-τ' return where τ is a lag, and τ = 1, 2, …,100.

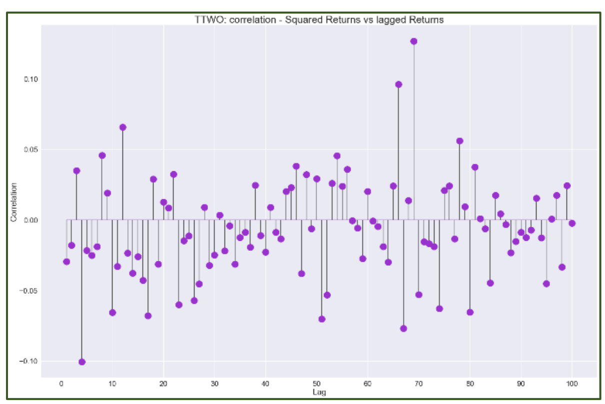
$$\rho = Corr(R_t^2, R_{t-\tau})$$

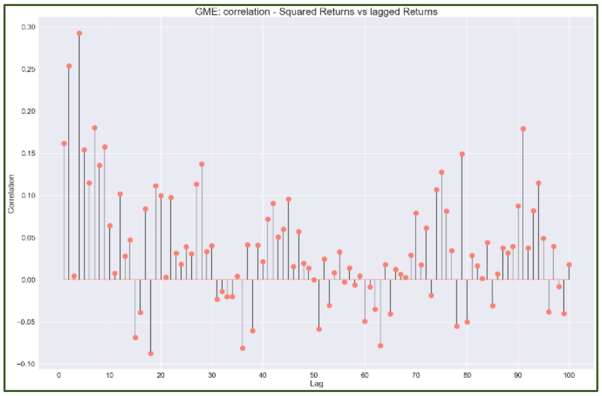
The correlations are computed for the selected four stocks and major US equity indices as well.

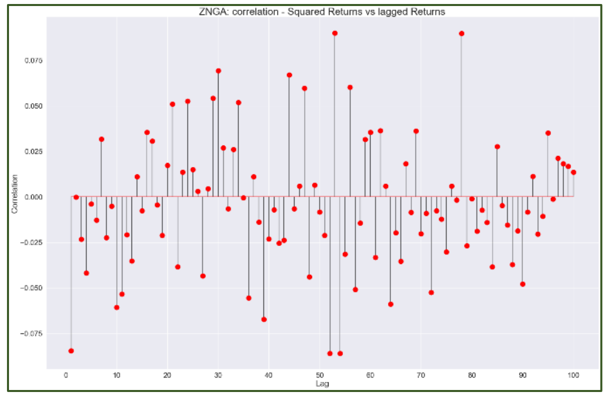
Test Results:

Individual Stocks:

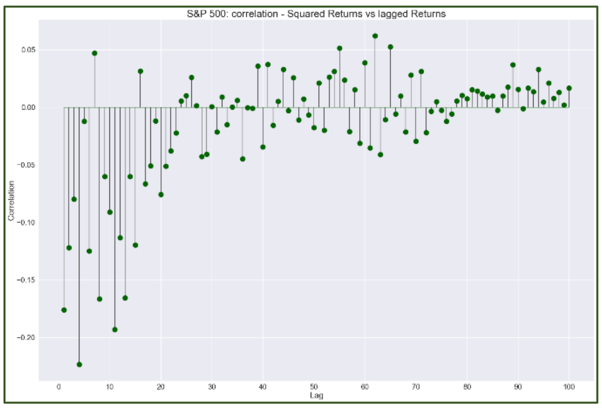


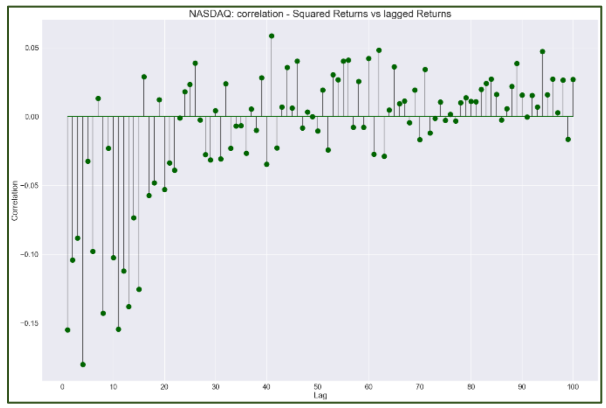


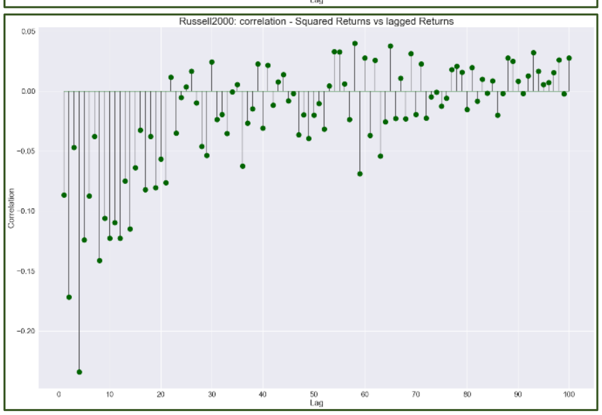


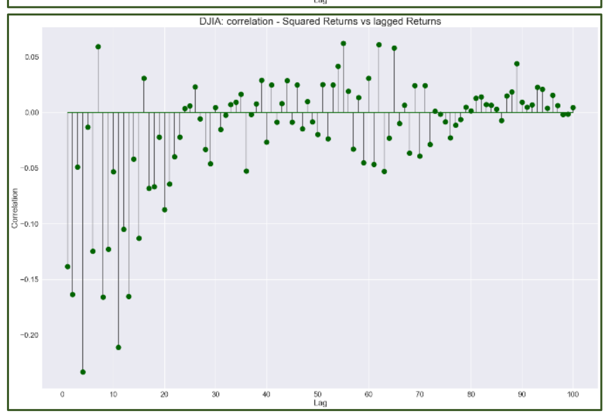


Equity Indices:









Observations:

For both individual stocks and equity indices, the correlation between present squared return and lagged return is negative for smaller lags and then tends to become zero for larger lags. There is an exception in the case of GME (GameStop) because the increase in volatility was due to the price increase which happened to the GME short squeeze in January 2021.

Skewness and Kurtosis Test:

Stylized Facts Addressed:

- Return series are leptokurtic or heavily tailed.
- The stock market exhibits occasional large drops but not equally large up moves: the returns distribution is negatively skewed. Other markets as FX tend to show less evidence for skewness.

The Skewness, Kurtosis and the JB Test statistic of the return timeseries for the selected stocks are computed. Skewness and Kurtosis of the Return timeseries calculated from various FX timeseries like “USDGBP”, “USDEUR”, “USDJPY” and “USDCHF” are also computed for comparison.

