Correctness Guarantee for the Composition of Adaptive Cruise Control and Lane Keeping and Adaptive Cruise Control

X. Xu, J.W. Grizle, P. Tabuada, and A.D. Ames

Presented by: Sergio García-Vergara

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What are Barrier Certificates?

- Also known as barrier functions, are used to verify temporal properties of a set without having to compute the system's reachable set.
- Recall **Lyapunov functions**: in control theory, these are linear functions used to prove the stability of a dynamical system.
- Let $\dot{x} = f(x)$ be the dynamics of a system with an equilibrium point at $x = 0$. The existence of a function $\boldsymbol{V(x) > 0 \, \forall x \neq 0}$ such that $\boldsymbol{\dot{V}(x) \leq 0}$, guarantees the stability of the equilibrium point (asymptotical stability if $\dot{V}(x) < 0$).

What are Barrier Certificates?

- Are used to design a family of control solutions that guarantee some nice properties about a set. Namely, **its forward invariance**.
- Given dynamics $\dot{x} = f(x)$ and a trajectory $x(t, x_o)$, set S is called *forward invariant* if:

$$\boxed{\text{for every } x_o \in S \text{ implies that } \boldsymbol{x(t, x_o) \in S \; \forall t = 0}.}$$

- Nice because we can define a *safety set* and guarantee that our system will always meet its constraints.

Definition: Barrier Certificates

- Consider a system with dynamics $\dot{x} = f(x)$ and a closed set

$$C = \{x \in \mathbb{R}^n \mid h(x) \geq 0\}$$

for some continuously differentiable function $h : \mathbb{R}^n \mapsto \mathbb{R}$.

- If there exists a constant $\gamma > 0$ and a set D with $C \subseteq D \subset \mathbb{R}^n$ such that:

$$\dot{\boldsymbol{h}}(\boldsymbol{x}) \geq -\gamma \boldsymbol{h}(\boldsymbol{x}), \ \forall \boldsymbol{x} \in \boldsymbol{D}$$

then:

the function $h(x)$ is called a **barrier function**.

General Theorem

- Consider an affine control system:

$$\dot{x} = f(x) + g(x)u$$

  with $x \in \mathbb{R}^n$ and $u \in \mathbb{R}^m$.

- Given a set $C = \{x \in \mathbb{R}^n \mid h(x) \geq 0\}$ for a continuously differentiable function h. If h is a control barrier function then any controller $\boldsymbol{u(x)} \in \boldsymbol{K_{zcbf}(x)}$ will render the set C **forward invariant**.

$$\boxed{K_{zcbf}(x) = \{u \in U \mid L_f h(x) + L_g h(x)u + \gamma h(x) \geq 0\}}$$

Problem Formulation

- Control objectives:
  - **LK**: Control the steering of an autonomous car to maintain lane centering.
  - **ACC**: maintain a safe following distance when a preceeding vehicle is driving at a lower speed.
- Safety constraints for LK and ACC are expressed in terms of set invariance.
- Controlled invariant sets are used to encode both the correct behavior of the closed-loop system and a set of feedback control laws that will achieve it.

Lane Keeping: Dynamics

- Lateral-yaw model:

$$\dot{\boldsymbol{x}}_1 = f_1(\boldsymbol{x}_1, v_f) + g_1(\boldsymbol{x}_1)u_1 + \Delta f_1(d)$$
$$\boldsymbol{x}_1 := (y, v, \Delta\Psi, r)'$$

  - y: lateral displacement from the center of the lane
  - v: lateral velocity
  - $\Delta\Psi$: yaw angle deviation in road-fixed coordinates
  - r: yaw rate

  - $u_1 = \delta_f$: steering angle of the front wheels
  - d: desired yaw rate (computed from road curvature)

Lane Keeping: Safety Constraints

- Main constraint: Keep the car within its lane (i.e. constrain the lateral displacement):

$$|y| \leq y_m$$

- Set of hard constraints for LK:

$$\boxed{\begin{aligned}\mathcal{X}_{LK} :=\{&\boldsymbol{x}_1 \in \mathbb{R}^4 \mid \\ &|y| \leq y_m, |v| \leq v_m, |\Delta\Psi| \leq \Delta\Psi_m, |r| \leq r_m\}\end{aligned}}$$

- Optional soft constraint: set an upper bound for the lateral acceleration to respect the driver's comfort.

