Theory *of* Finance

Solution Sheet on Problem Set 4

Fixed Income

Deadline: 15.12.2021

¨Solved by: Sandro Gassner

<table>
<tr><th>Task</th><th></th><th>Points Earned</th></tr>
<tr><td>1. Interest Rate Calculus
a)
Price of Zero Bond
(4 points)</td><td>The price of a 3 year zero-coupon bond with a face value of $100M is $ 96’943’788.</td><td></td></tr>
<tr><td>b)
Price of Coupon Bond
(4 points)</td><td>The face value of a 3 year bond with coupon payment of $5M yearly is $ 84’784’820.</td><td></td></tr>
<tr><td>c)
Yield-to-Maturity
(8 points)</td><td>YTM for zero coupon bond: 1.04% (equal to yield curve)
YTM for coupon paying bond: 1.02%</td><td></td></tr>
<tr><td>d)
Cont. and simple rates
(4 points)</td><td>We used the transformations from the lecture notes 16.6 for the calculations, where Y is the yield to maturity, $\tilde{Y}$ is the simple interest rate and y is the continuously compounded rate:

$$y = \ln(1+Y)$$
$$\tilde{Y} = [(1+Y)^m - 1]/m$$

The following table shows the continuous and simple interest rates for both bonds:

<table><tr><th></th><th>Simple</th><th>Continuous</th></tr><tr><td>Zero coupon bond</td><td>1.0519%</td><td>1.0357%</td></tr><tr><td>Coupon paying bond</td><td>1.0305%</td><td>1.0149%</td></tr></table></td><td></td></tr>
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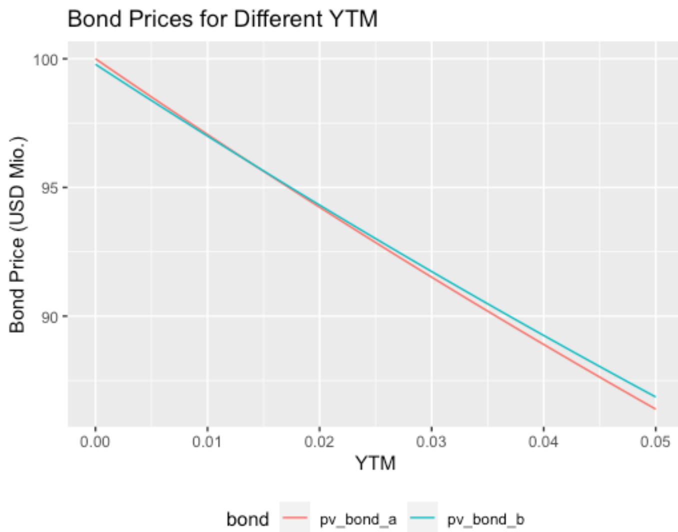
e)
Durations
(8 points)

The following table shows the Dollar, adjusted and Macaulay Duration for both bonds:

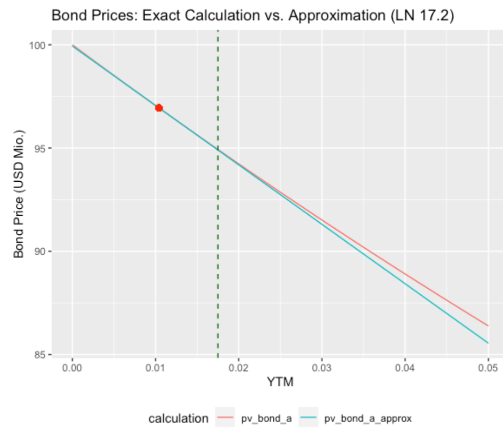
	Dollar	Adjusted	Macaulay
Zero coupon bond	287’825’994	2.9690	3
Coupon paying bond	273’237’996	2.8185	2.8473

f)
price-yield plot 1
(8 points)

The following graph shows the bond price for different YTM rates (Bond A = 3Y-Zero-Bond, Bond B = 3Y-Coupon-Bond):



