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Sean Gasiorowski

$HH \rightarrow b\bar{b}b\bar{b}$ or How I Learned to Stop Worrying and Love the QCD Background

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Abstract

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Insert abstract here

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GLOSSARY

388 ARGUMENT: replacement text which customizes a L^AT_EX macro for each particular usage.

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390 Five years is both a short time and a long time – many things have happened and many
391 have stayed the same. I certainly know much more physics than I did at the outset, but also
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415 squad, for a huge amount of hard work on the non-resonant, and of course the $HH \rightarrow 4\text{beers}$
416 team, many of whom have already been mentioned, but who deserve an extra shout out for
417 keeping things fun even during stressful times.

418 The physics is done, the rest is paperwork. Let us begin.

419

DEDICATION

420

To family, both given and found

421

Chapter 1

422

THE STANDARD MODEL OF PARTICLE PHYSICS

423 The Standard Model of Particle Physics (SM) is a monumental historical achievement,
 424 providing a formalism with which one may describe everything from the physics of everyday
 425 experience to the physics that is studied at very high energies at the Large Hadron Collider
 426 (Chapter 3). In this chapter, we will provide a brief overview of the pieces that go into the
 427 construction of such a model. The primary focus of this thesis is searches for pair production
 428 of Higgs bosons decaying to four b -quarks. Consequently, we will pay particular attention
 429 to the relevant pieces of the Higgs Mechanism, as well as the theory behind searches at a
 430 hadronic collider.

431 **1.1 Introduction: Particles and Fields**

432 What is a particle? The Standard Model describes a set of fundamental, point-like, objects
 433 shown in Figure 1.1. These objects have distinguishing characteristics (e.g., mass and spin).
 434 These objects interact in very specific ways. The set of objects and their interactions result
 435 in a set of observable effects, and these effects are the basis of a field of experimental physics.

436 The effects of these objects and their interactions are familiar as fundamental forces:
 437 electromagnetism (photons, electrons), the strong interaction (quarks, gluons), the weak
 438 interaction (neutrinos, W and Z bosons). Gravity is not described in this model, as the
 439 weakest, with effects most relevant on much larger distance scales than the rest. However,
 440 the description of these other three is powerful – verifying and searching for cracks in this
 441 description is a large effort, and the topic of this thesis.

442 The formalism for describing these particles and their interactions is that of quantum field
 443 theory. Classical field theory is most familiar in the context of, e.g., electromagnetism – an

444 electric field exists in some region of space, and a charged point-particle experiences a force
445 characterized by the charge of the point-particle and the magnitude of the field at the location
446 of the point-particle in spacetime. The same language translates to quantum field theory.
447 Here, particles are described in terms of quantum fields in some region of spacetime. These
448 fields have associated charges which describe the forces they experience when interacting
449 with other quantum fields. Most familiar is electric charge – however this applies to e.g., the
450 strong interaction as well, where quantum fields have an associated *color charge* describing
451 behavior under the strong force.

452 Particles are observed to behave in different ways under different forces. These behaviors
453 respect certain *symmetries*, which are most naturally described in the language of group
454 theory. The respective fields, charges, and generators of these symmetry groups are the basic
455 pieces of the SM Lagrangian, which describes the full dynamics of the theory. In the following,
456 we will build up the basic components of this Lagrangian. The treatment presented here relies
457 heavily on Jackson's Classical Electrodynamics [2] for the build-up, and Thomson's Modern
458 Particle Physics [3] for the rest, with reference to Srednicki's Quantum Field Theory [4], and
459 some personal biases and interjections.

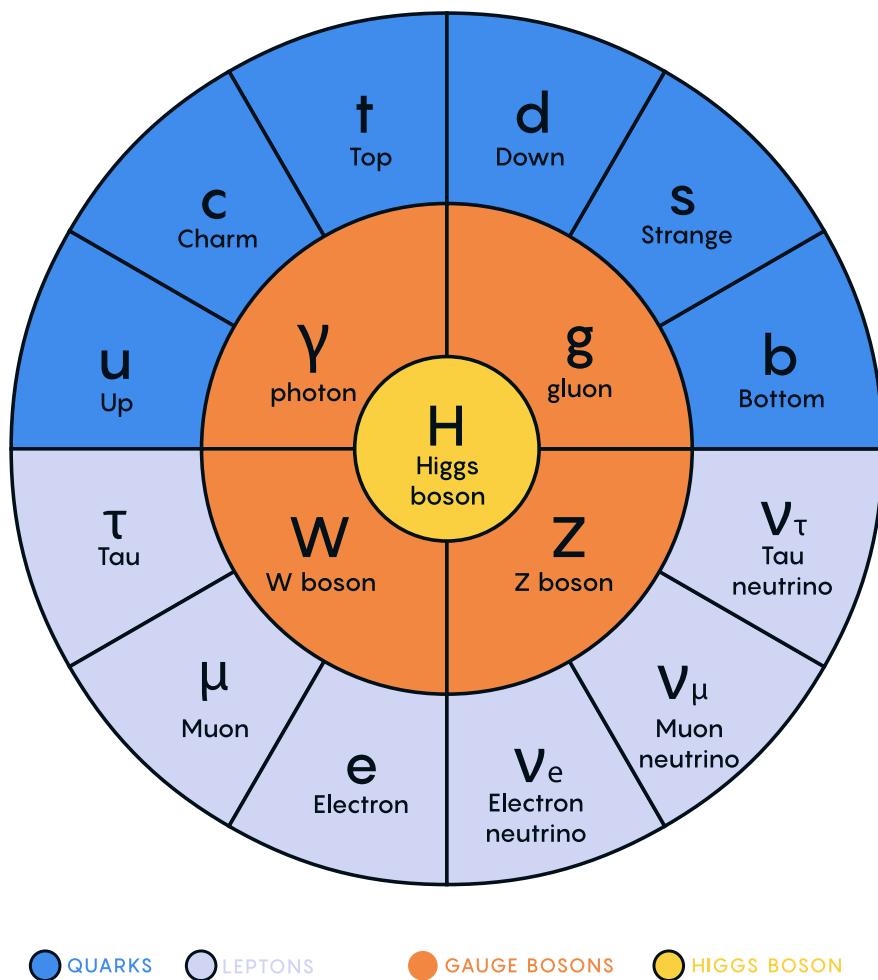


Figure 1.1: Diagram of the elementary particles described by the Standard Model [1].

⁴⁶⁰ **1.2 Quantum Electrodynamics**

Classical electrodynamics is familiar to the general physics audience: electric (\vec{E}) and magnetic (\vec{B}) fields are used to describe behavior of particles with charge q moving with velocity \vec{v} , with forces described as $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$. Hints at some more fundamental properties of electric and magnetic fields come via a simple thought experiment: in a frame of reference moving along with the particle at velocity \vec{v} , the particle would appear to be standing still, and therefore have no magnetic force exerted. Therefore a *relativistic* formulation of the theory is required. This is most easily accomplished with a repackaging: the fundamental objects are no longer classical fields but the electric and magnetic *potentials*: ϕ and \vec{A} respectively, with

$$\vec{E} = -\nabla\phi - \frac{\partial\vec{A}}{\partial t} \quad (1.1)$$

$$\vec{B} = \nabla \times \vec{A} \quad (1.2)$$

It is then natural to fully repackage into a relativistic *four-vector*: $A^\mu = (\phi, \vec{A})$. Considering $\partial^\mu = (\frac{\partial}{\partial t}, \nabla)$, the x components of these above two equations become:

$$E_x = -\frac{\partial\phi}{\partial x} - \frac{\partial A_x}{\partial t} = -(\partial^0 A^1 - \partial^1 A^0) \quad (1.3)$$

$$B_x = \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} = -(\partial^2 A^3 - \partial^3 A^2) \quad (1.4)$$

⁴⁶¹ where we have used the sign convention $(+, -, -, -)$, such that $\partial^\mu = (\frac{\partial}{\partial x_0}, -\nabla)$.

This is naturally suggestive of a second rank, antisymmetric tensor to describe both the electric and magnetic fields (the *field strength tensor*), defined as:

$$F^{\alpha\beta} = \partial^\alpha A^\beta - \partial^\beta A^\alpha \quad (1.5)$$

Defining a four-current as $J_\mu = (q, \vec{J})$, with q standard electric charge, \vec{J} standard electric current, conservation of charge may be expressed via the continuity equation

$$\partial_\mu J^\mu = 0 \quad (1.6)$$

and all of classical electromagnetism may be packaged into the Lagrangian density:

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - J^\mu A_\mu. \quad (1.7)$$

462 This gets us partway to our goal, but is entirely classical - the description is of classical
 463 fields and point charges, not of quantum fields and particles. To reframe this, let us go back
 464 to the zoomed out view of the particles of the Standard Model. Two of the most familiar
 465 objects associated with electromagnetism are electrons: spin-1/2 particles with charge e , mass
 466 m , and photons: massless spin-1 particles which are the "pieces" of electromagnetic radiation.

467 We know that electrons experience electromagnetic interactions with other objects. Given
 468 this, and the fact that such interactions must be transmitted *somewhat* between e.g. two
 469 electrons, it seems natural that these interactions are facilitated by electromagnetic radiation.
 470 More specifically, we may think of photons as *mediators* of the electromagnetic force. It
 471 follows, then, that a description of electromagnetism on the level of particles must involve a
 472 description of both the "source" particles (e.g. electrons), the mediators (photons), and their
 473 interactions. Further, this description must be (1) relativistic and (2) consistent with the
 474 classically derived dynamics described above.

The beginnings of a relativistic description of spin-1/2 particles is due to Paul Dirac, with the famous Dirac equation:

$$(i\gamma^\mu \partial_\mu - m)\psi = 0 \quad (1.8)$$

where ∂_μ is as defined above, ψ is a Dirac *spinor*, i.e. a four-component wavefunction, m is the mass of the particle, and γ^μ are the Dirac gamma matrices, which define the algebraic structure of the theory. For the following, we also define a conjugate spinor,

$$\bar{\psi} = \psi^\dagger \gamma^0 \quad (1.9)$$

which satisfies the conjugate Dirac equation

$$\bar{\psi}(i\gamma^\mu \partial_\mu - m) = 0 \quad (1.10)$$

475 where the derivative acts to the left.

The Dirac equation is the dynamical equation for spin-1/2, but we'd like to express these dynamics via a Lagrangian density. Further, to have a relativistic description, we'd like to

have this be density be Lorentz invariant. These constraints lead to a Lagrangian of the form

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi \quad (1.11)$$

⁴⁷⁶ where the Euler-Lagrange equation exactly recovers the Dirac equation.

The question now becomes how to marry the two Lagrangian descriptions that we have developed. Returning for a moment to classical electrodynamics, we know that the Hamiltonian for a charged particle in an electromagnetic field is described by

$$H = \frac{1}{2m}(\vec{p} - q\vec{A})^2 + q\phi. \quad (1.12)$$

Comparing this to the Hamiltonian for a free particle, we see that the modifications required are $\vec{p} \rightarrow \vec{p} - q\vec{A}$ and $E \rightarrow E - q\phi$. Using the canonical quantization trick of identifying \vec{p} with operator $-i\nabla$ and E with operator $i\frac{\partial}{\partial t}$, this identification becomes

$$i\partial_\mu \rightarrow i\partial_\mu - qA_\mu \quad (1.13)$$

Allowing for the naive substitution in the Dirac Lagrangian:

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu(\partial_\mu + iqA_\mu) - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}. \quad (1.14)$$

⁴⁷⁷ where the source term may be interpreted as coming from the Dirac fields themselves, namely,

⁴⁷⁸ $-q\bar{\psi}\gamma^\mu\psi A_\mu$.

Setting $q = e$ here (as appropriate for the case of an electron), and defining $D_\mu \equiv \partial_\mu + ieA_\mu$, this may then be written in the form

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}. \quad (1.15)$$

⁴⁷⁹ which is exactly the quantum electrodynamics Lagrangian.

⁴⁸⁰ We have swept a few things under the rug here, however. Recall that the general form
⁴⁸¹ of a Lagrangian is conventionally $\mathcal{L} = T - V$, where T is the kinetic term, and thus ought
⁴⁸² to contain a derivative with respect to time (c.f. the standard $\frac{1}{2}m\frac{\partial x}{\partial t}$ familiar from basic
⁴⁸³ kinematics). More particularly, given the definition of conjugate momentum as $\partial\mathcal{L}/\partial\dot{q}$ for

484 $\mathcal{L}(q, \dot{q}, t)$ and $\dot{q} = \frac{\partial q}{\partial t}$, any field q which has no time derivative in the Lagrangian has 0
485 conjugate momentum, and thus no dynamics.

486 Looking at this final form, there is an easily identifiable kinetic term for the spinor fields
487 (just applying the D_μ operator). However trying to identify something similar for the A fields,
488 one comes up short – the antisymmetric nature of $F^{\mu\nu}$ term means that there is no time
489 derivative applied to A^0 .

490 What does this mean? A^μ is a four component object, but it would appear that only three
491 of the components have dynamics: we have too many degrees of freedom in the theory. This
492 is the principle behind *gauge symmetry* – an extra constraint on A^μ (a *gauge condition*) must
493 be defined such that a unique A^μ defines the theory and satisfies the condition. However,
494 we are free to choose this extra condition – the physics content of the theory should be
495 independent of this choice (that is, it should be *gauge invariant*).

To ground this a bit, let us return to basic electric and magnetic fields. These are physical quantities that can be measured, and are defined in terms of potentials as

$$\vec{E} = -\nabla\phi - \frac{\partial\vec{A}}{\partial t} \quad (1.16)$$

$$\vec{B} = \nabla \times \vec{A}. \quad (1.17)$$

496 It is easy to show, for any scalar function λ , that $\nabla \times \nabla\lambda = 0$. This implies that the physical
497 \vec{B} field is invariant under the transformation $\vec{A} \rightarrow \vec{A} + \nabla\lambda$ for any scalar function λ .

498 Under the same transformation of \vec{A} , the electric field \vec{E} becomes $-\nabla\phi - \frac{\partial\vec{A}}{\partial t} - \frac{\partial\nabla\lambda}{\partial t} =$
499 $-\nabla(\phi + \frac{\partial\lambda}{\partial t}) - \frac{\partial\vec{A}}{\partial t}$, such that, for the \vec{E} field to be unchanged, we must additionally apply
500 the transformation $\phi \rightarrow \phi - \frac{\partial\lambda}{\partial t}$.

This set of transformations to the potentials that leave the physical degrees of freedom invariant is expressed in our four vector notation naturally as

$$A_\mu \rightarrow A_\mu - \partial_\mu \lambda \quad (1.18)$$

501 where $A_\mu = (\phi, -\vec{A})$ with our sign convention. It should be noted that this function λ is an
502 arbitrary function of *local* spacetime, and thus expresses invariance of the physics content

503 under a local transformation.

Let us return to the Lagrangian for QED. In particular, focusing on the free Dirac piece

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi \quad (1.19)$$

we note that if we apply a local transformation of the form $\psi \rightarrow e^{iq\lambda(x)}\psi$ (and correspondingly $\bar{\psi} \rightarrow \bar{\psi}e^{-iq\lambda(x)}$, by definition), the Lagrangian becomes

$$\bar{\psi}e^{-iq\lambda(x)}(i\gamma^\mu \partial_\mu - m)e^{iq\lambda(x)}\psi = \bar{\psi}e^{-iq\lambda(x)}(i\gamma^\mu \partial_\mu)e^{iq\lambda(x)}\psi - m\bar{\psi}\psi. \quad (1.20)$$

As $\partial_\mu(e^{iq\lambda(x)}\psi) = iq e^{iq\lambda(x)}(\partial_\mu \lambda(x))\psi + e^{iq\lambda(x)}\partial_\mu \psi$, this becomes

$$\bar{\psi}(i\gamma^\mu(\partial_\mu + iq\partial_\mu \lambda(x)) - m)\psi. \quad (1.21)$$

Thus, the free Dirac Lagrangian on its own is not invariant under this transformation. We may note, however, that on interaction with an electromagnetic field, as described above, this transformed Lagrangian may be packaged as:

$$\bar{\psi}(i\gamma^\mu(\partial_\mu + iq\partial_\mu \lambda(x) + iqA_\mu) - m)\psi = \bar{\psi}(i\gamma^\mu(\partial_\mu + iq(A_\mu + \partial_\mu \lambda(x))) - m)\psi. \quad (1.22)$$

504 since by the arguments above, the physics content of the Lagrangian is invariant under the
505 transformation $A_\mu \rightarrow A_\mu - \partial_\mu \lambda$, we may directly make this transformation, and remove this
506 extra $\partial_\mu \lambda(x)$ term. It is straightforward to verify that the $\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$ term is invariant under
507 this same transformation of A_μ , so we may say that the QED Lagrangian is invariant under
508 local transformations of the form $\psi \rightarrow e^{iq\lambda(x)}\psi$.

509 These arguments illuminate some important concepts which will serve us well going forward.
510 First, while we have remained grounded in the “familiar” physics of electromagnetism for the
511 above, arguments of the “top down” variety would lead us to the exact same conclusions.
512 That is, suppose we wanted to construct a theory of spin-1/2 particles that was invariant
513 under local transformations of the form $\psi \rightarrow e^{iq\lambda(x)}\psi$. More broadly, we could say that we
514 desire this theory to be invariant under local $U(1)$ transformations, where $U(1)$ is exactly
515 this group, under multiplication, of complex numbers with absolute value 1. By very similar

516 arguments as above, we would see that, to achieve invariance, this theory would necessitate
517 an additional degree of freedom, A_μ , with the exact properties that are familiar to us from
518 electrodynamics. These arguments based on symmetries are extremely powerful in building
519 theories with a less familiar grounding, as we will see in the following.

Second, we defined this quantity $D_\mu \equiv \partial_\mu + ieA_\mu$ above, seemingly as a matter of notational convenience. However, from the latter set of arguments, such a packaging takes on a new power: by explicitly including this gauge field A_μ which transforms in such a way as to keep invariance under a given transformation, the invariance is immediately more manifest. That is, to pose the $U(1)$ invariance in a more zoomed out way, under the transformation $\psi \rightarrow e^{iq\lambda(x)}\psi$, while

$$\bar{\psi}\partial_\mu\psi \rightarrow \bar{\psi}(\partial_\mu + iq\partial_\mu\lambda(x))\psi \quad (1.23)$$

with the extra term that gets canceled out by the gauge transformation of A_μ ,

$$\bar{\psi}D_\mu\psi \rightarrow \bar{\psi}D_\mu\psi \quad (1.24)$$

520 where this transformation is already folded in. This repackaging, called a *gauge covariant*
521 *derivative* is much more immediately expressive of the symmetries of the theory.

522 Finally, to emphasize how fundamental these gauge symmetries are to the corresponding
523 theory, let us examine the additional term needed for $U(1)$ invariance, $q\bar{\psi}\gamma^\mu A_\mu\psi$. While a
524 first principles examination of Feynman rules is beyond the scope of this thesis, it is powerful
525 to note that this is expressive of a QED vertex: the $U(1)$ invariance of the theory and the
526 interaction between photons and electrons are inextricably tied together.

527 1.3 An Aside on Group Theory

528 Quantum electrodynamics is very familiar and well covered, and provides (both historically
529 and in this thesis) a nice bridge between “standard” physics and the language of symmetries
530 and quantum field theory. However, now that we are acquainted with the language, we
531 may set up to dive a bit deeper. To begin, let us look again at the $U(1)$ group that is so
532 fundamental to QED. We have expressed this via a set of transformations on our Dirac spinor

objects, ψ , of the form $e^{iq\lambda(x)}$. Note that such transformations, though they are local (i.e. a function of spacetime) are purely *phase* transformations. Relatedly, $U(1)$ is an Abelian group, meaning that group elements commute.

To set up language to generalize beyond $U(1)$, note that we may equivalently write $U(1)$ elements as $e^{ig\vec{\alpha}(x)\cdot\vec{T}}$, $\vec{\alpha}(x)$ and \vec{T} and are vectors in the space of *generators* of the group, with each $\alpha^a(x)$ an associated scalar function to generator t^a , and g is some scalar strength parameter. Of course this is a bit silly for $U(1)$, which has a single generator, and thus reduces to the transformation we discussed above. However, this becomes much more useful for groups of higher degree, with more generators and degrees of freedom.

To discuss these groups in a bit more detail, note that $U(n)$ is the unitary group of degree n , and corresponds to the group of $n \times n$ unitary matrices (that is, $U^\dagger U = UU^\dagger = 1$). Given that group elements are $n \times n$, this means that there are n^2 degrees of freedom: n^2 generators are needed to characterize the group.

For $U(1)$, this is all consistent with what we have said above – the group of 1×1 unitary matrices have a single generator, and the phases we identify above clearly satisfy unitarity. Note that these degrees of freedom for the gauge group also characterize the number of gauge bosons we need to satisfy the local symmetry: for $U(1)$, we need one gauge boson, the photon.

Of relevance for the Standard Model are also the special unitary groups $SU(n)$. These are defined similarly to the unitary groups, with the additional requirement that group elements have determinant 1. This extra constraint removes 1 degree of freedom: groups are characterized by $n^2 - 1$ generators.

In particular, we will examine the groups $SU(2)$ in the context of the weak interaction, with an associated $2^2 - 1 = 3$ gauge bosons (cf. the W^\pm and Z bosons), and $SU(3)$, with an associated $3^2 - 1 = 8$ gauge bosons (cf. gluons of different flavors). Note that these groups are non-Abelian (2×2 or 3×3 matrices do not, in general, commute), leading to a variety of complications. However, both of these theories feature interactions with spin-1/2 particles, with transformations of a very similar form: $\psi \rightarrow e^{ig\vec{\alpha}(x)\cdot\vec{T}}\psi$, and the general framing of the arguments for QED will serve us well in the following.

561 **1.4 Quantum Chromodynamics**

562 In some sense, the simplest extension the development of QED is quantum chromodynamics
563 (QCD). QCD is a theory in which, once the basic dynamics are framed (a non-trivial task!)
564 the group structure becomes apparent. The quark model, developed by Murray Gell-Mann [5]
565 and George Zweig [6], provided the fundamental particles involved in the theory, and had
566 great success in explaining the expanding zoo of experimentally observed hadronic states.

567 Some puzzles were still apparent – the Δ^{++} baryon, e.g., is composed of three up quarks,
568 u , with aligned spins. As quarks are fermions, such a state should not be allowed by the
569 Pauli exclusion principle. The existence of such a state in nature implies the existence of
570 another quantum number, and a triplet of values, called *color charge* was proposed by Oscar
571 Greenberg [7]. With these pieces in place, the structure becomes more apparent, as elucidated
572 by Han and Nambu [8].

573 Let us reason our way to the symmetries using color charge. Experimentally, we know
574 that there is this triplet of color charge values r, g, b (the “plus” values, cf. electric charge)
575 and correspondingly anti-color charge $\bar{r}, \bar{g}, \bar{b}$ (the “minus” values). Supposing that the force
576 behind QCD (the *strong force*) is, similar to QED, interactions between fermions mediated
577 by gauge bosons (quarks and gluons respectively), we can start to line up the pieces.

578 What color charge does a gluon have? Similarly to electric charge, we may associate
579 particles with color charge, anti-particles with anti-color charge. Notably, free particles
580 observed experimentally are colorless (have no color charge). Thus, in order for charge to
581 be conserved throughout such processes, this already implies that there are charged gluons.
582 Further, examining color flow diagrams such as *TODO: insert*, it is apparent first that a
583 gluon has not one but two associated color charges and second that these two must be one
584 color charge and one anti-color charge.

585 Counting up the available types of gluons, then, we come up with nine. Six of mixed
586 color type: $r\bar{b}, r\bar{g}, b\bar{r}, b\bar{g}, g\bar{b}$, and $g\bar{r}$, and three of same color type: $r\bar{r}, g\bar{g}$, and $b\bar{b}$. In practice,
587 however, these latter three are a bit redundant: all express a colorless gluon, which, if we

588 could observe this as a free particle, would be indistinguishable from each other. The *color*
 589 *singlet* state is then a mix of these, $\frac{1}{\sqrt{3}}(r\bar{r} + g\bar{g} + b\bar{b})$, leaving two unclaimed degrees of
 590 freedom, which may be satisfied by the linearly independent combinations $\frac{1}{\sqrt{2}}(r\bar{r} - g\bar{g})$ and
 591 $\frac{1}{\sqrt{6}}(r\bar{r} + g\bar{g} - 2b\bar{b})$.

592 We thus have an octet of color states plus a colorless singlet state. If this colorless singlet
 593 state existed, however, we would be able to observe it, not only via interactions with quarks,
 594 but as a free particle. Since do not observe this in nature, this restricts us to 8 gluons. The
 595 simplest group with a corresponding 8 generators is $SU(3)$. Under the assumption that
 596 $SU(3)$ is the local gauge symmetry of the strong interaction, we may proceed in a similar
 597 way as we did for QED. The gauge transformation is $\psi \rightarrow e^{ig_S \vec{\alpha}(x) \cdot \vec{T}} \psi$, where \vec{T} is an eight
 598 component vector of the generators of $SU(3)$, often expressed via the Gell-Mann matrices,
 599 λ^a , as $t^a = \frac{1}{2}\lambda^a$, and the spinor ψ represents the fields corresponding to quarks.

600 This $SU(3)$ symmetry exactly expresses the color structure elucidated above – the Gell-
 601 Mann matrices are an equivalent presentation of the color combinations described above.
 602 Proceeding by analogy to QED, gauge invariance is achieved by introducing eight new degrees
 603 of freedom, G_μ^a , which are the gauge fields corresponding to the gluons, with the gauge
 604 covariant derivative then analogously taking the form $D_\mu \equiv \partial_\mu + ig_S G_\mu^a t^a$.

Recall from the QED derivation that the field strength tensor, $F^{\mu\nu}$ is a rank two antisymmetric tensor which is manifestly gauge invariant and which describes the physical dynamics of the A_μ field. We would like to analogously define a term for the gluon fields. Repackaging this QED tensor, it is apparent that

$$[D_\mu, D_\nu] = D_\mu D_\nu - D_\nu D_\mu \quad (1.25)$$

$$= (\partial_\mu + iqA_\mu)(\partial_\nu + iqA_\nu) - (\partial_\nu + iqA_\nu)(\partial_\mu + iqA_\mu) \quad (1.26)$$

$$= \partial_\mu \partial_\nu + iq\partial_\mu A_\nu + iqA_\mu \partial_\nu + (iq)^2 A_\mu A_\nu - (\partial_\nu \partial_\mu + iq\partial_\nu A_\mu + iqA_\nu \partial_\mu + (iq)^2 A_\nu A_\mu) \quad (1.27)$$

$$= iq(\partial_\mu A_\nu - \partial_\nu A_\mu) + (iq)^2 (A_\mu A_\nu - A_\nu A_\mu) \quad (1.28)$$

$$= iq(\partial_\mu A_\nu - \partial_\nu A_\mu) + (iq)^2 [A_\mu, A_\nu]. \quad (1.29)$$

We proceed through this derivation to highlight that, in the specific case of QED, with its Abelian $U(1)$ gauge symmetry, the field commutator vanishes, leaving exactly the definition of $F_{\mu\nu}$ as described above, i.e.,

$$F_{\mu\nu} = \frac{1}{iq}[D_\mu, D_\nu]. \quad (1.30)$$

We may proceed to define an analogous field strength term for G_μ^a in a similar way:

$$G_{\mu\nu} = \frac{1}{ig_S}[D_\mu, D_\nu] \quad (1.31)$$

This has an extremely nice correspondence, but is complicated by the non-Abelian nature of $SU(3)$, with

$$G_{\mu\nu} = \partial_\mu(G_\nu^a t^a) - \partial_\nu(G_\mu^a t^a) + ig_s[G_\mu^a t^a, G_\nu^a t^a]. \quad (1.32)$$

in which the field commutator term is non-zero. In particular (since each term is summing over a , so we may relabel) as

$$[G_\mu^a t^a, G_\nu^b t^b] = [t^a, t^b]G_\mu^a G_\nu^b \quad (1.33)$$

and as $[t^a, t^b] = if^{abc}t^c$ for the Gell-Mann matrices, where f^{abc} are the structure constants of $SU(3)$, we have

$$G_{\mu\nu} = \partial_\mu(G_\nu^a t^a) - \partial_\nu(G_\mu^a t^a) - g_s f^{abc} t^c G_\mu^a G_\nu^b \quad (1.34)$$

$$= t^a(\partial_\mu G_\nu^a - \partial_\nu G_\mu^a - f^{bca}G_\mu^b G_\nu^c) \quad (1.35)$$

$$= t^a G_{\mu\nu}^a \quad (1.36)$$

for $G_{\mu\nu}^a = \partial_\mu G_\nu^a - \partial_\nu G_\mu^a - f^{abc}G_\mu^b G_\nu^c$.

This gives the component of the field strength corresponding to a particular gauge field a , where the first two terms have the familiar form of the QED field strength, while the last term is new, and explicitly related to the group structure via the f^{abc} constants. In terms of the physics content of the theory, this latter term gives rise to a gluon *self-interaction*, a distinguishing feature of QCD.

Similarly as in QED, a Lorentz invariant combination of field strength tensors may be made as $G_{\mu\nu}G^{\mu\nu}$. However, this is not manifestly gauge invariant. Under a gauge transformation

- 613 U , the covariant derivative behaves as $D^\mu \rightarrow UD^\mu U^{-1}$, corresponding to $G^{\mu\nu} \rightarrow UG^{\mu\nu}U^{-1}$.
614 The cyclic property of the trace thus ensures the gauge invariance of $\text{tr}(G_{\mu\nu}G^{\mu\nu})$, which we
615 will write as $G_{\mu\nu}^a G_a^{\mu\nu}$ with the implied sum over generators a .

Packaging up the theory, it is tempting to copy the form of the QED Lagrangian, with the identifications we have made above:

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi - \frac{1}{4}G_{\mu\nu}^a G_a^{\mu\nu}. \quad (1.37)$$

However this is not quite correct due to the $SU(3)$ nature of the theory. In terms of the physics, the Dirac fields ψ have associated color charge, which must interact appropriately with the G_μ fields. Mathematically, the generators t^a are 3×3 matrices, while the ψ are four component spinors. Adding a color index to the Dirac fields, i.e., ψ_i where i runs over the three color charges, and similarly indexing the generators t_{ij}^a , we may then express the $SU(3)$ gauge covariant derivative component-wise as

$$(D_\mu)_{ij} = \partial_\mu \delta_{ij} + ig_S G_\mu^a t_{ij}^a \quad (1.38)$$

- 616 where δ_{ij} is the Kronecker delta, as ∂_μ does not participate in the $SU(3)$ structure.

The Lagrangian then becomes

$$\mathcal{L} = \bar{\psi}_i(i(\gamma^\mu D_\mu)_{ij} - m\delta_{ij})\psi_j - \frac{1}{4}G_{\mu\nu}^a G_a^{\mu\nu}. \quad (1.39)$$

- 617 and we have constructed QCD.

618 1.5 The Weak Interaction

- 619 One of the first theories of the weak interaction was from Enrico Fermi [9], in an effort to
620 explain beta decay, a process in which an electron or positron is emitted from an atomic
621 nucleus, resulting in the conversion of a neutron to a proton or proton to a neutron respectively.
622 Fermi's hypothesis was of a direct interaction between four fermions. However, in the advent of
623 QED, it is natural to wonder if a theory based on mediator particles and gauge symmetries
624 applies to the weak force as well. The modern formulation of such a theory is due to Sheldon

625 Glashow, Steven Weinberg, and Abdus Salam [10], and is what we will describe in the
626 following.

627 Considering emission of an electron, Fermi's theory involves an initial state neutron that
628 transitions to a proton with the emission of an electron and a neutrino. This transition
629 gives a hint that something slightly more complicated is happening than in QED: there is an
630 apparent mixing between particle types.

631 Now, with the assumption there are mediators for such an interaction, we further know
632 from beta decay and charge conservation that there must be at least two such degrees of
633 freedom: e.g. one that decays to an electron and neutrino (W^-) and one that decays to a
634 positron and neutrino (W^+). From consideration of the process $e^+e^- \rightarrow W^+W^-$, it turns
635 out that with just these two degrees of freedom, the cross section for this process increases
636 without limit as a function of center-of-mass energy, ultimately violating unitarity (more
637 W^+W^- pairs come out than e^+e^- pairs go in). This is resolved with a third, neutral degree
638 of freedom, the Z boson, whose contribution interferes negatively, regulating this process.

639 This leads to three degrees of freedom for the gauge symmetry of the weak interactions, so
640 we thus need a theory which is locally invariant under transformations of a group with three
641 generators. The simplest such choice is $SU(2)$. We may follow a very similar prescription as
642 for QED and QCD: $SU(2)$ has three generators, which implies the existence of three gauge
643 bosons, call them W_μ^k . The gauge transformation may be expressed as $\psi \rightarrow e^{ig_W \vec{\alpha}(x) \cdot \vec{T}} \psi$, where
644 in this case the generators are for $SU(2)$, which may be written in terms of the familiar Pauli
645 matrices: $\vec{T} = \frac{1}{2}\vec{\sigma}$. The structure constants for $SU(2)$ are the antisymmetric Levi-Civita
646 tensor, so the corresponding gauge covariant derivative is $D_\mu \equiv \partial_\mu + ig_W W_\mu^k t^k$, and the field
647 strength tensor is $W_{\mu\nu}^k = \partial_\mu W_\nu^k - \partial_\nu W_\mu^k - \epsilon^{ijk} W_\mu^k W_\nu^k$.

The corresponding Lagrangian would thus be

$$\mathcal{L} = \bar{\psi}_i (i(\gamma^\mu D_\mu)_{ij} - m\delta_{ij}) \psi_j - \frac{1}{4} W_{\mu\nu}^k W_k^{\mu\nu} \quad (1.40)$$

648 where indices i and j run over $SU(2)$ charges.

649 On considering some of the details, the universe unfortunately turns out to be a bit

more complicated. However, this still provides a useful starting place for elucidating the theory of weak interactions. First off, let us consider the particle content, namely, what do the Dirac fields correspond to? This is still a theory of fermionic interactions with gauge bosons. However, we might notice that the fermion content of this theory is both a) broader than QCD, as we know experimentally (cf. beta decay) that both quarks and leptons (e.g. electrons) participate in the weak interaction and b) this fermion content seemingly has a large overlap with QED. In terms of the gauge bosons, we know that at both W^+ and W^- are electrically charged – this means that we expect some interaction of the weak theory with electromagnetism.

However, before diving deeper into this apparent connection between the weak interaction and QED, let us focus on the gauge symmetry. In QCD, the $SU(3)$ content of the theory is expressed via a contraction of color indices – the theory allows for transitions between quarks of one color and quarks of another. Thinking similarly in terms of $SU(2)$ transitions, the beta decay example is already fruitful – there is a transition between an electron and its corresponding neutrino, as well as between two types of quark. In particular, for the case of neutron (with quark content udd) and proton (with quark content udu), the weak interaction provides for a transition from down to up quark.

Such $SU(2)$ dynamics are described via a quantity called *weak isospin*, denoted I_W with third component $I_W^{(3)}$, and can be thought of in a very similar way as color charge in QCD (i.e. as the charge corresponding to the weak interaction). Since $SU(2)$ is 2×2 , there are two such charge states for the fermions, denoted as $I_W^{(3)} = \pm\frac{1}{2}$. This means that the bosons must have $I_W = 1$ such that, by sign convention corresponding to electric charge, the W^+ boson has $I_W^{(3)} = +1$, the Z boson has $I_W^{(3)} = 0$, and the W^- boson has $I_W^{(3)} = -1$.

From conservation of electric charge, this means that transitions involving a W^\pm are between particles that differ by ± 1 in both weak isospin $I_W^{(3)}$ and electric charge. We may thus line up all such doublets as:

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}, \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}, \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}, \begin{pmatrix} u \\ d' \end{pmatrix}, \begin{pmatrix} c \\ s' \end{pmatrix}, \begin{pmatrix} t \\ b' \end{pmatrix} \quad (1.41)$$

673 with the top corresponding to the lower weak isospin and electric charge particles, and the
674 lower quark entries (d' , etc) corresponding to the weak quark eigenstates (which are related
675 to the mass eigenstates by the CKM matrix *TODO: more detail*). Similar doublets may be
676 constructed for the corresponding anti-particles.

The fundamental structuring of these transitions around both electric and weak charge is again indicative of a natural connection. However, nature is again a bit more complicated than we have described. This is because the weak interaction is a *chiral* theory. For massless particles, chirality is the same as the perhaps more intuitive *helicity*. This describes the relationship between a particle's spin and momentum: if the spin vector points in the same direction as the momentum vector, helicity is positive (the particle is “right-handed”), and if the two point in opposite directions, the helicity is negative (the particle is “left-handed”). More concretely:

$$H = \frac{\vec{s} \cdot \vec{p}}{|\vec{s} \cdot \vec{p}|}. \quad (1.42)$$

For massive particles, this generalizes a bit – in the language of Dirac fermions that we have developed, we define projection operators

$$P_R = \frac{1}{2}(1 + \gamma^5) \quad \text{and} \quad P_L = \frac{1}{2}(1 - \gamma^5) \quad (1.43)$$

677 for right and left-handed chiralities respectively – acting on a Dirac field with such operators
678 projects the field onto the corresponding chiral state.

Experimentally, this pops up via parity violation and the famous $V - A$ theory. For the scope of this thesis, it is sufficient to say that the weak interaction is only observed to take place for left-handed particles (and correspondingly, right-handed anti-particles). We therefore modify the theory stated above by projecting all fermions participating in the weak interaction onto respective chiral states – in particular, the $SU(2)$ gauge symmetry only acts on left-handed particles and right-handed anti-particles. We therefore modify the theory appropriately, denoting the chiral projected gauge symmetry as $SU(2)_L$, and similarly for the

Dirac fields. In particular, the weak isospin doublets listed above must now be left-handed:

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L, \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L, \begin{pmatrix} u \\ d' \end{pmatrix}_L, \begin{pmatrix} c \\ s' \end{pmatrix}_L, \begin{pmatrix} t \\ b' \end{pmatrix}_L \quad (1.44)$$

679 and right-handed particle states are placed in singlets and assigned 0 charge under $SU(2)_L$
680 ($I_W = I_W^{(3)} = 0$).

With all of these assignments, let us revisit our guess at the form of the weak interaction Lagrangian. First, dwelling on the kinetic term $\bar{\psi}_i(i(\gamma^\mu D_\mu)_{ij}\psi_j)$, we note that the assigning of left-handed fermions to isospin doublets and right-handed fermions to isospin singlets allows us to remove explicit $SU(2)$ indices by treating these as the fundamental objects, that is, for a single *generation* of fermions, we may write:

$$\bar{Q}i\gamma^\mu D_\mu Q + \bar{u}i\gamma^\mu D_\mu u + \bar{d}i\gamma^\mu D_\mu d + \bar{L}i\gamma^\mu D_\mu L + \bar{e}i\gamma^\mu D_\mu e \quad (1.45)$$

681 for left-handed doublets Q and L for quarks and electron fields respectively and right handed
682 singlets u and d for up and down quark fields and e for electrons.

More concisely, and summing over the three generations of fermions, we may write

$$\sum_f \bar{f}i\gamma^\mu D_\mu f \quad (1.46)$$

683 where the f are understood to run over the fermion chiral doublets and singlets as above.

This then leaves our Lagrangian as

$$\mathcal{L} = \sum_f \bar{f}i\gamma^\mu D_\mu f - \frac{1}{4}W_{\mu\nu}^k W_k^{\mu\nu} \quad (1.47)$$

$$= \sum_f \bar{f}\gamma^\mu(i\partial_\mu - \frac{1}{2}g_W W_\mu^k \sigma_k)f - \frac{1}{4}W_{\mu\nu}^k W_k^{\mu\nu}, \quad (1.48)$$

684 where we have expanded the covariant derivative for clarity. You may note that we have
685 dropped the mass term in the equation above – we will discuss this in detail in just a moment.

First, however, we return to the above comment about fermion content – we neglected to include the sum over fermions in our QED derivation for simplicity. However, all of the

fermions considered in the discussion of the weak interaction have an electric charge (except for the neutrinos). It would be nice to repackage the theory into a coherent *electroweak* theory. This is fairly straightforward when considering the gauge approach – from the discussion above we should expect the electroweak gauge group to be something like $SU(2) \times U(1)$, with four corresponding gauge bosons. Consider a gauge theory with group $SU(2)_L \times U(1)_Y$ – that is, the same weak interaction as discussed previously, but a new $U(1)_Y$ gauge group for electromagnetism, with transformations defined as

$$\psi \rightarrow e^{ig' \frac{Y}{2} \lambda(x)} \psi \quad (1.49)$$

686 with *weak hypercharge* Y .

Similarly to our discussion of QED, we may write the $U(1)_Y$ gauge field as B_μ , and interactions with the Dirac fields take the form $g' \frac{Y}{2} \gamma^\mu B_\mu \psi$. The relationship between this hypercharge and new B_μ field and classical electrodynamics is not so obvious – however it is convenient to parametrize as

$$\begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix} = \begin{pmatrix} \cos \theta_W & \sin \theta_W \\ -\sin \theta_W & \cos \theta_W \end{pmatrix} \begin{pmatrix} B_\mu \\ W_\mu^3 \end{pmatrix} \quad (1.50)$$

687 where A_μ and Z_μ are the physical fields, and we pick W_μ^3 as the neutral weak boson.

688 Note that in the $SU(2)_L \times U(1)_Y$ theory, the Lagrangian must be invariant under all of
689 the local gauge transformations. In particular, this means that the hypercharge must be the
690 same for fermion fields in each weak doublet to preserve $U(1)_Y$ invariance. This gives insight
691 into the relation between the charges of $SU(2)_L \times U(1)_Y$ and electric charge. In particular
692 we know that the hypercharge, Y , of e^- ($I_W^{(3)} = -\frac{1}{2}$) and ν_e ($I_W^{(3)} = +\frac{1}{2}$) is the same.

Supposing that $Y = \alpha I_W^{(3)} + \beta Q$, we must have $-\alpha \frac{1}{2} - \beta = \alpha \frac{1}{2} \implies \beta = -\alpha$. Therefore, choosing an overall scaling from convention,

$$Y = 2(Q - I_W^{(3)}). \quad (1.51)$$

693 Some of these particular forms are best understood in the context of the Higgs mechanism
694 – we will return to this discussion below.

695 **1.6 The Higgs Potential and the SM**

696 In the above, we have neglected a discussion of masses. However there are several things to
697 sort out here. In the first place, we know experimentally that the weak interactions occur
698 over very short ranges at low energies (e.g., why Fermi's effective four fermion interaction was
699 such a good description). This is consistent with massive W^\pm and Z bosons (and indeed, this
700 is seen experimentally). However, requiring local gauge invariance forbids mass terms in the
701 Lagrangian. In the simple $U(1)$ QED example, such a term would have the form $\frac{1}{2}m_\gamma^2 A_\mu A^\mu$,
702 which is not invariant under the transformation $A_\mu \rightarrow A_\mu - \partial_\mu \lambda$, and similar arguments hold
703 for gauge bosons in the electroweak theory and QCD.

Similar issues are encountered with fermions – in the electroweak theory above, the gauge symmetries are separated into left and right handed chirality via doublet and singlet states. This means that a mass term would need to be separated as well. Such a term would have the form:

$$m\bar{f}f = m(\bar{f}_L + \bar{f}_R)(f_L + f_R) \quad (1.52)$$

$$= m(\bar{f}_L f_L + \bar{f}_L f_R + \bar{f}_R f_L + \bar{f}_R f_R) \quad (1.53)$$

$$= m(\bar{f}_L f_R + \bar{f}_R f_L) \quad (1.54)$$

704 where we have used that $f_{L,R} = P_{L,R}f$, $\bar{f}_{L,R} = \bar{f}P_{R,L}$, and $P_R P_L = P_L P_R = 0$. As left
705 and right-handed particles transform differently under $SU(2)_L$, this is manifestly not gauge
706 invariant.

707 The question then becomes: how do we include particle masses while preserving the
708 gauge properties of our theory? The answer, due to Robert Brout and François Englert [11],
709 Peter Higgs [12], and Gerald Guralnik, Richard Hagen, and Tom Kibble [13] comes via the
710 Higgs mechanism, which we will describe in the following. Importantly for this thesis, this
711 mechanism predicts the existence of a physical particle, the Higgs boson, and a particle
712 consistent with the Higgs boson was seen by both ATLAS [14] and CMS [15] in 2012.

To explain the Higgs, we focus first on generating masses for the electroweak gauge bosons.

Consider adding two complex scalar fields ϕ^+ and ϕ^0 to the Standard Model embedded in a weak isospin doublet ϕ . We may write the doublet as

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix} \quad (1.55)$$

⁷¹³ where we explicitly note the four available degrees of freedom.

The Lagrangian for such a doublet takes the form

$$\mathcal{L} = (\partial_\mu \phi)^\dagger (\partial^\mu \phi) - V(\phi) \quad (1.56)$$

where V is the corresponding potential. Considering the particular form

$$V(\phi) = \mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2 \quad (1.57)$$

⁷¹⁴ we may notice that this has some interesting properties. Considering, as illustration, a similar
⁷¹⁵ potential for a real scalar field, $\mu^2 \chi^2 + \lambda \chi^4$, taking the derivative and setting it equal to 0
⁷¹⁶ yields extrema when $\chi = 0$ and $(\mu^2 + 2\lambda\chi^2) = 0 \implies \chi^2 = -\frac{\mu^2}{2\lambda}$. For $\mu^2 > 0$, there is a
⁷¹⁷ unique minimum at $\chi = 0$, and for $\mu^2 < 0$ there are degenerate minima at $\chi = \pm\sqrt{-\frac{\mu^2}{2\lambda}}$.
⁷¹⁸ Note that we take $\lambda > 0$, otherwise the only minima in the theory are trivial.

The same simple calculus for the complex Higgs doublet above yields degenerate minima for $\mu^2 < 0$ at

$$\phi^\dagger \phi = \frac{1}{2}(\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2) = \frac{v}{2} = -\frac{\mu^2}{2\lambda} \quad (1.58)$$

However, though there is this degenerate set of minima, there can only be a single *physical* vacuum state (we say that the symmetry is *spontaneously broken*). Without loss of generality, we may align our axes such that the physical vacuum state is at

$$\langle 0 | \phi | 0 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \quad (1.59)$$

⁷¹⁹ where we have explicitly chosen a real, non-zero vacuum expectation value for the neutral
⁷²⁰ component of the Higgs doublet to maintain a massless photon, as we shall see. Physically,
⁷²¹ however, this makes sense - the vacuum is not electrically charged.

The vacuum is a classical state – we want a quantum one. We may express fluctuations about this nonzero expectation value via an expansion as $v + \eta(x) + i\xi(x)$. However, renaming of fields is only meaningful for the non-zero vacuum component - we thus have:

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ v + \eta(x) + i\phi_4 \end{pmatrix}. \quad (1.60)$$

where we may expand the Lagrangian listed above:

$$\mathcal{L} = (\partial_\mu \phi)^\dagger (\partial^\mu \phi) - \mu^2 \phi^\dagger \phi - \lambda(\phi^\dagger \phi)^2. \quad (1.61)$$

It is an exercise in algebra to plug in the expansion about v into this Lagrangian: first expanding the potential

$$V(\phi) = \mu^2 \phi^\dagger \phi + \lambda(\phi^\dagger \phi)^2 \quad (1.62)$$

$$= \mu^2 \left(\sum_i \phi_i(x)^2 + (v + \eta(x))^2 \right) + \lambda \left(\sum_i \phi_i(x)^2 + (v + \eta(x))^2 \right) \quad (1.63)$$

$$= -\frac{1}{4} \lambda v^4 + \lambda v^2 \eta^2 + \lambda v \eta^3 + \frac{1}{4} \lambda \eta^4 \quad (1.64)$$

$$+ \frac{1}{2} \lambda \sum_{i \neq j} \phi_i^2 \phi_j^2 + \lambda v \eta \sum_i \phi_i(x)^2 + \frac{1}{2} \lambda \eta^2 \sum_i \phi_i(x)^2 + \frac{1}{4} \sum_i \phi_i(x)^4 \quad (1.65)$$

where the sums are over the $i \in 1, 2, 4$, that is, the fields with 0 vacuum expectation, and we have used the definition $\mu^2 = -\lambda v^2$.

Within this potential, we note a quadratic term in $\eta(x)$ which we may identify with a mass, namely $m_\eta = \sqrt{2\lambda v^2}$, whereas the ϕ_i are massless. These ϕ_i are known as *Goldstone bosons*, and correspond to quantum fluctuations along the minimum of the potential. Of particular note for this thesis are the interaction terms $\lambda v \eta^3$ and $\frac{1}{4} \lambda \eta^4$, expressing trilinear and quartic self-interactions of the η field.

Expanding the kinetic term

$$(\partial_\mu \phi)^\dagger (\partial^\mu \phi) = \frac{1}{2} \sum_i (\partial_\mu \phi_i)(\partial^\mu \phi_i) + \frac{1}{2} (\partial_\mu(v + \eta(x)))(\partial^\mu(v + \eta(x))) \quad (1.66)$$

$$= \frac{1}{2} \sum_i (\partial_\mu \phi_i)(\partial^\mu \phi_i) + \frac{1}{2} (\partial_\mu \eta)(\partial^\mu \eta) \quad (1.67)$$

⁷²⁹ in a similar way, completing the story of three massless degrees of freedom (Goldstone bosons)
⁷³⁰ and one massive one.

Now, this doublet is embedded in an $SU(2)_L \times U(1)$ theory, so we would like to preserve that gauge invariance. This is achieved in the same way as for the Dirac fields, with the introduction of the electroweak gauge covariant derivative such that the Lagrangian for the Higgs doublet and the electroweak bosons is just

$$\mathcal{L} = (D_\mu \phi)^\dagger (D^\mu \phi) - \mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2 - \frac{1}{4} W_{\mu\nu}^k W_k^{\mu\nu} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \quad (1.68)$$

⁷³¹ with $D_\mu = \partial_\mu + ig_W W_\mu^k t^k + ig' \frac{Y}{2} B_\mu$.

We note that it is convenient to pick a gauge such that the Goldstone fields do not appear in the Lagrangian, upon which we may identify the field $\eta(x)$ with the physical Higgs field, $h(x)$. The field mass terms then very apparently come via the covariant derivative, namely, as

$$W_\mu^k \sigma^k + B_\mu = \begin{pmatrix} W_\mu^3 + B_\mu & W_\mu^1 - iW_\mu^2 \\ W_\mu^1 + iW_\mu^2 & -W_\mu^3 + B_\mu \end{pmatrix} \quad (1.69)$$

we may then write

$$D_\mu \phi = \frac{1}{2\sqrt{2}} \begin{pmatrix} 2\partial_\mu + ig_W W_\mu^3 + ig' Y B_\mu & ig_W W_\mu^1 + \frac{1}{2} g_W W_\mu^2 \\ ig_W W_\mu^1 - g_W W_\mu^2 & 2\partial_\mu - ig_W W_\mu^3 + ig' Y B_\mu \end{pmatrix} \begin{pmatrix} 0 \\ v + h \end{pmatrix} \quad (1.70)$$

$$= \frac{1}{2\sqrt{2}} \begin{pmatrix} ig_W (W_\mu^1 - iW_\mu^2)(v + h) \\ (2\partial_\mu - ig_W W_\mu^3 + ig' Y B_\mu)(v + h) \end{pmatrix} \quad (1.71)$$

⁷³² As identified above, $Y = 2(Q - I_W^{(3)})$. The Higgs has 0 electric charge, and the lower doublet
⁷³³ component has $I_W^{(3)} = -\frac{1}{2}$, yielding $Y = 1$.

Computing $(D_\mu \phi)^\dagger (D^\mu \phi)$, then, yields

$$\frac{1}{8} g_W^2 (W_\mu^1 + iW_\mu^2)(W^{\mu 1} - iW^{\mu 2})(v + h)^2 + \frac{1}{8} (2\partial_\mu + ig_W W_\mu^3 - ig' B_\mu)(2\partial^\mu - ig_W W^{\mu 3} + ig' B^\mu)(v + h)^2 \quad (1.72)$$

and extracting terms quadratic in the fields gives

$$\frac{1}{8} g_W^2 v^2 (W_{\mu 1} W^{\mu 1} + W_{\mu 2} W^{\mu 2}) + \frac{1}{8} v^2 (g_W W_\mu^3 - g' B_\mu)(g_W W^{\mu 3} - g' B^\mu) \quad (1.73)$$

meaning that W_μ^1 and W_μ^2 have masses $m_W = \frac{1}{2}g_W v$. The neutral boson case is a bit more complicated. Writing the corresponding term as

$$\frac{1}{8}v^2 \begin{pmatrix} W_\mu^3 & B_\mu \end{pmatrix} \begin{pmatrix} g_W^2 & -g_W g' \\ -g_W g' & g'^2 \end{pmatrix} \begin{pmatrix} W^{\mu 3} \\ B^\mu \end{pmatrix} \quad (1.74)$$

we note that we must diagonalize this mass matrix to get the physical mass eigenstates. Doing so in the usual way yields eigenvalues 0 , $g'^2 + g_W^2$, thus corresponding to $m_\gamma = 0$ and $m_Z = \frac{1}{2}v\sqrt{g'^2 + g_W^2}$, with physical fields as the (normalized) eigenvectors

$$A_\mu = \frac{g' W_\mu^3 + g_W B_\mu}{\sqrt{g_W^2 + g'^2}} \quad (1.75)$$

$$Z_\mu = \frac{g_W W_\mu^3 - g' B_\mu}{\sqrt{g_W^2 + g'^2}} \quad (1.76)$$

From this form, the angular parametrization of the physical fields is very apparent, namely, defining

$$\tan \theta_W = \frac{g'}{g_W}, \quad (1.77)$$

these equations may be written in terms of the single parameter θ_W as

$$A_\mu = \cos \theta_W B_\mu + \sin \theta_W W_\mu^3 \quad (1.78)$$

$$Z_\mu = -\sin \theta_W B_\mu + \cos \theta_W W_\mu^3 \quad (1.79)$$

and, notably, from the above equations,

$$\frac{m_W}{m_Z} = \cos \theta_W. \quad (1.80)$$

To get the mass terms from Equation 1.72, we extracted those terms quadratic in fields, i.e., the v^2 terms within $(v + h)^2$. However there are also terms of the form VVh and $VVhh$ that arise, which describe the Higgs interactions with the corresponding vector bosons $V = W^\pm, Z$. Namely, identifying physical W bosons as

$$W^\pm = \frac{1}{\sqrt{2}}(W^1 \mp iW^2) \quad (1.81)$$

we may express the first term of Equation 1.72 as

$$\frac{1}{4}g_W^2 W_\mu^- W^{+\mu} (v + h)^2 = \frac{1}{4}g_W^2 v^2 W_\mu^- W^{+\mu} + \frac{1}{2}g_W^2 v W_\mu^- W^{+\mu} h + \frac{1}{4}g_W^2 W_\mu^- W^{+\mu} h^2 \quad (1.82)$$

with the first term corresponding to the mass term $m_W = \frac{1}{2}g_W v$, and the second two terms corresponding to hW^+W^- and hhW^+W^- vertices. Of particular note is the coupling strength

$$g_{HWW} = \frac{1}{2}g_W^2 v = g_W m_W \quad (1.83)$$

⁷³⁴ which is proportional to the W mass – an analysis with the form of the physical Z boson
⁷³⁵ finds that the coupling g_{HZZ} is also proportional to the Z mass.

The Higgs coupling to fermions (in particular to quarks) is of particular interest for this thesis. We showed above that a naive introduction of a mass term

$$m\bar{f}f = m(\bar{f}_L f_R + \bar{f}_R f_L) \quad (1.84)$$

⁷³⁶ is manifestly not gauge invariant because right and left handed particles transform differently
⁷³⁷ under $SU(2)_L$. However, because the Higgs is constructed via an $SU(2)_L$ doublet, ϕ , writing
⁷³⁸ a fermion doublet as L and conjugate \bar{L} , it is apparent that $\bar{L}\phi$ is invariant under $SU(2)_L$.

Combining with the right handed singlet, R , creates a term invariant under $SU(2)_L \times U(1)_Y$, $\bar{L}\phi R$ (and correspondingly $(\bar{L}\phi R)^\dagger$), such that we may include Yukawa [16] terms

$$\mathcal{L}_{Yukawa} = -g_f \left[\begin{pmatrix} \bar{f}_1 & \bar{f}_2 \end{pmatrix}_L \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} f_R + \bar{f}_R \begin{pmatrix} \phi^{+*} & \phi^{0*} \end{pmatrix} \begin{pmatrix} f_1 \\ f_2 \end{pmatrix}_L \right] \quad (1.85)$$

⁷³⁹ where g_f is a corresponding Yukawa coupling, f_1 and f_2 have been used to denote components
⁷⁴⁰ of the left-handed doublet and f_R the corresponding right-handed singlet.

After spontaneous symmetry breaking, with the gauge as described above to remove the Goldstone fields, the Higgs doublet becomes

$$\phi(x) = \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix} \quad (1.86)$$

giving rise to terms such as

$$-\frac{1}{\sqrt{2}}g_f v(\bar{f}_{2L}\bar{f}_R + \bar{f}_R f_{2L}) - \frac{1}{\sqrt{2}}g_f h(\bar{f}_{2L}\bar{f}_R + \bar{f}_R f_{2L}) \quad (1.87)$$

where we have kept the subscript f_{2L} to emphasize that these terms *only* impact the lower component of the left-handed doublet because of the 0 in the upper component of the Higgs doublet. Leaving this aside for a second, we note that the first term has the form of the desired mass term above (identifying f_{2L} to f_L) while the second term describes the coupling of the fermion to the physical Higgs field. The corresponding Yukawa coupling may be chosen to be consistent with the observed fermion mass, namely

$$g_f = \sqrt{2} \frac{m_f}{v} \quad (1.88)$$

such that

$$\mathcal{L}_f = -m_f \bar{f}f - \frac{m_f}{v} \bar{f}fh. \quad (1.89)$$

⁷⁴¹ Notably here, the fermion coupling to the Higgs boson scales with the mass of the fermion, a fact that is extremely relevant for this thesis analysis.

