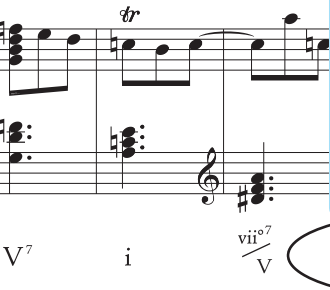
Set Theory



ROMAN NUMERALS WORK FOR CHORDS BUILT FROM *THIRDS.* BUT WHAT IF WE WANT TO CATEGORIZE *EVERY POSSIBLE COMBINATION OF NOTES?*

ROMAN NUMERALS *CAN'T KEEP UP!*

ROMAN NUMERAL ANALYSIS IS *GREAT* FOR *TONAL MUSIC,* BUT IT DOESN'T HELP MUCH WITH STUFF LIKE *ATONAL SERIALISM.*

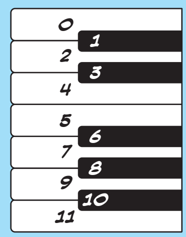
ONE OF THE MOST BASIC CHARACTERISTICS OF ANY CHORD IS HOW *CONSONANT* OR *DISSONANT* IT IS... SOMETHING THAT DEPENDS ENTIRELY ON WHICH *INTERVALS* ARE PRESENT IN THAT CHORD!

THE GOOD NEWS IS THAT *SET THEORY* DOES EXACTLY THAT! THE *BAD NEWS:* SET THEORY IS **MATH!**

PITCH CLASS SET

THE FIRST STEP TO ANALYZE A CHORD USING SET THEORY IS TO THINK ABOUT THE *PITCHES* IT CONTAINS. THIS IS *MATH,* SO INSTEAD OF USING LETTER NAMES WE'LL USE *NUMBERS...* WHERE *C* IS ALWAYS *ZERO.*

TAKE THOSE NUMBERS, REMOVE ANY *DUPLICATES,* AND LIST THEM IN *BRACKETS* LIKE THIS: *[1,2,3].*



= [1,2,8]

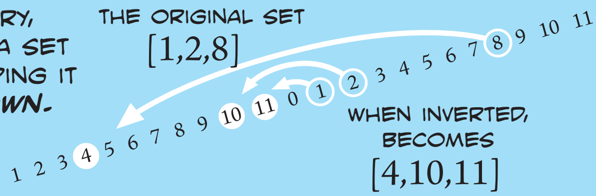
IN THIS CHORD, *G SHARP* AND *A FLAT* ARE CONSIDERED THE SAME: *ENHARMONICS* AND *OCTAVES* DON'T MATTER!

INVERSION

IN SET THEORY, *INVERTING* A SET MEANS FLIPPING IT *UPSIDE-DOWN.*

THE ORIGINAL SET

[1,2,8]



WHEN INVERTED, BECOMES

[4,10,11]

WE CAN DO THIS WITH *MATH* BY TAKING ALL *NON-ZERO* NUMBERS AND *SUBTRACTING THEM FROM 12.*

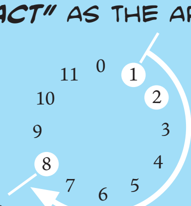
	12	12	12
ORIGINAL:	[− 1 ,	− 2 ,	− 8]
INVERSION:	[11 ,	10 ,	4]

NORMAL FORM

THE *NORMAL FORM* OF A SET IS THE *MOST COMPACT ORDERING* OF THE SET. WE DEFINE *"MOST COMPACT"* AS THE ARRANGEMENT WITH THE *SMALLEST INTERVALS!*

IT'S EASIEST TO DO THIS BY THINKING OF THE PITCHES IN A *CIRCLE* AND MEASURING THE *DISTANCE AROUND!*

JUST MAKE SURE TO ALWAYS MEASURE GOING *CLOCKWISE.*



[1,2,8]: 1 2 3 4 5 6 7 8

[2,8,1]: 2 3 4 5 6 7 8 9 10 11 0 1

[8,1,2]: 8 9 10 11 0 1 2 ⟵ *NORMAL FORM!*

PRIME FORM

TO FIND A SET'S *PRIME FORM,* FIND THE *MOST COMPACT* OF A SET'S *NORMAL FORM* AND THE NORMAL FORM OF *ITS INVERSION.* THEN *TRANSPOSE* THAT SET SO IT STARTS ON *ZERO!*

NORMAL FORM: [8,1,2]: 8 9 10 11 0 1 2

NORMAL FORM OF INVERSION: [10,11,4]: 10 11 0 1 2 3 4

THESE SETS SPAN THE *SAME DISTANCE...* SO TO DECIDE WHICH IS *MOST COMPACT,* WE COMPARE THE *NEXT LARGEST INTERVAL* IN EACH SET!

LASTLY, WE *TRANSPOSE IT* SO IT STARTS ON *ZERO:* 0 1 2 3 4 5 6

SO THE *PRIME FORM* OF [1,2,8] IS $\boxed{[0,1,6]}$*!*

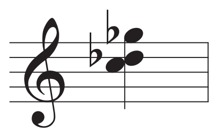
SO *SET THEORY* IS TELLING US THAT THESE TWO SETS HAVE SOMETHING *IMPORTANT* IN COMMON. *WHAT IS IT?*

LET'S TALLY UP ALL THE *INTERVALS* IN OUR ORIGINAL SET. (AND *INVERT* ANY INTERVALS LARGER THAN A *TRITONE* AND *SIMPLIFY* ANY *ENHARMONICS!*)

[1,2,8]

D–A♭	d5	TT
D–C♯	M7	m2
D–G♯	A4	TT
A♭–C♯	A3	P4
A♭–G♯	A7	P1
C♯–G♯	P5	P4

NOW LET'S DO THE SAME THING FOR THE SET IN ITS *PRIME FORM!*



[0,1,6]

C–D♭	m2	m2
C–G♭	d5	TT
D♭–G♭	P4	P4

PRIME FORM IS A WAY TO DESCRIBE *ANY SET* BY ITS *BASIC INTERVALS!*