Histograms of the standardized stock returns and FX returns along with the Normal probability density function is also plotted to check for deviations from normality.

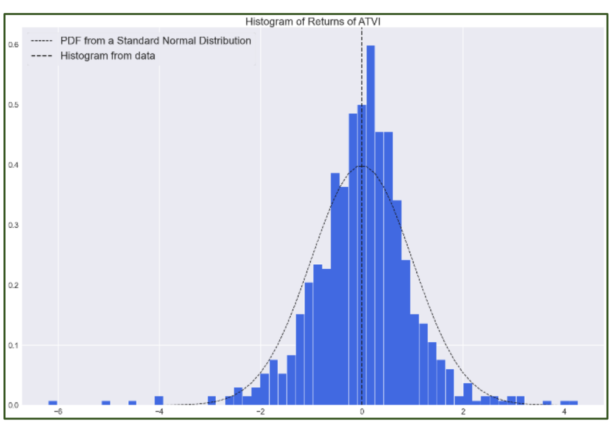
Test Results:

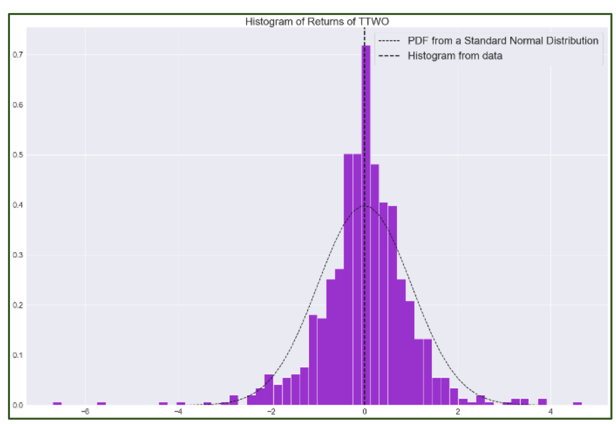
Stock Ticker	Skewness	Excess Kurtosis	JB Statistic	p-value
ATVI	-0.5002	4.5796	682.1197	0.0
TTWO	-0.5628	5.9415	1135.8684	0.0
GME	0.7010	42.5855	56590.7394	0.0
ZNGA	0.6373	5.9431	1147.7284	0.0

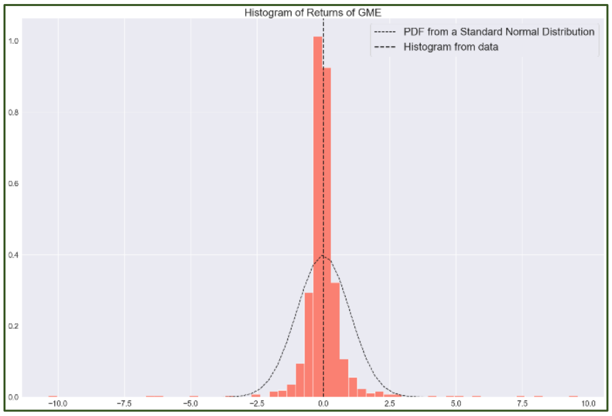
FX Rate	Skewness	Excess Kurtosis	JB Statistic	p-value
USDGBP	0.1511	4.7162	730.4935	0.0
USDEUR	0.3493	2.2711	184.6666	0.0
USDJPY	1.0036	10.2714	3582.5619	0.0
USDCHF	0.1522	2.0454	139.8725	0.0

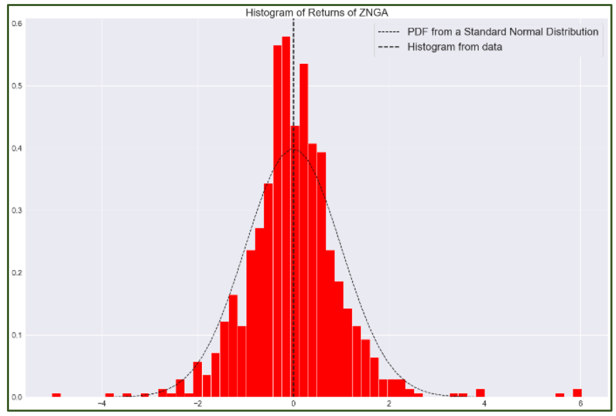
The histograms for the standardized stock returns and FX rate returns are plotted below to better visualize the deviation from normality:

Individual Stocks:

