

TEL411 – Digital Image Processing

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Assignment 7

Due date: Sunday, November 29, 2020

Haar Wavelet Transform

The Haar filter was proposed in 1909 by Alfréd Haar. It is the simplest possible wavelet transform. Haar filter relies on **averaging** and **differencing** values of a matrix making a bid for producing another sparse or nearly sparse matrix (Note: a sparse matrix is a matrix in which a large portion of its entries are 0 and for that reason it can be stored in an efficient manner, leading to smaller file sizes).

Use the following algorithm to build the Haar Wavelet Transform.

1. 1D Haar Analysis

- a. Consider the following input signal

$$A = [88 \ 88 \ 89 \ 90 \ 92 \ 94 \ 96 \ 97]$$

- b. Group the coefficients

$$A_G = [88 \ 88][89 \ 90][92 \ 94] [96 \ 97]$$

- c. Compute the average of each group
- d. Compute the difference of each value from the average value
- e. Store the outcome of steps (c) and (d) in a new vector

$$A_{haar} = [88 \ 89.5 \ 93 \ 96.5 \ 0 \ -0.5 \ -1 \ -0.5]$$

2. 1D Haar Synthesis

- a. Consider the outcome of Haar transform

$$A_{haar} = [88 \ 89.5 \ 93 \ 96.5 \ 0 \ -0.5 \ -1 \ -0.5]$$

- b. Upsample the average values

$$A_{up} = [88 \ 88 \ 89.5 \ 89.5 \ 93 \ 93 \ 96.5 \ 96.5]$$

- c. Use the differences to reconstruct the initial signal

$$A_{rec} = \left[\begin{array}{cccc} \overbrace{88 \ 88}^{\pm 0} & \overbrace{89.5 \ 89.5}^{\pm 0.5} & \overbrace{93 \ 93}^{\pm 1} & \overbrace{96.5 \ 96.5}^{\pm 0.5} \end{array} \right]$$

- d. Verify that $A_{rec} = A$

$$A = A_{rec} = [88 \ 88 \ 89 \ 90 \ 92 \ 94 \ 96 \ 97]$$

Note 1: The above process could be implemented as a convolution between the input signal and a set of low-pass L and highpass filters H resulting in averaging and differencing the values of each group.

Note 2: For a 2D input signal we apply the lowpass and high-pass filters to each row and the transpose of L and H to each column.

Note 3: For a multiscale decomposition we iterate steps 1.b – 1.e considering as input signal only the average values.

What to turn in

You should turn in only your code with adequate comments for each step of the analysis and synthesis.