As one could expect, the prices of both bonds fall with a rise in interest rates. As we can see in the plot, the price of the zero-coupon bond falls more drastically compared to the coupon paying bond. This means, it is better to hold a coupon bond if we expect interest rate levels to rise in the future. This can be explained by the fact, that for the zero-bond, we only get the face value back at maturity in 3 years and for the coupon bond, we get coupons in the nearer future that hence are discounted less. This is also reflected in the lower durations of the coupon bond in exercise 1e), what means the price is less sensitive to changes in the interest rate.

g) price-yield plot 2 (8 points)	The following graph shows the price calculations of the 3Y-Zero-Coupon Bond using the exact calculation and the duration approximation described in LN 17.2. The change of the price, ΔP, due to a small change in the yield, $\Delta\theta$, is approximately $$\Delta P \approx \frac{dP(\theta)}{d\theta} \times \Delta\theta. \qquad (17.2)$$  The starting point of the approximation is the bond price for a yield to maturity of 1.04% (red dot). By looking at the graph we observe that the approximation curve is generally below the “exact” calculation. This underestimation becomes bigger, as $\Delta\ \theta$ becomes bigger. For our case of an interest rate hike of 75 bps (green dashed line) the difference of the exact calculation and the approximation is likely to be neglectable (approximately \$30’000). This corresponds to an approximation error of less than 0.05% and shows that the approximation performs well.	
2. Duration hedge a) Which bond to use? (6 points)	**Hedging:** Generally, hedging works best by choosing hedging instruments that are similar in terms of duration and cash flows. If they perfectly match, we could eliminate fluctuations in the portfolio price, as their prices move in opposite directions with respect to interest rate changes. In our case, to hedge a three-year zero-coupon bond liability, we would optimally choose a bond that matches duration and cash flows. Therefore, we would choose to hold the three-year zero-coupon bond in our portfolio as a hedge.	

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<td>b)
Naïve hedge
(10 points)</td>
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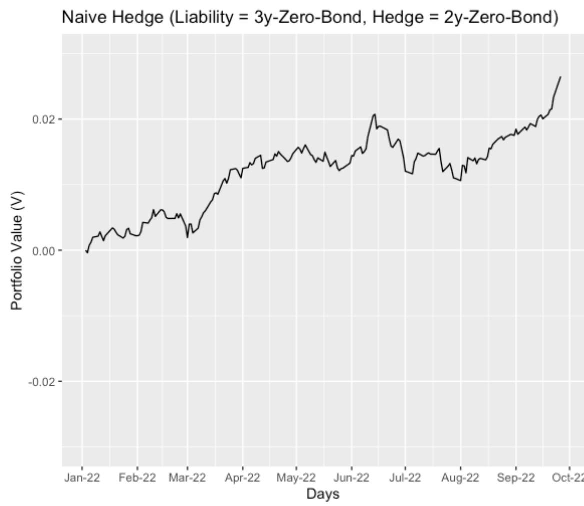
The plot below shows the portfolio value over time using a naïve hedge strategy:



As we can see in the plot, the price of the portfolio increases over the time. This means, we have a duration mismatch. This makes the overall portfolio sensitive to interest rate changes. In our case we face increasing interest rates over the periods so that the liability decreases more in value than the hedge. This causes our portfolio to increase in value and we face gains.

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<td>c)
Duration hedge
(10 points)</td>
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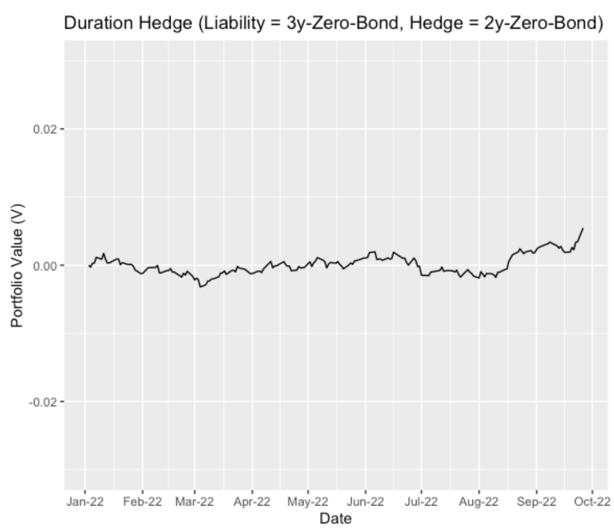
The plot below shows the portfolio value over time using a duration hedge strategy:



As we can see, the price of the portfolio remains much more stable at zero compared to the naïve hedging approach. The portfolio is much less sensitive to interest changes. Even though we had different maturities we were able to perform better with the duration hedging strategy compared to the naïve approach.

