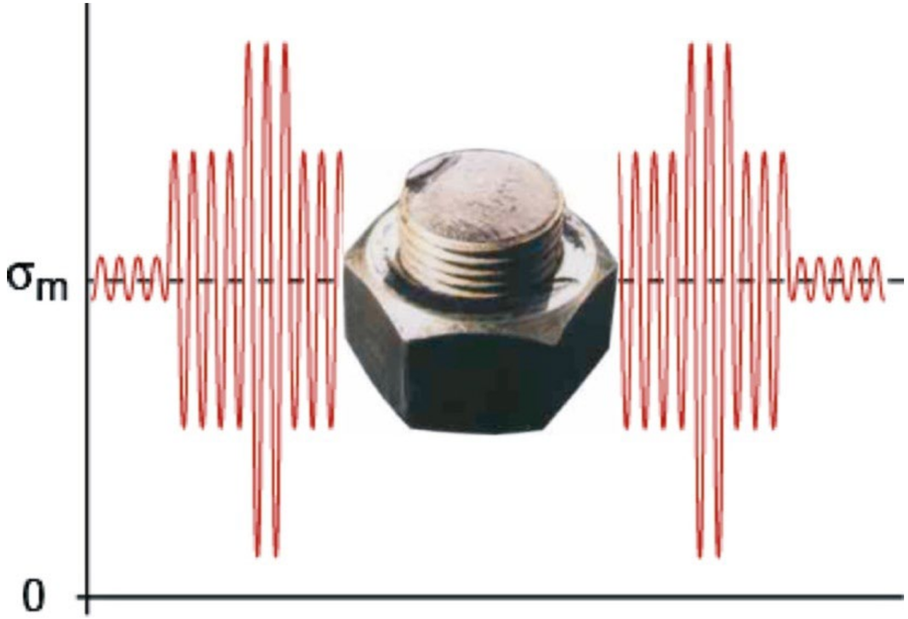
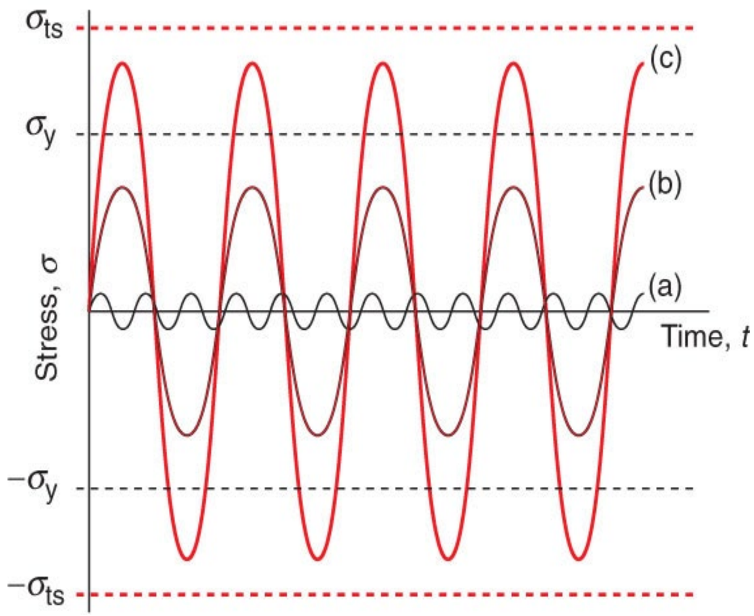
Fatigue of materials

Fatigue is failure due to cyclic loading. A microscopic crack can initiate from a flaw in the material or surface scratch, grow to
a macroscopic size, and finally lead to complete failure of an engineering component. If a relatively large flaw is present, the
fatigue life may be greatly reduced.



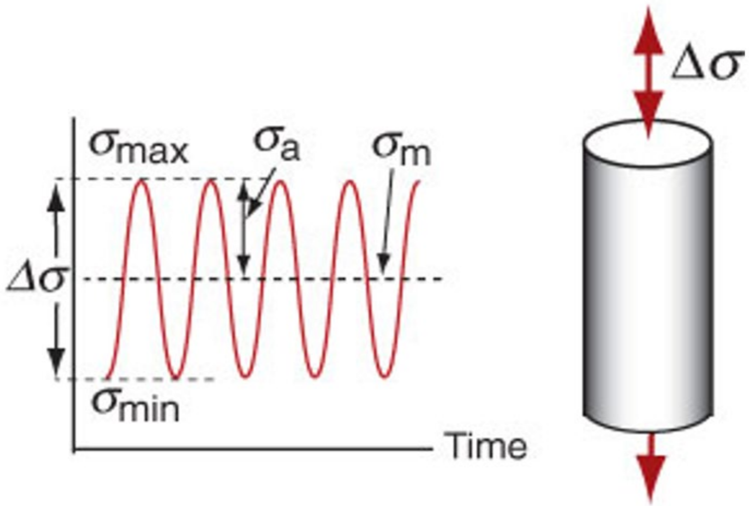
Types of Cyclic Loading



(a) Low amplitude acoustic vibration
(b) High-cycle fatigue: cycling below the yield strength
(c) Low-cycle fatigue: cycling above the yield strength but below the the tensile strength

