GOVERNMENT OF TAMIL NADU

STANDARD SEVEN

MATHEMATICS

Term - III

Volume-2

A publication under Free Textbook Programme of Government of Tamil Nadu

Department of School Education

Untouchability is Inhuman and a Crime

Government of Tamil Nadu

First Edition - 2019

Revised Edition - 2022

(Published under New syllabus in Trimester Pattern)

NOT FOR SALE

Content Creation

State Council of Educational Research and Training

Printing & Publishing

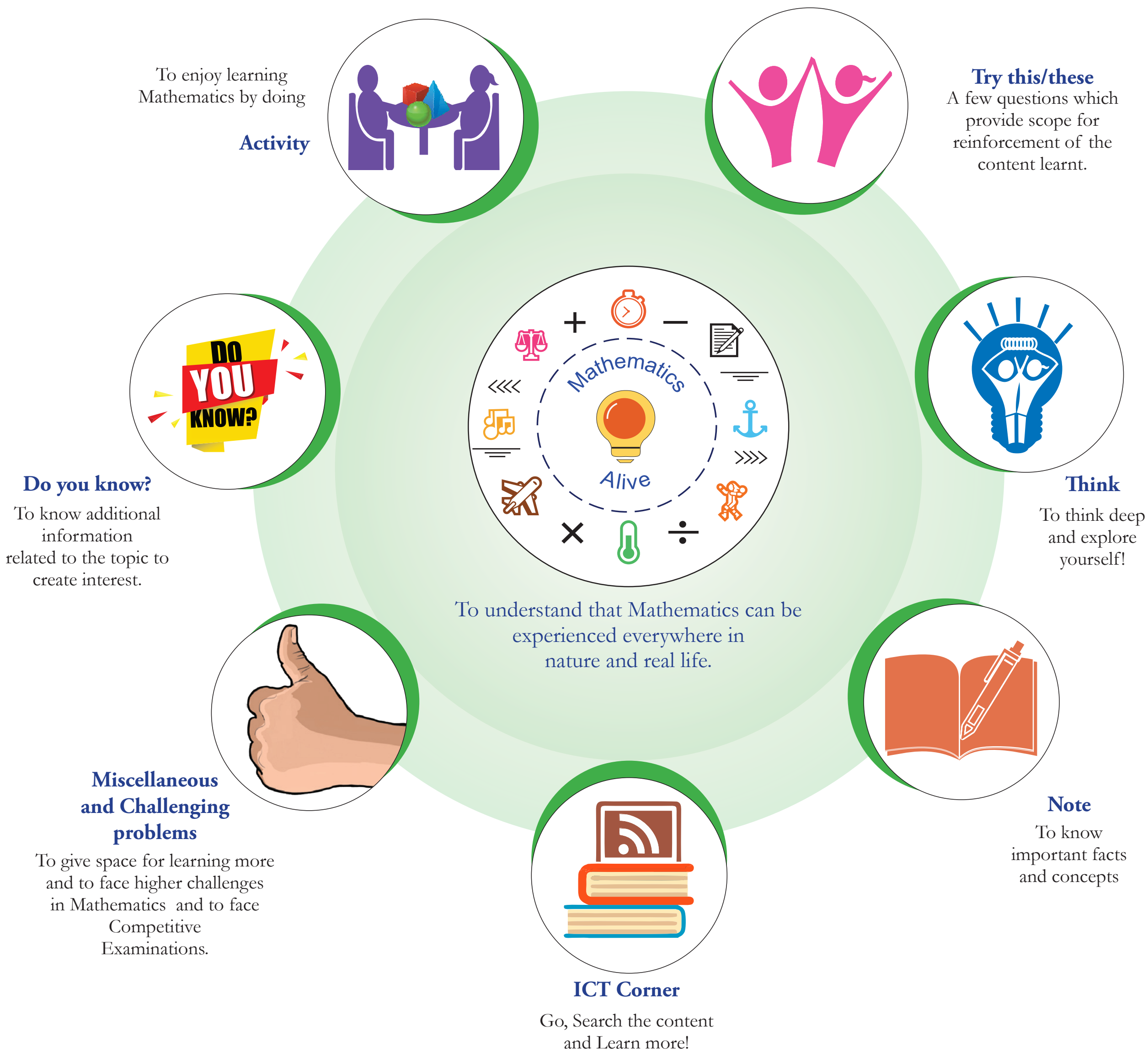
Tamil NaduTextbook and Educational Services Corporation

www.textbooksonline.tn.nic.in

Mathematics is a unique symbolic language in which the whole world works and acts accordingly. This text book is an attempt to make learning of Mathematics easy for the students community.

Mathematics is not about numbers, equations, computations or algorithms; it is about understanding

— William Paul Thurston



The main goal of Mathematics in School Education is to mathematise the child's thought process. It will be useful to know how to mathematise than to know a lot of Mathematics.

CONTENTS

E-book

Assessment

Chapter 1

NUMBER SYSTEM

Learning Objectives

- To know the rounding of decimal numbers.
- To understand the four fundamental operations on decimal numbers.

Recap-Decimal numbers

We have studied about decimal numbers in the previous term. Let us recall them now by answering the following questions.

Try these

1. Represent the fraction $\frac{1}{4}$ in decimal form.
2. What is the place value of 5 in $63.2\underline{5}7$.
3. Identify the digit in the tenth place of 75.036.
4. Express the decimal number 3.75 as a fraction.
5. Write the decimal number for the fraction $5\frac{1}{5}$.
6. Identify the bigger number : 0.567 or 0.576.
7. Compare 3.30 and 3.03 and identify the smaller number.
8. Put the appropriate sign (<, >, =).
   $2.57 \ \square \ 2.570$
9. Arrange the following decimal numbers in ascending order.
   5.14, 5.41, 1.54, 1.45, 4.15, 4.51.

1.1 Introduction

Mani went to market to purchase vegetables. The price details of five different vegetables on that day are shown in the following table.

Fig. 1.1

S.No.	Name of the vegetable	Price Per *kg* (in ₹)
1	Ladies finger	40.00
2	Brinjal	42.75
3	Beans	91.50
4	Carrot	90.50
5	Bell Pepper	100.00

- Mani wanted to round of each price to the nearest rupees.
- How will you decide on rounding each number?

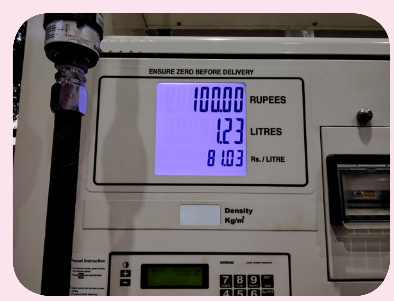
1.2 Rounding of Decimals

Rounding of decimals would be really useful for estimating amounts of money, duration of time, measure of distances and many other physical quantities. Let us learn the same here.

MATHEMATICS ALIVE - Decimals in real life

Weighing machine

Price tag



Petrol pump

You can round decimals just in the same way as you round whole numbers.

To round a decimal:

1. First underline the digit that is to be rounded. Then look at the digit to the right of the underlined digit.
2. If that digit is less than 5, then the underlined digit remains the same.
3. If that digit is greater than or equal to 5, add 1 to the underlined digit.
4. After rounding of, ignore all the digits after the underlined digit.

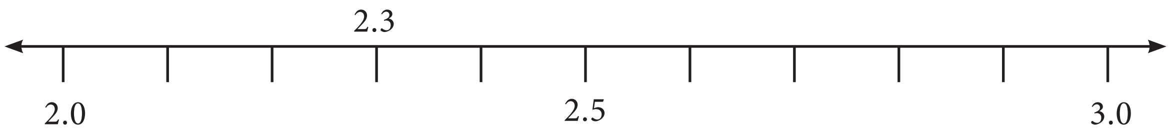
Example 1.1 Round 2.367 to the nearest whole number.

Solution

Underline the digit to be rounded - 2.367

Since the digit next to the underlined digit is 3 which is less than 5, the underlined digit 2 remains the same.

Also look at the number line shown below :



On the number line, we observe that 2.3 is closer to 2.0 than 3.0

Hence, the rounded value of 2.367 to the nearest whole number is 2.

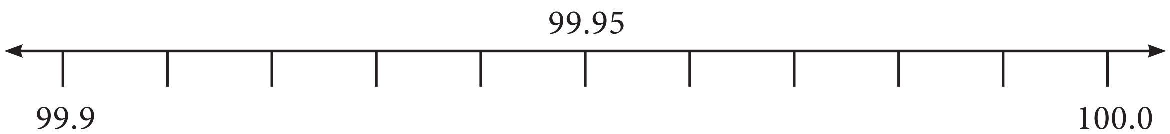
Example 1.2 Round 99.95 to the nearest tenth place.

Solution

Underline the digit to be rounded 99.9**5.**

Since the digit right to the tenth place is 5, we add 1 to the tenth place (underlined digit) of 99.95 and we get 100.0.

Look at the number line shown below:



So, the rounded value of 99.95 is 100.0.

Example 1.3 Round 52.6583 upto 2 places of decimal.

Solution

Round 52.6583 upto 2 places of decimal means round to the nearest hundredths place.

Underlining the digit in the hundredth place of 52.6583 gives 52.6**5**83.

We observe that the digit after the hundredth place value is 8 which is more than 5. Therefore, we should add 1 to the underlined digit. Hence, we get 52.66.

So the rounded value of 52.6583 upto 2 places of decimal is 52.66.

Example 1.4 Round 189.0007 upto 3 places of decimal.

Solution:

Round 189.0007 upto 3 places of decimal means rounding to the nearest thousandth place.

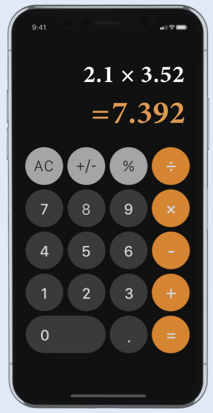
Underlining the digit in the thousandth place of 189.0007 gives $189.00\underline{0}7$

In $189.00\underline{0}7$ we observe that the digit next to the thousandth place value is 7, which is greater than 5.

Therefore, we should add 1 to the underlined digit. Hence, we get 189.001.

So the rounded value of 189.0007 upto 3 places of decimal is 189.001

DO YOU KNOW?



Nowadays calculators, computers and even mobile phones were used for multiplication and division of decimals. But, actually multiplication of decimals using logarithm tables will be more useful and handy.

Exercise 1.1

1. Round each of the following decimals to the nearest whole number.

(i) 8.71 (ii) 26.01 (iii) 69.48

(iv) 103.72 (v) 49.84 (vi) 101.35

(vii) 39.814 (viii) 1.23

2. Round each decimal number to the given place value.

(i) 5.992 to tenth place

(ii) 21.805 to hundredth place

(iii) 35.0014 to thousandth place

3. Round the following decimal numbers upto 1 places of decimal.

(i) 123.37 (ii) 19.99 (iii) 910.546

4. Round the following decimal numbers upto 2 places of decimal.

(i) 87.755 (ii) 301.513 (iii) 79.997

5. Round the following decimal numbers upto 3 places of decimal.

(i) 24.4003 (ii) 1251.2345 (iii)61.00203

1.3 Operations on Decimal Numbers

Already we are familiar with decimal numbers. We know how to represent a decimal number as a fraction and the place values of digits. Now, let us learn the operations on decimal numbers.

1.3.1 Addition and Subtraction of Decimal Numbers

Iniya has purchased notebooks for ₹46.50 and a pencil box for ₹16.50. How much will she get as balance if she paid ₹100 to the shop keeper?

Price of a note book = ₹46.50 ; Price of a pencil box= ₹16.50

To find the amount to be paid, we have to add the price of both the items.

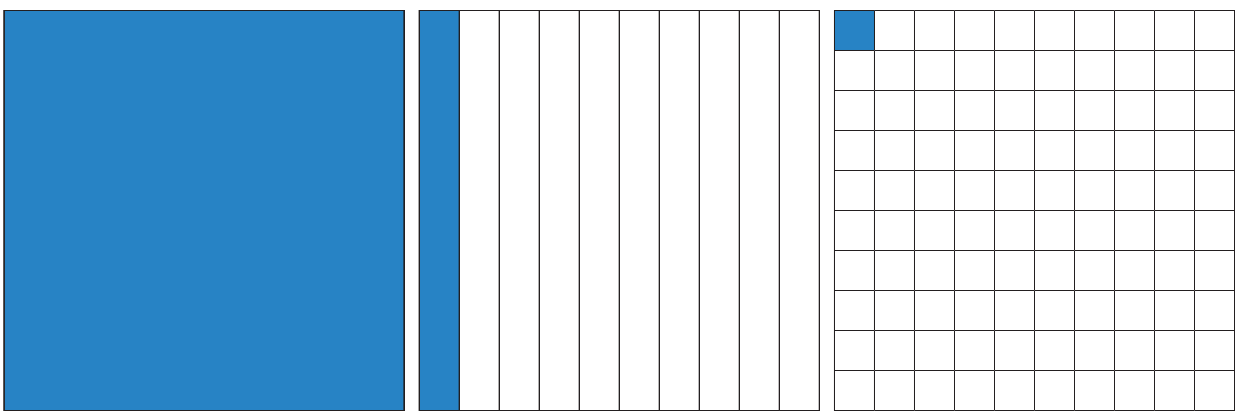
To get the balance amount we have to subtract the total expense from ₹100. To know the total expenses and the balance money, we need to understand addition and subtraction of decimals.

Addition and subtraction of decimals through models

Decimal grid or area models can be used to understand the process of addition and subtraction using decimal numbers.

(i) Grid model

We see below the grids to represent the decimal numbers 1.0, 0.1 and 0.01.



One whole, 10 by 10 Grid represents 1 or 1.0

Each column represents one tenth or 0.1

Each square represents one hundredth or 0.01

Having these grids let us try to do addition and subtraction of decimal numbers.

Example 1.5 Find the sum of 0.16 and 0.77 using decimal grid models.

Solution

Here, $0.16 = \frac{16}{100}$ and $0.77 = \frac{77}{100}$

First shade the region 0.16 and then shade 0.77.

That is, first shade 16 squares out of 100 and then shade 77 more squares, which means total shaded region is 93 squares out of 100.

The total shaded area is the sum.

So, $0.16 + 0.77 = 0.93$.

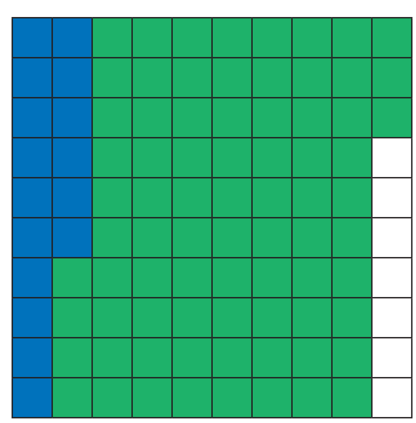


Fig. 1.2

Example 1.6 Find 0.52 – 0.08 using decimal grid models.

Solution

Here $0.52 = \frac{52}{100}$ and $0.08 = \frac{8}{100}$. First shade the region 0.52 then cross out 0.08, which is $\frac{8}{100}$ from the shaded area. The left out shaded region without cross marks is the difference. So, $0.52 - 0.08 = 0.44$.

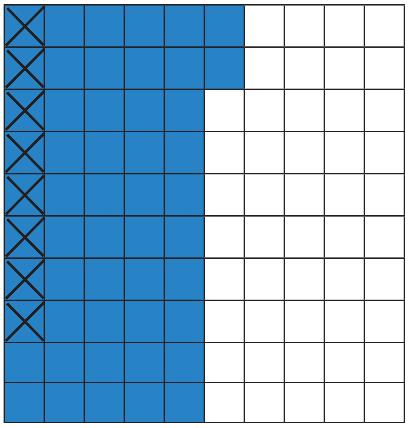


Fig. 1.3

Example 1.7 Find the value of 0.72 – 0.51 by using grids.

Solution

Take a square of 100 boxes. Shade 72 boxes to represent 0.72.

Then strike out 51 boxes out of 72 shaded boxes to subtract 0.51 from 0.72.

The left over shaded boxes represent the required value.

Therefore, 0.72 – 0.51 = 0.21.

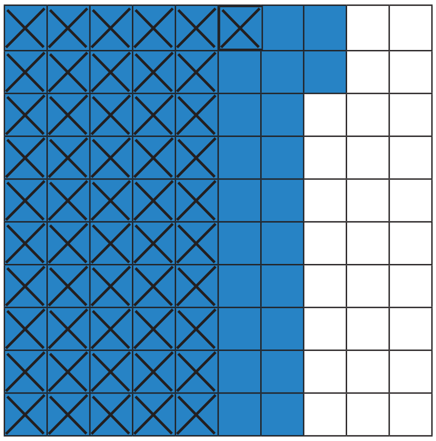


Fig. 1.4

Try these

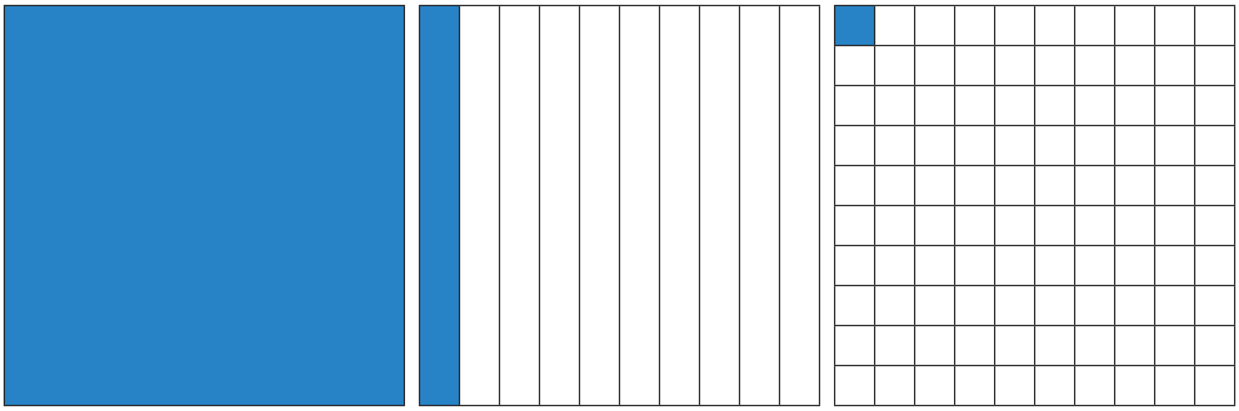
Find the following using grid models:

(i) 0.83 + 0.04

(ii) 0.35 – 0.09

(ii) Area model

The whole number (unit place) which is a part of decimals represents a square area and $\frac{1}{10}^{\text{th}}$ part of this square area which is a thin rectangular strip represents the tenth place of the decimal (0.1) and $\frac{1}{100}^{\text{th}}$ part of this square area which is a smaller square represents the hundredth place value (0.01) and the same process will be continued for the next place and so on.