As we said above, these terms *only* impact the lower component of the left-handed doublet. The inclusion of terms for the upper component is accomplished via the introduction of a Higgs conjugate doublet, defined as

$$\phi_c = -i\sigma_2\phi^* = \begin{pmatrix} -\phi^{0*} \\ \phi^- \end{pmatrix}. \quad (1.90)$$

⁷⁴³ The argument proceeds similarly to the above, with similar results for couplings and masses of upper components.

⁷⁴⁵ 1.7 The Standard Model: A Summary

After all of the above, we may write the Standard Model as a theory with a local $SU(3) \times SU(2)_L \times U(1)_Y$ gauge symmetry, described by the Lagrangian

$$\mathcal{L} = \sum_f \bar{f}i\gamma^\mu D_\mu f - \frac{1}{4} \sum_{gauges} F_{\mu\nu}F^{\mu\nu} + (D_\mu\phi)^\dagger(D^\mu\phi) - \mu^2\phi^\dagger\phi - \lambda(\phi^\dagger\phi)^2 \quad (1.91)$$

where $D_\mu = \partial_\mu + ig_W W_\mu^k t^k + ig' \frac{Y}{2} B_\mu + ig_S G_\mu^a t^a$, in addition to the Yukawa terms, which we write generally as

$$\mathcal{L}_{Yukawa} = - \sum_{f,\phi=\phi_c,-\phi_c} y_f (\bar{f}\phi f + (\bar{f}\phi f)^\dagger) \quad (1.92)$$

746 with the sum running over running over appropriate chiral fermion and Higgs doublets.

747 The $SU(2)_L \times U(1)_Y$ subgroup is spontaneously broken to a $U(1)$ symmetry, lending mass
748 to the associated gauge bosons and fermions. Of relevance for this thesis is the resulting
749 physical Higgs field, with a predicted trilinear self-interaction and associated coupling λv ,
750 related to the experimentally observed Higgs boson mass by $m_H = \sqrt{2\lambda v^2}$, as well as the fact
751 that the strength of the Higgs coupling to fermions scales proportionally with the fermion
752 mass.

753 The Standard Model has been monumentally successful, with predictions consistent across
754 many varied experimental cross-checks. This thesis participates in one such cross check.
755 However, the Standard Model is notably not a complete theory of the universe – there is
756 no inclusion of gravity, for instance, though a consistent description may be provided with
757 the introduction of a spin-2 particle. Neutrino oscillations demonstrate that neutrinos have
758 mass, but right-handed neutrinos have not been observed, leading to questions about whether
759 there is a different mechanism to provide neutrinos with mass than that described above.
760 Cosmology tells us that dark matter exists, but there is no corresponding particle within the
761 Standard Model. This thesis therefore also participates in searches for physics beyond the
762 Standard Model. We will provide a sketch of the relevant theories in the following chapter,
763 though a detailed theoretical discussion is beyond the scope of this work.

764

Chapter 2

765

DI-HIGGS PHENOMENOLOGY AND PHYSICS BEYOND THE STANDARD MODEL

766

767 This thesis focuses on searches for di-Higgs production in the $b\bar{b}b\bar{b}$ final state. In this
 768 chapter, we will provide a brief overview of the practical theoretical information motivating
 769 such searches. Though the searches test for physics beyond the Standard Model, particularly
 770 in the search for resonances, the goal of the experimental results is to be somewhat agnostic
 771 to particular theoretical frameworks. An in depth treatment of such models is therefore
 772 beyond the scope of this thesis, though we will attempt to provide a grounding for the models
 773 that we consider.

774 **2.1 Intro to Di-Higgs**

775 Di-Higgs searches can be split into two major theoretical categories: *resonant searches*, in
 776 which a physical resonance is produced that subsequently decays into two Higgs bosons, and
 777 a *non-resonant searches* in which no physical resonance is produced, but where the HH
 778 production cross section has a contribution from an exchange of a *virtual* or *off-shell* particle.

779 The focus of this thesis is gluon initiated processes – in the case of di-Higgs this is
 780 termed gluon-gluon fusion (ggF). HH production may also occur via vector boson fusion [17].
 781 However the cross section for such production is significantly smaller. Representative Feynman
 782 diagrams are shown in for gluon-gluon fusion non-resonant production in Figure 2.1.

783 As shown in Chapter 1, the Higgs coupling to fermions scales with particle mass. As the
 784 top quark has a mass of 173 GeV, whereas the H has a mass of 125 GeV, such that $H \rightarrow t\bar{t}$ is
 785 kinematically disfavored, $H \rightarrow b\bar{b}$ is the dominant fermionic Higgs decay mode, and, in fact,
 786 the dominant overall decay mode, with a branching fraction of around 58 %. The dominant

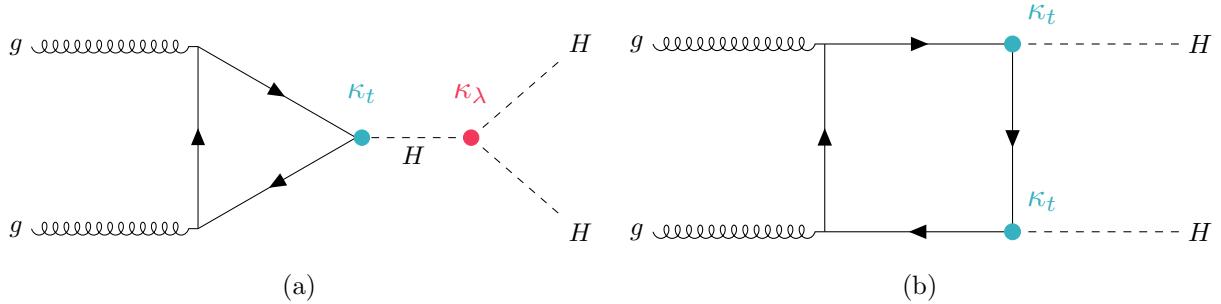


Figure 2.1: Dominant contributing diagrams for non-resonant gluon-gluon fusion production of HH . κ_λ and κ_t represent variations of the Higgs self-coupling and coupling to top quarks respectively, relative to that predicted by the Standard Model.

⁷⁸⁷ top quark Yukawa coupling to the H does play a role in H production, however – gluon-gluon
⁷⁸⁸ fusion is dominated by processes including a top loop.

The single H properties translate to HH production, with $HH \rightarrow b\bar{b}b\bar{b}$ accounting for around 34 % of all HH decays. The H H branching fractions are shown in Figure 2.2.

791 2.2 Resonant HH Searches

Resonant di-Higgs production is predicted in a variety of extensions to the Standard Model.
In particular, this thesis presents searches for both spin-0 and spin-2 resonances. The decay
of spin-1 resonances to two identical spin-0 bosons is prohibited, as the final state must
correspondingly be symmetric under particle exchange, but this process would require orbital
angular momentum $\ell = 1$, and thus an anti-symmetric final state. Each model considered
here is implemented in a particular theoretical context, but set up experimental results for
generic searches.

The spin-2 signal considered is implemented within the bulk Randall-Sundrum (RS) model [18, 19], which features spin-2 Kaluza-Klein gravitons, G_{KK}^* , that are produced via gluon-fusion and which may decay to a pair of Higgs bosons. The model predicts such

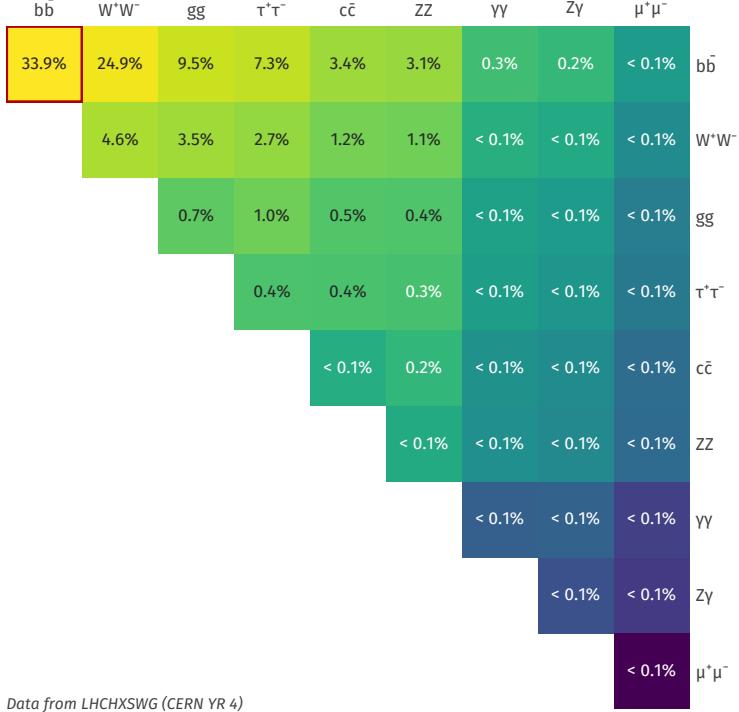


Figure 2.2: Illustration of dominant HH branching ratios. $HH \rightarrow b\bar{b}b\bar{b}$ is the most common decay mode, representing 34 % of all HH events produced at the LHC.

gravitons as a consequence of warped extra dimensions, and is correspondingly parametrized by a value $c = k/\overline{M}_{\text{Pl}} = 1$, where k describes a curvature scale for the extra dimension and \overline{M}_{Pl} is the Planck mass. The model considered here has $c = 1.0$. However, this model was considered in the early Run 2 HH analyses [20], and was excluded across much of the relevant mass range.

The primary theoretical focus of this work is therefore the spin-0 result, which is implemented as a generic resonance with width below detector resolution. Scalar resonances are interesting, for instance, in the context of two Higgs doublet models [21], which posit the existence of a second Higgs doublet. This leads to the existence of five scalar particles in the Higgs sector – roughly, two complex doublets provide eight degrees of freedom, three of which

812 are “eaten” by the electroweak bosons, leaving five degrees of freedom which may correspond
813 to physical fields.

814 **2.3 Non-resonant HH Searches**

Non-resonant HH production is predicted by the Standard Model via the trilinear coupling discussed above, as well as via production in a fermion loop. More explicitly, after electroweak symmetry breaking, we have

$$\mathcal{L}_{SM} \supset -\lambda v^2 h^2 - \lambda v h^3 - \frac{1}{4} \lambda h^4 \quad (2.1)$$

$$= -\frac{1}{2} m_H^2 - \lambda_{HHH}^{SM} v h^3 - \lambda_{HHHH}^{SM} h^4 \quad (2.2)$$

where $m_H = \sqrt{2\lambda v^2}$ so that

$$\lambda_{HHH}^{SM} = \frac{m_H^2}{2v^2}. \quad (2.3)$$

815 The mass of the SM Higgs boson has been experimentally measured to be 125 GeV [22],
816 and the vacuum expectation value $v = 246$ GeV has a precise determination from the muon
817 lifetime [23]. This coupling is therefore precisely predicted in the Standard Model, such that
818 an observed deviation from this prediction would be a clear sign of new physics.

819 The relevant diagrams for non-resonant HH production are shown in Figure 2.1. Notably,
820 the diagrams *interfere* with each other, which can be easily seen by counting the fermion
821 lines. A detailed theoretical discussion is provided by, e.g. [24].

For the searches presented here, the quark couplings to the Higgs are considered to be consistent with the Standard Model value, with measurements of the dominant top Yukawa coupling left to more sensitive direct measurements, e.g. from $t\bar{t}$ final states [25]. Variations of the trilinear coupling away from the Standard Model are considered, however. Such variations are parametrized via

$$\kappa_\lambda = \frac{\lambda_{HHH}}{\lambda_{HHH}^{SM}} \quad (2.4)$$

822 where λ_{HHH} is a varied coupling, whereas λ_{HHH}^{SM} is the Standard Model prediction, given by
823 As this variation only impacts the *triangle* diagram, significant and interesting effects are

824 observed due to the interference. Examples of the impact of this tradeoff on the di-Higgs
825 invariant mass are shown in *TODO: include plot*. Generally speaking, the triangle diagram
826 contributes more at low mass, while the box diagram contributes more at high mass, and for
827 positive values of κ_λ , more events are predicted at low mass, whereas for negative values of
828 κ_λ , more events are predicted at high mass.

829

Chapter 3

830

EXPERIMENTAL APPARATUS

831 What machines must we build to examine the smallest pieces of the universe? The famous
 832 equation $E = m$ provides that to create massive particles, we need to provide enough energy.
 833 In order to give kinematic phase space to the types of processes that are examined in this
 834 thesis (and many others besides), a system must be created in which there is enough energy
 835 to (at bare minimum), overcome kinematic thresholds: if you want to search for HH decays,
 836 you should have at least 250 GeV ($= 2 \times m_H$) to work with. It is not enough to simply induce
 837 such processes, however. These processes need to be captured in some way, emitted energy
 838 and particles must be characterized and identified, and in the end all of this information must
 839 be put into a useful and useable form such that selections can be made, statistics can be run,
 840 and a meaningful statement can be made about the universe. In this chapter, we describe the
 841 machines behind the physics, namely the Large Hadron Collider and the ATLAS experiment.

842 **3.1 The Large Hadron Collider**

843 The Large Hadron Collider is a particle accelerator near Geneva, Switzerland, operating
 844 at a center of mass energy $\sqrt{s} = 13$ TeV. In broad scope, it is a ring with a 27 kilometer
 845 circumference. Hadrons (usually protons or heavy ions) move in two counter-circulating
 846 beams, which are made to collide at four collision points at various points on the ring. These
 847 four collision points correspond to the four detectors placed around the ring: two “general
 848 purpose” experiments: ATLAS and CMS; LHCb, focused primarily on flavor physics; and
 849 ALICE, focused primarily on heavy ions.

850 For proton-proton collisions, the focus of this thesis, the acceleration chain proceeds as
 851 follows: first, an electric field strips hydrogen of its electrons, creating protons. A linear

852 accelerator, LINAC 2, accelerates protons to 50 MeV. The resulting beam is injected into
 853 the Proton Synchrotron Booster (PSB), which pushes the protons to 1.4 GeV, and then the
 854 Proton Synchrotron, which brings the beam to 25 GeV.

855 Protons are then transferred to the Super Proton Synchrotron (SPS), which ramps up
 856 the energy to 450 GeV. Finally, the protons enter the LHC itself, bringing the beam up to
 857 6.5 TeV. *TODO: cite: <https://home.cern/science/accelerators/accelerator-complex>*

858 While there is, of course, much that goes into the Large Hadron Collider development and
 859 operation, perhaps two of the most fundamental ideas are (1) how are the beams directed
 860 and manipulated and (2) what do we mean when we say “protons are accelerated”. These
 861 questions both are directly answered by pieces of hardware, namely (1) magnets and (2)
 862 radiofrequency (RF) cavities.

863 One of fundamental components of the LHC is a large set of superconducting niobium-
 864 titanium magnets. These are cooled by liquid helium to achieve superconducting temperatures,
 865 and there are several types with very specific purposes. The obvious first question with a
 866 circular accelerator is how to keep the particle beam moving around in that circle. This job
 867 is done via a set of dipole magnets placed around the *beam pipes*: the tubes containing the
 868 beam. These are designed such that the magnetic field in the center of the beam pipe runs
 869 perpendicular to the velocity of the charged particles, providing the necessary centripetal
 870 force for the synchrotron motion.

871 A proton beam is not made of a single proton, however, but of many protons, grouped
 872 into a series of *bunches*. As all of these are positively charged, if unchecked, these bunches
 873 would become diffuse and break apart. What we want is a stable beam with tightly clustered
 874 protons to maximize the chance of a high energy collision. Such clustering is done via a series
 875 of quadropole magnets, with field distributed as in *TODO: grab image from General Exam.*
 876 Alternating sets of quadropoles provide the necessary forces for a tight, stable beam. While
 877 these are the two major components of the LHC magnet system, it is not the full story –
 878 higher order magnets are used to correct for small imperfections in the beam *TODO: expand.*

879 Magnetic fields do no work, however, so the magnet system is unable to do the job of the

actual acceleration. This is accomplished via a set of radiofrequency (RF) cavities. Within these cavities, an electric field is made to oscillate (switch direction) at a precise rate. These rates interact with the beam via in RF *buckets*, with bunches corresponding to groups of protons that fill a given bucket. The timing is such that protons will always experience an accelerating voltage, corresponding to the 25 ns bunch spacing used at the LHC.

A nice property of this bucket/bunch configuration is that there is some self-correction – there is some finite spread in the grouping of particles. If a particle arrives too early, it will experience some decelerating voltage; if too late, it will experience a higher accelerating voltage.

3.2 The ATLAS Experiment

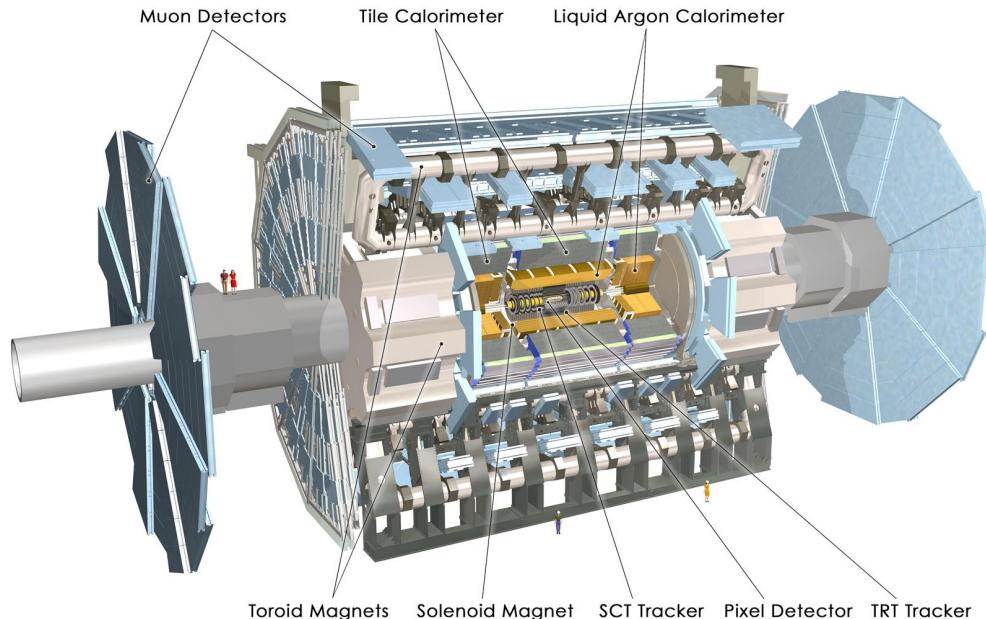


Figure 3.1: Diagram of the ATLAS detector [26]

This thesis focuses on searches done with the ATLAS experiment. As mentioned, this is one of two “general purpose” experiments at the LHC, by which we mean there is a very large and

892 broad variety of physics done within the experimental collaboration. This broad physics focus
 893 has a direct relation to the design of the ATLAS detector [27], pictured in Figure 3.1, which
 894 is composed of a sophisticated set of subsystems designed to fully characterize the physics of
 895 a given high energy particle collision. It consists of an inner tracking detector surrounded
 896 by a thin superconducting solenoid, electromagnetic and hadronic calorimeters, and a muon
 897 spectrometer incorporating three large superconducting toroidal magnets. The ATLAS
 898 detector covers nearly the entire solid angle around the collision point, fully characterizing
 899 the “visible” components of a collision and allowing for indirect sensitivity to particles that
 900 do not interact with the detector (e.g. neutrinos) via “missing” energy (roughly momentum
 901 balance). We will go through the design and physics contribution of each of the detector
 902 components in the following. A schematic of how various particles interact with the detector
 903 is shown in Figure 3.2.

904 3.2.1 ATLAS Coordinate System

905 Of relevance for the following discussion, as well as for the analysis presented in Chapter 7,
 906 is the ATLAS coordinate system. ATLAS uses a right-handed coordinate system with its
 907 origin at the nominal interaction point (IP) in the center of the detector and the z -axis along
 908 the beam pipe. The x -axis points from the IP to the centre of the LHC ring, and the y -axis
 909 points upwards. Cylindrical coordinates (r, ϕ) are used in the transverse plane, ϕ being the
 910 azimuthal angle around the z -axis. The pseudorapidity is defined in terms of the polar angle
 911 θ as $\eta = -\ln \tan(\theta/2)$. Angular distance is measured in units of $\Delta R \equiv \sqrt{(\Delta\eta)^2 + (\Delta\phi)^2}$.
 912 These coordinates are shown in Figure 3.3.

913 3.2.2 Inner Detector

914 The purpose of the inner detector is the reconstruction of the trajectory of charged particles,
 915 called *tracking*. This is accomplished primarily through the collection of electrons displaced
 916 when a charged particle passes through a tracking detector. By setting up multiple layers of

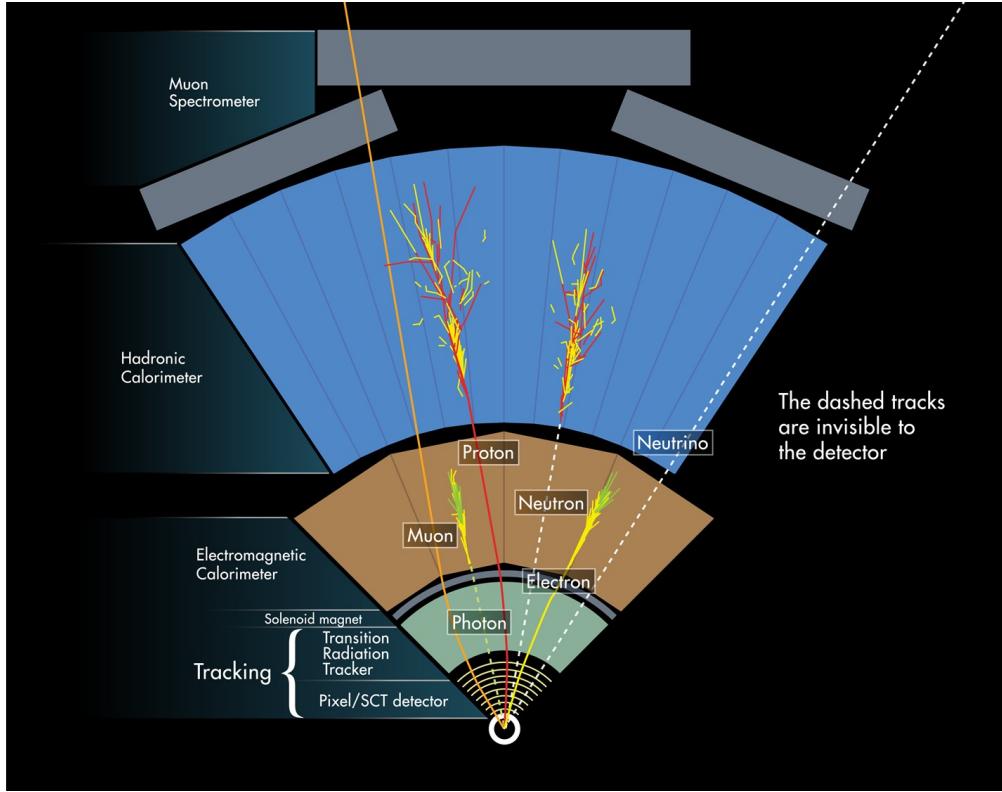


Figure 3.2: Cross section of the ATLAS detector showing how particles interact with various detector components [28]

such detectors, such that a given particle leaves a signature, known as a “hit”, in each layer, the trajectory of the particle may be inferred via “connecting the dots” between these hits.

The raw trajectory of a particle only provides positional information. However, the trajectory of a charged particle in a known magnetic field additionally provides information on particle momentum and charge via the curvature of the corresponding track (cf. $\vec{F} = q\vec{v} \times \vec{B}$). The inner detector system is therefore surrounded by a solenoid magnet, providing a 2 T magnetic field along the z -axis (yielding curvature in the transverse $x - y$ plane).

The inner detector provides charged particle tracking in the range $|\eta| < 2.5$ via a series of detector layers. The innermost of these is the high-granularity silicon pixel detector which typically provides four measurements per track, with the first hit in the insertable B-layer

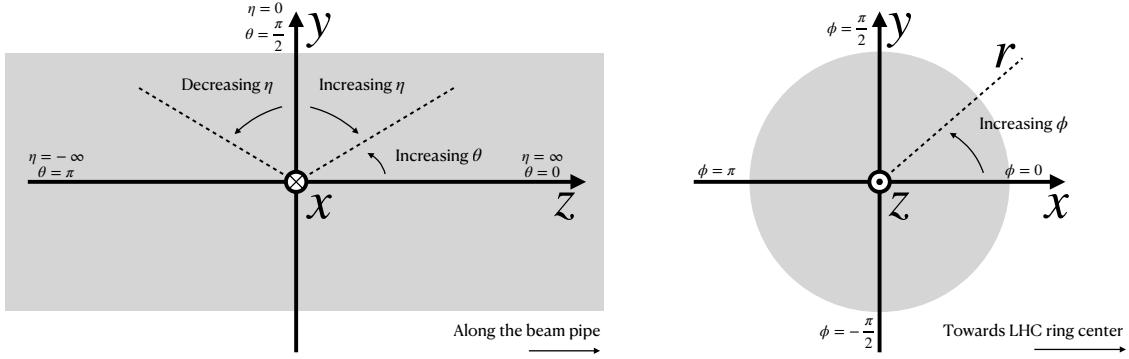


Figure 3.3: 2D projections of the ATLAS coordinate system

927 (IBL) installed before Run 2 [29, 30]. This is very close to the interaction point with a
 928 high degree of positional information, and is therefore very important for e.g. b -tagging (see
 929 Chapter 5). It is followed by the silicon microstrip tracker (SCT), which usually provides
 930 eight measurements per track. This is lower granularity, but similar in concept to the pixel
 931 detector.

932 Both of these silicon detectors are complemented by the transition radiation tracker
 933 (TRT), which extends the radial track reconstruction within the range $|\eta| < 2.0$. This is
 934 a different design, composed of *drift tubes*, i.e. straws filled with Xenon gas with a wire
 935 in the center, but similarly collects electrons displaced by ionizing particles. In addition,
 936 the TRT includes materials with widely varying indices of refraction, which leads to the
 937 production of transition radiation, namely radiation produced by a charged particle passing
 938 through an inhomogeneous medium. The energy loss on such a transition is proportional
 939 to the Lorentz factor $\gamma = E/m$ – correspondingly, lighter particles (e.g. electrons) tend to
 940 lose more energy and emit more photons compared to heavier particles (e.g. pions). In the
 941 detector, this corresponds to a larger fraction of hits (typically 30 in total) above a given

942 high energy-deposit threshold for electrons, providing particle identification information.

943 *3.2.3 Calorimeter*

944 Surrounding the inner detector in ATLAS is the calorimeter. The principle of the calorimeter
945 is to completely absorb the energy of a produced particle in order to measure it. However,
946 a pure block of absorber does not provide much information about the particle interaction
947 with the material. The ATLAS calorimeter therefore has a *sampling calorimeter* structure,
948 namely, layers of absorber interspersed with layers of sensitive material, giving the calorimeter
949 “stopping power” while allowing detailed measurement of the resulting particle shower and
950 corresponding deposited energy.

951 The ATLAS calorimetersystem covers the pseudorapidity range $|\eta| < 4.9$, and is primarily
952 composed of two components, an electromagnetic calorimeter, designed to measure particles
953 which primarily interact via electromagnetism (e.g. photons and electrons), and a hadronic
954 calorimeter, designed to measure particles which interact via the strong force (e.g. pions,
955 other hadrons). We will return to the differences between these in a moment.

956 In ATLAS, the electromagnetic calorimeter covers the region of $|\eta| < 3.2$, and uses
957 lead for the absorbers and liquid-argon for the sensitive material. It is high granularity
958 and, geometrically, has two components: the “barrel”, which covers the cylindrical body of
959 the detector volume and the “endcap”, covering the ends. An additional thin liquid-argon
960 presampler covers $|\eta| < 1.8$ to correct for energy loss in material upstream of the calorimeters.

961 The hadronic calorimeter is composed of alternating steel and plastic scintillator tiles,
962 segmented into three barrel structures within $|\eta| < 1.7$, in addition to two copper/liquid-argon
963 endcap calorimeters.

964 The solid angle coverage is completed with forward copper/liquid-argon and tungsten/liquid-
965 argon calorimeter modules optimized for electromagnetic and hadronic energy measurements
966 respectively.

967 *3.2.4 Muon Spectrometer*

968 While muons interact electromagnetically, they are around 200 times heavier than electrons
 969 ($m_\mu = 106 \text{ MeV}$, while $m_e = 0.510 \text{ MeV}$). Therefore, electromagnetic interactions with ab-
 970 sorbers in the calorimeter are not sufficient to stop them, and, as they do not interact via the
 971 strong force, hard scattering with nuclei is rare. A dedicated system for muon measurements
 972 is therefore required.

973 The muon spectrometer (MS) is the outermost layer of ATLAS and is designed for this
 974 purpose. It is composed of three parts: a set of triggering chambers, which detect if there is
 975 a muon and provide a coordinate measurement, in conjunction with high-precision tracking
 976 chambers, which measure the deflection of muons in a magnetic field to measure muon
 977 momentum, similar to the inner detector solenoid. The magnetic field is generated by the
 978 superconducting air-core toroidal magnets, with a field integral between 2.0 and 6.0 T m
 979 across most of the detector. The toroid magnetic field runs roughly in a circle in the $x - y$
 980 plane around the beam line, leading to muon curvature along the z-axis.

981 The precision tracking system covers the region $|\eta| < 2.7$ via three layers of monitored
 982 drift tubes, and is complemented by cathode-strip chambers in the forward region, where the
 983 background is highest. The muon trigger system covers the range $|\eta| < 2.4$ with resistive-plate
 984 chambers in the barrel, and thin-gap chambers in the endcap regions.

985 *3.2.5 Triggering*

986 During a typical run of the LHC, there are roughly 1 billion collisions in ATLAS per second
 987 (1 GHz), corresponding to a 40 MHz bunch crossing rate. *TODO: cite: <https://cds.cern.ch/record/1457044/file>*
 988 Saving the information from all of them is not only unnecessary, but infeasible. The ATLAS
 989 trigger system provides a sophisticated set of selections to filter the collision data and only
 990 keep those collision events useful for downstream analysis.

991 These events are selected by the first-level trigger system, which is implemented in custom
 992 hardware, and accepts events at a rate below 100 kHz. Selections are then made by algorithms

993 implemented in software in the high-level trigger [31], reducing this further, and, in the end,
994 events are recorded to disk at much more manageable rate of about 1 kHz.

995 An extensive set of ATLAS software [32] is open source, including the software used for
996 real and simulated data reconstruction and analysis and that used in the trigger and data
997 acquisition systems of the experiment.

998 3.2.6 Particle Showers and the Calorimeter

999 The design of the ATLAS detector is directly tied to the physics it is trying to detect. Of these,
1000 possibly the most non-trivial distinction is in the calorimeter design. It is therefore useful to
1001 discuss in more detail the various properties of electromagnetic and hadronic interactions
1002 with material, and how these correspond to the particle showers measured by the detector
1003 described above.

1004 Electromagnetic showers in ATLAS predominantly occur via bremsstrahlung, or “braking
1005 radiation”, and electron-positron pair production. This proceeds roughly as follows: an electron
1006 entering a material is deflected by the electromagnetic field of a heavy nucleus. This results in
1007 the radiation of a photon. That photon produces an electron-positron pair, and the process
1008 repeats, resulting in a shower structure. At each step, characterized by *radiation length*, X_0 ,
1009 the number of particles approximately doubles and the average particle energy decreases by
1010 approximately a factor of two. *TODO: Include nice Thomson image*

Note that bremsstrahlung and pair production only dominate in specific energy regimes, with other processes taking over depending on particle energy. For electrons, bremsstrahlung only dominates for higher energies, as low energy electrons will form ions with the atoms of the material. The point where the rates for the two processes are equal is called the *critical energy*, and is roughly

$$E_c \approx \frac{800 \text{ MeV}}{Z} \quad (3.1)$$

1011 where Z is the nuclear charge. From a similar analysis of rates, we may see that the
1012 bremsstrahlung rate is inversely proportional to the square of the mass of the particle. This

₁₀₁₃ explains why muons do not shower in a similar way, as the rate of bremsstrahlung is suppressed
₁₀₁₄ by $(m_e/m_\mu)^2$ relative to electrons.

For lead, the absorber used for the ATLAS electromagnetic calorimeter, which has $Z = 82$, this critical energy is therefore around 10 MeV. Electrons resulting from LHC collisions are of a 1.3×10^3 GeV scale. With the approximation of a reduction in particle energy by a factor of two every radiation length, the number of radiation lengths before the critical energy is reached is

$$x = \frac{\ln(E/E_c)}{\ln 2} \quad (3.2)$$

₁₀₁₅ such that for a 100 GeV shower in lead, $x \sim 13$. The radiation length for lead is around
₁₀₁₆ 0.56 cm, such that an electromagnetic shower could be expected to be captured within 10 cm
₁₀₁₇ of lead.

₁₀₁₈ Electromagnetic showers are therefore characterized by depositing much of their energy
₁₀₁₉ within a small region of space. As we show below (Chapter 4) though electromagnetic
₁₀₂₀ showering is not deterministic, the large number of particles and the restricted set of processes
₁₀₂₁ involved means that the shower development as a whole is very similar between individual
₁₀₂₂ electromagnetic showers of the same energy.

₁₀₂₃ For completeness, note as well that pair production dominates for photons of energy greater
₁₀₂₄ than around 10 MeV, whereas for lower energies (below around 1 MeV), the photoelectric
₁₀₂₅ effect, namely atomic photon absorption and electron emission, dominates.

₁₀₂₆ Hadronic showers are distinguished by the fact that they interact strongly with atomic
₁₀₂₇ nuclei. They are correspondingly more complex because (1) they involve a wider variety
₁₀₂₈ of processes than electromagnetic showers, and (2) these processes have a wide variety of
₁₀₂₉ associated length scales. Because these are heavier than electrons (e.g. protons and charged
₁₀₃₀ pions) bremsstrahlung is suppressed, but ionization interactions with the electrons will cause
₁₀₃₁ these particles to lose energy as they pass through the material. Hadronic showering occurs
₁₀₃₂ on interaction with atomic nuclei. This may lead to production of, e.g. both charged (π^\pm)
₁₀₃₃ and neutral (π^0) pions. The π^0 lifetime is much much shorter than that of the charged pions
₁₀₃₄ (around a factor of 10^8), and immediately decays to two photons, starting an electromagnetic

shower, as described above. The longer lived π^\pm travel further in the detector before experiencing another strong interaction with more particles produced, also with varying lifetimes and decay properties.

It is therefore immediately apparent that hadronic showers are more complex than electromagnetic ones (electromagnetic showers can be a subset of the hadronic!), and therefore much more variable from shower to shower. The length scales involved are also significantly larger due to the reliance on nuclear interactions, characterized by length λ_I , which is around 17 cm for iron (used in the ATLAS hadronic calorimeter). This motivates the calorimeter design, and results in the properties demonstrated in Figure 3.2.

1044

Chapter 4

1045

SIMULATION

1046 Simulated physics samples are a core piece of the physics output of the Large Hadron
 1047 Collider, providing a map from a physics theory into what is observed in our detector. This
 1048 is crucial for searches for new physics, where simulation is necessary to describe what a given
 1049 signal model looks like, but also extremely valuable for describing the physics of the Standard
 1050 Model, providing detailed predictions of background processes for use in everything from
 1051 designing simple cuts to training multivariate discriminators. Broadly, simulation can be split
 1052 into two stages: *event generation*, in which physics theory is used to generate a description of
 1053 particles present after a proton-proton collision, and *detector simulation*, which passes this
 1054 particle description through a simulation of the detector material, providing a view of the
 1055 physics event as it would be seen in ATLAS data. Such simulation is often called Monte Carlo
 1056 in reference to the underlying mathematical framework, which relies on random sampling.

1057 **4.1 Event Generation**

1058 A variety of tools are used to simulate various aspects of event generation. MADGRAPH [33]
 1059 is commonly used for the generation of the “hard scatter” event, i.e., two protons collide
 1060 and some desired physics process happens. In practice, this is not quite as simple as two
 1061 quarks or gluons interacting. Protons are composed of three “valence” quarks with various
 1062 momenta interacting with each other via exchange of gluons, but also a sea of virtual gluons
 1063 which may decay into other quarks. A hard scatter event is therefore characterized by
 1064 the corresponding particle level diagrams, but additionally by a set of *parton distribution*
 1065 *functions* (PDFs), which describe the probability to find constituent quarks or gluons at
 1066 carrying various momenta at a given energy scale (often written Q^2). Such PDFs are measured

1067 experimentally *TODO: cite* and the selection of a “PDF set” and a given physics process
 1068 characterizes the hard scatter. Depending on the model being considered and the particular
 1069 theoretical constraints, processes are often simulated at either leading (LO) or next to leading
 1070 order (NLO), corresponding to the order of the perturbative expansion (i.e. tree level or 1
 1071 loop diagrams). Various additional tools are developed for such NLO calculations, including
 1072 POWHEG Box v2 [34–36], which is used for this thesis.

1073 The hard scatter is not the only component of a given collider event, however. Incoming
 1074 and outgoing particles are themselves very energetic and may radiate particles along their
 1075 trajectory. In particular, gluons, which have a self-interaction term as described in Chapter 1,
 1076 may be radiated, which subsequently themselves radiate gluons or decay to quarks which can
 1077 also radiate gluons, in a whole mess of QCD that both contributes to the particle content
 1078 of a collider event and is not directly described by the hard scatter. This cascade, called
 1079 a *parton shower*, has a dedicated set of simulation tools, commonly HERWIG 7 [37][38] and
 1080 PYTHIA 8 [39], which interface with tools such as MADGRAPH for simulation.

1081 Due to color confinement (Chapter 1), quarks and gluons cannot be observed free particles,
 1082 but rather undergo a process called hadronization, in which they are grouped into colorless
 1083 hadrons (e.g. *mesons*, consisting of one quark and one antiquark). In simulation, this is also
 1084 handled with HERWIG 7 and PYTHIA 8.

1085 The physics of *b*-quarks is quite important for a variety of searches for new physics and
 1086 measurements of the Standard Model, including this thesis work *TODO: ref flavor tagging*
 1087 *sec?*. Correspondingly, the decay of “heavy flavor” particles (e.g. *B* and *D* mesons, containing
 1088 *b* and *c* quarks respectively) has been very well studied, and a dedicated simulation tool,
 1089 EVTGEN [40], is used for such processes.

1090 *TODO: add nice parton shower image*

1091 4.2 Detector Simulation

1092 Event generation provides a full description of the particle content of a given collider event.
 1093 In reality, however, we do not have access to such a description, and must rely on physical

1094 detectors to collect information about said particle content. The design and components of
 1095 the ATLAS detector are described in Chapter 3. Simulation of this detector quickly becomes
 1096 complicated – there are a variety of different materials and subdetectors, each with particular
 1097 configurations and resolutions. Interactions of particles with the detector materials can cause
 1098 showering, and such showers must be simulated and characterized.

1099 In ATLAS, the GEANT4 [41] simulation toolkit is used for detailed simulation of the
 1100 ATLAS detector, often referred to as *full simulation*. The method can be thought of as
 1101 proceeding step by step as a particle moves through the detector, simulating the interaction
 1102 of the material at each stage, and following each branch of each resulting shower with a
 1103 similarly detailed step by step simulation.

1104 This type of simulation is very computationally intensive, especially in the calorimeter,
 1105 which has a high density of material, leading to an extremely large set of material interactions
 1106 to simulate. There is correspondingly a large effort within ATLAS to develop techniques to
 1107 decrease the computational load – these techniques will be of increasing importance for Run
 1108 3 and the HL-LHC *TODO: include classic budget plot*.

1109 The fast simulation used for this thesis, AtlFast-II [42], is one such technique, which uses
 1110 a parametrized simulation of the calorimeter, called FastCaloSim, in conjunction with full
 1111 simulation of the inner detector, to achieve an order of magnitude speed up in simulation
 1112 time. This parametrized simulation uses a simplified detector geometry, in conjunction with
 1113 a simulation of particle shower development based on statistical sampling of distributions
 1114 from fully simulated events, to massively speed up simulation time and computational load.

1115 Such a speed up comes at a bit of a cost in performance. In particular, the modeling of
 1116 jet substructure (see Chapter 5) historically has been an issue for FastCaloSim. The ATLAS
 1117 authorship qualification work supporting this thesis is an effort to improve such modeling,
 1118 and is part of a suite of updates being considered for a new fast simulation targeting Run 3.
 1119 We briefly describe this work in the following.

1120 **4.3 Correlated Fluctuations in FastCaloSim**

1121 A variety of developments have been made to FastCaloSim, improving on the version used for
 1122 AtlFast-II. This new fast calorimeter simulation [43] is largely based on two components: one
 1123 which describes the *total energy* deposited in each calorimeter layer as a shower moves from
 1124 the interaction point outward, and one which describes the *shape*, i.e., the pattern of energy
 1125 deposits, of a shower in each respective calorimeter layer. Both methods are parametrizations
 1126 of the full simulation, and therefore are considered to be performing well if they are able
 1127 to reproduce corresponding full simulation distributions. Of course, directly sampling from
 1128 a library of showers would identically reproduce such distributions – however a statistical
 1129 sampling of various shower *properties* provides much more generality in the simulation.

1130 For the simulation of total energy in each given layer, the primary challenge is that such en-
 1131 ergy deposits are highly correlated. The new FastCaloSim thus relies on a technique called Prin-
 1132 cipal Component Analysis (PCA) *TODO: cite <https://root.cern.ch/doc/master/classTPrincipal.html>*
 1133 to de-correlate the layers, aiding parametrisation.

1134 The PCA chain transforms N energy inputs into N Gaussians and projects these Gaussians
 1135 onto the eigenvectors of the corresponding covariance matrix. This results in N de-correlated
 1136 components, as the eigenvectors are orthogonal. The component of the PCA decomposition
 1137 with the largest corresponding eigenvalue is then used to define bins, in which showers
 1138 demonstrate similar patterns of energy deposition across the calorimeter layers. To further
 1139 de-correlate the inputs, the PCA chain is repeated on the showers within each such bin. This
 1140 full process is reversed for the particle simulation. A full description of the method can be
 1141 found in [43].

1142 Modeling of the lateral shower shape makes use of 2D histograms filled with GEANT4
 1143 hit energies in each layer and PCA bin. Binned in polar $\alpha - R$ coordinates in a local plane
 1144 tangential to the surface of the calorimeter system, these histograms represent the spatial
 1145 distribution of energy deposits for a given particle shower. Such histograms are constructed
 1146 for a number of Geant4 events, and the histograms for each event are normalized to total

1147 energy deposited in the given layer. The average of these histograms is then taken (what is
 1148 called here the “average shape”).

1149 In simulation, these average shape histograms are used as probability distributions, from
 1150 which a finite number of equal energy hits are drawn. This finite drawing of hits induces
 1151 a statistical fluctuation about the average shape which is tuned to match the expected
 1152 calorimeter sampling uncertainty.

1153 As an example, the intrinsic resolution of the ATLAS Liquid Argon calorimeter has a
 1154 sampling term of $\sigma_{\text{samp}} \approx 10\%/\sqrt{E}$ [44]. The number of hits to be drawn for each layer, $N_{\text{hits}}^{\text{layer}}$,
 1155 is thus taken from a Poisson distribution with mean $1/\sigma_{\text{samp}}^2$, where the energy assigned to
 1156 each hit is then just $E_{\text{hit}} = \frac{E_{\text{layer}}}{N_{\text{hits}}^{\text{layer}}}$. This induces a fluctuation of the order of $10\%/\sqrt{E_{\text{bin}}}$ for
 1157 each bin in the average shape.

1158 Figure 4.1 shows a comparison of energy and weta2 [45], defined as the energy weighted
 1159 lateral width of a shower in the second electromagnetic calorimeter layer, for 16 GeV photons
 1160 simulated with the new FastCaloSim and with full GEANT4 simulation. The agreement is
 1161 quite good, with FastCaloSim matching the Geant4 mean to within 0.3 and 0.03 percent
 respectively. Similar results are seen for other photon energies and η points.

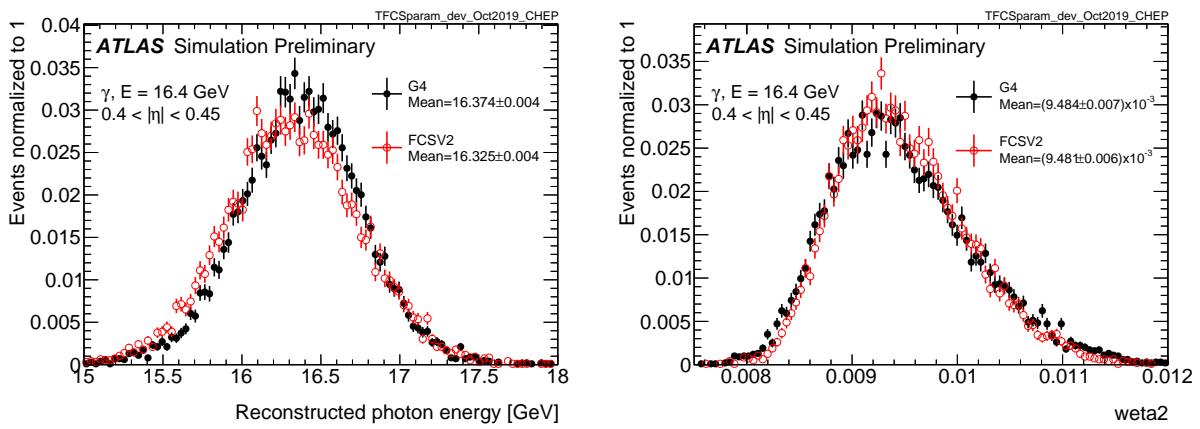


Figure 4.1: Energy and lateral shower width variable, weta2, for 16 GeV photons with full simulation (G4) and FastCaloSimV2 (FCSV2) [43].

1163 4.3.1 *Fluctuation Modeling*

1164 Figure 4.2 shows the ratio of calorimeter cell energies for single GEANT4 photon and pion
 1165 events to the corresponding cell energies in their respective average shapes. While the photon
 1166 event is quite close to the corresponding average, the pion event shows a deviation from the
 1167 average which is much larger and has a non-trivial structure, reflecting the different natures
 1168 of electromagnetic and hadronic showering.

1169 While the shape parametrization described above is thus sufficient for describing electro-
 1170 magnetic showers, we will demonstrate below that it is not sufficient for describing hadronic
 1171 showers (Figures 4.5 and 4.6). We therefore present and validate methods to improve this
 1172 hadronic shower modeling.

1173 Two methods for modeling deviations from the average shape have been studied: (1)
 1174 a neural network based approach using a Variational Autoencoder (VAE) [46] and (2) a
 1175 map through cumulative distributions to an n -dimensional Gaussian. With both methods,
 1176 the shape simulation then proceeds as described in Section ??, with the drawing of hits
 1177 according to the average shape. However, these hits no longer have equal energy, but have
 1178 weights applied to increase or decrease their energy depending on their spatial position.
 1179 This application of weights is designed to mimic a realistic shower structure and to encode
 1180 correlations between energy deposits.

1181 Both methods are trained on ratios of energy in binned units called voxels. This voxelization
 1182 is performed in the same polar $\alpha - R$ coordinates as the average shape, with a 5 mm core in
 1183 R and 20 mm binning thereafter. There are a total of 8 α bins from 0 to 2π and 8 additional
 1184 R bins from 5 mm to 165 mm. The 5 mm core is filled with the average value of core voxels
 1185 across the 8 α bins when creating the parametrisation. However, during simulation, each of
 1186 these 8 core bins is treated independently. The outputs of both methods mimic these energy
 1187 ratios and are used in the shape simulation as the weights described above. In contrast to
 1188 an approach based on, e.g., calorimeter cells, using voxels allows for flexibility in tuning the
 1189 binning used in creating the parametrisation. Further, due to their relatively large size, using

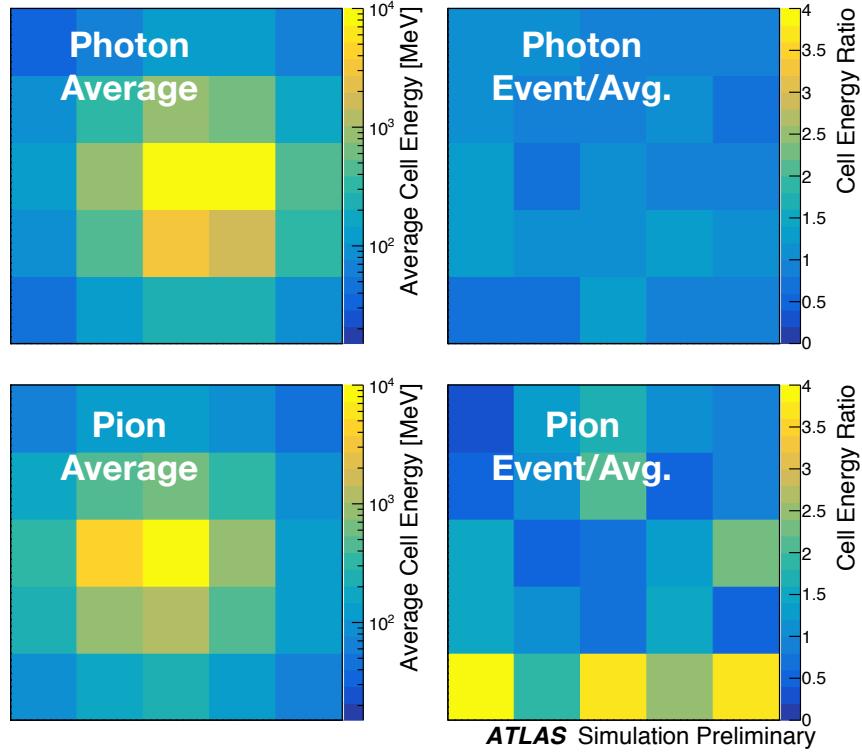


Figure 4.2: Example of photon and pion average shapes in 5×5 calorimeter cells. The left column shows the average shape over a sample of 10000 events, while the right column shows the energy ratio, in each cell, of single GEANT4 events with respect to this average. The photon ratios are all close to 1, while the pion ratios show significant deviation from the average.

1190 calorimeter cells is subject to “edge effects”, where the splitting of energy between cells has a
 1191 non-trivial effect on the observed energy ratio. The binning used here is of the order of half
 1192 of a cell size, mitigating this effect.

1193 The Gaussian method operates by using cumulative distributions to map GEANT4 energy
 1194 ratios to a multidimensional Gaussian distribution. New events are generated by randomly
 1195 sampling from this Gaussian distribution.

1196 For the VAE method, a system of two linked neural networks is trained to generate events.

1197 The first “encoder” neural network maps input GEANT4 energy ratios to a lower dimensional
 1198 latent space. A second “decoder” neural network then samples from that latent space and
 1199 tries to reproduce the inputs. In simulation, events are generated by taking random samples
 1200 from the latent space and passing them through the trained decoder.

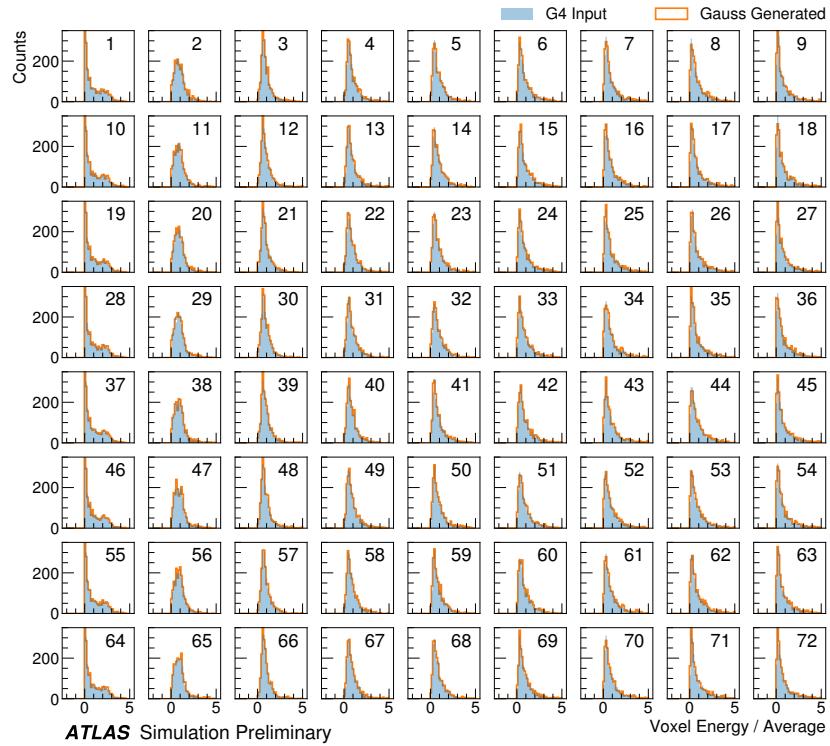


Figure 4.3: Distribution of the ratio of voxel energy in single events to the corresponding voxel energy in the average shape, with GEANT4 events in blue and Gaussian model events in orange, for 65 GeV central pions in EMB2. Moving top to bottom corresponds to increasing α , left to right corresponds to increasing R , with core voxels numbered 1, 10, 19, Agreement is quite good across all voxels. Results are similar for the VAE method.

1201 Figure 4.3 shows the distributions of input GEANT4 and Gaussian method generated
 1202 energy ratios in the grid of voxels. Figure 4.4 shows the correlation coefficient between the
 1203 center voxel from $\alpha = 0$ to $2\pi/8$ for input GEANT4 and the Gaussian and VAE fluctuation

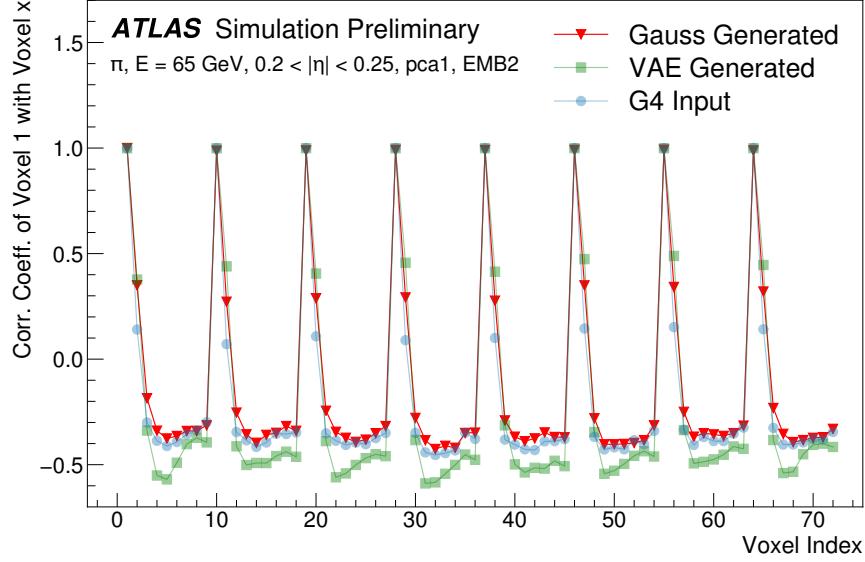


Figure 4.4: Correlation coefficient of ratios of voxel energy in single events to the corresponding voxel energy in the average shape, examined between the core bin from $\alpha = 0$ to $2\pi/8$ and each of the other voxels. The periodic structure represents the binning in α , and the increasing numbers in each of these periods correspond to increasing R , where the eight points with correlation coefficient 1 are the eight core bins. Both the Gaussian and VAE generated toy events are able to reproduce the major correlation structures for 65 GeV central pions in EMB2.

1204 methods. Agreement is good throughout.