High-cycle fatigue loading is most significant in engineering terms

Cyclic loading

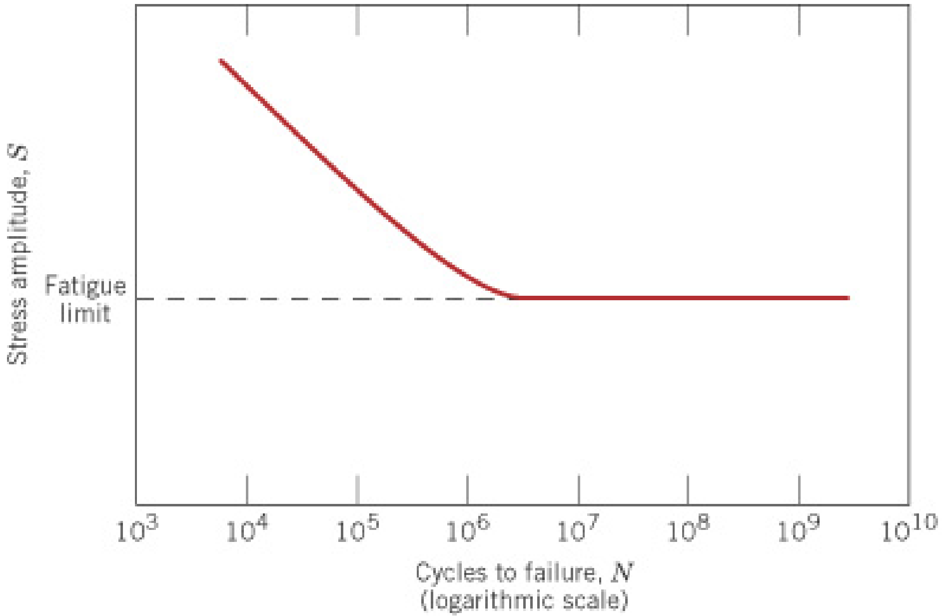


$$\sigma_a = \frac{\Delta\sigma}{2} = \frac{\sigma_{max} - \sigma_{min}}{2}$$

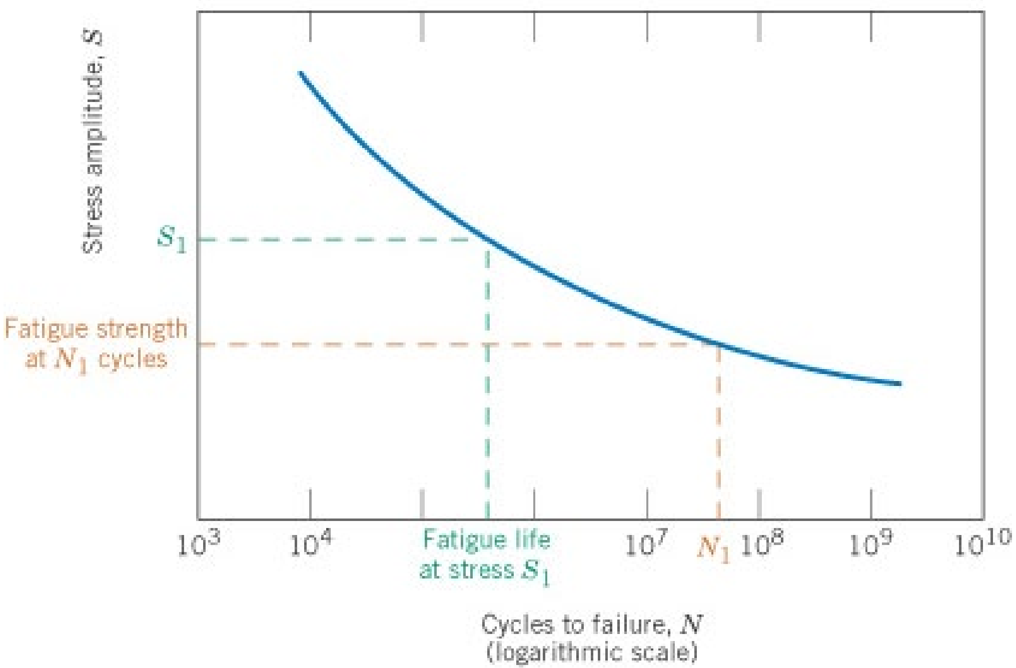
$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

$$R = \frac{\sigma_{min}}{\sigma_{max}}$$

Stress-life curves from fatigue tests



Fatigue limit behavior, as in some steels

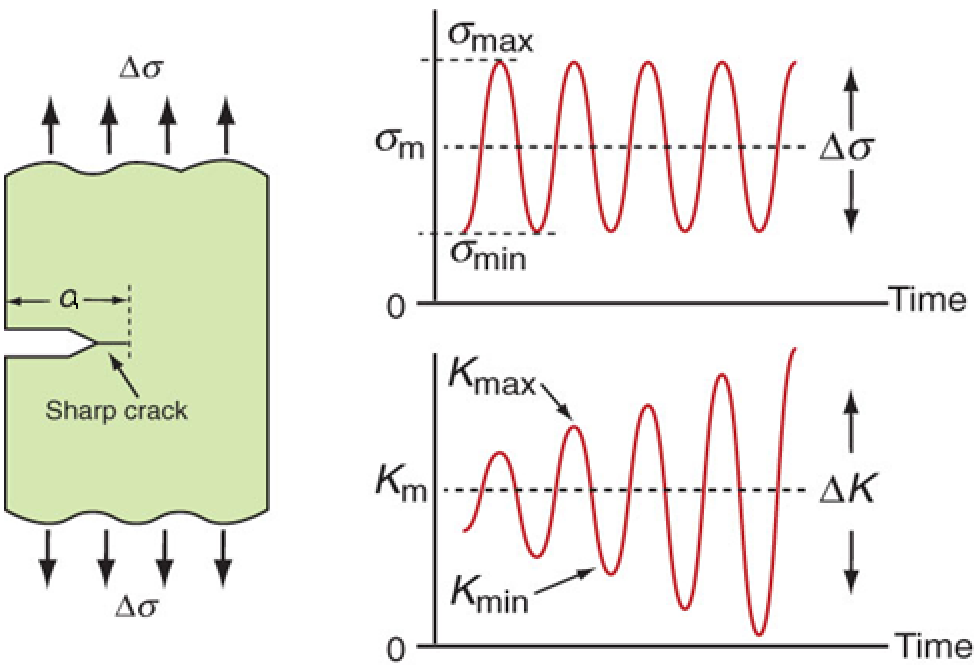


No fatigue limit, as in aluminum alloys

Fatigue Loading of Cracked Components

$$K = Y\sigma\sqrt{\pi a}$$

Fatigue crack growth is studied by cyclically loading specimens containing a sharp crack



