Theory *of* Finance

Solution Sheet for Problem Set 1

Return Calculations, Portfolio Choice and Mean-Variance Frontier

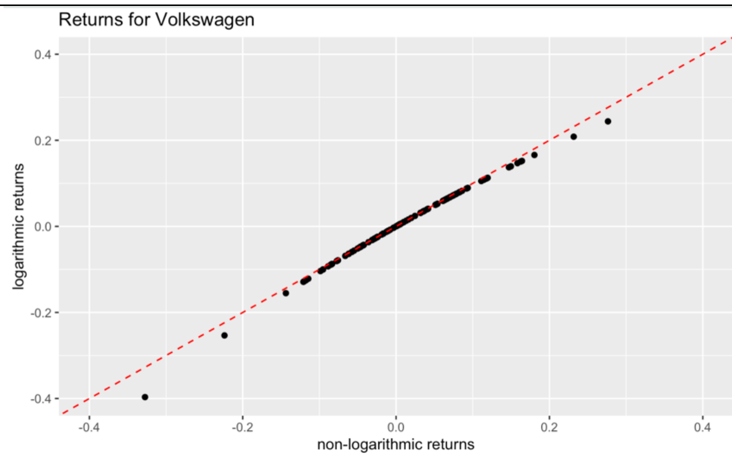
Deadline: 13.10.2022

Solved by: ______________________________

<table>
<tr><th>Task</th><th></th><th>Points Earned</th></tr>
<tr><td>1. Return Calculation
a) Comparison of annualized measures
(6 points)</td><td>

Stock <chr>	mean_return <dbl>	annual_mean_return <dbl>	sd <dbl>	annual_sd <dbl>
BAYER	0.0011	0.0132	0.0740	0.2562
BMW	0.0066	0.0794	0.0727	0.2517
DEUTSCHE.TELEKOM	0.0069	0.0832	0.0622	0.2153
EON	−0.0001	−0.0016	0.0706	0.2447
RWE	0.0044	0.0529	0.0917	0.3176
SAP	0.0103	0.1232	0.0648	0.2245
VOLKSWAGEN	0.0104	0.1251	0.0878	0.3040

</td><td></td></tr>
<tr><td>b) Comparison of logarithmic and non-logarithmic returns
(9 points)</td><td>



As return is closely centered around 0 and is therefore mostly stable, it is reasonable to either use log or non-log return as an approximation for the other. The approximation for larger value is less precise. However, as there are only four datapoints with higher differences, it is still reasonable to use the approximation in this case.

</td><td></td></tr>
<tr><td>c) Annualized measures for equally weighted portfolio
(4 points)</td><td>The annualized mean return and annualized standard deviation of an equally weighted portfolio consisting of the given assets are the following:
E(R)= 0.0679
Var(R) = 0.1735</td><td></td></tr>
</table>

d) Zero-cost portfolio rationale, expectations and performance (8 points)

A zero-cost portfolio is an extreme case where assets are borrowed and sold (short) to buy other assets on the market. There are three rationales for zero-cost portfolios:

1) In this exercise shorting one firm (e.g., BMW) while going long on another firm (e.g., VW) within the same industry at the same time can have a positive effect on the returns especially if VW firm is seen as the winner and BMW as the loser which increases/decreases their stock price. Therefore, we can extract the returns on BMW's decreasing stock price as well as VW's increasing stock price.
2) Additionally, by borrowing and selling one stock to buy another, we can theoretically participate in the market without any initial investment. (Regulations and fees are neglected in this example)
3) Lastly, zero-cost portfolio can also be used to profit from arbitrage possibilities.

However, stock prices might move in different directions then predicted thus resulting in the shorted stock increasing in value and invested stock decreasing in value. Therefore, with the analogy of 1), when BMW increases in value and VW decreases in value, the loses are amplified as we must buy back BMW at a higher rate than initially borrowed and sold. Furthermore, VW has less value thus selling VW will not be enough to cover the short on BWM. Additionally, with short positions, investor can have infinite losses as the price of BMW can theoretically appreciate infinitively.

