Theory *of* Finance

Solution Sheet for Problem Set 3

Asset Pricing Models & Portfolio Choice

Deadline: 01.12.2022

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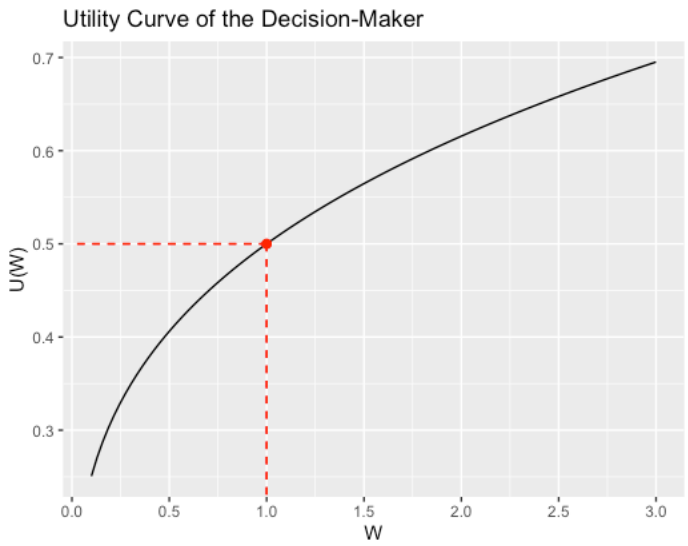
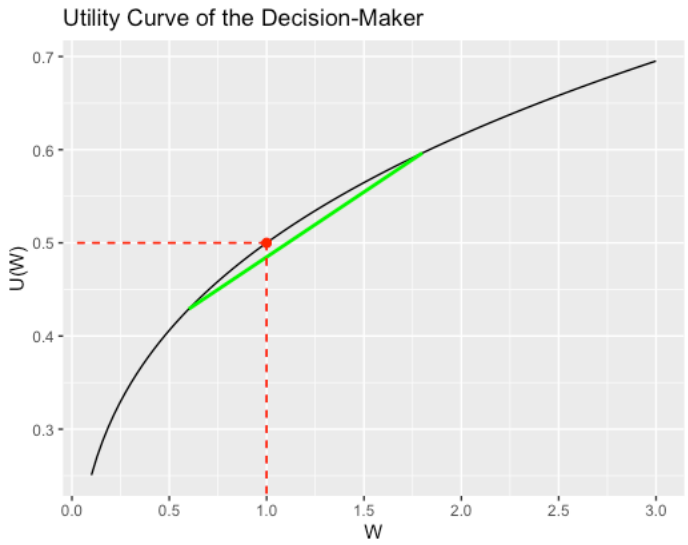
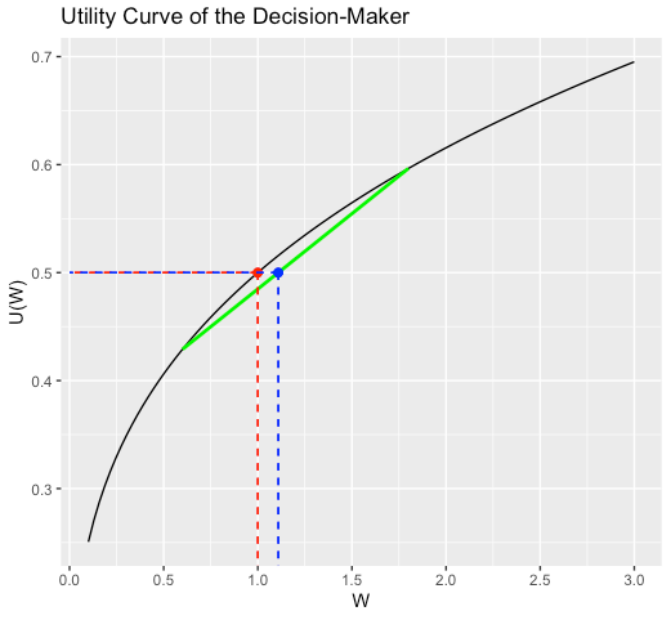
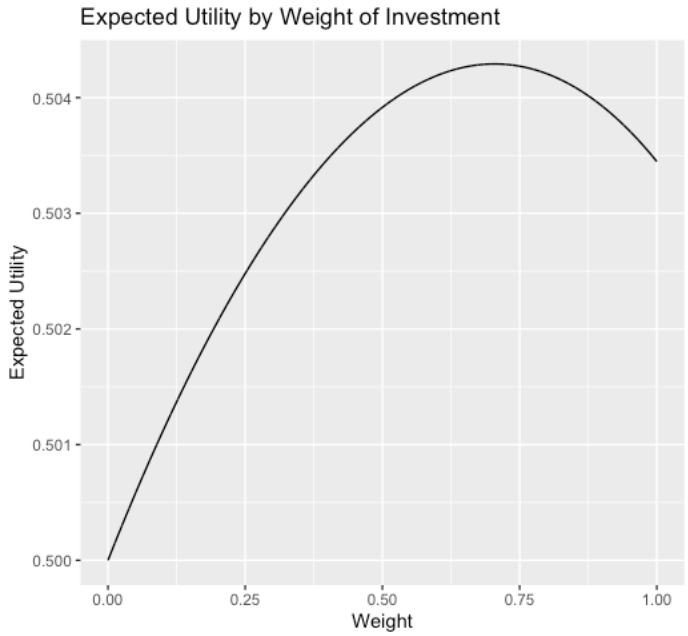
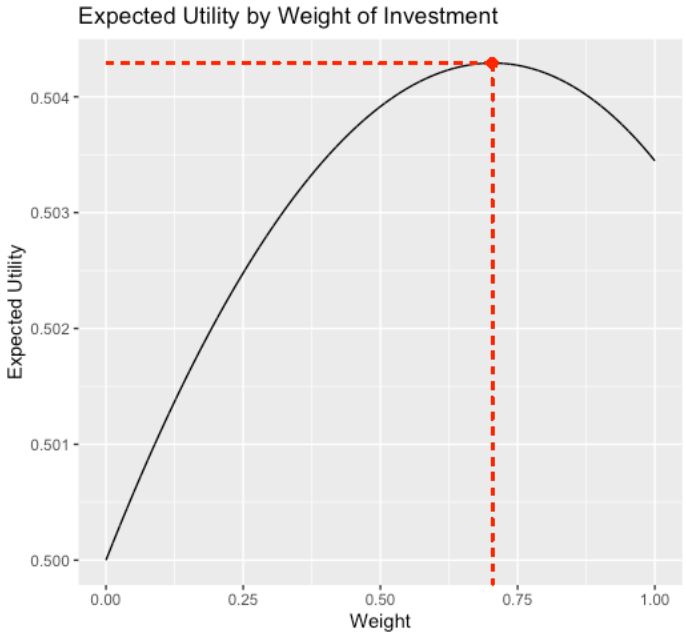
Task		Points Earned
1. Utility-Based Portfolio Choice a) Plot utility curve (6 points)	$$U(W) = \left(\frac{-W^{1.2-k}}{1-k}\right)^{-1}$$ **Plotting the utility function at an interval of W = [0.1, 3] and k=1.5 above yields the following graph:**  The red dot at W=1 and U(W)=0.5 marks the initial wealth W=1	