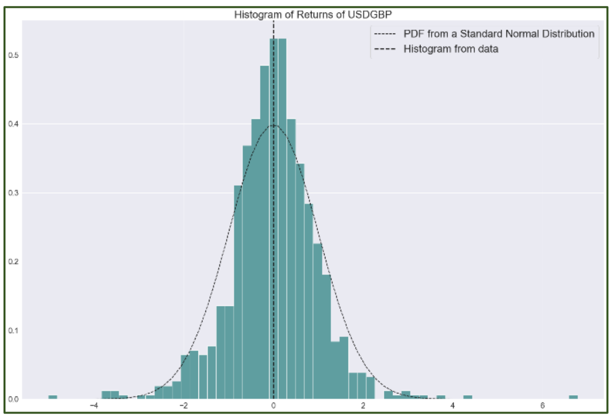


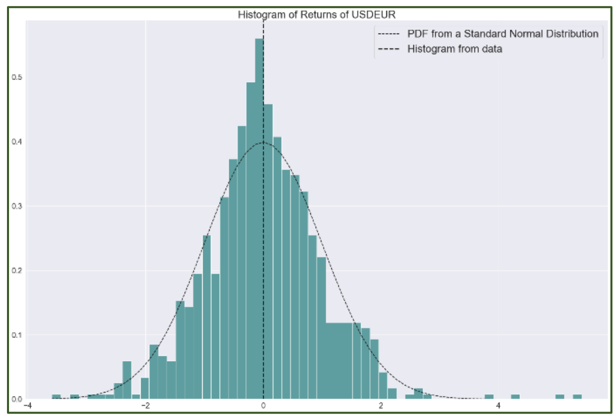


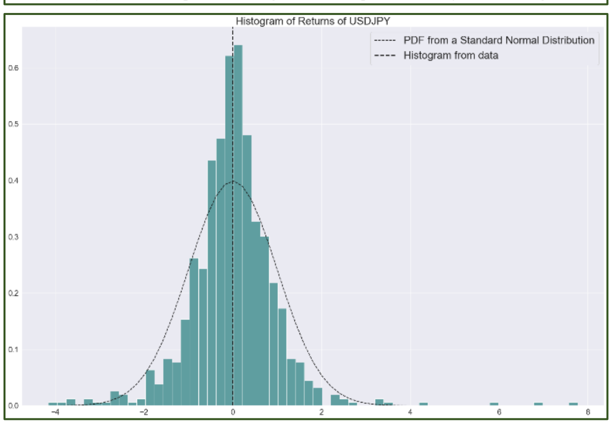


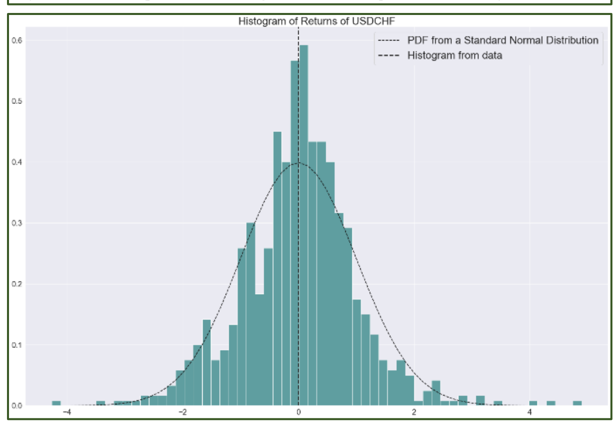


FX Rates:









Observations:

All Stocks have kurtosis greater than 3 and are therefore Leptokurtic meaning the distribution of stock returns have fatter tails when compared to the Normal distribution.

All Stocks also show more skewness when compared to the skewness of FX Returns.

The JB test also show that the distribution of stock returns shows huge deviation from normality.

Multivariate Correlations:

Stylized Facts Addressed:

- Multivariate series show little evidence of cross correlations, except for contemporaneous returns.
- Multivariate series of absolute returns show profound evidence of cross correlations.
- Correlations between series (contemporaneous returns) vary over time.

The correlation matrix for the contemporaneous returns for all the four selected stocks is computed:

Stock	ATVI	TTWO	GME	ZNGA
ATVI	1.0000	0.6523	0.0221	0.5285
TTWO	0.6523	1.0000	0.0339	0.4753
GME	0.0221	0.0339	1.0000	0.0005
ZNGA	0.5285	0.4753	0.0005	1.0000

Multivariate Lagged Returns Correlation Analysis:

The multivariate correlations for lagged returns are computed. The correlation of time ‘t’ returns of stock ‘x’ and time ‘t-τ’ returns for another stock ‘y’ is computed.

$$\rho_{xy}^{\tau} = Corr\left(R_t^x, R_{t-\tau}^y\right)$$

Similar exercise is repeated for absolute lagged returns. The correlation of time ‘t’ absolute returns of one stock ‘x’ and time ‘t-τ’ absolute returns for another stock ‘y’ is computed.

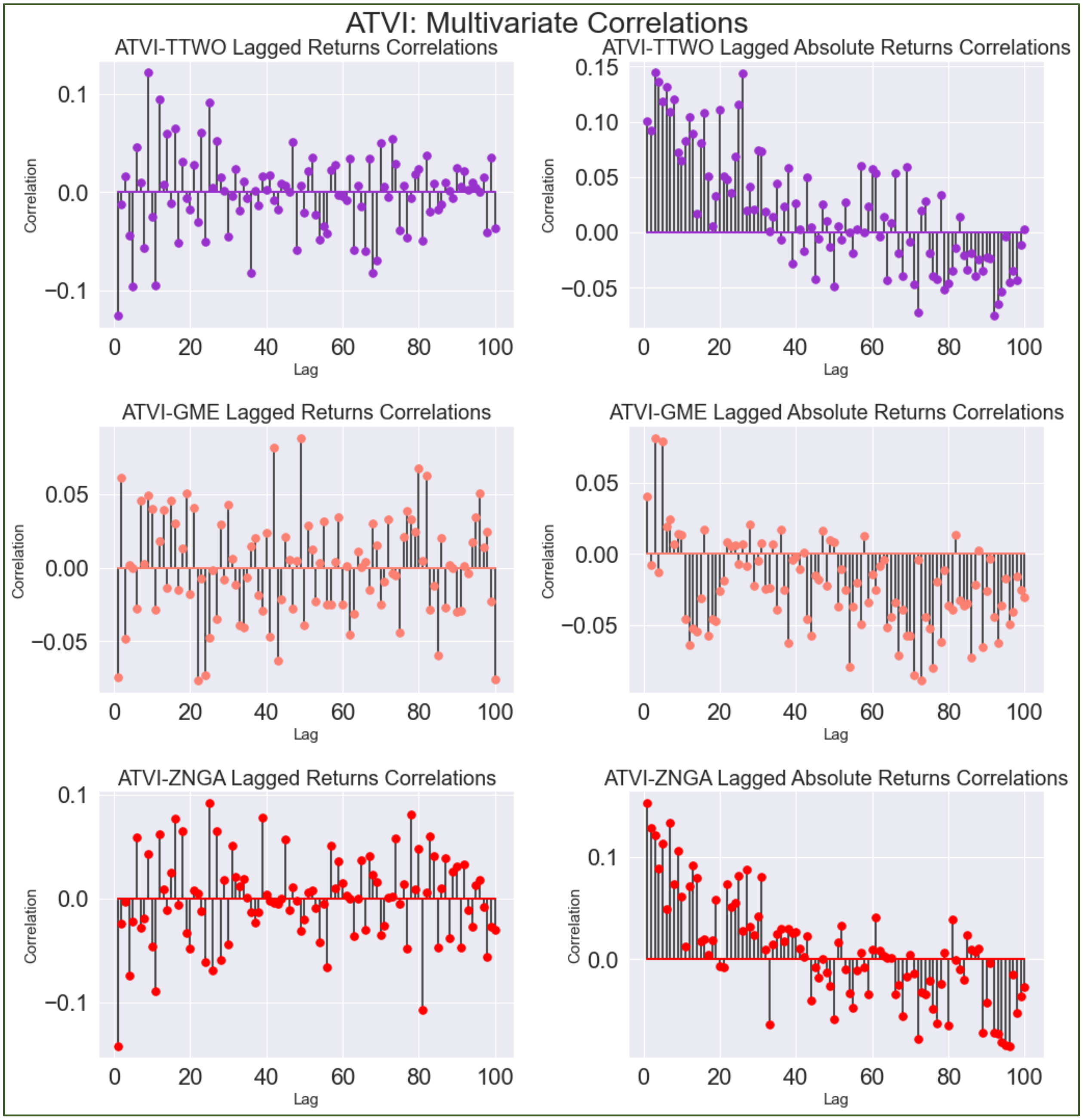
$$\hat{\rho}_{xy}^{\tau} = Corr\left(|R_t^x|, |R_{t-\tau}^y|\right)$$

Where $x \neq y$ and $\tau = 1, 2, \ldots, 100$ (i.e., the test is run for up to 100 days lagged returns).

