Assignment-7 : The Laplace Transform

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Abstract

Linear Time-Invariant systems are of interest in EE, and tools such as the Laplace transform, Fourier transform aid the analysis of such systems, particularly those governed by Linear constant coefficient differential equations which are a central part of LTI systems. Python provides a signals toolbox which shall aid the process of analysis. The problems being considered deal with rational transforms thus simplifying the analysis.

1 Spring System

Consider a spring system which is governed by the input output relationship :

$$\ddot{x} + 2.25x = f(t) \tag{1}$$

The above equations can be solved in the Laplace domain and X(s) can be inverted to obtain x(t). The above equation-1 in s-domain reads :

$$X(s) * (s^2 + 2.25) = F(s) \tag{2}$$

$$X(s) = \frac{(s + 0.5)}{((s + 0.5)^2 + 2.25)(s^2 + 2.25)} \tag{3}$$

The above equation is inverted using 'sp.impusle' and the plot of x(t) is given in fig. 1. When a

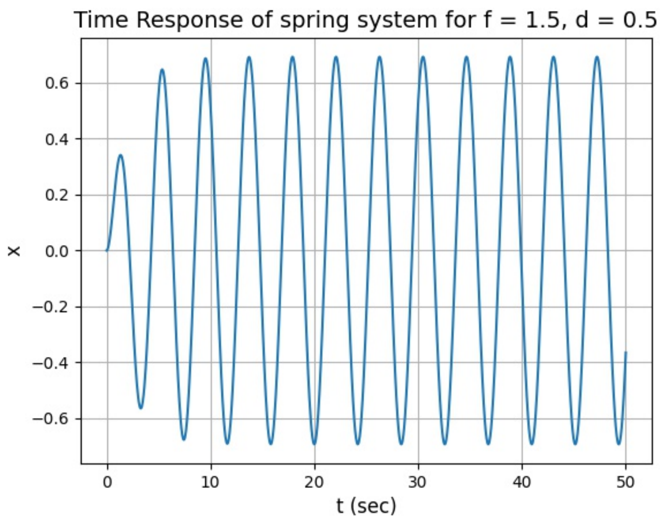

Figure 1: Time domain response for f = 1.5 and d = 0.5

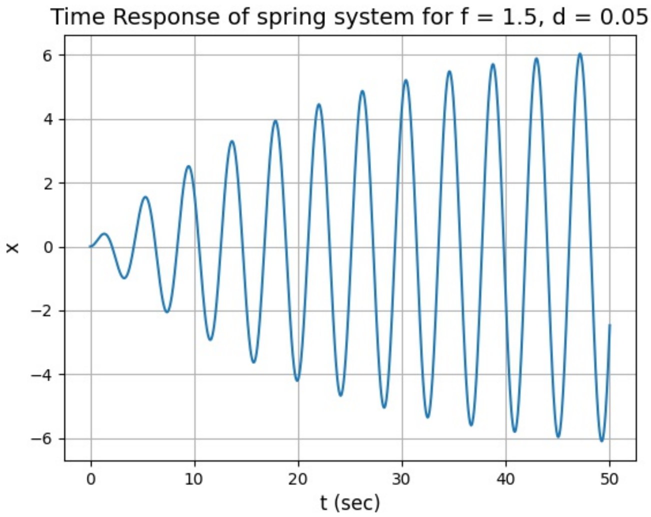

Figure 2: Time domain response for f = 1.5 and d = 0.05

smaller decay is used, the output x(t) has been presented in fig 2. Clearly from both the figures it can be inferred that when decay factor is smaller, the curve takes more time to settle.

By the assumption that given system is LTI, the transfer function of the system is given by :

$$H(s) = \frac{1}{s^2 + 2.25} \tag{4}$$

Now we have the impusle response, for varied frequencies employed in f(t) while keeping the decay rate at 0.05, continuous time convolution is performed with h(t) for each signal, using 'sp.lism'. The set of plots obtained are presented below. Clearly the natural frequency of the impulse response is 1.5. When an input is applied it gives rise to system modes and natural modes in the output, particularly when the input frequency is same as natural frequency of impulse response, both the modes coincide thus giving a phenomena of resonance, as a result of which the amplitude is the greatest when f = 1.5, while the curves on the either side of f = 1.5 behave similarly, for say f = 1.4, 1.6.