1.0 **0.1** **0.01**

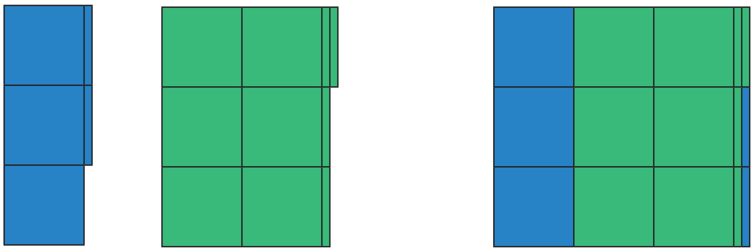
Having these square and rectangular area let us try to do addition and subtraction of decimal numbers.

Note

$\frac{1}{100}^{th}$ part is the same as $\frac{1}{10}^{th}$ of $\frac{1}{10}$.

Example 1.8 Add 3.2 + 6.4.

Solution



3.2 6.4 9.6

Here 3.2 is represented in Blue colour and 6.4 is represented in Green colour. Hence, the sum of 3.2 and 6.4 is 9.6.

Example 1.9 Subtract 7.5 – 3.4 .

Solution

First represent the decimal number 7.5 using 7 squares and 5 rectangular strips. Cross out 3 squares from 7 squares and 4 rectangular strips from 5 rectangular strips to get the difference (see *Fig. 1.5*).

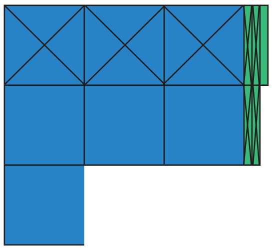


Fig. 1.5

Hence, 7.5 – 3.4 = 4.1.

Try these

Using the area models solve the following:

(i) $1.2 + 3.5$

(ii) $3.5 - 2.3$

(iii) Place value grid model

So far we have discussed grid models to do addition and subtraction of decimal numbers. Earlier we have studied representation of decimal numbers in place value tables. Let us use the same representation for addition and subtraction of decimal numbers.

For example, while adding 4.83 and 1.67, we have

Decimal No.	Ones	Tenth	Hundredth
4.38	4	3	8
1.67	1	6	7
6.05	6	0	5

Therefore, 4.38 + 1.67 = 6.05.

Example 1.10 Add the following: (i) 30.9 + 52.73 (ii) 25.67 + 33.856

Solution

(i) 30.9 + 52.73

Let us use the place value grid.

Decimal No.	Tens	Ones	Tenth	Hundredth
30.9	3	0	9	0
52.73	5	2	7	3
83.63	8	3	6	3

(Since the digits in the decimal place of 52.73 is 2 and 30.9 is 1, we should add 0 at the hundredth place of 30.9 to equalise the digits in the decimal place)

Therefore, 30.9 + 52.73 = 83.63.

(ii) 25.67 + 33.856

Let us use the place value grid.

Decimal No.	Tens	Ones	Tenth	Hundredth	Thousandth
25.67	2	5	6	7	0
33.856	3	3	8	5	6
59.526	5	9	5	2	6

Therefore, 25.67 + 33.856 = 59.526.

Note

Adding zeros at the right end of decimal digits will not change the value of the number.

Example 1.11 Everyday Malar travels 1.820 *km* by bus and 295 *m* by walk to reach the school. Find the distance of school from her house in *km*.

Solution

Distance travelled by bus =1.820 *km*

Distance covered by walk = 0.295 *km*

Total distance = 1.820 + 0.295

=2.115 *km*

$1000\ m = 1\ km;\ 1\ m = \frac{1}{1000}\ km$

Hence, $295\ m = \frac{295}{1000}\ km$

$= 0.295\ km$

Therefore, the school is situated at a distance of 2.115 *km* from her house.

Example 1.12 Subtract 2.85 from 4.97.

Solution

4.97 – 2.85 = ?

Let us use the place value grid.

Decimal No.	Ones	Tenth	Hundredth
4.97	4	9	7
2.85	2	8	5
2.12	2	1	2

Therefore, 4.97 – 2.85 = 2.12.

Example 1.13 Subtract 3.09 from 12.7.

Solution

12.7 – 3.09 = ?

Let us use the place value grid.

Decimal No.	Tens	Ones	Tenth	Hundredth
12.7	1	2	7	0
3.09		3	0	9
9.61		9	6	1

Therefore, 12.7 – 3.09 = 9.61.

Note

1. We can equalize the decimal digits of given numbers by adding zero at the right end of a decimal number.
2. Zeros are added at the right end of decimal digits of a decimal number that are to be added or subtracted.

Example 1.14 Subtract 32.042 from 86.9.

Solution

	86.900
(–)	32.042
	54.858

Therefore, 86.9 – 32.042 = 54.858.

Try this

Complete the magic square in such a way that rows, columns and diagonals give the same sum 1.5.

0.8		0.6
	0.5	
0.4		

Example 1.15 Naren bought 7.4 *kg* of mangoes. On the way home, he gave 4.650 *kg* of mangoes to his sister's family. Find the weight of the remaining mangoes.

Solution

Mangoes bought by Naren = 7.4 *kg*

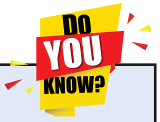
Mangoes given to Naren's sister = 4.650 *kg*

Weight of remaining mangoes = 7.400 – 4.650

= 2.750 *kg*

Fig. 1.6

Therefore, the weight of the remaining mangoes is 2.750 *kg*.



We use decimals every day, while dealing with money, weight, length etc. Decimal numbers are used in situations where more accuracy is required.

Exercise 1.2

1. Add by using grid 0.51 + 0.25
2. Add the following by using place value grid.
   (i) 25.8 + 18.53 (ii) 17.4 + 23.435
3. Find the value of 0.46 – 0.13 by grid model.
4. Subtract the following by using place value grid.
   (i) 6.567 from 9.231 (ii) 3.235 from 7
5. Simplify: 23.5 – 27.89 + 35.4 – 17.
6. Sulaiman bought 3.350 *kg* of Potato, 2.250 *kg* of Tomato and some Onions. If the weight of the total items are 10.250 *kg*, then find the weight of Onions?
7. What should be subtracted from 7.1 to get 0.713?
8. How much is 35.6 *km* less than 53.7 *km*?
9. Akilan purchased a geometry box for ₹25.75, a pencil for ₹3.75 and a pen for ₹17.90. He gave ₹50 to the shopkeeper. What amount did he get back?
10. Find the perimeter of an equilateral triangle with a side measuring 3.8 *cm*.

Objective type questions

11. 1.0 + 0.83=?
    (i) 0.17 (ii) 0.71 (iii) 1.83 (iv) 1.38
12. 7.0 – 2.83 = ?
    (i) 3.47 (ii) 4.17 (iii) 7.34 (iv) 4.73

13. Subtract 1.35 from 3.51

(i) 6.21 (ii) 4.86 (iii) 8.64 (iv) 2.16

14. Sum of two decimals is 4.78. If one decimal is 3.21, then the other one is

(i)1.57 (ii) 1.75 (iii)1.59 (iv) 1.58

15. The difference of two decimals is 86.58 and one of the decimal is 42.31 Find the other one

(i) 128.89 (ii) 128.69 (iii) 128.36 (iv) 128.39

1.3.2 Multiplication of Decimal Numbers

Mathan wants to buy a shirt material which costs ₹75.50 per metre. He needs 1.5 metre to stitch a shirt. How much does he have to pay? Here we need to multiply 75.50 and 1.5. In real life, there are many situations where we need to multiply decimal numbers.

(i) Decimal multiplication through models

Let us try to understand decimal multiplication using grid model.

Let us find 0.1×0.1.

$0.1 = \frac{1}{10}$. Therefore, $0.1 \times 0.1 = \frac{1}{10} \times \frac{1}{10}$

That is, $\frac{1}{10}^{\text{th}}$ of $\frac{1}{10}$.

$\frac{1}{10}$

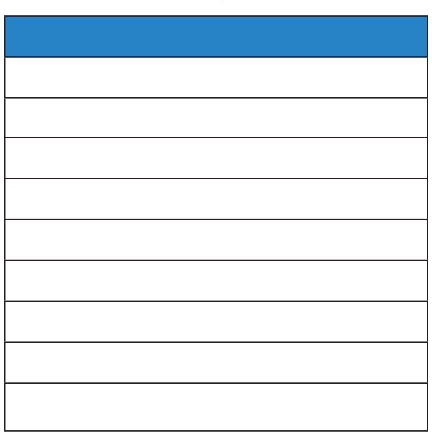


Fig. 1.7

$\frac{1}{10}^{\text{th}}$ of $\frac{1}{10} = \frac{1}{100}$

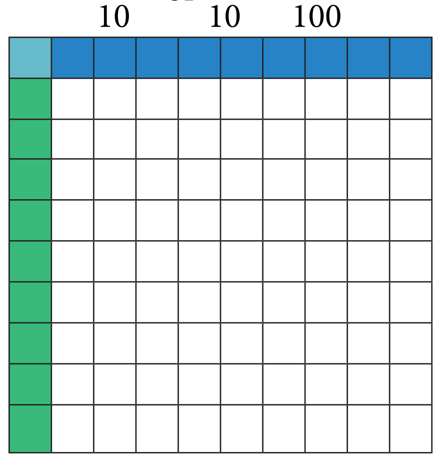


Fig. 1.8

Shade horizontally $\frac{1}{10}$ by blue colour (*Fig. 1.7*). Shade vertically $\frac{1}{10}$ by green colour (*Fig. 1.8*).

Then $\frac{1}{10}^{\text{th}}$ of $\frac{1}{10}$ is the common portion, that is $\frac{1}{100}^{\text{th}}$.

Therefore, $\frac{1}{10} \times \frac{1}{10} = \frac{1}{100} = 0.01$

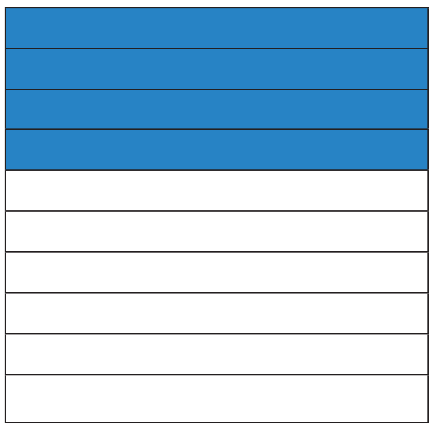
Hence, $0.1 \times 0.1 = 0.01$.

Example 1.16 Find the value of 0.3×0.4

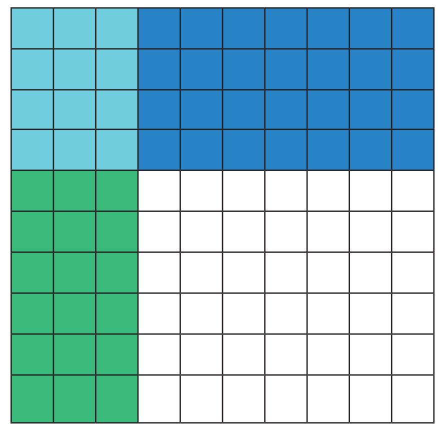
Solution

First shade 4 rows of the grid in blue colour to represent 0.4. Shade 3 columns of the grid in green colour to represent 0.3 of 0.4. Now 12 squares represents the common portion. This represents 12 hundredth or 0.12. Hence $0.3 \times 0.4 = 0.12$.

$\frac{4}{10}$



$\frac{3}{10}^{th}$ of $\frac{4}{10} = \frac{12}{100}$



Note

The number of decimal digits in 0.12 is two. So, we can conclude that the number of decimal digits in the product of two decimal numbers is equal to the sum of the number of decimal digits that are multiplied.

Area model

We have already learnt about the area model in the addition and subtraction of decimal numbers. In the same way we are going to multiply the decimal numbers. Now we shall see an example.

Example 1.17 Multiply 3.2 and 2.1

Solution

Let us try to represent the product of decimal numbers (3.2×2.1) as the area of a rectangle. Let us consider a rectangular portion as shown in *Fig. 1.9*.

Fig. 1.9

The rectangular portion is split into 3 wholes and 2 tenth along it's length (*Fig. 1.10*).

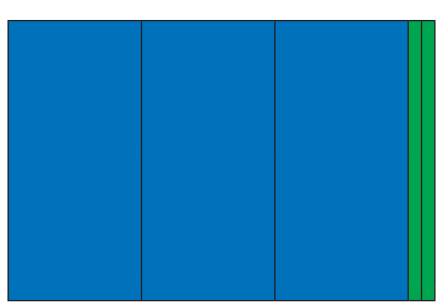


Fig. 1.10

Since, 3 wholes and 2 tenth is multiplied with 2.1, we split the same area into 2 wholes and 1 tenth along its breadth (*Fig. 1.11*).

Here, each row contains 3 wholes and 2 tenth. Each column contains 2 wholes and 1 tenth. The entire area model represents 6 wholes, 7 tenth and 2 hundredth.

Therfore, $3.2 \times 2.1 = 6.72$

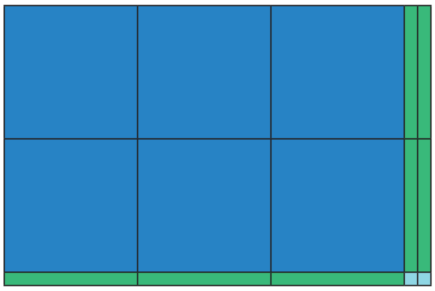


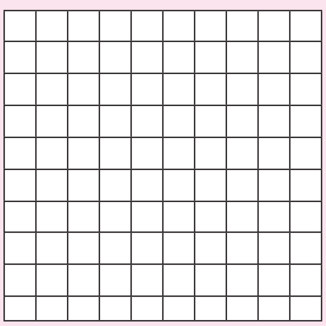
Fig. 1.11

Think

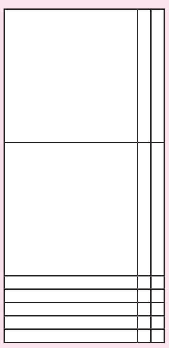
How are the products 2.1×3.2 and 21×32 alike? How are they different.

Try these

(1) Shade the grid to multiply 0.3×0.6.



(2) Use the area model to multiply 1.2×2.5



Example 1.18 Multiply the following

(i) 2.3×1.4 (ii) 5.6×3.2

2.3	← 1 decimal place
× 1.4	← 1 decimal place
92	
23	
3.22	← 2 decimal place(s)

Solution

(i) 2.3×1.4

First, let us multiply 23×14

$23 \times 14 = 322$

Now, $2.3 \times 1.4 = 3.22$.

(ii) 5.6×3.2

First, let us multiply 56×32

$56 \times 32 = 1792$

Now, $5.6 \times 3.2 = 17.92$.

5.6	← 1 decimal place
× 3.2	← 1 decimal place
112	
168	
17.92	← 2 decimal place(s)

Example 1.19 Latha purchased a churidhar material of 3.75 *m* at the rate of ₹62.50 per metre. Find the amount to be paid.

Solution

Cost of churidhar material = ₹62.50 per metre

Length of churidhar material = 3.75 *m*

Amount to be paid = 3.75 × 62.50

= ₹ 234.3750

= ₹ 234.38 (rounded to two decimals).

	62.50	← 2 decimal place
×	3.75	← 2 decimal place
	31250	
	43750	
	18750	
	234.3750	← 4 decimal place(s)

Example 1.20 The length and breadth of a rectangle is 23.5 *cm* and 1.5 *cm* respectively. Find the area of the rectangle.

Solution

Area of a rectangle = $l \times b$ *sq. units*

Here, l=23.5 *cm*, b=1.5 *cm*.

Area of the rectangle = 23.5 × 1.5

= 35.25 *sq.cm.*

	23.5	← 1 decimal place
×	1.5	← 1 decimal place
	1175	
	235	
	35.25	← 2 decimal place(s)

(ii) Multiplication of Decimal Numbers by 10, 100 and 1000

We have studied about conversion of decimals into fractions in the first term. Consider 45.6 and 4.56.

Expressing these decimal numbers into fractions, we get

$$45.6 = 40 + 5 + \frac{6}{10} = 45 + \frac{6}{10} = \frac{456}{10}$$

Now, $4.56 = 4 + \frac{5}{10} + \frac{6}{100} = \frac{456}{100}$

Comparing the two fractions, we see that if there is one digit after the decimal point, then the denominator is 10 and if there are two digits after the decimal point, then the denominator is 100 and so on.

Let us see what happens if a decimal number is multiplied by 10, 100 and 1000.

Observe the following table and complete it.

Try these

$2.35 \times 10 = \frac{235}{100} \times 10 = \frac{235}{10} = 23.5$	$7.63 \times 10 =$ ______	$63.237 \times 10 =$ ______
$2.35 \times 100 = \frac{235}{100} \times 100 = 235 = 235.0$	$7.63 \times 100 =$ ______	$63.237 \times 100 =$ ______
$2.35 \times 1000 = \frac{235}{100} \times 1000 = 2350.0$	$7.63 \times 1000 =$ ______	$63.237 \times 1000 =$ ______
$0.6 \times 10 = \frac{6}{10} \times 10 = 6$	$0.6 \times 100 = \frac{6}{10} \times 100$ = ______	$0.6 \times 1000 = \frac{6}{10} \times 1000$ = ______

Can you observe any pattern in the above table? There is a pattern in the shift of decimal points of the products in the table. In $2.35 \times 10 = 23.5$ the digits are the same, that is 2, 3, 5. Observe 2.35 and 23.5. To which side has the decimal point been shifted, right or left? The decimal point is shifted to the right by one place. Also note that 10 is one zero followed by 1.