b) Utility of investment opportunity (6 points)	 The green line represents the expected utility from the lottery: p*U(1.8)+(1-p)*U(0.6) with p $\in$ [0,1] Due to the risk-aversion of the decision maker, the utility of the expected value of the lottery is higher as the expected utility from the lottery.	
c) p that makes investment decision indifferent (4 points)	At p=0.4242… the decision-maker is indifferent between participating and not participating in the lottery.	
d) Expected wealth at optimal p. Interpretation of results (8 points)		

	The blue dot marks the expected wealth and their expected utility from the lottery. We observe that the blue dot is below the utility function of the decision-maker thus conclude E(U(W)) < U(E(W)). This suggests that the decision-maker is risk averse. Furthermore, we can calculate the certainty equivalent (P) and risk premium (RP). P = 1 E(X) = 1.1091 This means that the decision maker is willing to sell the lottery with E(X) = 1.1091 to have certain amount of wealth W=1. In other words: The decision maker is willing to pay a risk-premium of 0.1091 to avoid the lottery.	
e) Plot expected utility as a function of w (6 points)	 We once again see that in terms of expected utility, the investor is always better off to assign some weights to the financial lottery compared to the initial expected utility for w $\in$ (0,1]. However, we also observe that there is weight which maximizes the expected utility for the investor.	

f) Optimal w and interpretation of results (6 points)



The weight = 0.7045 maximizes the expected utility of the decision maker. For this weight, the expected utility is = 0.5042908.