1205 Validation of the Gaussian and VAE fluctuation methods was performed within FastCaloSimV2.

1206 Figure 4.5 shows the energy ratio of cells for a given simulation to the corresponding cells in
 1207 the average shape as a function of the distance from the shower center. The mean for all
 1208 simulation methods is expected to be around 1, with deviation from the average (the RMS
 1209 fluctuation) shown by the error bars. The Gaussian method RMS (red) and VAE method
 1210 RMS (green) both match the GEANT4 RMS (yellow) better than the case without correlated
 1211 fluctuations (blue) for a variety of energies, η points, and layers, often reproducing 80 – 100 %

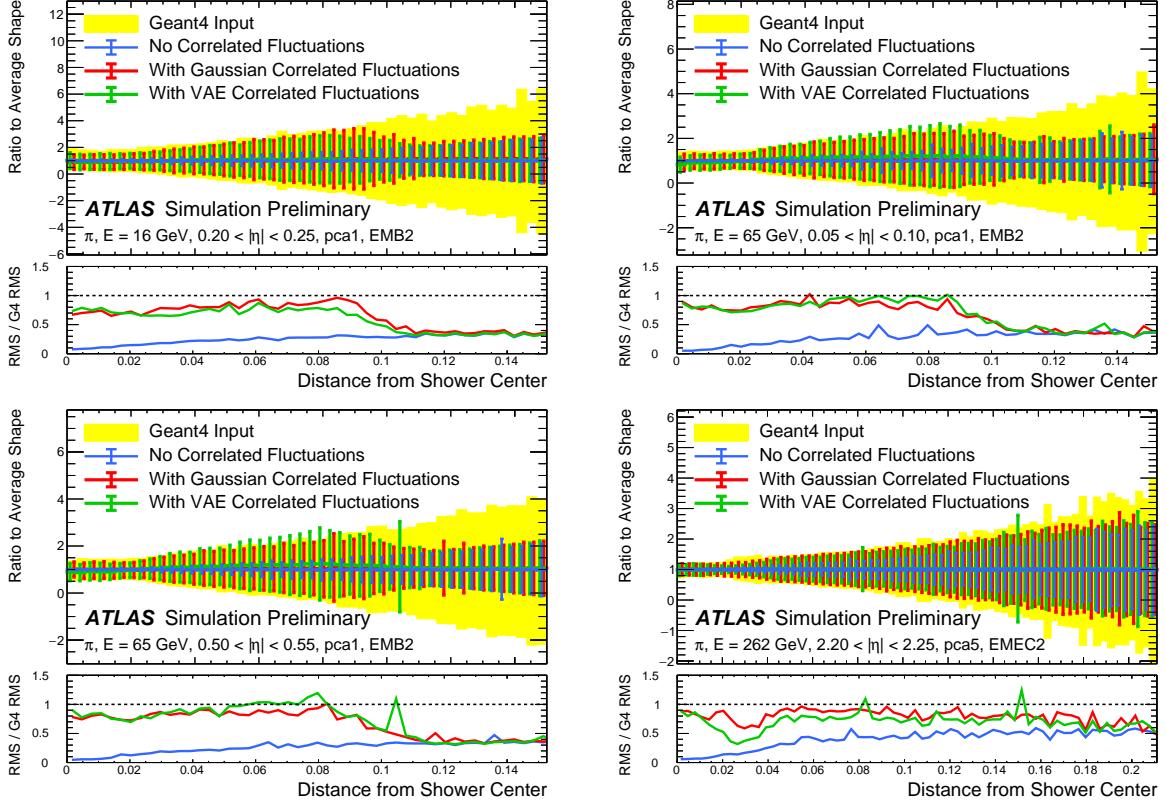


Figure 4.5: Comparison of the RMS fluctuations about the average shape with the Gaussian fluctuation model (red), the VAE fluctuation model (green), and without correlated fluctuations (blue) for a range of pion energies, η points, and layers.

of the GEANT4 RMS magnitude, compared to the 5 – 30% observed in the no correlated fluctuations case.

Figure 4.6 shows the result of a simulation with full ATLAS reconstruction for 65 GeV central pions with the Gaussian fluctuation model. The simulation with the Gaussian fluctuation model demonstrates improved modeling of several shape variables relative to baseline FastCaloSimV2, reproducing the distributions of events simulated with GEANT4.

The new fast calorimeter simulation is a crucial part of the future of simulation for the ATLAS Experiment at the LHC. The per event simulation time of the full detector with GEANT4,

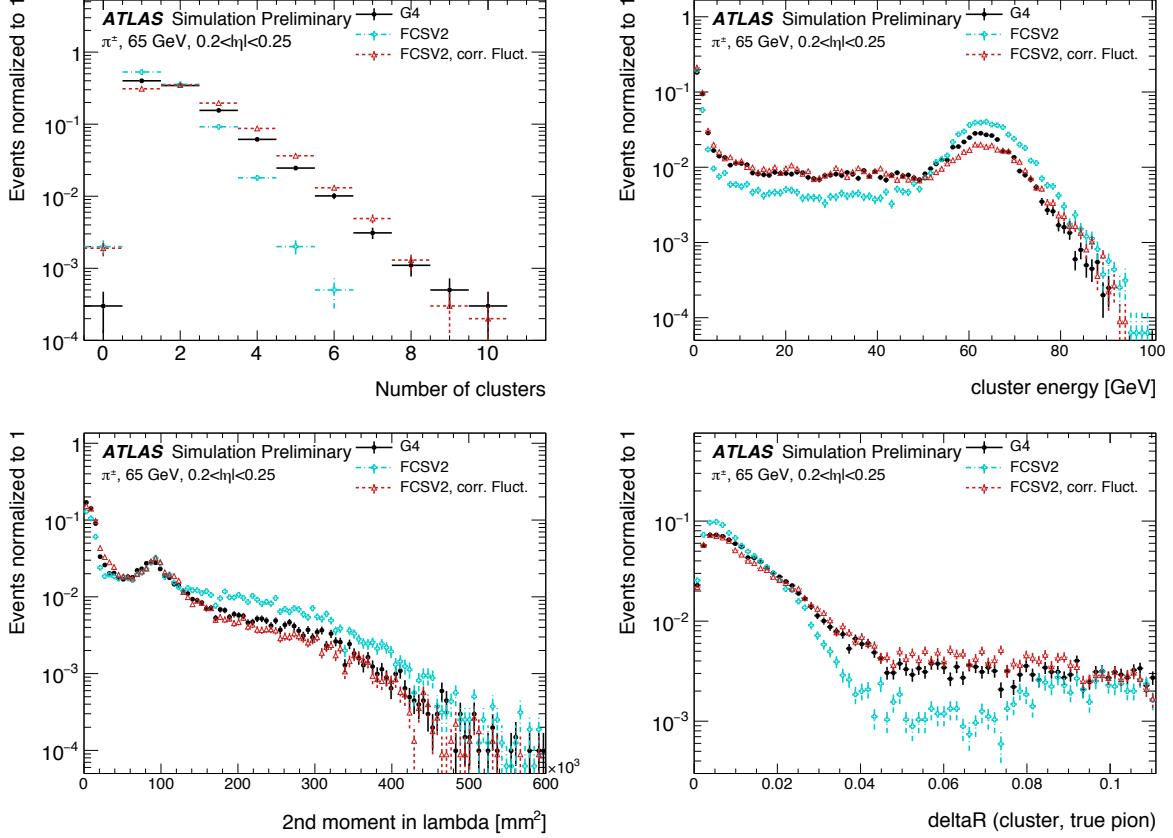


Figure 4.6: Comparison of the Gaussian fluctuation model to the default FCSV2 version and to G4 simulation, using pions of 65 GeV energy and $0.2 < |\eta| < 0.25$. With the correlated fluctuations, several shape variables demonstrate improved modeling.

1220 calculated over 100 $t\bar{t}$ events, is 228.9 s. Using FastCaloSim for the calorimeter simulation
 1221 reduces this to 26.6 s, of which FastCaloSim itself is only 0.015 s. Good physics modeling is
 1222 achieved, the correlated fluctuations method shows good proof of concept improvement for
 1223 the modeling of hadronic showers.

1224

Chapter 5

1225

RECONSTRUCTION

1226 Chapter 3 discusses how a proton-proton collision may be captured by a physical detector
 1227 and turned into data that may be stored and analyzed. Chapter 4 discusses the simulation
 1228 of this same process. At this most basic level, however, the ATLAS detector is only a
 1229 machine for turning particles into a set of electrical signals, albeit in a very sophisticated,
 1230 physics motivated way. This chapter discusses the step of turning these electrical signals into
 1231 objects which may be identified with the underlying physics processes, and therefore used to
 1232 make statements about what occurred within a given collision event. This process is termed
 1233 *reconstruction*, and we will focus particularly on jets and flavor tagging, as the most relevant
 1234 pieces for this thesis work.

1235

5.1 Jets

1236 As discussed in Chapters 3 and 4, the production of particles with color charge from a
 1237 proton-proton interaction is complicated both by parton showering and by confinement: a
 1238 quark produced from a hard scatter is not seen as a quark, but rather, as a spray of particles
 1239 with a variety of hadrons in the final state, which subsequently shower upon interaction with
 1240 the calorimeter in a complicated way.

1241 For hard scatter electrons, photons, or muons on the other hand, the picture is much
 1242 clearer: there is no parton showering, and each has a distinct signature in the detector:
 1243 photons have no tracks and a very localized calorimeter shower, electrons are associated
 1244 with tracks and are similarly localized in the calorimeter, and muons are associated with
 1245 tracks, pass through the calorimeter due to their large mass, and leave signatures in the muon
 1246 spectrometer.

1247 Jets are a tool to deal with the messiness of quarks and gluons. The basic concept is to
 1248 group the multitude of particles produced by a quark or gluon decay into a single object. Such
 1249 an object then has associated properties, including a four-vector, which may be identified
 1250 with the corresponding initial state particle. In practice a variety of information from the
 1251 ATLAS detector is used for such a reconstruction. The analysis considered in this thesis uses
 1252 particle flow jets [47], which combines information from both the tracker and the calorimeter,
 1253 where the combined objects may be identified with underlying particles. However, jets built
 1254 from clusters of calorimeter cells [48] as well as from charged particle tracks [49] have also
 1255 been used very effectively.

1256 A variety of algorithms are used to associate detector level objects to a given jet. The
 1257 most commonly used in ATLAS is the anti- k_T algorithm [50], which is a successor to the
 1258 k_T algorithm, among others [51], and develops as follows. Both algorithms are sequential
 1259 recombination algorithms, which begin with the smallest distance, d_{ij} between considered
 1260 objects (e.g. particles or intermediate groupings of particles). If d_{ij} is less than a parameter
 1261 d_{iB} (B for “beam”) object i is combined with object j , the distance d_{ij} is recomputed, and
 1262 the process repeats. This proceeds until $d_{ij} \geq d_{iB}$, at which point the jet is “complete” and
 1263 removed from the list of considered objects.

The definitional difference between k_T and anti- k_T is these distance parameters. In general
 form, these are defined as

$$d_{ij} = \min(p_{Ti}^{2p}, p_{Tj}^{2p}) \frac{\Delta R_{ij}^2}{R^2} \quad (5.1)$$

$$d_{iB} = p_{Ti}^{2p} \quad (5.2)$$

1264 where p_{Ti} is the transverse momentum of object i , ΔR_{ij} is the angular distance between
 1265 objects i and j , R is a radius parameter, and p controls the tradeoff between the p_T and
 1266 angular distance terms. For the k_T algorithm $p = 1$; for the anti- k_T algorithm, $p = -1$. This
 1267 is a simple change, but results in significantly different behavior.

The anti- k_T algorithm can be understood as follows: for a single high p_T particle (p_{T1})
 surrounded by a bunch of low p_T particles, the low p_T particles will be clustered with the

high p_T one if

$$d_{1j} = \frac{1}{p_{T1}^2} \frac{\Delta R_{1j}^2}{R^2} < \frac{1}{p_{T1}^2} \quad (5.3)$$

$$\implies \Delta R_{1j} < R. \quad (5.4)$$

1268 Therefore, a single high p_T particle (p_{T1}) surrounded by a bunch of low p_T particles results in
 1269 a perfectly conical jet. This shape may change with the presence of other high momentum
 1270 particles, but the key feature of the dynamics is that the jet shape is determined by high p_T
 1271 objects due to the $\frac{1}{p_T}$ nature of this definition. In contrast, the k_T algorithm results in jets
 1272 influenced by low momentum particles, which results in a less regular shape. This property,
 1273 of regular jet shapes determined by high momentum objects, as well as demonstrated good
 1274 practical performance, makes the anti- k_T algorithm the favored jet algorithm in ATLAS.

1275 Because jets are composed of multiple objects, a useful property of jets is jet *substructure*,
 1276 that is, acknowledging that jets are composite objects, analyzing the structure of a given
 1277 jet to infer physics information. This leads to the use of *subjets*; that is, after running jet
 1278 clustering, often to create a “large-R”, $R = 1.0$ anti- k_T jet, a smaller radius jet clustering
 1279 algorithm is run within the jet. Subjets are often chosen using the k_T algorithm, which again
 1280 is sensitive to lower momentum particles, with $R = 0.2$ or 0.3 . For the boosted version of this
 1281 thesis analysis, such a strategy is used, in which the subjets are *variable radius* and depend
 1282 on the momentum of the (proto)jet in question. Beyond this thesis work, substructure is
 1283 used in a large variety of analyses, with a set of associated variables and tools developed for
 1284 exploiting this structure *TODO: Cite some?*.

1285 5.2 Flavor Tagging

1286 For this this thesis, the physics process being considered is $HH \rightarrow b\bar{b}b\bar{b}$. From the previous
 1287 section, we know that the standard practice is to identify these b quarks (or, rather, the
 1288 resulting B hadrons, due to confinement) with jets – in our case, these b -*jets* are $R=0.4$
 1289 anti- k_T particle flow jets (see Chapter 7). However, not all jets produced at the LHC are
 1290 from B hadrons; rather, there are a variety of different types of jets corresponding to different

flavors of quarks. These are often classified as light jets (from u , d , or s quarks, or gluons) or as other *heavy flavor* jets, e.g. c -jets, involving c quarks. Distinguishing between these different categories is a very active area of work in ATLAS, termed *flavor tagging*, with much focus on *b-tagging* in particular, that is, the identification of jets from B hadron decays. We here briefly describe the techniques used for flavor tagging in ATLAS.

What distinguishes a b -jet from any other jet? This question is fundamental to the design of the various b -tagging algorithms, and has two major answers: (1) B hadrons have long lifetimes, and (2) B hadrons have large masses. It is most illustrative to compare the B hadron properties to a common light meson, e.g. π^0 , the neutral pion, with quark content $\frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$. B hadrons have lifetimes around 1.5 ps, corresponding to a decay length $c\tau \approx 0.45$ mm. In contrast, π^0 has a lifetime of 8.4×10^{-5} ps, which is around 20,000 times shorter! Theoretically, this comes from CKM suppression of the b to c transition *TODO: check*, which dominates the B decay modes. Experimentally, this difference pops up as shown in Figure 5.1 – light flavor initiated jets decay almost immediately at the proton-proton interaction point, whereas b -jets are distinguished by a displaced secondary vertex, corresponding to the 5 mm decay length calculated above. This displaced vertex falls short of the detector itself, but may be inferred from larger transverse (perpendicular to beam) and longitudinal (parallel to beam) impact parameters of the resulting tracks, termed d_0 and z_0 respectively.

Coming to the mass, B mesons have masses of around 5.2 GeV, whereas the π^0 mass is around 0.134 GeV, (around 40 times lighter). This higher mass gives access to a larger decay phase space, leading to a high multiplicity for b -jets (average of 5 charged particles per decay).

One final distinguishing feature of B hadrons is their *fragmentation function*, a function describing the production of an observed final state. For B hadrons, this is particularly “hard”, with the B hadrons themselves contributing to an average of around 75 % of the b -jet energy. Thus, the identification of b -jets with B hadrons is, in some sense, descriptive.

We have contrasted b -jets and light jets, demonstrating that there are several handles available for making this distinction. c -jets are slightly more similar to b -jets, but the same

1319 handles still apply – c -hadron lifetimes are between 0.5 and 1 ps, a factor of 2 smaller than B
1320 hadrons. Their mass is around 1.9 GeV, 2 to 3 times smaller than B hadrons, and c -hadrons
1321 contribute to an average of around 55 % of c -jet energy. Therefore, while the gap is slightly
1322 smaller, a distinction may still be made.

1323 The ATLAS flavor tagging framework [53] relies on developing a suite of “low-level”
1324 taggers, which use a variety of information about tracks and vertices as inputs. The output
1325 of these lower level taggers are then fed into a higher level tagger, which aggregates these
1326 results into a high level discriminant. Each of these taggers is described below.

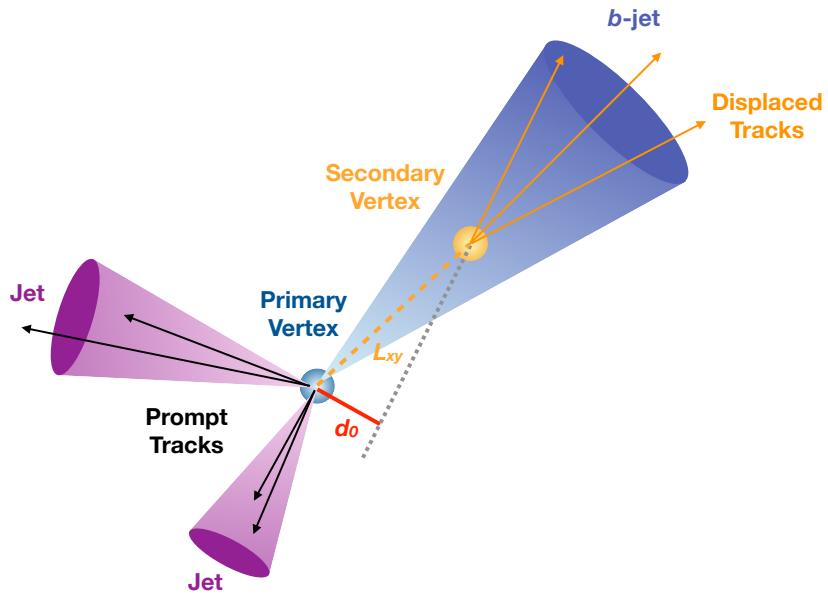


Figure 5.1: Illustration of an interaction producing two light jets and one b -jet in the transverse plane. While the light jets decay “promptly”, coinciding with the primary vertex of the proton-proton interaction, the longer lifetime of B hadrons leads to a secondary decay vertex, displaced from the primary vertex by length L_{xy} . This is typically a few mm, and therefore is not directly visible in the detector, but leads to a large transverse impact parameter, d_0 , for the resulting tracks. [52]

1327 5.2.1 IP2D/3D

1328 IP2D and IP3D are taggers based on the large track impact parameter (IP) nature of B
 1329 hadron decays. Both are based on histogram templates derived from Monte Carlo simulation,
 1330 which are used as probability density functions to construct log-likelihood discriminants.
 1331 IP2D incorporates just the transverse impact parameter information using 1D histogram
 1332 templates, whereas IP3D uses both transverse and longitudinal impact parameters in a 2D
 1333 template, which accounts for correlations. Importantly, these are *signed* impact parameters,
 1334 with sign based on the angle between the impact parameter and the considered jet – positive
 1335 impact parameters are consistent with a track extrapolation in front of the jet (angle between
 1336 impact parameter line and jet $< 90^\circ$), and therefore more consistent with tracks originating
 1337 from a displaced decay.

1338 Rather than using the impact parameters directly, an impact parameter *significance*
 1339 is used which incorporates an uncertainty on the impact parameter – tracks with a lower
 1340 uncertainty but the same impact parameter will contribute more in the calculation. This
 1341 signed significance is what is used to sample from the PDF templates, with the resulting
 1342 discriminants the sum of probability ratios between given jet hypotheses over tracks associated
 1343 to a given jet, namely $\sum_{i=1}^N \log \frac{p_b}{p_{light}}$ between b -jet and light jet hypotheses, where p_b and
 1344 p_{light} are the per-track probabilities. Similar discriminants are defined between b - and c -jets
 1345 and c and light jets. *TODO: show distributions?*

1346 5.2.2 SV1

1347 SV1 is an algorithm which aims to find a secondary vertex (SV) in a given jet. Operating
 1348 on all vertices associated with a considered jet, the algorithm discards tracks based on a
 1349 variety of cleaning requirements. It then proceeds to first construct all two-track vertices,
 1350 and then iterates over all the tracks involved in these two track vertices to try to fit a single
 1351 secondary vertex, which would then be identified with the secondary vertex from the b or c
 1352 hadron decay. This fit proceeds by evaluating a χ^2 for the association of a track and vertex,

removing the track with the largest χ^2 , and iterating until the χ^2 is acceptable and the vertex has an invariant mass of less than 6 GeV (for consistency with b or c hadron decay).

A variety of discriminating variables may then be constructed, including (1) invariant mass of the secondary vertex, (2) number of tracks associated with the secondary vertex, (3) number of two-track vertices, (4) energy fraction of the tracks associated to the secondary vertex (relative to all of the tracks associated to the jet), and various metrics associated with the secondary vertex position and decay length, including (5) transverse distance between the primary and secondary vertex, (6) distance between the primary and secondary vertex (7) distance between the primary and secondary vertex divided by its uncertainty, and (8) ΔR between the jet axis and the direction of the secondary vertex relative to the primary vertex.

While all eight of these variables are used as inputs to the higher level taggers, the number of two-track vertices, the vertex mass, and the vertex energy fraction are additionally used with 3D histogram templates to evaluate flavor tagging performance by constructing log-likelihood discriminants, similar to the procedure for IP2D/3D.

5.2.3 JetFitter

Rather than focusing on a particular aspect of the B hadron or D hadron decay topology (e.g impact parameter or secondary vertex), the third low level tagger, JETFITTER [54], tries to reconstruct the full decay chain, including all involved vertices. This is structured around a Kalman filter formalism [55], and has the strong underlying assumption that all tracks which stem from B and D hadron decay must intersect a common flight path. This assumption provides significant constraints, allowing for the reconstruction of vertices from even a single track, reducing the number of degrees of freedom in the fit, and allowing the use of “downstream” information, e.g., compatibility of tracks with a $B \rightarrow D$ -like decay. The constructed topology, including primary vertex location and B -hadron flight path, is iteratively updated over tracks associated to a given jet, and a variety of discriminating variables related to the resulting topology and reconstructed decay are used as inputs to the high level taggers.

1380 *5.2.4 RNNIP*

1381 The IP2D and IP3D algorithms rely on per-track probabilities, and the final discriminating
 1382 variables (and inputs to the higher level taggers) are sums (products) over these independently
 1383 considered quantities. In practice, however, the tracks are not independent – this is merely a
 1384 simplifying assumption to allow for the use of a binned likelihood, as treatment of all of the
 1385 interdependencies in such a framework quickly becomes intractable. To address this issue, a
 1386 recurrent neural network-based algorithm, RNNIP [56], is used, which takes as input a variety
 1387 of per-track variables, including the signed impact parameter significances (as in IP3D) as
 1388 well as track momentum fraction relative to the jet and ΔR between the track and the jet.
 1389 RNNs are sequence-based, and vectors of input variables corresponding to tracks for a given
 1390 jet are ordered by magnitude of transverse impact parameter significance and then passed
 1391 to the network, which outputs class probabilities corresponding to b-jet, c-jet, light-jet, and
 1392 τ -jet hypotheses. Such a procedure allows the network to learn interdependencies between
 1393 tracks, improving performance.

1394 *5.2.5 MV2 and DL1*

1395 Outputs from the above taggers are combined into high level taggers to aggregate all of the
 1396 information and improve discriminating power relative to the respective individual taggers as,
 1397 as shown in Figure 5.2. These high level taggers are primarily in two forms: MV2, which
 1398 uses a Boosted Decision Tree (BDT) for this aggregation, and DL1, which uses a deep neural
 1399 network. For the baseline versions of these taggers, only inputs from IP2D, IP3D, SV1, and
 1400 JetFitter are used. The tagger used for this thesis analysis, DL1r, additionally incorporates
 1401 RNNIP, demonstrating improved performance over the baseline DL1, as shown in Figure 5.3.
 1402 All high level taggers also include jet p_T and $|\eta|$.

DL1 offers a variety of improvements over MV2. Rather than a single discriminant output, as with MV2, DL1 has a multidimensional output, corresponding to probabilities for a jet to be a *b*-jet, *c*-jet, or light jet. This allows the trained network to be used for both *b*- and *c*-jet

tagging. The final discriminant for b -tagging with DL1 correspondingly takes the form

$$D_{\text{DL1}} = \ln \left(\frac{p_b}{f_c \cdot p_c + (1 - f_c) \cdot p_{\text{light}}} \right) \quad (5.5)$$

where p_b , p_c , and p_{light} are the output b , c , and light jet probabilities, and f_c corresponds to an effective c -jet fraction, which may be tuned to optimize performance.

DL1 further includes an additional set of JETFITTER input variables relative to MV2 which correspond to c -tagging – notably properties of secondary and tertiary vertices, as would be seen in a $B \rightarrow D$ decay chain.

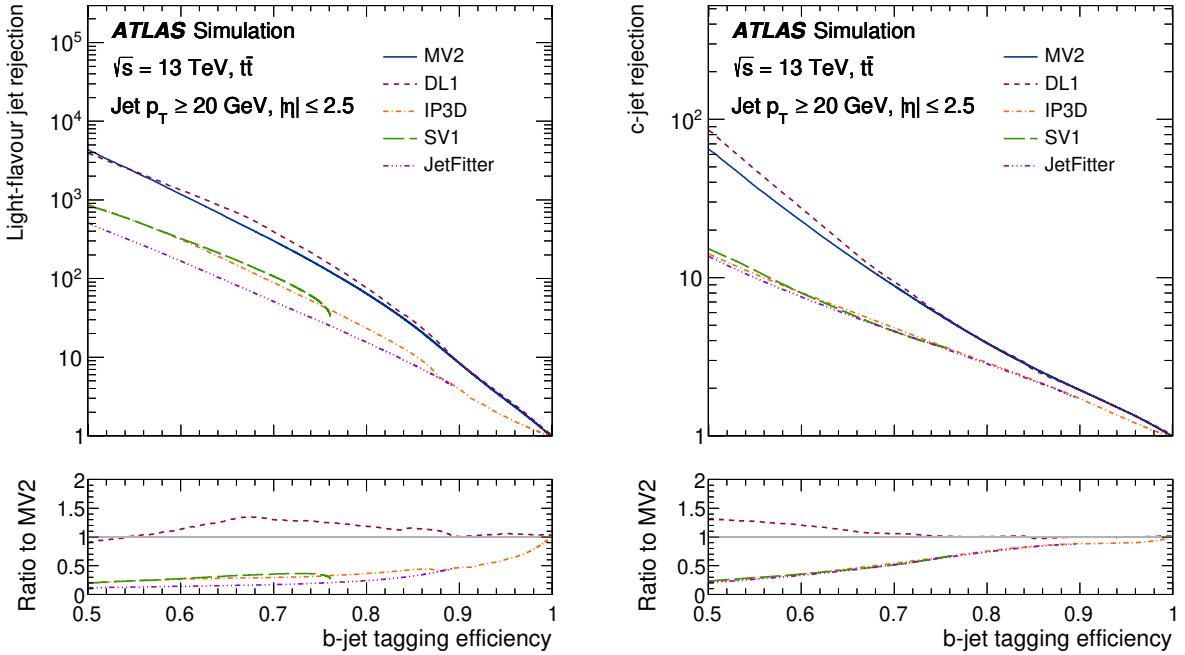


Figure 5.2: Performance of the various low and high level flavor tagging algorithms in $t\bar{t}$ simulation, demonstrating the tradeoff between b -jet efficiency and light and c -jet rejection. The high level taggers demonstrate significantly better performance than any of the individual low level taggers, with DL1 offering slight improvements over MV2 due to the inclusion of additional input variables.

Figure 5.2 shows a comparison of the performance of the various taggers. The b -tagging performance of DL1 and MV2 is found to be similar, with some improvements in light jet and c -jet rejection from the additional variables used in DL1. The performance of these high level taggers additionally is seen to be significantly better than any of the individual low level ones, even in regimes where only a single low level tagger is relevant (such as high b -tagging efficiencies, where SV1 and JETFITTER are limited by selections on tracks entering the respective algorithms).

The inclusion of RNNIP offers a significant improvement on top of baseline DL1, as shown in Figure 5.3, strongly motivating the choice of DL1r for this thesis.

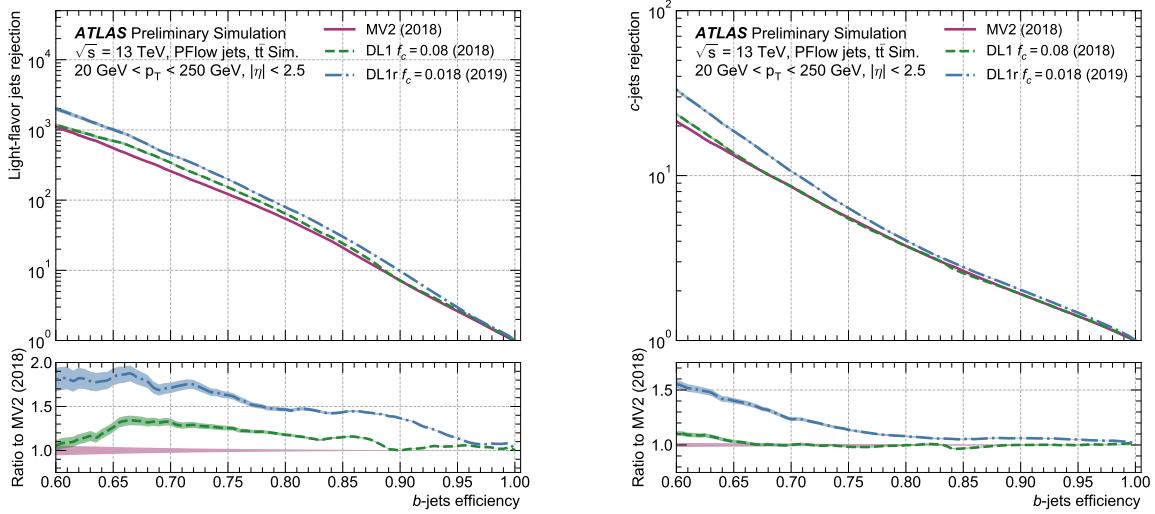


Figure 5.3: Performance of the MV2, DL1, and DL1r algorithms in $t\bar{t}$ simulation, demonstrating the tradeoff between b -jet efficiency and light and c -jet rejection. f_c controls the importance of c -jet rejection in the discriminating variable, and values shown have been optimized separately for each DL1 configuration. DL1r demonstrates a significant improvement in both light and c jet rejection over MV2 and DL1. [57]

1417 *5.2.6 Some Practical Notes*

1418 The b -tagging metrics presented in Figures 5.2 and 5.3 correspond to evaluating a tradeoff
1419 between b -jet efficiency and light jet and c -jet rejection. In this case, b -jet efficiency is defined
1420 such that, e.g. for a 77 % efficiency, 77 % of the real b -jets will be tagged as such. Somewhat
1421 counterintuitively, this means that a lower b -jet efficiency corresponds to a more aggressive
1422 (“tighter”) selection on the discriminating variable, while a higher b -jet efficiency corresponds
1423 to a less aggressive (“looser”) cut (100 % efficiency means no cut). Light and c jet efficiencies
1424 are defined similarly, with rejection defined as 1/ the corresponding efficiency.

1425 In ATLAS, the respective b -tagging efficiencies are used to define various b -tagging working
1426 points. The working point used for the nominal b -jet identification in this thesis is 77 % with
1427 DL1r. A loosened (less aggressive) selection at the 85 % working point is additionally used.
1428 See Chapter 7 for further details.

1429

Chapter 6

1430

THE ANATOMY OF AN LHC SEARCH

1431 In this thesis so far, we have set the theoretical foundation for the work carried out at the
 1432 LHC. We have described how one may translate between this theoretical foundation and what
 1433 we are actually able to observe with the ATLAS detector. We have further stepped through
 1434 the process of simulating production of specific physics processes and their appearance in
 1435 our detector, allowing us to describe how a hypothetical physics model would be seen in
 1436 our experiment. The question then becomes: all of these pieces are on the table, what do
 1437 we do with them? This chapter attempts to answer exactly that, setting up a roadmap for
 1438 assembling these pieces into a statement about the universe.

1439 ***6.1 Object Selection and Identification***

1440 As described in Chapter 5, there is a complicated set of steps for going from electrical signals
 1441 in a detector to physics objects.

1442 ***6.2 Defining a Signal Region***

1443 ***6.3 Background Estimation***

1444 ***6.4 Uncertainty Estimation***

1445 ***6.5 Hypothesis Testing***

1446

Chapter 7

1447

SEARCH FOR PAIR PRODUCTION OF HIGGS BOSONS IN THE $b\bar{b}b\bar{b}$ FINAL STATE

1448

1449 This chapter presents two complementary searches for pair production of Higgs bosons
 1450 in the final state. Such searches are separated based on the signal models being considered:
 1451 resonant production, in which a new spin-0 or spin-2 particle is produced and decays to two
 1452 Standard Model Higgs bosons, and non-resonant production, which is sensitive to the value
 1453 of the Higgs self-coupling λ_{HHH} . Further information on the theory behind both channels
 1454 can be found in Chapter 2.

1455

While the searches face many similar challenges and proceed (in broad strokes) in a very
 similar manner, separate optimizations are performed to maximize the respective sensitivities
 for these two very different sets of signal hypotheses. More particularly, resonant signal
 hypotheses are (1) very peaked in values of the mass of the HH candidate system near
 the value of the resonance mass considered and (2) considered across a very broad range of
 signal mass hypotheses. The resonant searches are therefore split into resolved and boosted
 topologies based on Lorentz boost of the decay products, with the resolved channel as one of
 the primary focuses of this thesis. Further, several analysis design decisions are made to allow
 for sensitivity to a broad range of masses – in particular, though sensitivity is limited at lower
 values of m_{HH} relative to other channels *TODO: Combination, bbyy* due to the challenging
 background topology, retaining and properly reconstructing these low mass events allows the
 $b\bar{b}b\bar{b}$ channel to retain sensitivity up until the kinematic threshold at 250 GeV.

1467

In contrast, non-resonant signal hypotheses are quite broad in m_{HH} , and have a much
 more limited mass range, with Standard Model production peaking near 400 GeV, and the
 majority of the analysis sensitivity able to be captured with a resolved topology. Even for

1468

1469

1470 Beyond the Standard Model signal hypotheses, which may have more events at low m_{HH} ,
 1471 the non-resonant nature of the production allows the $b\bar{b}b\bar{b}$ channel to retain sensitivity while
 1472 discarding much of the challenging low mass background. Such freedom allows for decisions
 1473 which focus on improved background modeling for the middle to upper HH mass regime,
 1474 resulting in improved modeling and smaller uncertainties than would be obtained with a
 1475 more generic approach.

1476 Both searches are presented in the following, with emphasis on particular motivations for,
 1477 and consequences of, the various design decisions involved for each respective set of signal
 1478 hypotheses.

1479 The analyses improve upon previous work ?? in several notable ways. The resonant search
 1480 leverages a Boosted Decision Tree (BDT) based pairing algorithm, offering improved HH
 1481 pairing efficiency over a broad mass spectrum. The non-resonant adopts a different approach,
 1482 with a simplified algorithm based on the minimum angular distance (ΔR) between jets in
 1483 a Higgs candidate. Such an approach very efficiently discards low mass background events,
 1484 resulting in an easier to estimate background with reduced systematic uncertainties.

1485 A particular contribution of this thesis is the background estimation, which uses a novel,
 1486 neural network based approach, offering improved modeling over previous methods, as well
 1487 as the ability to model correlations between observables. While all aspects of the analysis of
 1488 course contribute to the final result, the author of this thesis wishes to emphasize that the
 1489 background estimate, with the corresponding uncertainties and all other associated decisions,
 1490 really is the core of the $HH \rightarrow b\bar{b}b\bar{b}$ analysis – the development of this procedure, and all
 1491 associated decisions, is similarly the core of this thesis work.

1492 ATLAS has performed a variety of searches in complementary decay channels as well, no-
 1493 tably in the $b\bar{b}W^+W^-$ [58], $b\bar{b}\tau^+\tau^-$ [59], $W^+W^-W^+W^-$ [60], $b\bar{b}\gamma\gamma$ [61], and $W^+W^-\gamma\gamma$ [62]
 1494 final states, which were combined along with $b\bar{b}b\bar{b}$ in [20].

1495 CMS has also performed searches for resonant production of Higgs boson pairs in the
 1496 $b\bar{b}b\bar{b}$ final state (among others) at $\sqrt{s} = 8$ TeV [63] and $\sqrt{s} = 13$ TeV [64]. CMS have also
 1497 performed a combination of their searches in the $b\bar{b}b\bar{b}$, $b\bar{b}\tau^+\tau^-$, $b\bar{b}\gamma\gamma$, and $b\bar{b}VV$ channels

1498 in [65].

1499 This analysis also benefits from improvements to ATLAS jet reconstruction and calibration,
1500 and flavour tagging [53]. In particular, this analysis benefits from the introduction of particle
1501 flow jets [47]. These make use of tracking information to supplement calorimeter energy
1502 deposits, improving the angular and transverse momentum resolution of jets by better
1503 measuring these quantities for charged particles in those jets.

1504 The analysis also benefits from the new DL1r ATLAS flavour tagging algorithm. Whereas
1505 the flavour tagging algorithm used in the previous analysis (MV2) used a boosted decision
1506 tree (BDT) to combine the output of various low level algorithms, DL1r (and the baseline
1507 DL1 algorithm) uses a deep neural network to do this combination. In addition to the low
1508 level algorithms used as inputs to MV2, DL1 includes a variety of additional variables used
1509 for c -tagging. DL1r further incorporates RNNIP, a recurrent neural network designed to
1510 identify b -jets using the impact parameters, kinematics, and quality information of the tracks
1511 in the jets, while also taking into account the correlations between the track features.

1512 The overall analysis sensitivity further benefits from a factor of ~ 4.6 increase in integrated
1513 luminosity.

1514 7.1 Data and Monte Carlo Simulation

1515 Both the resonant and non-resonant searches are performed on the full ATLAS Run 2 dataset,
1516 consisting of $\sqrt{s} = 13\text{ TeV}$ proton-proton collision data taken from 2016 to 2018 inclusive.
1517 Data taken in 2015 is not used due to a lack of trigger jet matching information and b -jet
1518 trigger scale factors. The integrated luminosity collected and usable in this analysis¹ was:

1519 • 24.6 fb^{-1} in 2016

1520 • 43.65 fb^{-1} in 2017

1520¹approximately 9 fb^{-1} of data was collected but could not be used in this analysis due to an inefficiency in the b -jet triggers at the start of 2016 [66]

- 1521 • 57.7 fb^{-1} in 2018

1522 This gives a total integrated luminosity of 126 fb^{-1} . This is lower than the 139 fb^{-1} ATLAS
 1523 collected during Run 2 [67] due to the inefficiency described in footnote 1 as well as the
 1524 3.2 fb^{-1} of 2015 data which is unused due to the trigger scale factor issue mentioned above.

1525 In this analysis, Monte Carlo samples are used purely for modelling signal processes. The
 1526 background is strongly dominated by events produced by QCD multijet processes, which
 1527 are difficult to correctly model in simulation. This necessitates the use of a data-driven
 1528 background modelling technique, which is described in Section 7.6.

1529 The scalar resonance signal model is simulated at leading order in α_s using MADGRAPH
 1530 [33]. Hadronization and parton showering are done using HERWIG 7 [37][38] with EVTGEN [40],
 1531 and the nominal PDF is NNPDF 2.3 LO. In practice this is implemented as a two Higgs
 1532 doublet model where the new neutral scalar is produced through gluon fusion and required
 1533 to decay to a pair of SM Higgs bosons. The heavy scalar is assigned a width much smaller
 1534 than detector resolution, and the other 2HDM particles do not enter the calculation.

1535 Scalar samples are produced at resonance masses between 251 and 900 GeV and the
 1536 detector simulation is done using AtlFast-II [42]. In addition the samples at 400 GeV and
 1537 900 GeV are also fully simulated to verify that the use of AtlFast-II is acceptable. For higher
 1538 masses, as well as for the boosted analysis, samples are produced between 1000 and 5000 GeV,
 1539 and the detector is fully simulated. As discussed in Chapter 4, an outstanding issue with
 1540 AtlFast-II is the modeling of jet substructure. While such variables are not used for the
 1541 resolved analysis, the boosted analysis begins at 900 GeV, motivating the different detector
 1542 simulation in these two regimes.

1543 The spin-2 resonance signal model is also simulated at LO in α_s using MADGRAPH.
 1544 Hadronization and parton showering are done using PYTHIA 8 [39] with EVTGEN, and the
 1545 nominal PDF is NNPDF 2.3 LO. In practice this is implemented as a Randall-Sundrum
 1546 graviton with $c = 1.0$.

1547 Spin-2 resonance samples are produced at masses between 251 and 5000 GeV, and these

1548 samples are all produced with full detector simulation.

1549 For the non-resonant search, samples are produced at values of $\kappa_\lambda = 1.0$ and 10.0, and are
1550 simulated using Powheg Box v2 generator [34–36] at next-to-leading order (NLO), with full
1551 NLO corrections with finite top mass, using the PDF4LHC [68] parton distribution function
1552 (PDF) set. Parton showers and hadronization are simulated with Pythia 8.

1553 Alternative ggF samples are simulated at NLO using Powheg Box v2, but instead using
1554 Herwig 7 [69] for parton showering and hadronization. The comparison between these two
1555 is used to assess an uncertainty on the parton showering.

1556 **7.2 Triggers and Object Definitions**

1557 To maximize analysis sensitivity, a combination of multi- b -jet triggers is used. Due to the use
1558 of events with two b -tagged jets in the background estimate, such triggers have a maximum
1559 requirement of two b -tagged jets. For the resonant analysis, a combination of triggers of
1560 various topologies is used, namely

1561 • 2b + HT, which requires two b -tagged jets and a minimum value of of H_T , defined to
1562 be the scalar sum of p_T across all jets in the event.

1563 • 2b + 2j, which requires two b -tagged jets and two other jets matching some kinematic
1564 requirements

1565 • 2b + 1j, which requires two b -tagged jets and one other jet matching some kinematic
1566 requirements

1567 • 1b, which requires one b -tagged jet

1568 Due to minimal contributions from some of these triggers for the Standard Model non-resonant
1569 signal, a simplified strategy relying entirely on 2b + 1j and 2b + 2j triggers is used for the
1570 non-resonant search.

1571 While the use of multiple triggers is beneficial for analysis sensitivity, it comes with some
 1572 complications. Namely, a set of scale factors must be assigned to simulated events account
 1573 for trigger inefficiencies in data *TODO: check*. Because these scale factors may differ between
 1574 triggers, the use of multiple triggers becomes complicated: an event may pass more than one
 1575 trigger, while trigger scale factors are only provided for individual triggers.

1576 To simplify this calculation, a set of hierarchical offline selections is applied, closely
 1577 mimicking the trigger selection. Based on these selections, events are sorted into categories
 1578 (*trigger buckets*), after which the decision of a *single trigger* is checked.

1579 The resonant search applies such categorization in the following way, with selections
 1580 considered in order:

- 1581 1. If the leading jet is b -tagged with $p_T > 325 \text{ GeV}$, the event is in the $1b$ trigger category.
- 1582 2. Otherwise, if the leading jet is not b -tagged, but has $p_T > 168.75 \text{ GeV}$, the event is in
 1583 the $2b + 1j$ trigger category.
- 1584 3. If neither of the first two selections pass, if the scalar sum of jet p_T s, $H_T > 900 \text{ GeV}$,
 1585 the event falls into the $2b + HT$ trigger category.
- 1586 4. Events that do not pass any of the above offline selections are in the $2b + 2j$ trigger
 1587 category.

1588 Corresponding triggers are then checked in each category, and the final set of events consists
 1589 of those events that pass the trigger decision in their respective categories.

1590 For the resonant search, the $2b + 1j$ and $2b + 2j$ triggers are the dominant categories,
 1591 containing roughly 26 % and 49 % of spin-2 events, evaluated on MC16d samples with
 1592 resonance masses between 300 and 1200 GeV. Notably, the $1b$ trigger efficiency is largest at
 1593 high ($> 1 \text{ TeV}$) resonance masses.

1594 For the non-resonant search, it was noted that the $1b$ trigger has minimal contribution,
 1595 while the $2b + HT$ events are largely captured by the $2b + 2j$ trigger. Therefore, for, a

1596 simplified scheme is considered, with selections:

- 1597 1. If the 1st leading jet has $p_T > 170 \text{ GeV}$ and the 3rd leading jet has $p_T > 70 \text{ GeV}$, the
1598 event is in the $2b + 1j$ trigger category.
- 1599 2. Otherwise, the event is in the $2b + 2j$ trigger category.

1600 **7.3 Analysis Selection**

1601 After the trigger selections of Section 7.2, a variety of selections on the analysis objects are
1602 made, with the goal of (1) reconstructing a HH -like topology and (2) suppressing contributions
1603 from background processes.

1604 Both analyses begin with a common pre-selection, requiring at least four $R = 0.4$ anti- k_T
1605 jets with $|\eta| < 2.5$ and $p_T > 40 \text{ GeV}$. The $|\eta| < 2.5$ requirement is necessary for b -tagging
1606 due to the coverage of the ATLAS tracking detector (see Chapter 3) *TODO: check*, while the
1607 $p_T > 40 \text{ GeV}$ requirement is motivated by the trigger thresholds *TODO: mention low pT*. At
1608 least two of the jets passing this pre-selection are required to be b -tagged, and additional
1609 b -tagging requirements are made to define the following regions:

- 1610 • “2 b Region”: require exactly two b -tagged jets, used for background estimation
- 1611 • “4 b Region”: require at least (but possibly more) four b -tagged jets, used as a signal
1612 region for both resonant and non-resonant searches

1613 The non-resonant analysis additionally defines two 3 b regions:

- 1614 • “3 $b+1$ loose Region”: require exactly three b -tagged jets which pass the 77 % b-tagging
1615 working point (nominal) and one additional jet that fails the 77 % b-tagging working
1616 point but passes the *looser* 85 % b-tagging working point. Used as a signal region for
1617 the non-resonant search.

- 1618 • “3 b +1 fail Region”: complement of 3 b +1 loose. Require exactly three b -tagged jets
 1619 which pass the 77 % b-tagging working point, but require that none of the remaining jets
 1620 pass the 85 % b-tagging working point. Used as a validation region for the non-resonant
 1621 search.

1622 After these requirements, four jets are chosen, ranked first by b -tagging requirement and then
 1623 by p_T (e.g. for the 2 b region, the jets chosen are the two b -tagged jets and the two highest p_T
 1624 non-tagged jets; for the 4 b region, the jets are the four highest p_T b -tagged jets). To match
 1625 the topology of a $HH \rightarrow b\bar{b}b\bar{b}$ event, these four jets are then *paired* into *Higgs candidates*: the
 1626 four jets are split into two sets of two, and each of these pairs is used to define a reconstructed
 1627 object that is a proxy for a Higgs in a HH event.

1628 For four jets there are three possible pairings. For signal events, a correct pairing can be
 1629 identified (provided all necessary jets pass pre-selection) using the truth information of the
 1630 Monte Carlo simulation, and such information may be used to design/select an appropriate
 1631 pairing algorithm. This is only part of the story, however. The vast majority of the events in
 1632 data do *not* include a real HH decay (this is a search for a reason!), either because the event
 1633 originates from a background process (e.g. for 4 b events), or because the selection is not
 1634 designed to maximize the signal (e.g. 2 b events). As the pairing is part of the selection, it must
 1635 still be run on such events, such that various algorithms which achieve similar performance
 1636 in terms of pairing efficiency may have vastly different impacts in terms of the shape of the
 1637 background and the biases inherent in the background estimation procedure. The interplay
 1638 between these two facets of the pairing is an important part of the choices made for this
 1639 analysis.

1640 A comparison of different shapes due to three different paring strategies is shown in Figure
 1641 7.1.

1642 *7.3.1 Resonant Pairing Strategy*

1643 For the resonant analysis, a Boosted Decision Tree (BDT) is used for the pairing. The boosted
1644 decision tree is given the total separation between the two jets in each of the two pairs (ΔR_1
1645 and ΔR_2), the pseudo-rapidity separation between the two jets in each pair ($\Delta\eta_1$ and $\Delta\eta_2$),
1646 and the angular separation between the two jets in each pair in the $x - y$ plane ($\Delta\phi_1$ and
1647 $\Delta\phi_2$). The total separations (ΔR_s) are provided in addition to the components in order to
1648 avoid requiring the boosted decision tree to reconstruct these variables in order to use them.
1649 For these variables, pair 1 is the pair with the highest scalar sum of jet p_{T} s, and pair 2 the
1650 other pair.

1651 The boosted decision tree is also parameterized on the di-Higgs mass (m_{HH}) by providing
1652 this as an additional feature. Since the boosted decision tree is trained on correct and
1653 incorrect pairings in signal events, there will be exactly one correct pairing and two incorrect
1654 pairings in the training set for each m_{HH} value present in that set. As a result, this variable
1655 cannot, in itself, distinguish a correct pairing from an incorrect pairing, and therefore the
1656 inclusion of this variable simply serves to parameterize the BDT on m_{HH} ².

1657 The boosted decision tree was trained on one quarter of the total AFII simulated scalar
1658 MC statistics, using the Gradient-based One Side Sampling (GOSS) algorithm which allows
1659 rapid training with very large datasets. A preselection was applied requiring events to have
1660 four jets with a p_{T} of at least 35 GeV. Note that this is a looser requirement than the 40 GeV
1661 used in the analysis selection, and is meant to increase the available statistics for events with
1662 low m_{HH} and to ensure a better performance as a function of that variable. Events were also
1663 required to have four distinct jets that could be geometrically matched (to within $\Delta R \leq 0.4$)
1664 to the b -quarks. The events used to train the BDT were not included when the analysis was
1665 run on these signal simulations. The boosted decision tree was constructed with the following
1666 hyperparameters:

1667 `min_data_in_leaf=50,`

²That is, the conditions placed on the other variables by the BDT vary with m_{HH} .

1668 num_leaves=180,
 1669 learning_rate=0.01

1670 These hyperparameters were optimized using a Bayesian optimization procedure [70].
 1671 Three fold cross-validation was used to perform this optimization without the need for an
 1672 additional sample, while avoiding over-training on signal events.

1673 *7.3.2 Non-resonant Pairing Strategy*

1674 For the non-resonant analysis, a simpler pairing algorithm is used, which proceeds as follows:
 1675 in a given event, Higgs candidates for each possible pairing are sorted by the p_T of the vector
 1676 sum of constituent jets. The angular separation, ΔR is computed between jets in the each of
 1677 the leading Higgs candidates, and the pairing with the smallest separation (ΔR_{jj}) is selected.
 1678 This method will be referred to as $\min \Delta R$ in the following.

1679 The primary motivation for the use of this pairing in the non-resonant search is a *smooth*
 1680 *mass plane*: by efficiently discarding low mass events, $\min \Delta R$ removes the background peak
 1681 present in the resonant search while maintaining good pairing efficiency for the Standard
 1682 Model non-resonant signal. This facilitates a background estimate with small kinematic bias
 1683 – the region in which the background estimate is derived is more similar to the signal region.

1684 Along with discarding low mass background, there is a corresponding loss of low mass
 1685 signal. This predominantly impacts points away from the Standard Model (see Figure 7.2),
 1686 but, because the $4b$ channel has the strongest contribution near the Standard Model and
 1687 because of the large low mass background present with other pairing methods, the impact on
 1688 analysis sensitivity is mitigated. The $\min \Delta R$ pairing is thus adopted for the non-resonant
 1689 search.

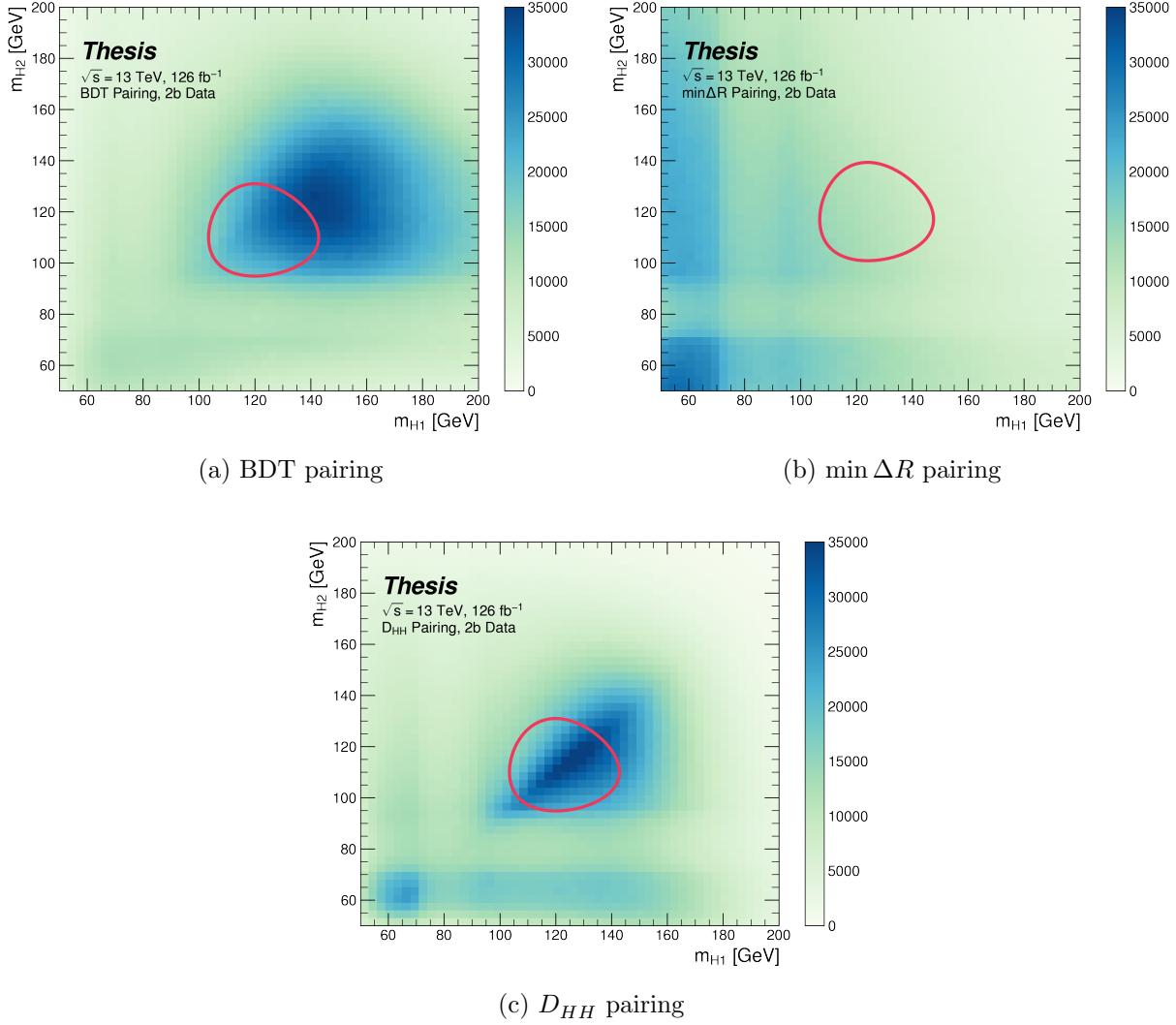


Figure 7.1: Comparison of m_{H1} vs m_{H2} planes for the full Run 2 2b dataset with different pairings. As evidenced, this choice significantly impacts where events fall in this plane, and therefore which events fall into the various kinematic regions defined in this plane (see Section 7.5). Respective signal regions are shown for reference, with the $\min \Delta R$ signal region shifted slightly up and to the right to match the non-resonant selection. Note that the band structure around 80 GeV in both m_{H1} and m_{H2} is introduced by the top veto described in Section 7.4.

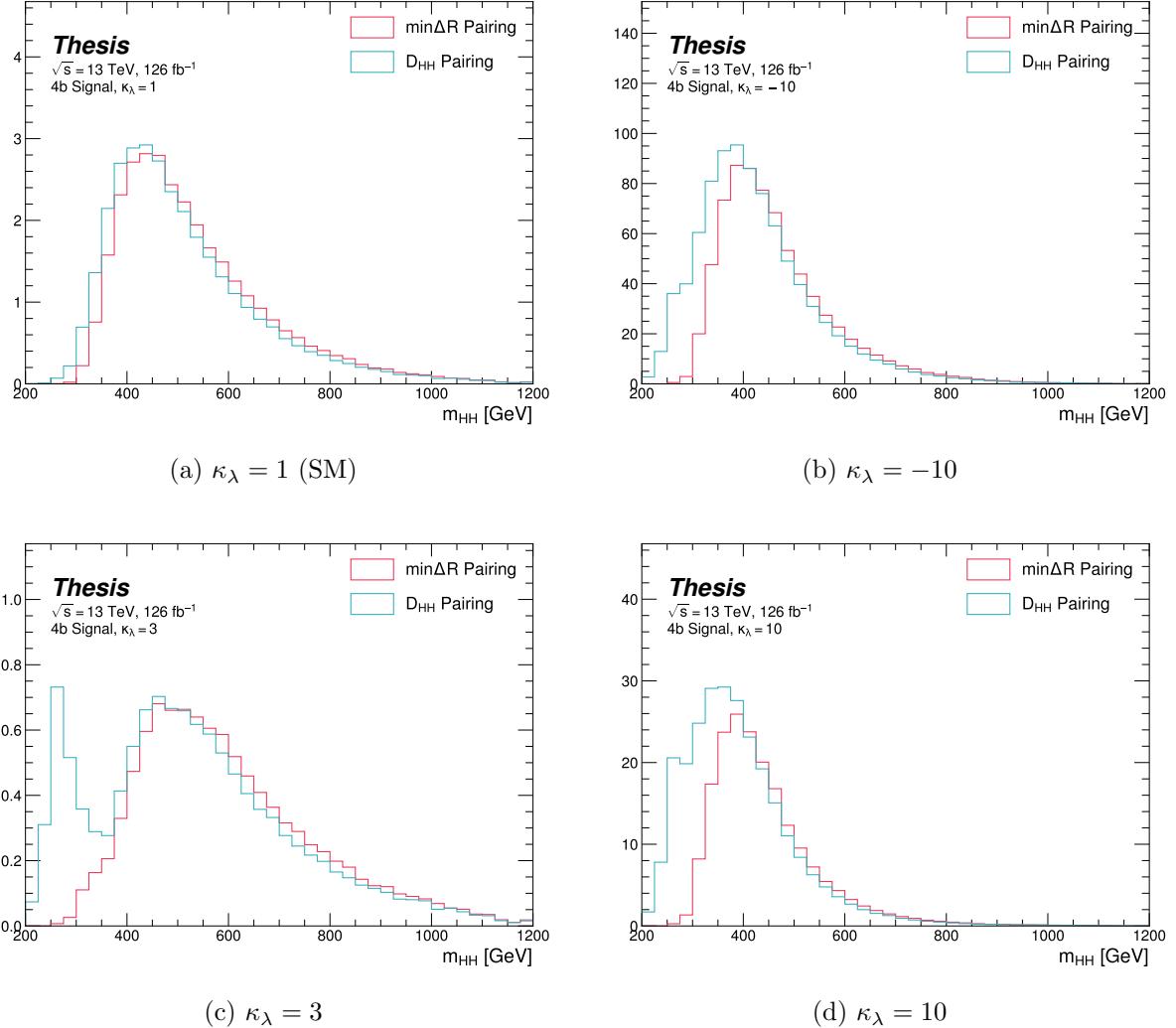


Figure 7.2: Comparison of signal distributions in the respective signal regions for the $\min \Delta R$ and D_{HH} pairing for various values of the Higgs trilinear coupling in the respective signal regions. The distributions are quite similar at the Standard Model point, but for other variations, $\min \Delta R$ does not pick up the low mass features.