In $2.35 \times 100 = 235.0$, observe 2.35 and 235. To which side and by how many digits has the decimal point been shifted? The decimal point has been shifted to the right by two places. Note that 100 is two zeros followed by 1.

Similarly in $2.35 \times 1000 = 2350.0$, we see that the decimal point has been shifted to the right by three places by adding one more digit 0 to the number 235. Note that 1000 is three zeros followed by 1.

So we conclude that when a decimal number is multiplied by 10, 100 or 1000, the digits in the product are same as in the decimal number but the decimal point in the product is shifted to the right by as many places as there are zeros followed by 1.

Try these

Find

(i) 9.13×10

(ii) 9.13×100

(iii) 9.13×1000

Based on these observations we can now say,

$0.02 \times 10 = 0.2$; $0.02 \times 100 = 2$ and $0.02 \times 1000 = 20$.

Can you find the value of following?

$2.76 \times 10 = ?$

$2.76 \times 100 = ?$

$2.76 \times 1000 = ?$

Example 1.21 Find the value of the following

(i) 3.26×10 (ii) 3.26×100 (iii) 3.26×1000

(iv) 7.01×10 (v) 7.01×100 (vi) 7.01×1000

Solution

(i) $3.26 \times 10 = 32.6$

(ii) $3.26 \times 100 = 326.0$

(iii) $3.26 \times 1000 = 3260.0$

(iv) $7.01 \times 10 = 70.1$

(v) $7.01 \times 100 = 701.0$

(vi) $7.01 \times 1000 = 7010.0$

Example 1.22 A concessional entrance ticket for students to visit a zoo is ₹12.50. How much has to be paid for 20 tickets?

Solution

Cost of one ticket = ₹12.50

Amount to be paid for 20 students = 12.50×20 = ₹250.00

We have already discussed about the multiplication of decimal numbers by 10, 100 and 1000. In the same way we can find patterns for multiplying decimal numbers by 0.1, 0.01 and 0.001. Observe the following.

$$12.3 \times 0.1 = \frac{123}{10} \times \frac{1}{10} = \frac{123}{100} = 1.23$$

$$12.3 \times 0.01 = \frac{123}{10} \times \frac{1}{100} = \frac{123}{1000} = 0.123$$

$$12.3 \times 0.001 = \frac{123}{10} \times \frac{1}{1000} = \frac{123}{10000} = 0.0123$$

From the above multiplication we can conclude that, when multiplying by

- 0.1, the decimal point moves one place left.
- 0.01, the decimal point moves two places left.
- 0.001, the decimal point moves three places left.

Zeros may be added as required when multiplied by 0.1, 0.01 and 0.001 as mentioned above.

Try these

Complete the following table :

11.6×0.1	$\frac{116}{10} \times \frac{1}{10}$	–	–
11.6×0.01	–	–	–
11.6×0.001	–	–	–

Exercise 1.3

1. Find the product of the following
   (i) 0.5×3 (ii) 3.75×6 (iii) 50.2×4
   (iv) 0.03×9 (v) 453.03×7 (vi) 4×0.7
2. Find the area of the parallelogram whose base is 6.8 *cm* and height is 3.5 *cm*.
3. Find the area of the rectangle whose length is 23.7 *cm* and breadth is 15.2 *cm*.
4. Multiply the following
   (i) 2.57×10 (ii) 0.51×10 (iii) 125.367×100 (iv) 34.51×100
   (v) 62.735×100 (vi) 0.7×10 (vii) 0.03×100 (viii) 0.4×1000
5. A wheel of a baby cycle covers 49.7 *cm* in one rotation. Find the distance covered in 10 rotations.
6. A picture chart costs ₹1.50. Radha wants to buy 20 charts to make an album. How much does she have to pay?

7. Find the product of the following.

(i) 3.6×0.3 (ii) 52.3×0.1 (iii) 537.4×0.2

(iv) 0.6×0.06 (v) 62.2×0.23 (vi) 1.02×0.05

(vii) 10.05×1.05 (viii) 101.01×0.01 (ix) 100.01×1.1

Objective type questions

8. 1.07×0.1 _________

(i) 1.070 (ii) 0.107 (iii) 10.70 (iv) 11.07

9. $2.08 \times 10 =$ _________

(i) 20.8 (ii) 208.0 (iii) 0.208 (iv) 280.0

10. A frog jumps 5.3 *cm* in one jump. The distance travelled by the frog in 10 jumps is _________

(i) 0.53 *cm* (ii) 530 *cm* (iii) 53.0 *cm* (iv) 53.5 *cm*

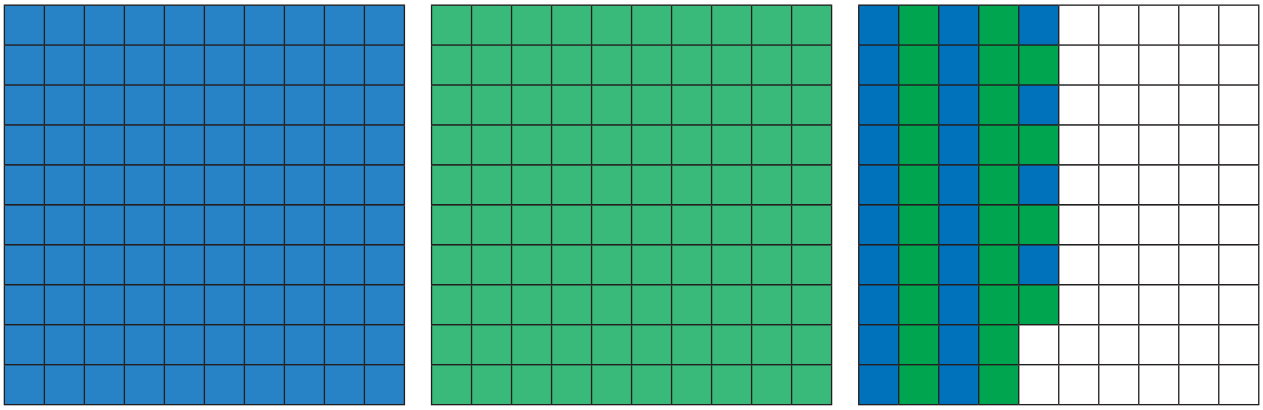
1.3.3 Division of Decimal Numbers

A bale of shirt material consists of 24.75 *m*. A shop-keeper wants to make shirt bits each of 1.5 *m* outfit. How many bits can be made? To find the answer, we have to divide 24.75 by 1.5.

In real life, there are many situations where we need to divide decimal numbers.

(i) Decimal Division through models

- Use decimal grids to find the quotient, as you have already learnt to represent decimals in the grid form.
- Consider 2.48 in the grid form.
- To find $2.48 \div 2$, shade grids to show the decimal number 2.48 by 2. Since division means equal sharing. Seperate the shaded region into 2 equal parts as shown below.

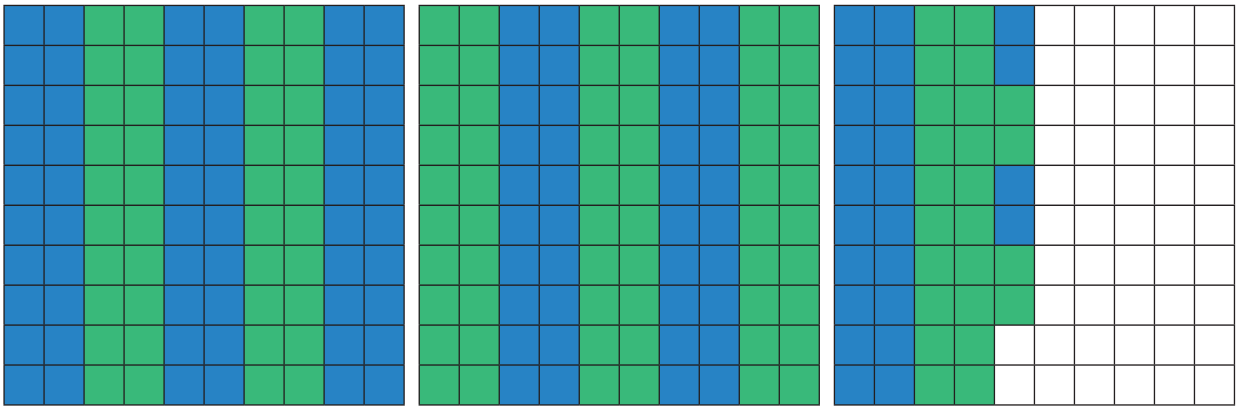


Here each colour represents the quotient (i.e) 1.24.

Hence, $2.48 \div 2 = 1.24$.

Let us do the division of 2.48 by 2 in the other way.

Observe the following figure.



Consider 2 wholes, when you divide 2 wholes equally in groups of two's, each group gets 5 such groups which represents 1 whole.

Next consider the tenth, taking four tenth and dividing it in group of two's, we get two such groups and each group represents two tenth.

Finally, dividing 8 hundredth in group of two's, each group has 4 hundredth.

Therefore 2.48 ÷ 2 = 1 whole 2 tenth 4 hundredth

= 1.24.

Example 1.23 Divide 6.3 by 3 using area model.

Solution

The decimal number 6.3 is shown in *Fig. 1.12.*

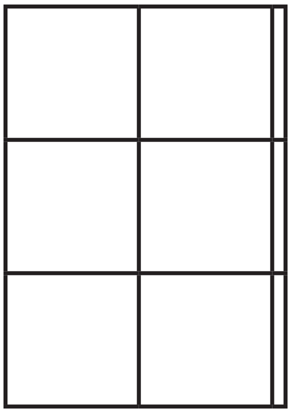


Fig. 1.12

Since 6.3 is divided by 3, we seperate the areas into 3 equal groups using three different colours as in *Fig.1.13*.

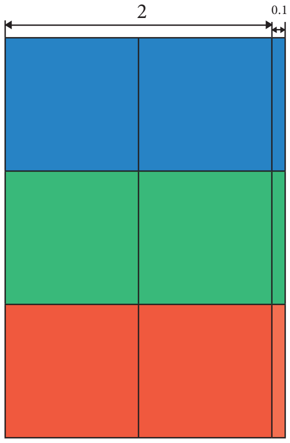


Fig. 1.13

Each group to represents 2.1, which is the quotient.

Therefore, 6.3 ÷3 = 2.1.

(ii) Division by 10, 100 and 1000

Let us learn how to divide decimal numbers by 10, 100 and 1000.

For example, consider 51.7 ÷ 10

Now $51.7 = \frac{517}{10}$

Therefore, $\frac{51.7}{10} = \frac{517}{10} \times \frac{1}{10} = \frac{517}{100} = 5.17$

Similarly, $51.7 \div 100 = \frac{517}{10} \times \frac{1}{100} = \frac{517}{1000} = 0.517$

Also $51.7 \div 1000 = \frac{517}{10} \times \frac{1}{1000} = \frac{517}{10000} = 0.0517$

When decimal numbers are divided by powers of 10, it can be noted that the decimal number so obtained contains the same number of decimal digits as that of zeros in powers of 10.

Let us see whether there is a pattern for dividing numbers by 10, 100 and 1000.

Observe the following and complete it.

Try these

$36.7 \div 10 = 3.67$	$436.7 \div 10 = 43.67$	$2.3 \div 10 = 0.23$	$27.17 \div 10 =$ ____
$36.7 \div 100 = 0.367$	$436.7 \div 100 =$ ____	$2.3 \div 100 =$ ____	$27.17 \div 10 =$ ____
$36.7 \div 1000 = 0.0367$	$436.7 \div 1000 =$ ____	$2.3 \div 1000 =$ ____	$27.17 \div 1000 =$ ____

Take. 36.7 ÷ 10 = 3.67. In 36.7 and 3.67, the digits are the same namely, 3, 6 and 7, but the decimal point has been shifted to the left by one place in the obtained decimal number after division. Note that 10 is one zero followed by 1.

Similarly, in 36.7 ÷ 100 = 0.367, the decimal point has been shifted to the left by two places in the obtained decimal number after division. Note that 100 is two zeros followed by 1.

Similarly, in 36.7 ÷ 1000 = 0.0367, the decimal point has been shifted to the left by three places in the obtained decimal number after division. Note that 1000 is three zeros followed by 1.

Hence we can conclude that while dividing a decimal number by 10, 100 and 1000, the digits of the number (Dividend) and the obtained decimal number after division are the same but the decimal point in the obtained decimal number after division is shifted to the left by as many places as there are zeros followed by 1.

Try these

Divide the following

(i) 17.237 √ 10

(ii) 17.237 √ 100

(iii) 17.237 √ 1000

Therefore, we can find $2.68 \div 10 = 0.268$;

$2.68 \div 100 = 0.0268$; $2.68 \div 1000 = 0.00268$.

(iii) Division of a decimal number by a whole number

Let us divide a decimal number by a whole number.

Consider the following.

(i) $7.6 \div 2$

$$7.6 \div 2 = \frac{7.6}{2}$$

But $$7.6 = \frac{76}{10}$$

Therefore, $$7.6 \div 2 = \frac{76}{10} \times \frac{1}{2} = \frac{1}{10} \times \frac{76}{2}$$

$$= \frac{1}{10} \times 38 = \frac{38}{10} = 3.8$$

$$\begin{array}{r} 3.8 \\ 2\overline{)\,7.6} \\ 6 \\ \hline 16 \\ 16 \\ \hline 0 \end{array}$$

(ii) Consider $7.26 \div 2$

Now $$7.26 = \frac{726}{100}$$

$$7.26 \div 2 = \frac{726}{100} \times \frac{1}{2} = \frac{1}{100} \times \frac{726}{2}$$

$$= \frac{1}{100} \times 363 = \frac{363}{100} = 3.63$$

$$\begin{array}{r} 3.63 \\ 2\overline{)\,7.26} \\ 6 \\ \hline 12 \\ 12 \\ \hline 6 \\ 6 \\ \hline 0 \end{array}$$

(iii) $9.6 \div 8$.

Now $$9.6 = \frac{96}{10}$$

$$9.6 \div 8 = \frac{96}{10} \times \frac{1}{8}$$

$$= \frac{1}{10} \times \frac{96}{8}$$

$$= \frac{1}{10} \times 12$$

$$= \frac{12}{10} = 1.2$$

$$\begin{array}{r} 1.2 \\ 8\overline{)\,9.6} \\ 8 \\ \hline 16 \\ 16 \\ \hline 0 \end{array}$$

Note

We have to estimate to check that the quotient is reasonably correct. Round 9.6 to 10 and divide by 8. Since 8 goes into 10 once with a small reminder, the answer 1.2 is reasonably correct.

From the above three cases, we observe that we can also divide the number without considering the decimal point first and then the number of decimal digits of the quotient has to be made equivalent to the number of decimal digits of the dividend.

For example, take 52.16 ÷ 4

Now, let us divide 5216 by 4.

$$\frac{5216}{4} = 1304$$

Now there are 2 decimal digits in the dividend. So the quotient also will have two decimal digits.

Therefore, 52.16 ÷ 4 = 13.04.

Try these

Find the value of the following.

(i) $46.2 \div 3 = ?$

(ii) $71.6 \div 4 = ?$

(iii) $23.24 \div 2 = ?$

(iv) $127.35 \div 9 = ?$

(v) $47.201 \div 7 = ?$

Example 1.24 Sugarcane juice of 1.5 *l* has to be shared equally among five members. What is the share of each member?

Solution

Quantity of sugarcane juice = 1.5 *l*

Number of persons to be shared = 5

Individual share $= \frac{1.5}{5} = 0.3\ l$

Therefore, the individual share is 0.3 litres.

Fig. 1.14

(iv) Division of a Decimal Number by another Decimal Number

Kavitha wants to distribute groundnut balls to all her classmates on her birthday. She has ₹61.50 in her piggy bank. If each groundnut ball costs ₹1.50, how many balls she can get with that amount? In real life, there are situations in which we need to divide a decimal number by another decimal number.

(i) Consider, 15.5 ÷ 0.5

$$15.5 \div 0.5 = \frac{15.5}{0.5} = \frac{\left(\frac{155}{10}\right)}{\left(\frac{5}{10}\right)} = \frac{155}{10} \times \frac{10}{5} = 31$$

Thus, $\frac{15.5}{0.5} = 31.$

(ii) Let us consider 62.5 ÷ 0.25

Now $62.5 \div 0.25 = \frac{62.5}{0.25}$

Since $62.5 = \frac{625}{10}$ and $0.25 = \frac{25}{100}$ we get,

$$\frac{62.5}{0.25} = \frac{\left(\frac{625}{10}\right)}{\left(\frac{25}{100}\right)} = \frac{625}{10} \times \frac{100}{25} = \frac{625 \times 10}{25} = \frac{6250}{25} = 250.$$

(iii) Consider another example 2.25 ÷ 0.9

Now $2.25 \div 0.9 = \dfrac{2.25}{0.9}$

Since $2.25 = \dfrac{225}{100}$ and $0.9 = \dfrac{9}{10}$

we get,

$$\frac{2.25}{0.9} = \frac{\left(\frac{225}{100}\right)}{\left(\frac{9}{10}\right)} = \frac{225}{100} \times \frac{10}{9} = \frac{225}{9 \times 10} = \frac{225}{90} = 2.5.$$

Hence, we can conclude that when both dividend and divisor has same number of decimal digits the division is as simple as the usual division of two natural numbers. When dividend and divisor has different number of decimal digits, first express both the divisor and the dividend in fractional form and then divide.

Try these

Divide the following.