Adaptive Cruise Control: Dynamics

- Point-mass model:

$$\dot{\boldsymbol{x}}_2 = f_2(\boldsymbol{x}_2) + g_2(\boldsymbol{x}_2)u_2 + \Delta f_2(vr, a_L)$$
$$\boldsymbol{x}_2 := (v_f, v_l, D)'$$

  - v_f: following car's speed
  - v_l: lead car's speed
  - D: distance between the two cars

  - $u_2 = F_w$: longitudinal force developed by the wheels
  - F_r: aerodynamic drag
  - a_L: overall acceleration/deceleration of the lead car

Adaptive Cruise Control: Safety Constraints

- Main constraint: the controlled vehicle should maintain a safe distance from the lead car. Paper uses this formulation:

$$\boxed{D \geq \tau_d v_f + D_o}$$

where τ_d is the desired time headway and D_o is the minimal distance between cars when they are fully stopped.

- Soft constraint: achieve a desired speed v_d set by the driver:

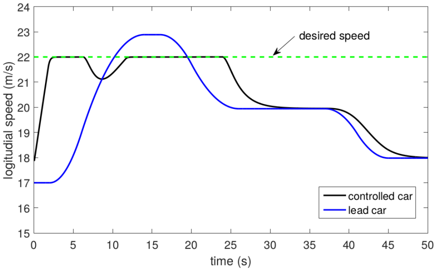
$$\lim_{t\to\infty} v_f(t) - v_d = 0$$

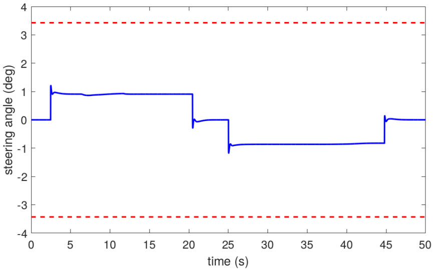
Methodology

- **GOAL:** Barrier certificates seek a function whose sub-level sets are all invariant, without the difficult task of computing the system's reachable set.

1. Construct a controlled invariant set that encodes the safety specifications.
2. Construct a feedback law that ensures that trajectories of the controlled system are confined within the set.

Simulation Results





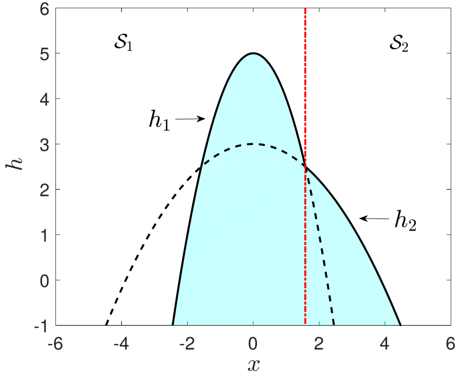
Conclusions

- Longitudinal force and steering angle are generated by solving quadratic programs.
- The safety constraints are hard constraints that are enforced by confining the states of the vehicle within determined controlled-invariant sets.
  - The performance objectives are soft constraints that can be overridden when they are in conflict with safety.
- Any control laws respecting the contracts given for LK and ADD will guarantee safety of the closed-loop system when the two modules are activated simultaneously.

Questions?

Thank You!

Example: Barrier Certificates



- Functions: $h_1(x) = -x^2 + 5$ and $h_2(x) = -0.2x^2 + 3$
- Partitions: $S_1 = (-\infty, \sqrt{10}/2]$ and $S_2 = [\sqrt{10}/2, \infty)$

$$\boxed{\text{Safe set: } C := \{x | h(x) \geq 0\} \text{ is the shaded area.}}$$

Example: Barrier Certificates

Algorithm 1 Synthesis of Control Barrier Functions for LK

Input: $y_m, \nu_m, \Delta\psi_m, r_m, \hat{\delta}_f, d_{\max}, \bar{v}, \underline{v}, Q, R, \gamma, \varepsilon, \rho_0, p, \alpha, \beta$
Output: $\kappa, \hat{h}_{lk}(\mathbf{x}_1), u(\mathbf{x}_1, d, v_f)$

Solve for the LQR gain K and solve $(\mathcal{P}_0)$
while $(\mathcal{P}_0)$ is not feasible **do**
    Modify Q, R, γ, ρ_0 and solve $(\mathcal{P}_0)$
end while
converged = false
while $\neg$ converged **do**
    Fix $\hat{h}_{lk}$, find u, s_i, κ and maximize κ by solving $(\mathcal{P}_1)$
    Fix u, find $\hat{h}_{lk}, s_i, \kappa$ and maximize κ by solving $(\mathcal{P}_2)$
    if $|\kappa^{new} - \kappa^{old}| \leq$ some threshold **then**
        converged = true
    end if
end while

Figure: Algorithm for the Synthesis of Control Barrier Functions for LK.