2. Portfolio Choice

a) Maximum Sharpe ratio and Sharpe ratio evaluation (8 points)

Descriptive statistics (0.1%):

Stock <chr>	excess_return <dbl>	sd <dbl>	sharp_ratio <dbl>
BAYER	9.862123e-05	0.07395085	0.001333605
BMW	5.615102e-03	0.07265041	0.077289342
DEUTSCHE.TELEKOM	5.929585e-03	0.06215833	0.095394858
EON	−1.129471e-03	0.07063704	−0.015989787
RWE	3.411629e-03	0.09168423	0.037210644
SAP	9.263085e-03	0.06481144	0.142923604
VOLKSWAGEN	9.428598e-03	0.08775777	0.107438896

Given a risk-free rate of return of 0.1% and maximization of Sharpe ratio, we would invest into SAP.

Descriptive statistics (1%):

Stock <chr>	excess_return <dbl>	sd <dbl>	sharp_ratio <dbl>
BAYER	−0.0089013788	0.07395085	−0.120368858
BMW	−0.0033848978	0.07265041	−0.046591588
DEUTSCHE.TELEKOM	−0.0030704152	0.06215833	−0.049396683
EON	−0.0101294713	0.07063704	−0.143401692
RWE	−0.0055883710	0.09168423	−0.060952371
SAP	0.0002630851	0.06481144	0.004059238
VOLKSWAGEN	0.0004285984	0.08775777	0.004883879

With a risk-free rate of return of 1%, Volkswagen would now be the asset of choice when maximizing the Sharpe ratio.

This is due to the standard deviation of VW being larger than SAP which scales the risk-free rate less. From the formular below we observe that a higher $Std\ R_i^e$ will decrease the term: $\frac{ER_f}{Std\ R_i^e}$ thus affect the Sharpe ratio less negatively.

$$SR = \frac{E\ R_i^e}{Std\ R_i^e} = \frac{E\ R_i}{Std\ R_i^e} - \frac{ER_f}{Std\ R_i^e}$$

b) Highest return portfolio including transaction costs
(12 points)

transaction_fee	portfolio_1	portfolio_2	portfolio_3
<dbl>	<dbl>	<dbl>	<dbl>
2000	0.0886	0.0967	0.0394
3000	0.0786	0.0767	0.0094

After two years, portfolio 2 (RWE & BMW) yields the highest expected return. Therefore, we would choose to invest into portfolio 2 to maximize the expected return.

c) Highest return portfolio including higher transaction costs
(4 points)

transaction_fee	portfolio_1	portfolio_2	portfolio_3
<dbl>	<dbl>	<dbl>	<dbl>
2000	0.0886	0.0967	0.0394
3000	0.0786	0.0767	0.0094

With transaction costs increasing to CHF 3000, portfolio 1 (RWE) yields the highest return after 2 years. Thus, we would choose to invest into portfolio 1.

d) Risk aversion parameter
(8 points)

$$\mathrm{E}\,U(R_p) = \mathrm{E}\,R_p - \frac{k}{2}\,\mathrm{Var}(R_p),\ \text{where}$$

$$\begin{aligned} R_p &= vR_i + (1-v)R_f \\ &= vR_i^e + R_f. \end{aligned}$$

Given the utility function above, with the weights being 80% in SAP and 20% in the risk-free rate, k = 2.7675 maximizes the expected utility.

3. Mean-Variance Frontier and Portfolio Choice

a) Minimum variance portfolio for BMW & VOLKSWAGEN and RWE & VOLKSWAGEN
(10 points)

The minimum variance portfolio weights, annualized expected return and standard deviation for a portfolio containing **BMW and Volkswagen** is the following:

```
Portfolio expected return:     0.08684403
Portfolio standard deviation:  0.2492141
Portfolio weights:
       BMW VOLKSWAGEN
    0.8324     0.1676
```

The minimum variance portfolio weights, annualized expected return and standard deviation for a portfolio containing **RWE and Volkswagen** is the following:

```
Portfolio expected return:     0.09053187
Portfolio standard deviation:  0.2293591
Portfolio weights:
       RWE VOLKSWAGEN
    0.4759     0.5241
```

From 1a we obtain the following table:

Stock <chr>	mean_return <dbl>	annual_mean_return <dbl>	sd <dbl>	annual_sd <dbl>
BAYER	0.0011	0.0132	0.0740	0.2562
BMW	0.0066	0.0794	0.0727	0.2517
DEUTSCHE.TELEKOM	0.0069	0.0832	0.0622	0.2153
EON	−0.0001	−0.0016	0.0706	0.2447
RWE	0.0044	0.0529	0.0917	0.3176
SAP	0.0103	0.1232	0.0648	0.2245
VOLKSWAGEN	0.0104	0.1251	0.0878	0.3040

1) Comparing the two portfolios to the individual stocks themselves (BMW, RWE and Volkswagen) in 1a) we observe that the two portfolios were able to reduce their standard deviation thus have less risk. Furthermore, we can see that expected returns lies somewhere between the two assets of the portfolio. Therefore, the portfolios are able to retain a good level of profit while minimizing risk.

2) The portfolio containing RWE and Volkswagen retained a better annualized expected return and standard deviation compared to the other portfolio thus benefited more from diversification.

3) The main difference between the two portfolios is that one contains BMW and the other one RWE in addition to Volkswagen. Therefore, this difference caused the portfolios to have different weights, returns and standard deviations.

b) Minimum variance and tangent portfolio weights and evaluation (8 points)

Minimum Variance Portfolio (annualized returns and sd) :

```
Portfolio expected return:     0.06537739
Portfolio standard deviation:  0.165565
Portfolio weights:
 BAYER    BMW    DTE    EON    RWE    SAP     VW
0.0805 0.0960 0.2632 0.2702 0.0091 0.2512 0.0297
```

Tangency Portfolio (annualized returns and sd) :

```
Portfolio expected return:     0.3044734
Portfolio standard deviation:  0.3875548
Portfolio weights:
  BAYER     BMW     DTE     EON     RWE     SAP      VW
-0.8895 -0.4294  0.9307 -1.2285  0.7409  1.2721  0.6036
```

The difference is simply due to the fact that the tangency portfolio maximizes the Sharpe ratio while the minimum variance portfolio minimizes the variance. Therefore, the two portfolios have different optimization problem and thus have different weights to achieve their optimum. This is graphically illustrated in 3d/e.

c) Performance of the two efficient portfolios
(5 points)

Minimum Variance Portfolio (annualized returns and sd):

```
Portfolio expected return:     0.06537739
Portfolio standard deviation:  0.165565
Portfolio weights:
 BAYER    BMW    DTE    EON    RWE    SAP     VW
0.0805 0.0960 0.2632 0.2702 0.0091 0.2512 0.0297
```

Tangency Portfolio (annualized returns and sd):

```
Portfolio expected return:     0.3044734
Portfolio standard deviation:  0.3875548
Portfolio weights:
  BAYER     BMW     DTE     EON     RWE     SAP      VW
-0.8895 -0.4294  0.9307 -1.2285  0.7409  1.2721  0.6036
```

Stock <chr>	mean_return <dbl>	annual_mean_return <dbl>	sd <dbl>	annual_sd <dbl>
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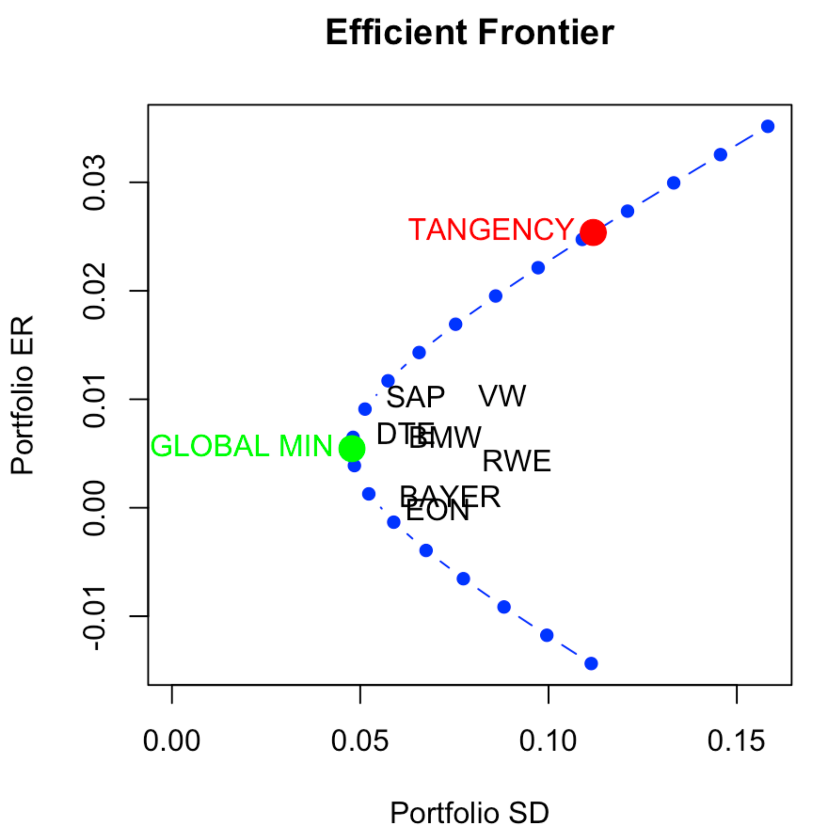
Combining the minimum variance with the tangency portfolio allows the investor to allocate his/her fund according to his/her preferences. Therefore, by shifting the weights the investor can decrease variance or/and increase performance. Furthermore, the combination of tangency and minimum variance portfolio allows the investor to replicate payoff and risk structure of all efficient portfolios. (Two-fund separation theorem).