(i) $\dfrac{9.25}{0.25}$ (ii) $\dfrac{8.6}{4.3}$

(iii) $\dfrac{44.1}{0.21}$ (iv) $\dfrac{9.6}{1.2}$

Example 1.25 A car covers 16.8 *km* in 0.21 hours. What is the distance covered by the car in one hour.

Solution

The distance covered by the car in one hour $= \dfrac{16.8}{0.21}$

Multiply the divisor by 100 to make it a whole number. Hence multiply the dividend also by 100.

Therefore, $\dfrac{16.8 \times 100}{0.21 \times 100} = \dfrac{1680}{21} = 80$.

Hence, the distance covered by the car in one hour is 80 *km*.

Example 1.26 The perimeter of a regular polygon is 17.5 *cm*. Each side measures 2.5 *cm*. Find the number of sides of the polygon.

Solution

Since it is given that the polygon is a regular polygon, all sides will be equal.

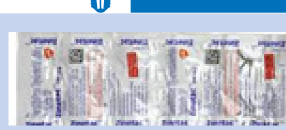
Therefore, the number of sides of a polygon $= \dfrac{\text{perimeter}}{\text{one side}}$

$= \dfrac{17.5}{2.5} = \dfrac{175}{25} = 7$ (Equal number of decimal digits in both dividend and divisor)

Hence, the polygon has 7 sides.

Think

The price of a tablet strip containing 30 tablets is ₹ 22.63 Then how will you find the price of each tablet?



Exercise 1.4

1. Simplify the following.

   (i) 0.6 √ 3 (ii) 0.90 √ 5 (iii) 4.08 √ 4 (iv) 21.56 √ 7

   (v) 0.564 √ 6 (vi) 41.36 √ 4 (vii) 298.2 √ 3

2. Simplify the following.

   (i) 5.7 √ 10 (ii) 93.7 ÷ 10 (iii) 0.9 √ 10 (iv) 301.301 ÷ 10

   (v) 0.83 √ 10 (vi) 0.062 √ 10

3. Simplify the following.

   (i) 0.7 √ 100 (ii) 3.8 √ 100 (iii) 49.3 √ 100 (iv) 463.85 ÷ 100

   (v) 0.3 √ 100 (vi) 27.4 √ 100

4. Simplify the following.

   (i) 18.9 √ 1000 (ii) 0.87 √ 1000 (iii) 49.3 ÷ 1000 (iv) 0.3 √ 1000

   (v) 382.4 √ 1000 (vi) 93.8 √ 1000

5. Simplify the following.

   (i) 19.2 √ 2.4 (ii) 4.95 √ 0.5 (iii) 19.11 √ 1.3 (iv) 0.399 √ 2.1

   (v) 5.4 ÷ 0.6 (vi) 2.197 √ 1.3

6. Divide 9.55 *kg* of sweet among 5 children. How much will each child get?

7. A vehicle covers a distance of 76.8 *km* for 1.2 *litre* of petrol. How much distance will it cover for one *litre* of petrol?

8. Cost of levelling a land at the rate of ₹15.50 *sq. ft* is ₹10,075. Find the area of the land.

9. The cost of 28 books are ₹1506.4. Find the cost of one book.

10. The product of two numbers is 40.376. One number is 14.42. Find the other number.

Objective type questions

11. $5.6 \div 0.5 = ?$

    (i) 11.4 (ii) 10.4 (iii) 0.14 (iv) 11.2

12. $2.01 \div 0.03 = ?$

    (i) 6.7 (ii) 67.0 (iii) 0.67 (iv) 0.067

13. $0.05 \div 0.5 = ?$

    (i)0.01 (ii) 0.1 (iii) 0.10 (iv) 1.0

Exercise 1.5

Miscellaneous Practice problems

1. Malini bought three ribbons of length 13.92 *m*, 11.5 *m* and 10.64 *m*. Find the total length of the ribbons?

2. Chitra has bought 10 *kg* 35 *g* of ghee for preparing sweets. She used 8 *kg* 59 *g* of ghee. How much ghee will be left?

3. If the capacity of a milk can is 2.53 *l*, then how much milk is required to fill 8 such cans?

4. A basket of orange weighs 22.5 *kg*. If each family requires 2.5 *kg* of orange, how many families can share?

5. A baker uses 3.924 *kg* of sugar to bake 10 cakes of equal size. How much sugar is used in each cake?

6. Evaluate: (i) 26.13×4.6 (ii) $3.628 + 31.73 - 2.1$

7. Murugan bought some bags of vegetables. Each bag weighs 20.55 *kg*. If the total weight of all the bags is 308.25 *kg*, how many bags did he buy?

8. A man walks around a circular park of distance 23.761 *m*. How much distance will he cover in 100 rounds?

9. How much 0.0543 is greater than 0.002?

10. A printer can print 15 pages per minute. How many pages can it print in 4.6 minutes?

Challenge Problems

11. The distance travelled by Prabhu from home to Yoga centre is 102 *m* and from Yoga centre to school is 165 *m*. What is the total distance travelled by him in kilometres (in decimal form)?

12. Anbu and Mala travelled from A to C in two different routes. Anbu travelled from place A to place B and from there to place C. A is 8.3 *km* from B and B is 15.6 *km* from C. Mala travelled from place A to place D and from there to place C. D is 7.5 *km* from A and C is 16.9 *km* from D. Who travelled more and by how much distance?

13. Ramesh paid ₹97.75 per hour for a taxi and he used 35 hours in a week. How much he has to pay totally as taxi fare for a week?

14. An Aeroplane travelled 2781.20 *kms* in 6 hours. Find the average speed of the aeroplane in *km/hr*.

15. Kumar's car gives 12.6 *km* mileage per litre. If his fuel tank holds 25.8 litres then how far can he travel?

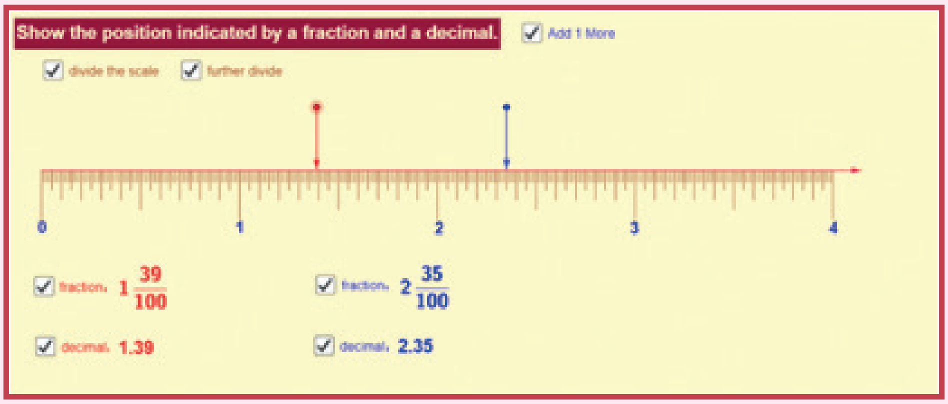
Summary

- To round a decimal
    - First underline the digit that is to be rounded. Then look at the digit to the right of the underlined digit.
    - If that digit is less than 5, then the underlined digit remains the same.
    - If that digit is greater than or equal to 5, add 1 to the underlined digit.
    - After rounding of, ignore all the digits after the underlined digit.
- Adding zeros at the right end of decimal digits will not change the value of the number.
- Zeros are added at the right end of decimal digits of a decimal number that are to be added or subtracted.
- The number of decimal digits in the product of two decimal numbers is equal to the sum of decimal digits that are multiplied.
- When a decimal number is multiplied by 10, 100 or 1000, the digits in the product are same as in the decimal number but the decimal point in the product is shifted to the right by as many places as there are zeros followed by 1.
- When a decimal number is divided by 10, 100 and 1000, the digits of the number (Dividend) and the obtained decimal number after division are the same but the decimal point in the obtained decimal number after division is shifted to the left by as many places as there are zeros followed by 1.

ICT Corner

Number system

Expected outcome



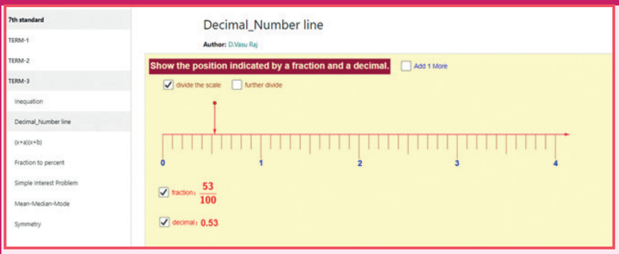
Step – 1

Open the Browser and type the URL Link given below (or) Scan the QR Code. GeoGebra work sheet named “Decimal_NumberLine” will open. Click on “divide the scale” and “further divide”

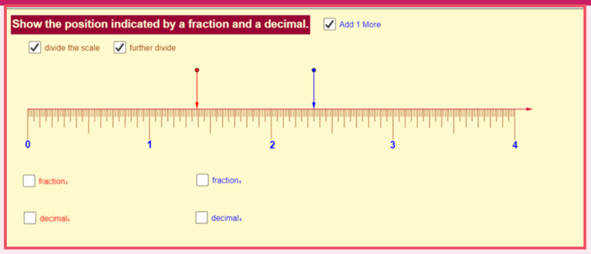
Step - 2

Move the arrows pointing downwards and click check box, see the fraction and decimals of the position. Also click “Add 1 more” to the given decimal number and check.

Step 1



Step 2



Browse in the link
Decimal Number line: https://www.geogebra.org/m/f4w7csup#material/nezvwyk6

or Scan the QR Code.

Chapter 2

PERCENTAGE AND SIMPLE INTEREST

Learning Objectives

- To understand the meaning of per cent.
- To convert a fraction into percentage and vice-versa.
- To convert a decimal number into percentage and vice-versa.
- To solve problems on percentage.
- To find simple interest by formula.
- To apply simple interest formula in different situations.

2.1 Introduction

We have already been introduced to the concepts such as 'Ratio and proportion', unitary method and its use in solving day-to-day application problems. Also, ratio has been explained as a method of comparison by division. One of the most common methods to compare two quantities is by using percentage.

Situation

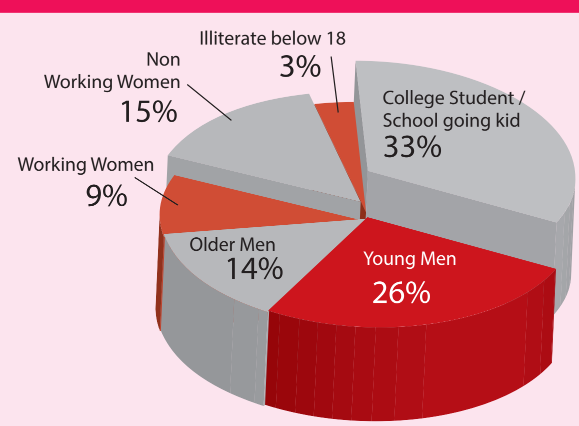
Geetha scored 475 marks out of 600 and Seetha scored 425 out of 500. Can we conclude Geetha has scored higher marks than Seetha? Is it right? Whom do you think has done better?

We cannot decide who has done better by just comparing the marks, they have scored because the maximum marks in both the cases are different.

To get an answer for these situations, we use "Percentage". We are going to see about "percentage" in this chapter.

MATHEMATICS ALIVE-Percentage in real Life

Advertisement of a shop



Internet user demographic profile

Per cent is derived from the Latin word 'Per centum' meaning 'per hundred'. Per cent is denoted by the symbol '%' and means hundredth too. That is 1% means 1 out of hundred or one hundredth which can be written as $1\% = \frac{1}{100} = 0.01$. It is read as 1 per cent.

In the same way, 50% means 50 out of hundred or fifty hundredth. That is $50\% = \frac{50}{100} = 0.50$

80% means 80 out of hundred or eighty hundredth. That is $80\% = \frac{80}{100} = 0.80$

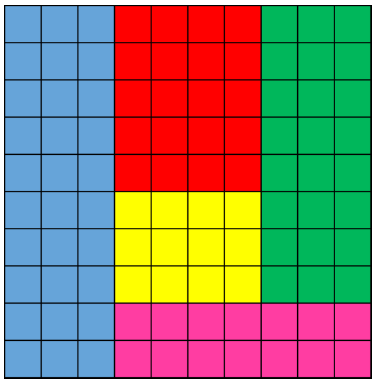
20% means 20 out of hundred or twenty hundredth. That is $20\% = \frac{20}{100} = 0.20$

To understand this let us do the following activity.

Activity

Take a 10 × 10 square grid to recall the previous knowledge on fraction. The grid is shaded using 5 different colours. The particulars related to blue colour shaded portion shown in the grid is given in the table below. Observe the grid and complete the table.

Colour	Number of Squares	Fraction	Percentage
Blue	30	$\frac{30}{100}$	30%
Red			
Yellow			
Green			
Pink			



From this we can understand that *percentage can be written as a fraction with denominator hundred.*

Try these

Find the percentage of children whose scores fall in different categories given in table below.

Category	Number of students	Fraction	Percentage
Below 60	25		
60 – 80	23		
81 – 90	42		
91 – 99	9		
Centum	1		
Total	100		

In all these examples, the total number of items add upto 100. Can we calculate those percentage of items if the total number do not add upto 100? Yes. We can find the percentage of items. In such cases we need to convert the given fractions to their equivalent fraction with denominator 100.

For example consider a 5×10 square grid.

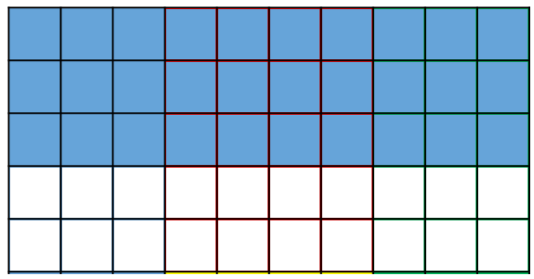


Fig 2.1

In the Fig 2.1, the blue shaded portion of the grid represents the fraction $\frac{30}{50}$. Which is equal to $\frac{60}{100}$ or 0.60 or 0.6 or 60%.

Try these

There are 50 students in class VII of a school. The number of students involved in these activities are:

Scout – 7	Red Ribbon Club – 6	Junior Red Cross – 9	Green Force – 3
Sports – 14	Cultural activity – 11		

Find the percentage of students who involved in various activities.

2.1.1 Converting Fraction to Percentage

All numbers which are represented using numerator and denominator are fractions. They can have any number as a denominator. If the denominator of the fraction is hundred then it can be very easily expressed as a percentage. Let us try to convert different fraction to percentage.

Example 2.1 Write $\frac{1}{5}$ as per cent.

Solution

We have $\frac{1}{5} = \frac{1}{5} \times \frac{100}{100} = \frac{1}{5} \times 100\% = \frac{100}{5}\% = 20\%$.

Example 2.2 Convert $\frac{7}{4}$ to per cent.

Solution

We have $\frac{7}{4} = \frac{7}{4} \times \frac{100}{100} = \frac{7}{4} \times 100\% = \frac{700}{4}\% = 175\%$.

Example 2.3 Out of 20 beads, 5 beads are red. What is the percentage of red.

Solution

We have $\frac{5}{20} = \frac{5}{20} \times \frac{100}{100} = \frac{5}{20} \times 100\% = \frac{500}{20}\% = 25\%$.

Example 2.4 Convert the fraction $\frac{23}{30}$ as per cent.

Solution

We have $\frac{23}{30} = \frac{23}{30} \times \frac{100}{100} = \frac{23}{30} \times 100\% = 76\frac{2}{3}\%$

From these examples we see that the percentage of proper fractions are less than 100 and that of improper fractions are more than 100.

Try these

Convert the fractions as percentage.

(i) $\frac{1}{20}$ (ii) $\frac{13}{25}$ (iii) $\frac{45}{50}$ (iv) $\frac{18}{5}$ (v) $\frac{27}{10}$ (vi) $\frac{72}{90}$

2.1.2 Converting percentage as fraction

A percentage is a number or ratio expressed as a fraction of 100. Here, let us try to convert different percentage to fraction.

Example 2.5 Write the following percentage into fraction.

(i) 60% (ii) 125% (iii) $\frac{3}{5}\%$ (iv) $\frac{15}{10}\%$ (v) $28\frac{1}{3}\%$

Solution

(i) $60\% = \frac{60}{100} = \frac{6}{10} = \frac{3}{5}$

(ii) $125\% = \frac{125}{100} = \frac{5}{4}$

(iii) $\frac{3}{5}\% = \frac{\frac{3}{5}}{100} = \frac{3}{500}$

(iv) $\frac{15}{10}\% = \frac{\frac{15}{10}}{100} = \frac{\frac{3}{2}}{100} = \frac{3}{200}$

(v) $28\frac{1}{3}\% = \frac{28\frac{1}{3}}{100} = \frac{\frac{85}{3}}{100} = \frac{17}{60}$

Try these

Convert the following percentage as fractions.

(i) 50% (ii) 75% (iii) 250%

(iv) $30\frac{1}{5}\%$ (v) $\frac{7}{20}\%$ (vi) 90%

Example 2.6 In a survey one out of five people said they preferred a particular brand of soap. Convert it into percentage?

Solution

Fraction $= \frac{1}{5}$

Percentage $= \frac{1}{5} \times 100\% = 20\%$

Example 2.7 75 students from a Government High school appeared for S.S.L.C. examination. 72 of them are declared passed in the examination. Find the percentage of students passed.

Solution

Total number of students = 75

Number of students declared passed = 72

$$\text{Percentage} = \frac{72}{75} \times 100\%$$

$$= \frac{24}{25} \times 100\%$$

$$= 24 \times 4\%$$

$$= 96\%.$$

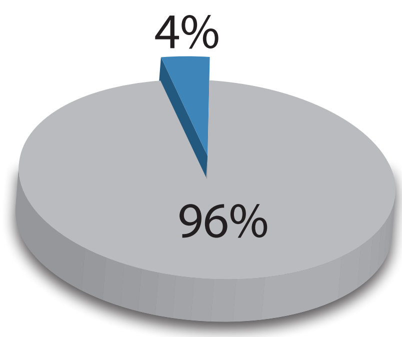


Fig 2.2

Note

All the parts of the whole when added together gives the whole or 100%. So out of 2 parts if we are given 1 part we can definitely find the other part. That is in the above example, if 96% of the students has passed means, 96 out of 100 has passed and the remaining (100-96)% 4% have failed.