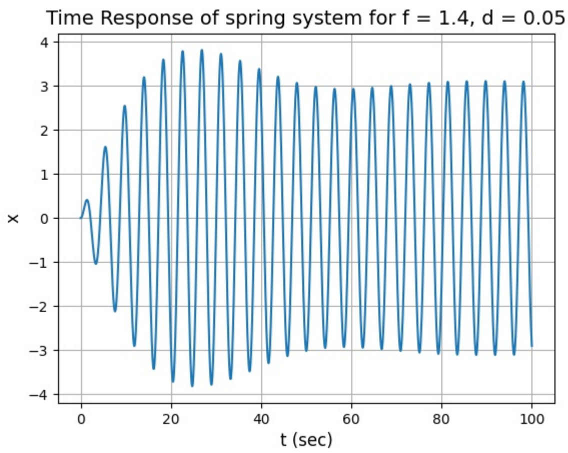

Figure 3: Time response for f = 1.4

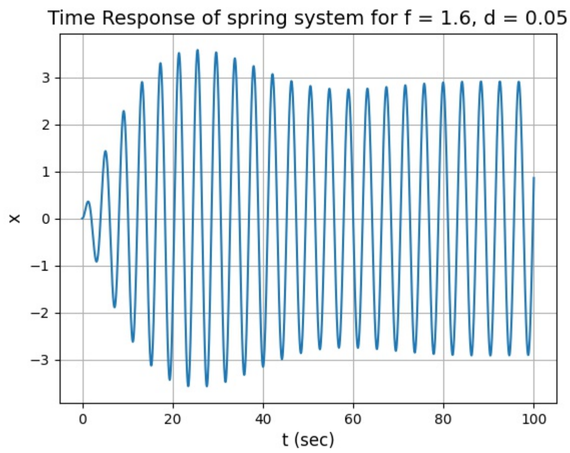

Figure 4: Time response for f = 1.6

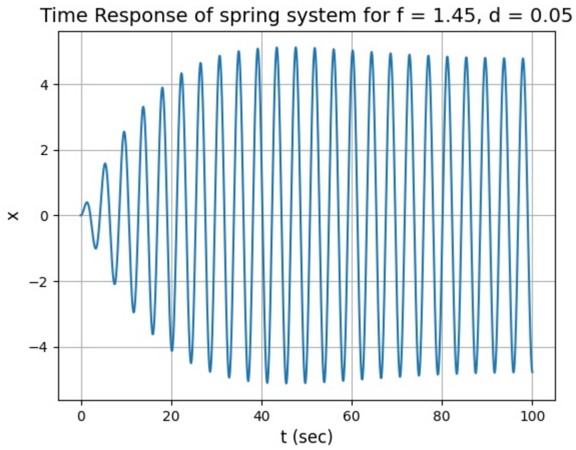

Figure 5: Time response for f = 1.45

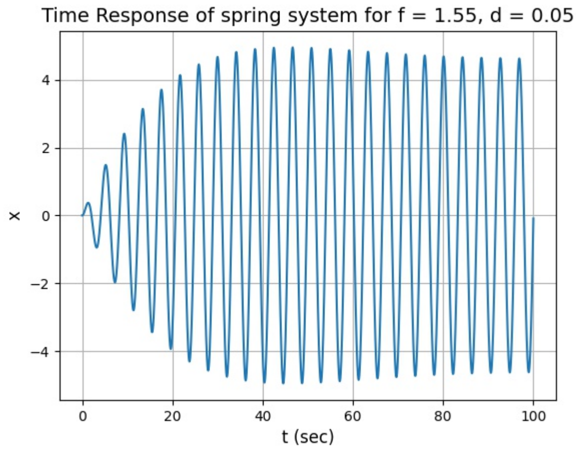

Figure 6: Time response for f = 1.55

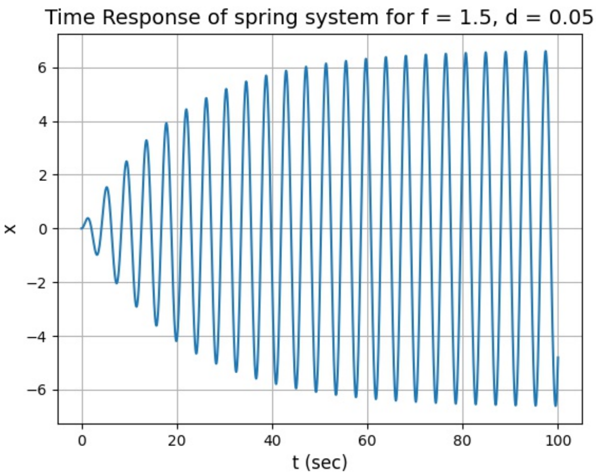

Figure 7: Time response for f = 1.5

2 Coupled spring system

Consider the following set of coupled differential equations with initial conditions as $\dot{x}(0) = 0, \dot{y}(0) = 0, x(0) = 1, y(0) = 0$.

$$\ddot{x} + (x - y) = 0 \tag{5}$$

and

$$\ddot{y} + 2(y - x) = 0 \tag{6}$$