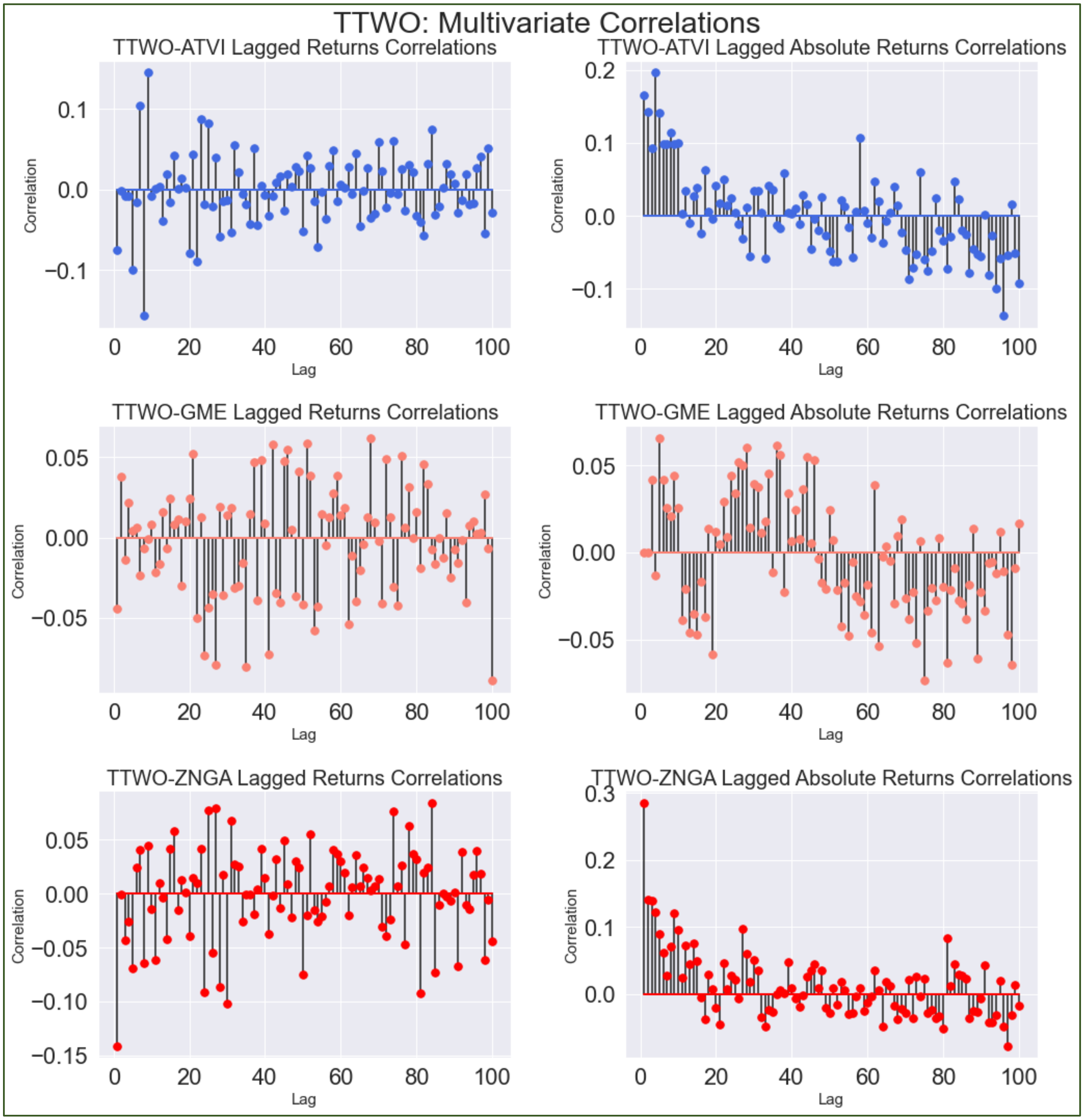
The correlations vs lag plots are then plotted for each stock to check for significant correlations and patterns.

Test Result:

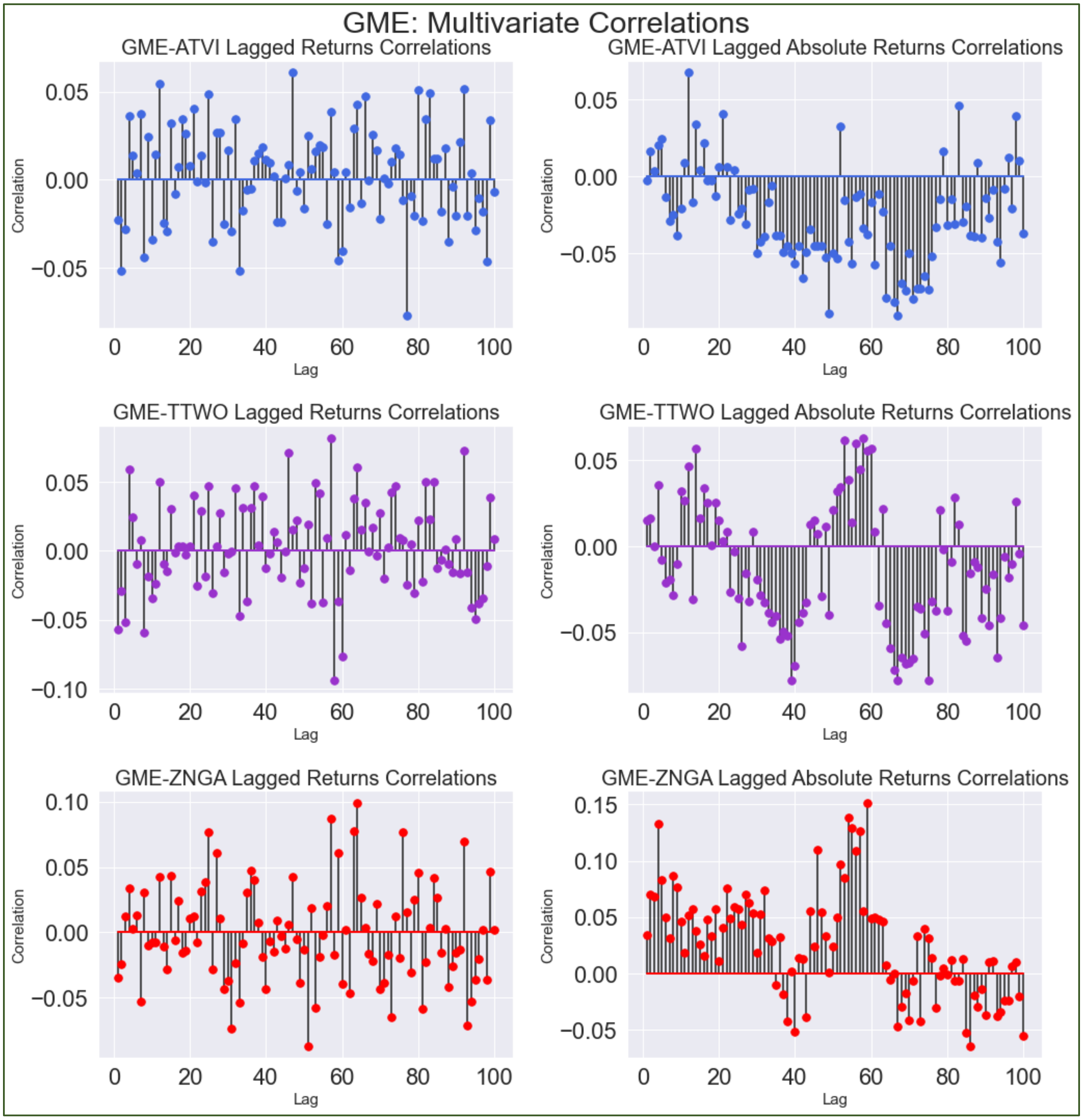
(Results in the next page)