Example 2.8 In a class of 50 students if 28 are girls and 22 are boys then express boys and girls in percentage.

Solution

Let us find the percentage of boys and girls. It is given in the form of table below.

	Number of students	Fraction	Make denominator as 100	Percentage
Girls	28	$\frac{28}{50}$	$\frac{28}{50} \times \frac{100}{100} = \frac{56}{100}$	56%
Boys	22	$\frac{22}{50}$	$\frac{22}{50} \times \frac{100}{100} = \frac{44}{100}$	44%
Total	50			100%

To find the percentage of boys and girls we can also use unitary method or multiply both numerator and denominator by a same number which makes denominator 100.

Example 2.9 There are 560 students in a school. Out of 560 students, 320 are boys. Find the percentage of girls in that school.

Solution

Total number of students = 560

Number of boys = 320

Number of girls = 560 – 320 = 240

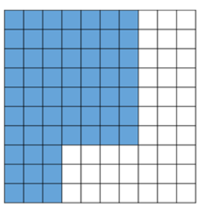
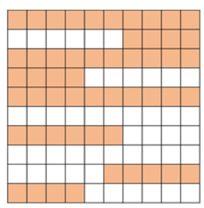
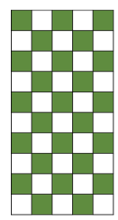
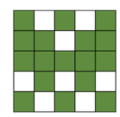
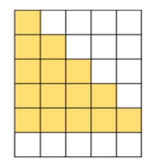
$$\text{Percentage} = \frac{240}{560}\times 100\% = \frac{24}{56}\times 100\%$$
$$= \frac{3}{7}\times 100\% = \frac{300}{7}\%$$
$$= 42.86\%$$

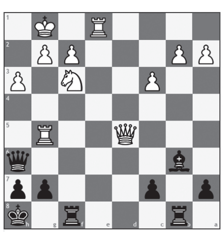
Think

1. What is the difference between 0.01 and 1%?
2. In a readymade shop there will be a board showing upto 50% off. Most of the people will realize that everything is half of its original price, Is that true?

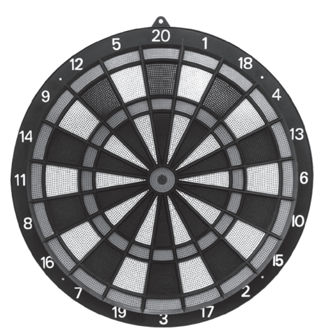
Exercise 2.1

1. In each of the following grid, find the number of coloured squares and express it as a fraction, decimal and percentage.

(i)

(ii)

(iii)

(iv)

(v)




2. A picture of chess board is given. (i) Find the percentage of the white coloured squares. (ii) Find the percentage of grey coloured squares. (iii) Find the percentage of the squares that have the pieces and (iv) The squares that do not have the pieces.



3. A picture of dart board is given. Find the percentage of white coloured portion and black coloured portion.
4. Write each of the following fraction as percentage.

   (i) $\frac{36}{50}$ (ii) $\frac{81}{30}$ (iii) $\frac{42}{56}$

   (iv) $2\frac{1}{4}$ (v) $1\frac{3}{5}$
5. Anbu scored 436 marks out of 500 in his exams. What was the percentage he scored?
6. Write each of the following percentage as fraction.

   (i) 21% (ii) 93.1% (iii) 151% (iv) 65% (v) 0.64%
7. Iniyan bought 5 dozen eggs. Out of that 5 dozen eggs, 10 eggs are rotten. Express the number of good eggs as percentage.
8. In an election, Candidate X secured 48% of votes. What fraction will represent his votes?
9. Ranjith's total income was ₹7,500. He saved 25% of his total income. Find the amount saved by him.

Objective type Questions

10. Thendral saved one fourth of her salary. Her savings percentage is

(i) $\frac{3}{4}\%$ (ii) $\frac{1}{4}\%$ (iii) 25% (iv) 1%

11. Kavin scored 15 out of 25 in a test. The percentage of his marks is

(i) 60% (ii) 15% (iii) 25% (iv) 15/25

12. 0.07% is

(i) $\frac{7}{10}$ (ii) $\frac{7}{100}$ (iii) $\frac{7}{1000}$ (iv) $\frac{7}{10,000}$

2.1.3 Converting Decimals to Percentage

We have seen how to convert fractions into per cent. Let us now learn how to convert decimals into per cent.

Example 2.10 Convert the given decimals to percentage.

(i) 0.85 (ii) 0.05 (iii) 0.3 (iv) 0.025 (v) 2.25

Solution:

(i) $0.85 = 0.85 \times 100\% = \frac{85}{100} \times 100\% = 85\%$

(ii) $0.05 = \frac{5}{100} \times 100\% = 5\%$

(iii) $0.3 = \frac{3}{10} \times 100\% = 30\%$

(iv) $0.025 = \frac{25}{1000} \times 100\% = \frac{25}{10}\% = \frac{5}{2}\%$ or 2.5%

(v) $2.25 = \frac{225}{100} \times 100\% = 225\%$

Try these

Convert these decimals to percentage.

(i) 0.25 (ii) 0.07 (iii) 0.8 (iv) 0.375 (v) 3.75

2.1.4 Converting Percentages to Decimals

We have seen conversion of decimals to percentage. We can also do the reverse process, that is when the percentage is given we can convert them to decimals.

Example 2.11 Convert the given percentage to decimals.

(i) 58% (ii) 8% (iii) 30% (iv) 120% (v) 1.25%

Solution:

(i) $58\% = \frac{58}{100} = 0.58$

(ii) $8\% = \frac{8}{100} = 0.08$

(iii) $30\% = \frac{30}{100} = 0.3$

(iv) $120\% = \frac{120}{100} = 1.2$

(v) $1.25\% = \frac{1.25}{100} = 0.0125$

From the above examples we see that to convert percentage to decimals we first convert it into fraction and get the solution.

Try these

Write these percentage as decimals.

(i) 3% (ii) 25% (iii) 80% (iv) 67% (v) 17.5% (vi) 135% (vii) 0.5%

Example 2.12 Malar bought 1.75 *m* of fabric from a roll of 25 *m*. Express the fabric bought in terms of percentage?

Solution

Total length of the fabric $= 25\,m$

Length of the fabric bought $= 1.75\,m$

Percentage of the fabric bought $= \frac{1.75}{25} \times \frac{100}{100} = \frac{175}{25 \times 100} = \frac{7}{100} = 7\%$

Area as Percentage

Percentages also help us to estimate the area.

Example 2.13 How many per cent in the *Fig. 2.3* is shaded as blue?

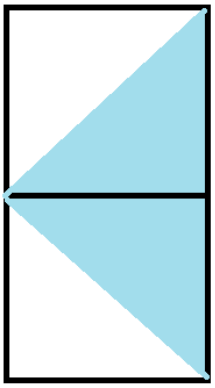


Fig. 2.3

Solution

The fraction in the Fig. 2.3, that is shaded $= \frac{2}{4} = \frac{1}{2}$.

That is half of the Fig. 2.3 is shaded blue.

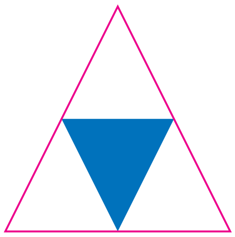
So, the percentage of shaded portion $= \frac{1}{2} \times 100\% = 50\%$

Thus, 50% of the Fig. 2.3 is shaded.

Exercise 2.2

1. Write each of the following percentage as decimal.
   (i) 21% (ii) 93.1% (iii) 151% (iv) 65% (v) 0.64%.
2 Convert each of the following decimal as percentage
   (i) 0.282 (ii) 1.51 (iii) 1.09 (iv) 0.71 (v) 0.858
3. In an examination a student scored 75% of marks. Represent the given percentage in decimal form?
4. In a village 70.5% people are literate. Express it as a decimal.
5. Scoring rate of a batsman is 86%. Write his strike rate as decimal.
6. The height of a flag pole in a school is 6.75*m*. Express it as percentage.

7. The weights of two chemical substances are 20.34 g and 18.78 g. Write the difference in percentage.
8. Find the percentage of shaded region in the following figure.



Objective type Questions

9. Decimal value of 142.5% is
   (i) 1.425 (ii) 0.1425 (iii) 142.5 (iv) 14.25
10. The percentage of 0.005 is
   (i) 0.005% (ii) 5% (iii) 0.5% (iv) 0.05%
11. The percentage of 4.7 is
   (i) 0.47% (ii) 4.7% (iii) 47% (iv) 470%

2.2 Percentage in Real Life

We have seen how percentage are used in comparison of quantities. We also learnt to convert fractions and decimals to percentage and vice-versa.

Now we shall see some situations that use percentage in real life such as 5% of income is allotted for saving; 20% of children's picture book is coloured green; a book distributor gets 10% of profit on every book sold by him. What can we conclude from these situations.

Percentage as a value

Example 2.14 There are 50 students in a class. If 14% are absent on a particular day, find the number of students present in the class.

Solution

Number of students absent on a particular day $= 14\ \%\ of\ 50$

$$= \frac{14}{100} \times 50 = 7$$

Therefore, the number of students present $= 50 - 7 = 43$ students.

Example 2.15 Kuralmathi bought a raincoat and saved ₹25 with discount of 20%. What was the original price of the raincoat?

Solution

Let the price of the raincoat (in ₹) be P. So 20% of $P = 25$

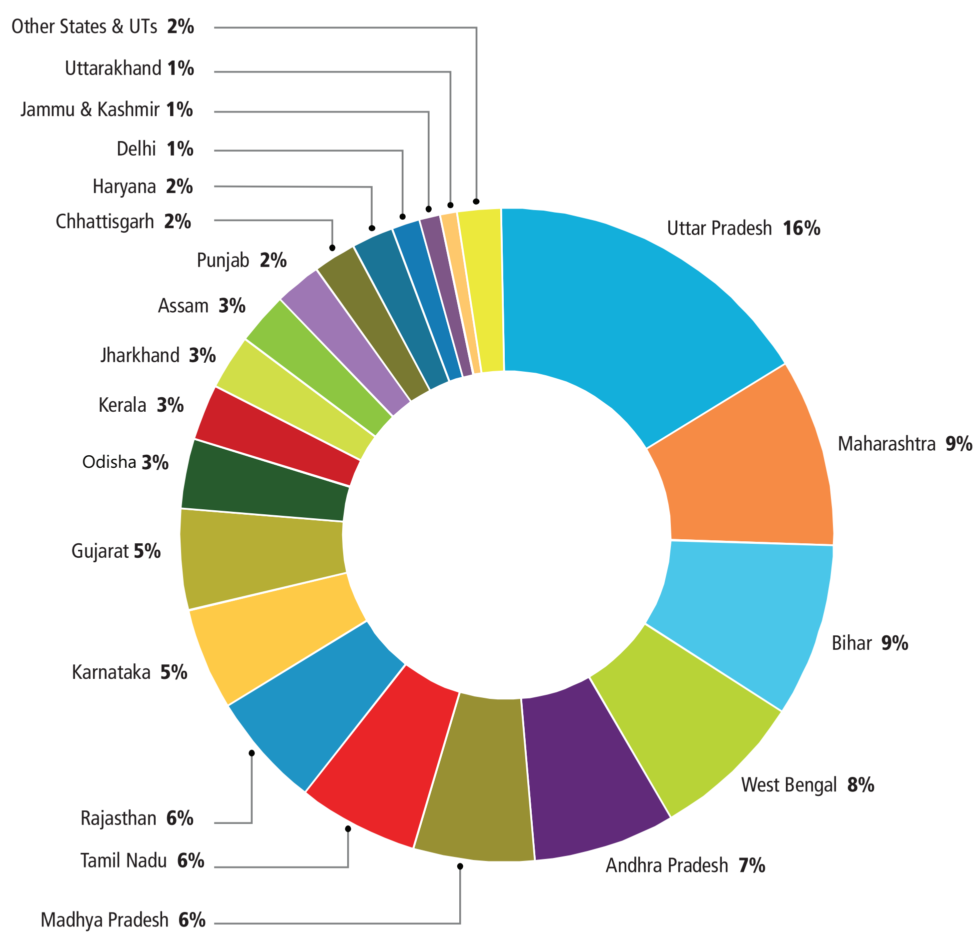
$$\frac{20}{100} \times P = 25$$

$$P = \frac{25 \times 100}{20} = 125$$

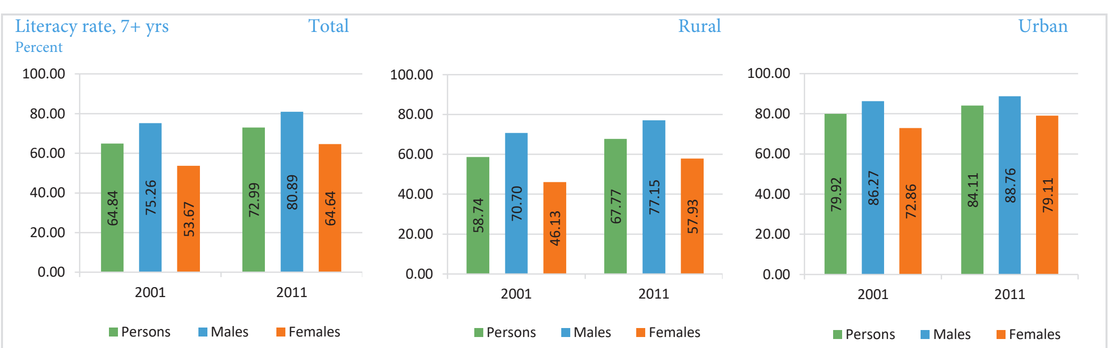
Therefore, the original price of the raincoat is ₹125.

DO YOU KNOW?

Population share of States and Union Territories, India: 2011



India



Total	2001	2011	Rural	2001	2011	Urban	2001	2011
Persons	64.84	72.99	Persons	58.74	67.77	Persons	79.92	84.11
Males	75.26	80.89	Males	70.70	77.15	Males	86.27	88.76
Females	53.67	64.64	Females	46.13	57.93	Females	72.86	79.11

http://censusindia.gov.in/2011-prov-results/data_files/india/

Example 2.16 An alloy contains 26 % of copper. What quantity of alloy is required to get 260 g of copper ?

Solution:

Let the quantity of alloy required be Q g

Then 26 % of Q =260 g

$$\frac{26}{100} \times Q = 260\ g$$

$$Q = \frac{260 \times 100}{26}\ g$$

$$Q = \frac{26000}{26}\ g$$

$$Q = 1000\ g$$

Therefore, the required quantity of alloy is 1000 g.

Ratios as percentage

Sometimes ingredients used to prepare food can be represented in the form of ratio. Let us see some examples.

Example 2.17 Kuzhal's mother makes dosa by mixing the batter made from 1 portion of Black gram with 4 portions of rice. Represent each of the ingredients used in the batter as percentage.

Solution

Representing ingredients used in the batter as ratio, we get, rice : Black gram $= 4:1$

Now, the total number of parts is $4+1=5$.

That is, $\frac{4}{5}$ portion of rice is mixed with $\frac{1}{5}$ portion of Black gram.

Thus, the percentage of rice would be $\frac{4}{5} \times 100\% = \frac{400}{5}\% = 80\%$

The percentage of Black gram would be $\frac{1}{5} \times 100\% = \frac{100}{5}\% = 20\%$

Example 2.18 A family cleans a house for pongal celebration by dividing the work in the ratio 1:2:3. Express each portion of work as percentage.

Solution

The total number of parts of the work is $1+2+3=6$

That is, the work is divided into 3 portions as $\frac{1}{6}, \frac{2}{6}$ and $\frac{3}{6}$.

Thus, the percentage of $\frac{1}{6}^{th}$ portion of work would be $\frac{1}{6} \times 100\% = \frac{100}{6}\% = 16\frac{2}{3}\%$

Similarly, the percentage of $\frac{2}{6}^{th}$ portion of work would be $\frac{2}{6} \times 100\% = \frac{200}{6}\% = 33\frac{1}{3}\%$

Similarly, the percentage of $\frac{3}{6}^{th}$ portion of work would be $\frac{3}{6}\times 100\% = \frac{300}{6}\% = 50\%$

Increase or decrease as Percentage

There are situations where we need to know the increase or decrease of a certain quantity as percentage. Let us see few examples.

Example 2.19 During Aadi sale the price of shirt decreased from ₹90 to ₹50. What is the percentage of decrease.

Solution

Original price = the price of the shirt before Aadi month

Amount of change = the decrease in the price = 90 – 50 = ₹40

Therefore, the percentage of decrease $= \frac{\text{Amount of change}}{\text{Original amount}} \times 100$

$$= \frac{40}{90}\times 100 = \frac{400}{9}$$

$$= 44\frac{4}{9}\%$$

Example 2.20 The number of literate persons in a city increased from 5 lakhs to 8 lakhs in 5 years. What is the percentage of increase?

Solution

Original amount = the number of literate persons initially = 5 lakhs

Amount of change = increase in the number of literate persons = 8 – 5 = 3 lakhs

Therefore, the percentage of increase $= \frac{\text{Amount of change}}{\text{Original amount}} \times 100$

$$= \frac{3}{5}\times 100 = 60\%$$

Try this

Level of water in a tank is increased from 35 litres to 50 litres in 2 minutes, what is the percentage of increase?

Profit or Loss as a Percentage

We have learnt already profit and loss of items. Now we will see how a profit or loss can be converted to percentage. That is, to find the profit % or loss %, we will see some examples.

Example 2.21 A shopkeeper bought a chair for ₹325 and sold it for ₹350. Find the profit percentage.

Solution

Profit per cent $= \frac{\text{Profit}}{\text{C.P}} \times 100$

$$= \frac{25}{325} \times 100 = \frac{100}{13} = 7\frac{9}{13}\%.$$

Example 2.22 A T-shirt bought for ₹110 is sold at ₹90. Find the loss percentage.