1690 **7.4 Background Reduction and Top Veto**

1691 Choosing a pairing of the four b-tagged jets fully defines the di-Higgs candidate system used
 1692 for each event in the remainder of the analysis chain. A requirement of $|\Delta\eta_{HH}| < 1.5$ on this
 1693 di-Higgs candidate system mitigates QCD multijet background.

1694 Figure ?? illustrates this variable in the validation region (see Section ??). It demonstrates
 1695 that this selection rejects only a small fraction of signal, but a significant fraction of data
 1696 (which, in the validation region, is a good approximation of pure background).

1697 In order to mitigate the hadronic $t\bar{t}$ background, a top veto is then applied, removing
 1698 events consistent with a $t \rightarrow b(W \rightarrow q_1\bar{q}_2)$ decay.

1699 The jets in the event are separated into *HC jets* which are the four jets used to build the
 1700 Higgs candidates, and *non-*HC jets**, the other jets (passing the p_T and $|\eta|$ requirements) in
 1701 the event.

1702 W candidates are built by forming all possible pairs of all jets in each event. With n jets,
 1703 there are $\binom{n}{2}$ such pairs. t candidates are then built by pairing each W candidate with each
 1704 HC jet (for $4\binom{n}{2}$ combinations). Note that all jets in a t candidate must be distinct (i.e. a
 1705 HC jet may not be used both on its own and in a W candidate).

With m_t denoting the invariant mass of the t candidate, and m_W the invariant mass of
 the W candidate, the quantity

$$X_{Wt} = \sqrt{\left(\frac{m_W - 80.4 \text{ GeV}}{0.1 \cdot m_W}\right)^2 + \left(\frac{m_t - 172.5 \text{ GeV}}{0.1 \cdot m_t}\right)^2} \quad (7.1)$$

1706 is constructed for each combination.

1707 Events are then vetoed if the minimum X_{Wt} over all combinations is less than 1.5.

1708 The same definitions and procedures are used for both the resonant and non-resonant
 1709 analyses. However, for the non-resonant search, the top candidates considered for X_{Wt} have
 1710 the additional requirement that the jet used for the b is b -tagged. While this is identical to
 1711 the resonant analysis by definition for $4b$ events, it does change the set of events considered in
 1712 lower tag regions, in particular for the $2b$ events considered in the derivation of the background

¹⁷¹³ estimate. Such a change is found to reduce the impact of background systematics by increasing

¹⁷¹⁴ $2b$ support in the high X_{Wt} kinematic region. *TODO: Insert plot*

¹⁷¹⁵ **7.5 Kinematic Region Definition**

As has been mentioned, an important piece of the analysis is the plane defined by the two Higgs candidate masses (the *Higgs candidate mass plane*). After the selection described above, a signal region is defined by requiring $X_{HH} < 1.6$, where:

$$X_{HH} = \sqrt{\left(\frac{m(H_1) - c_1}{0.1 \cdot m(H_1)}\right)^2 + \left(\frac{m(H_2) - c_2}{0.1 \cdot m(H_2)}\right)^2} \quad (7.2)$$

¹⁷¹⁶ with $m(H_1)$, $m(H_2)$ the leading and subleading Higgs candidate masses, c_1 and c_2 correspond
¹⁷¹⁷ to the center of the signal region, and the denominator provides a Higgs candidate mass
¹⁷¹⁸ dependent resolution of 10 %. For consistency with the HH decay hypothesis, c_1 and c_2
¹⁷¹⁹ are nominally (125 GeV, 125 GeV). However, these are allowed to vary due to energy loss,
¹⁷²⁰ with specific values chosen described below. The selection of these values is one of several
¹⁷²¹ significant differences between the regions defined for the resonant and non-resonant search.
¹⁷²² We describe both below.

¹⁷²³ **7.5.1 Resonant Kinematic Regions**

¹⁷²⁴ For the resonant analysis, the signal region is centered at (120 GeV, 110 GeV) to account for
¹⁷²⁵ energy loss leading to the Higgs masses being under-reconstructed. *TODO: insert signal*
¹⁷²⁶ *location plot?* Note that leading and subleading Higgs candidates are defined according to
¹⁷²⁷ the *scalar sum* of constituent jet p_T .

For the background estimation, two regions are defined which are roughly concentric around the signal region: a *validation region* which consists of those events not in the signal region, but which do pass

$$\sqrt{(m(H_1) - 1.03 \times 120 \text{ GeV})^2 + (m(H_2) - 1.03 \times 110 \text{ GeV})^2} < 30 \text{ GeV} \quad (7.3)$$

and a *control region* whcih consists of those events not in the signal or validation regions, but which do pass

$$\sqrt{(m(H_1) - 1.05 \times 120 \text{ GeV})^2 + (m(H_2) - 1.05 \times 110 \text{ GeV})^2} < 45 \text{ GeV} \quad (7.4)$$

1728 For simplicity, the SR/VR/CR definitions from the early Run 2 paper [71] were chosen
1729 for the resonant analysis, but were found to be close to optimal.

1730 *7.5.2 Non-resonant Kinematic Regions*

1731 For the non-resonant analysis the signal region is centered at (124 GeV, 117 GeV), corre-
1732 sponding to the means of *correctly paired* Standard Model signal events. The shape of the
1733 signal region (other than this change of center) was found to remain optimal.

1734 For the non-resonant search, leading and subleading Higgs candidates are defined according
1735 to the *vector sum* of constituent jet p_T , more closely corresponding to the $1 \rightarrow 2$ decay
1736 assumption behind the min ΔR pairing algorithm.

1737 Two areas for improvement were identified in the resonant analysis, which will be dis-
1738 cussed in more detail below: *signal contamination* of the validation region (which impacts
1739 the uncertainty assessed due to extrapolation) and *large nuisance parameter pulls* on this
1740 uncertainty, corresponding to a rough assumption that the validation region is closer to the
1741 signal region in the mass plane, and so offers a better estimate of the signal region.

To mitigate these two issues, a redesign of the control and validation regions was performed for the non-resonant analysis. The outer boundary defined by the shifted resonant control region:

$$\sqrt{(m(H_1) - 1.05 \times 124 \text{ GeV})^2 + (m(H_2) - 1.05 \times 117 \text{ GeV})^2} < 45 \text{ GeV} \quad (7.5)$$

1742 is kept, roughly corresponding to combining the regions used for the resonant analysis. In
1743 order to assess the variation of the background estimate, two sets of regions are desired, so
1744 this combined region is split into *quadrants*, that is, divided into four pieces along axes that
1745 intersect with the signal region center. To avoid kinematic bias, quadrants on opposite sides
1746 of the signal region are paired, with these pairs corresponding to the non-resonant control
1747 and validation regions.

1748 The particular orientation of the regions is chosen such that region centers align with the
1749 leading and subleading Higgs candidate masses, corresponding to a set of axes rotated at

1750 45° , with the “top” and “bottom” quadrants together comprising the control region, and the
 1751 other set (“left” and “right”) the validation region.

1752 This design of regions includes a set of events closer to the signal region in the mass plane,
 1753 leveraging the assumption that these events are more similar to signal region events, while also
 1754 including events further away from the signal region, mitigating signal contamination. This
 1755 region selection is found to have good performance in alternate validation regions. *TODO:*
 1756 *add more studies probs*

1757 7.5.3 Discriminating Variable

1758 The discriminant used for the resonant analysis is *corrected m_{HH}* . This variable is calculated
 1759 by re-scaling the Higgs candidate four vectors such that each $m_H = 125$ GeV. These re-scaled
 1760 four-vectors are then summed, and their invariant mass is the corrected m_{HH} . These re-scaled
 1761 four-vectors are not used for any other purpose. The effect of this correction, which sharpens
 1762 the m_{HH} peak and correctly centres it, is shown in Figure ??.

1763 For the non-resonant analysis, due to the broad nature of the signal in m_{HH} , such a
 1764 correction is not as motivated, and, indeed, is found to have very minimal impact. The
 1765 uncorrected m_{HH} (just referred to as m_{HH}) is therefore used as a discriminant. To maximize
 1766 sensitivity, the non-resonant analysis additionally uses two variables for categorization: $\Delta\eta_{HH}$,
 1767 an angular variable which, along with m_{HH} , fully characterizes the HH system *TODO:*
 1768 *ref costhetastar paper?*, and X_{HH} , the variable used for the signal region definition, which
 1769 leverages the peaked structure of the signal in the $(m(H_1), m(H_2))$ plane to split the signal
 1770 extraction fit into lower and higher purity regions (highest purity near $X_{HH} = 0$, the center
 1771 of the signal region). Distributions of these variables are shown in *TODO: plots*

1772 **7.6 Background Estimation**

1773 After the event selection described above there are two major backgrounds, QCD and $t\bar{t}$.
1774 A very similar approach is used for both the resonant and the non-resonant analyses, with
1775 some small modifications. This approach is notably fully data-driven, which is warranted due
1776 to the flexibility of the estimation method, as well as the high relative proportion of QCD
1777 background ($> 90\%$), and allows for the use of machine learning methods in the construction
1778 of the background estimate. However, it sacrifices an explicit treatment of the $t\bar{t}$ component.
1779 Performance of the background estimate on the $t\bar{t}$ component is checked explicitly *TODO:*
1780 *add plots*, and minimal impact due to $t\bar{t}$ mismodeling is seen.

1781 Contributions of single Higgs processes and ZZ are found to be negligible, and the
1782 Standard Model HH background is found to have no impact on the resonant search.

1783 The foundation of the background estimate lies in the derivation of a reweighting function
1784 which matches the kinematics of events with exactly two b -tagged jets to those of events in
1785 the higher tagged regions (events with three or four b -tagged jets). The reweighting function
1786 and overall normalization are derived in the control region. Systematic bias of this estimate
1787 is assessed in the validation region.

1788 For the resonant analysis, the systematic bias is a bias due to extrapolation: the validation
1789 region lies between the control and signal regions. For the non-resonant analysis, the bias
1790 instead comes from different possible interpolations of the signal region kinematics – given the
1791 choice of nominal estimate, the validation region is a conceptually equivalent, but maximally
1792 different, signal region estimate.

1793 **7.6.1 The Two Tag Region**

1794 Events in data with exactly two b -tagged jets are used for the data driven background
1795 estimate. The hypothesis here is that, due to the presence of multiple b -tagged jets, the
1796 kinematics of such events are similar to the kinematics of events in higher b -tagged regions (i.e.
1797 events with three and four b -tagged jets, respectively), and any differences can be corrected

1798 by a reweighting procedure. The region with three b -tagged jets is split into two b -tagging
 1799 regions, with the $3b + 1$ loose region used as an additional signal region (see Section *TODO:*
 1800 *Add ref*). The lower tagged $3b$ component ($3b + 1$ fail, as described in Section ??) is reserved
 1801 for validation of the background modelling procedure. Events with fewer than two b -tagged
 1802 jets are not used for this analysis, as they are relatively more different from the higher tag
 1803 regions.

1804 The nominal event selection requires at least four jets in order to form Higgs candidates.
 1805 For the four tag region, these are the four highest p_T b -tagged jets. For the three tag regions,
 1806 these jets are the three b -tagged jets, plus the highest p_T jet satisfying a loosened b -tagging
 1807 requirement. Similarly, and following the approach of the resonant analysis, the two tag region
 1808 uses the two b -tagged jets and the two highest p_T non-tagged jets to form Higgs candidates.
 1809 Combinatoric bias from selection of different numbers of b -tagged jets is corrected as a part
 1810 of the kinematic reweighting procedure through the reweighting of the total number of jets in
 1811 the event. In this way, the full event selection may be run on two tagged events.

1812 7.6.2 Kinematic Reweighting

1813 The set of two tagged data events is the fundamental piece of the data driven background
 1814 estimate. However, kinematic differences from the four tag region exist and must be corrected
 1815 in order for this estimate to be useful. Binned approaches based on ratios of histograms
 1816 have been previously considered [71], [17], but are limited in their handling of correlations
 1817 between variables and by the “curse of dimensionality”, i.e. the dataset becomes sparser and
 1818 sparser in “reweighting space” as the number of dimensions in which to reweight increases,
 1819 limiting the number of variables used for reweighting. This leads either to an unstable fit
 1820 result (overfitting with finely grained bins) or a lower quality fit result (underfitting with
 1821 coarse bins).

1822 Note that even machine learning methods such as Boosted Decision Trees (BDTs), may
 1823 suffer from this curse of dimensionality, as the depth of each decision tree used is limited
 1824 by the available statistics after each set of corresponding selections (cf. binning in a more

1825 sophisticated way), limiting the expressivity of the learned reweighting function.

1826 To solve these issues, a neural network based reweighting procedure is used here. This
1827 is a truly multivariate approach, allowing for proper treatment of variable correlations. It
1828 further overcomes the issues associated with binned approaches by learning the reweighting
1829 function directly, allowing for greater sensitivity to local differences and helping to avoid the
1830 curse of dimensionality.

1831 *Neural Network Reweighting*

Let $p_{4b}(x)$ and $p_{2b}(x)$ be the probability density functions for four and two tag data respectively across some input variables x . The problem of learning the reweighting function between two and four tag data is then the problem of learning a function $w(x)$ such that

$$p_{2b}(x) \cdot w(x) = p_{4b}(x) \quad (7.6)$$

from which it follows that

$$w(x) = \frac{p_{4b}(x)}{p_{2b}(x)}. \quad (7.7)$$

This falls into the domain of density ratio estimation, for which there are a variety of approaches. The method considered here is modified from [72, 73], and depends on a loss function of the form

$$\mathcal{L}(R(x)) = \mathbb{E}_{x \sim p_{2b}}[\sqrt{R(x)}] + \mathbb{E}_{x \sim p_{4b}}\left[\frac{1}{\sqrt{R(x)}}\right]. \quad (7.8)$$

where $R(x)$ is some estimator dependent on x and $\mathbb{E}_{x \sim p_{2b}}$ and $\mathbb{E}_{x \sim p_{4b}}$ are the expectation values with respect to the 2b and 4b probability densities. A neural network trained with such a loss function has the objective of finding the estimator, $R(x)$, that minimizes this loss. It is straightforward to show (Appendix ??) that

$$\arg \min_R \mathcal{L}(R(x)) = \frac{p_{4b}(x)}{p_{2b}(x)} \quad (7.9)$$

1832 which is exactly the form of the desired reweighting function.

In practice, to avoid imposing explicit positivity constraints, the substitution $Q(x) \equiv \log R(x)$ is made. The loss function then takes the equivalent form

$$\mathcal{L}(Q(x)) = \mathbb{E}_{x \sim p_{2b}}[\sqrt{e^{Q(x)}}] + \mathbb{E}_{x \sim p_{4b}}\left[\frac{1}{\sqrt{e^{Q(x)}}}\right], \quad (7.10)$$

with solution

$$\arg \min_Q \mathcal{L}(Q(x)) = \log \frac{p_{4b}(x)}{p_{2b}(x)}. \quad (7.11)$$

1833 Taking the exponent then results in the desired reweighting function.

1834 Note that similar methods for density ratio estimation are available *TODO: cite*, e.g. from

1835 a more standard binary cross-entropy loss. However, these were found to perform no better
1836 than the formulation presented here.

1837 *Variables and Results*

1838 The neural network is trained on a variety of variables sensitive to two vs. four tag differences.

1839 To help bring out these differences, the natural logarithm of some of the variables with a
1840 large, local change is taken. The set of training variables used for the resonant analysis is

1841 1. $\log(p_T)$ of the 4th leading Higgs candidate jet

1842 2. $\log(p_T)$ of the 2nd leading Higgs candidate jet

1843 3. $\log(\Delta R)$ between the closest two Higgs candidate jets

1844 4. $\log(\Delta R)$ between the other two Higgs candidate jets

1845 5. Average absolute value of Higgs candidate jet η

1846 6. $\log(p_T)$ of the di-Higgs system.

1847 7. ΔR between the two Higgs candidates

1848 8. $\Delta\phi$ between the jets in the leading Higgs candidate

1849 9. $\Delta\phi$ between the jets in the subleading Higgs candidate

1850 10. $\log(X_{Wt})$, where X_{Wt} is the variable used for the top veto

1851 11. Number of jets in the event.

1852 The non-resonant analysis uses an identical set of variables with two notable changes

1853 1. The definition of X_{Wt} differs from the resonant definition (as described in Section
1854 *TODO: ref*)

1855 2. An integer encoding of the two trigger categories is used as an input (variable which
1856 takes on the value 0 or 1 corresponding to each of the two categories). This was found
1857 to improve a mismodeling near the tradeoff in m_{HH} of the two buckets.

1858 The neural network used for both resonant and non-resonant reweighting has three densely
1859 connected hidden layers of 50 nodes each with ReLU activation functions and a single node
1860 linear output. This configuration demonstrates good performance in the modelling of a variety
1861 of relevant variables, including m_{HH} , when compared to a range of networks of similar size.

1862 In practice, a given training of the reweighting neural network is subject to variation
1863 due to training statistics and initial conditions. An uncertainty is assigned to account for
1864 this (Section 7.7), which relies on training an ensemble of reweighting networks [74]. To
1865 increase the stability of the background estimate, the median of the predicted weight for each
1866 event is calculated across the ensemble. This median is then used as the nominal background
1867 estimate. This approach is indeed seen to be much more stable and to demonstrate a better
1868 overall performance than a single arbitrary training. Each ensemble used for this analysis
1869 consists of 100 neural networks, trained as described in Section 7.7.

1870 The training of the ensemble used for the nominal estimate is done in the kinematic
1871 Control Region. The prediction of these networks in the Signal Region is then used for the
1872 nominal background estimate. In addition, a separate ensemble of networks is trained in the

1873 Validation Region. The difference between the prediction of the nominal estimate and the
 1874 estimate from the VR derived networks in the Signal Region is used to assign a systematic
 1875 uncertainty. Further details on this systematic uncertainty are shown in Section 7.7. Note
 1876 that although the same procedure is used for both Control and Validation Region trained
 1877 networks, only the median estimate from the VR derived reweighting is used for assessing a
 1878 systematic – no additional “uncertainty on the uncertainty” from VR ensemble variation is
 1879 applied.

1880 Each reweighted estimate is normalized such that the reweighted $2b$ yield matches the $4b$
 1881 yield in the corresponding training region. Note that this applies to each of the networks used
 1882 in each ensemble, where the normalization factor is also subject to the procedure described in
 1883 Section 7.7. As the median over these normalized weights is not guaranteed to preserve this
 1884 normalization, a further correction is applied such that the $2b$ yield, after the median weights
 1885 are applied, matches the $4b$ yield in the corresponding training region. As no preprocessing
 1886 is applied to correct for the class imbalance between $2b$ and $4b$ events entering the training,
 1887 this ratio of number of $4b$ events ($n(4b)$) over number of $2b$ events ($n(2b)$) is folded into the
 1888 learned weights. Correspondingly, the set of normalization factors described above is near 1
 1889 and the learned weights are centered around $n(4b)/n(2b)$ (roughly 0.01 over the full dataset).
 1890 This normalization procedure applies for all instances of the reweighting (e.g. those used for
 1891 validations in Section ??), with appropriate substitutions of reweighting origin (here $2b$) and
 1892 reweighting target (here $4b$).

1893 Note that, due to different trigger and pileup selections during each year, the reweighting
 1894 is trained on each year separately. An approach of training all of the years together with a
 1895 one-hot encoding was explored *TODO: reference study*, but was found to have minimal benefit
 1896 over the split years approach, and in fact to increase the systematic bias of the corresponding
 1897 background estimate. Because of this, and because trigger selections for each year significantly
 1898 impact the kinematics of each year, such that categorizing by year is expected to reflect
 1899 groupings of kinematically similar events and to provide a meaningful degree of freedom in
 1900 the signal extraction fit, the split-year approach is kept.

1901 The control region closure for the 2018 dataset is shown for the resonant search in Figures
 1902 7.3 through 7.11 and for the non-resonant search in Figures 7.21 through 7.29 for 4b and
 1903 Figures 7.39 through 7.47 for 3b1l. The impact of this control region derived reweighting
 1904 on the validation region is shown in Figures 7.12 through 7.20 for the resonant search and
 1905 Figures 7.30 through 7.38 for 4b and Figures 7.48 through 7.56 for 3b1l for the non-resonant
 1906 search. Generally good performance is seen, with some occasional mis-modeling. For the
 1907 resonant search, this is most notable in the case of individual jet p_T . Such mis-modeling
 1908 may be corrected by including the variables in the input set, but this was found to not
 1909 improve the modeling of m_{HH} , and so is not done here. This mis-modeling is notable for the
 1910 non-resonant search in the leading Higgs candidate jet p_T , and is a direct consequence of the
 1911 trigger category input, which improves modeling of m_{HH} . Results are similar for other years,
 1912 but are not included here for brevity.

1913 One other salient feature of the non-resonant plots is the distributions of m_{H1} and m_{H2} ,
 1914 which emphasize the quadrant region definitions – the control region has a peak around
 1915 125 GeV in m_{H1} , which may be thought of as “signal region-like”, motivating this alignment,
 1916 though consequently the distribution of m_{H2} is quite bimodal. The reverse is true for the
 1917 validation region.

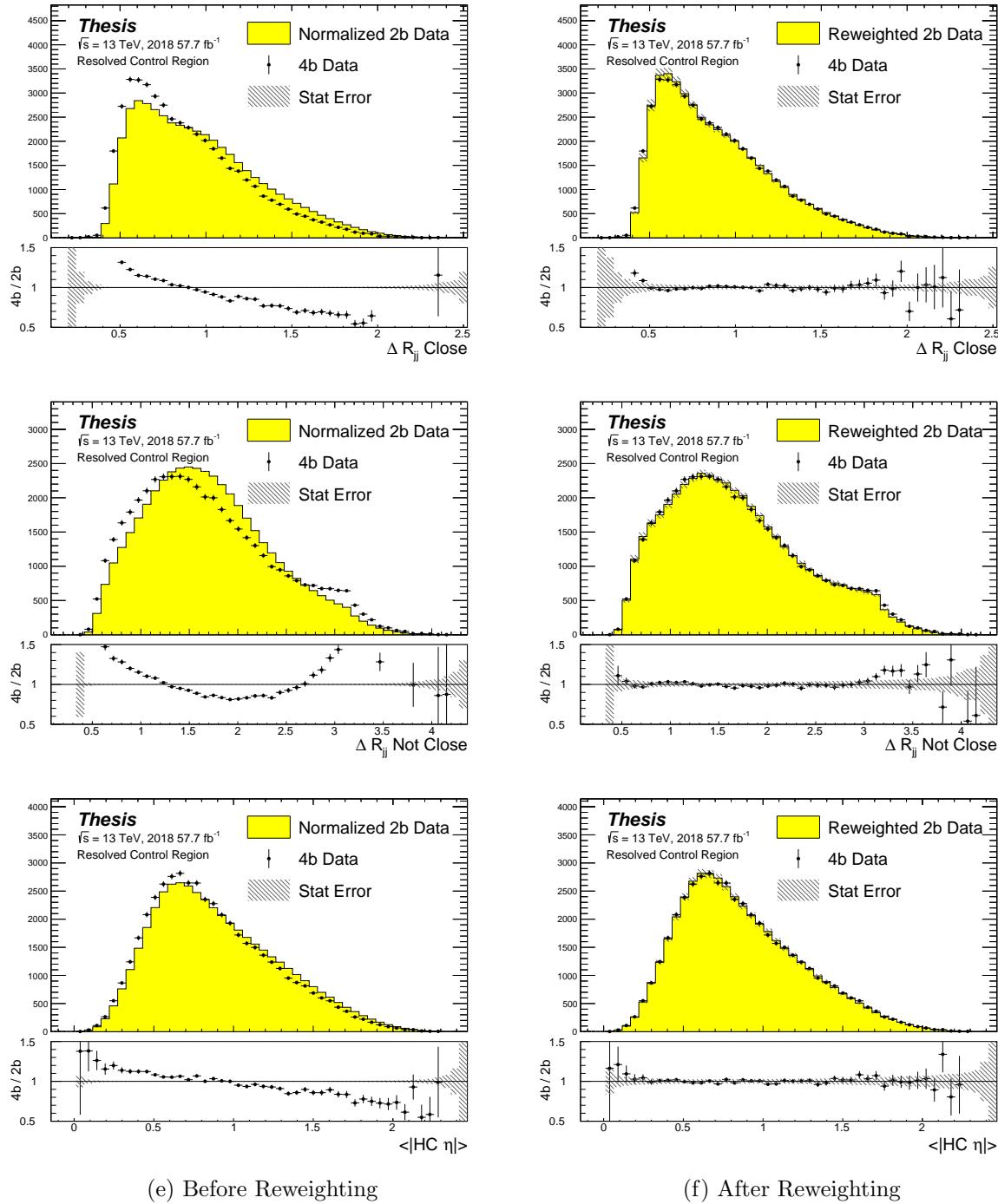


Figure 7.3: **Resonant Search:** Distributions of ΔR between the closest Higgs Candidate jets, ΔR between the other two, and average absolute value of HC jet η before and after CR derived reweighting for the 2018 Control Region.

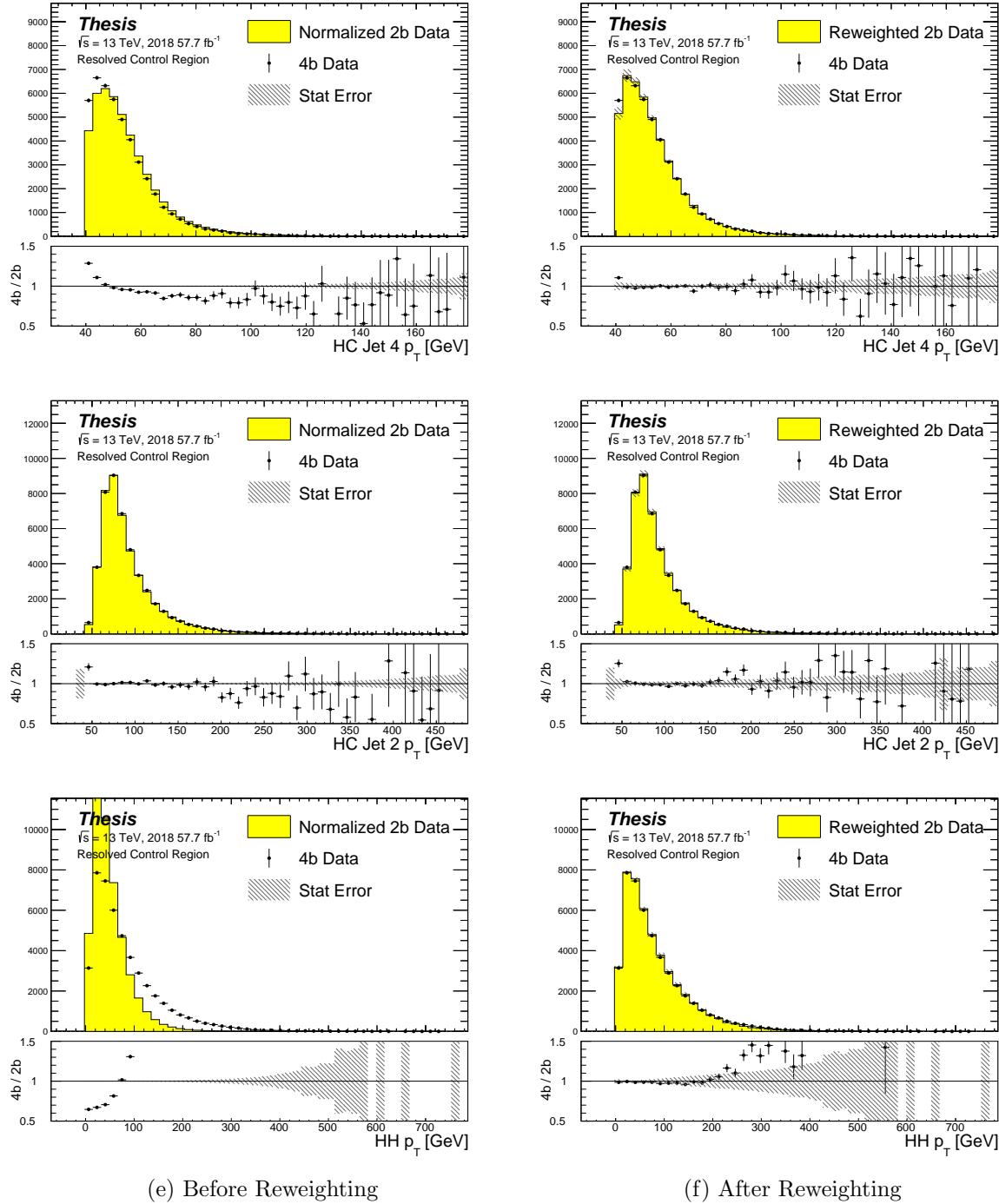


Figure 7.4: **Resonant Search:** Distributions of p_T of the 2nd and 4th leading Higgs Candidate jets and the p_T of the di-Higgs system before and after CR derived reweighting for the 2018 Control Region.

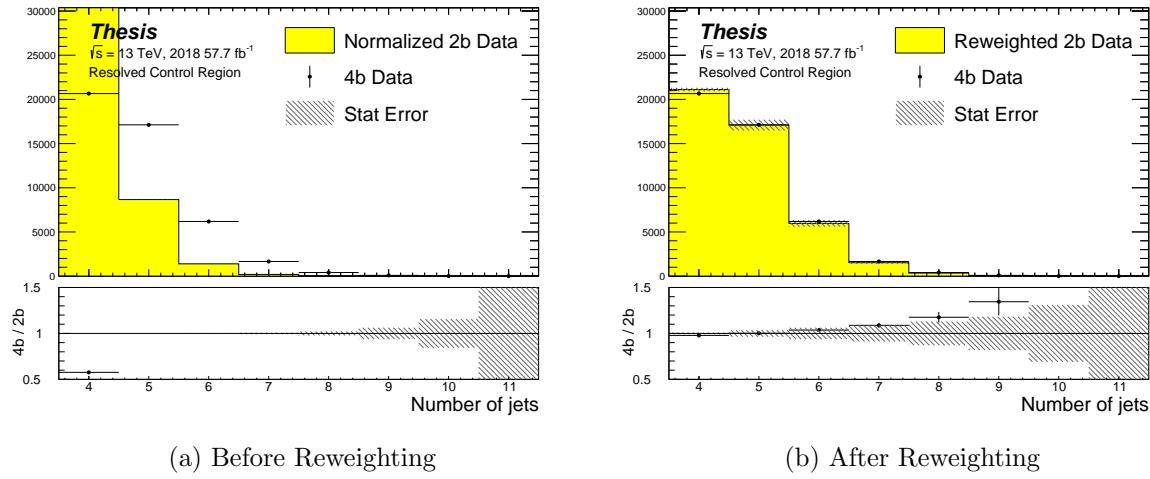


Figure 7.5: Resonant Search: Distributions of the number of jets before and after CR derived reweighting for the 2018 Control Region. A minimum of 4 jets is required in each event in order to form Higgs candidates.

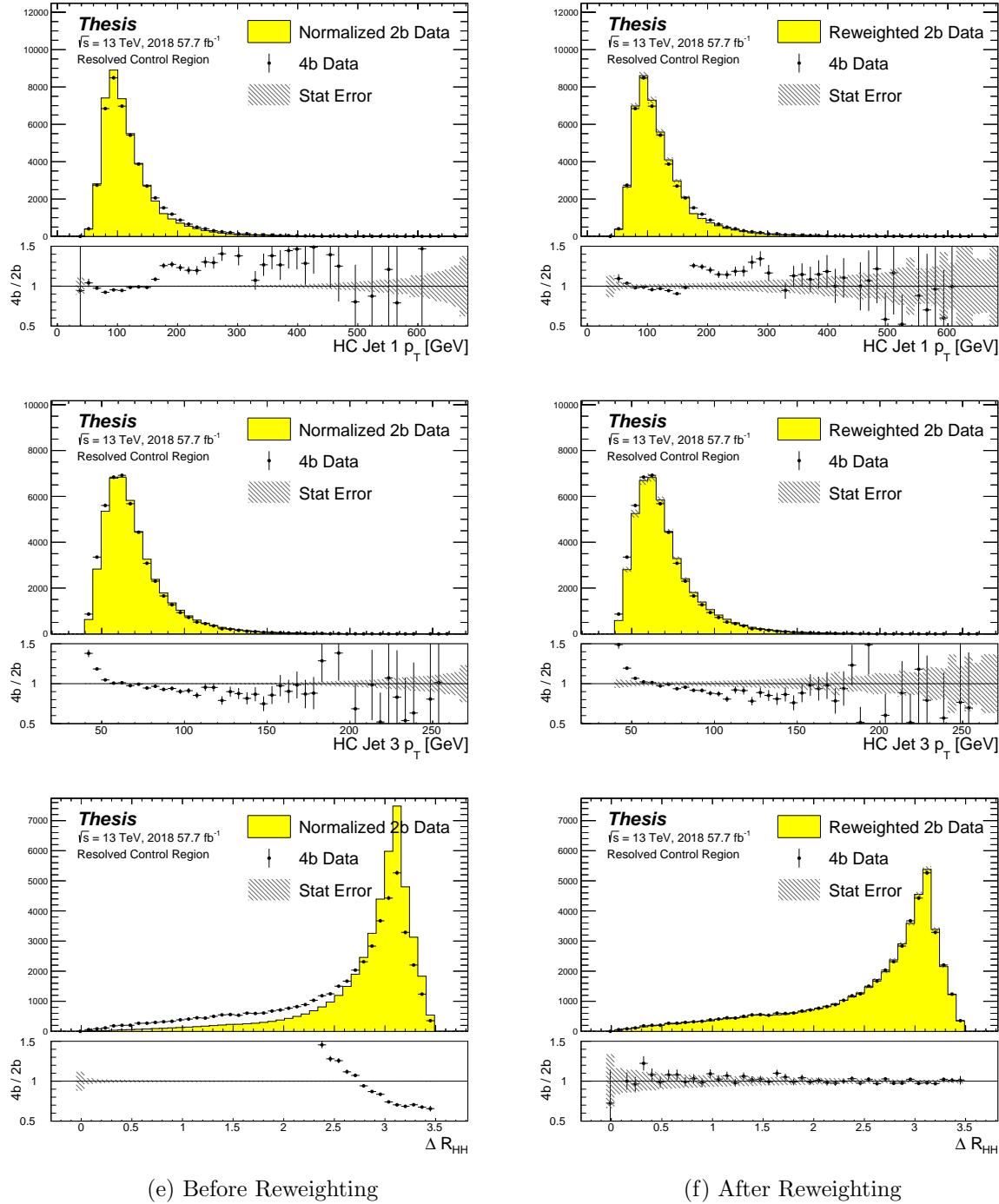


Figure 7.6: **Resonant Search:** Distributions of p_T of the 1st and 3rd leading Higgs Candidate jets and ΔR between Higgs candidates before and after CR derived reweighting for the 2018 Control Region.

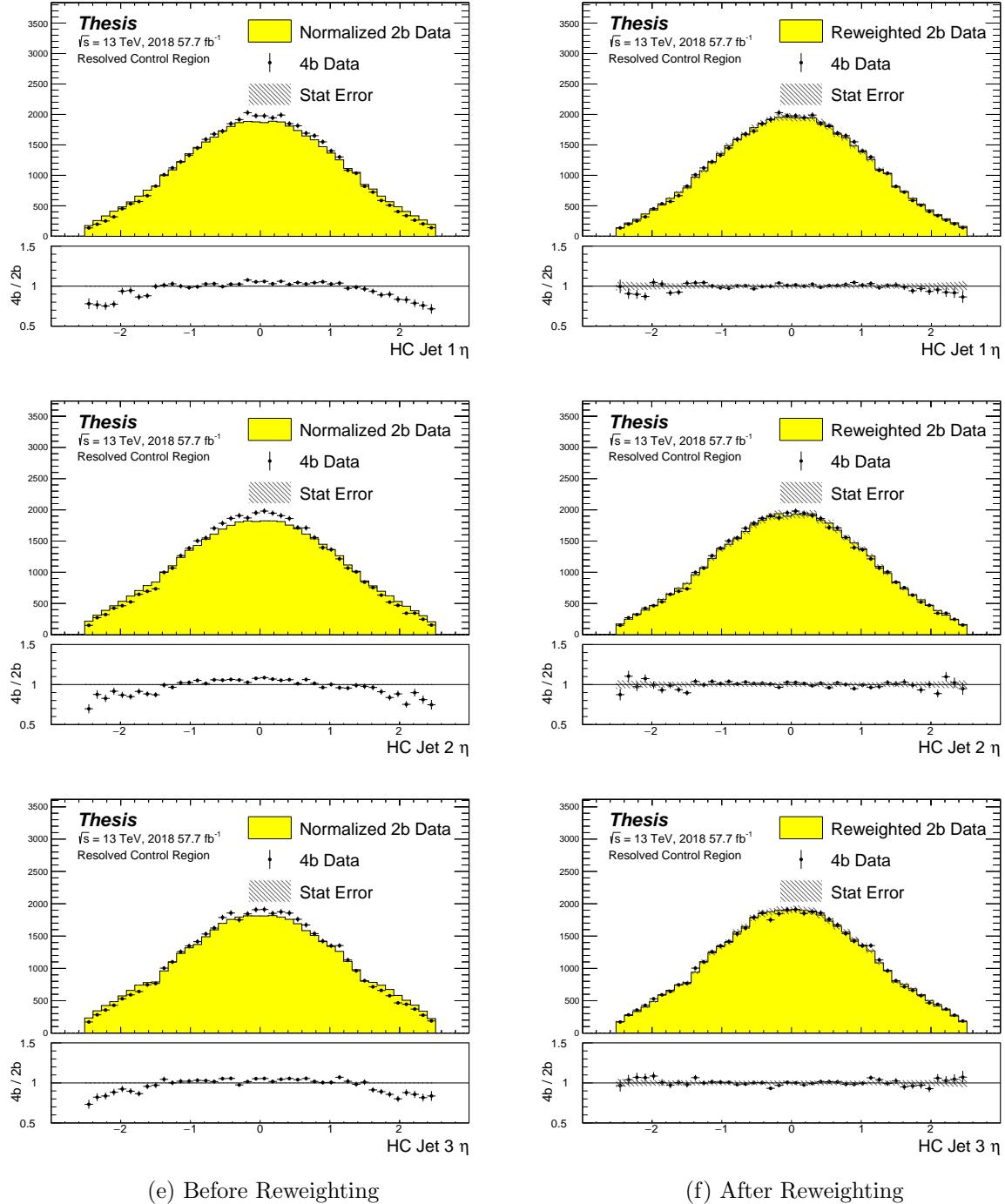


Figure 7.7: **Resonant Search:** Distributions of η of the 1st, 2nd, and 3rd leading Higgs Candidate jets before and after CR derived reweighting for the 2018 Control Region.

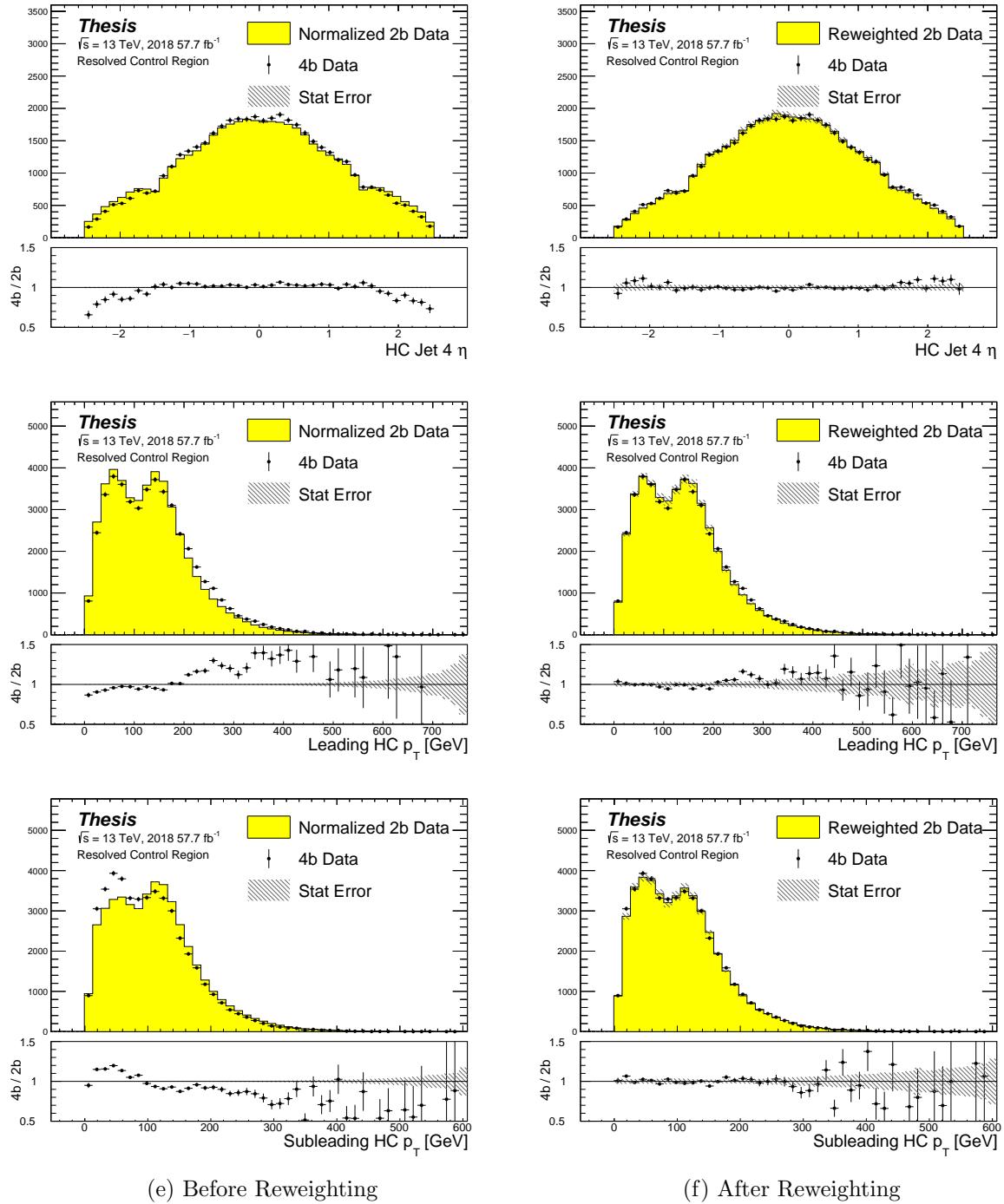


Figure 7.8: **Resonant Search:** Distributions of η of the 4th leading Higgs Candidate jet and the p_T of the leading and subleading Higgs candidates before and after CR derived reweighting for the 2018 Control Region.

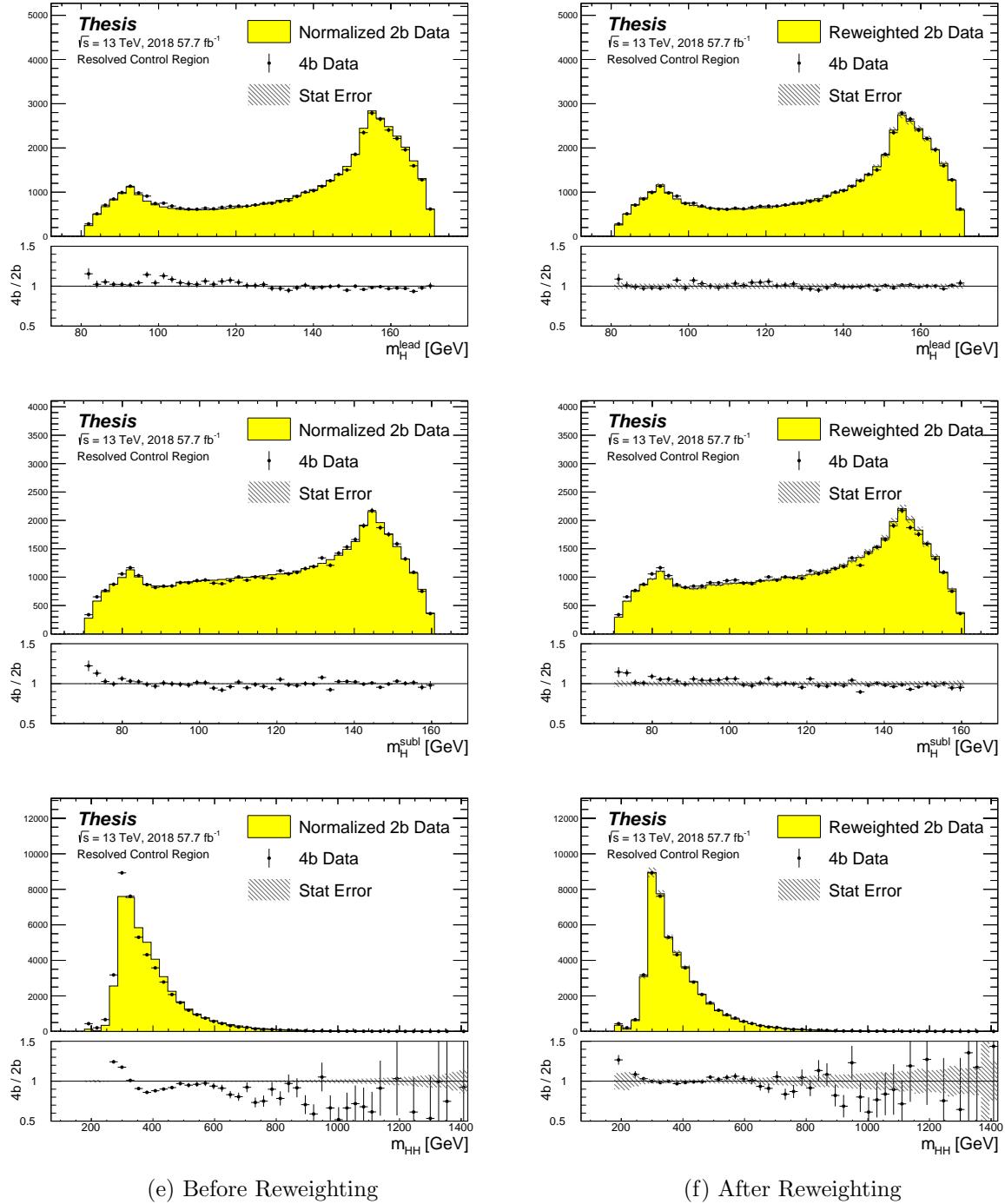


Figure 7.9: Resonant Search: Distributions of mass of the leading and subleading Higgs candidates and of the di-Higgs system before and after CR derived reweighting for the 2018 Control Region.

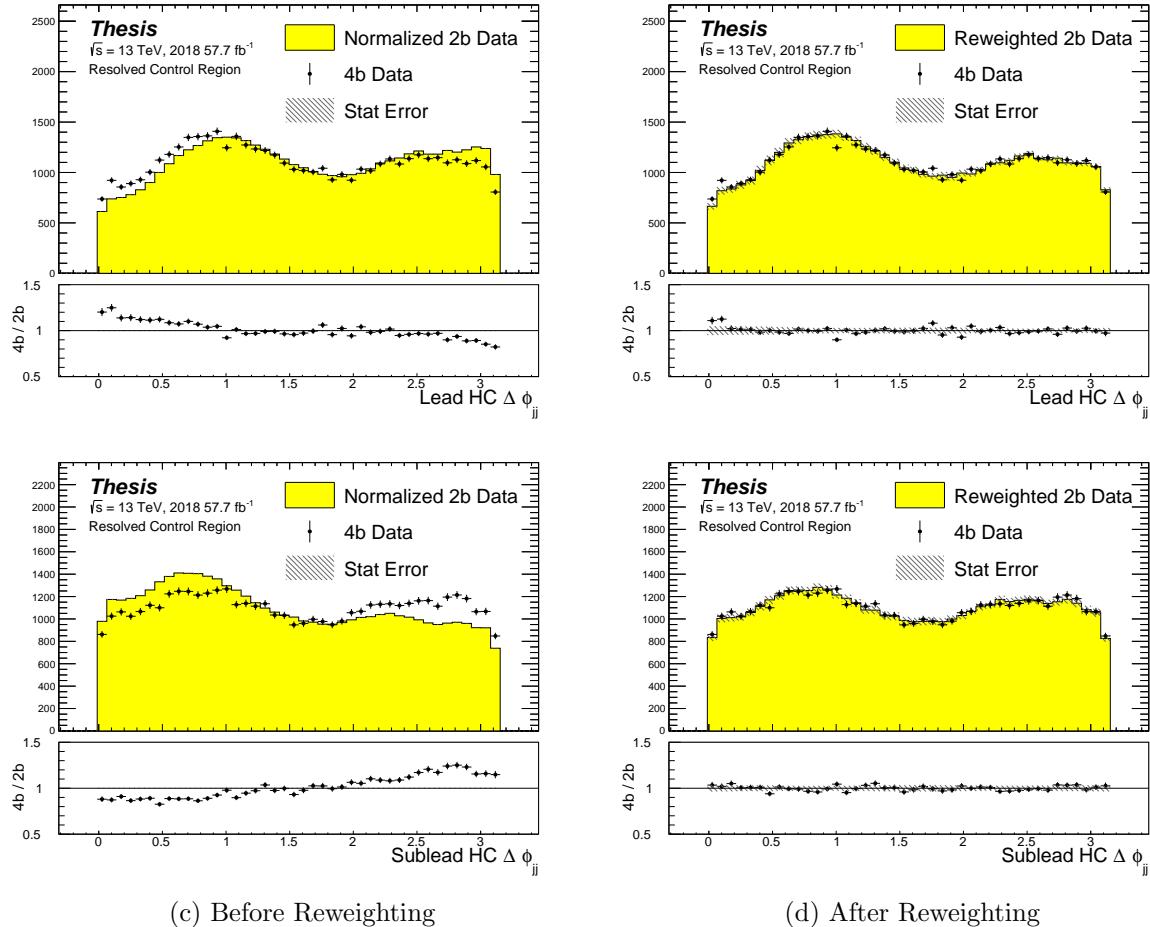


Figure 7.10: **Resonant Search:** Distributions of $\Delta\phi$ between jets in the leading and subleading Higgs candidates before and after CR derived reweighting for the 2018 Control Region.

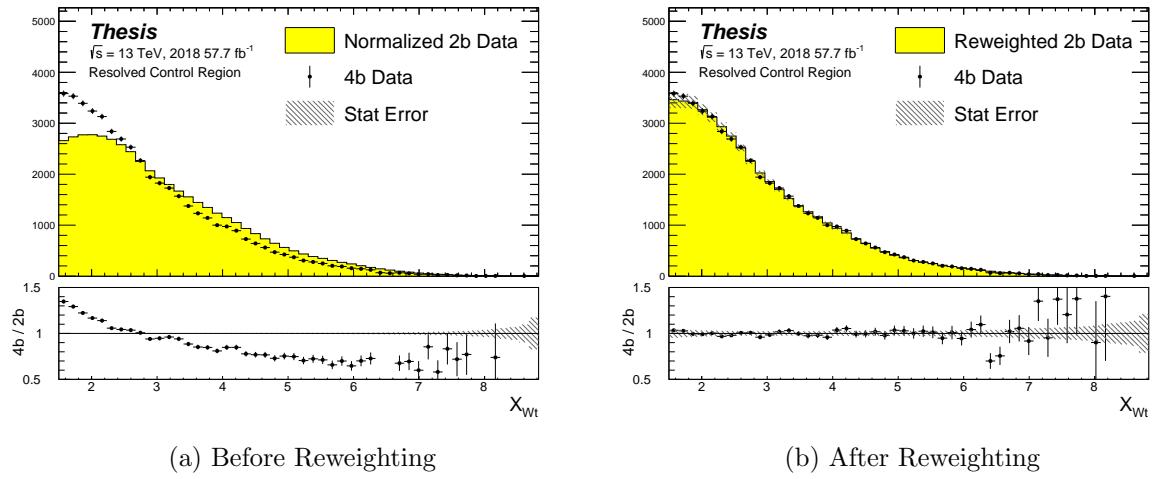


Figure 7.11: **Resonant Search:** Distributions of the top veto variable, X_{Wt} , before and after CR derived reweighting for the 2018 Control Region. Reweighting is done after the cut on this variable is applied

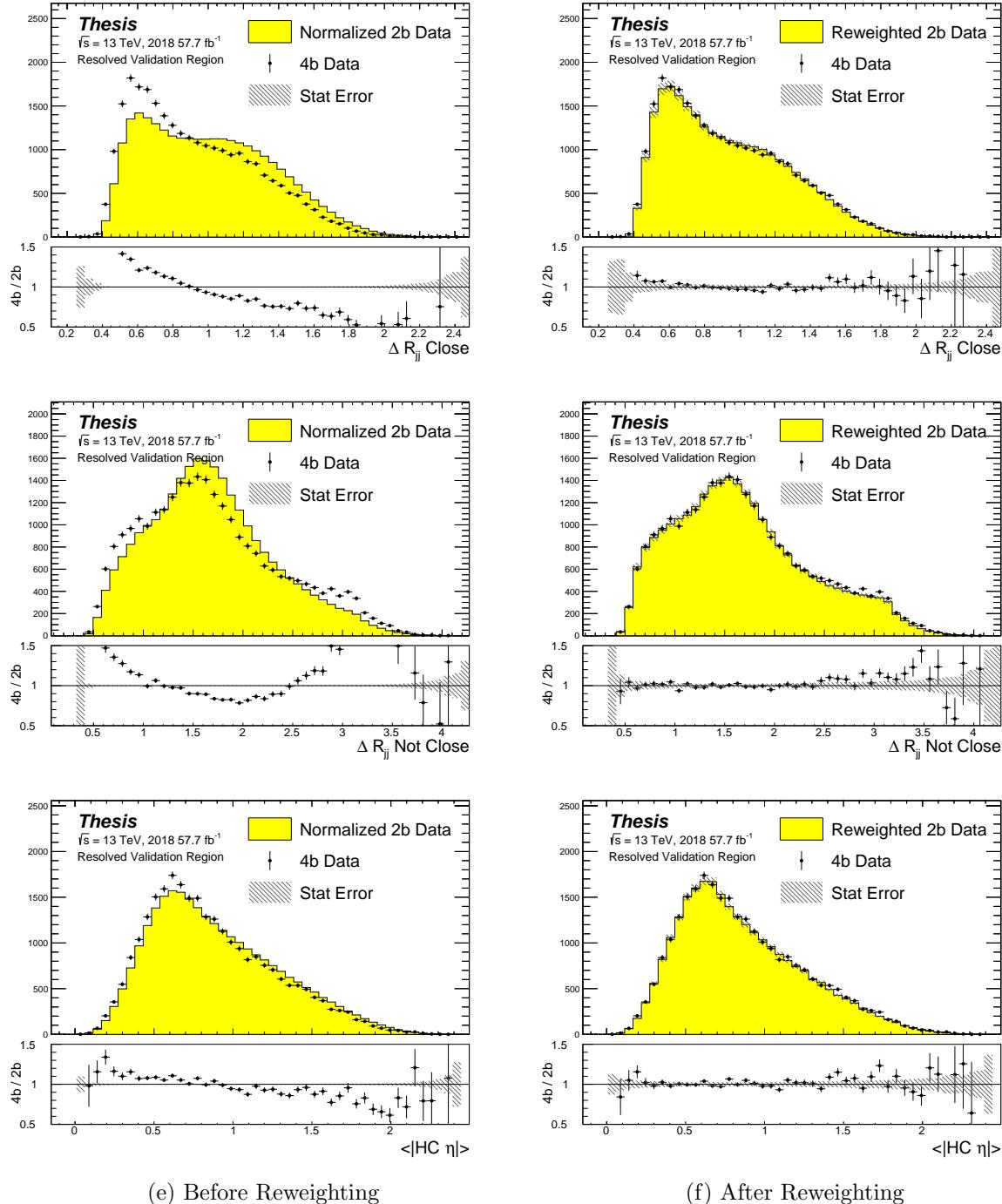


Figure 7.12: **Resonant Search:** Distributions of ΔR between the closest Higgs Candidate jets, ΔR between the other two, and average absolute value of HC jet η before and after CR derived reweighting for the 2018 Validation Region.