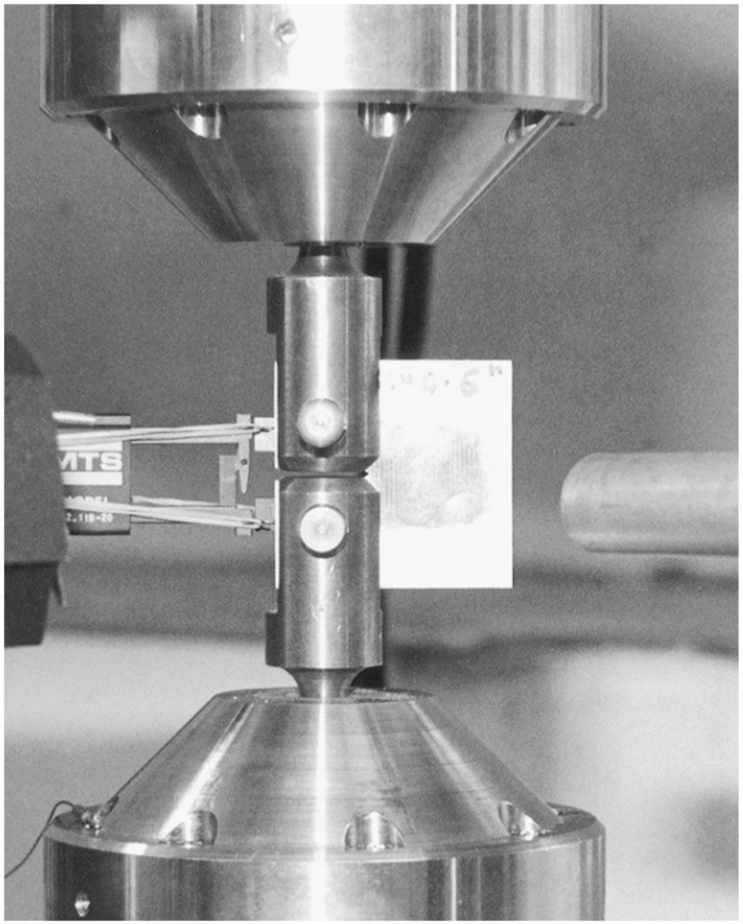
Cyclic stress intensity range

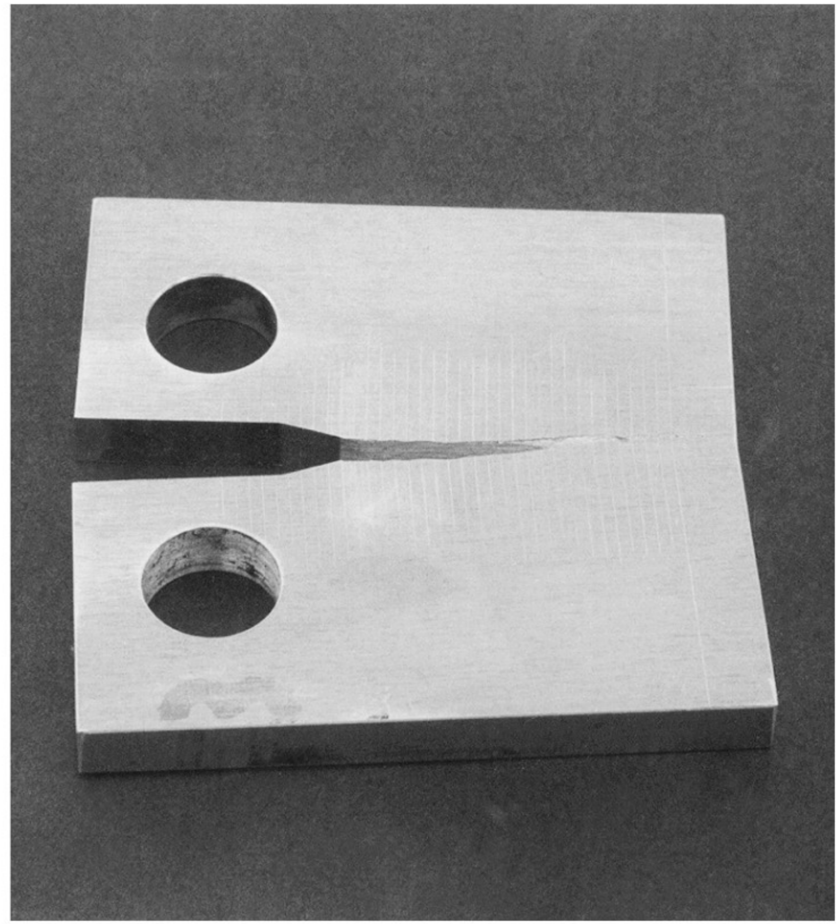
$$\Delta K = K_{max} - K_{min} = \Delta\sigma\sqrt{\pi a}$$

The range ΔK increases with time under constant cyclic stress because the crack grows in length