The above equations when translated to Laplace domain and when solved linearly gives decoupled X(s) and Y(s), given by the expressions :

$$X(s) = \frac{s^2 + 2}{s^3 + 3s} \tag{7}$$

$$Y(s) = \frac{2}{s^3 + 3s} \tag{8}$$

The y(t) and x(t) obtained for $0 < t < 20$ have same frequency, as inferred from the plot below.

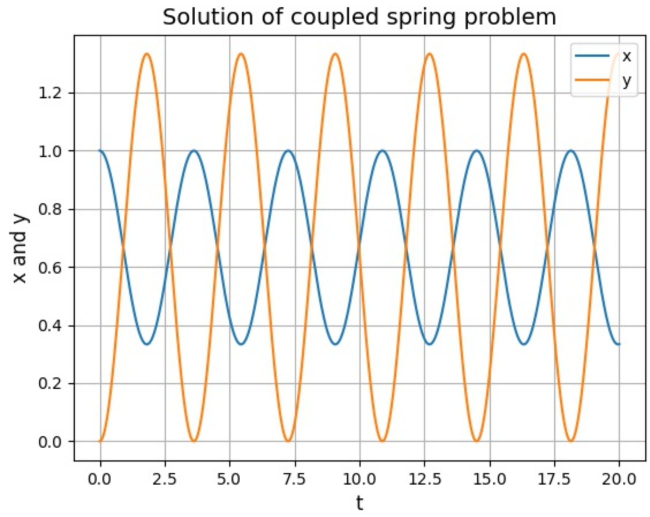

Figure 8: Decoupled time domain solution

3 RLC Circuit

The transfer function of the series RLC circuit is given below. This particular circuit acts as a second order real low pass filter.

$$H(s) = \frac{1}{s^2LC + sRC + 1} \tag{9}$$

The bode magnitude and phase response is plotted using 'H.bode' and the resultant plots obtained are presented.