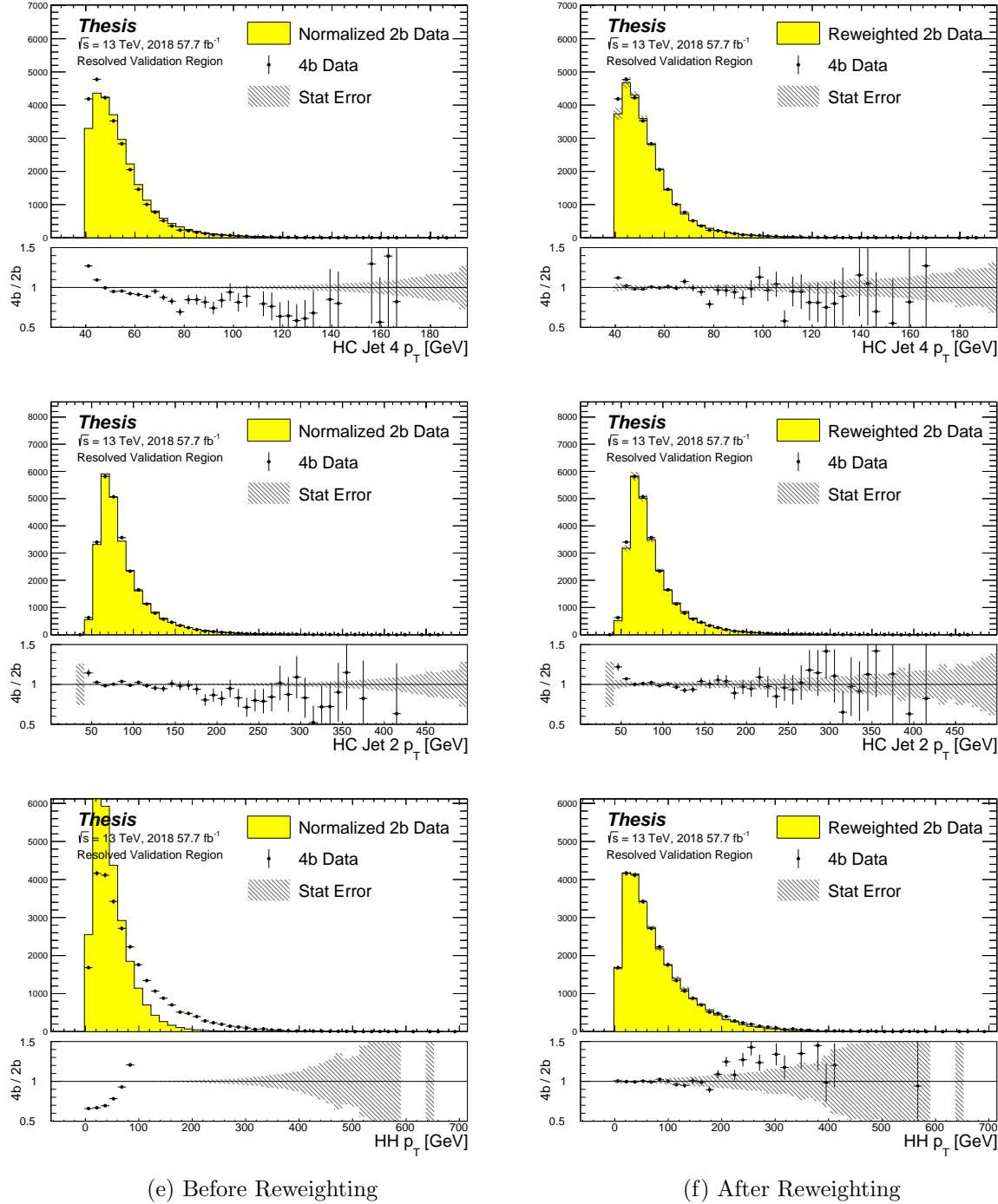


Figure 7.13: **Resonant Search:** Distributions of p_T of the 2nd and 4th leading Higgs Candidate jets and the p_T of the di-Higgs system before and after CR derived reweighting for the 2018 Validation Region.

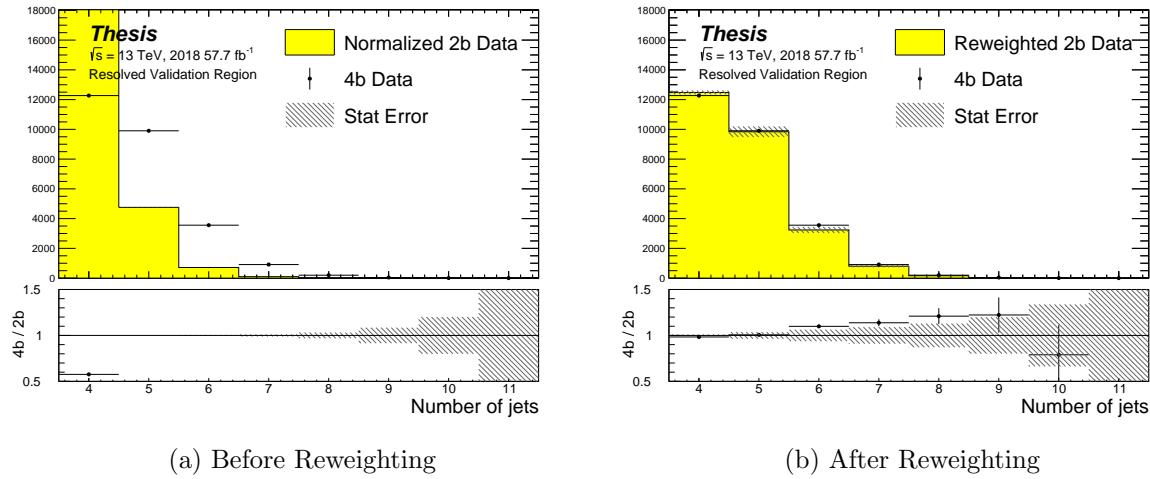


Figure 7.14: **Resonant Search:** Distributions of the number of jets before and after CR derived reweighting for the 2018 Validation Region. A minimum of 4 jets is required in each event in order to form Higgs candidates.

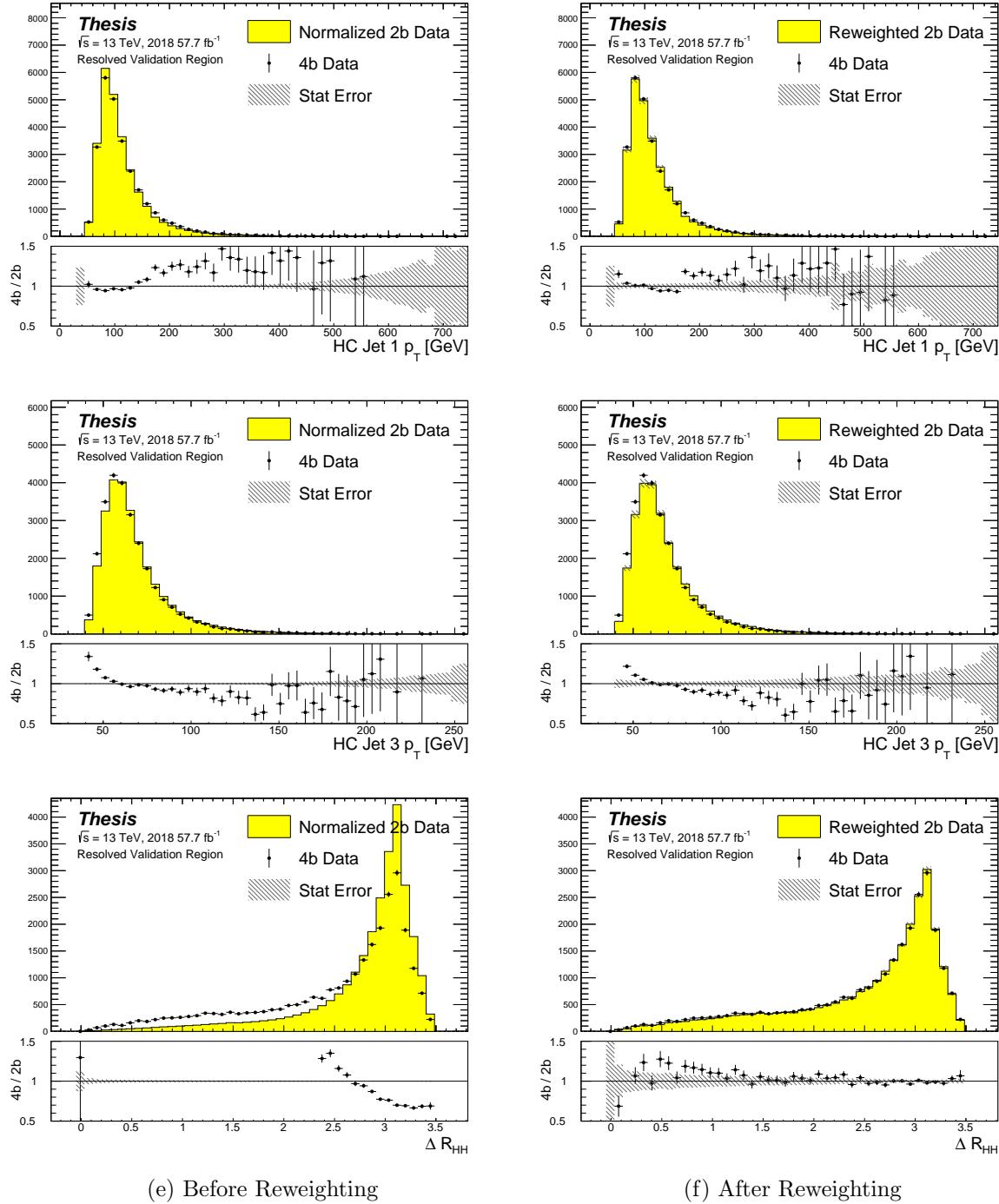


Figure 7.15: **Resonant Search:** Distributions of p_T of the 1st and 3rd leading Higgs Candidate jets and ΔR between Higgs candidates before and after CR derived reweighting for the 2018 Validation Region.

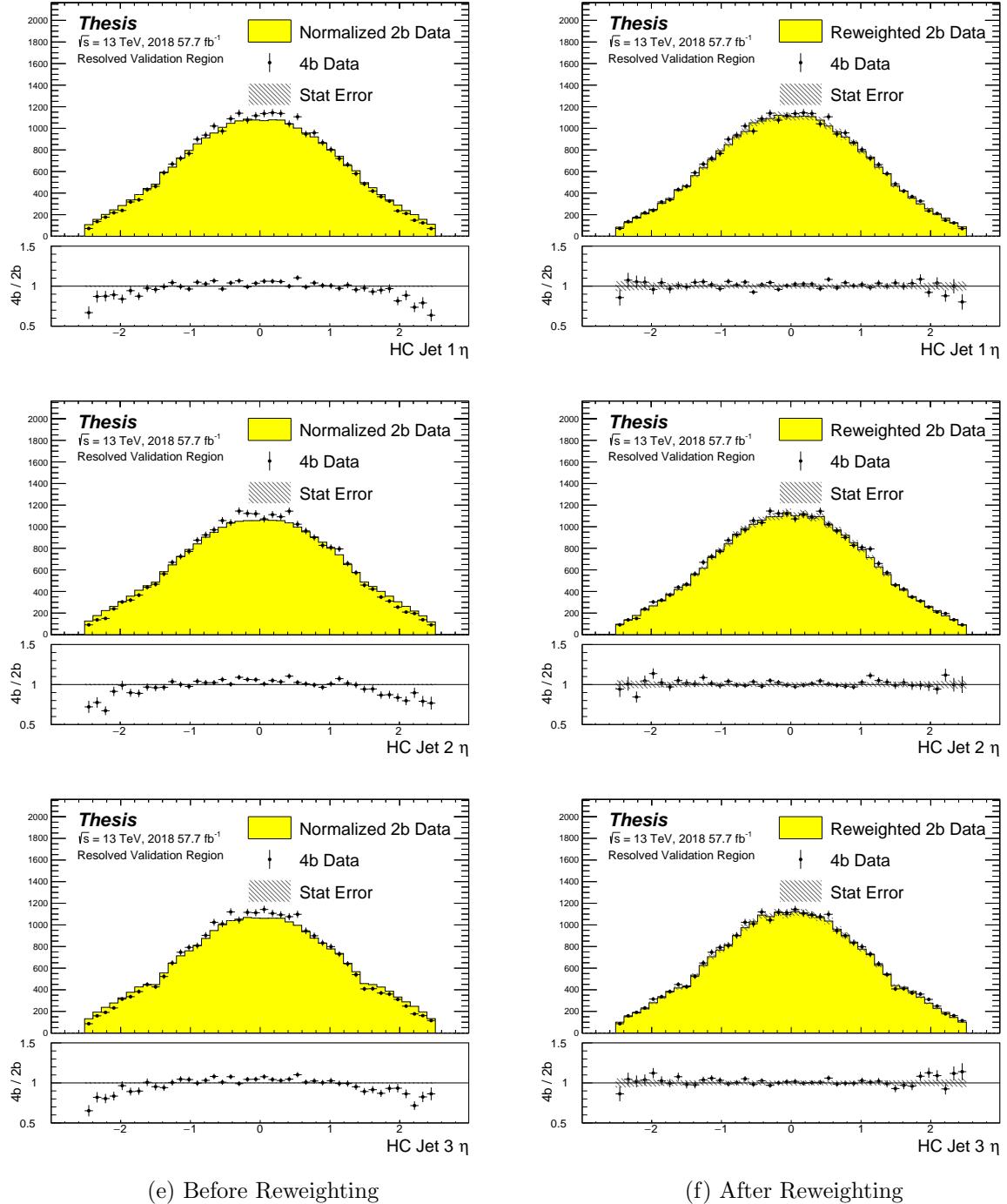


Figure 7.16: **Resonant Search:** Distributions of η of the 1st, 2nd, and 3rd leading Higgs Candidate jets before and after CR derived reweighting for the 2018 Validation Region.

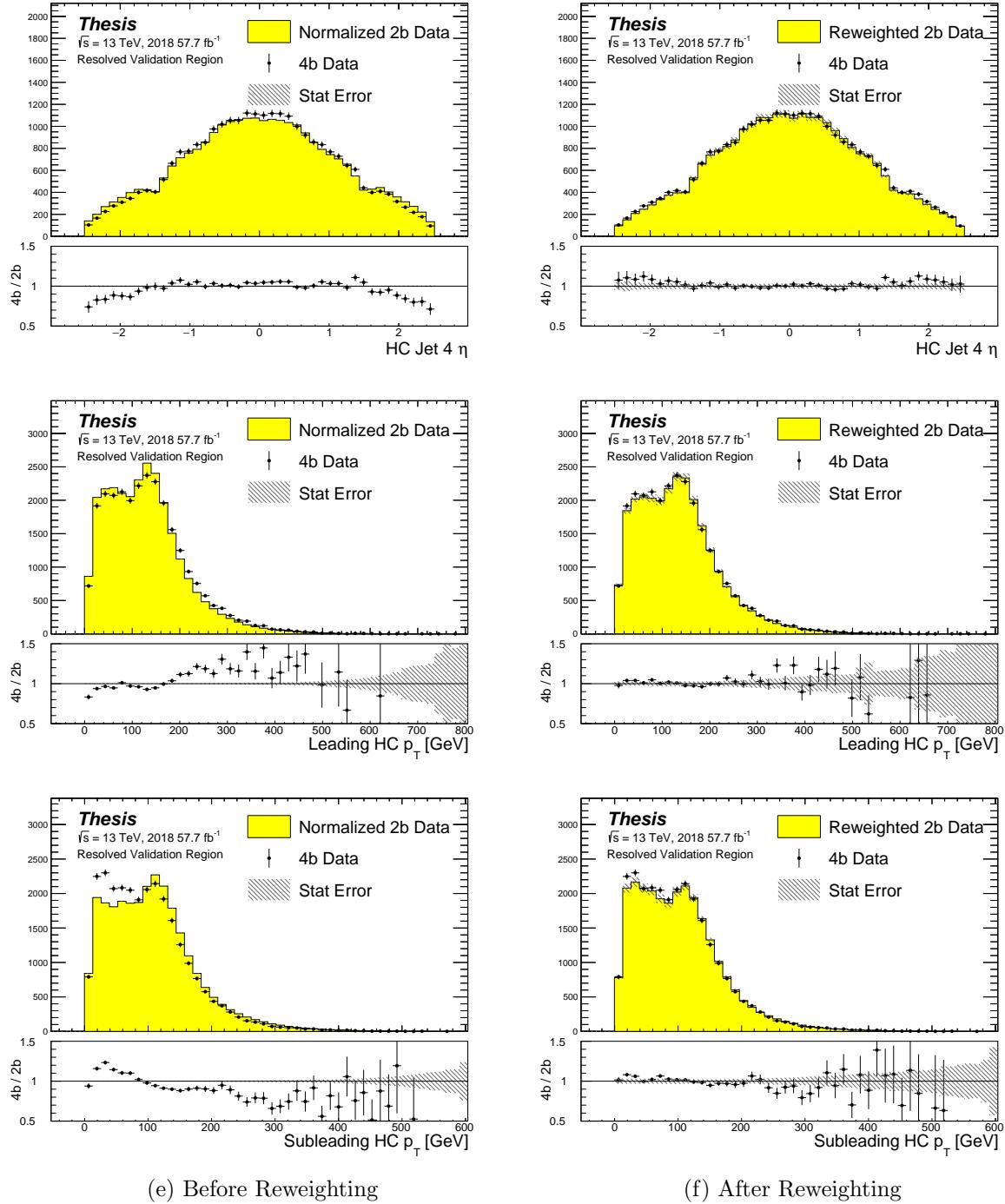


Figure 7.17: **Resonant Search:** Distributions of η of the 4th leading Higgs Candidate jet and the p_T of the leading and subleading Higgs candidates before and after CR derived reweighting for the 2018 Validation Region.

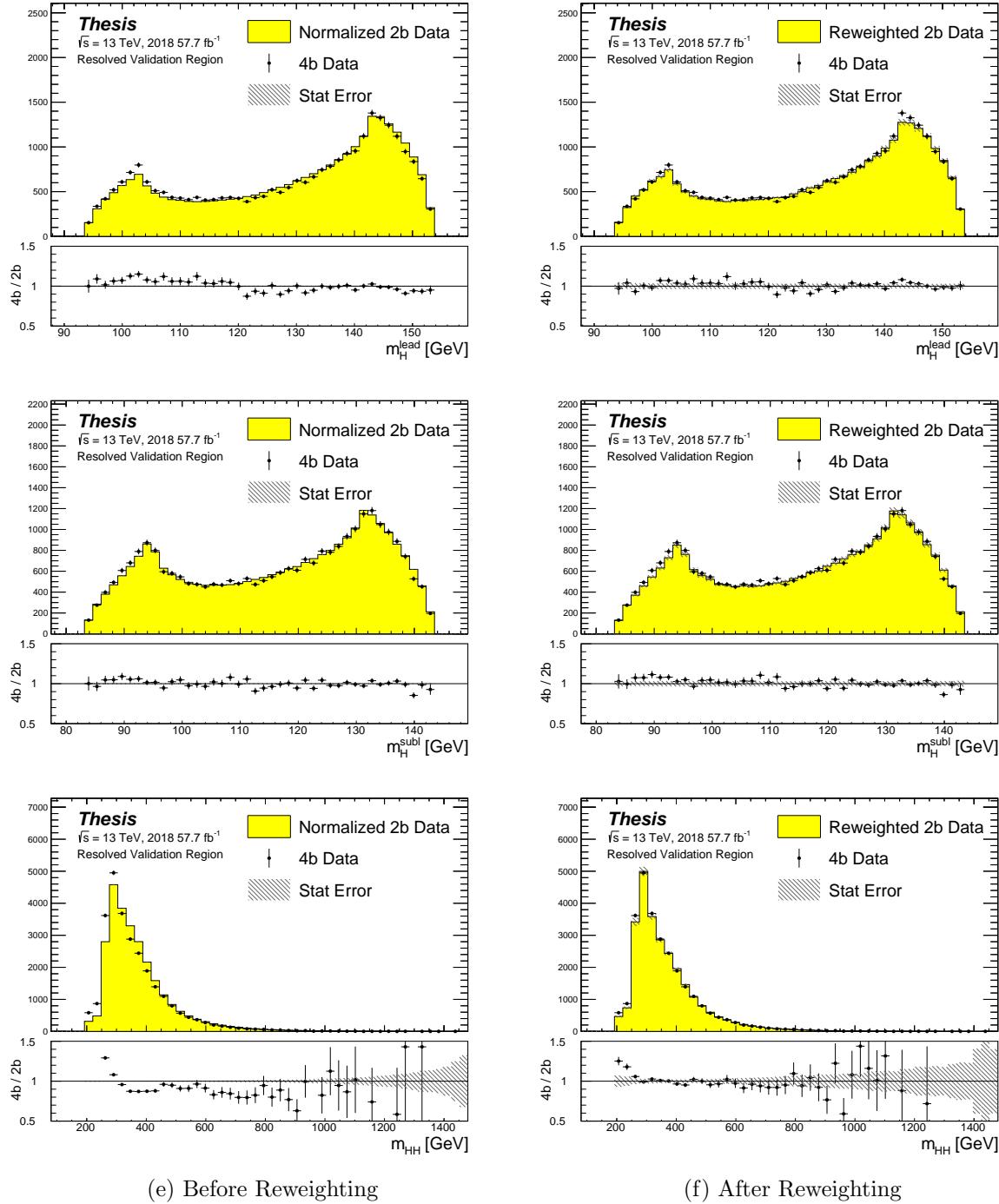


Figure 7.18: **Resonant Search:** Distributions of mass of the leading and subleading Higgs candidates and of the di-Higgs system before and after CR derived reweighting for the 2018 Validation Region.

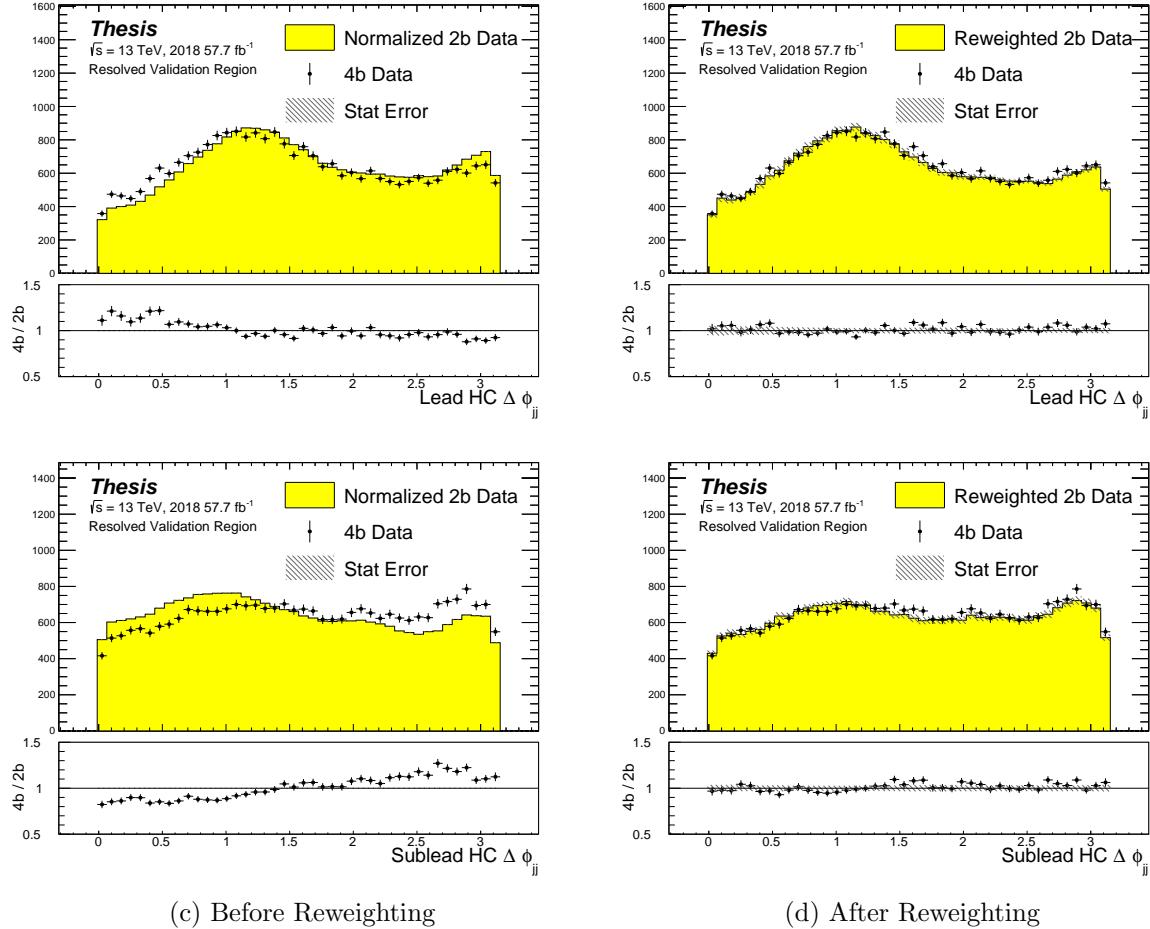


Figure 7.19: **Resonant Search:** Distributions of $\Delta\phi$ between jets in the leading and subleading Higgs candidates before and after CR derived reweighting for the 2018 Validation Region.

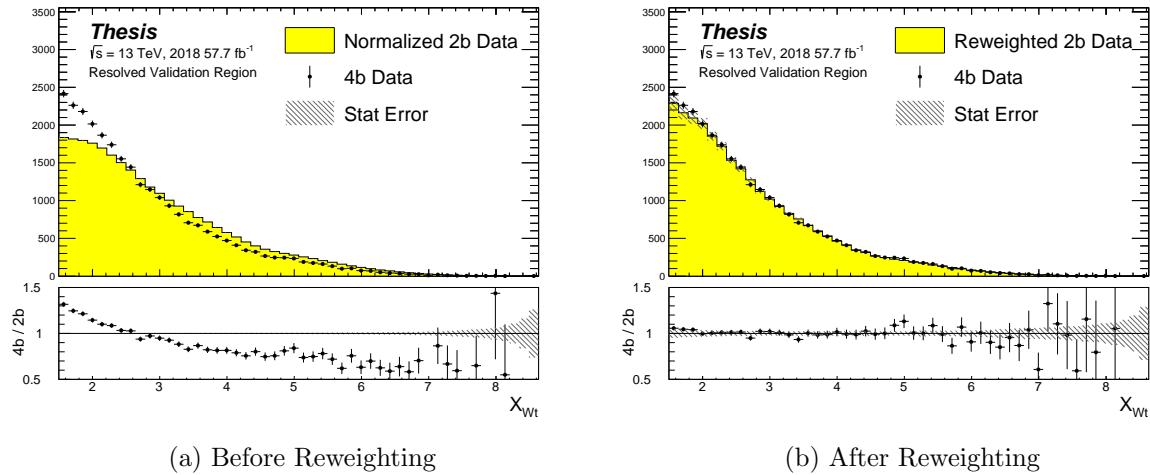


Figure 7.20: **Resonant Search:** Distributions of the top veto variable, X_{Wt} , before and after CR derived reweighting for the 2018 Validation Region. Reweighting is done after the cut on this variable is applied

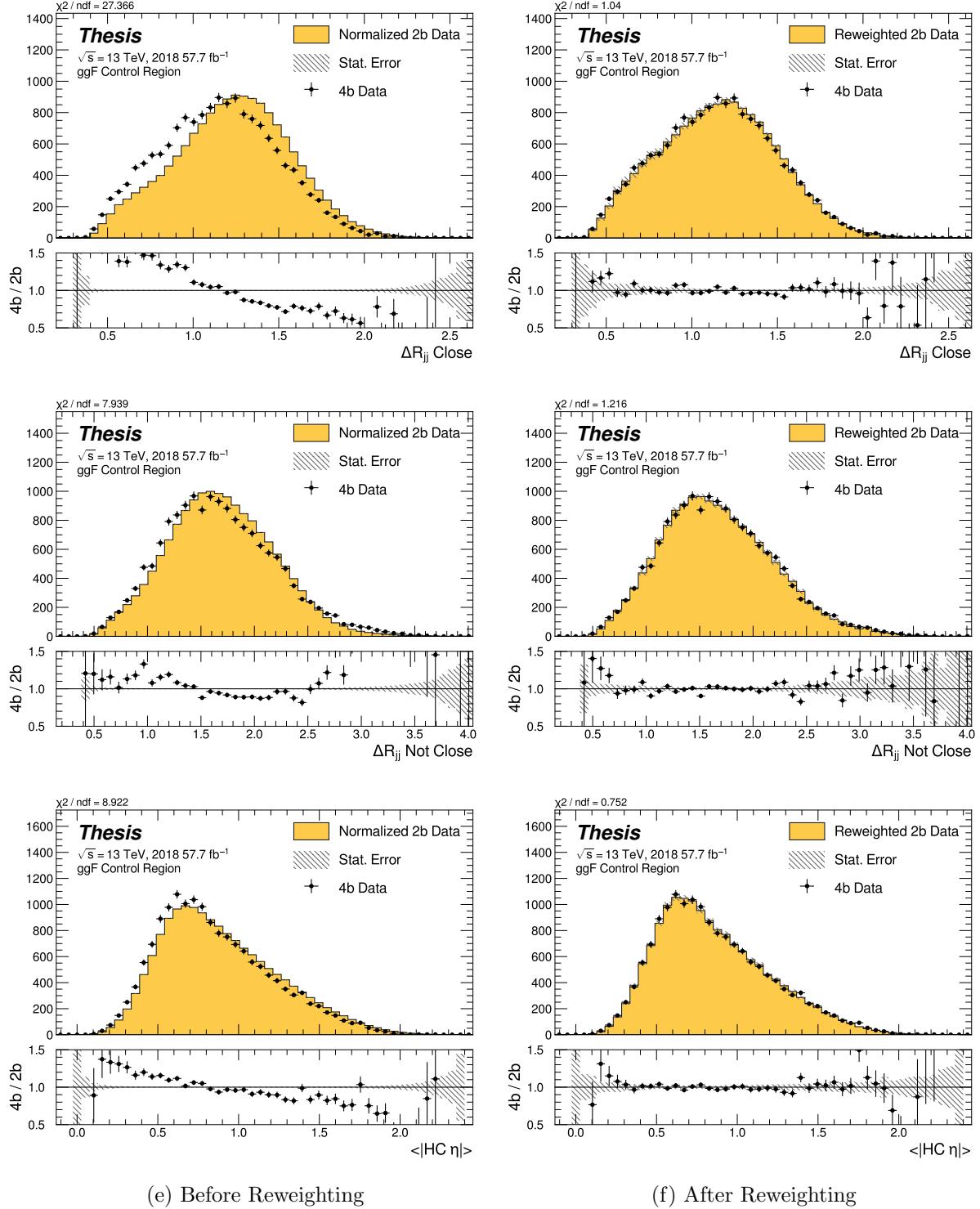


Figure 7.21: **Non-resonant Search (4b):** Distributions of ΔR between the closest Higgs Candidate jets, ΔR between the other two, and average absolute value of HC jet η before and after CR derived reweighting for the 2018 4b Control Region.

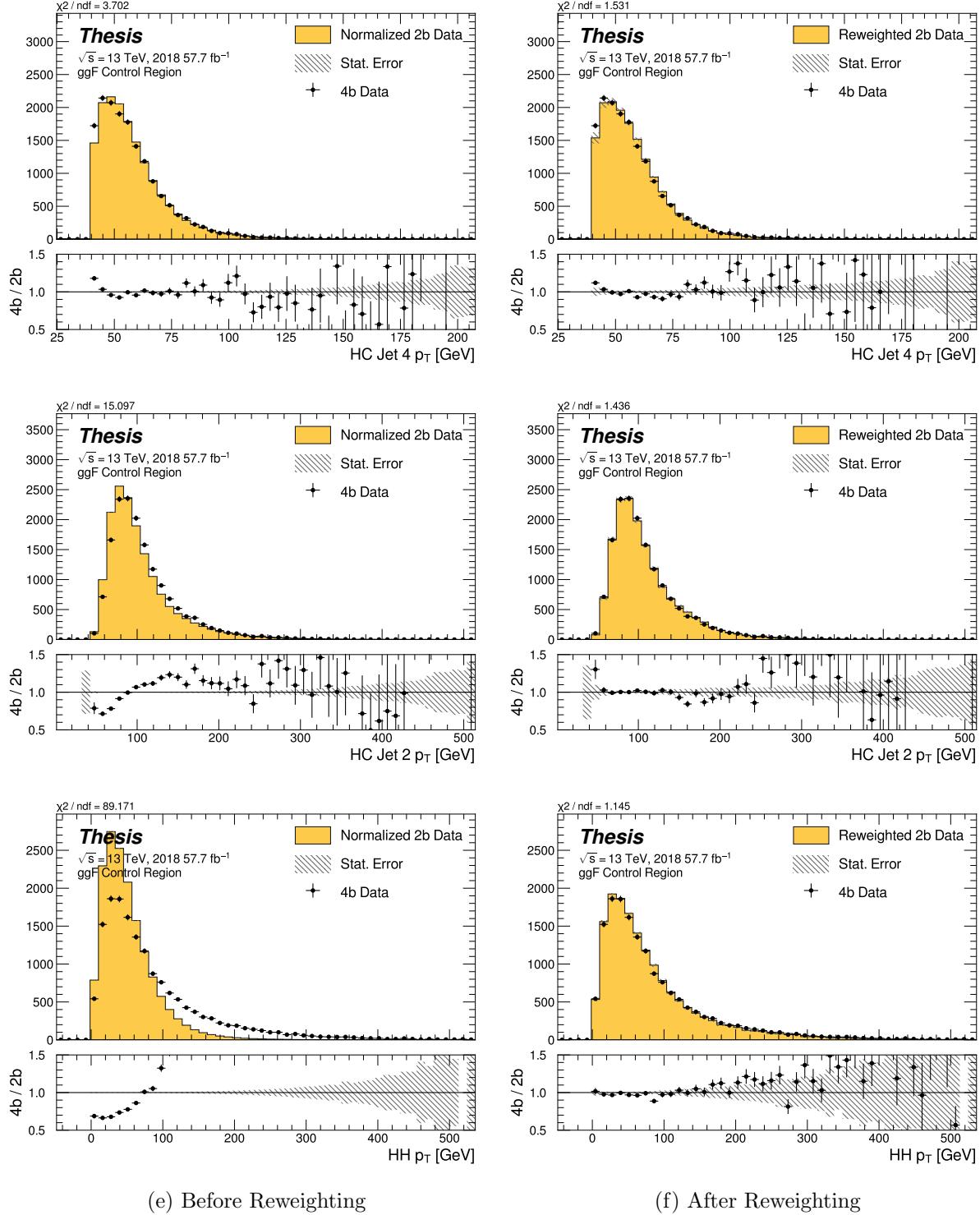


Figure 7.22: **Non-resonant Search (4b):** Distributions of p_T of the 2nd and 4th leading Higgs Candidate jets and the p_T of the di-Higgs system before and after CR derived reweighting for the 2018 4b Control Region.

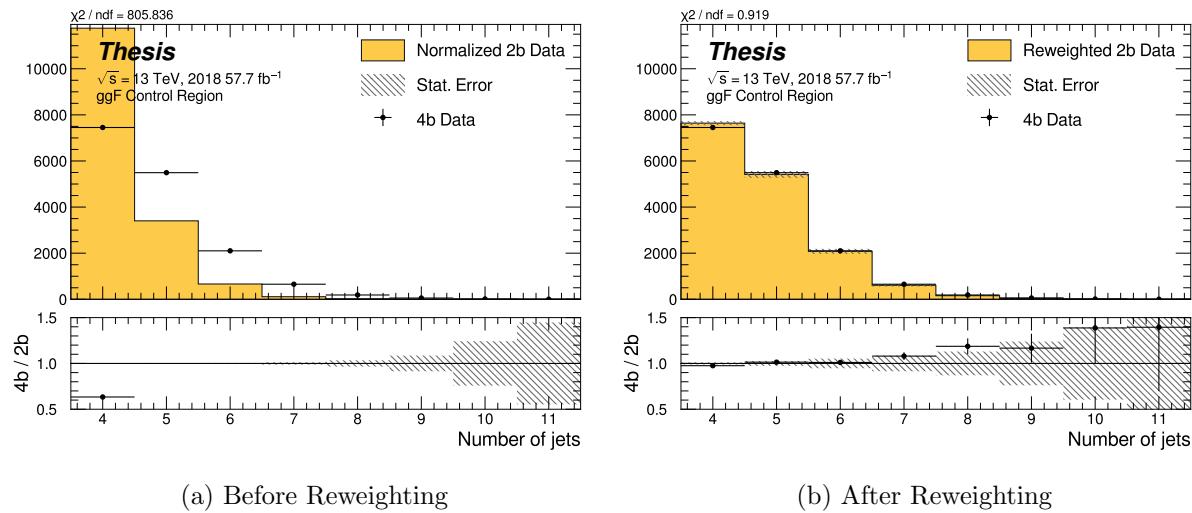


Figure 7.23: **Non-resonant Search (4b):** Distributions of the number of jets before and after CR derived reweighting for the 2018 4b Control Region. A minimum of 4 jets is required in each event in order to form Higgs candidates.

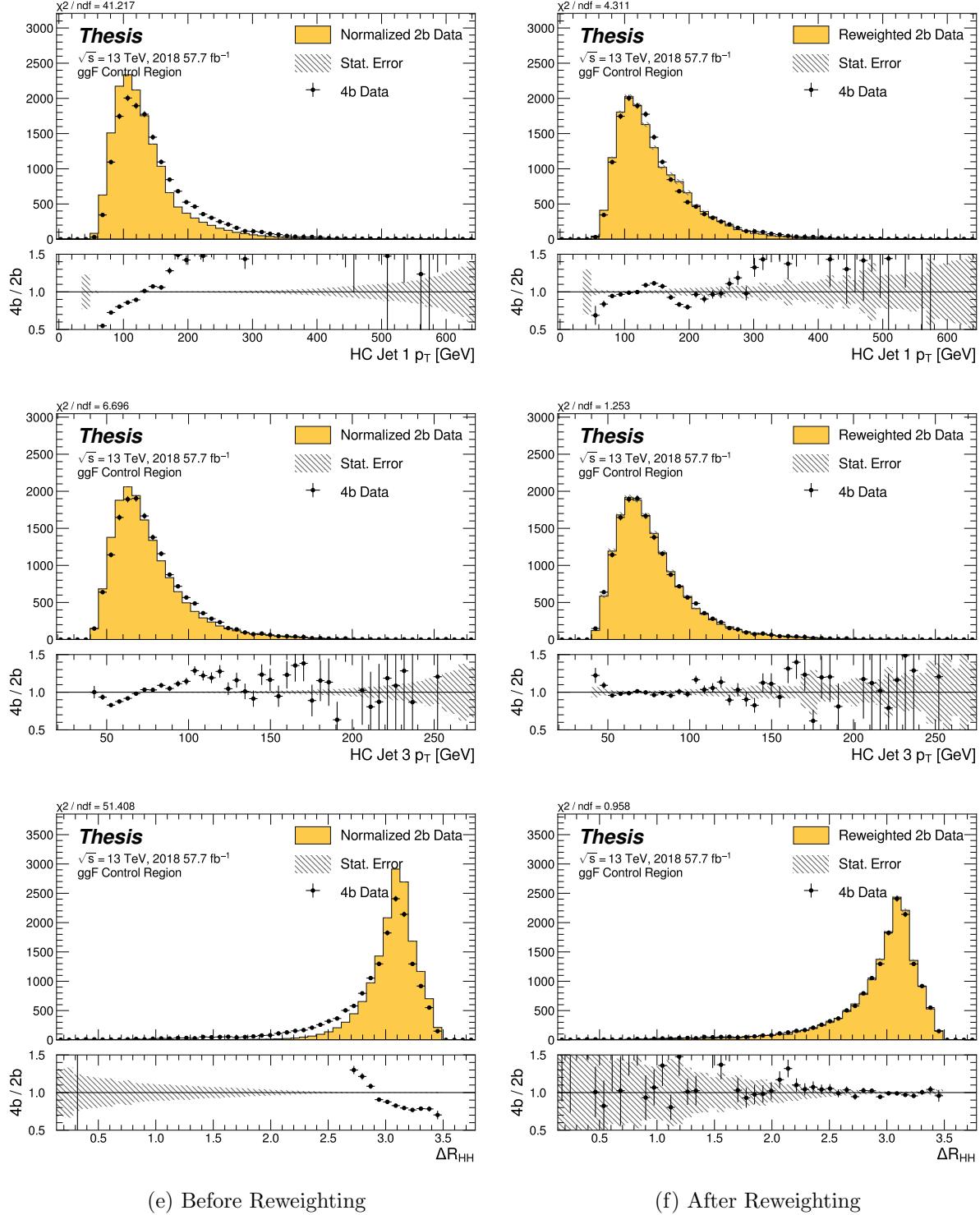


Figure 7.24: **Non-resonant Search (4b):** Distributions of p_T of the 1st and 3rd leading Higgs Candidate jets and ΔR between Higgs candidates before and after CR derived reweighting for the 2018 4b Control Region.

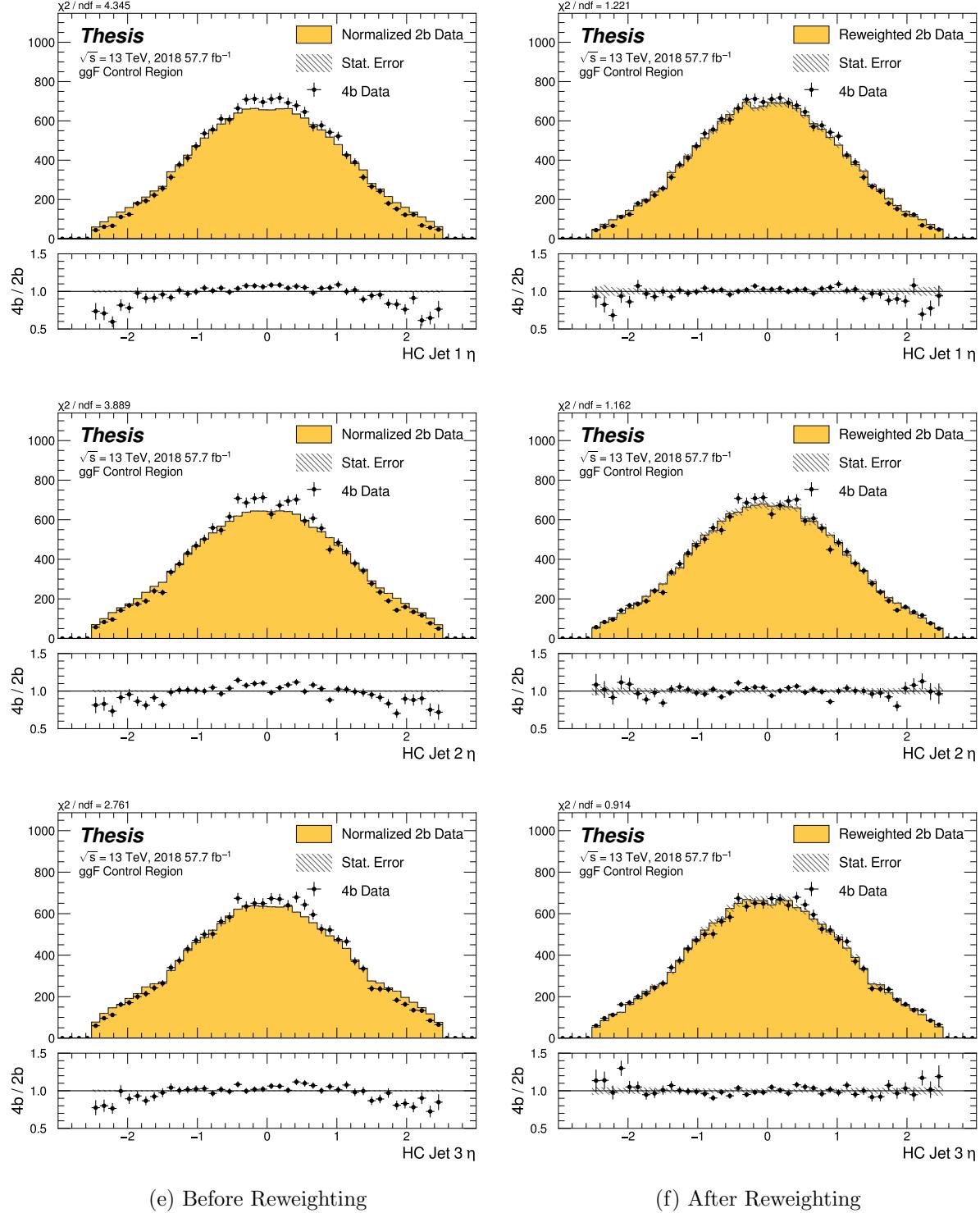


Figure 7.25: **Non-resonant Search (4b):** Distributions of η of the 1st, 2nd, and 3rd leading Higgs Candidate jets before and after CR derived reweighting for the 2018 4b Control Region.

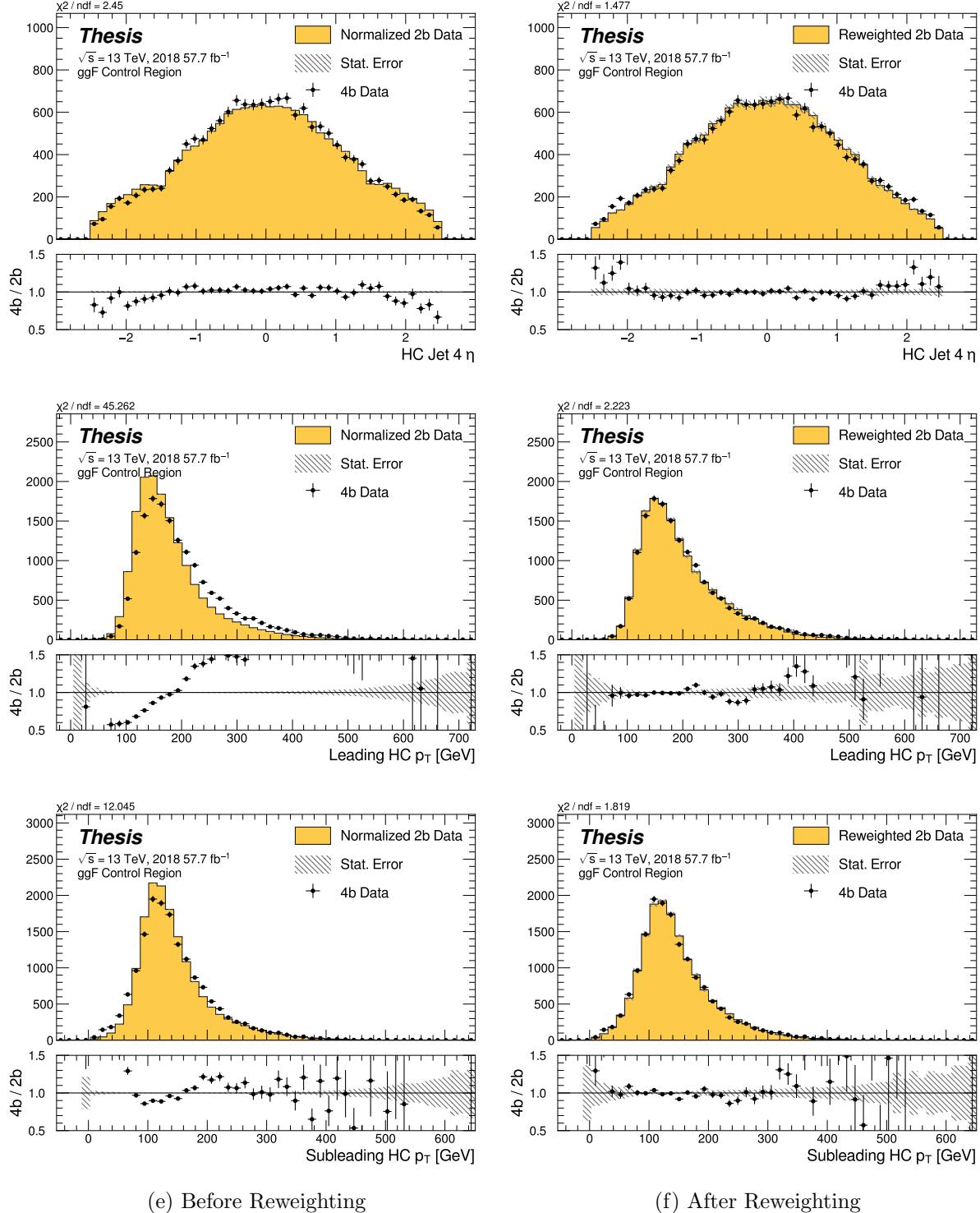


Figure 7.26: **Non-resonant Search (4b):** Distributions of η of the 4th leading Higgs Candidate jet and the p_T of the leading and subleading Higgs candidates before and after CR derived reweighting for the 2018 4b Control Region.

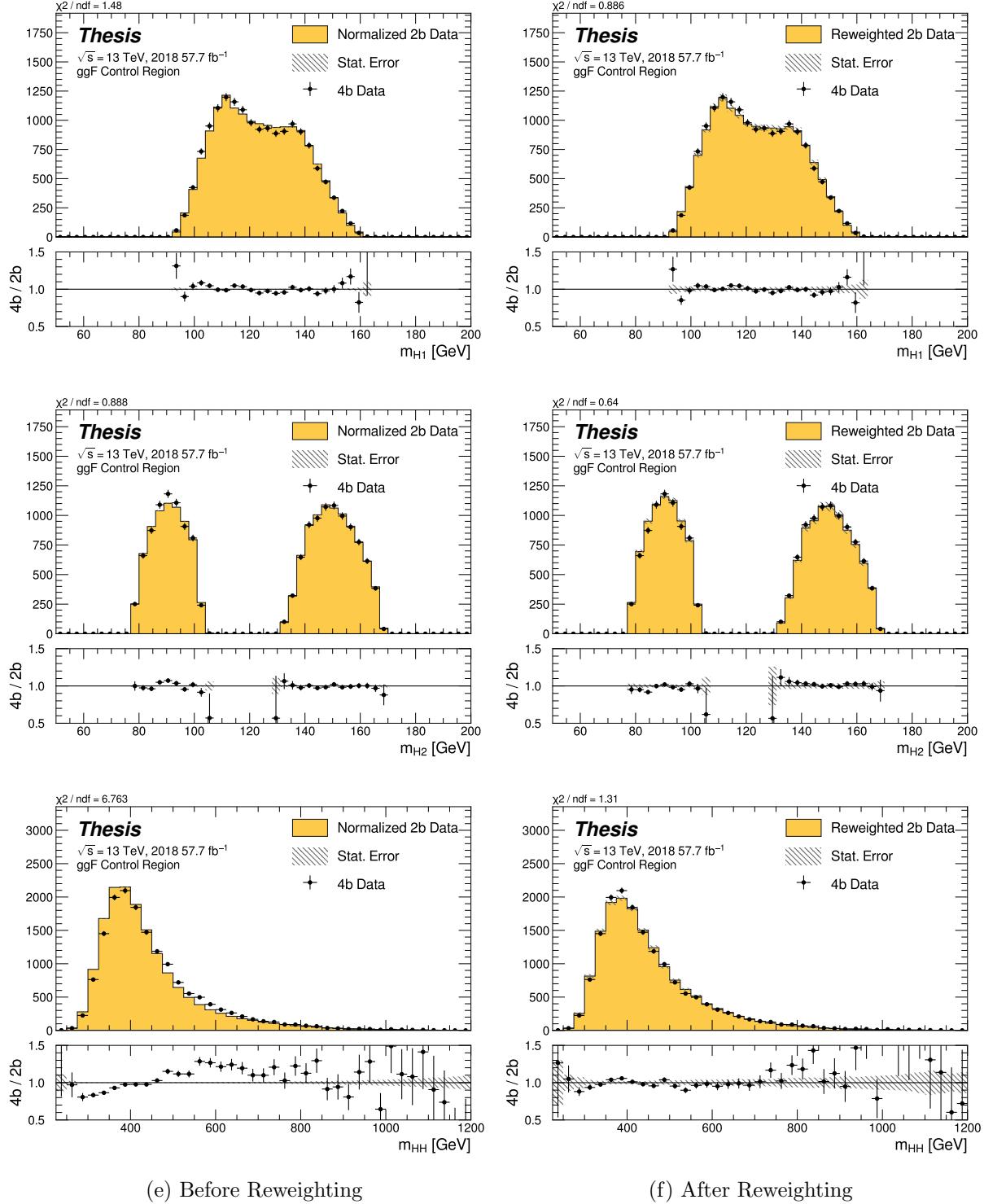


Figure 7.27: **Non-resonant Search (4b):** Distributions of mass of the leading and subleading Higgs candidates and of the di-Higgs system before and after CR derived reweighting for the 2018 4b Control Region.

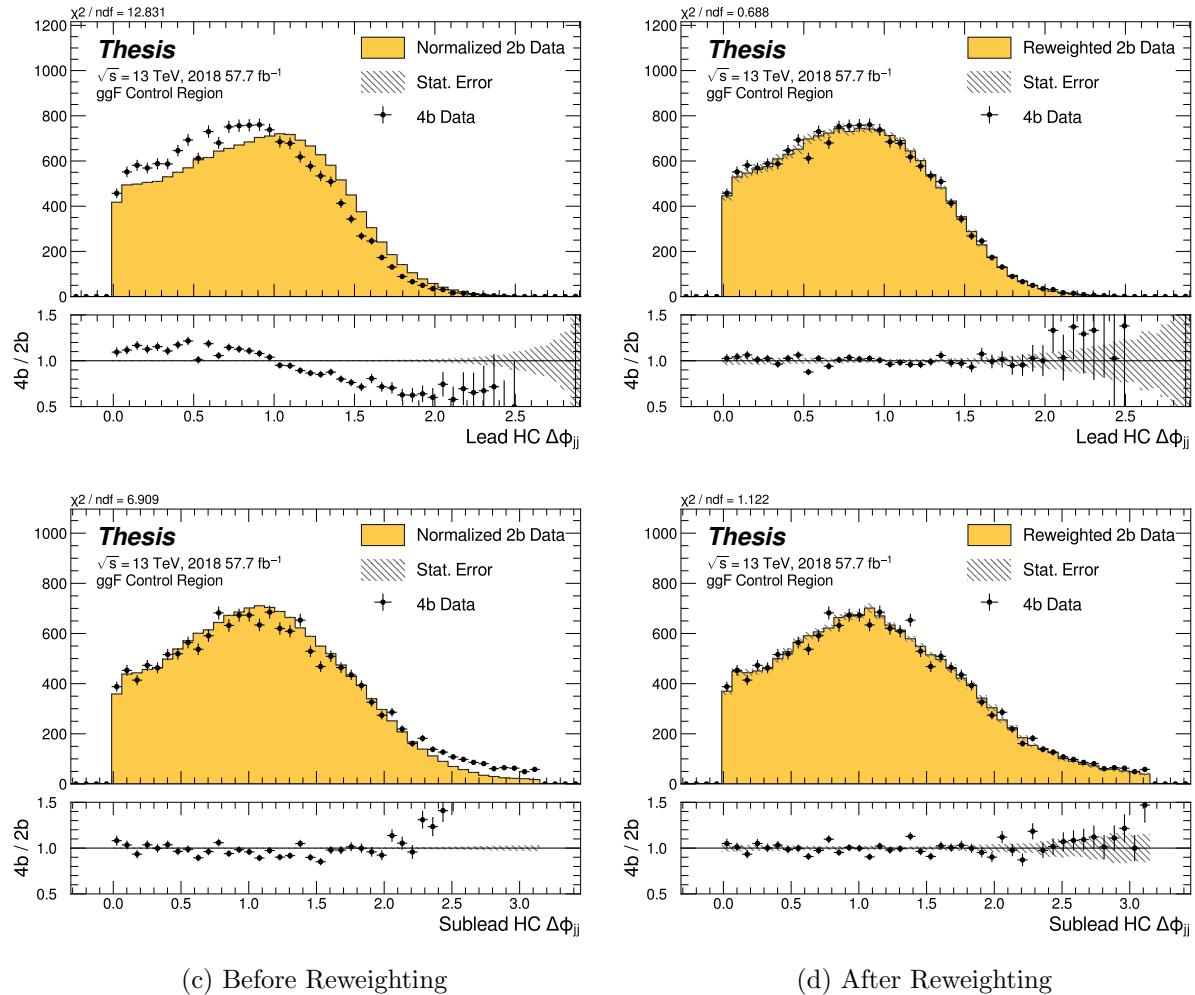


Figure 7.28: **Non-resonant Search (4b):** Distributions of $\Delta\phi$ between jets in the leading and subleading Higgs candidates before and after CR derived reweighting for the 2018 4b Control Region.

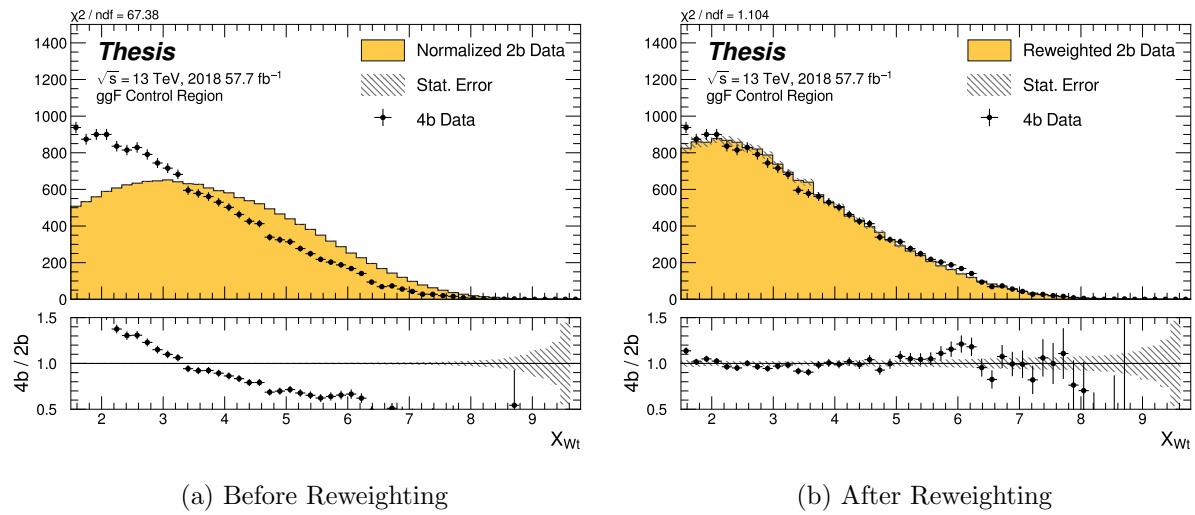


Figure 7.29: **Non-resonant Search (4b):** Distributions of the top veto variable, X_{Wt} , before and after CR derived reweighting for the 2018 4b Control Region. Reweighting is done after the cut on this variable is applied.