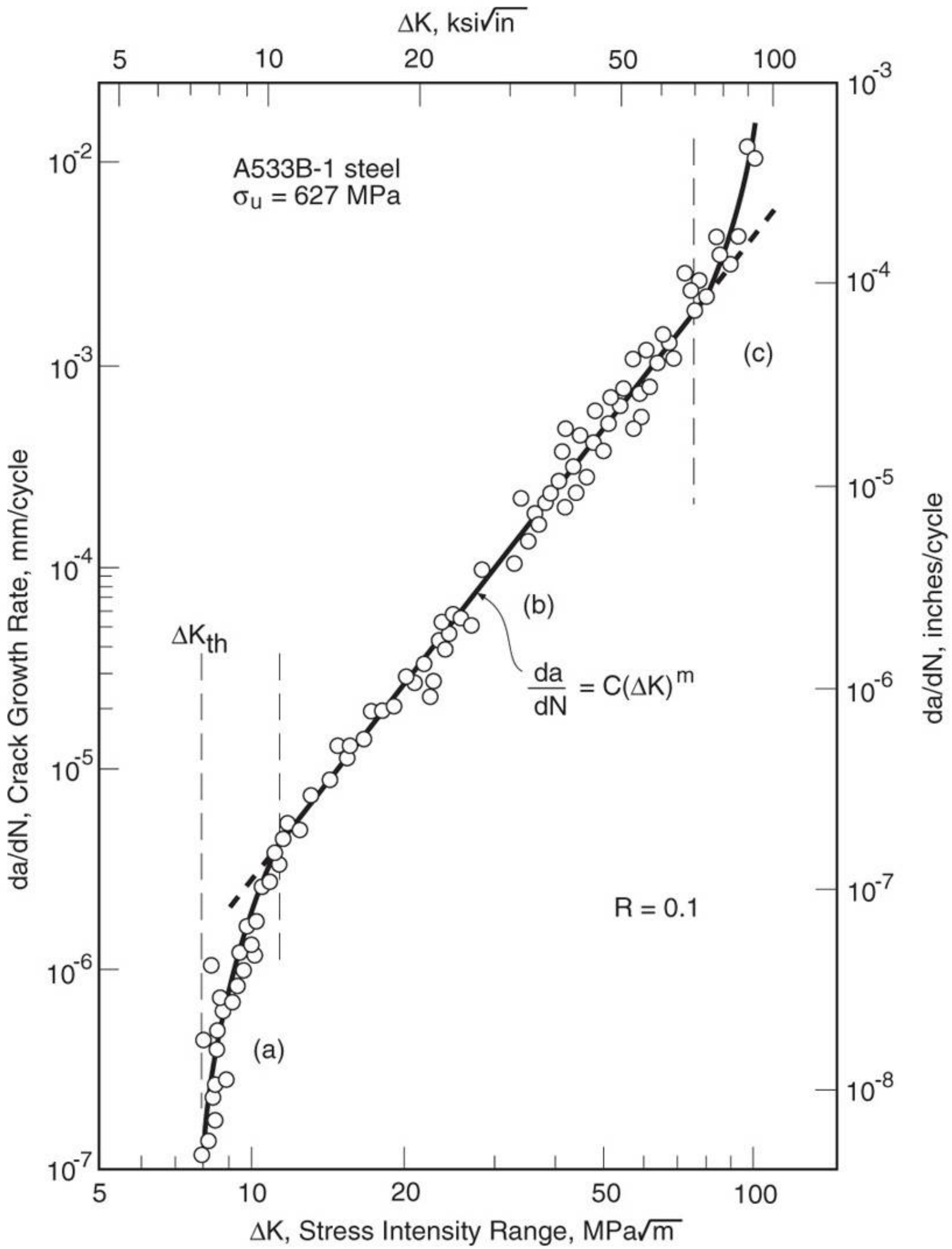
Fatigue crack growth rate test

Cyclic loading is applied to a material sample so as to grow a crack, with the increasing length of the crack being measured.





From: *Mechanical Behavior of Materials: Engineering Methods for Deformation, Fracture, and Fatigue*, 3rd edition, by Norman E. Dowling. ISBN 0-13-186312-6.



Crack growth behavior

When measured over a wide range, crack growth rates *da/dN* become very slow at low ΔK, and at high ΔK transition to unstable behavior due to the beginning of brittle fracture or gross yielding. A power relationship applies except near either extreme.

$$\frac{da}{dN} = C(\Delta K)^m$$

$m \approx 3$ for metals

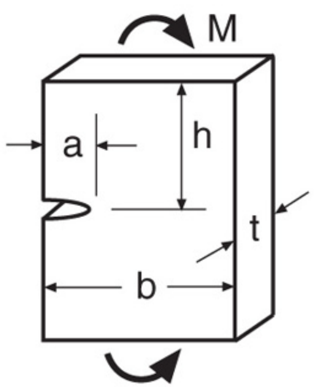
From: *Mechanical Behavior of Materials: Engineering Methods for Deformation, Fracture, and Fatigue*, 3rd edition, by Norman E. Dowling. ISBN 0-13-186312-6.

Example Problem: A polystyrene ruler is loaded as a cantilever with a cyclic force that varies from 0 to 10 N.

(a) What depth of transverse crack is needed at the base of the cantilever for fast fracture to occur when the end force is 10 N?

(b) If the ruler has an initial transverse scratch of depth 0.1 mm, how many cycles of the force will it take before the ruler breaks?

For this problem assume n = 4 and C = 5 x 10^{-6} when $\Delta\sigma$ is in MPa.



Single-edge-cracked plate in bending

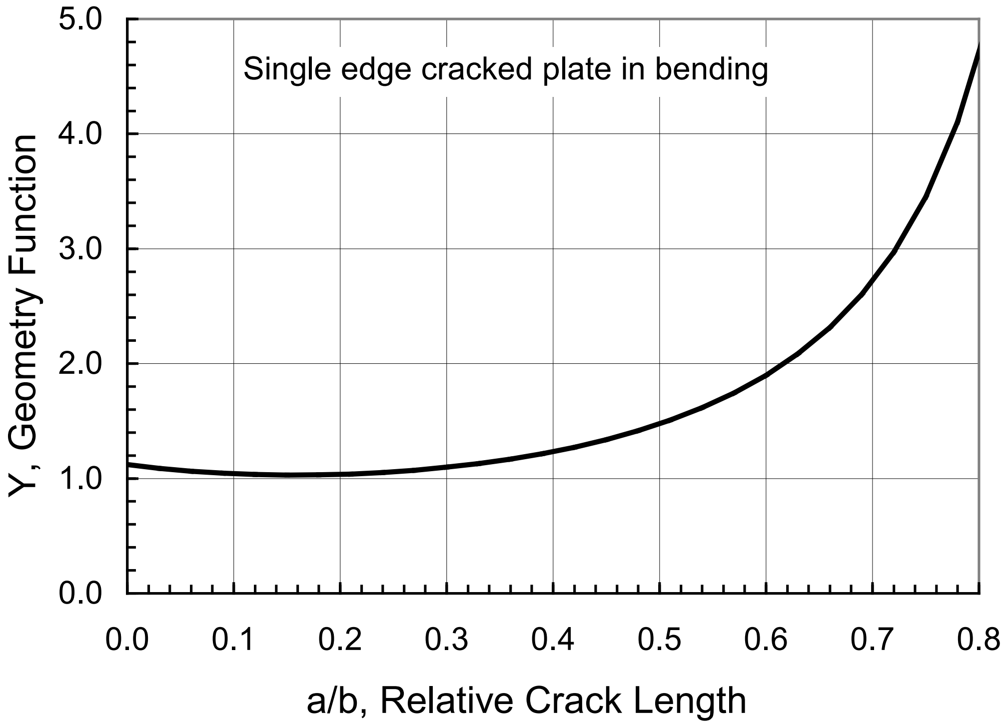
$$K = Y\sigma\sqrt{\pi a}$$

$$\boxed{\sigma = \frac{6M}{b^2 t}}$$

$$Y \approx 1.12 \quad (\text{within } 10\%,\ a/b \le 0.4)$$

$$\text{For any } \beta = \frac{\pi a}{2b} \ \text{(radians)}:$$

$$Y = \sqrt{\frac{1}{\beta}\tan\beta}\left(\frac{0.923 + 0.199(1-\sin\beta)^4}{\cos\beta}\right)$$



From: Tada, H., P. C. Paris, and G. R. Irwin. 2000. *The Stress Analysis of Cracks Handbook*, 3rd ed., ASME Press, American Society of Mechanical Engineers, New York, NY.

