Linear Combinations of Two Signals

Wednesday, 15 September 2021 12:04 pm

A signal $g(t)$ that is a linear combination of two periodic signals $x_1(t)$ with fundamental period T_1 and $x_2(t)$ with fundamental period T_2

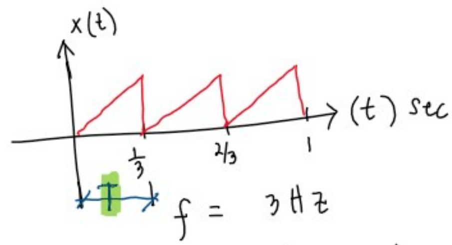
$$g(t) = a x_1(t) + b x_2(t)$$

is periodic IFF

$$\frac{T_1}{T_2} = \frac{m}{n} = \text{rational number}$$

The fundamental period of $g(t)$ is given by $nT_1 = mT_2$ provided that the values of m and n are chosen such that the greatest common divisor (gcd) between m and n is 1.

Example: Determine if the ff signals are periodic. If yes, determine the fundamental period



$f = 3\text{ Hz}$

$T = \frac{1}{f} = \frac{1}{3}$

a) $g_1(t) = 3\sin(\underset{\omega}{4\pi t}) + 7\cos(\underset{\omega}{3\pi t})$

b) $g_2(t) = 3\sin(4\pi t) + 7\cos(10t)$

c) $x_1(t) = \sin 10\pi t$ $T = \frac{1}{5}$ periodic

ω = angular freq

$\boxed{\omega = 2\pi f}$

d) $x_2(t) = \sin 20\pi t$ $T = \frac{1}{10}$ periodic

e) $x_3(t) = \sin 3t$ $T = \frac{2\pi}{3}$ not periodic

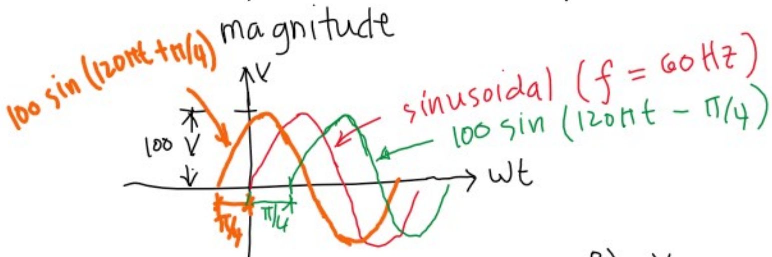
f) $x_4(t) = x_1(t) + x_2(t)$

g) $x_5(t) = x_1(t) + x_3(t)$

general eqn for a sine/cosine wave

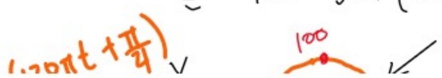
$$v = V_m \sin(\underbrace{\omega t}_{2\pi f} + \theta)$$

V_m: magnitude; θ: phase angle

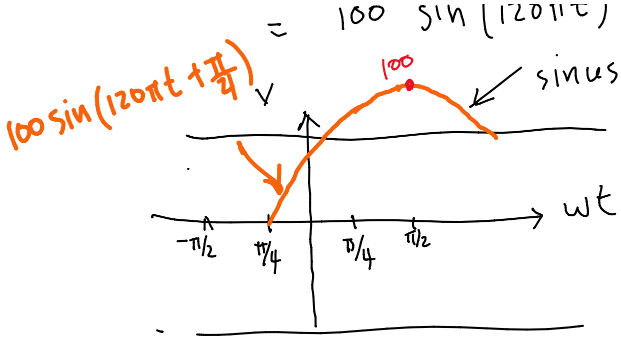


$V = 100\sin(120\pi t + 0°)$ V

$= 100\sin(120\pi t)$



$= 100 \sin(120\pi t)$



a) $g_1(t) = 3\sin(\underbrace{4\pi}_{\omega_1} t) + 7\cos(\underbrace{3\pi}_{\omega_2} t)$

$w = 2\pi f \quad ; \quad T = \frac{1}{f} \quad ; \quad f = \frac{1}{T}$

$w = \frac{2\pi}{T}$

$\boxed{\therefore \; T = \frac{2\pi}{w}}$

$T_1 = \frac{2\pi}{4\pi} = \frac{1}{2} \qquad T_2 = \frac{2\pi}{3\pi} = 2/3$

$\frac{T_1}{T_2} = \frac{1/2}{2/3}$

$\frac{T_1}{T_2} = \frac{3}{4}$ $\therefore$ periodic

$4T_1 = 3T_2 \qquad 4T_1 = 4\left(\frac{1}{2}\right) = 2\,s$

$3T_2 = 3\left(2/3\right) = 2s$

b) $g_2(t) = 3\sin(\underbrace{4\pi}_{\omega_1} t) + 7\cos(\underbrace{10}_{\omega_2} t)$

$\boxed{T = \frac{2\pi}{w}}$

$T_1 = \frac{2\pi}{4\pi} = \frac{1}{2} \qquad T_2 = \frac{2\pi}{10} = \pi/5$

$\frac{T_1}{T_2} = \frac{1/2}{\pi/5} = \frac{5}{2\pi}$ not a rationale fraction

$\therefore$ not periodic

c)

f) $x_4(t) = x_1(t) + x_2(t)$

$= \sin(\underbrace{10\pi}_{\omega_1} t) + \sin(\underbrace{20\pi}_{\omega_2} t)$

$T_1 = \frac{2\pi}{10\pi}$ $\quad$ $T_2 = \frac{2\pi}{20\pi}$

$= \frac{1}{5}$ $\quad$ $= \frac{1}{10}$

$\frac{T_1}{T_2} = \frac{1/5}{1/10}$

$\frac{T_1}{T_2} = \frac{2}{1}$ $\therefore$ periodic $\quad T_4 = \frac{1}{5}\,s$

$T_1 = 2T_2$

$\frac{1}{5} = 2\left(\frac{1}{10}\right) \approx \frac{1}{5}$

$T_1 = \frac{2\pi}{4} = \frac{\pi}{2}$ $\quad$ $T_2 = \frac{2\pi}{8} = \frac{\pi}{4}$

$\frac{T_1}{T_2} = \frac{\pi/2}{\pi/4} = \frac{\pi}{2} \cdot \frac{4}{\pi} = 2$ $\quad$ $T_1 = 2T_2$

$\boxed{\therefore T = \pi/2}$