Solution

Cost price of T-shirt is ₹110 and Selling price is ₹90. So, the loss is ₹20

Hence, for ₹100 the loss is $\frac{20}{110} \times 100 = \frac{200}{11} = 18\frac{2}{11}\%$.

Example 2.23 An item was bought at ₹200 and sold at a loss of 4%. What is its selling price?

Solution

To find the Loss,

$$\text{Loss per cent} = \frac{\text{Loss}}{\text{C.P}} \times 100$$

$$4\% = \frac{\text{Loss}}{\text{C.P}} \times 100$$

$$4\% = \frac{\text{Loss}}{200} \times 100$$

Loss = ₹8.

$$\text{S.P} = \text{C.P} - \text{Loss}$$
$$= 200 - 8$$
$$= 192$$

Hence the selling price of the item is ₹192.

The world's population is growing by 1.10 % per year.
50.4 % of the world's population is male and 49.6 % is female.

Exercise 2.3

1. 14 out of the 70 magazines at the bookstore are comedy magazines. What percentage of the magazines at the bookstore are comedy magazines?
2. A tank can hold 50 litres of water. At present, it is only 30 % full. How many litres of water will fill the tank, so that it is 50 % full?
3. Karun bought a pair of shoes at 25% discount sale. If the amount he paid was ₹1000, then find the marked price.
4. An agent of an insurance company gets a commission of 5% on the basic premium he collects. What will be the commission earned by him if he collects ₹4800?
5. A biology class examined some flowers in a local Grass land. Out of the 40 flowers they saw, 30 were perennials. What percentage of the flowers were perennials?

6. Ismail ordered a collection of beads. He received 50 beads in all. Out of that 15 beads were brown. Find the percentage of brown beads?
7. Ramu scored 20 out of 25 marks in English, 30 out of 40 marks in Science and 68 out of 80 marks in mathematics. In which subject his percentage of marks is best?
8. Peter requires 50% to pass. If he gets 280 marks and falls short by 20 marks, what would have been the maximum marks of the exam?
9. Kayal scored 225 marks out of 500 in revision test 1 and 265 out of 500 marks in revision test 2. Find the percentage of increase in her score.
10. Roja earned ₹ 18,000 per month. She utilized her salary in the ratio 2:1:3 for education, savings and other expenses respectively. Express her usage of income in percentage.

2.3 Simple Interest

Selvam said that her sister took a loan from the bank to do her higher studies. The loan money that she borrowed from the bank is known as Sum or Principal. The borrower takes some time period to return the money to the bank. To use the money during a particular period of time, the borrower has to pay an additional amount to the bank. This is known as Interest. So to repay the money borrowed, the borrower has to add the principal and the interest.

That is, Amount = Principal+Interest

Interest is generally given in per cent for a period of 1 year per annum. Suppose the bank gives an amount of ₹ 100 at an interest rate of ₹ 8, it is written as 8% per year or per annum or in short 8% p.a. (per annum).

It means on every ₹ 100 borrowed, ₹ 8 is the required interest, to be paid for every one year. To understand this let us consider an example.

Selvam takes a loan of ₹ 10000 at 15% per year as rate of interest. Let us find the interest he has to pay at the end of 1 year.

Sum borrowed = ₹ 10000

Rate of interest = 15 % per year

This means if ₹ 100 is borrowed he has to pay ₹ 15 as interest.

So, for the borrowed amount of ₹ 10000, the interest for one year would be

$$\frac{15}{100}\times 10000 = ₹\,1500$$

So at the end of 1 year, he has to give an amount of = 10000 + 1500 = ₹ 11500.

Now we can write a general relation to find interest for one year. Take P as the principal or sum and r % as Rate per cent per annum. On every ₹ 100 borrowed the interest paid is ₹ r. Therefore, on ₹ P borrowed the interest paid for 1 year would be $\frac{P\times r}{100}$. If the amount is borrowed for more than 1 year then the interest is calculated for the total period during which the money is kept. This way of calculating interest for the total time period for the same Principal is known as simple interest.

We know that on a Principal of ₹P at r % rate of interest per year, the interest paid for 1 year is $\frac{P \times r}{100}$. Hence the interest 'I' paid for 'n' years would be $\frac{P \times n \times r}{100}$ or $\frac{Pnr}{100}$.

The amount we have to pay at the end of 'n' years is $A = P + I$.

Example 2.24 Find the simple interest on ₹25,000 at 8% per annum for 3 years?

Solution

Here, the Principal (P) = ₹25,000

Rate of interest (r) = 8% per annum

Time (n) = 3 years

$$\text{Simple Interest (I)} = \frac{Pnr}{100}$$

$$= \frac{25000 \times 3 \times 8}{100} = 6000$$

Hence, Simple Interest (I) is ₹6,000.

Try these

1. Arjun borrowed a sum of ₹5,000 from a bank at 5% per annum. Find the interest and amount to be paid at the end of three year.
2. Shanti borrowed ₹6,000 from a Bank for 7 years at 12% per annum. What amount will clear off her debt?

Example 2.25 Kumaravel has paid simple interest on a certain sum for 2 years at 10% per annum is ₹750. Find the sum.

Solution

Given the rate of interest (r) = 10% per annum

Time (n) = 2 years

$$\text{We know that Simple Interest (I)} = \frac{Pnr}{100}$$

$$750 = \frac{P \times 2 \times 10}{100}$$

$$\text{Therefore, Principal (P)} = \frac{750 \times 100}{2 \times 10} = 3750$$

Therefore, Kumaravel has borrowed a sum of ₹3,750.

Example 2.26 In what time will ₹ 5,600 amount to ₹ 6,720 at 6% per annum?

Solution:

Principal (P) = ₹ 5,600

Rate (r) = 6% per annum

Amount = ₹ 6,720

Amount = principal + interest

Interest = Amount – Principal

= 6720 – 5600 = 1120

$$\text{We know that Simple Interest (I)} = \frac{Pnr}{100}$$

$$1120 = \frac{5600 \times 6 \times n}{100}$$

$$\text{Therefore, } n = \frac{1120 \times 100}{5600 \times 6} = 3\frac{1}{3} \text{ years.}$$

Example 2.27 Sathish kumar borrowed ₹ 52,000 from a money lender at a particular rate of simple interest. After 4 years, he paid ₹ 79,040 to settle his debt. At what rate of interest he borrowed the money?

Solution

Principal (P) = ₹ 52,000

Time (n) = 4 years

Interest = Amount – Principal

$= 79,040 - 52,000 = 27,040$

$$\text{We find the Simple Interest (I)} = \frac{Pnr}{100}$$

$$\text{Therefore, } 27040 = \frac{52000 \times r \times 4}{100}$$

$$\text{Rate of interest he borrowed } (r) = \frac{27040 \times 100}{52000 \times 4} = 13\%.$$

Example 2.28 A sum of ₹ 46,000 was lent out at simple interest and at the end of 1 year and 9 months, the total amount was ₹ 52,440. Find the rate of interest per year.

Solution

A = P + I

I = A – P

= 52440 – 46000

= ₹ 6,440

r = ?

$$1 \text{ Year and 9 months} = 1\frac{9}{12} = 1\frac{3}{4} = \frac{7}{4}$$

$$\text{we know that, } I = \frac{Pnr}{100}$$

$$\text{Therefore, } 6440 = \frac{46000 \times r \times \frac{7}{4}}{100}$$

$$6440 = 46000 \times r \times \frac{7}{4} \times \frac{1}{100}$$

$$r = \frac{6440 \times 4 \times 100}{46000 \times 7}$$

$$= 8\%.$$

Example 2.29 A principal becomes ₹ 10,050 at the rate of 10% in 5 years. Find the principal.

Solution

A = ₹ 10,050

n = 5 years

r = 10%

P = ?

For calculating principal with the given data, we proceed as follows.
We know that,

$$I = \frac{Pnr}{100}$$

$$A = P + I$$

$$A = P + \frac{Pnr}{100}$$

$$A = P\left(1 + \frac{nr}{100}\right)$$

$$\text{Therefore, } 10,050 = P\left(1 + \frac{10 \times 5}{100}\right)$$

$$= P\left(1 + \frac{50}{100}\right)$$

$$= P\left(\frac{150}{100}\right)$$

$$= P\left(\frac{3}{2}\right)$$

$$\text{Therefore, } P = 10,050 \times \frac{2}{3} = 3350 \times 2 = 6700$$

Hence, Principal = ₹ 6,700.

Think

In simple interest, a sum of money doubles itself in 10 years. In how many years it will get triple itself.

Exercise 2.4

1. Find the simple interest on ₹ 35,000 at 9% per annum for 2 years?
2. Aravind borrowed a sum of ₹ 8,000 from Akash at 7% per annum. Find the interest and amount to be paid at the end of two years.
3. Sheela has paid simple interest on a certain sum for 4 years at 9.5% per annum is ₹ 21,280. Find the sum.
4. Basha borrowed ₹ 8,500 from a bank at a particular rate of simple interest. After 3 years, he paid ₹ 11,050 to settle his debt. At what rate of interest he borrowed the money?
5. In What time will ₹ 16,500 amount to ₹ 22,935 at 13% per annum?
6. In what time will ₹ 17800 amount to ₹ 19936 at 6% per annum?
7. A sum of ₹ 48,000 was lent out at simple interest and at the end of 2 years and 3 months the total amount was ₹ 55,560. Find the rate of interest per year.
8. A principal becomes ₹ 17,000 at the rate of 12% in 3 years. Find the principal.

Objective type Questions.

9. The interest for a principle of ₹ 4,500 which gives an amount of ₹ 5,000 at end of certain period is

   (i) ₹ 500 (ii) ₹ 200 (iii) 20% (iv) 15%

10. Which among the following is the simple interest for the principle of ₹ 1,000 for one year at the rate of 10% interest per annum?

    (i) ₹ 200 (ii) ₹ 10 (iii) ₹ 100 (iv) ₹ 1,000

11. Which among the following rate of interest yields an interest of ₹ 200 for the principle of ₹ 2,000 for one year.

    (i) 10% (ii) 20% (iii) 5% (iv) 15%

Exercise 2.5

Miscellaneous Practice problems

1. When Mathi was buying her flat she had to put down a deposit of $\frac{1}{10}$ of the value of the flat. What percentage was this?
2. Yazhini scored 15 out of 25 in a test. Express the marks scored by her in percentage.
3. Out of total 120 teachers of a school 70 were male. Express the number of male teachers as percentage.
4. A cricket team won 70 matches during a year and lost 28 matches and no results for two matches. Find the percentage of matches they won.
5. There are 500 students in a rural school. If 370 of them can swim, what percentage of them can swim and what percentage cannot?

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6. The ratio of Saral's income to her savings is 4 : 1. What is the percentage of money saved by her?

7. A salesman is on a commission rate of 5%. How much commission does he make on sales worth ₹ 1,500?

8. In the year 2015 ticket to the world cup cricket match was ₹ 1,500. This year the price has been increased by 18%. What is the price of a ticket this year?

9. 2 is what percentage of 50?

10. What percentage of 8 is 64?

11. Stephen invested ₹ 10,000 in a savings bank account that earned 2% simple interest. Find the interest earned if the amount was kept in the bank for 4 years.

12. Riya bought ₹ 15,000 from a bank to buy a car at 10% simple interest. If she paid ₹ 9,000 as interest while clearing the loan, find the time for which the loan was given.

13. In how much time will the simple interest on ₹ 3,000 at the rate of 8% per annum be the same as simple interest on ₹ 4,000 at 12% per annum for 4 years?

Challenge Problems

14. A man travelled 80 *km* by car and 320 *km* by train to reach his destination. Find what percent of total journey did he travel by car and what per cent by train?

15. Lalitha took a math test and got 35 correct and 10 incorrect answers. What was the percentage of correct answers?

16. Kumaran worked 7 months out of the year. What percentage of the year did he work?

17. The population of a village is 8000. Out of these, 80% are literate and of these literate people, 40% are women. Find the percentage of literate women to the total population?

18. A student earned a grade of 80% on a math test that had 20 problems. How many problems on this test did the student answer correctly?

19. A metal bar weighs 8.5 *kg*. 85% of the bar is silver. How many kilograms of silver are in the bar?

20. Concession card holders pay ₹ 120 for a train ticket. Full fare is ₹ 230. What is the percentage of discount for concession card holders?

21. A tank can hold 200 litres of water. At present, it is only 40 % full. How many litres of water to fill in the tank, so that it is 75 % full?

22. Which is greater $16\frac{2}{3}$ or $\frac{2}{5}$ or 0.17 ?

23. The value of a machine depreciates at 10% per year. If the present value is ₹ 1,62,000, what is the worth of the machine after two years.

24. In simple interest, a sum of money amounts to ₹ 6,200 in 2 years and ₹ 6,800 in 3 years. Find the principal and rate of interest.

25. A sum of ₹ 46,900 was lent out at simple interest and at the end of 2 years, the total amount was ₹ 53,466.Find the rate of interest per year.

26. Arun lent ₹ 5,000 to Balaji for 2 years and ₹ 3,000 to Charles for 4 years on simple interest at the same rate of interest and received ₹ 2,200 in all from both of them as interest. Find the rate of interest per year.

27. If a principal is getting doubled after 4 years, then calculate the rate of interest. (Hint : Let P = ₹ 100).

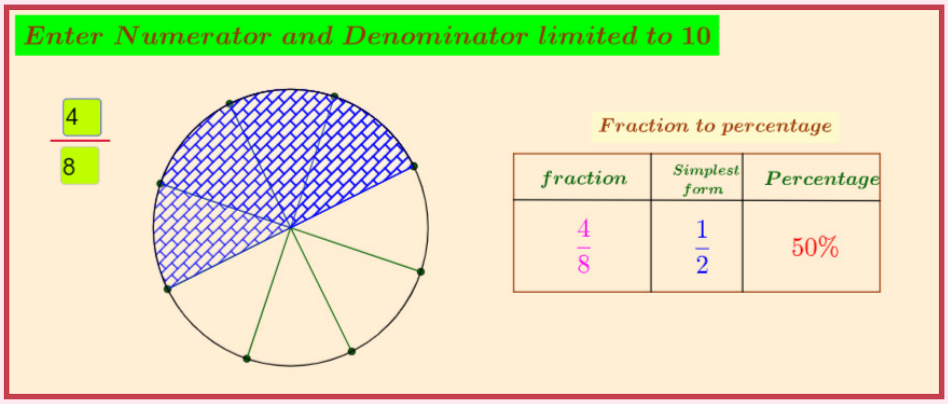
Summary

- Percentage is a fraction with denominator hundred.
- To convert a fraction as percentage, multiply the numerator and denominator of the fraction by 100.
- To convert a percentage as fraction, write it is as fraction with denominator 100.
- To convert decimals into percentage, multiply the given decimals by 100%
- Principal is the money borrowed or lent.
- Interest is the additional money given by the borrower to use the principal for a certain period of time
- Rate of interest is the percentage of the principal paid every year.
- Time is the period for which the money is borrowed or lent.
- Amount is the total money returned by the borrower to the lender after a certain period of time. It is found by using Amount = Principal + Interest.
- Simple interest can be calculated by using $\frac{P \times n \times r}{100}$, where P – Principal, r – Rate of Interest, n – Time.

ICT Corner

Percentage and Simple interest

Expected outcome



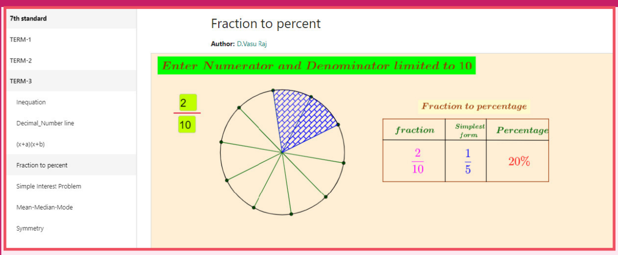
Step – 1

Open the Browser and type the URL Link given below (or) Scan the QR Code. GeoGebra work sheet named "Fraction to Percent" will open. Enter the numerator and denominator upto 10 and check the fraction and percentage.

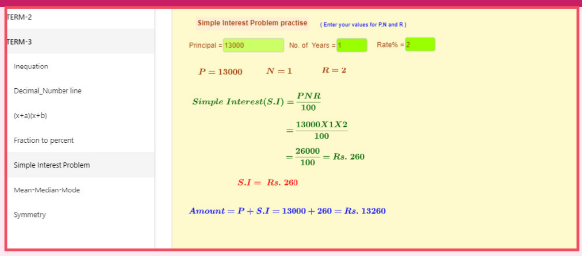
Step - 2

Click on "Simple interest problem" in the left side and enter P, N and R values in the box given. Find the Simple Interest and Amount and check your answer.

Step 1



Step 2



Browse in the link

Fraction to percent: https://www.geogebra.org/m/f4w7csup#material/frbmnsrw

or Scan the QR Code.

Chapter

3

ALGEBRA

Learning Objectives

- To understand the following identities through geometrical proof
  - $(x+a)(x+b) = x^2 + x(a+b) + ab$
  - $(a+b)^2 = a^2 + 2ab + b^2$
  - $(a-b)^2 = a^2 - 2ab + b^2$ and
  - $(a+b)(a-b) = a^2 - b^2$.
- Able to apply the identities in numerical problems.
- Able to recognise expressions that are factorisable.
- To represent inequalities in one variable, graphically.

3.1 Introduction to Identities

In earlier classes, we have learnt to construct algebraic expressions using exponential notations. For example, $x^2 + 3x + 2$ is an algebraic expression in the variable x. This can also be written as an equation $x^2 + 3x = -2$.

By substituting numerical values for x, we can verify this equation.

If $x = -2$, then L.H.S $= x^2 + 3x = (-2)^2 + 3(-2)$

$= 4 - 6$

$= -2 =$ R.H.S

Hence, this equation is true when $x = -2$.

If $x = -1$, then L.H.S $= x^2 + 3x = (-1)^2 + 3(-1)$

$= 1 - 3$

$= -2 =$ R.H.S

Hence, this equation is true when $x = -1$.