Example Problem: A polystyrene ruler is loaded as a cantilever with a cyclic force that varies from 0 to 10 N.

(a) What depth of transverse crack is needed at the base of the cantilever for fast fracture to occur when the end force is 10 N?

$$K_{\mathrm{Ic}} = 10^6 \ \mathrm{Pa} \sqrt{m}$$
$$Y = 1.1$$

$$\sigma = \frac{6\,F\,L}{w\,t^2} = 28.4 \ \mathrm{MPa}$$

$$a_{\mathrm{crit}} = \frac{K_{\mathrm{Ic}}^2}{\pi\,Y^2\,\sigma^2} = 0.33 \ \mathrm{mm}$$

$$\sigma = \frac{6\,F\,L}{w\,t^2} = \frac{6 \times 10 \times 0.25}{0.025 \times 0.0046^2} = 28.4 \ \mathrm{MPa}$$

(b) If the ruler has an initial transverse scratch of depth 0.1 mm, how many cycles of the force will it take before the ruler breaks?

For this problem assume n = 4 and C = 5 x 10^{-6} when $\Delta\sigma$ is in MPa.

$$\frac{da}{dN} = C\,\Delta K^n = C\left(Y\,\Delta\sigma\,\sqrt{\pi a}\right)^n$$

$$\Delta\sigma = \sigma_{\max} - \sigma_{\min} = \sigma_{\max}$$

$$\int_0^{N_f} dN = \frac{1}{C\,Y^n\,\sigma_{\max}^n\,\pi^{n/2}} \int_{a_i}^{a_{\text{crit}}} a^{-(n/2)}\,da$$

$$N_f = \frac{1}{C\,Y^n\,\sigma_{\max}^n\,\pi^{n/2}\left(1-\frac{n}{2}\right)}\left(a_{\text{crit}}^{1-n/2} - a_i^{1-n/2}\right)$$

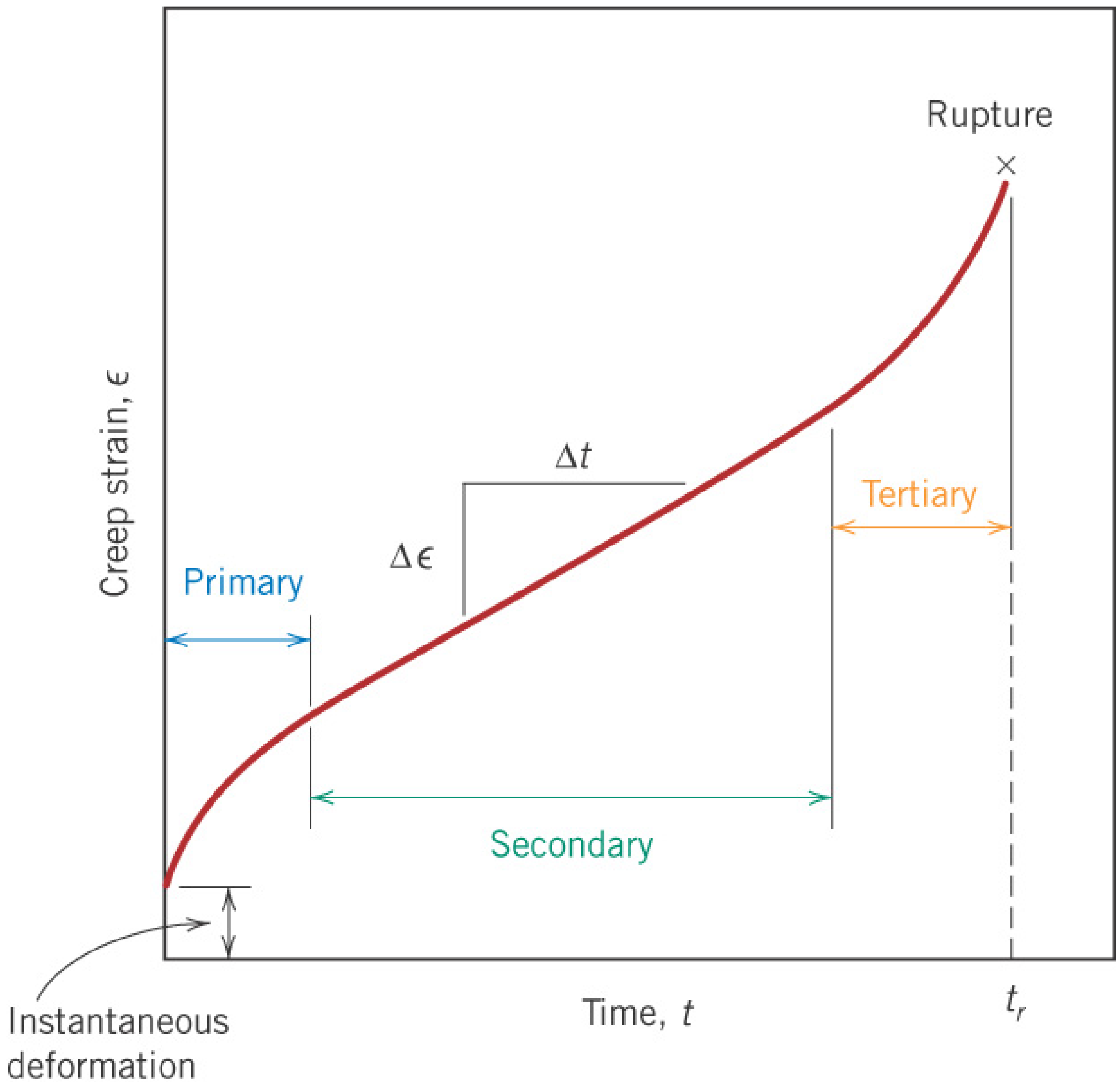
$$= 330,000 \text{ cycles}$$

Creep

Creep is deformation that proceeds gradually with time. Such behavior is more prevalent at higher temperatures. In crystalline materials (metals and ceramics) it becomes significant above 0.3 to 0.6T_m, depending on the material, and it occurs in polymers at room temperature.