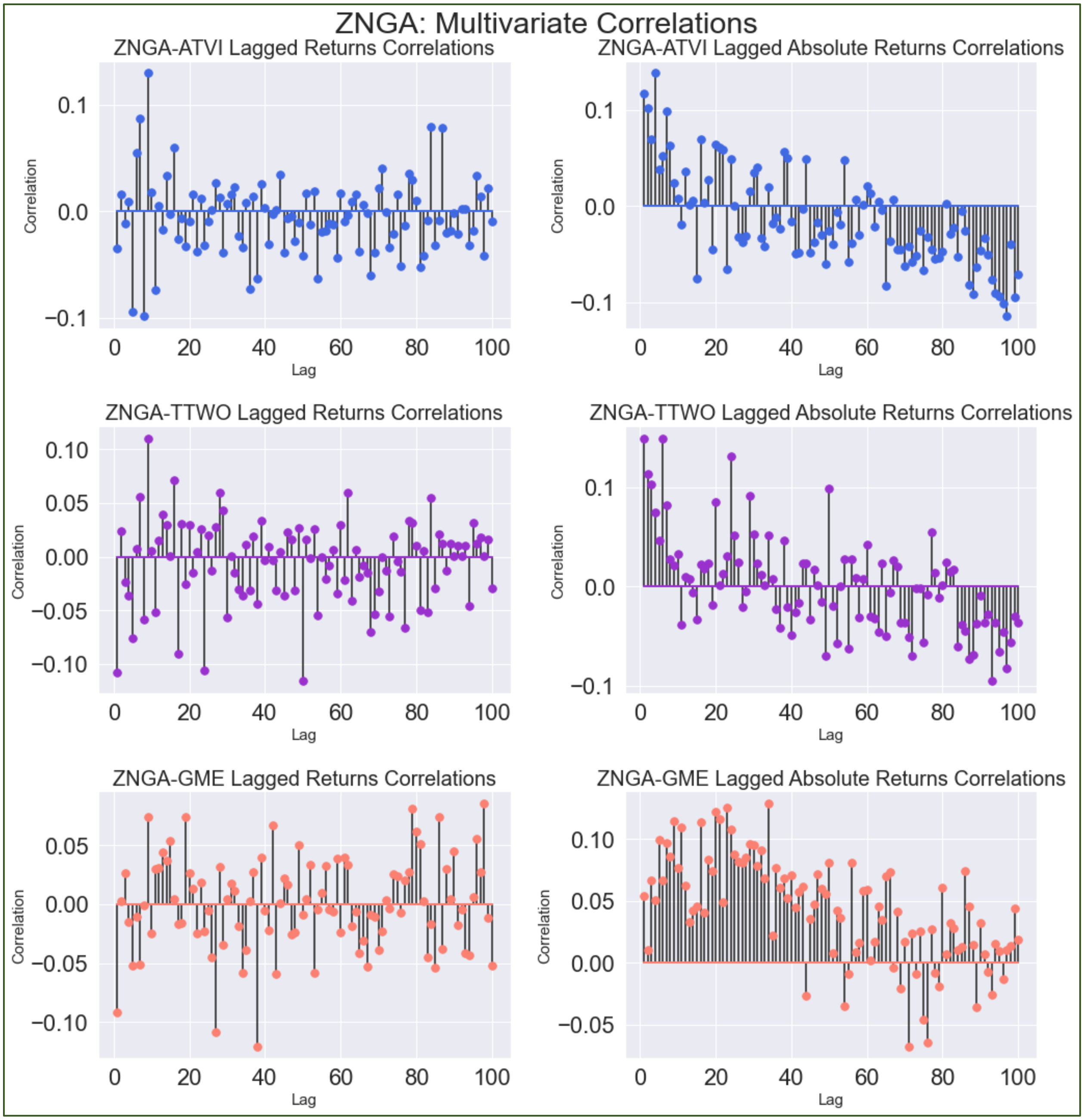


ATVI: Multivariate Correlations
ATVI-TTWO Lagged Returns Correlations
ATVI-TTWO Lagged Absolute Returns Correlations
ATVI-GME Lagged Returns Correlations
ATVI-GME Lagged Absolute Returns Correlations
ATVI-ZNGA Lagged Returns Correlations
ATVI-ZNGA Lagged Absolute Returns Correlations
Correlation
Lag



TTWO: Multivariate Correlations
TTWO-ATVI Lagged Returns Correlations
TTWO-ATVI Lagged Absolute Returns Correlations
TTWO-GME Lagged Returns Correlations
TTWO-GME Lagged Absolute Returns Correlations
TTWO-ZNGA Lagged Returns Correlations
TTWO-ZNGA Lagged Absolute Returns Correlations
Correlation
Lag


GME: Multivariate Correlations
GME-ATVI Lagged Returns Correlations
GME-ATVI Lagged Absolute Returns Correlations
GME-TTWO Lagged Returns Correlations
GME-TTWO Lagged Absolute Returns Correlations
GME-ZNGA Lagged Returns Correlations
GME-ZNGA Lagged Absolute Returns Correlations
Correlation
Lag



Observations:

The lagged correlations for returns are generally in [-0.15,0.15] and do not show any pattern.

The lagged correlations for absolute returns turn out to be positive for lag $\tau = 1$ and then generally move towards zero as the lag increases.

To check if the correlations vary with time, we slice the three-year timeseries data for all the stocks into 6 parts and compute the correlation matrix for the 6 non-overlapping parts and high changes in correlations along the slices are marked in amber.