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<td>d)
Hedge comparison
(6 points)</td>
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Standard deviations:

Naïve hedging standard deviation: 0.000893
Duration hedging standard deviation: 0.000486

Interpretation:

By comparing the standard deviation (SD) between the hedging strategies, we observe that the duration hedging has a smaller SD. Conclusively, this signals that the duration hedging performed better as there are less variation in the portfolio value thus less volatility.

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3. Empirics & Premia

a)
Empirical properties
(8 points)</td>
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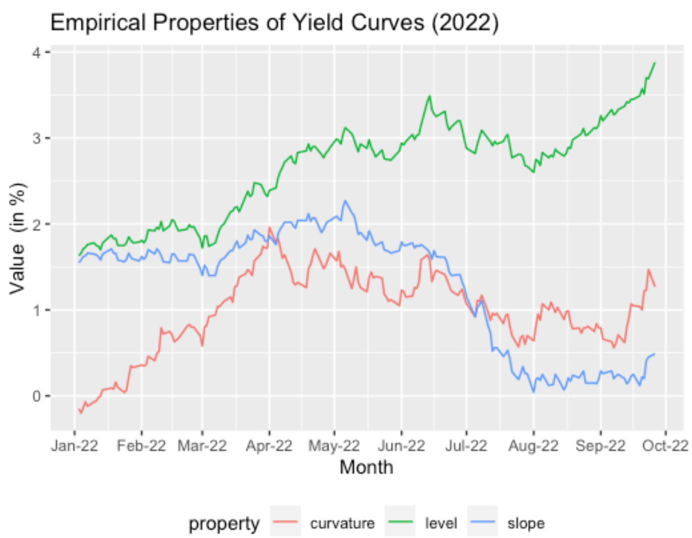
The following graph shows the level, slope and curvature of the interest date:



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<td>b)
Correlation analysis
(8 points)</td>
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The following tables shows the correlations:

Table: Correlations of Changes in Empirical Yield Properties

<table>
<tr><th></th><th>delta_level</th><th>delta_slope</th><th>delta_curvature</th></tr>
<tr><td>delta_level</td><td>1.0000</td><td>0.7113</td><td>0.3287</td></tr>
<tr><td>delta_slope</td><td>0.7113</td><td>1.0000</td><td>0.4326</td></tr>
<tr><td>delta_curvature</td><td>0.3287</td><td>0.4326</td><td>1.0000</td></tr>
</table>

Generally, we observe a correlation between the level, slope, and curvature. The correlation is expected as these empirical properties consists of the same yield rates. This can be observed by their definition:

$$\text{Level} = y(\text{10-years})$$

$$\text{Slope} = y(\text{10-years}) - y(\text{3-months})$$

$$\text{Curvature} = [y(\text{2-years}) - y(\text{3-months})] - [y(\text{10-years}) - y(\text{2-years})]$$

Surprisingly, the correlation of the change in curvature and level is positive. As it enters negatively in the formula, we would expect the correlation to be negative.

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c)
Correlations of
Premia
(8 points)

Correlation of different Risk Premia:

Table: Correlations of Different Risk Premia

	term_rp	e_inflation_rp	default_rp
term_rp	1.0000	0.6094	-0.0708
e_inflation_rp	0.6094	1.0000	0.0318
default_rp	-0.0708	0.0318	1.0000

The term risk premium is the premium that investors demand for holding bonds with longer maturities, as they are subject to more uncertainty. The expected inflation risk premium (IRP) is the premium investors demand for having the risk of inflation and the default risk premium is the premium for the additional risk for holding a BBB corporate bond compared to a less risky government bond.

As we can see, the term risk premium and the IRP are strongly correlated. This follows from the fact that the risk of holding a bond long-term also includes the risk of inflation. The default risk premium is whether strongly correlated with the term risk premium nor the IRP. In fact, the correlation is almost zero.