Creep rupture occurs when a material gradual tears apart as a result of excessive creep deformation.

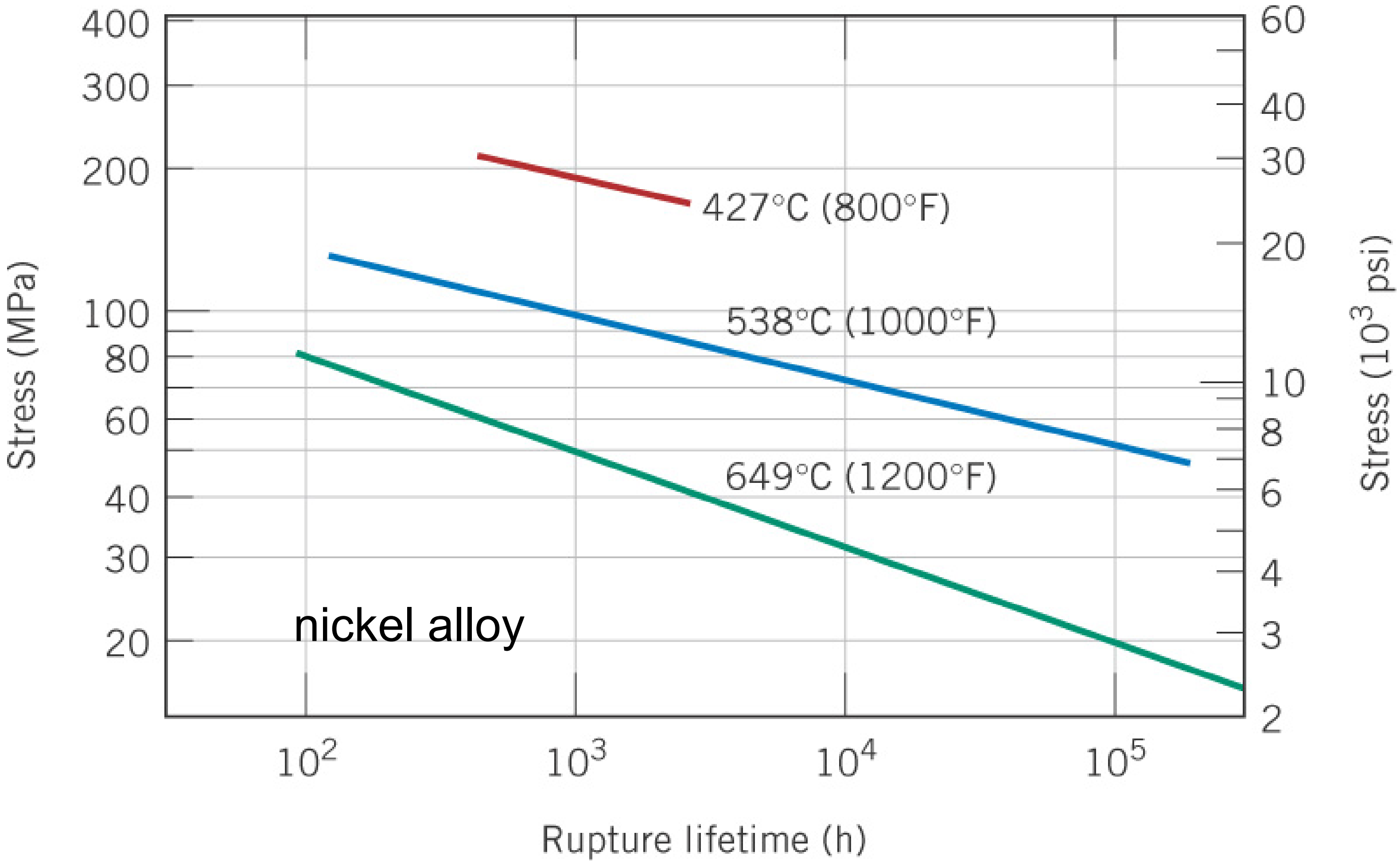
Stages of creep

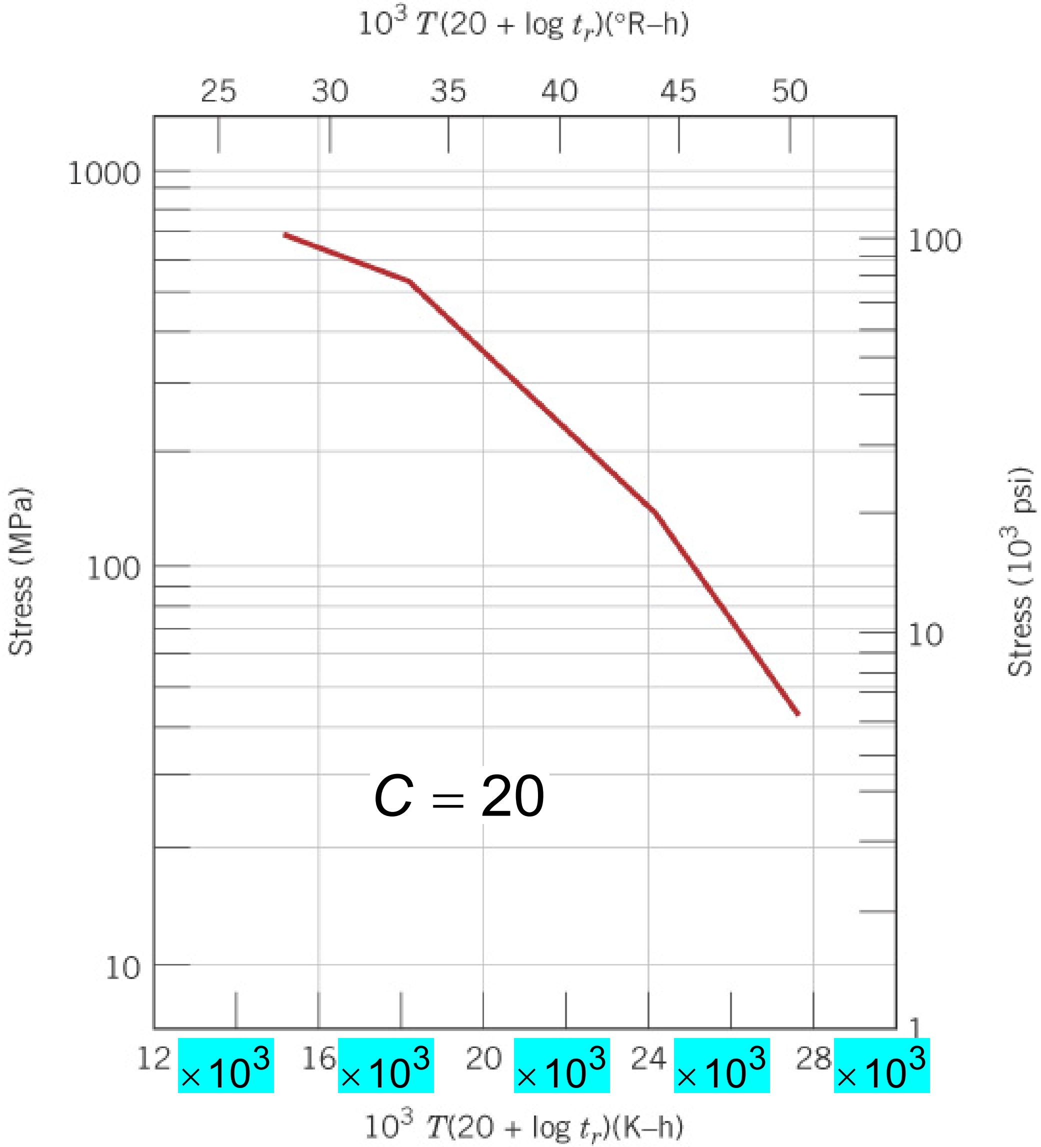


In a creep test at a given temperature, stress is held constant and the increase in strain is measured.

$$\dot{\varepsilon}_s = \frac{\Delta\varepsilon}{\Delta t}$$

Stress vs. rupture life curves





$$P_{LM} = T(\log t_r + C)\,, \quad \mathrm{K} \cdot \log(\mathrm{h})$$

Larsen-Miller plot