Test Result:

Slice 1 Correlations:

Stock	ATVI	TTWO	GME	ZNGA
ATVI	1.0000	0.6289	0.0355	0.5232
TTWO	0.6289	1.0000	0.0052	0.4443
GME	0.0355	0.0052	1.0000	-0.0516
ZNGA	0.5232	0.4443	-0.0516	1.0000

Slice 2 Correlations:

Stock	ATVI	TTWO	GME	ZNGA
ATVI	1.0000	0.7064	0.2505	0.4326
TTWO	0.7064	1.0000	0.1570	0.4527
GME	0.2505	0.1570	1.0000	0.0796
ZNGA	0.4326	0.4527	0.0796	1.0000

Slice 3 Correlations:

Stock	ATVI	TTWO	GME	ZNGA
ATVI	1.0000	0.5482	0.1196	0.4693
TTWO	0.5482	1.0000	0.0158	0.5155
GME	0.1196	0.0158	1.0000	0.0781
ZNGA	0.4693	0.5155	0.0781	1.0000

Slice 4 Correlations:

Stock	ATVI	TTWO	GME	ZNGA
ATVI	1.0000	0.4696	0.2125	0.4896
TTWO	0.4696	1.0000	0.2799	0.5065
GME	0.2125	0.2799	1.0000	0.1108
ZNGA	0.4856	0.5065	0.1108	1.0000

Slice 5 Correlations:

Stock	ATVI	TTWO	GME	ZNGA
ATVI	1.0000	0.7523	0.0854	0.7027
TTWO	0.7523	1.0000	0.2205	0.6053
GME	0.0854	0.2205	1.0000	0.1432
ZNGA	0.7027	0.6053	0.1432	1.0000

Slice 6 Correlations:

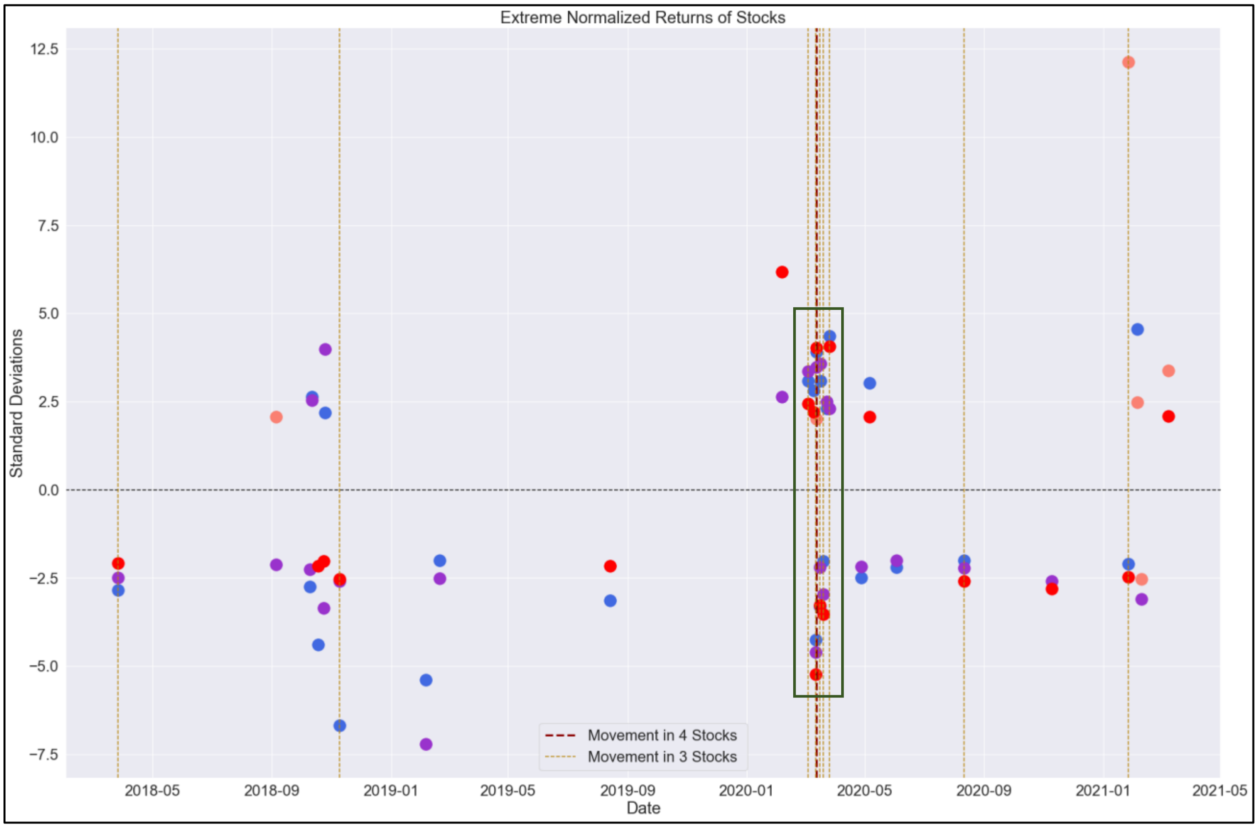
Stock	ATVI	TTWO	GME	ZNGA
ATVI	1.0000	0.6373	-0.1346	0.5547
TTWO	0.6373	1.0000	-0.0968	0.3409
GME	-0.1346	-0.0968	1.0000	-0.0946
ZNGA	0.5547	0.3409	-0.0946	1.0000

Observations:

The correlation matrix changes along the 6 slices of the time series suggesting that the correlation among stocks varies with time.

Extreme Returns Occurrence Test:

The return data for all the stocks is normalized using the historical mean and the standard deviation of returns. A normalized return Z is defined to be extreme if |Z| > 2 as two-standard deviations cover 95% of a normally distributed data (the definition is arbitrary). Such occurrences are filtered and plotted.



Scatter Points Legend:

ATVI: ▆ TTWO: ▆ GME: ▆ ZNGA: ▆

Observations:

Boxed area in graph indicates that there was a period during which there were multiple instances of extreme returns occurring in different stocks at the same time. This period is specifically during March 2020, when the 2020 stock market crash (Coronavirus Crash) occurred.

Observation	Number of Samples/Dates	Percentage
Total number of Return data points	757	100%
Number of dates when **at least two** stocks shoe extreme returns	30	3.96%

Number of dates when only **two** stocks show extreme returns	20	2.64%
Number of dates when **three** stocks show extreme returns	9	1.19%
Number of dates when all **four** stocks show extreme returns	1	0.13%

PART 2: Choice of the portfolio of stocks

Part 1: Portfolio Optimization

"Today" Date: 12th March 2021

You are given 100 MM dollars to invest in stock market hoping for a positive return with investment horizon of 6 months. We initially test whether the stocks have a drift using return data for 3 year and 6 months. We calculate the mean return for these horizons and check if it is statistically significant from 0.