From the graph above, we observe that allocating any fraction of the wealth into the security yields a higher expected utility than not investing at all. This is mainly due to the strong positive expected value of the security. However, at some point, increasing the weight reduces the expected utility due to the decreasing marginal return of wealth and increase of risk.

2. Factor Investing

a) Calculation of performance measures (6 points)

We obtain the following metrics for the portfolios:

Table: Equal-Weighted Portfolios

factor	annual_return	annual_sd	sharpe_ratio
investment	0.0536	0.0558	0.9616
low_risk	0.0291	0.1189	0.2450
momentum	0.0615	0.1049	0.5868
profitability	0.0183	0.0909	0.2009
short_term_reversal	0.0575	0.0422	1.3621
size	0.0321	0.1050	0.3054
value	0.0537	0.0964	0.5569

Table: Value-Weighted Portfolios

factor	annual_return	annual_sd	sharpe_ratio
investment	0.0170	0.0675	0.2522
low_risk	0.0113	0.1180	0.0961
momentum	0.0356	0.1062	0.3353
profitability	0.0217	0.0710	0.3058
short_term_reversal	0.0099	0.0410	0.2418
size	0.0180	0.0918	0.1966
value	0.0270	0.0912	0.2956

In regards of risk-adjusted returns, the best strategy with the highest **Sharpe ratio** is the **equally weighted short-term reversal** portfolio.

From the given performance metrices, the best performing in regards of **annual return** is the **equally weighted momentum** portfolio.

b) CAPM regression (10 points)

Running a CAPM regression yields the following results for beta, alpha and their p-value:

Equally weighted:

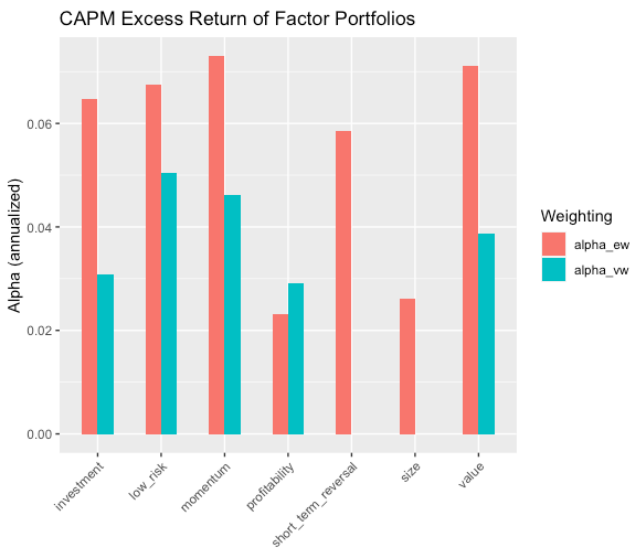
factor	annual_return	annual_sd	sharpe_ratio	alpha	p_value_alpha	beta	p_value_beta
investment	0.0536	0.0558	0.9616	0.0646	0.0000	-0.1404	0.0000
low_risk	0.0291	0.1189	0.2450	0.0674	0.0000	-0.4876	0.0000
momentum	0.0615	0.1049	0.5868	0.0730	0.0000	-0.1466	0.0000
profitability	0.0183	0.0909	0.2009	0.0232	0.0338	-0.0632	0.0026
short_term_reversal	0.0575	0.0422	1.3621	0.0585	0.0000	-0.0126	0.1952
size	0.0321	0.1050	0.3054	0.0260	0.0393	0.0770	0.0015
value	0.0537	0.0964	0.5569	0.0712	0.0000	-0.2234	0.0000