From tests done at various stresses σ and temperatures T, each giving a rupture time t_r , plot σ versus P_{LM}, where:

$$P_{LM} = T(\log t_r + C)$$

t_r in **hours**, σ in **MPa**,
T in kelvin, K = °C + 273

Horizontal scale:
P_{LM} = 12,000, 16,000, etc.

A Larsen-Miller parameter plot gives stress as a function of the quantity P_{LM} that combines temperature and rupture time into one variable. Given such a plot, if any two of t_r, σ, and T are known, the third one can be calculated.

$$P_{LM} = T(\log t_r + C)$$

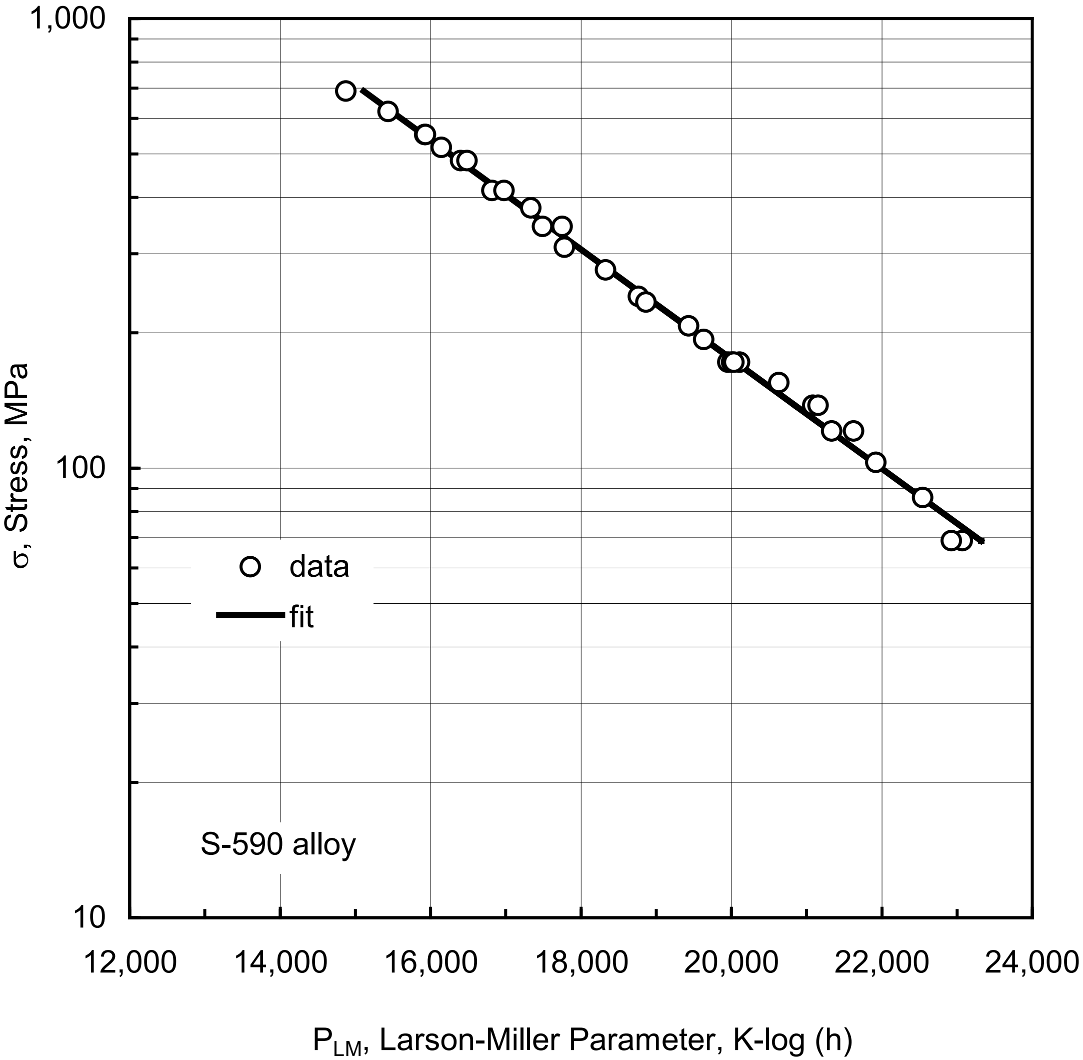
t_r in hours, σ in MPa, T in Kelvin, K = °C + 273