But when $x = 1$, L.H.S $= x^2 + 3x = (1)^2 + 3(1)$

$= 1 + 3$

$= 4 \neq$ R.H.S

Thus, this equation is not true when $x = 1$.

Thus, $x^2 + 3x = -2$ is an equation which is true only when x takes the values –1 and –2. Hence, an equation is true only for certain values of the variable in it.

It is not true for all values of the variables.

Now, consider the algebraic expression, $(a+b)^2 = a^2 + 2ab + b^2$. Let us try to find the values of the expression for the given values of a and b.

For example, when $a = 3$ and $b = 5$,

L.H.S = $(a+b)^2 = (3+5)^2 = 8^2 = 8 \times 8 = 64$

R.H.S = $a^2 + 2ab + b^2 = 3^2 + (2 \times 3 \times 5) + 5^2 = 9 + 30 + 25 = 64$

Thus, for $a = 3$ and $b = 5$, L.H.S = R.H.S

Similarly, when $a = 4$ and $b = 7$,

L.H.S = $(a+b)^2 = (4+7)^2 = 11^2 = 121$

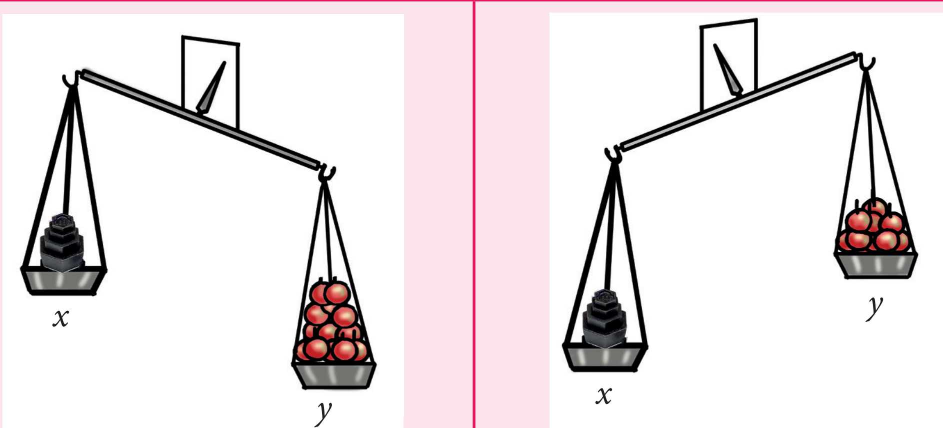
R.H.S = $a^2 + 2ab + b^2 = 4^2 + 2 \times 4 \times 7 + 7^2 = 16 + 56 + 49 = 121$

Also, for $a = 4$ and $b = 7$, L.H.S = R.H.S

Thus, we shall find that for any value of 'a' and 'b' L.H.S = R.H.S. Such an equality, which is true for every value of the variable in it is called an identity. Thus, we observe that the equation $(a+b)^2 = a^2 + 2ab + b^2$ is an identity.

In general, algebraic equalities which hold true for all the values of the variables are called **Identities**. Let us see the basic identities with geometrical proof.

MATHEMATICS ALIVE - Inequalities in Real Life



$x < y$	$x > y$

3.2 Geometrical Approach to Multiplication of Monomials

We have already learnt that the operation of multiplication can be modelled in different ways. The one that we use in this unit is the representation of multiplication as a product table that is similar to area.

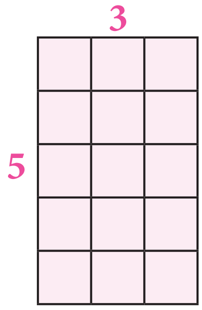


Fig. 3.1

For example, the product 5×3 can be represented as shown in *Fig. 3.1*, which has five rows and three columns and it comprises 15 small squares. From *Fig. 3.2*, it is also clear that the product of 5×3 is the same as the product of 3×5 [Since, multiplication is commutative].

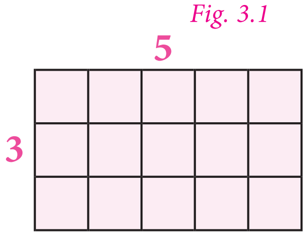
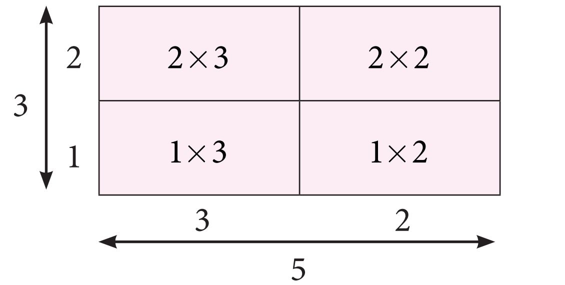


Fig. 3.2

Here, multiplication is represented using grid model. The same can be represented using area model.

The area model helps us to understand multiplication by decomposing the area of large rectangle into areas of smaller rectangles. Also the same example which is discussed above may be decomposed as



Think

Is it the only way to decompose the numbers representing length and breadth? Discuss.

$$5\times 3=(2\times 3)+(2\times 2)+(1\times 3)+(1\times 2)$$
$$=6+4+3+2$$
$$=15$$

This decomposition model is very useful when we are finding the product of large numbers.

Note

It can be practiced not to draw the area models in proportions to the numbers, since it stimulates abstract representation.

Let us try to extend this concept of multiplication for variables.

Take few tiles in rectangular shape of length 'x' and breadth 'y'. Also the area of one tile is xy ($\boxed{xy}$).

Arrange them as shown in *Fig. 3.3* and try to find its area.

xy	xy	xy
xy	xy	xy

Fig. 3.3

In *Fig. 3.3*, 6 tiles create a rectangular shape. Area of each tile is xy, hence the area of the shape is $6\times xy=6xy$. ...(1)

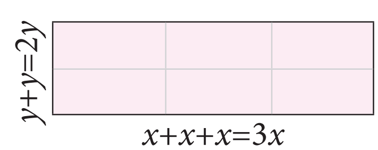


Fig. 3.4

The same area can also be found out by taking the length of the rectangular shape (*Fig. 3.4*) as $x+x+x=3x$ and breadth as $y+y=2y$.

Hence, its area = $3x\times 2y$. ...(2)

Also, the same area ($6xy$) can be represented by taking the length of the rectangular shape as $y+y=2y$ and the breadh as $x+x+x=3x$.

Then the area of the rectangular shape (*Fig. 3.5*) so obtained = $2y\times 3x$. ...(3)

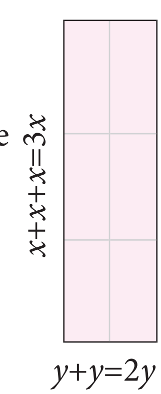
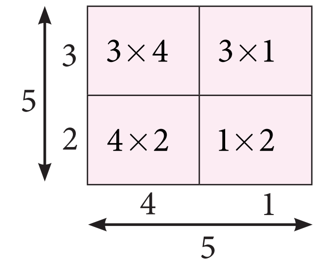


Fig. 3.5

Since all the areas are same, from (1), (2) and (3) we get,

$$6xy = 3x \times 2y = 2y \times 3x.$$

In our earlier discussion, we saw the geometrical representation of 5×3 (*Fig. 3.1*). Here we shall replace the second number 3 with 5. We get, 5×5.



We get the square number $5^2=25$, that may be represented as

$$5^2=(3\times4)+(1\times3)+(4\times2)+(1\times2)$$
$$=12+3+8+2$$
$$=25$$

Extending the same for variables in *Fig. 3.5*, the rectangular area is made into a square by making the length equal to breadth, that is $2y$ is taken as $3x$ which becomes the side of the square.

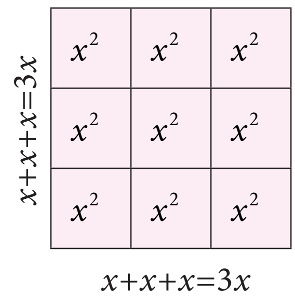


Fig. 3.6

Now in *Fig. 3.6* the sides are $3x$ and area of the square is $3x \times 3x$. ...(4)

Also, each area is x^2 and since there are 9 small squares the area of whole square is $9x^2$. ...(5)

Since both the areas [(4) and (5)] are equal, we get $3x\times3x=9x^2$.

Let us find the product of x and $(a+b)$. Look at the following figure.

In the *Fig. 3.7*, the two rectangles are combined together to form a new rectangle. Breadth of each of the rectangle are x and their lengths are a and b. So, the length and breadth of the new rectangle are $(a+b)$ and x respectively.

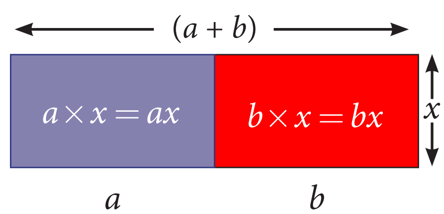


Fig. 3.7

Thus,

Area of Bigger rectangle = Area of rectangle 1 + Area of rectangle 2

Therefore, $\left(a+b\right)\times x=\left(a\times x\right)+\left(b\times x\right)$ [Distributive property]

$$=ax+bx.$$

Now, we can conclude that the multiplication of two or more monomials, as given below.

Note

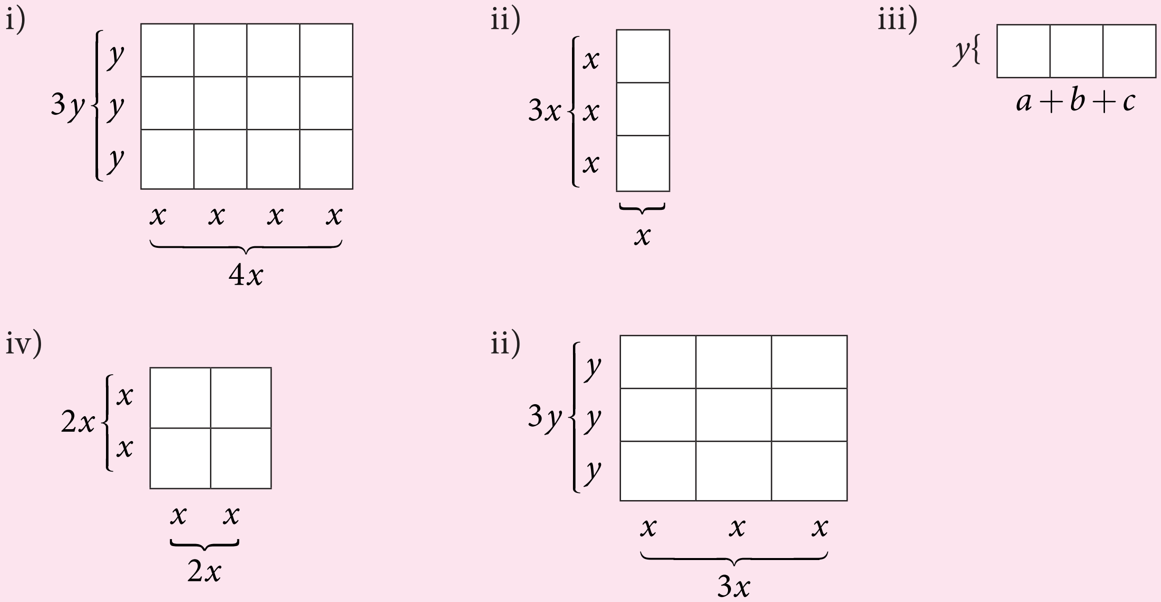
Product of monomials is also a monomial.

1. If the variables are same in both the monomials then,
    - multiply the numerical co-efficient of the two monomials, separately.
    - multiply the same variable by using the product rule of exponents, that is, $a^m\times a^n=a^{m+n}$. For example, $x\times x=x^1\times x^1=x^{1+1}=x^2$.

    Also, $3x^2\times2x^3=(3\times2)(x^2\times x^3)=6\times x^{2+3}=6x^5$.
2. If the variables are different in both the monomials, then multiply them by expressing it as a product of the variables. For example, $5x\times4y=(5\times4)\times(x\times y)=20xy$.

Try these

1. Observe the following figures and try to find its area, geometrically. Also verify the same by multiplication of monomial.



2. Let the length and breadth of a tile be x and y respectively. Using such tiles construct as many rectangles as you can and find out the length and breadth of the rectangles so formed such that its area is

(i) $12xy$ (ii) $8xy$ (iii) $9xy$

3.3 Geometrical proof of Identities

By using this concept of multiplication of monomials, let us try to prove the identities geometrically, which are very much useful in solving algebraic problems.

3.3.1 Identity-1: $(x+a)(x+b)=x^2+x(a+b)+ab$

Consider four regions. One region is square shaped with dimension 3×3 (Grey). Also, the other three regions are rectangle in shape with dimensions 4×3 (yellow), 3×2 (red) and 4×2 (Blue).

Arrange these four regions to form a rectangular shape as shown in the *Fig. 3.8*.

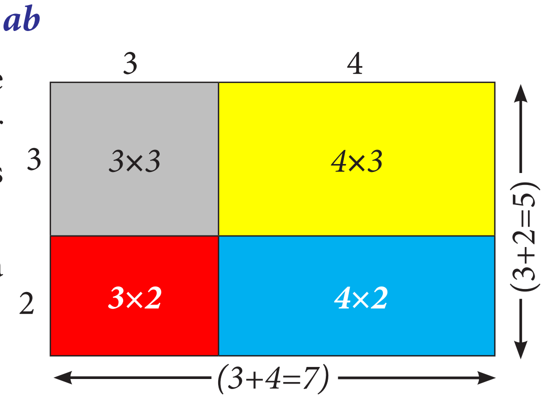


Fig. 3.8

By observing *Fig. 3.8*, we can note that,

Area of the bigger rectangle = Area of a square (Grey) + Area of Three rectangles

$$(3+4)(3+2)=(3\times3)+(4\times3)+(3\times2)+(4\times2)$$

$$(3+4)(3+2)=(3\times3)+3\times(4+2)+(4\times2) \qquad ...(1)$$

Where, LHS is $(3+4)(3+2)=7\times5=35$

RHS is $(3\times3)+3(4+2)+(4\times2)=9+(3\times6)+8$

$=9+18+8=35$

Therefore, LHS = RHS.

In the similar way as explained above, let us check for another set of four regions as shown in the *Fig. 3.9.*

> **Note**
>
> We know that,
>
> Area of rectangle
>
> $= length\times breadth$
>
> $=l\times b$
>
> Also, area of square
>
> $= side\times side$
>
> $=a\times a=a^2$

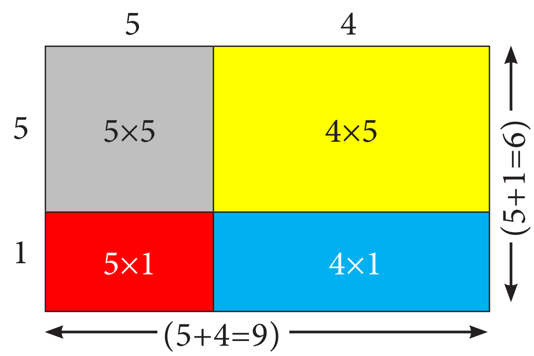


Fig. 3.9

By observing *Fig. 3.9*, we can note that,

Area of the bigger rectangle = Area of a square (Grey) + Area of Three rectangles

$$(5+4)\ (5+1) = (5\times5) + (5\times1) + (5\times4) + (1\times4)$$

$$(5+4)\ (5+1) = (5\times5) + 5(1+4) + (1\times4) \qquad ...(2)$$

Where, LHS is $(5+1)(5+4)=6\times9=54$

RHS is $5^2+5(1+4)+(1\times4)=25+(5\times5)+4$

$=25+25+4=54$

Therefore, LHS = RHS.

Thus, equation (1) and (2) is true for given set of any three values.

By generalising those three values as 'x', 'a' and 'b'

we get, $(x+a)(x+b)=(x\times x)+x(a+b)+(a\times b)$

That is, $(x+a)(x+b)=x^2+x(a+b)+ab$

Hence, $(x+a)(x+b)=x^2+x(a+b)+ab$ is an identity.

Now, let us prove this identity, geometrically.

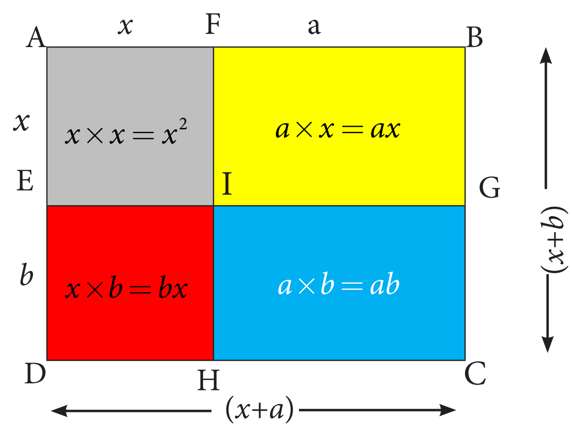


Fig. 3.10

Let one side of a rectangle be $(x+a)$ and the other side be $(x+b)$ units.

Then, the total area of the rectangle ABCD=length ×breadth=$(x+a)(x+b)$ ...(3)

From the Fig. 3.10, we can see that the

Area of the rectangle ABCD = area of the square AFIE + area of the rectangle FBGI + area of the rectangle EIHD + area of the rectangle IGCH

$$= x^2 + ax + bx + ab$$

$$= x^2 + x(a+b) + ab \qquad ...(4)$$

From (3) and (4) we get, $(x+a)(x+b) = x^2 + x(a+b) + ab$

Hence, $(x+a)(x+b) = x^2 + x(a+b) + ab$ is an identity.