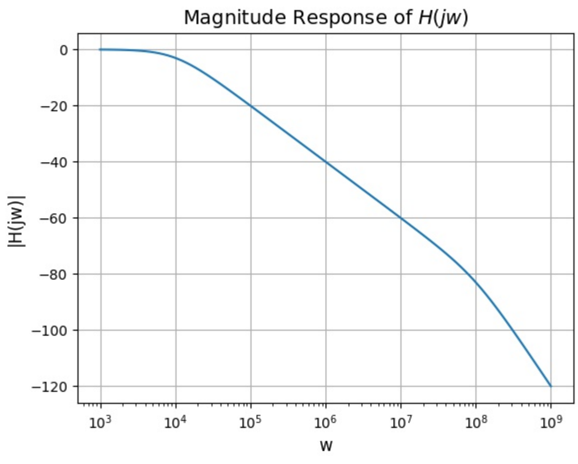

Figure 9: Magnitude response

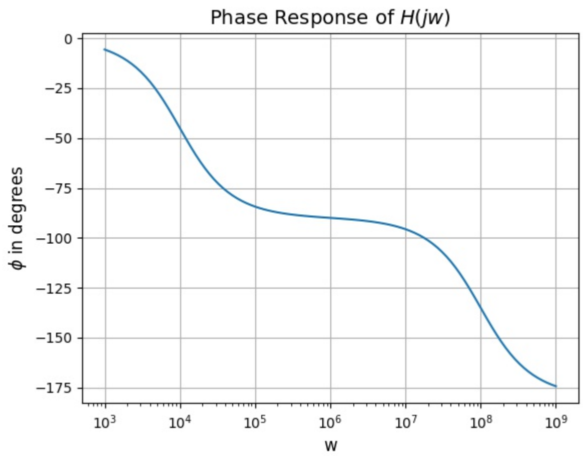

Figure 10: Phase response

As inferred from the plots the magnitude response falls as we move to higher frequencies and thus this arrangement acts as a low pass filter. When the input $V_1(t)$ is applied to the system, the output which is nothing but convolution in time domain with h(t) and multiplication in frequency domain, is obtained using 'sp.lism'. The input $V_1(t)$ is given by :

$$V_i(t) = (cos(10^3t) - cos(10^6t))u(t)$$

The plots presented below of the output are for two cases $0 < t < 10ms$ and $0 < t < 30\mu s$ respectively.

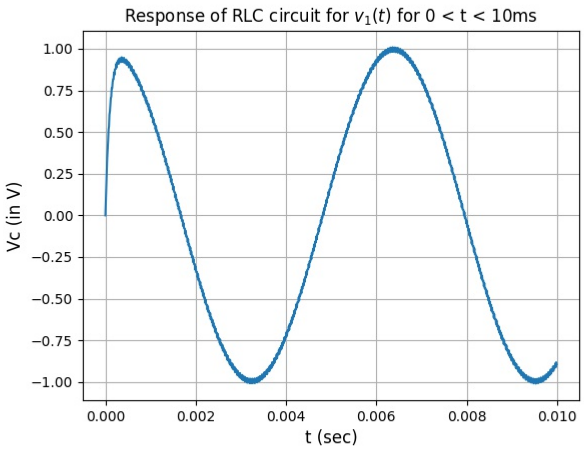

Figure 11: Output for $0 < t < 10ms$

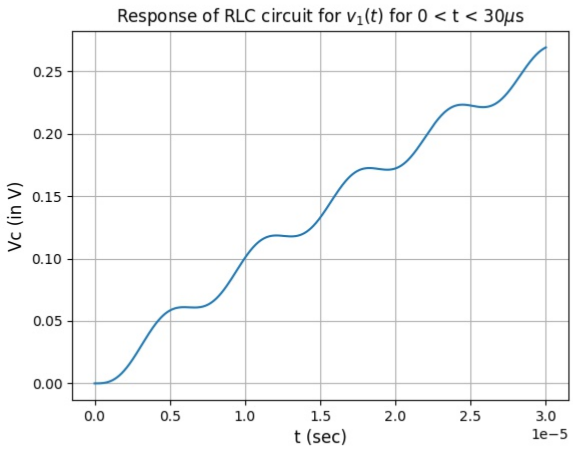

Figure 12: Output for $0 < t < 30\mu s$

The above circuit acts as a low pass filter, as result the gain of component with f = 1000 is almost 1, while the component with f = 10^6 is attenuated, thus making the former as a dominant component, this accounts for the difference in both the graphs, where in the second graph it should have been a rapidly oscillating sinusoid if both the components have been amplified equally, and the graph in ms scale is almost similar to that of the one with only lower frequency component being considered as high frequency term is attenuated.

4 Conclusion

The Laplace transform provides an effective way for circuit analysis, as well as solving differential equations. The python signal toolbox further amplifies the ease with which these systems are being analysed. The LCCDE equations forming a sub-class of LTI systems posses rational transfer functions which are simpler to analyse. The spring system considered highlights on the relation of settling time and decay factor as well as resonance. The Low pass filter modelled by a RLC circuit provides a good estimate of the ideal one. The Laplace domain representation reduces the complexity of the equations and provides a better representation.