Rather than reading a graph, it is convenient to fit the data to a polynomial as follows

$$P_{LM} = b_0 + b_1 x + b_2 x^2 + b_3 x^3 , \qquad x = \log \sigma$$

$$\log t_r = \frac{P_{LM}}{T} - C$$

Example fitted data

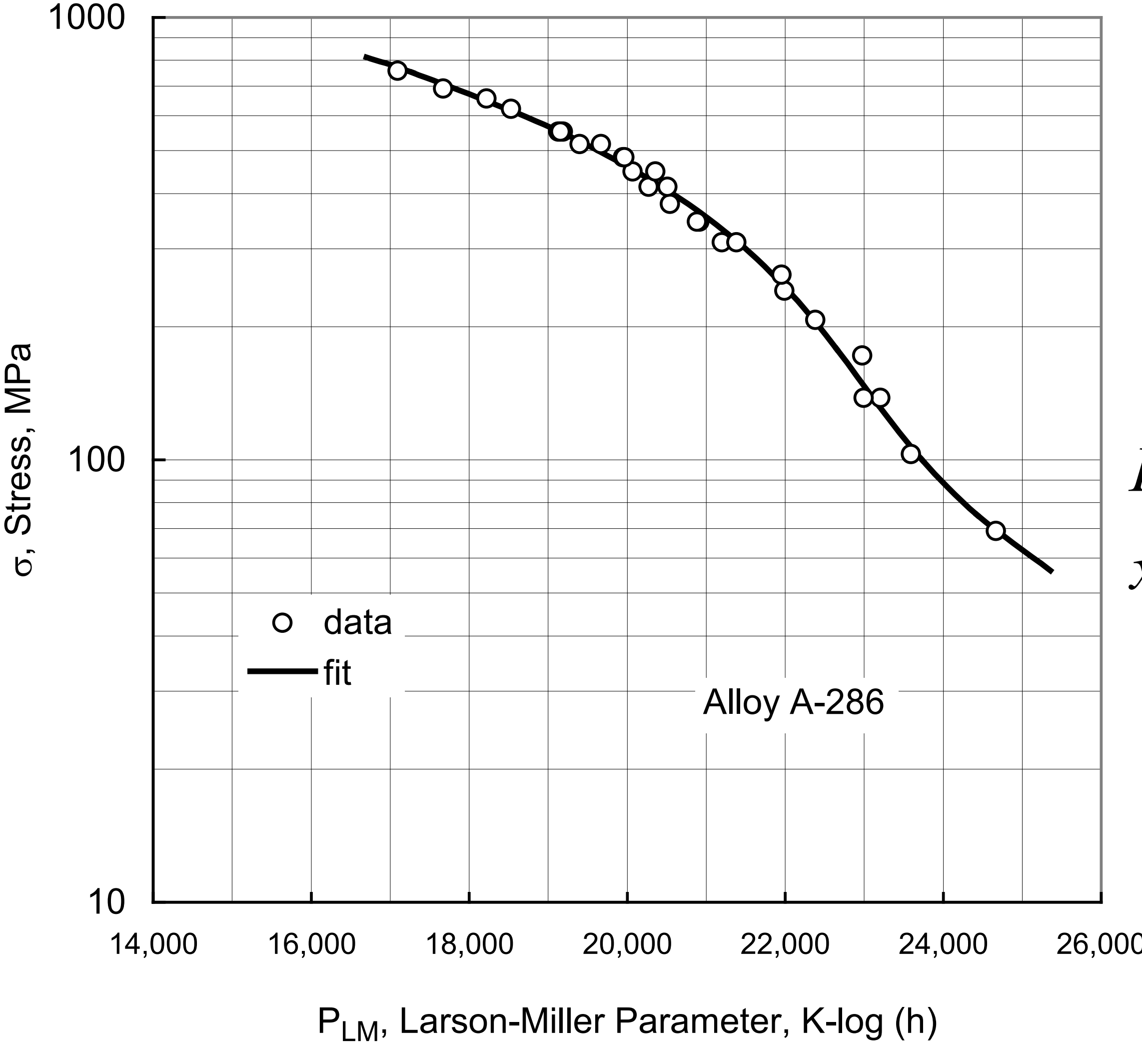


$$P_{LM} = T(\log t_r + C)$$

$$P_{LM} = b_0 + b_1 x + b_2 x^2 + b_3 x^3$$
$$x = \log \sigma$$

C, log (h)	b_0	b_1	b_2	b_3
17	38,405	–8206	0	0

Another example



$$P_{LM} = T(\log t_r + C)$$

$$P_{LM} = b_0 + b_1 x + b_2 x^2 + b_3 x^3$$

$$x = \log \sigma$$

C, log (h)	b_0	b_1	b_2	b_3
20	116,400	–120,500	53,460	–8,188

Design Example 9.2

Alloy S-590 is subjected to a stress of 140 MPa at 800°C. Estimate the time to creep rupture.

Solution: Use the data fit just given to calculate the P_{LM} value for the given stress, and from this and the given temperature, calculate the rupture time.

C, log (h)	b_0	b_1	b_2	b_3
17	38,405	–8206	0	0

$$P_{LM} = b_0 + b_1 x + b_2 x^2 + b_3 x^3 , \qquad x = \log\sigma$$

$$P_{LM} = 38{,}405 - 8206 \log(140) = 20{,}794$$

$$P_{LM} = T(\log t_r + C)$$

$$\log t_r = \frac{P_{LM}}{T} - C = \frac{20{,}794}{800 + 273} - 17 = 2.379$$

$$t_r = 10^{2.379} = 239\,\text{hours} = 10.0\,\text{days} \qquad \triangleleft$$