For the 3-year timeseries, the test has already been performed for showing the stylized fact: The standard deviation of returns completely dominates the mean of returns at short horizons: it is impossible to reject statistically zero mean.

The results for the 6 months horizon are shown below:

6 month trend	ATVI	TTWO	GME	ZNGA
Average Return	0.121%	0.078%	2.618%	0.209%
Variance of Return	0.00031	0.00038	0.03663	0.00052
95% lower bound (based on sample mean and standard deviation of return)	-3.421%	-3.814%	-35.660%	-4.360%
Presence of upward drift	No	No	No	No

The tests failed to conclude above zero returns at 95% confidence. However, all the stocks have average positive daily returns, with GME at the highest at 2.6% and earns more than the risk-free rate of 0.06% p.a.

As the stocks earn more than the risk-free rate on an average, we decided to go with a minimum variance portfolio to reduce the risk. If the weights of stocks in the portfolio is w and matrix of individual stock returns and covariance matrix of the stocks are denoted by μ and ∑ respectively, then the portfolio expected return and standard deviation are given by:

$$E[r_{portfolio}] = w^T \mu$$

$$\sigma_{portfolio} = \sqrt{w^T \Sigma w}$$

We select portfolio weights vector 'w' such that $\sigma_{portfolio}$ is minimized. The mean vector μ (daily return) and the covariance matrix ∑ for our portfolio of four stocks are given by (computed using 3-year timeseries):

μ vector:

Stock	ATVI	TTWO	GME	ZNGA
Mean Return	0.0484%	0.0727%	0.2960%	0.1570%

∑ covariance:

Stock/Stock	ATVI	TTWO	GME	ZNGA
ATVI	0.0004607	0.0003114	0.0000421	0.0002334
TTWO	0.0003114	0.0004946	0.0000670	0.0002175

GME	0.0000421	0.0000670	0.0078811	0.0000010
ZNGA	0.0002334	0.0002175	0.0000010	0.0004234

The optimal weights are:

ATVI	TTWO	GME	ZNGA
0.2654	0.2504	0.0353	0.4488

The minimum portfolio Expected Return and Standard Deviation (daily) are 0.112% and 1.751% respectively.

Total Portfolio Value: $100 MM

The number of stocks for a stock 'x' with weight w_x and price P_x in the portfolio is given by:

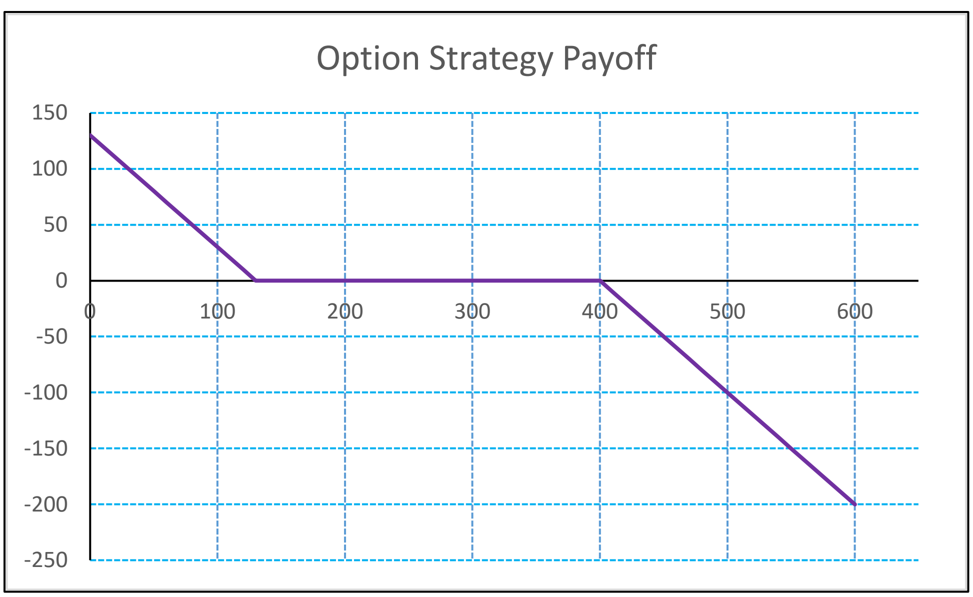
$$N_x = \frac{w_x V}{P_x}$$

The number of stocks in the portfolio is:

Stock	Price	Weight	Number of Stocks
ATVI	93.04	0.2654	285,294.66
TTWO	170.25	0.2504	147,095.86
GME	264.5	0.0353	13,342.45
ZNGA	10.16	0.4488	4,417,720.00

Part 2: Adding Options to Portfolio

The option strategy for our portfolio is a zero-cost collar on our most volatile portfolio stock, GME (volatility highlighted in red in the covariance matrix computed in sub-part 1). To do so, we used market option prices to find two out of the money options, a call and a put with the same price (We used mid-price for the sake of the assignment). By going long the put and shorting the call, we hedge against unpredictable drops in the stock price which is important, seeing that GME has been extremely volatile due to squeeze manipulations. The cost of setting up the strategy is also zero as the price is the same for both the call and put option. The tradeoff is that we cap our gains.



After doing so, we calculated Option Greeks on the options we used.

The details for the option are provided below:

The risk-free rate is taken from the yield of the US Treasury bill maturing in approximately 6 months i.e., 0.06%

Parameter	Call	Put
Underlying Stock	GME	GME
Price	50.225	50.225
Strike	400	130
Today Date	12th March, 2021	12th March, 2021
Contract Maturity Date	15th October, 2021	15th October, 2021
Implied Volatility	160.41%	177.56%
Dividend Yield	0.80%	0.80%

(Dividend Yield for GME sourced from: https://a2-finance.com/en/issuers/gamestop/dividends)

We use the Black Scholes model to compute the greeks (Delta and Vega) of the chosen options:

$$Delta\ (\Delta) = \ e^{-qT}\Phi(d_1)$$

$$Vega\ (\nu) = Se^{-qT}\phi(d_1)\sqrt{T}$$

The Sensitivities for one option are as follows:

Sensitivity	Call	Put
Delta	0.57664	-0.11654
Vega	0.75806	1.50881

The number of options to use in the strategy is determined such that the delta of portfolio with respect to GME is made to zero. Let the number of options to be used be 'N'.

$$13342.45 * (1) + N(-0.57664 - 0.11654) = 0$$

$$N = 19{,}248.24$$

The Final Portfolio is:

Component	Number	Price	Total Value
ATVI Stock	285,294.66	93.04	26,543,815.48
TTWO Stock	147,095.86	170.25	25,043,071.00
GME Stock	13,342.45	264.5	3,529,078.31
ZNGA Stock	4,417,720.00	10.16	44,884,035.21
GME short call @ 400	-19,248.24	50.225	-966,742.86
GME long put @ 130	19,248.24	50.225	966,742.86
		Total Portfolio Value	**100,000,000.00**