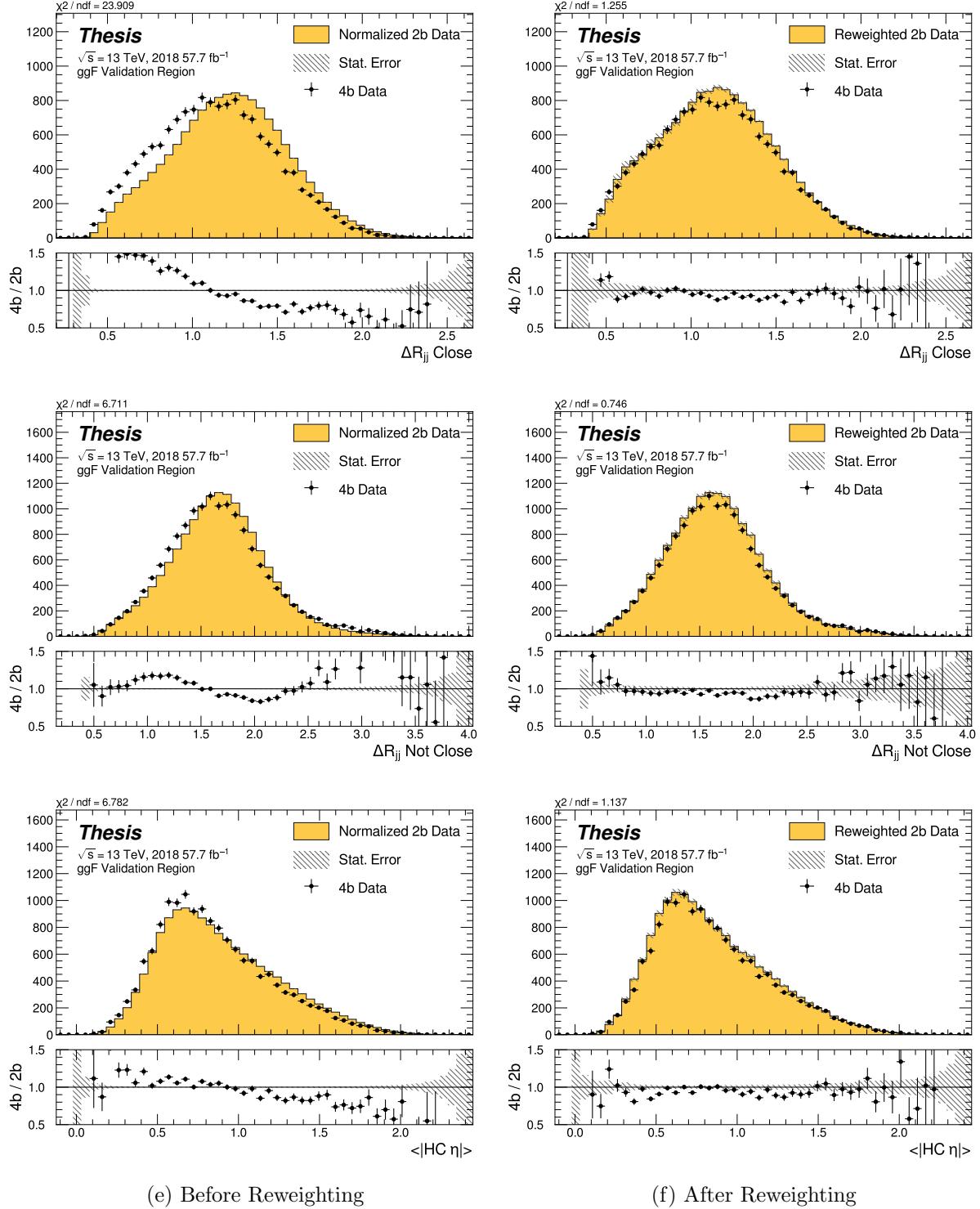


Figure 7.30: **Non-resonant Search (4b):** Distributions of ΔR between the closest Higgs Candidate jets, ΔR between the other two, and average absolute value of HC jet η before and after CR derived reweighting for the 2018 4b Validation Region.

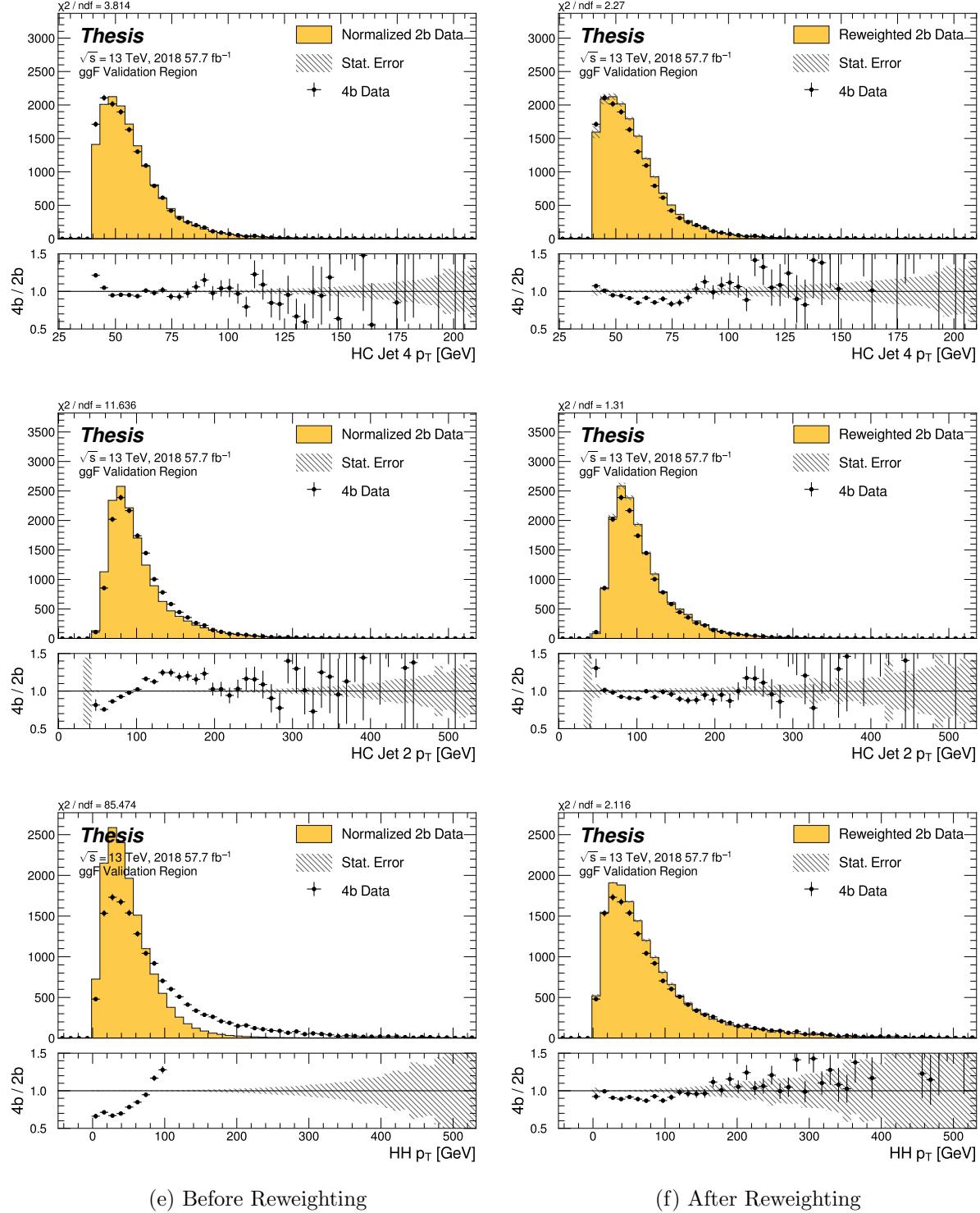


Figure 7.31: **Non-resonant Search (4b):** Distributions of p_T of the 2nd and 4th leading Higgs Candidate jets and the p_T of the di-Higgs system before and after CR derived reweighting for the 2018 4b Validation Region.

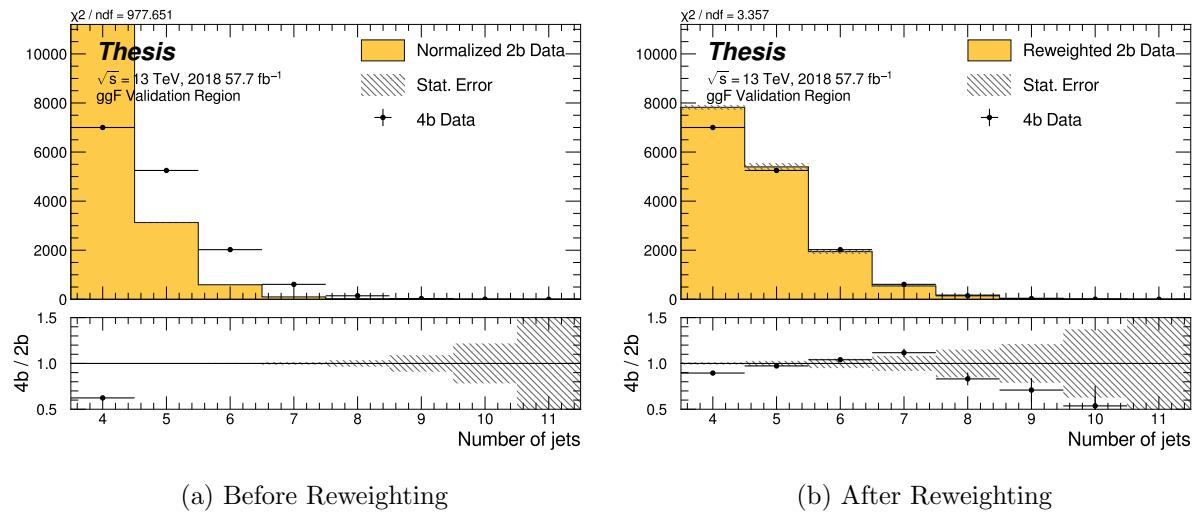


Figure 7.32: **Non-resonant Search (4b):** Distributions of the number of jets before and after CR derived reweighting for the 2018 4b Validation Region. A minimum of 4 jets is required in each event in order to form Higgs candidates.

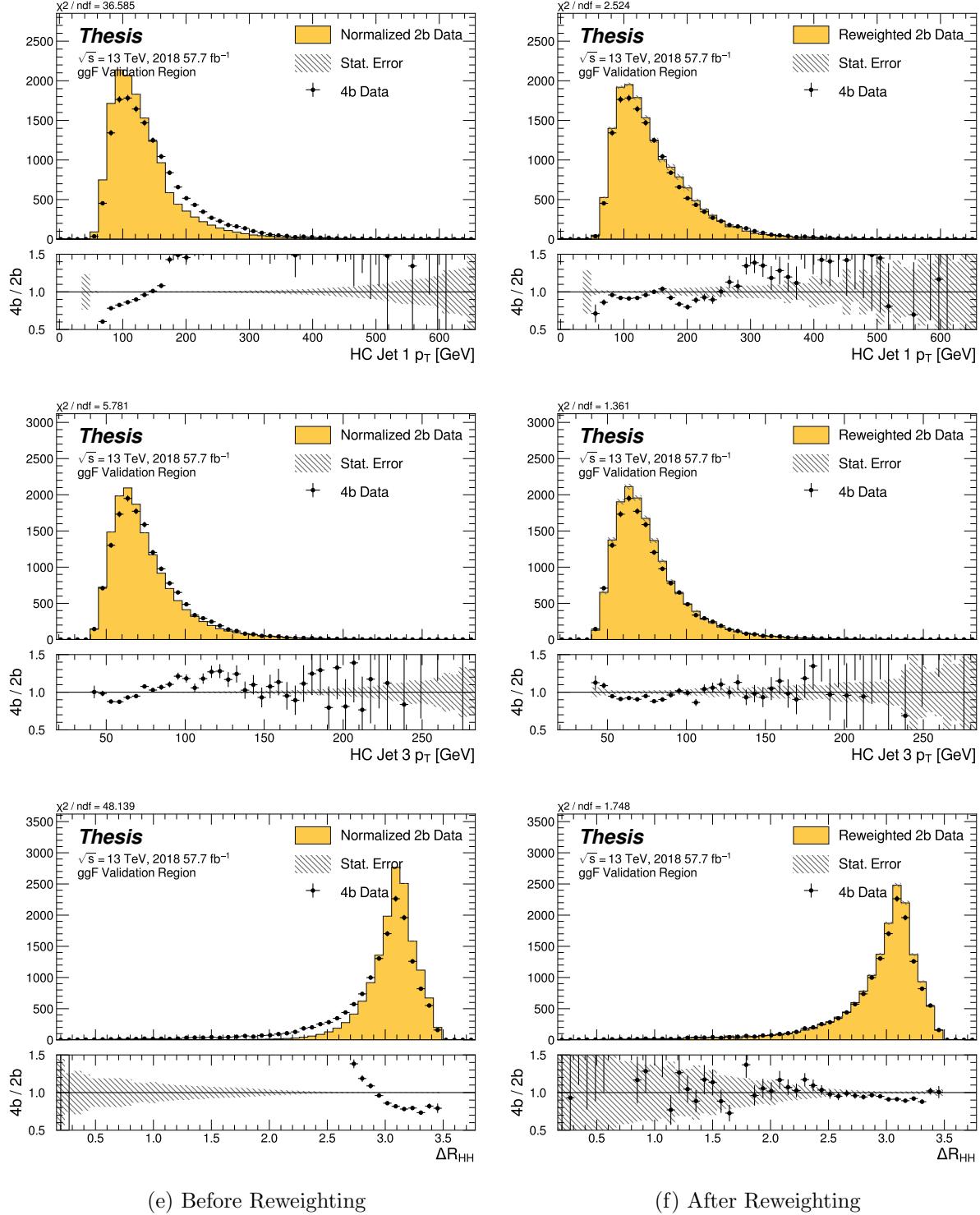


Figure 7.33: **Non-resonant Search (4b):** Distributions of p_T of the 1st and 3rd leading Higgs Candidate jets and ΔR between Higgs candidates before and after CR derived reweighting for the 2018 4b Validation Region.

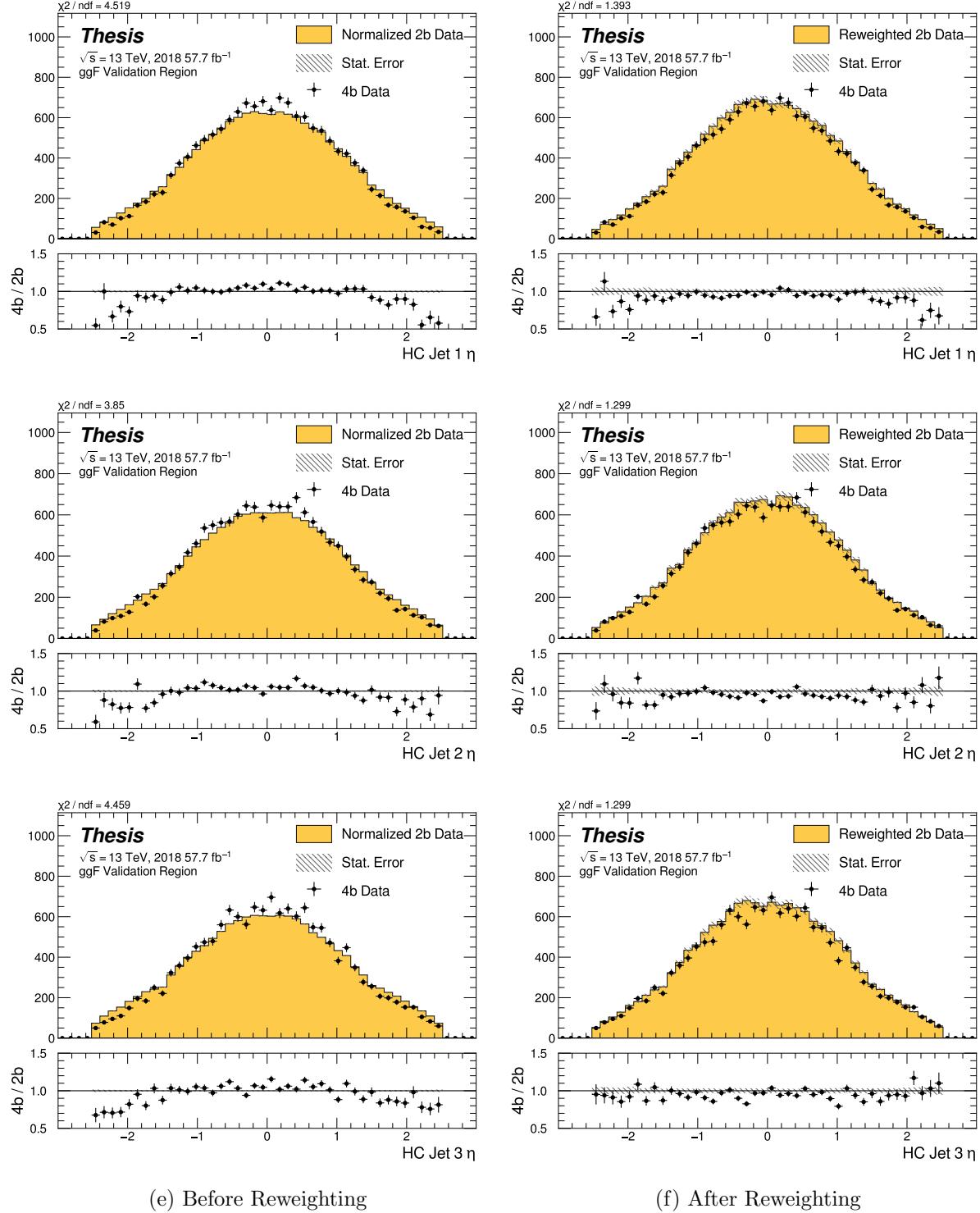


Figure 7.34: **Non-resonant Search (4b):** Distributions of η of the 1st, 2nd, and 3rd leading Higgs Candidate jets before and after CR derived reweighting for the 2018 4b Validation Region.

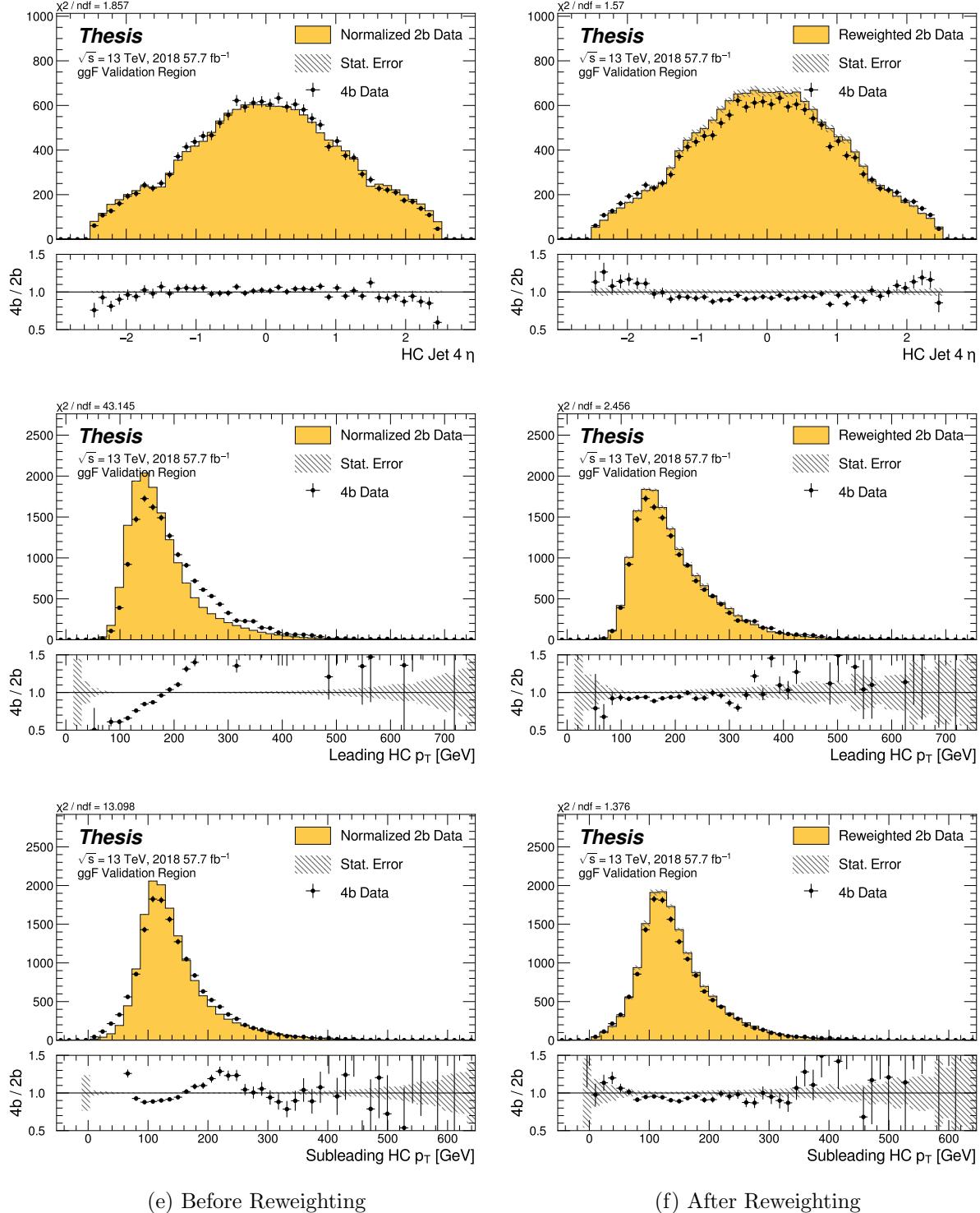


Figure 7.35: **Non-resonant Search (4b):** Distributions of η of the 4th leading Higgs Candidate jet and the p_T of the leading and subleading Higgs candidates before and after CR derived reweighting for the 2018 4b Validation Region.

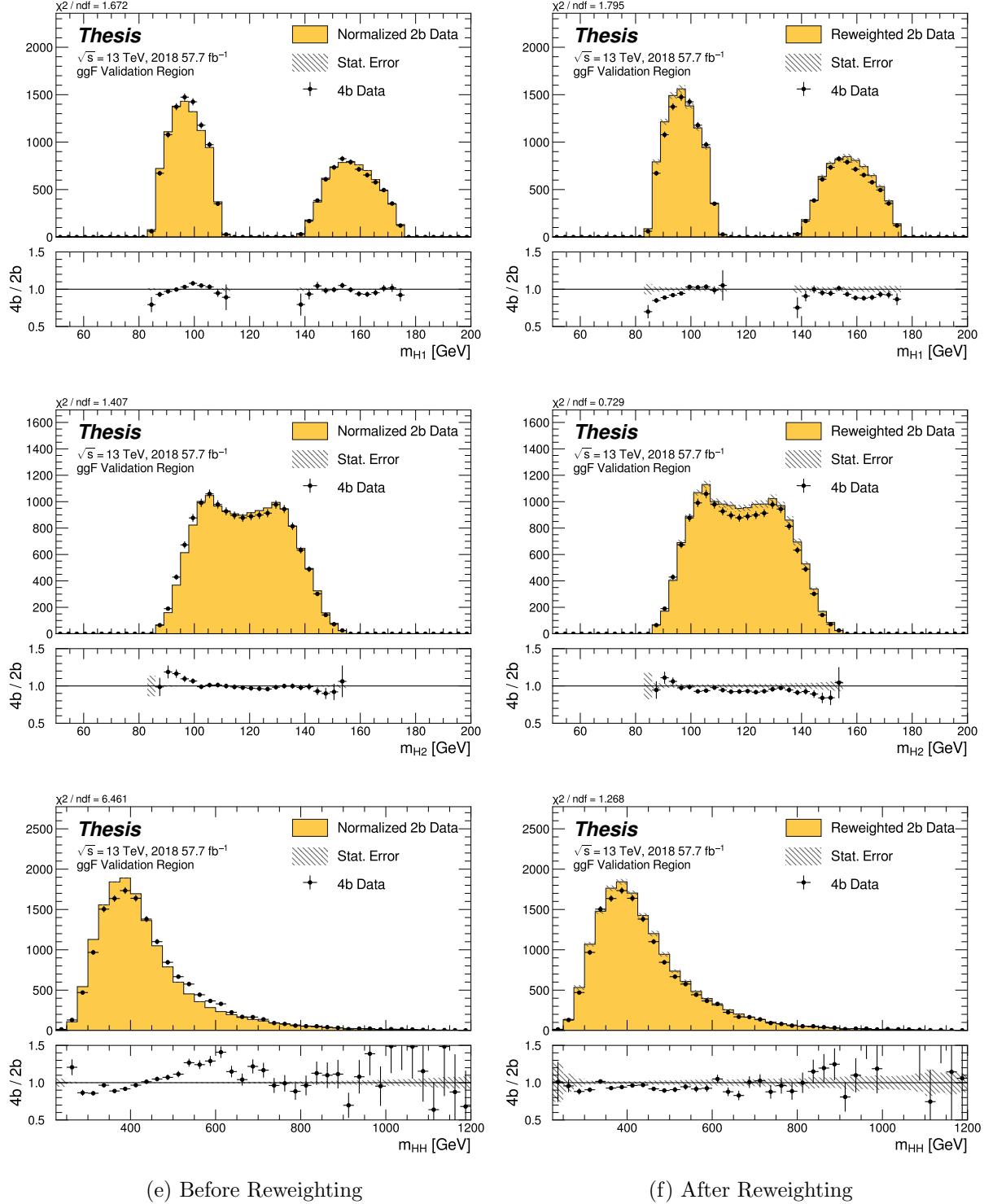


Figure 7.36: **Non-resonant Search (4b):** Distributions of mass of the leading and subleading Higgs candidates and of the di-Higgs system before and after CR derived reweighting for the 2018 4b Validation Region.

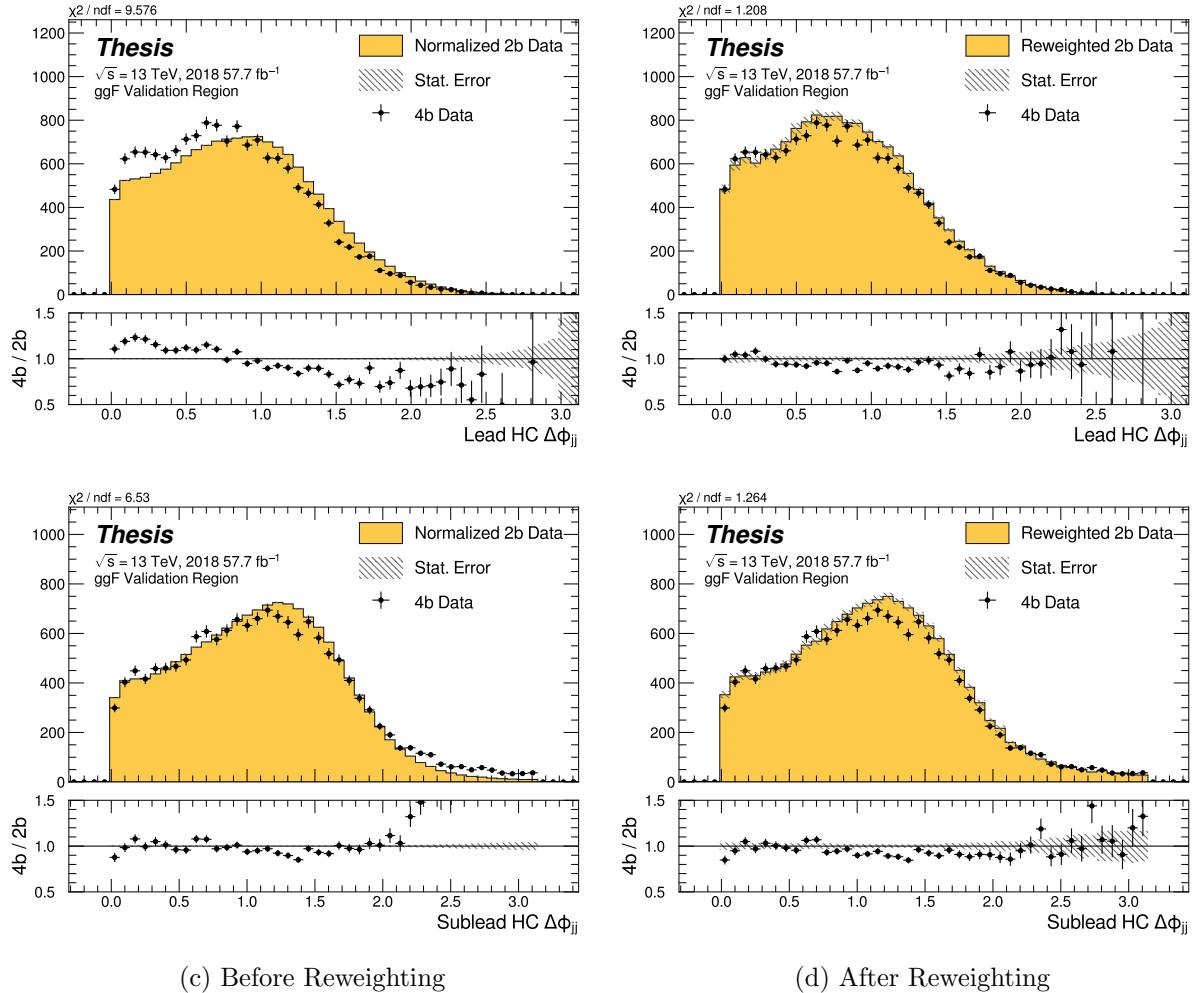


Figure 7.37: **Non-resonant Search (4b):** Distributions of $\Delta\phi$ between jets in the leading and subleading Higgs candidates before and after CR derived reweighting for the 2018 4b Validation Region.

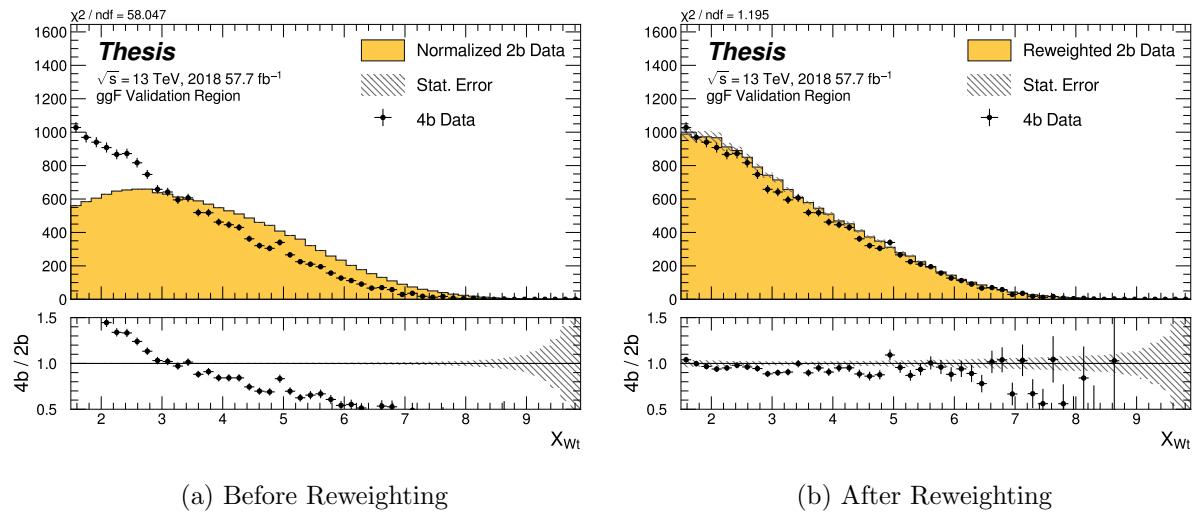


Figure 7.38: **Non-resonant Search (4b)**: Distributions of the top veto variable, X_{Wt} , before and after CR derived reweighting for the 2018 4b Validation Region. Reweighting is done after the cut on this variable is applied.

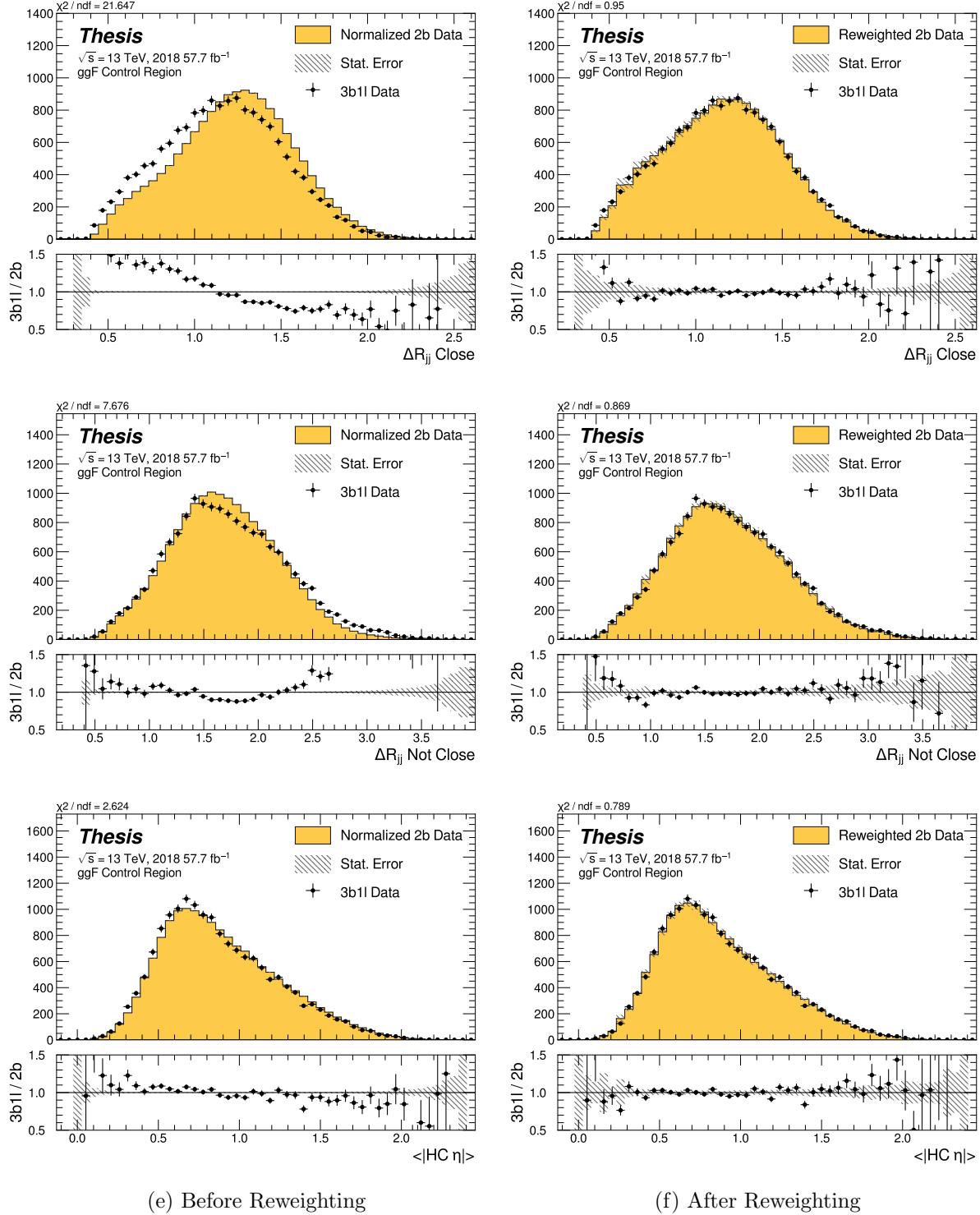


Figure 7.39: **Non-resonant Search (3b1l):** Distributions of ΔR between the closest Higgs Candidate jets, ΔR between the other two, and average absolute value of HC jet η before and after CR derived reweighting for the 2018 3b1l Control Region.

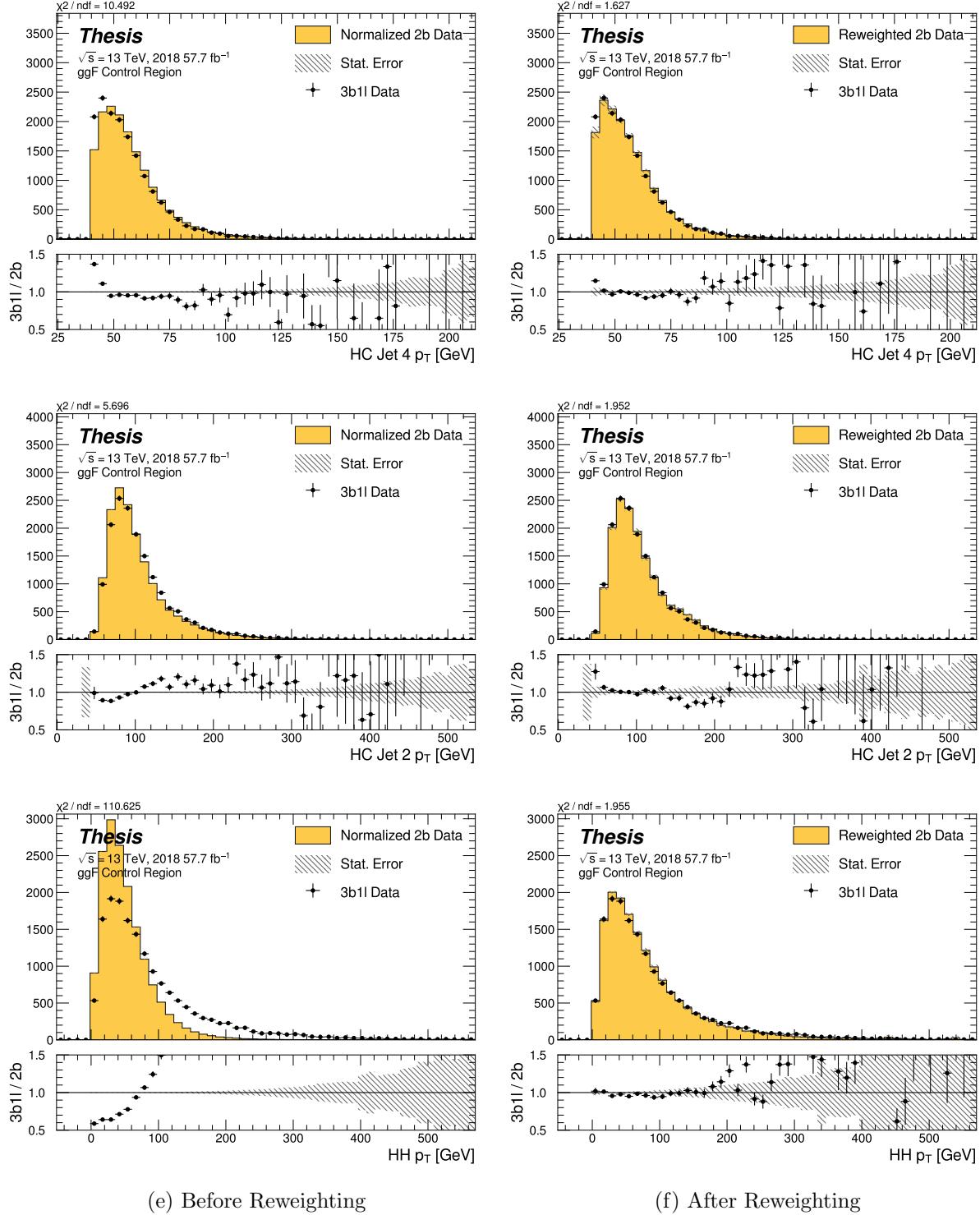


Figure 7.40: **Non-resonant Search (3b1l):** Distributions of p_T of the 2nd and 4th leading Higgs Candidate jets and the p_T of the di-Higgs system before and after CR derived reweighting for the 2018 3b1l Control Region.

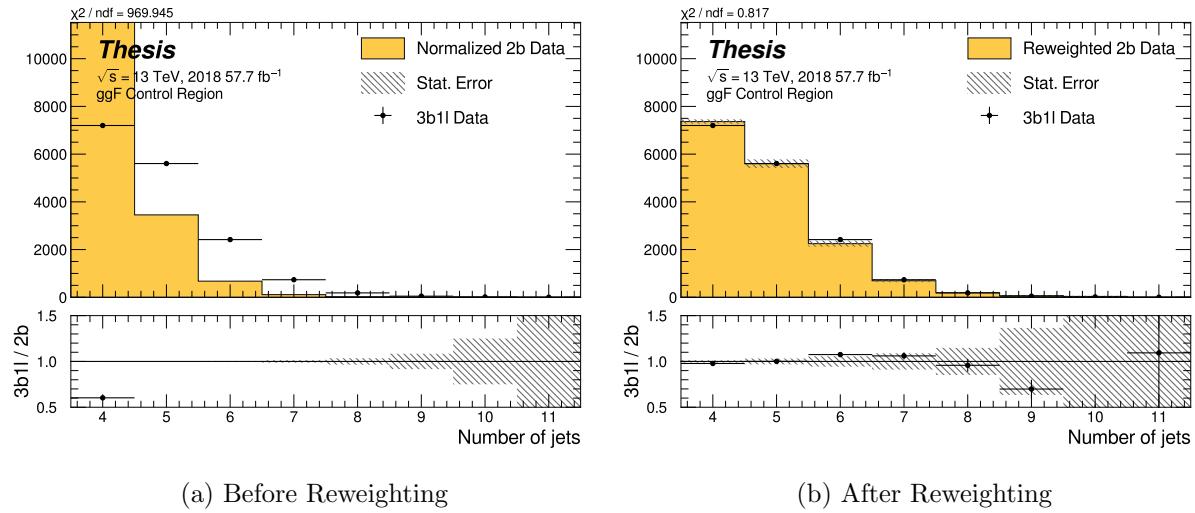


Figure 7.41: **Non-resonant Search (3b1l):** Distributions of the number of jets before and after CR derived reweighting for the 2018 3b1l Control Region. A minimum of 4 jets is required in each event in order to form Higgs candidates.

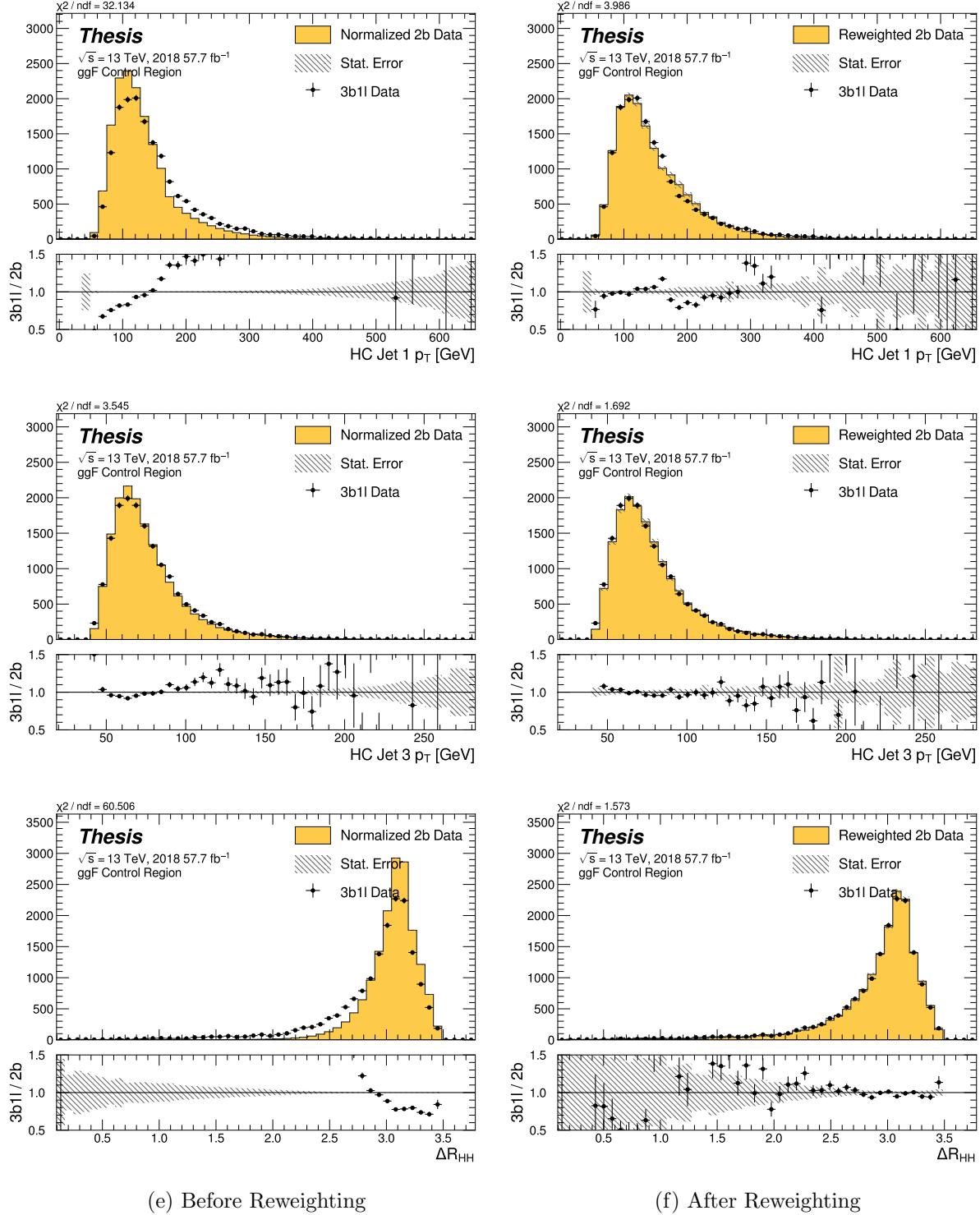


Figure 7.42: **Non-resonant Search (3b1l):** Distributions of p_T of the 1st and 3rd leading Higgs Candidate jets and ΔR between Higgs candidates before and after CR derived reweighting for the 2018 3b1l Control Region.

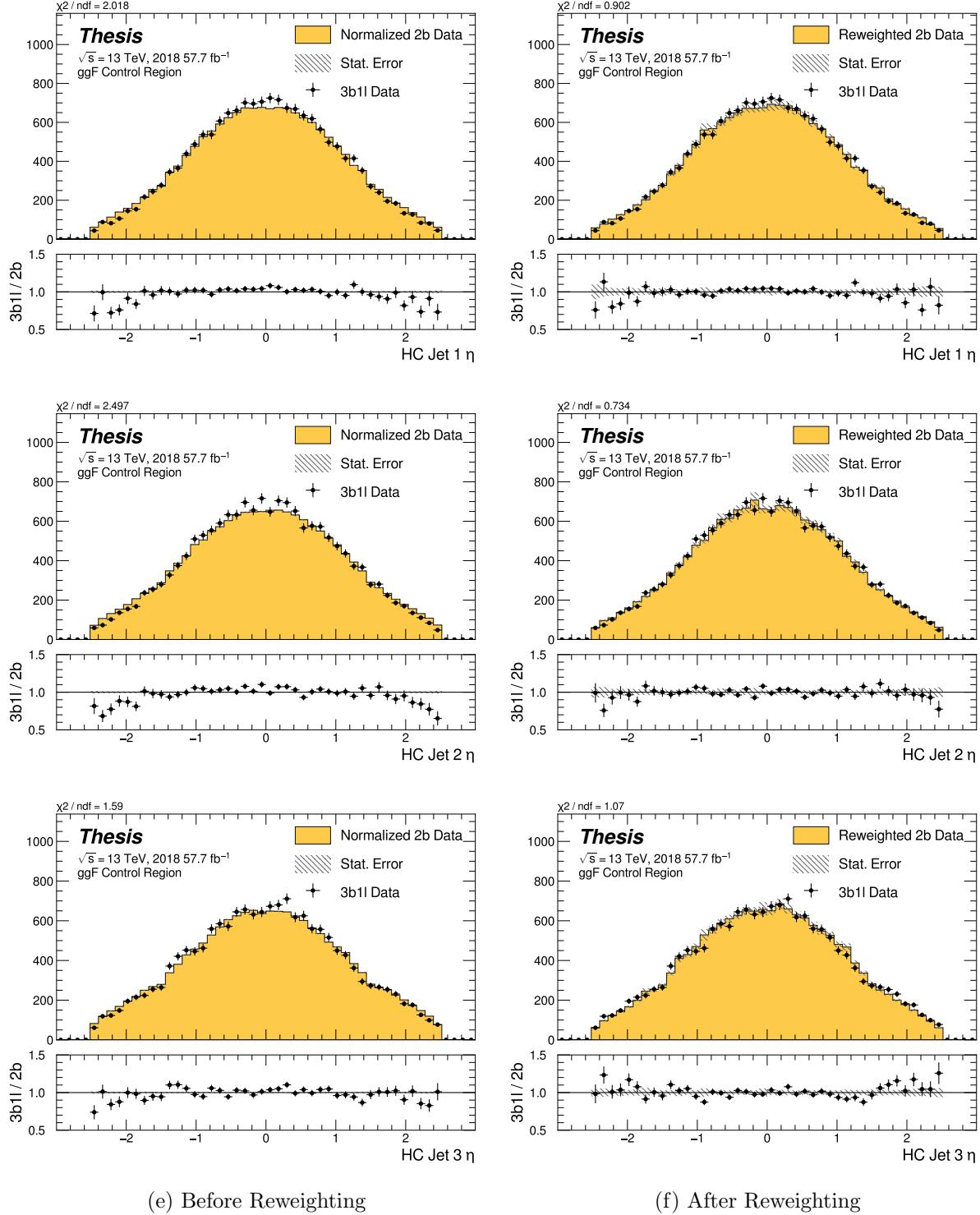


Figure 7.43: **Non-resonant Search (3b1l):** Distributions of η of the 1st, 2nd, and 3rd leading Higgs Candidate jets before and after CR derived reweighting for the 2018 3b1l Control Region.

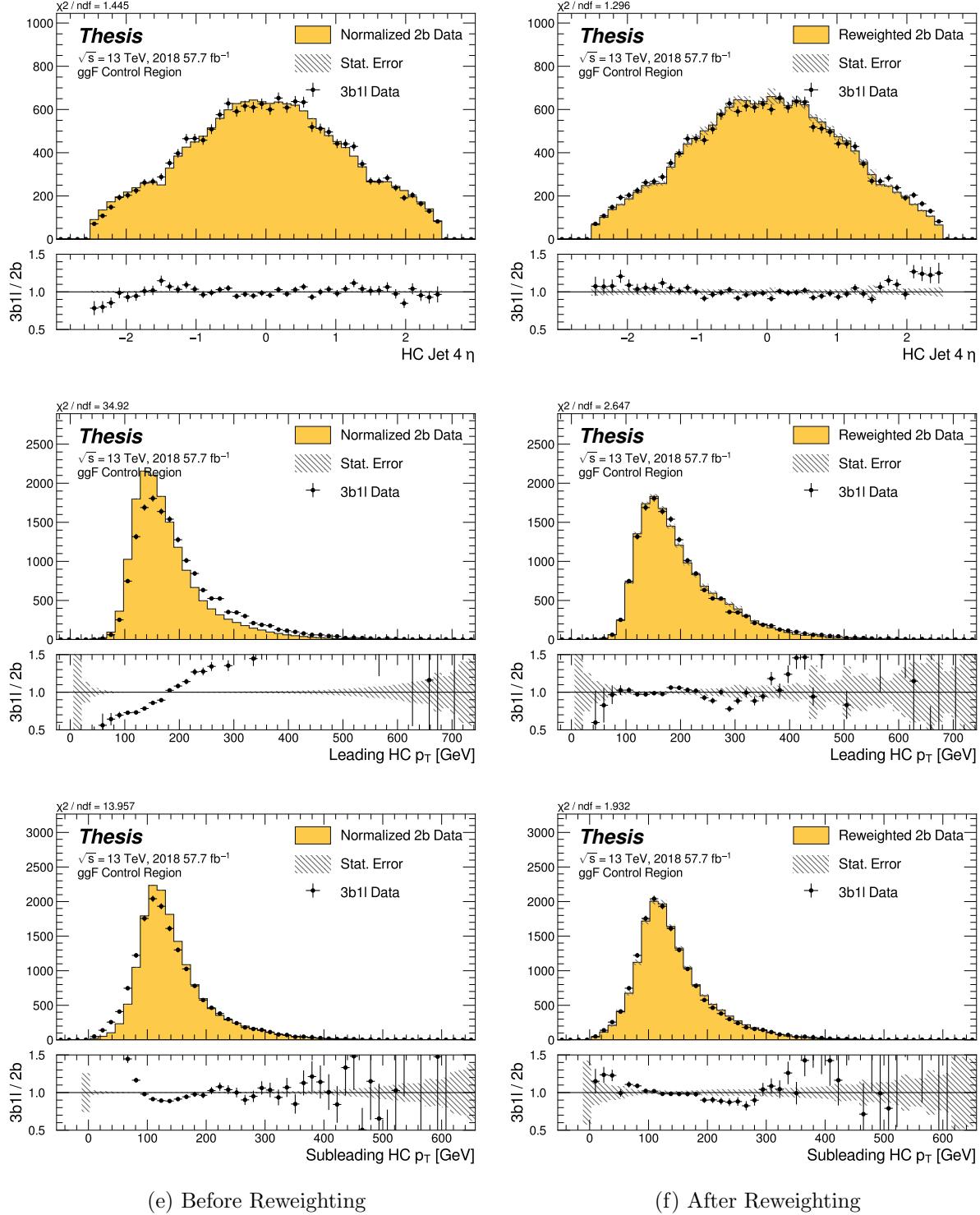


Figure 7.44: **Non-resonant Search (3b1l):** Distributions of η of the 4th leading Higgs Candidate jet and the p_T of the leading and subleading Higgs candidates before and after CR derived reweighting for the 2018 3b1l Control Region.

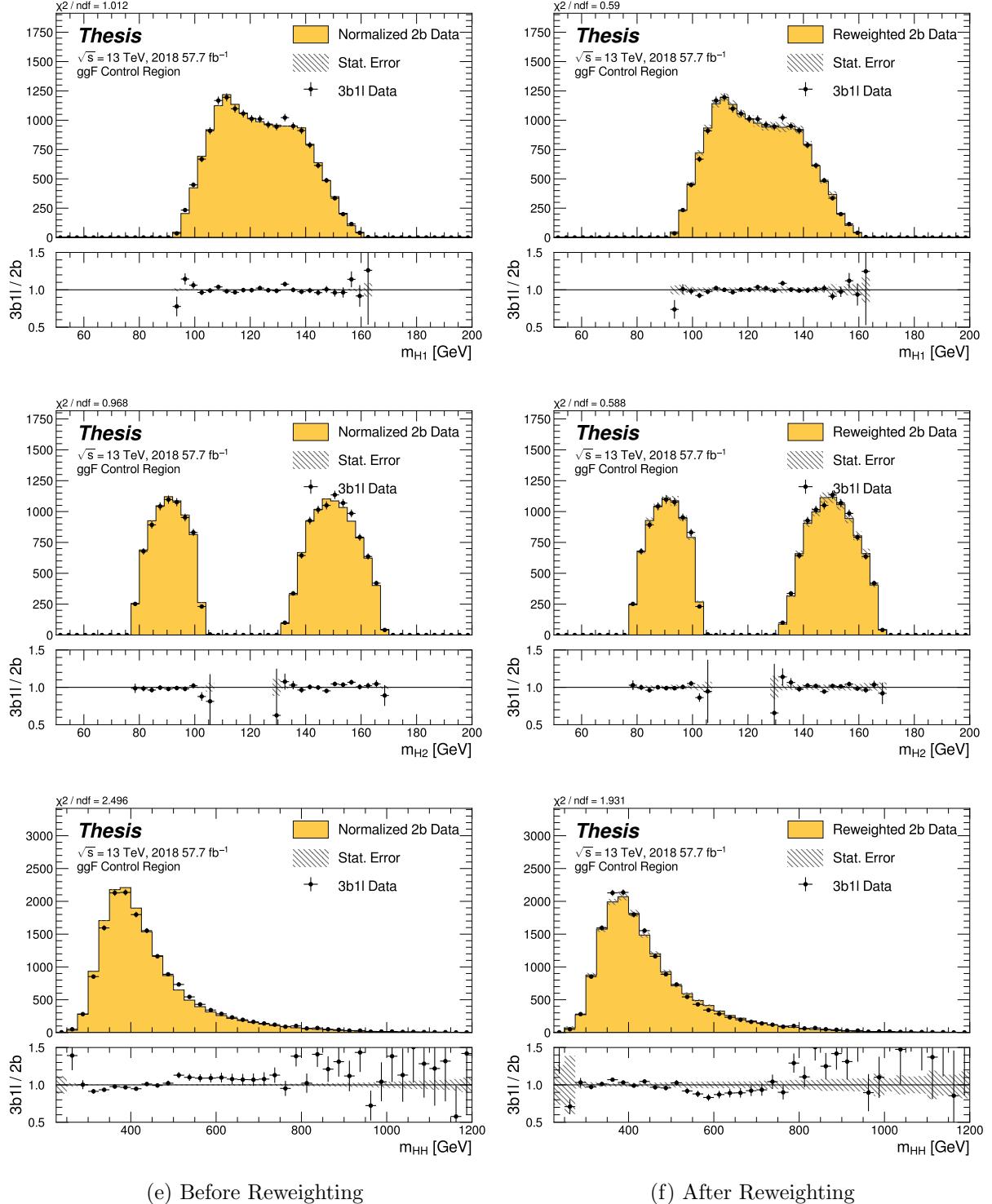


Figure 7.45: **Non-resonant Search (3b1l):** Distributions of mass of the leading and sub-leading Higgs candidates and of the di-Higgs system before and after CR derived reweighting for the 2018 3b1l Control Region.