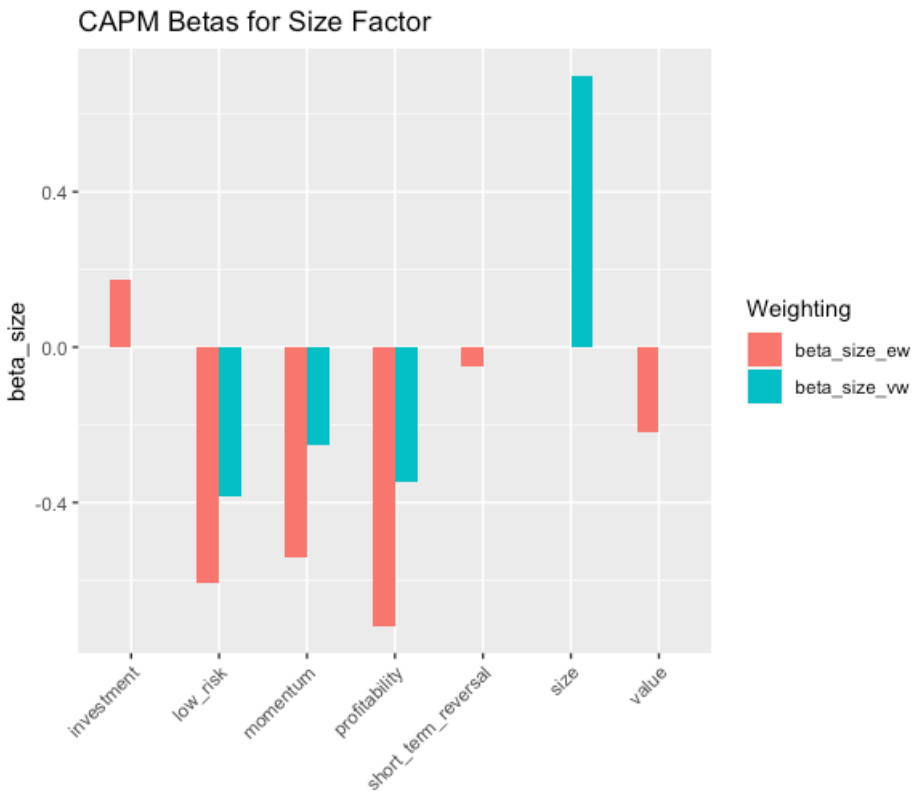
Value weighted:

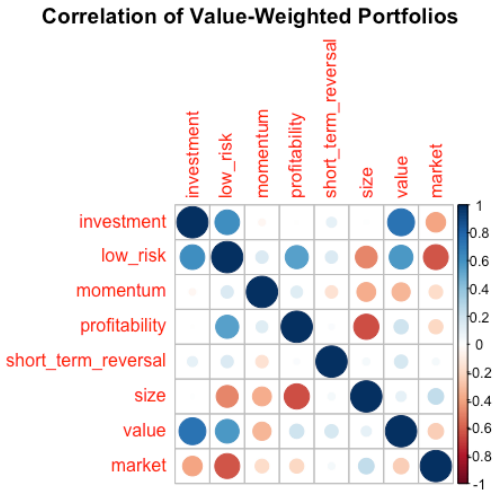
factor	annual_return	annual_sd	sharpe_ratio	alpha	p_value_alpha	beta	p_value_beta
investment	0.0170	0.0675	0.2522	0.0309	0.0000	-0.1767	0.0000
low_risk	0.0113	0.1180	0.0961	0.0504	0.0000	-0.4974	0.0000
momentum	0.0356	0.1062	0.3353	0.0462	0.0003	-0.1348	0.0000
profitability	0.0217	0.0710	0.3058	0.0292	0.0005	-0.0957	0.0000
short_term_reversal	0.0099	0.0410	0.2418	0.0090	0.0701	0.0120	0.2063
size	0.0180	0.0918	0.1966	0.0061	0.5697	0.1521	0.0000
value	0.0270	0.0912	0.2956	0.0387	0.0003	-0.1497	0.0000

We observe that the **equally weighted momentum portfolio** possesses the highest alpha, indicating the highest abnormal return compared to the other portfolios. Furthermore, the alpha is also statistically significant at 5% due to the p-value = 0. This is the case for all equally-weighted portfolios but not for all value-weighted portfolios (short term reversal & size).

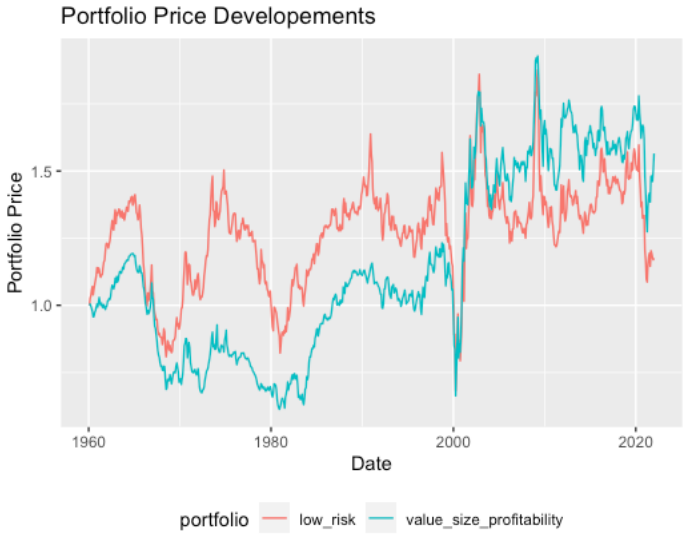
Generally, the equally-weighted portfolios seem to perform better when it comes to alpha (except profitability). This is in line with our findings in exercise 2a), where we saw, that equally-weighted portfolios had higher risk-adjusted returns based on the sharpe