d) Mean-variance frontier plot (12 points)		

<table>
<tr>
<td>e) Inclusion of capital market line in the plot (6 points)</td>
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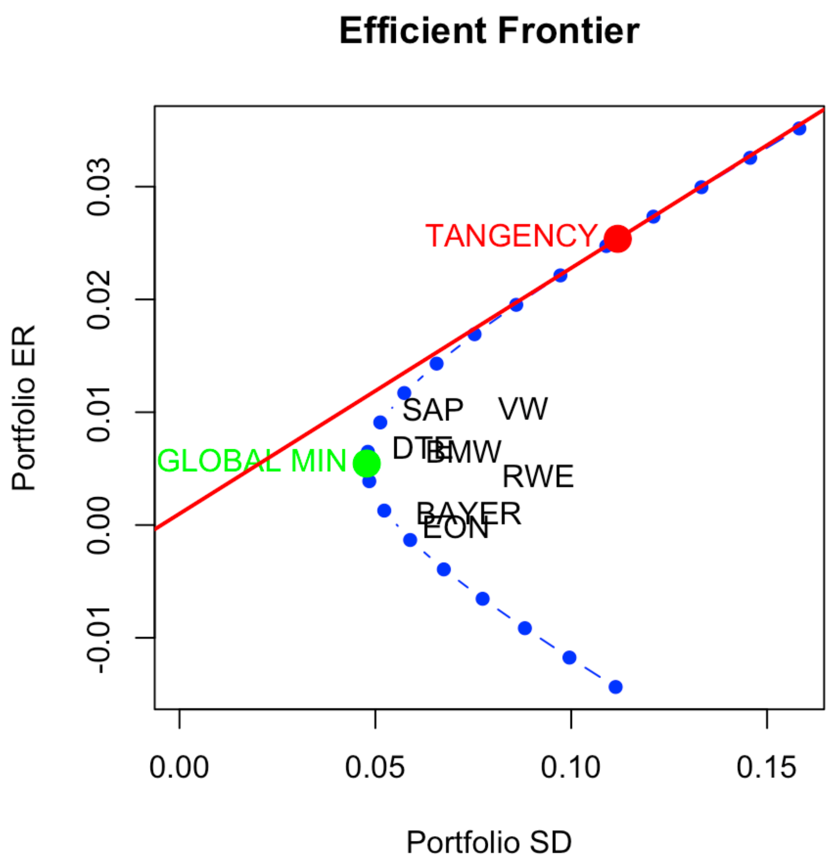

An investor who can invest into the risk-free asset and tangency portfolio will earn at least the same Sharpe ratio as someone who can split his/her money into the seven stocks because the linear combination between the risk-free asset and tangency portfolio allows the investor to graphically move on the CML thus always have a Shape ratio of the tangency portfolio. As the tangency portfolio has the highest achievable Sharpe ratio for the second investor who can only invest into the seven stocks, the first investor will always have at least the same Sharpe ratio.
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