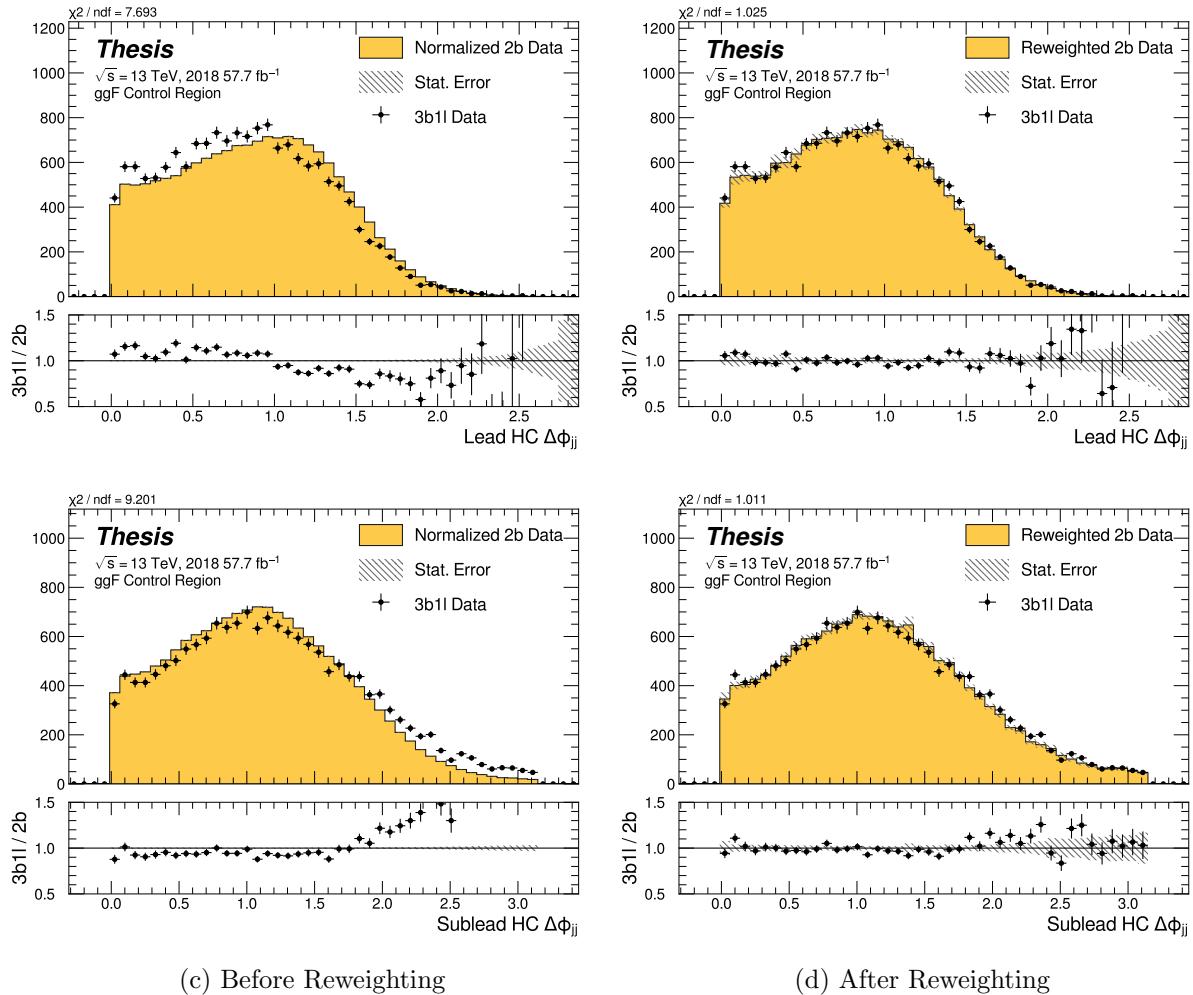


Figure 7.46: **Non-resonant Search (3b1l):** Distributions of $\Delta\phi$ between jets in the leading and subleading Higgs candidates before and after CR derived reweighting for the 2018 3b1l Control Region.

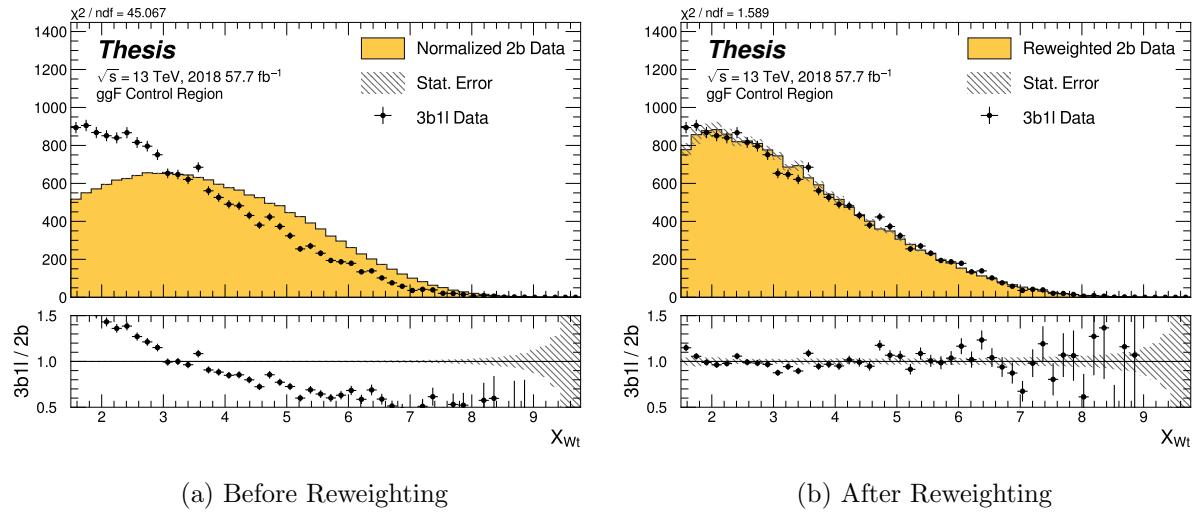


Figure 7.47: **Non-resonant Search (3b1l):** Distributions of the top veto variable, X_{Wt} , before and after CR derived reweighting for the 2018 3b1l Control Region. Reweighting is done after the cut on this variable is applied.

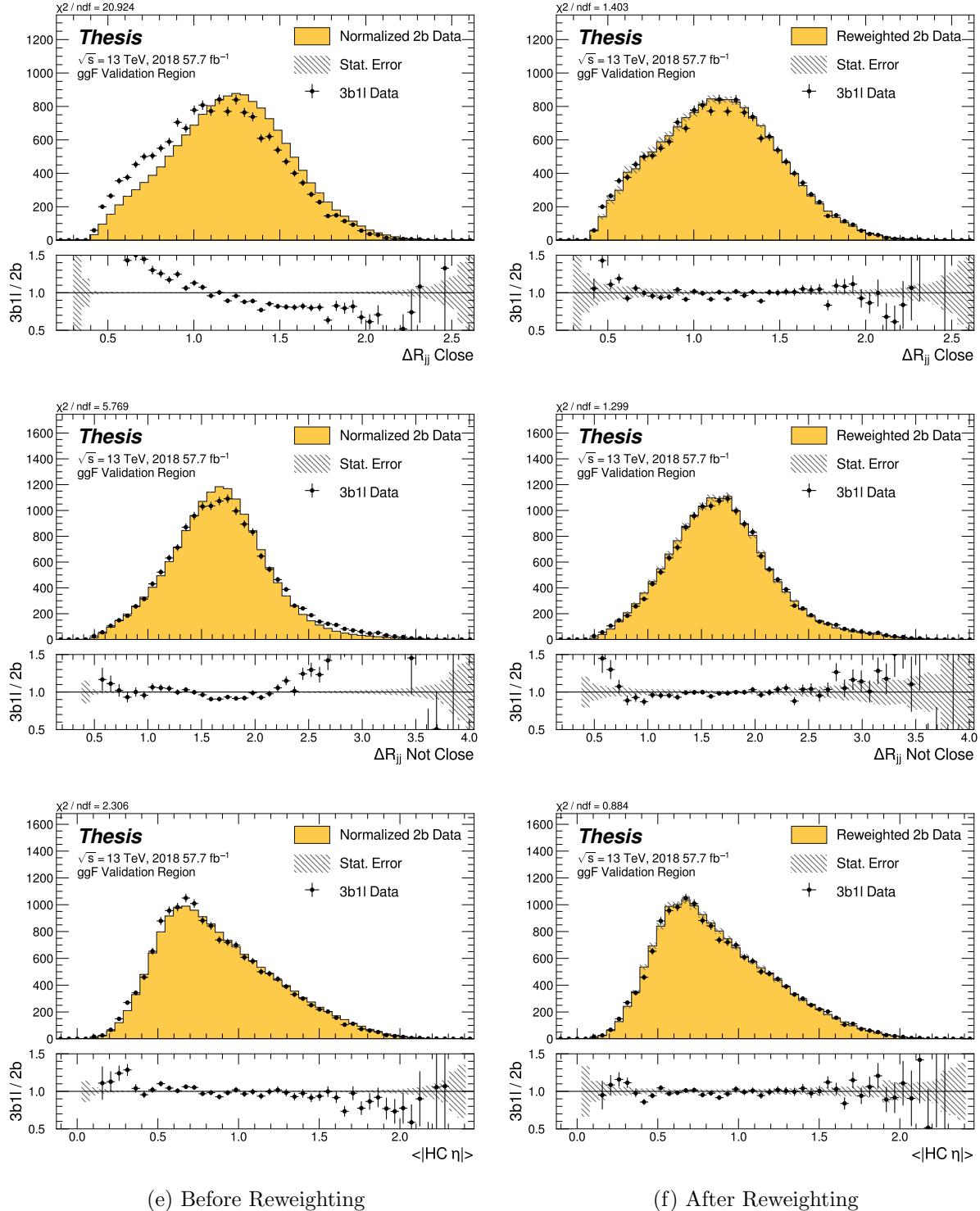


Figure 7.48: **Non-resonant Search (3b1l):** Distributions of ΔR between the closest Higgs Candidate jets, ΔR between the other two, and average absolute value of HC jet η before and after CR derived reweighting for the 2018 3b1l Validation Region.

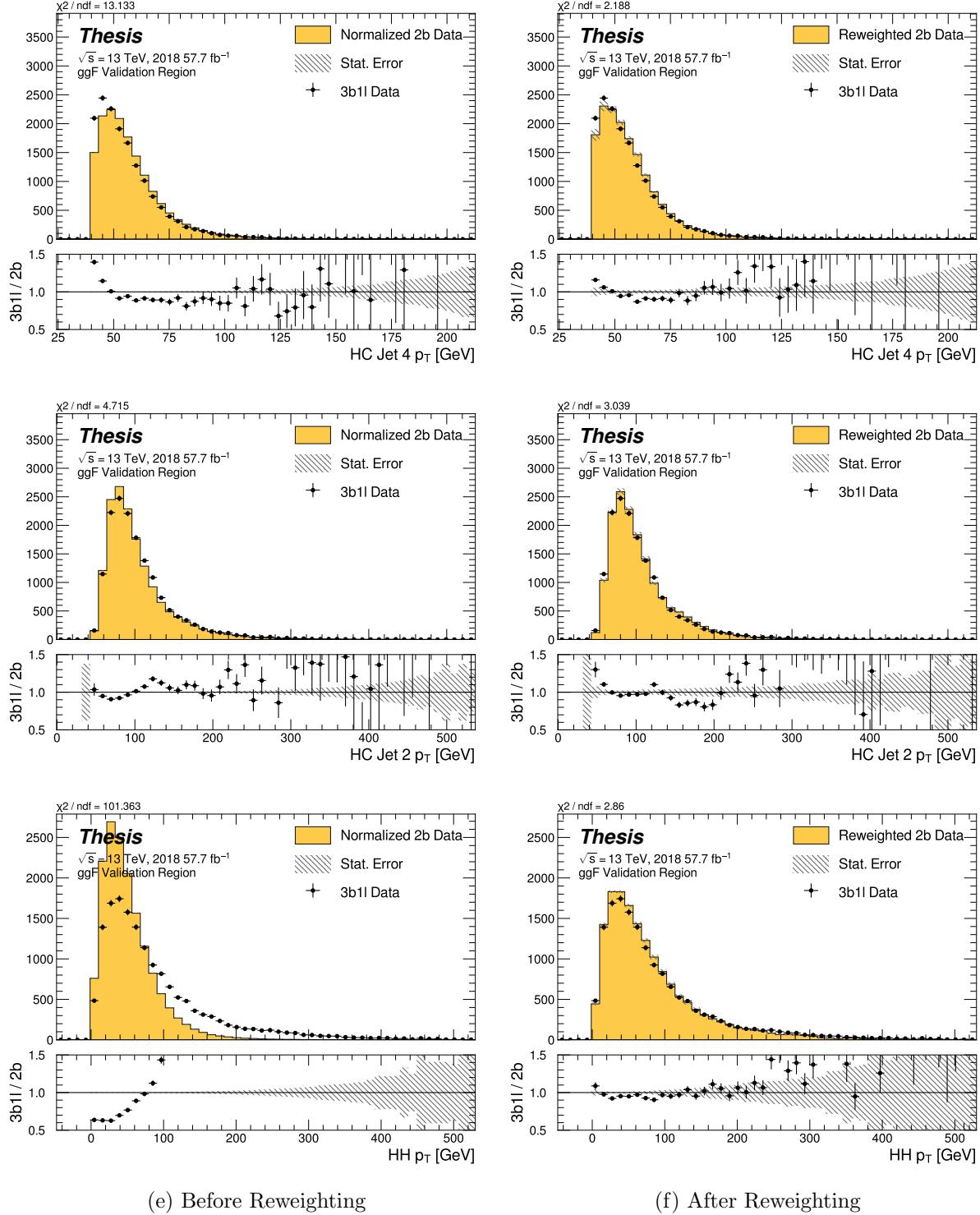


Figure 7.49: **Non-resonant Search (3b1l):** Distributions of p_T of the 2nd and 4th leading Higgs Candidate jets and the p_T of the di-Higgs system before and after CR derived reweighting for the 2018 3b1l Validation Region.

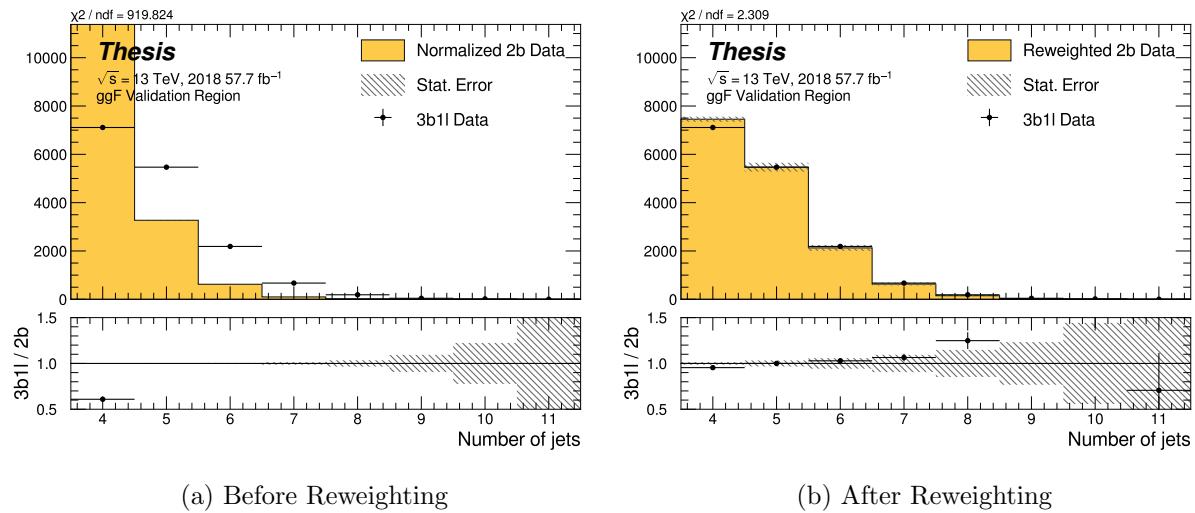


Figure 7.50: **Non-resonant Search (3b1l):** Distributions of the number of jets before and after CR derived reweighting for the 2018 3b1l Validation Region. A minimum of 4 jets is required in each event in order to form Higgs candidates.

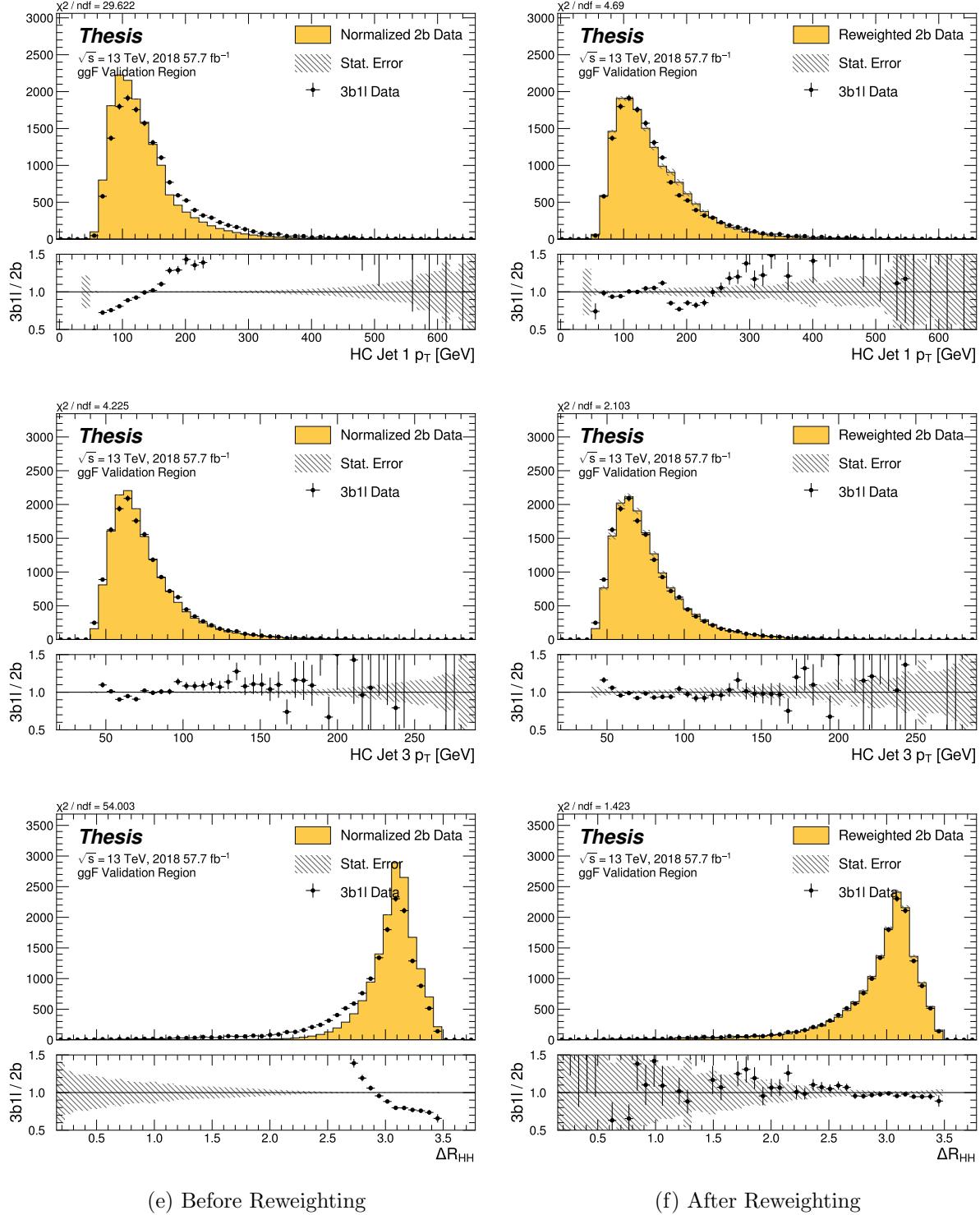


Figure 7.51: **Non-resonant Search (3b1l):** Distributions of p_T of the 1st and 3rd leading Higgs Candidate jets and ΔR between Higgs candidates before and after CR derived reweighting for the 2018 3b1l Validation Region.

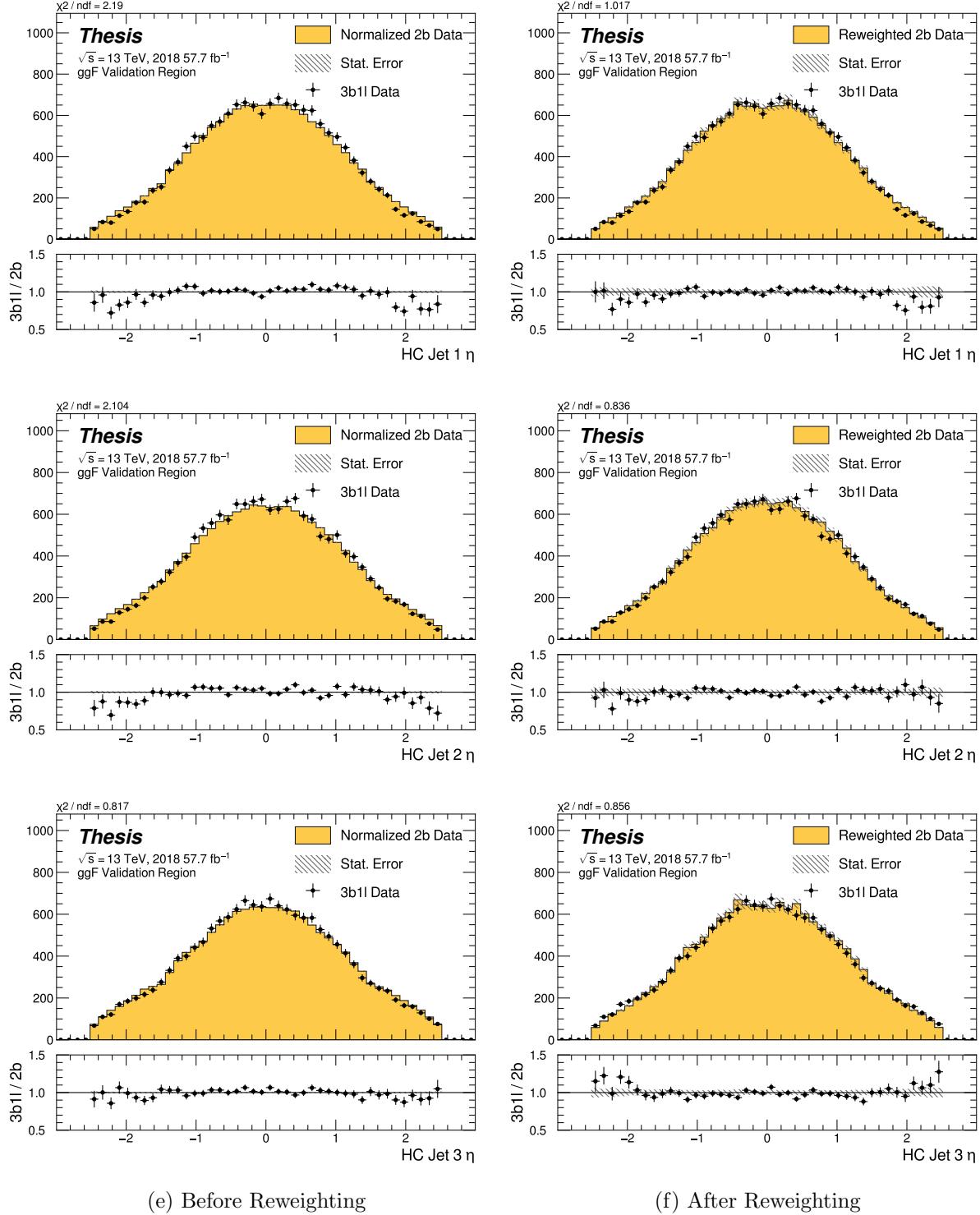


Figure 7.52: **Non-resonant Search (3b1l):** Distributions of η of the 1st, 2nd, and 3rd leading Higgs Candidate jets before and after CR derived reweighting for the 2018 3b1l Validation Region.

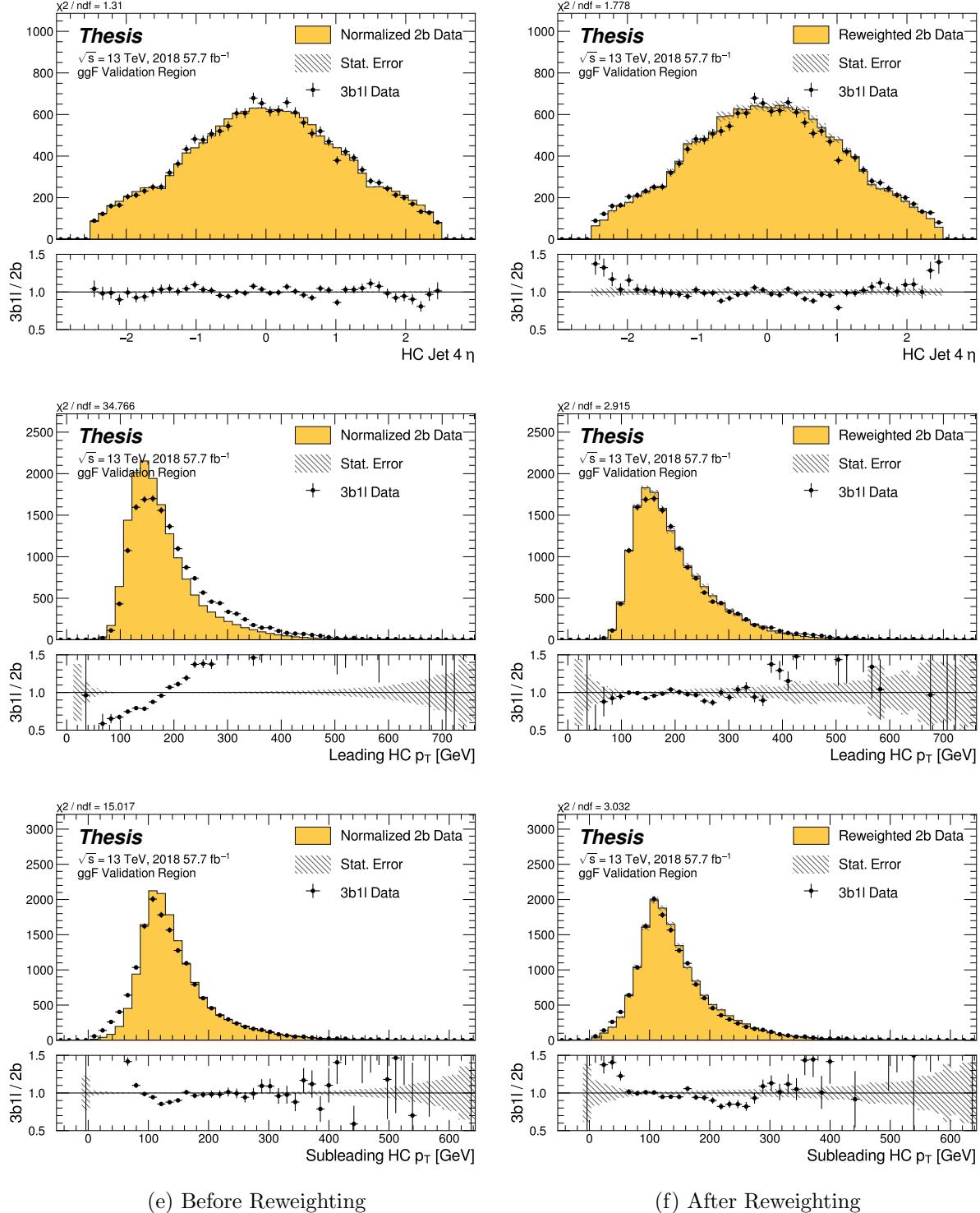


Figure 7.53: **Non-resonant Search (3b1l):** Distributions of η of the 4th leading Higgs Candidate jet and the p_T of the leading and subleading Higgs candidates before and after CR derived reweighting for the 2018 3b1l Validation Region.

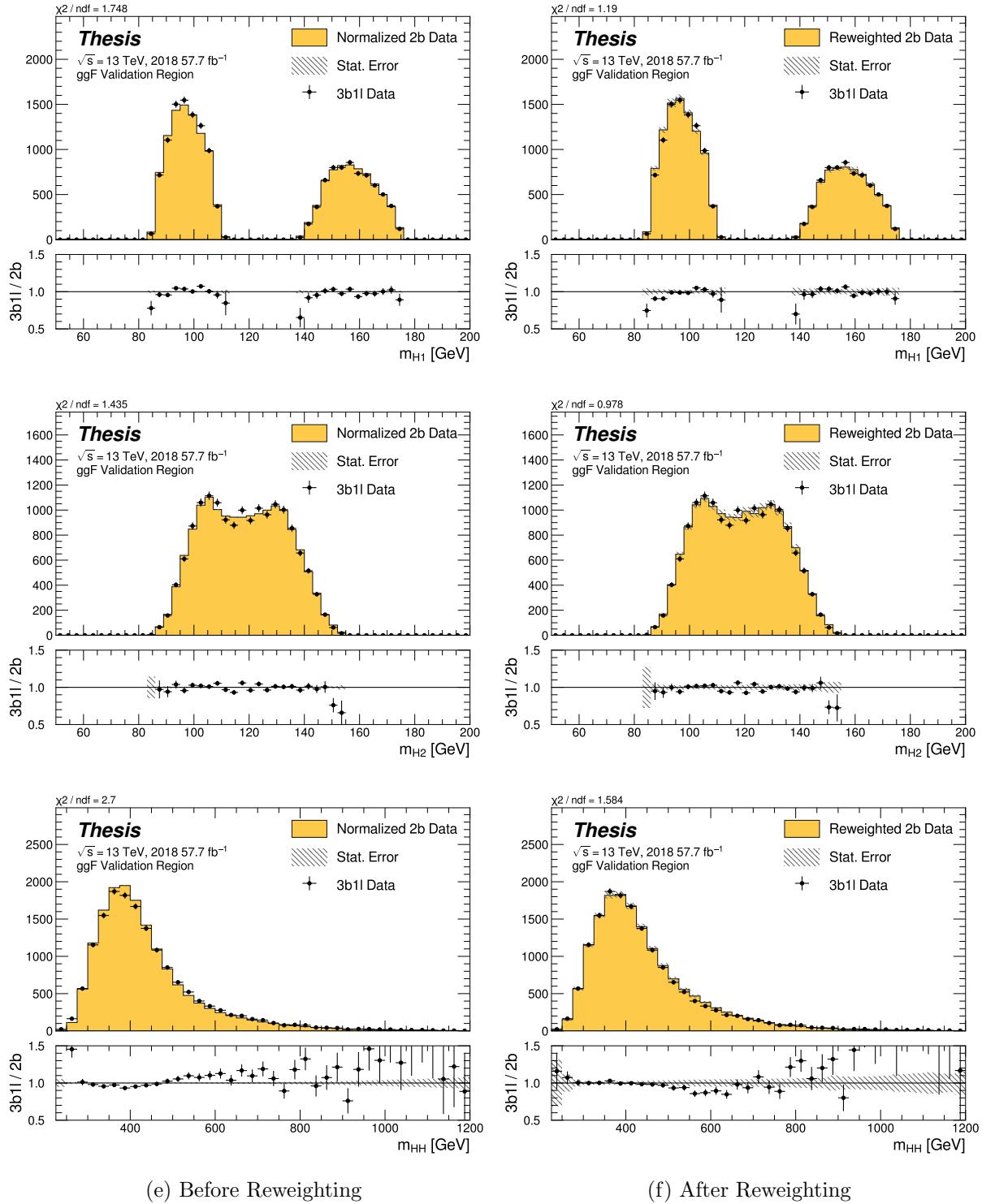


Figure 7.54: **Non-resonant Search (3b1l)**: Distributions of mass of the leading and sub-leading Higgs candidates and of the di-Higgs system before and after CR derived reweighting for the 2018 3b1l Validation Region.

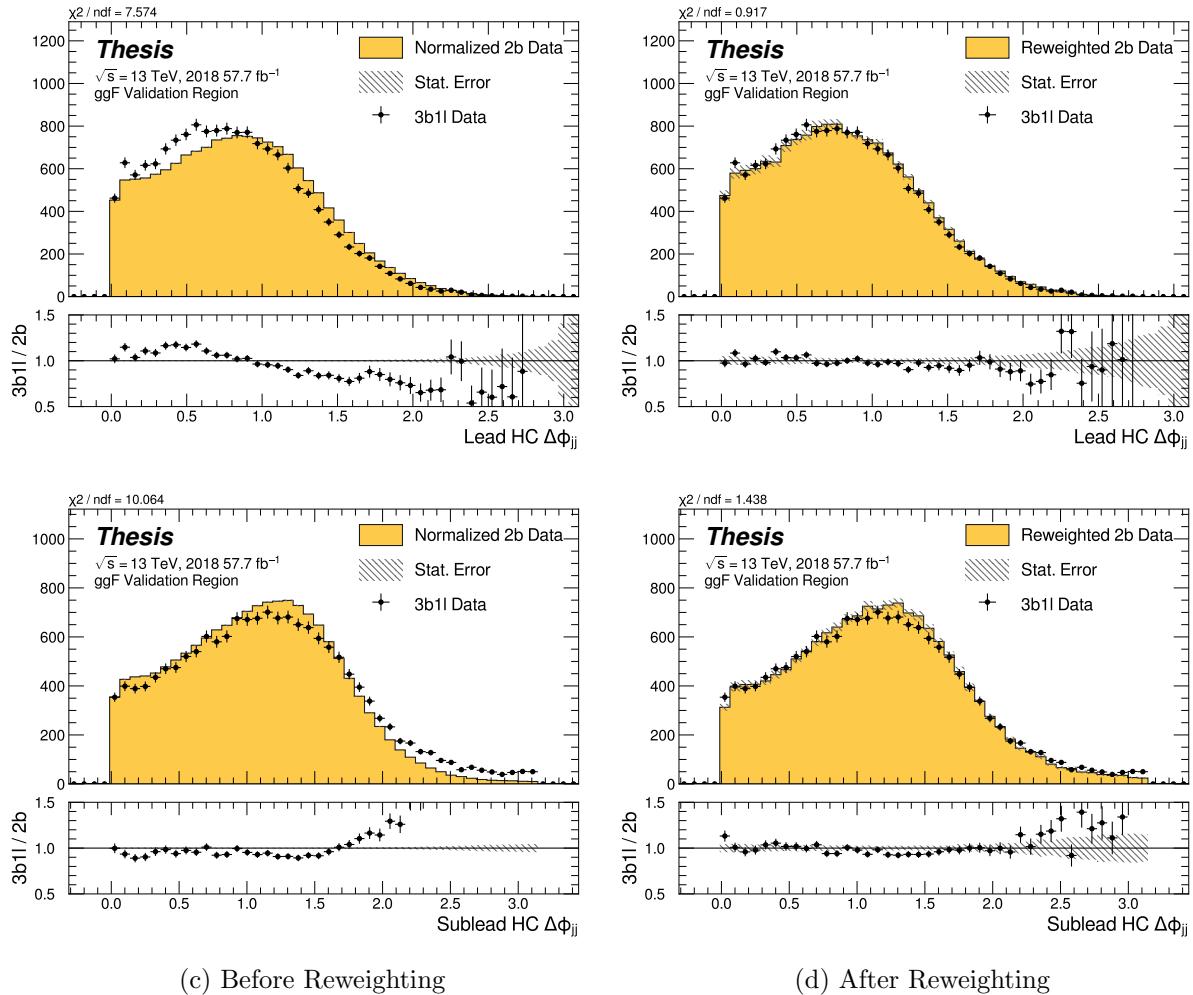


Figure 7.55: **Non-resonant Search (3b1l):** Distributions of $\Delta\phi$ between jets in the leading and subleading Higgs candidates before and after CR derived reweighting for the 2018 3b1l Validation Region.

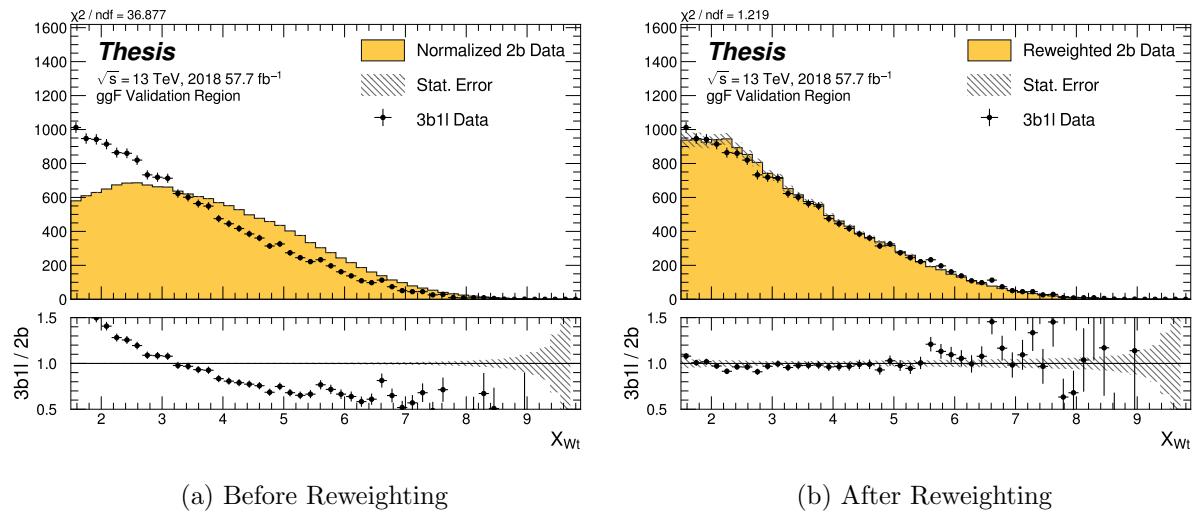


Figure 7.56: **Non-resonant Search (3b1l):** Distributions of the top veto variable, X_{Wt} , before and after CR derived reweighting for the 2018 3b1l Validation Region. Reweighting is done after the cut on this variable is applied.

¹⁹¹⁸ **7.7 Uncertainties**

¹⁹¹⁹ A variety of uncertainties are assigned to account for known biases in the underlying methods,
¹⁹²⁰ calibrations, and objects used for this analysis. The largest such uncertainty is associated with
¹⁹²¹ the kinematic bias inherent in deriving the background estimate away from the signal region.
¹⁹²² However, a statistical biasing of this same estimate has an effect of a similar magnitude.
¹⁹²³ Additionally, due to the use of Monte Carlo for signal modelling and b -tagging calibration,
¹⁹²⁴ uncertainties related to mismodellings in simulation must also be accounted for. These
¹⁹²⁵ components, and their impact on this analysis, are described here in detail. Note that, while
¹⁹²⁶ the Poisson error (from 2b data statistics) is negligible relative to the bootstrap error in
¹⁹²⁷ the bulk of the distribution, it becomes relevant in the high m_{HH} tail. The final statistical
¹⁹²⁸ uncertainty used for the limit setting is therefore the sum (in quadrature) of these two
¹⁹²⁹ components.

¹⁹³⁰ **7.7.1 Statistical Uncertainties and Bootstrapping**

¹⁹³¹ There are two components to the statistical error for the neural network background estimate.
¹⁹³² The first is standard Poisson error, i.e., a given bin, i , in the background histogram has value
¹⁹³³ $n_i = \sum_{j \in i} w_j$, where w_j is the weight for an event j which falls in bin i . Standard techniques
¹⁹³⁴ then result in statistical error $\delta n_i = \sqrt{\sum_{j \in i} w_j^2}$, which reduces to the familiar \sqrt{N} Poisson error
¹⁹³⁵ when all w_j are equal to 1.

¹⁹³⁶ However, this procedure does not take into account the statistical uncertainty on the
¹⁹³⁷ w_j due to the finite training dataset. Due to the large size difference between the two tag
¹⁹³⁸ and four tag datasets, it is the statistical uncertainty due to the four tag training data that
¹⁹³⁹ dominates that on the background. A standard method for estimating this uncertainty is the
¹⁹⁴⁰ bootstrap resampling technique [75]. Conceptually, a set of statistically equivalent sets is
¹⁹⁴¹ constructed by sampling with replacement from the original training set. The reweighting
¹⁹⁴² network is then trained on each of these separately, resulting in a set of statistically equivalent
¹⁹⁴³ background estimates. Each of these sets is below referred to as a replica.

1944 In practice, as the original training set is large, the resampling procedure is able to
 1945 be simplified through the relation $\lim_{n \rightarrow \infty} \text{Binomial}(n, 1/n) = \text{Poisson}(1)$, which dictates that
 1946 sampling with replacement is approximately equivalent to applying a randomly distributed
 1947 integer weight to each event, drawn from a Poisson distribution with a mean of 1.

1948 Though the network configuration itself is the same for each bootstrap training, the
 1949 network initialization is allowed to vary. It should therefore be noted that the bootstrap
 1950 uncertainties implicitly capture the uncertainty due to this variation in addition to the
 1951 previously mentioned training set variation.

1952 The variation from this bootstrapping procedure is used to assign a bin-by-bin uncertainty
 1953 which is treated as a statistical uncertainty in the fit. Due to practical constraints, a
 1954 procedure for approximating the full bootstrap error band is developed which demonstrates
 1955 good agreement with the full bootstrap uncertainty. This procedure is described below.

1956 *Calculating the Bootstrap Error Band*

1957 The standard procedure to calculate the bootstrap uncertainty would proceed as follows: first,
 1958 each network trained on each bootstrap replica dataset would be used to produce a histogram
 1959 in the variable of interest. This would result in a set of replica histograms (e.g. for 100
 1960 bootstrap replicas, 100 histograms would be created). The nominal estimate would then be
 1961 the mean of bin values across these replica histograms, with errors set by the corresponding
 1962 standard deviation.

1963 In practice, such an approach is inflexible and demanding both in computation and in
 1964 storage, in so far as we would like to produce histograms in many variables, with a variety
 1965 of different cuts and binnings. This motivates a derivation based on event-level quantities.
 1966 However, due to non-trivial correlations between replica weights, simple linear propagation of
 1967 event weight variation is not correct.

1968 We therefore adopt an approach which has been empirically found to produce results
 1969 (for this analysis) in line with those produced by generating all of the histograms, as in the
 1970 standard procedure. This approach is described below. Note that, for robustness to outliers

1971 and weight distribution asymmetry, the median and interquartile range (IQR) are used for
1972 the central value and width respectively (as opposed to the mean and standard deviation).

1973 The components involved in the calculation have been mentioned in Section 7.6 and are
1974 as follows:

- 1975 1. Replica weight (w_i): weight predicted for a given event by a network trained on replica
1976 dataset i .
- 1977 2. Replica norm (α_i): normalization factor for replica i . This normalizes the reweighting
1978 prediction of the network trained on replica dataset i to match the correponding target
1979 yield.
- 1977 3. Median weight (w_{med}): median weight for a given event across replica datasets, used
1978 for the nominal estimate. Defined (for 100 bootstrap replicas) as

$$w_{med} \equiv \text{median}(\alpha_1 w_1, \dots, \alpha_{100} w_{100}) \quad (7.12)$$

- 1980 4. Normalization correction (α_{med}): normalization factor to match the predicted yield of
1981 the median weights (w_{med}) to the target yield in the training region.

1982 As mentioned in Section 7.6, the *nominal estimate* is constructed from the set of median
1983 weights and the normalization correction, i.e. $\alpha_{med} \cdot w_{med}$.

1984 For the bootstrap error band, a “varied” histogram is then generated by applying, for
1985 each event, a weight equal to the median weight (with no normalization correction) plus half
1986 the interquartile range of the replica weights: $w_{varied} = w_{med} + \frac{1}{2} \text{IQR}(w_1, \dots, w_{100})$.

1987 This varied histogram is scaled to match the yield of the nominal estimate. To account
1988 for variation of the nominal estimate yield, a normalization variation is calculated from the
1989 interquartile range of the replica norms: $\frac{1}{2} \text{IQR}(\alpha_1, \dots, \alpha_{100})$. This variation, multiplied into
1990 the nominal estimate, is used to set a baseline for the varied histogram described above.

Denoting $H(\text{weights})$ as a histogram constructed from a given set of weights, $Y(\text{weights})$ as the predicted yield for a given set of weights, the final varied histogram is thus:

$$H(w_{med} + \frac{1}{2} \text{IQR}(w_1, \dots, w_{100})) \cdot \frac{Y(\alpha_{med} w_{med})}{Y(w_{med} + \frac{1}{2} \text{IQR}(w_1, \dots, w_{100}))} + \frac{1}{2} \text{IQR}(\alpha_1, \dots, \alpha_{100}) \cdot H(\alpha_{med} w_{med}) \quad (7.13)$$

where the first term roughly describes the behaviour of the bootstrap variation across

the distribution of the variable of interest while the second term describes the normalization

variation of the bootstrap replicas.

The difference between the varied histogram and the nominal histogram is then taken to

be the bootstrap statistical uncertainty on the nominal histogram.

Figure 7.57 demonstrates how each of the components described above contribute to the

uncertainty envelope for the non-resonant 2017 Control Region and compares this approximate

band to the variation of histograms from individual bootstrap estimates. The error band

constructed from the above procedure is seen to provide a good description of the bootstrap

variation.

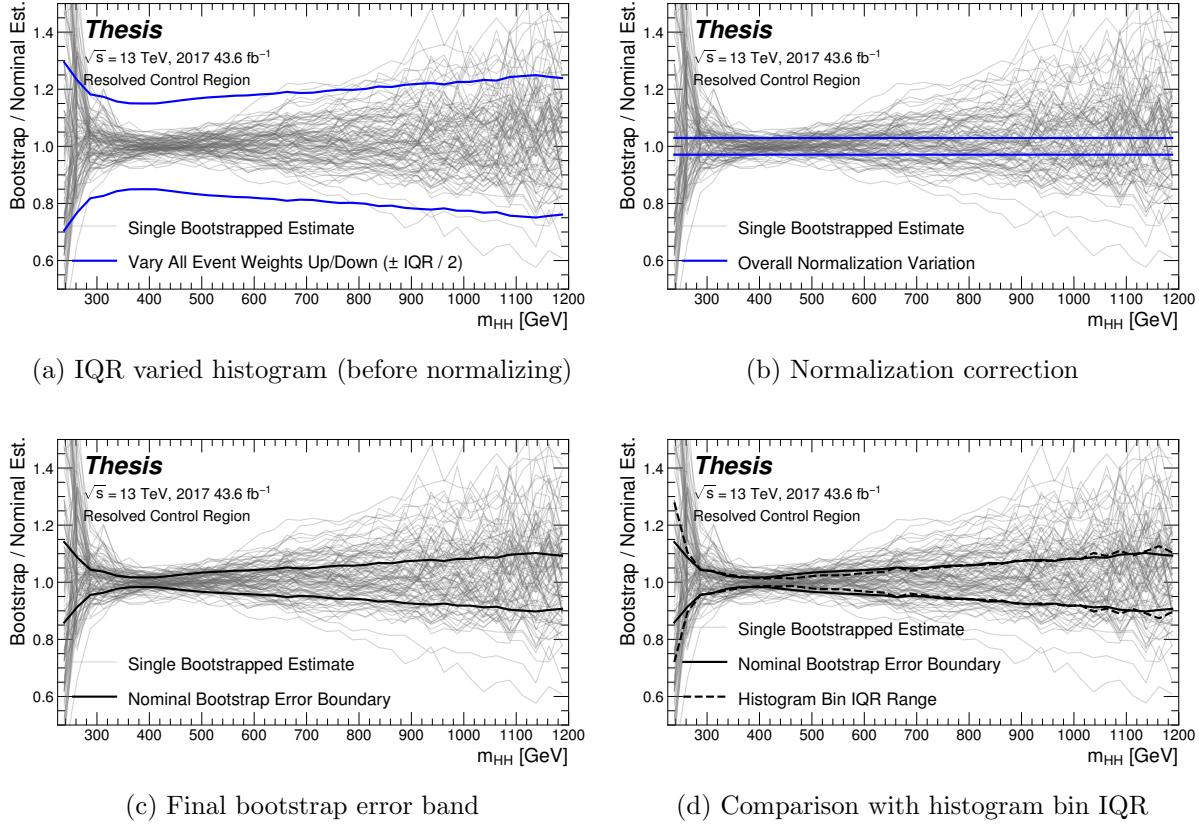


Figure 7.57: Illustration of the approximate bootstrap band procedure, shown as a ratio to the nominal estimate for the 2017 non-resonant background estimate. Each grey line is from the m_{HH} prediction for a single bootstrap training. Figure 7.57(a) shows the variation histograms constructed from median weight \pm the IQR of the replica weights. It can be seen that this captures the rough shape of the bootstrap envelope, but is not good estimate for the overall magnitude of the variation. Figure 7.57(b) demonstrates the applied normalization correction, and Figure 7.57(c) shows the final band (normalized Figure 7.57(a) + Figure 7.57(b)). Comparing this with the IQR variation for the prediction from each bootstrap in each bin in Figure 7.57(d), the approximate envelope describes a very similar variation.

2001 *7.7.2 Background Shape Uncertainties*

2002 To account for the systematic bias associated with deriving the reweighting function in the
2003 control region and extrapolating to the signal region, an alternative background model is
2004 derived in the validation region. Because of the fully data-driven nature of the background
2005 model, this is an uncertainty assessed on the full background. The alternative model and
2006 the baseline are consistent with the observed data in their training regions, and differences
2007 between the alternative and baseline models are used to define a shape uncertainty on the
2008 m_{HH} spectrum, with a two-sided uncertainty defined by symmetrizing the difference about
2009 the baseline.

2010 For the resonant analysis, this uncertainty is split into two components to allow for two
2011 independent variations of the m_{HH} spectrum: : a low- H_T and a high- H_T component, where
2012 H_T is the scalar sum of the p_T of the four jets constituting the Higgs boson candidates, and
2013 serves as a proxy for m_{HH} , while avoiding introducing a sharp discontinuity. The boundary
2014 value is 300 GeV. The low- H_T shape uncertainty primarily affects the m_{HH} spectrum below
2015 400 GeV (close to the kinematic threshold) by up to around 5%, and the high- H_T uncertainty
2016 mainly m_{HH} above this by up to around 20% relative to nominal. These separate m_{HH}
2017 regimes are by design – the H_T split is introduced to prevent low mass bins from constraining
2018 the high mass uncertainty and vice-versa.

2019 This was the *status quo* shape uncertainty decomposition from the Early Run 2 analysis.
2020 A decomposition in terms of orthogonal polynomials, which would provide increased flexibility,
2021 was also evaluated. This study revealed that both decompositions are able to account for the
2022 systematic deviations between four tag data and the background estimate (evaluated in the
2023 kinematic validation region), and produce almost identical limits. The simpler *status quo*
2024 decomposition is therefore kept.

2025 For the non-resonant analysis, the quadrant nature of the background estimation leads to
2026 a natural breakdown of the nuisance parameters: quadrants are defined in the signal region
2027 along the same axes as those used for the control and validation region definitions. Variations

2028 are then assessed in each of these signal region quadrants, corresponding to regions that
 2029 are “closer to” and “further away from” the nominal and alternate estimate regions, fully
 2030 leveraging the power of the two equivalent but systematically different estimates.

2031 Figure 7.58 shows an example of the variation in each H_T region for the 2018 resonant
 2032 analysis. Figure 7.59 shows the example quadrant variation for the 2018 4 b non-resonant
 analysis.

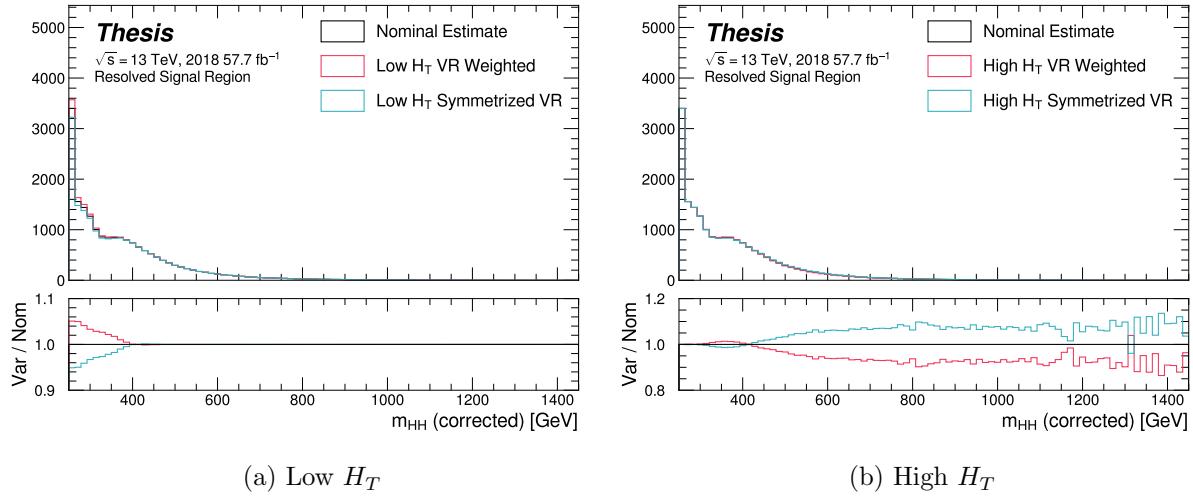
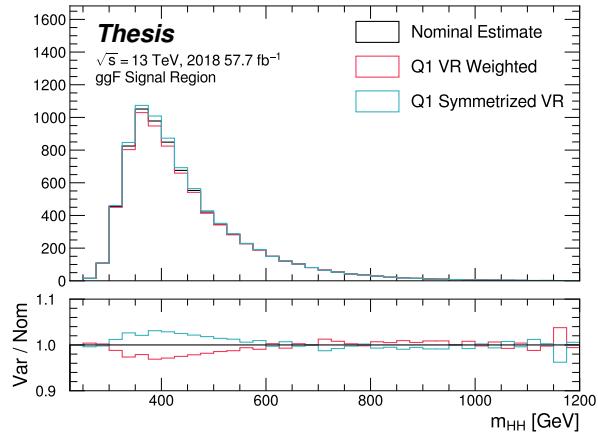
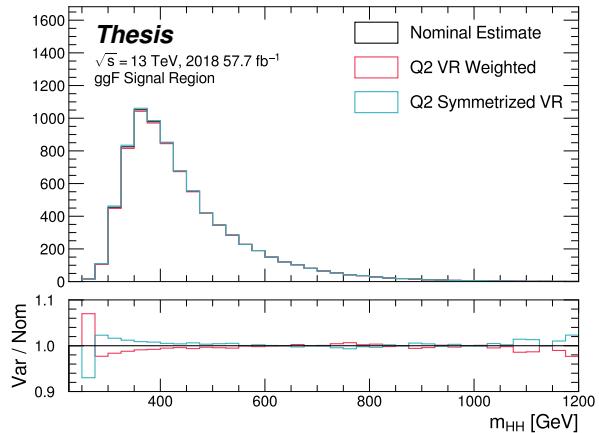


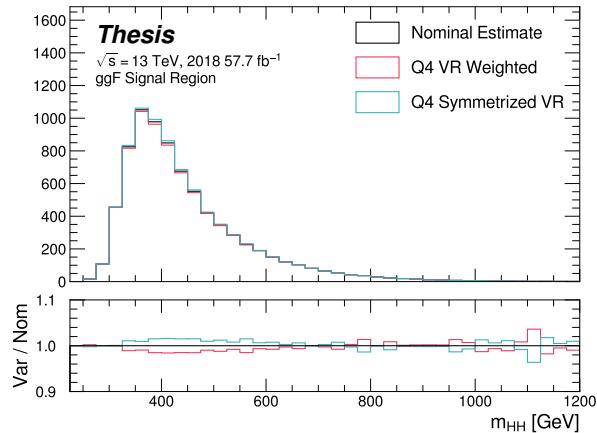
Figure 7.58: **Resonant Search:** Example of CR vs VR variation in each H_T region for 2018.
 The variation nicely factorizes into low and high mass components.



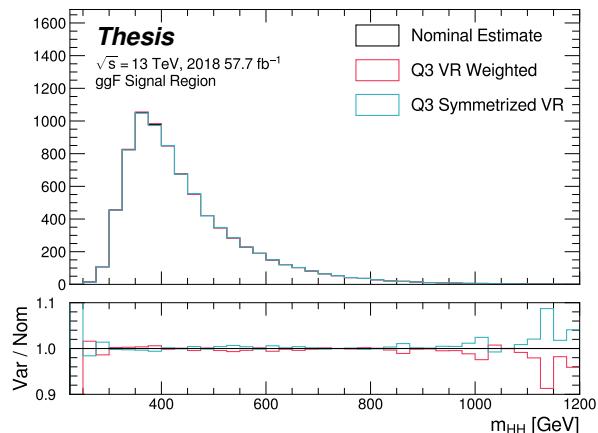
(a) Q1 (top)



(b) Q2 (left)



(c) Q4 (right)



(d) Q3 (bottom)

Figure 7.59: **Non-resonant Search (4b):** Example of CR vs VR variation in each signal region quadrant for 2018. Significantly different behavior is seen between quadrants, with the largest variation in quadrant 1 and the smallest in quadrant 4.