	ratio. Furthermore, it seems interesting that most of the betas are negative, what implies a negative correlation with the market return.	
c) Evaluation of market-neutrality of factors (6 points)	A market neutral portfolio tries to get a beta of 0 while maximizing alpha by implementing a long-short strategy. From the information obtained in a) and b) we observe that the beta is statistically significant different from zero, except for **short term reversal** in **equally weighted** as well as **value weighted** portfolios. For these two portfolios, the null hypothesis for beta cannot be rejected at 5% level. Therefore, we conclude that the short-term reversal seems to be the only true market neutral portfolio. Still most portfolio were able to get a relative low beta between -0.25 and 0.25.	
d) Bar plot of excess returns and interpretation (8 points)	The following graphs shows the excess returns of each portfolio:  We observe that in general, equally weighted portfolios (red bars) outperform their value weighted peers (blue bars) except for the profitability portfolio. The coefficients of the value-weighted short term reversal and size portfolio were not statistically significant at 5% and therefore they are not in the plot. The intuition behind this is that by giving all the assets an equal weight, the portfolio will put more emphasis on assets with smaller values e.g., “growth stocks” thus bear more risk but also higher possible returns. This is also supported the two tables from exercise 2b) where equally weighted portfolios have a higher standard deviation.	

e) Plot of size coefficients and interpretation (10 points)	 By including equal weighted size factor into the regression, we observe that most portfolio have negative size beta except for investment equally weighted portfolio and the value-weighted size portfolio. The latter one is obvious, as there is a high correlation between the equally-weighted and value-weighted returns of the size factor portfolios. The value-weighted investment, short term reversal and value coefficients are not included, as they are not significant at 5% level. Also, the equally-weighted size portfolio is not included, as this would lead to perfect multicollinearity in the OLS model. Generally, we see a correlation with the asset returns and size factor. This suggests a possible omitting bias when not including multiple factors. Consequently, omitting certain covariates will result in an alpha which is not equal to 0. (We see this in 1b)	

f) Correlation plot (4 points)	 If we look at the correlation plot, we observe high absolute values regarding low-risk and value, size and profitability. Additionally, we also see a strong correlation between low risk and investment as well as market. Our thoughts on the hypothesis are that considering multiple factors can lead to a better explainability of the low-risk portfolio return. Additionally, if the low-risk portfolio is composed of other factors, we might be able to additionally mimic the low-risk portfolio by investing into multiple portfolios, thus profit more from diversification.	
g) Rolling window monthly regression. Reporting of average alpha and significance (10 points)	Running a rolling-window monthly regression starting in January 1960 for the returns of the past 5 years yields the following results for regressing the low-risk portfolio on the returns of the value, size, and profitability portfolio: **Mean alpha**: -0.0002 **Annualized mean alpha**: -0.0022 Alpha was **85** times (in 11.42% of the cases) statistically significant at 95% confidence. With a small number of statistically significant alphas, the included factors tend to explain the returns of the low-risk portfolio quite well.	

h) Creation of factor mimicking portfolio. Reporting of performance measures, correlation and price plot
(10 points)
	Using the betas from exercise g) as weights, the portfolio yields the following metrics:
	
	`portfolio annual_return annual_sd`
	`value_size_profitability 0.0127 0.1044`
	`low_risk 0.0116 0.1241`
	
	The correlation between the two portfolios is equal to 0.8395	



Looking at the graph and the correlation of the two return series, the value-size-profitability portfolio (VSP) seems to replicate the returns of the low-risk portfolio quite well.

Looking at the performance metrics (annual return and annual sd) we observe that the VSP portfolio managed to get a higher return and lower standard deviation than the low-risk portfolio. Essentially, it managed to maintain better returns and lower risk by diversifying and combining multiple portfolio strategies.

Furthermore, by looking at the graph we observe that the VSP portfolio initially showed weaker results but was able to capture stronger returns after year 2000 eventually surpassing the low-risk portfolio.