The Delta's with respect to each stock is given in the table below:

Stock	Portfolio Delta w.r.t Stock
ATVI	285,294.66

TTWO	147,095.86
GME	0
ZNGA	4,417,720.00

PART 3: Measuring the Risk of the Portfolio

Part 1: Stress Tests

The stress test is performed by constructing a two-dimensional grid of volatility and prices:

$$\Delta\sigma = [-10\%, -5\%, 0\%, +5\%, +10\%]$$

$$\frac{\Delta S}{S} = [-40\%, -30\%, \dots, +30\%, +40\%]$$

The portfolio loss is computed using the sensitivities computed in Part 2: Choice of the portfolio of stocks. The sensitivities are presented again below:

Stock	Portfolio Delta w.r.t Stock (δ)
ATVI	285,294.66
TTWO	147,095.86
GME	0
ZNGA	4,417,720.00

Option	Portfolio Vega (v)
GME short call @ 400	-14,591.24
GME long put @ 130	29,042.02
Total Portfolio Vega w.r.t GME volatility	14,450.78

The portfolio loss in computed in the following manner using the sensitivities:

$$PnL = -\left[\sum_x \delta_x(\Delta S_x) + \sum_x \nu_x(\Delta\sigma_x)\right]_{x\,\epsilon\,\{ATVI,TTWO,GME,ZNGA\}}$$

All stock prices and all volatilities are shocked by the same amount.

The PnL values for different scenarios are presented below:

Column: Δσ Row: ΔS/S	-10%	-5%	0%	5%	10%
-40%	9,652,872.48	4,829,326.40	5,780.31	(4,817,765.77)	(9,641,311.86)
-30%	9,651,427.40	4,827,881.32	4,335.23	(4,819,210.85)	(9,642,756.94)
-20%	9,649,982.32	4,826,436.24	2,890.16	(4,820,655.93)	(9,644,202.01)
-10%	9,648,537.25	4,824,991.16	1,445.08	(4,822,101.01)	(9,645,647.09)
0%	9,647,092.17	4,823,546.08	-	(4,823,546.08)	(9,647,092.17)
+10%	9,645,647.09	4,822,101.01	(1,445.08)	(4,824,991.16)	(9,648,537.25)
+20%	9,644,202.01	4,820,655.93	(2,890.16)	(4,826,436.24)	(9,649,982.32)
+30%	9,642,756.94	4,819,210.85	(4,335.23)	(4,827,881.32)	(9,651,427.40)

+40%	9,641,311.86	4,817,765.77	(5,780.31)	(4,829,326.40)	(9,652,872.48)

Assuming equal weights $\omega_i = 1/N_{sc}$, N_{sc} being the number of scenarios i.e., 45. We calculated the risk as the maximum weighted loss:

$$L_{sc} = \max_i(\omega_i L_i)$$

The maximum loss (L) in our scenarios is **$9,652,872.48** (Scenario where the volatility decreases by 10% and the stocks decrease by a factor of 40%) and therefore the **maximum weighted loss** is **$214,508.28.**

Part 2: 1-day 95% VaR and ES

Linear Approximation and Joint Multivariate Distribution assumption:

The Stocks component in the Portfolio is given as follows:

Stock	Price	Number of Shares	Total Value
ATVI	93.04	285,294.66	$26,543,815.48
TTWO	170.25	147,095.86	$25,043,071.00
GME	264.5	13,342.45	$3,529,078.31
ZNGA	10.16	4,417,720.00	$44,884,035.21

For using the Linear Approximation and Joint Multivariate Distribution Assumptions to compute the Portfolio 95% VaR and Expected Shortfall, we initially convert the options in the portfolio to their equivalent Shares by using the option deltas.

$$Number\ of\ Equivalent\ Shares = \ \delta * Number\ of\ Options$$

The calculations for converting the option positions into equivalent stock positions is:

Option	Price	Number of Options	Delta of Options	Number of Equivalent Shares	Value of Equivalent Shares
GME short call @ 400	50.225	19,428.24	-0.5766	-11,099.32	-2,935,769.32
GME long put @ 130	50.225	19,428.24	-0.1165	-2,243.13	-593,308.99

The Final Adjusted Portfolio positions and weights are given by:

Portfolio Component	Total Adjusted Value	Weight	Expected Daily Return	Expected Daily % Loss
ATVI	26,543,815.48	0.2751	0.0484%	-0.0484%
TTWO	25,043,071.00	0.2596	0.0727%	-0.0727%
GME	-	0	0.2960%	-0.2960%
ZNGA	44,884,035.21	0.4653	0.1570%	-0.1570%
Portfolio	96,470,921.69	1	0.1053%	-0.1053%

The Covariance Matrix of the Stock returns (which is also equal to the covariance matrix of individual stock losses) is calculated from the 3-year historical return timeseries.

Covariance Matrix	ATVI	TTWO	GME	ZNGA
ATVI	0.00046070	0.00031139	0.00004208	0.00023342
TTWO	0.00031139	0.00049461	0.00006703	0.00021753
GME	0.00004208	0.00006703	0.00788106	0.00000097
ZNGA	0.00023342	0.00021753	0.00000097	0.00042343

The Dollar Value of Mean μ_L and Standard deviation σ_L of portfolio loss form the Portfolio adjusted total value V, weight vector w, individual stock expected losses vector μ and stock loss covariance matrix ∑ is given by:

$$\mu_L = V * w^T \mu$$

$$\sigma_L = V * \sqrt{w^T \Sigma w}$$

And the 95% Var and Expected Shortfall are given by:

$$VaR_\alpha = \mu_L + \sigma_L \Phi^{-1}(\alpha)$$

$$ES_\alpha = \mu_L + \sigma_L \frac{\phi(\Phi^{-1}(\alpha))}{1-\alpha}$$

Where α = 95%. The 95% VaR and Expected Shortfall for our portfolio from the above formulae are:

95% VaR	**$2,722,169.37**
Expected Shortfall	**$3,439,505.10**

Historical Simulation:

The 95% VaR and Expected Shortfall is computed from the 4-year historical daily Price change timeseries and the deltas of the portfolio. The PnL of the portfolio from a historical scenario 'I' is given by:

$$PnL_i = -\sum_x \delta_x (\Delta P_x)_i$$

And ΔP_i is given by:

$$\Delta P_i = P_i - P_{i-1}$$

Where x ϵ {ATVI, TTWO, GME, ZNGA}

This gives a strip of PnL values computed from the 4-year time series. The 95th percentile loss from this strip is the 1-day 95% VaR and the average of losses greater than or equal to the 95% VaR is the Expected Shortfall.

Risk Metric	Value
95% VaR	**$1,775,266.54**
ES	**$3,045,408.09**