2034 *7.7.3 Detector Modelling and Reconstruction Uncertainties*

2035 Detector modelling and reconstruction uncertainties account for Monte Carlo simulation not
 2036 being a faithful representation of real data as a result of mismodelling of the detector and
 2037 differential performance of algorithms on simulation compared to data. In this analysis they
 2038 consist of uncertainties related to jet properties, and uncertainties stemming from the flavour
 2039 tagging procedure. The background modelling in this analysis is fully data-driven. As a
 2040 result, these uncertainties are applied only to the signal simulation.

2041 The jet uncertainties are implemented as variations of the jet properties themselves. The
 2042 category reduction (with ~ 30 nuisance parameters) is used for jet energy scale uncertainties
 2043 and the FullJER configuration is used for jet energy resolution uncertainties (14 nuisance
 2044 parameters). This is to preserve the ability to meaningfully statistically combine the results
 2045 of this analysis with other di-Higgs analyses. The flavour tagging uncertainties meanwhile
 2046 are implemented as scale factors applied to the Monte Carlo event weights.

2047 A systematic related to the PtReco b -jet energy correction has been studied in the
 2048 $HH \rightarrow \gamma\gamma b\bar{b}$ analysis [76] and found to be negligible compared to JER. Following this
 2049 example, such a systematic is therefore neglected here.

2050 *7.7.4 Trigger Uncertainties*

2051 Trigger uncertainties stem from imperfect knowledge of the ratio between the efficiency of a
 2052 given trigger in data to its efficiency in Monte Carlo simulation. This ratio is applied as a
 2053 scale factor to all simulated events (as described in Section ??), with the systematic variations
 2054 produced by varying the scale factor up or down by one sigma.

2055 *7.7.5 Theoretical Uncertainties*

2056 The theoretical uncertainties on the acceptance times efficiency ($A \times \varepsilon$) are evaluated by
 2057 analysis of specially-generated, particle-level signal samples. The generation of these samples
 2058 follows the configuration of the baseline samples, but with modifications to probe the following

2059 theoretical uncertainties: uncertainties in the parton density functions (PDFs); uncertainties
 2060 due to missing higher order terms in the matrix elements; and uncertainties in the modelling
 2061 of the underlying event, which includes multi-parton interactions, of hadronic showers and of
 2062 initial and final state radiation.

2063 Uncertainties due to modelling of the parton shower and the underlying event (including
 2064 multi-parton interactions) are evaluated by switching the MC generator used. For the scalar
 2065 samples, this means switching from Herwig7.7.1.3 to Pythia 8.235. Figure ?? shows the
 2066 impact of these variations on the signal acceptance for two resonance masses: 500 GeV and
 2067 1 TeV, covering the range of the resolved analysis. No significant dependence on the variable
 2068 of interest, m_{HH} , is observed. The disagreement observed in the tails of $p_T(hh)$ and the
 2069 number of jets multiplicities is negligible with respect to the final signal acceptance. A 5%
 2070 flat systematic uncertainty is assigned to all signal samples, extracted from the acceptance
 2071 comparison for the full 4-tag selection, as seen in Figs. ?? and ??.

2072 To evaluate the potential effect of missing higher order terms in the matrix element, the
 2073 renormalization and factorization scales used in the signal generation were varied coherently
 2074 by factors of $0.5\times$ and $2\times$ for the signals. The alternative weights were generated as described
 2075 on the TWiki [here](#), applying on-the-fly variations using the ATLAS MadGraphControl
 2076 framework. These weights correspond to variations of the scales either together or separately
 2077 up and down by a factor of two. Seven-point scale variations are considered: $(\mu_R, \mu_F) = (0.5,$
 2078 $0.5), (1, 0.5), (0.5, 1), (1, 1), (2, 1), (1, 2), (2, 2)$. The scale uncertainties are combined by
 2079 taking an envelope of all of the uncertainties. These uncertainties are evaluated to be less
 2080 than $\pm 1\%$, thus neglected.

2081 PDF uncertainties are evaluated using the PDF4LHC15_nlo_mc set, which combined
 2082 CT14, MMHT14 and NNPDF3.0 PDF sets. The uncertainty is evaluated by calculating
 2083 the acceptance for each PDF replica. The standard deviation of these acceptance values
 2084 divided by the baseline acceptance is taken as the PDF uncertainty. For each mass point the
 2085 distribution of their corresponding ration is compatible with a Gaussian centered one. The
 2086 measured uncertainty in acceptance due to PDF uncertainties is less than $\pm 1\%$ across the

2087 full mass range considered for the analysis. For this reason, it is neglected in the statistical
2088 analysis described in Section ??.

2089 These uncertainties are implemented in the final statistical analysis as normalization
2090 uncertainties on the signals, with the value taken from the polynomial fit. This smooths out
2091 statistical fluctuations and allows interpolation between the generated mass points, if needed.

2092 The results for the non-resonant analysis presented here are preliminary and only include
2093 background systematics. However, these are expected to be by far the dominant uncertainties,
2094 and should therefore be reflective of the final results.

2095 **7.8 Background Validation**

2096 In addition to checking the performance of the background estimate in the control and
2097 validation regions, a variety of alternative selections are defined to allow for a full “dress
2098 rehearsal” of the background estimation procedure.

2099 Both the resonant and non-resonant analyses make use of a *reversed* $\Delta\eta$ region, in which
2100 the kinematic cut on $\Delta\eta_{HH}$ is reversed, so that events are required to have $\Delta\eta_{HH} > 1.5$.
2101 This is orthogonal to the nominal signal region and has minimal sensitivity, allowing for the
2102 comparison of the background estimate $4b$ data in the corresponding “signal region”. For
2103 this validation, a new reweighting is trained following nominal procedures, but entirely in the
2104 $\Delta\eta_{HH} > 1.5$ region.

2105 The non-resonant analysis additionally makes use of the $3b + 1$ fail region mentioned
2106 above, which again is orthogonal to the nominal signal regions and has minimal sensitivity.
2107 The reweighting in this case is between $2b$ and $3b + 1$ fail events rather than between $2b$
2108 and $3b + 1$ loose or $2b$ and $4b$. However, the kinematic selections of signal region events are
2109 otherwise identical, allowing for a complementary test of the background estimate.

2110 *TODO: Add shifted regions if they’re ready*

2111 Results are shown in *TODO: add figures*

2112 **7.9 Overview of Other $b\bar{b}b\bar{b}$ Channels**

2113 The results discussed above have been developed in conjunction with (1) a boosted channel for
2114 the resonant search and (2) a vector boson fusion (VBF) channel for the non-resonant search.
2115 Detailed discussions of these two channels are beyond the scope of this thesis. However,
2116 a combined set of results is presented below (*TODO: or will be combined for VBF?*). We
2117 therefore briefly summarize the analyses here.

2118 **7.9.1 Resonant: Boosted Channel**

2119 The boosted analysis selection targets resonance masses from 900 GeV to 5 TeV. In such
2120 events, H decays have a high Lorentz boost, such that the $b\bar{b}$ decays are very collimated. The
2121 resolved analysis fails to reconstruct such HH events, as the $R = 0.4$ jets start to overlap.

2122 The boosted analysis instead reconstructs H decays as large radius, $R = 1.0$ jets, with
2123 corresponding b -quarks identified with variable radius subjets, that is jets with a radius that
2124 scales as ρ/p_T , the p_T is that of the jet in question, and ρ is a fixed parameter, here chosen
2125 to be 30 GeV, which is optimized to maintain truth-level double b -labelling efficiency across
2126 the full range of Higgs jet p_T *TODO: cite: <https://cds.cern.ch/record/2268678>.*

2127 Due to limited boosted b -tagging efficiency *TODO: cite* and to maintain sensitivity even
2128 when b -jets are highly collimated, the boosted analysis is divided into three categories based
2129 on the number of b -tagged jets associated to each large radius jet:

- 2130 • 4 b category: two b -tagged jets in each
- 2131 • 2 $b - 1$ category: two b -tagged jets in one, one in the other
- 2132 • 1 $b - 1$ category: one b -tagged jet in each

2133 The analysis then proceeds in each of these categories. *TODO: what other boosted details?*
2134 The resolved and boosted channels are combined for resonance masses from 900 GeV to
2135 1.5 TeV inclusive. To keep the channels statistically independent, the boosted channel vetos

2136 events passing the resolved analysis selection.

2137 *7.9.2 Non-resonant: VBF Channel*

2138 The vector boson fusion channel is only considered for the non-resonant search. While the
2139 sensitivity is in general much more limited than the gluon-gluon fusion analysis due to the
2140 much smaller production cross section, VBF is sensitive to a variety of Beyond the Standard
2141 Model physics, both complementary and orthogonal to the theoretical scope of gluon-gluon
2142 fusion. *TODO: I'll probably mention more details in the pheno section*

2143 The VBF channel proceeds very similarly to the ggF, with the primary differences being
2144 the kinematic selections and the categorization.

2145 *TODO: fill in kinematics*

2146 Note that the background estimation is inherited from the resonant and ggF analyses, an
2147 ancillary, but significant, contribution of this thesis work.

2148 **7.10 m_{HH} Distributions**

2149 *7.10.1 Resonant Search*

2150 The final discriminant used for the resonant search is corrected m_{HH} . Histogram binning
2151 was optimized for the resonant search to be 84 equal width bins from 250 GeV to 1450 GeV,
2152 corresponding to a bin width of 14.3 GeV, and overflow events (events above 1450 GeV) are
2153 included in the last bin. A demonstration of the performance of the reweighting on this
2154 distribution is shown in Figure 7.60 for the control region and Figure 7.61 for the validation
2155 region. A background-only profile likelihood fit is run for the distribution in the
2156 signal region, and results with spin-0 signals overlaid are shown in Figure 7.62. Note that the
2157 plots show the sum across all years, but the signal extraction fit and background estimate
2158 are run with the years separately. Agreement is generally good throughout.

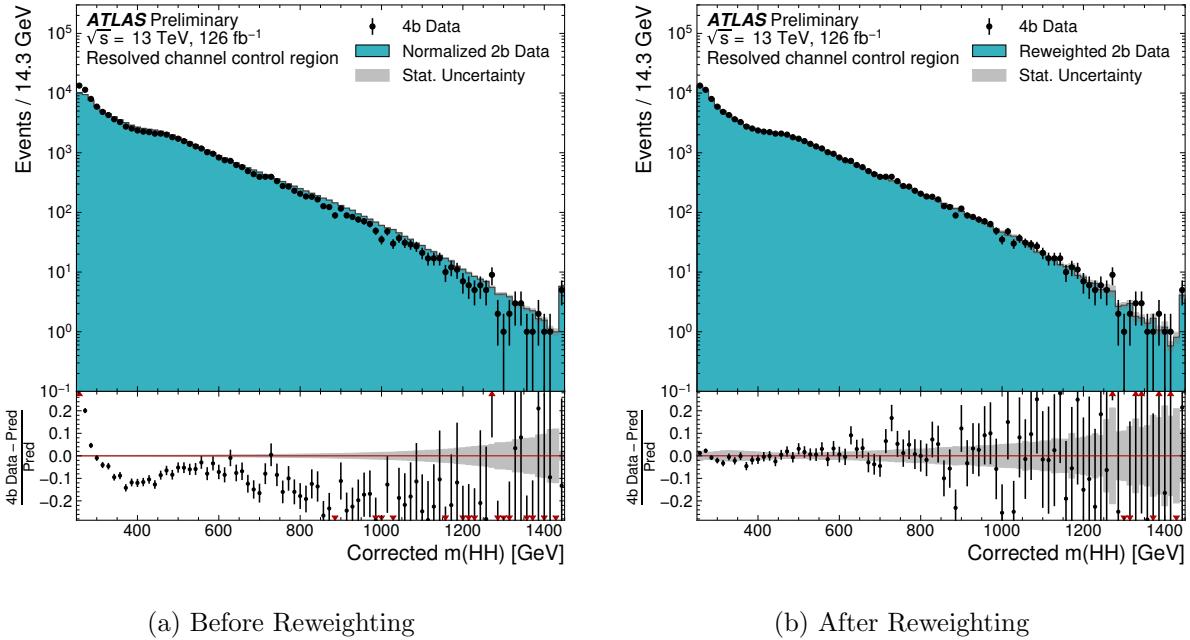


Figure 7.60: **Resonant Search:** Demonstration of the performance of the nominal reweighting in the control region on corrected m_{HH} , with Figure 7.60(a) showing $2b$ events normalized to the total $4b$ yield and Figure 7.60(b) applying the reweighting procedure. Agreement is much improved with the reweighting. Note that overall reweighted $2b$ yield agrees with $4b$ yield in the control region by construction.

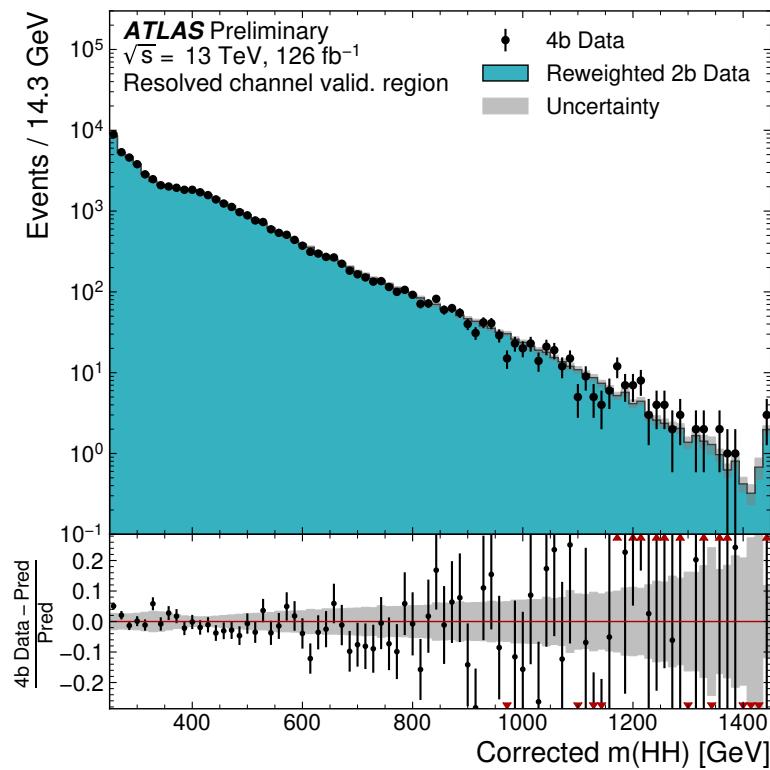


Figure 7.61: **Resonant Search:** Demonstration of the performance of the control region derived reweighting in the validation region on corrected m_{HH} . Agreement is generally good for this extrapolated estimate. Note that the uncertainty band includes the extrapolation systematic, which is defined by a reweighting trained in the validation region.

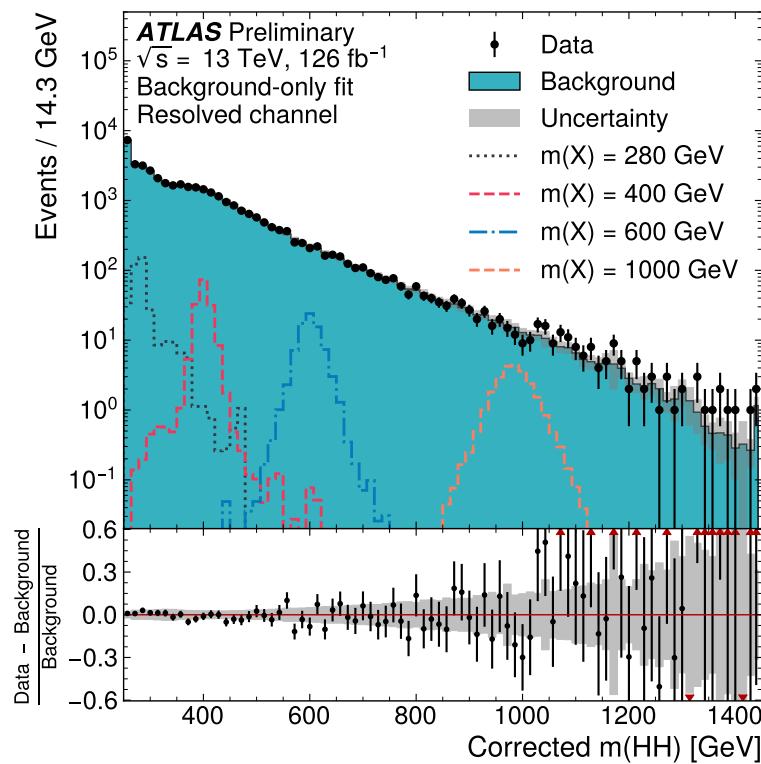


Figure 7.62: Signal region agreement of the background estimate and observed data after a background-only profile likelihood fit. The closure is generally quite good, though there is an evident deficit in the background estimate relative to the data for higher values of corrected m_{HH} .

2159 7.10.2 Non-resonant Search

As discussed above, the non-resonant search splits the signal extraction into two categories of $\Delta\eta_{HH}$ ($0 \leq \Delta\eta_{HH} < 0.75$ and $0.75 \leq \Delta\eta_{HH} < 1.5$), and two categories of X_{HH} ($0 \leq X_{HH} < 0.95$ and $0.95 \leq X_{HH} < 1.6$). To maintain reasonable statistics in each bin entering the signal extraction fit, a variable width binning is considered defined by a resolution parameter, r , and a set range in m_{HH} , where bin edges are determined iteratively as

$$b_{low}^{i+1} = b_{low}^i + r \cdot b_{low}^i, \quad (7.14)$$

2160 where b_{low}^i is the low edge of bin i . The parameters used here are $r = 0.08$ over a range
2161 from 280 GeV to 975 GeV, and underflow and overflow are included in the intial and final
2162 bin contents respectively. m_{HH} with no correction is used as the final discriminant in each
2163 category.

2164 A demonstration of the performance of the reweighting on distributions unrolled across
2165 categories is shown in Figure *TODO: insert* for the control region and Figure *TODO:*
2166 *insert* for the validation region. A background-only profile likelihood fit is run for the
2167 distribution in the signal region, and results with the Standard Model HH signal overlaid
2168 are shown for $4b$ in Figure 7.63 and $3b1l$ in Figure 7.64. Note that the plots show the sum
2169 across all years, but the signal extraction fit and background estimate are run with the years
2170 separately. All bins are normalized to represent a density of Events / 15 GeV.

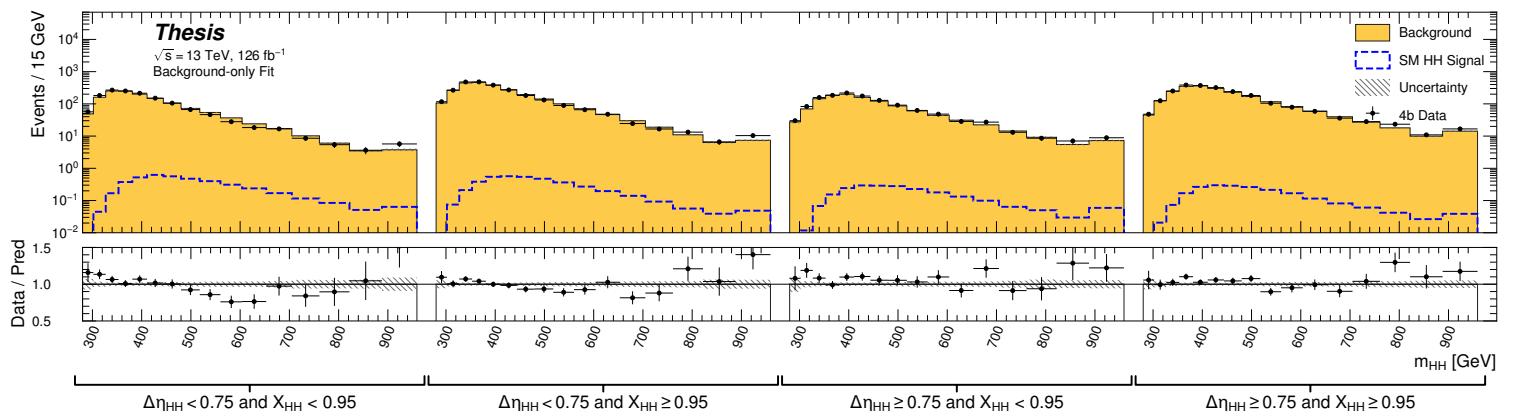


Figure 7.63: **Non-resonant Search:** Signal region agreement of the background estimate and observed data after a background-only profile likelihood fit for the 4b channels, with Standard Model signal overlaid for reference. Modeling is generally quite good near the Standard Model peak, but disagreements are seen at very low and high masses. A deficit is present in low $\Delta\eta_{HH}$ bins near 600 GeV.

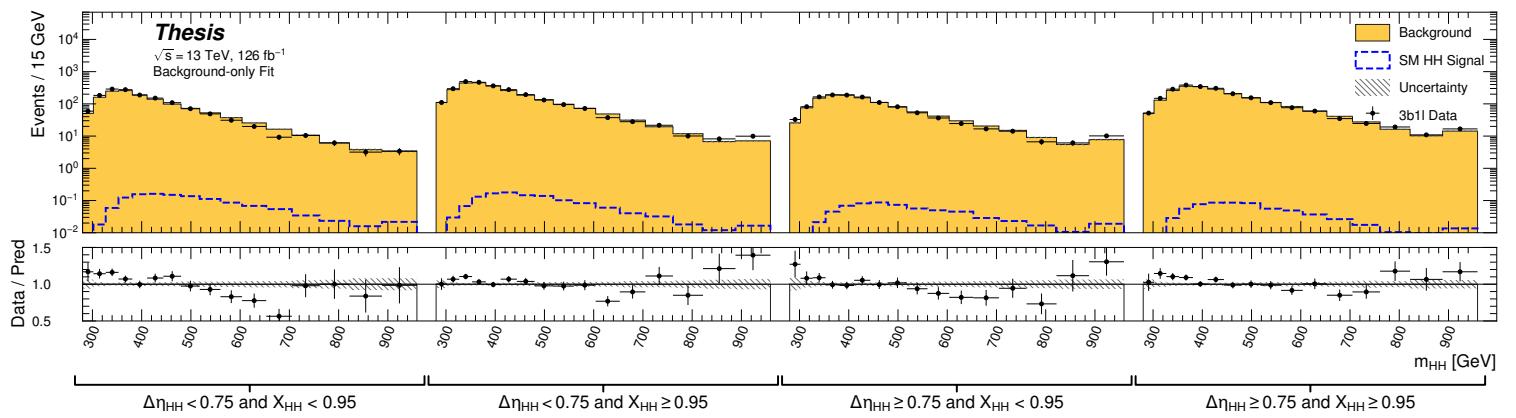


Figure 7.64: **Non-resonant Search:** Signal region agreement of the background estimate and observed data after a background-only profile likelihood fit for the $3b1l$ channels, with Standard Model signal overlaid for reference. Conclusions are very similar to the $4b$ channels, with generally good modeling near the Standard Model peak, but disagreements at very low and high masses. A deficit is present near 600 GeV.

2171 **7.11 Statistical Analysis**

2172 The resonant analysis is used to set a 95% confidence level upper limit on the $pp \rightarrow X \rightarrow$
2173 $HH \rightarrow b\bar{b}b\bar{b}$ and $pp \rightarrow G_{KK}^* \rightarrow HH \rightarrow b\bar{b}b\bar{b}$ cross-sections, while the non-resonant analysis
2174 is used to set a 95% confidence level upper limit on the $pp \rightarrow HH \rightarrow b\bar{b}b\bar{b}$ cross sections for
2175 a variety of values of the trilinear Higgs coupling.

2176 The upper limit is extracted using the CL_s method [77]. The test statistic used is q_μ [78],
2177 where μ is the signal strength, and θ represents the nuisance parameters. Due to the use of
2178 signals normalized to 1 fb, μ is also the signal cross-section in fb. A single hat represents the
2179 maximum likelihood estimate of a parameter, while $\hat{\theta}(x)$ represents the conditional maximum
2180 likelihood estimate of the nuisance parameters if the signal cross-section is fixed at x .

$$q_\mu = \begin{cases} -2 \ln \left(\frac{\mathcal{L}(\mu, \hat{\theta}(\mu))}{\mathcal{L}(\hat{\mu}, \hat{\theta})} \right) & \hat{\mu} \leq \mu \\ 0 & \hat{\mu} > \mu \end{cases} \quad (7.15)$$

2181 CL_s for some test value of μ is then defined by

$$CL_s = \frac{CL_{s+b}}{CL_b} = \frac{p(q_\mu \geq q_{\mu, \text{obs}} | s+b)}{p(q_\mu \geq q_{\mu, \text{obs}} | b)}, \quad (7.16)$$

2182 where the p -values are calculated in the asymptotic approximation [78], which is valid in
2183 the large sample limit.

2184 The signal cross-section μ fb is excluded at the 95% confidence level if $CL_s < 0.05$.

2185 **7.12 Results**

2186 Figure 7.65 shows the expected limit for the spin-0 and spin-2 resonant search. The resolved
2187 channel covers the range between 251 and 1500 GeV and is combined with the boosted channel
2188 between 900 and 1500 GeV. The boosted channel then extends to 3 TeV. The most significant
2189 excess is seen for a signal mass of 1100 GeV, with local significance of 2.6σ for the spin-0
2190 signal and 2.7σ for the spin-2 signal. This is reduced to 1.0σ and 1.2σ globally.

2191 The spin-2 bulk Randall-Sundrum model with $k/\bar{M}_{\text{Pl}} = 1$ is excluded for graviton masses
2192 between 298 and 1440 GeV.

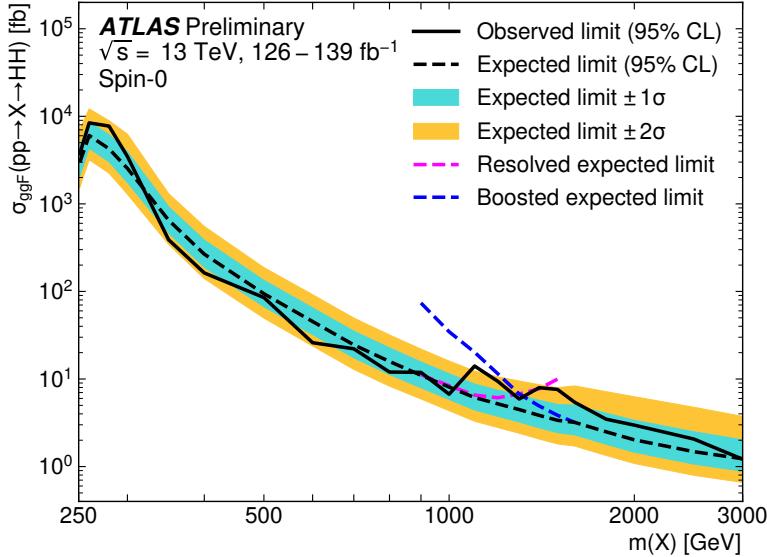
2193 Preliminary results are presented here for the gluon-gluon fusion non-resonant search,
2194 combining results from the $4b$ and $3b+1l$ signal regions in the 2×2 category scheme in $\Delta\eta_{HH}$
2195 and X_{HH} . These results will be further combined with a VBF channel as discussed, but this
2196 is left for future work. Results shown here include background systematics only. Limits are
2197 set for κ_λ values from -20 to 20 . The cross section limit for HH production is set at 140 fb
2198 (180 fb) observed (expected), corresponding to an observed (expected) limit of 4.4 (5.9) times
2199 the Standard Model prediction. κ_λ is constrained to be within the range $-4.9 \leq \kappa_\lambda \leq 14.4$
2200 observed ($-3.9 \leq \kappa_\lambda \leq 10.9$ expected). These results are shown in Figure 7.66.

2201 We note that this is a significant improvement over the early Run 2 result, which achieved
2202 an observed (expected) limit of 12.9 (20.7) times the Standard Model prediction. The dataset
2203 is 4.6 times larger, and a naive scaling of the early Run 2 result (Poisson statistics \implies a factor
2204 of $1/\sqrt{4.6}$) would predict an observed (expected) limit of 6.0 (9.7) times the Standard Model.
2205 The result of 4.4 (5.9) observed (expected) presented here is therefore both an improvement
2206 by a factor of 3 (3.5) over the previous result and also beats the statistical scaling by around
2207 30 (40) %, demonstrating the impact of the various analysis improvements presented here.
2208 We note again that these results do not include the complete set of uncertainties – however
2209 we expect the addition of the remaining uncertainties to have no more than a few percent
2210 impact.

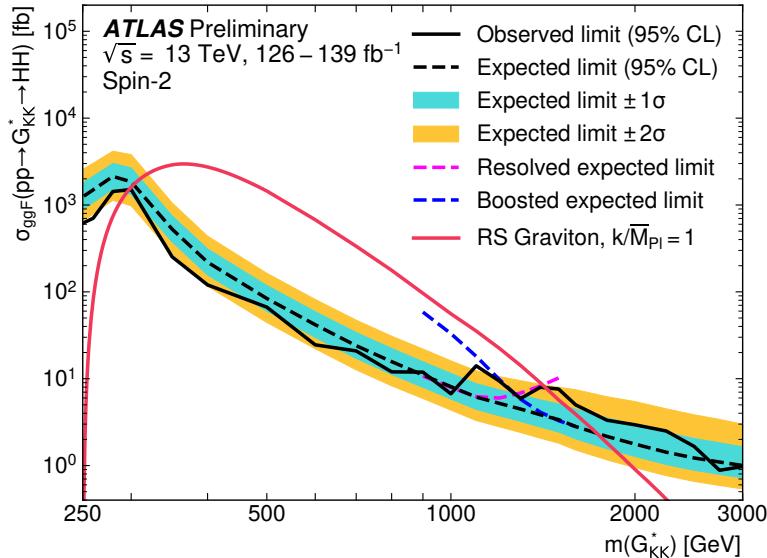
2211 The observed limits presented in Figure 7.66 are consistently above the 2σ band for values

of $\kappa_\lambda \geq 5$. As this analysis is optimized for points near the Standard Model, and as there is no excess present in more sensitive channels in this same region (e.g. $HH \rightarrow bb\gamma\gamma$), we do not believe this is a real effect, but is rather due to a mis-modeling of the background at low mass, where the min ΔR pairing has poor signal efficiency and the assumption of well behaved background in the mass plane breaks down. It was considered, but not implemented, for this analysis to impose a cut on m_{HH} near 350 or 400 GeV to avoid such an issue.

To check the impact of if we would have imposed such a cut, and to verify that the excess is due to the low mass regime, we therefore run the same set of limits without the low mass bins. In this case, we choose to simply drop the first few bins in m_{HH} such that everything else, including the higher mass bin edges, is kept the same. Due to the variable width binning, this corresponds to an m_{HH} cut of 381 GeV. The results of this check are shown in Figure 7.67, overlaid with the limits of Figure 7.66 for reference. With the m_{HH} cut imposed, there is a slight degradation in the expected limits for larger positive and negative values of κ_λ , but the points near the Standard Model are nearly identical. Further, the observed excess is significantly reduced, with observed limits for $\kappa_\lambda \geq 5$ now falling entirely within the expected 1σ band. Due to the preliminary nature of these results, further study is left for future work. However, we believe, in conjunction with the $HH \rightarrow bb\gamma\gamma$ results and our expectations about the difficulty of the background estimation at low mass, that this is demonstrative of a mismodeling rather than a real excess.



(a)



(b)

Figure 7.65: Expected (dashed black) and observed (solid black) 95% CL upper limits on the cross-section times branching ratio of resonant production for spin-0 ($X \rightarrow HH$) and spin-2 $G_{KK}^* \rightarrow HH$. The $\pm 1\sigma$ and $\pm 2\sigma$ ranges for the expected limits are shown in the colored bands. The resolved channel expected limit is shown in dashed pink and covers the range from 251 and 1500 GeV. It is combined with the boosted channel (dashed blue) between 900 and 1500 GeV. The theoretical prediction for the bulk RS model with $k/\bar{M}_{Pl} = 1$ [19] (solid red line) is shown, with the decrease below 350 GeV due to a sharp reduction in the $G_{KK}^* \rightarrow HH$ branching ratio. The nominal $H \rightarrow b\bar{b}$ branching ratio is taken as 0.582.

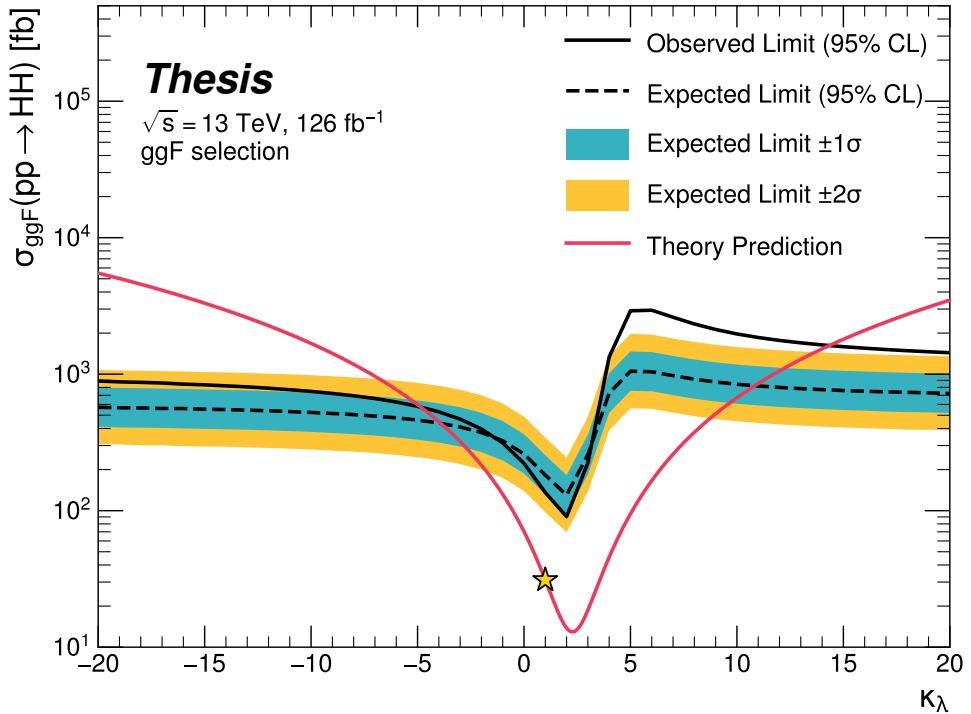


Figure 7.66: Expected (dashed black) and observed (solid black) 95% CL upper limits on the cross-section times branching ratio of non-resonant production for a range of values of the Higgs self-coupling, with the Standard Model value ($\kappa_\lambda = 1$) illustrated with a star. The $\pm 1\sigma$ and $\pm 2\sigma$ ranges for the expected limits are shown in the colored bands. The cross section limit for HH production is set at 140 fb (180 fb) observed (expected), corresponding to an observed (expected) limit of 4.4 (5.9) times the Standard Model prediction. κ_λ is constrained to be within the range $-4.9 \leq \kappa_\lambda \leq 14.4$ observed ($-3.9 \leq \kappa_\lambda \leq 10.9$ expected). The nominal $H \rightarrow b\bar{b}$ branching ratio is taken as 0.582. We note that the excess present for $\kappa_\lambda \geq 5$ is thought to be due to a low mass background mis-modeling, present due to the optimization of this analysis for the Standard Model point, and is not present in more sensitive channels in this same region (e.g. $HH \rightarrow bb\gamma\gamma$).

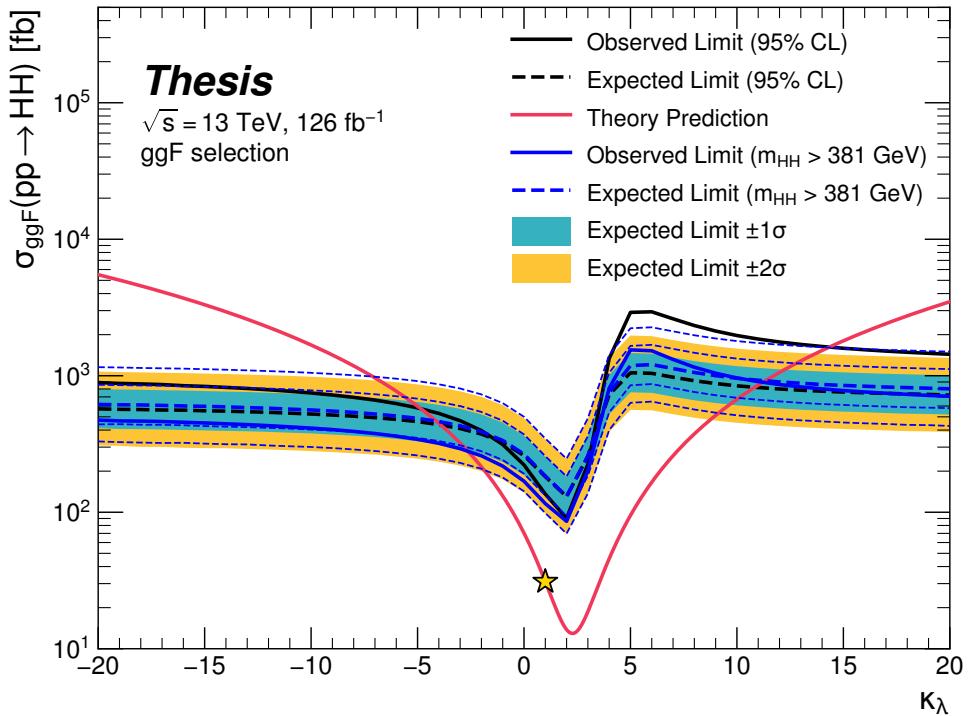


Figure 7.67: Comparison of the limits in Figure 7.66 with an equivalent set of limits that drop the m_{HH} bins below 381 GeV, with the value of 381 GeV determined by the optimized variable width binning. The expected limit band with this mass cut is shown in dashed blue, and the observed is shown in solid blue. The excess at and above $\kappa_\lambda = 5$ is significantly reduced, demonstrating that this is driven by low mass. Notably, there is minimal impact on the expected sensitivity with this m_{HH} cut, and the background modeling is known to be difficult for the low mass region.

2231 Chapter 8

2232 **FUTURE IDEAS FOR $HH \rightarrow b\bar{b}b\bar{b}$**

2233 The searches presented in this thesis make use of a large suite of sophisticated techniques,
2234 selected through careful study and validation. During this process, a variety of interesting
2235 directions for the $HH \rightarrow b\bar{b}b\bar{b}$ analysis were explored by this thesis author, in collaboration
2236 with a few others¹, but were not used due to a variety of constraints. We present two
2237 such interesting directions here, with the hope of encouraging further exploration of these
2238 techniques in future work.

2239 **8.1 pairAGraph: A New Method for Jet Pairing**

2240 As discussed in Chapter 7, one of the main problems to solve is the pairing of b -jets into
2241 Higgs candidates. Figure 7.1 demonstrates that the choice of the pairing method, while
2242 important for achieving good reconstruction of signal events, also significantly impacts the
2243 structure of non- HH events, leading to various biases in the background estimate. Evaluation
2244 of the pairing method therefore must take both of these factors into account. While we have
2245 presented some advantages in respective contexts for the pairing methods considered here,
2246 we of course would like to explore further improvements to this important component of the
2247 analysis.

2248 To that end, we note that all of the pairing methods considered here share a common
2249 feature: four jets are selected, and the pairing is some discrimination between the available
2250 three pairings of these four jets. For the methods used in this analysis, the jet selection
2251 proceeds via a simple p_T ordering, with b -tagged jets receiving a higher priority than non-

¹Notably Nicole Hartman (SLAC), who spearheaded much of the development and proof of concept work, in collaboration with Michael Kagan and Rafael Teixeira De Lima.

2252 tagged jets.

2253 With the advent of a variety of machine learning methods for dealing with a variable
2254 number of inputs *TODO: cite: RNNs, Deep Sets, GNNs, Transformers*, a natural place to
2255 improve on the pairing is to consider more than just four jets. The pairing and jet selection
2256 is then performed simultaneously, allowing for the incorporation of more event information in
2257 the pairing decision and the incorporation of jet correlation structure in the jet selection.

2258 In practice, the majority of $HH \rightarrow b\bar{b}b\bar{b}$ events have either four or five jets which pass the
2259 kinematic preselection, and any gain from this additional freedom would come from events
2260 with greater than or equal to five jets. However, this five jet topology is particularly exciting
2261 for scenarios such as events with initial state radiation (ISR), in which the $HH - > 4b$ jets
2262 are offset by a single jet with p_T similar in magnitude to that of the $HH - > 4b$ system.
2263 Such events have explicit event level information which is not encoded with the inclusion
2264 of only the $HH - > 4b$ jets, and are pathological if the ISR jet happens to pass b -tagging
2265 requirements.

2266 Additionally, with the use of lower tagged regions for background estimation and alternate
2267 signal regions, this extra flexibility in jet selection may provide a very useful bias – since the
2268 algorithm is trained on signal, the selected jets for the pairing will be the most “4b-like” jets
2269 available in the considered set.

2270 For the studies considered here, a transformer *TODO: cite Attention* based architecture
2271 is used. This is best visualized by considering the event as a graph with jets corresponding
2272 to nodes and edges corresponding to potential connections – for this reason, we term this
2273 algorithm “pairAGraph”. The approach is as follows: each jet, i , is represented by some
2274 vector of input variables, \vec{x}_i , in our case the four-vector information, (p_T, η, ϕ, E) of each jet,
2275 plus information on the b -tagging decision. A multi-layer perceptron (MLP) is used to create
2276 a latent embedding, $\mathbf{h}(\vec{x}_i)$, of this input vector.

To describe the relationship between various jets in the event, we then define a vector \vec{z}_i

for each jet as

$$\vec{z}_i = \sum_j w_{ij} \mathbf{h}(\vec{x}_j) \quad (8.1)$$

where j runs over all jets in the event (including $i = j$), the w_{ij} can be thought of as edge weights, and $\mathbf{h}(\vec{x}_j)$ is the latent embedding for jet j mentioned above.

Within this formula, both \mathbf{h} and the w_{ij} are learnable. To learn an appropriate latent mapping and set of edge weights, we define a similarity metric corresponding to each possible jet pairing:

$$\vec{z}_{1a} \cdot \vec{z}_{1b} + \vec{z}_{2a} \cdot \vec{z}_{2b} \quad (8.2)$$

where subscripts $1a$ and $1b$ correspond to the two jets in pair 1, $2a$ and $2b$ to the jets in pair 2 for a given pairing of four distinct jets.

This similarity metric is calculated for all possible pairings, which are then passed through a softmax *TODO: cite* activation function, which compresses these scores to between 0 and 1 with sum of 1, lending an interpretation as probability of each pairing.

In training, the ground truth pairing is set by *truth matching* jets to the b -jets in the HH signal simulation – a jet is considered to match if it is < 0.3 in ΔR away from a b -jet in the simulation record. Given this ground truth, a cross-entropy loss *TODO: cite* is used on the softmax outputs, and w_{ij} and \mathbf{h} are updated correspondingly. Training in such a way corresponds to updating w_{ij} and \mathbf{h} to maximize the similarity metric for the correct pairing.

In evaluation, the pairings with a higher score (and therefore higher softmax output) given the trained h and w_{ij} therefore correspond to the pairings that are most “ HH -like”. The maximum over these scores is therefore the pairing used as the predicted result from the algorithm.

Because the majority of $HH \rightarrow b\bar{b}b\bar{b}$ events have either four or five jets, it was found to be sufficient to only consider a maximum of 5 jets. Consideration of more is in principle possible, but the quickly expanding combinatorics leads to a rapidly more difficult problem. The jets considered are the five leading jets in p_T . Notably, this set of jets may include jets which are not b -tagged, even for the nominal $4b$ region – therefore events with 4 b -jets are

2298 not required to use all of them in the construction of Higgs candidates, in contrast to the
2299 other algorithms used in this thesis.

2300 **8.2 Background Estimation with Mass Plane Interpolation**

2301 The choice of a pairing algorithm that results in a smooth mass plane (such as $\min \Delta R$)
2302 opens up a variety of options for the background estimation. While the method based on
2303 reweighting of $2b$ events used for this thesis performs well and has been extensively studied
2304 and validated, it also relies on several assumptions. In particular, the reweighting is derived
2305 between e.g., $2b$ and $4b$ events *outside* of the signal region and then applied to $2b$ events *inside*
2306 the signal region, with the assumption that the $2b$ to $4b$ transfer function will be sufficiently
2307 similar in both regions of the mass plane. An uncertainty is assigned to account for the bias
2308 due to this assumption, but the extrapolation in the mass plane is never explicitly treated in
2309 the nominal estimate. While the approach of reweighting $2b$ events within the signal region
2310 does have the advantage of incorporating explicit signal region information (that is, the $2b$
2311 signal region events), the importance of the extrapolation bias motivates consideration of
2312 a method that operates within the $4b$ mass plane. This additionally removes the reliance
2313 on lower b -tagging regions, allowing for the use of, e.g. $3b$ triggers, and future-proofing the
2314 analysis against trigger bandwidth constraints in the low tag regions.

The method considered here relies on the following: for a given vector of input variables (event kinematics, etc), \vec{x} , the joint probability in the HH mass plane may be written as:

$$p(\vec{x}, m_{H1}, m_{H2}) = p(\vec{x}|m_{H1}, m_{H2})p(m_{H1}, m_{H2}) \quad (8.3)$$

2315 by the chain rule of probability. This means that the full dynamics of events in the HH mass
2316 plane may be described by (1) the conditional probability of considered variables \vec{x} , given
2317 values of m_{H1} and m_{H2} , and (2) the density of the mass plane itself.

2318 We present here an approach which uses normalizing flows *TODO: cite* to model the
2319 conditional probabilities of events in the mass plane and Gaussian processes to model the
2320 mass plane density. These models are trained in a region around, but not including, the

2321 signal region, and the trained models are then used to construct an *interpolated* estimate of
 2322 the signal region kinematics. This approach therefore explicitly treats event behavior within
 2323 the mass plane, avoiding the concerns associated with a reweighted estimate. Validation of
 2324 such a method, as well as assessing of closure and biases of the method, may be done in
 2325 alternate b -tagging or kinematic regions, notably the now unused $2b$ region, results of which
 2326 are shown below.

2327 *8.2.1 Normalizing Flows*

Normalizing flows model observed data $x \in X$, with $x \sim p_X$, as the output of an invertible,
 differentiable function $f : X \rightarrow Z$, with $z \in Z$ a latent variable with a simple prior probability
 distribution (often standard normal), $z \sim p_Z$. From a change of variables, given such a
 function, we may write

$$p_X(x) = p_Z(f(x)) \left| \det \left(\frac{d(f(x))}{dx} \right) \right| \quad (8.4)$$

2328 where $\left(\frac{d(f(x))}{dx} \right)$ is the Jacobian of f at x .

2329 The problem of normalizing flows then reduces to (1) choosing sets of f which are both
 2330 tractable and sufficiently expressive to describe observed data, and (2) optimizing associated
 2331 sets of functional parameters on observed data via maximum likelihood estimation using
 2332 the above formula. Sampling from the learned density is done by drawing from the latent
 2333 distribution $z \sim p_Z$ (cf. inverse transform sampling) – the corresponding sample is then
 2334 $x \sim p_X$ with $x = f^{-1}(z)$.

2335 A standard approach to the definition of these f is as a composition of affine transfor-
 2336 mations (e.g. RealNVP *TODO: cite*), that is, transformations of the form $\alpha z + \beta$, with α and β
 2337 learnable parameter vectors. This can roughly be thought of as shifting and squeezing the
 2338 input prior density in order to match the data density. However, this has somewhat
 2339 limited expressivity, for instance in the case of a multi-modal density.

This work thus instead relies on neural spline flows *TODO: cite: <https://arxiv.org/pdf/1906.04032.pdf>*
 in which the functions considered are monotonic rational-quadratic splines, which have an

analytic inverse. A rational quadratic function has the form of a quotient of two quadratic polynomials, namely,

$$f_j(x_i) = \frac{a_{ij}x_i^2 + b_{ij}x_{ij} + c_{ij}}{d_{ij}x_i^2 + e_{ij}x_i + f_{ij}} \quad (8.5)$$

with six associated parameters (a_{ij} through f_{ij}) per each piecewise bin j of the spline and each input dimension i . This is explicitly more flexible and expressive than a simple affine transformation, allowing, e.g., the treatment of multi-modality via the piecewise nature of the spline.

The rational quadratic spline is defined on an set interval. The transformation outside of this interval is set to the identity, with these linear tails allowing for unconstrained inputs. The boundaries between bins of the spline are set by coordinates scalled *knots*, with $K + 1$ knots for K bins – the two endpoints for the spline interval plus the $K - 1$ internal boundaries. The derivatives at these points are constrained to be positive for the internal knots, and boundary derivatives are set to 1 to match the linear tails.

The bin widths and heights are learnable ($2 \cdot K$ parameters) as are the internal knot derivatives ($K - 1$ parameters), and these $3K - 1$ ouputs of the neural network are sufficient to define a monotonic rational-quadratic spline which passes through each knot and has the given derivative value at each knot.

In the context of the $HH \rightarrow 4b$ analysis, a neural spline flow is used to model the four vector information of each Higgs candidate, conditional on their respective masses. The resulting flow is therefore five dimensional, with inputs $x = (p_{T,H1}, p_{T,H2}, \eta_{H1}, \eta_{H2}, \Delta\phi_{HH})$, where the ATLAS ϕ symmetry has been encdoded by assuming $\phi_{H1} = 0$. Conditional variables m_{H1} and m_{H2} are not modeled by the flow, but “come along for the ride”. A standard normal distribution in 5 dimensions is used for the underlying prior. Modeling of the four vectors was chosen in order to reduce bias from modeling m_{HH} directly.

The trained flow model then gives a model for $p(x|m_{H1}, m_{H2})$ which may be sampled from to reconstruct distributions of HH kinematics given values of m_{H1} and m_{H2} .

2363 8.2.2 Gaussian Processes

2364 The second piece of this background estimate is the modeling of the mass plane density,
2365 $p(m_{H1}, m_{H2})$. This is done using Gaussian process regression – note that a similar procedure
2366 is used to define a systematic in the boosted $4b$ analysis. Generally, Gaussian processes
2367 are a collection of random variables in which every finite collection of said variables is
2368 distributed according to a multivariate normal distribution. For the context of Gaussian
2369 process regression, what we consider is a Gaussian process over function space, that is, for a
2370 collection of points, x_1, \dots, x_N , the space of corresponding function values, $(f(x_1), \dots, f(x_N))$
2371 is Gaussian process distributed, that is, described by an N dimensional normal distribution
2372 with mean μ , covariance matrix Σ .

2373 For a single point, this would correspond to a function space described entirely by a
2374 normal distribution, with various samples from that distribution yielding various candidate
2375 functions. For multiple points, a covariance matrix describes the relationship between each
2376 pair of points – correspondingly, it is represented via a *kernel function*, $K(x, x')$. As, in
2377 practice, μ may always be set to 0 via a centering of the data, the kernel function fully defines
2378 the considered family of functions.

The considered family of functions describes a Bayesian *prior* for the data. This prior may be conditioned on a set of training data points (X_1, \vec{y}_1) . This conditional *posterior* may then be used to make predictions $\vec{y}_2 = f(X_2)$ at a set of new points X_2 . Because of the Gaussian process prior assumption, \vec{y}_1 and \vec{y}_2 are assumed to be jointly Gaussian. We may therefore write

$$\begin{pmatrix} \vec{y}_1 \\ \vec{y}_2 \end{pmatrix} \sim \mathcal{N} \left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} K(X_1, X_1) & K(X_1, X_2) \\ K(X_1, X_2) & K(X_2, X_2) \end{pmatrix} \right) \quad (8.6)$$

2379 where we have used that the kernel function is symmetric and assumed prior mean 0.

By standard conditioning properties of Gaussian distributions,

$$\vec{y}_2 | \vec{y}_1 \sim \mathcal{N}(K(X_2, X_1)K(X_1, X_1)^{-1}\vec{y}_1, K(X_2, X_2) - K(X_2, X_1)K(X_1, X_1)^{-1}K(X_1, X_2)) \quad (8.7)$$

2380 which is the sampling distribution for a Gaussian process given kernel K . In practice, the
 2381 mean of this sampling distribution is used as the function estimate, with an uncertainty from
 2382 the predicted variance at a given point.

The choice of kernel function has a very strong impact on the fitted curve, and must therefore be chosen to express the expected dynamics of the data. A common such choice is a radial basis function (RBF) kernel, which takes the form

$$K(x, x') = \exp\left(-\frac{d(x, x')^2}{2l^2}\right) \quad (8.8)$$

2383 where $d(\cdot, \cdot)$ is the Euclidean distance and $l > 0$ is a length scale parameter. Conceptually, as
 2384 distances $d(x, x')$ increase relative to the chosen length scale, the kernel smoothly dies off –
 2385 further away points influence each other less.

2386 Coming back to our case of the mass plane, the procedure runs as follows:

2387 1. A binned 2d histogram of the blinded mass plane is created in a window around the
 2388 “standard” analysis regions. Bins which have any overlap with the signal region are
 2389 excluded.

2390 2. A Gaussian process is trained using the bin centers, values as training points. The
 2391 scikit-learn implementation [79] is used, with RBF kernel with anisotropic length scale
 2392 (l is dimension 2). The length scale is initialized to $(50, 50)$ to cover the signal region,
 2393 and optimized by minimizing the negative log-marginal likelihood on the training data,
 2394 $-\log p(\vec{y}|\theta)$. Training data is centered and scaled to mean 0, variance 1, and a statistical
 2395 error is included in the fit.

2396 3. The Gaussian process is then used to predict the density $p(m_{H1}, m_{H2})$ in the signal
 2397 region. This may then be sampled from via an inverse transform sampling to generate
 2398 values (m_{H1}, m_{H2}) according to the density (specifically, according to the mean of the
 2399 Gaussian process posterior). Though in principle the Gaussian process sampling is not
 2400 limited to bin centers, this is kept for simplicity, with a uniform smearing applied within

2401 each sampled bin to approximate the continuous estimate, namely, if a bin is sampled
2402 from, the returned value is drawn uniformly at random within the sampled bin.

4. The sampling in the previous step can be arbitrary – to set the overall normalization, a Monte Carlo sampling of the Gaussian process is done to approximate the relative fraction of events predicted both inside (f_{in}) and outside (f_{out}) of the signal region, within the training box. The number of events outside of the signal region (n_{out}) is known, therefore, the number of events inside of the signal region, n_{in} , may be estimated as

$$n_{in} = \frac{n_{out}}{f_{out}} \cdot f_{in}. \quad (8.9)$$

2403 Note that the Monte Carlo sampling procedure is simply a set of samples of the Gaussian
2404 process from uniformly random values of m_{H1}, m_{H2} , and is the most convenient approach
2405 given the irregular shape of the signal region.

2406 This procedure results in a generated set of predicted m_{H1}, m_{H2} values for signal region
2407 background events, along with an overall yield prediction.

2408 8.2.3 The Full Prediction

2409 Given the normalizing flow parametrization of $p(x|m_{H1}, m_{H2})$ and the Gaussian process
2410 generation of $(m_{H1}, m_{H2}) \sim p(m_{H1}, m_{H2})$ and prediction of the signal region yield, all of the
2411 pieces are in place to construct an interpolation background estimate. Namely

- 2412 1. Gaussian process sampled (m_{H1}, m_{H2}) values are provided to the normalizing flow to
2413 predict the other variables for the Higgs candidate four-vectors. These are used to
2414 construct the HH system (notably $m_{HH}, \cos \theta^*$).
- 2415 2. These final distributions are normalized according to the predicted background yield.

2416 8.2.4 *Outstanding Points*

2417 While good performance is demonstrated from the nominal interpolated background estimate,
2418 various uncertainties must be assigned according to the various stages of the estimate. These
2419 notably include

- 2420 • Assessing a statistical uncertainty on the normalizing flow training (cf. bootstrap
2421 uncertainty).
- 2422 • Propagation of the Gaussian process uncertainty through the sampling procedure.
- 2423 • Validation of the resulting estimate and assessment of necessary systematic uncertainties
2424 (e.g. from validation region non-closure).

2425 These are all quite tractable, but some, especially the choice of an appropriate systematic
2426 uncertainty, are certainly not obvious and require detailed study. In this respect, the
2427 reweighting validation work of the non-resonant analysis is certainly quite useful as a starting
2428 place in terms of the available regions and their correspondence to the nominal $4b$ signal
2429 region.

2430

Chapter 9

2431

CONCLUSIONS

2432 This thesis has provided an overview of the Standard Model, with an emphasis on pair
2433 production of Higgs bosons and how this process may be used to both verify the Standard
2434 Model and to search for new physics. An overview of the Large Hadron Collider and the
2435 ATLAS detector has been provided, and the design and use of simulation infrastructure
2436 has been explained, including work to improve hadronic shower modeling in fast detector
2437 simulation. The translation of detector level information to analysis level information has
2438 been explained, with an emphasis on jets and the identification of B hadron decay. Finally,
2439 two searches for Higgs boson pair production have been presented, with a complete set of
2440 results for resonant production included, focusing on searches beyond the Standard Model,
2441 and a preliminary set of results for non-resonant production, targeting Standard Model
2442 production, with variations of the Higgs self-coupling. Two advanced techniques for the
2443 future of these analyses are further presented, along with proof-of-concept results.

2444

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