MA 511: Linear Algebra and its Applications
HW1: Summer 2023

Due: June 19, 2023

Question 1 Sketch these three lines and decide if the equations are solvable:

$$\begin{aligned} x + 2y &= 2 \\ x - y &= 2 \\ y &= 1 \end{aligned}$$

What happens if all right-hand sides are zero? Is there any nonzero choice of right-hand sides that allows the three lines to intersect at the same point?

Question 2 Use Gauss Elimination (show all elimination steps!) to solve

$$\begin{aligned} x + y + z &= 6 \\ x + 2y + 2z &= 11 \\ 2x + 3y - 4z &= 3. \end{aligned}$$

Question 3 Consider the system of linear equations

$$\begin{aligned} x + 2y + 3z &= 1 \\ 3x + 5y + 4z &= a \\ 2x + 3y + a^2 z &= 0. \end{aligned}$$

(a). For which values of a does the system have no solution?

(b). For which values of a does the system have infinitely many solutions?

(c). For which values of a does the system have exactly one solution?

Question 4

(a). Let $A = \begin{bmatrix} 1 & -3 \\ 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix}$. Compute AB and BA.

(b). Let $A = \begin{bmatrix} 2 & 3 \\ -1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 9 \\ -3 & k \end{bmatrix}$. For what value of k, if any, will make $AB = BA$?

Question 5 Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Suppose all a, b, c and d are nonzero numbers.

(a). Find two conditions on a, b, c and d that guarantee $A^2 = I$.

(b). Using part (a), find 2 examples of A such that $A^2 = I$.

Question 6 Use the Gauss-Jordan method to invert the following matrices.

(a). $A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

(b). $B = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$.

Question 7 Suppose $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 3 & 2 \\ 1 & -3 & 1 \end{bmatrix} = S + K$, where S is a symmetric matrix (i.e., $S^T = S$) and K is a skew symmetric matrix (i.e., $K^T = -K$). Find S and K.

Question 8 Factor A into LU, and write down the upper triangular system $Ux = c$ which appears after elimination, for

$$Ax = \begin{bmatrix} 2 & 3 & 3 \\ 0 & 5 & 7 \\ 6 & 9 & 8 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 5 \end{bmatrix}.$$

Question 9 Find the $PA = LDU$ factorization for

(a). $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 2 & 3 & 4 \end{bmatrix}$

(b). $B = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 1 & 1 \end{bmatrix}$.

Question 10 Find the L and U in the LU factorization for the symmetric matrix

$$A = \begin{bmatrix} a & a & a & a \\ a & b & b & b \\ a & b & c & c \\ a & b & c & d \end{bmatrix}.$$

Find four conditions on a, b, c, d to get $A = LU$ with four pivots.