Note

(i) In case if $b = -b$ then the identity is $(x+a)(x+(-b)) = x^2 + x(a+(-b)) + a(-b)$

$$(x+a)(x-b) = x^2 + x(a-b) - ab$$

(ii) If $a = -a$ then the identity is $(x+(-a))(x+b) = x^2 + x((-a)+b) + (-a)b$

$$(x-a)(x+b) = x^2 + x(b-a) - ab$$

(iii) If $a = -a$ *and* $b = -b$ then the identity is

$$(x+(-a))(x+(-b)) = x^2 + x((-a)+(-b)) + (-a)(-b)$$

$$(x-a)(x-b) = x^2 - x(a+b) + ab$$

Example 3.1 Simplify the following using the identity $(x+a)(x+b) = x^2 + x(a+b) + ab$:

(i) $(x + 3)(x + 5)$ (ii) $(y + 6)(y + 8)$ (iii) 43 x 36

Solution

(i) $(x + 3)(x + 5)$

Let us represent the expression geometrically, as shown in the *Fig. 3.11*.

In the rectangle with length $(x+5)$ and breadth $(x+3)$, we get,

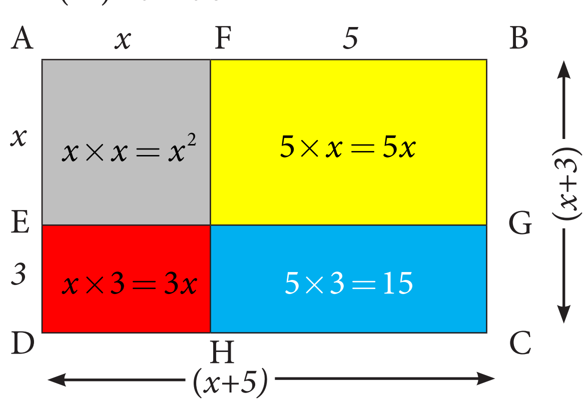


Fig. 3.11

Area of bigger rectangle = Area of a square + Area of three rectangles

Therefore,

$$(x+3)(x+5) = x^2 + 5x + 3x + 15$$

$$= x^2 + (5+3)x + 15$$

$$= x^2 + 8x + 15.$$

(ii) $(y + 6)(y + 8)$

Let us represent the expression geometrically, as shown in *Fig. 3.12*.

In the rectangle with length $(y+6)$ and breadth $(y+8)$ units, we get,

Area of bigger rectangle = Area of square + Area of three rectangles

Therefore,

$$(y+6)(y+8) = y^2 + 6y + 8y + 48$$

$$(y+6)(y+8) = y^2 + (6+8)y + 48$$

$$= y^2 + 14y + 48$$

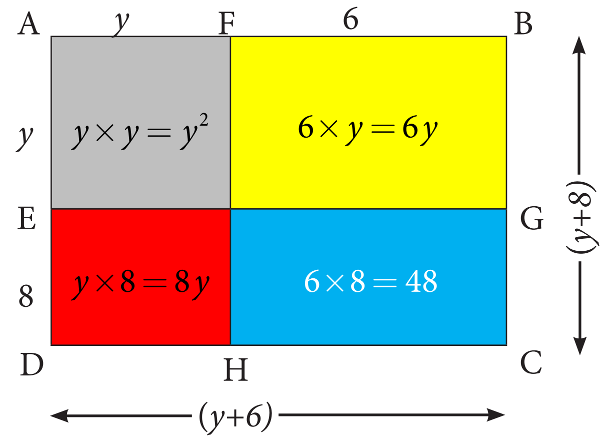


Fig. 3.12

(iii) $43 \times 36 = (40+3) \times (40-4)$

We know the identity

$$(x+a)(x+b) = x^2 + x(a+b) + ab$$

Taking, $x = 40$, $a=3$ and $b = -4$, we get

$$(40+3)(40-4) = 40^2 + 40(3-4) + 3(-4)$$

$$= 1600 + 40(-1) - 12$$

$$= 1600 - 40 - 12$$

$$= 1600 - 52$$

Therefore, $43 \times 36 = 1548$.

3.3.2 Identity-2: $(a+b)^2 = a^2 + 2ab + b^2$

Consider four regions. Two square shaped regions with the dimensions of 3 × 3 (Grey) and 2 × 2 (Blue). Also, there are two rectangle shaped regions and both are in the dimension of 3 × 2 (yellow).

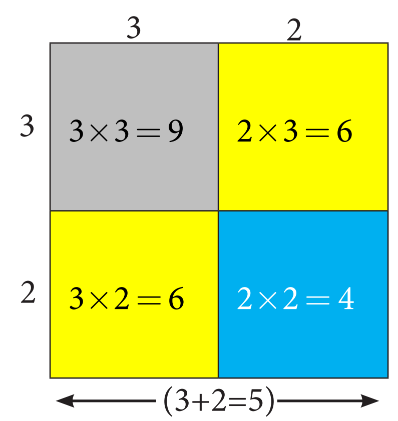


Fig. 3.13

Arrange them in a square shape as shown in the *Fig. 3.13*.

By observing the *Fig. 3.13*, we can note that

Area of the bigger square = Area of two small squares + Area of two rectangles.

$(3+2)^2 = 3^2 + (2\times3) + (3\times2) + 2^2 = 3^2 + (3\times2) + (3\times2) + 2^2$ [since, $2\times3 = 3\times2$]

Therefore, $(3+2)^2 = 3^2 + 2\times(3\times2) + 2^2$

where, L.H.S is $(3+2)^2 = 5^2 = 25$ ...(1)

R.H.S is $3^2 + 2\times(3\times2) + 2^2 = 9 + 12 + 4 = 25$. ...(2)

From (1) and (2), L.H.S = R.H.S.

Now, we can prove this for the variables a and b.

Let us take a square ABCD of side $(a+b)$, hence its area is $(a+b)(a+b) = (a+b)^2$

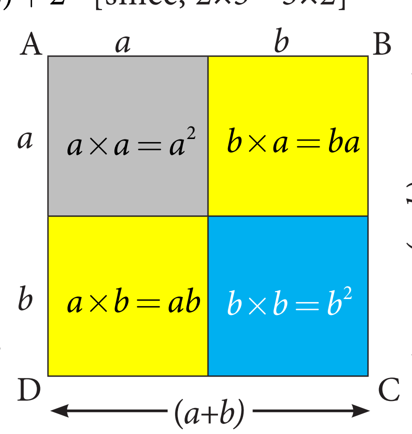


Fig. 3.14

By observing the *Fig. 3.14*,

Area of the bigger square ABCD = Area of two small squares (Grey and blue) + Area of two rectangles (yellow).

So, $(a+b)^2 = a^2 + ba + ab + b^2$

$$= a^2 + ab + ab + b^2 \qquad \text{(since, } ba = ab\text{)}$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

Hence proved.

Note

If, we substitute $b = a$ in the identity $(x+a)(x+b) = x^2 + x(a+b) + ab$, we get,

$$(x+a)(x+a) = x^2 + x(a+a) + a \times a$$

$$(x+a)^2 = x^2 + x(2a) + a^2$$

$(x+a)^2 = x^2 + 2ax + a^2$, which is similar to the identity $(a+b)^2 = a^2 + 2ab + b^2$.

Example 3.2 Simplify the following using the identity $(a+b)^2 = a^2 + 2ab + b^2$.

(i) $(2x+5)^2$ (ii) 21^2

Solution

(i) $(2x+5)^2$

Let the side of the square be $2x+5$ units. Then its area is $side \times side$, that is $(2x+5)^2$.

The geometrical representation of the given expression is as shown in *Fig. 3.15*.

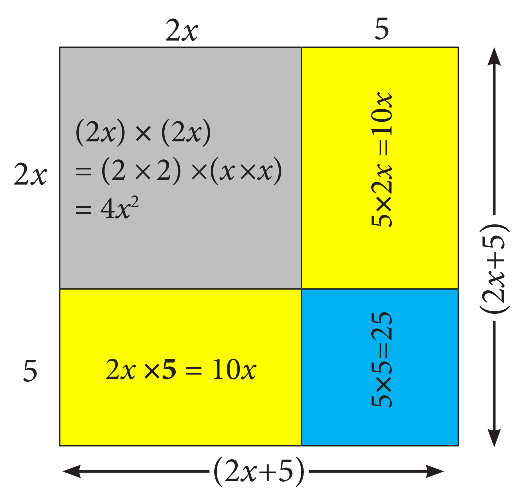


Fig. 3.15

Area of the bigger square } = Area of two squares + Area of two rectangles.

$$(2x+5)^2 = 4x^2 + 25 + 10x + 10x$$

$$= 4x^2 + (10+10)x + 25 \text{ (Adding like terms)}$$

$$= 4x^2 + 20x + 25.$$

(ii) 21^2

Let the side of the square be 21. Hence, its area is 21^2.

Consider 21^2 as $(20+1)^2$ which is one of the way to represent it geometrically as shown in *Fig. 3.16*.

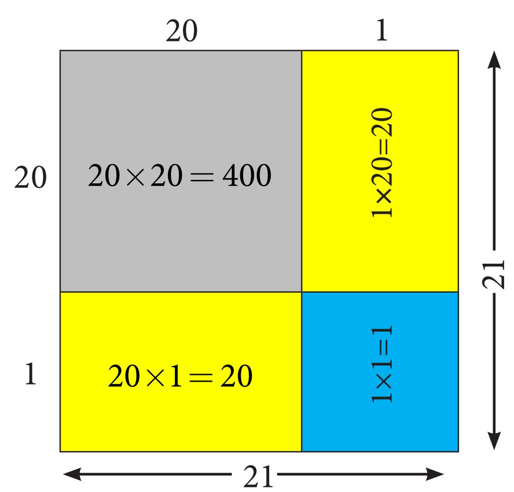


Fig. 3.16

Now,

Area of the bigger square } = Area of two squares + Area of two rectangles.

$$21^2 = 400+1+20+20=441.$$

Aliter method:

We know the identity, $(a+b)^2 = (a+b)(a+b)$

$= a^2 + 2ab + b^2$

	a	b
a	a^2	ab
b	ab	b^2

To find the value of 21^2,

We take, $21^2 = (20+1)^2$

$= (20+1)(20+1)$

Here, $a = 20$ and $b = 1$.

Therefore,

$a^2 + 2ab + b^2 = 20^2 + 2\times 20\times 1 + 1^2$

$= 400 + 40 + 1 = 441$.

3.3.3 Identity-3: $(a-b)^2 = a^2 - 2ab + b^2$

When we replace 'b' by '$-b$' in 'identity-2', we get a new identity.

	a	$-b$
a	a^2	$a\times -b = -ab$
$-b$	$a\times -b = -ab$	$-b\times -b = b^2$

We know that, $(a+b)^2 = a^2 + 2ab + b^2$

Taking 'b' as '$-b$' in 'identity-2', we get, $[a+(-b)]^2 = a^2 + 2a(-b) + (-b)^2$

$(a-b)^2 = a^2 - 2ab + (-b)(-b)$

Therefore, $(a-b)^2 = a^2 - 2ab + b^2$

Note that, when we change the sign of b, the sign of $2ab$ (second term) alone is changed.

Example 3.3 Using the identity $(a-b)^2 = a^2 - 2ab + b^2$, simplify the following:

(i) $(3x-5y)^2$ (ii) 47^2

Solution

(i) $(3x-5y)^2$

Put $a = 3x$ and $b = 5y$ in the identity $(a-b)^2 = a^2 - 2ab + b^2$, we get,

$(3x-5y)^2 = (3x)^2 - 2\times(3x)\times(5y) + (5y)^2$

$= 3^2\times x^2 - (2\times 3\times 5)xy + (5^2\times y^2)$

$= 9x^2 - 30xy + 25y^2$.

(ii) $47^2 = (50-3)^2$, substituting $a = 50$ and $b = 3$ in the identity $(a-b)^2 = a^2 - 2ab + b^2$, we get,

$(50-3)^2 = 50^2 - 2\times 50\times 3 + 3^2$

$= 2500 - 300 + 9$

$= 2509 - 300 = 2209$.

3.3.4 Identity-4: $(a + b)(a - b)= a^2-b^2$

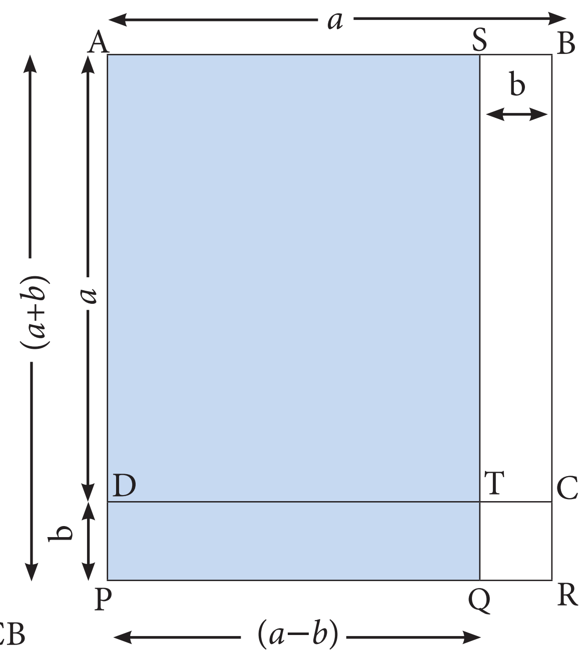


Fig. 3.17

In the given figure, AB = AD = a.

So, area of square ABCD $= a^2$.

Also, SB = DP = b.

Area of the rectangle SBCT = ab.

Similarly, area of the rectangle DPRC = ab.

Also, area of the square TQRC = b^2.

Area of the rectangle DPQT = $ab - b^2$.

Now, AS = PQ = $(a - b)$ and AP = SQ = $(a + b)$.

$$\left.\begin{array}{r}\text{Hence, area of}\\ \text{the rectangle}\\ \text{APQS (the shaded}\\ \text{rectangle)}\end{array}\right\} = \begin{array}{l}\text{area of square ABCD}\\ -\text{ area of rectangle STCB}\\ +\text{ area of rectangle DPQT}\end{array}$$

$$= a^2-ab + (ab-b^2)$$

$$= a^2-ab + ab - b^2$$

$$= a^2 - b^2$$

Hence, $(a + b)(a - b) = a^2 - b^2$

Example 3.4 Simplify by using the identity $(a + b)(a - b) = a^2 - b^2$.

(i) $(3x + 4)(3x - 4)$ (ii) 53×47.

Solution

(i) $(3x + 4)(3x - 4)$

Substitute, $a = 3x$ and $b = 4$ in the identity $(a+b)\times(a-b) = a^2 - b^2$, we get,

$(3x + 4)(3x - 4) = (3x)^2 - 4^2$

$(3^2 \times x^2) - 16 = 9x^2 - 16$.

(ii) $53 \times 47 = (50 + 3) \times (50 - 3)$.

Take, $a = 50$ and $b = 3$,

Substituting the values of 'a' and 'b' in the identity $(a + b)(a - b) = a^2 - b^2$), we get,

$= 50^2 - 3^2$

$= 2500 - 9$

$= 2491$.

Try this

Consider a square shaped paddy field with side of 48 m. A pathway with uniform breadth is surrounded the square field and the length of the outer side is 52 m. Can you find the area of the pathway by using identities?

3.4 Factorisation using identities

A factor of a number is the number that exactly divides a given number without leaving any remainder. For example, the factors of 4 are 1, 2 and 4; the factors of 12 are 1, 2, 3, 4, 6 and 12. And, the number can be expressed as product of its factors such as $12 = 1 \times 12$ or 2×6 or 3×4.

In the same way, algebraic expressions also have factors that divide the expression exactly. Given an algebraic expression, the **factors of that algebraic expression** are two or more expressions whose product is the given expression. For example, $x^2y = x \times x \times y$, hence the factors of the algebraic expression x^2y are x, x and y.

Consider the algebraic expression $x^2 - 4$.

Now, $x^2 - 4 = x^2 - 2^2 = (x + 2)(x - 2)$ [because $a^2 - b^2 = (a+b)(a-b)$]

Therefore, $(x + 2)$ and $(x - 2)$ are the factors of $x^2 - 4$.

Obviously, 1 is also a factor of $x^2 - 4$.

We know that, $x^2 = x \times x$. Similarly, $(x+5)^2 = (x+5)\times(x+5)$.

Thus, the factors of $(x + 5)^2$ are $(x + 5)$ and $(x + 5)$.

The process of writing an algebraic expression as the product of its factors is called factorisation.

We shall learn more to factorise algebraic expressions, using the basic identities.

Example 3.5 Express the following algebraic expressions as the product of its factors:
(i) ab^2 (ii) $-3pq^3$ (iii) $12m^2n^2p$

Solution

(i) $ab^2 = a \times b \times b$.

(ii) $-3pq^3 = -3 \times p \times q \times q \times q$.

(iii) $12\, m^2n^2p = 4 \times 3 \times m \times m \times n \times n \times p = 2 \times 2 \times 3 \times m \times m \times n \times n \times p$.

Example 3.6 Factorise by using the identity: $a^2 - b^2 = (a+b)(a-b)$

(i) $a^2 - 1$ (ii) $9k^2 - 25$ (iii) $64x^2 - 81y^2$

Solution

(i) $a^2 - 1 = a^2 - 1^2 = (a + 1)(a - 1)$ [since, $1^2 = 1 \times 1 = 1$]

Therefore, the factors of $a^2 - 1$ are $(a + 1)$ and $(a - 1)$.

(ii) $9k^2 - 25 = \left(3^2 \times k^2\right) - 5^2 = \left(3k\right)^2 - 5^2$ [since, $a^m \times b^m = (a \times b)^m$]

$= (3k + 5)(3k - 5)$ $\left[\text{since, } a^2 - b^2 = (a+b)(a-b)\right]$

Therefore, the factors of $9k^2 - 25$ are $(3k + 5)$ and $(3k - 5)$.

(iii) $64x^2 - 81y^2 = \left(8^2 \times x^2\right) - \left(9^2 \times y^2\right) = (8x)^2 - (9y)^2$

$= (8x + 9y)(8x - 9y)$

Therefore, the factors of $64x^2 - 81y^2$ are $(8x + 9y)$ and $(8x - 9y)$.

Example 3.7 Factorise $9x^2 + 30xy + 25y^2$

Solution

$$9x^2 + 30xy + 25y^2 = \left(3^2 \times x^2\right) + (2 \times 3 \times 5) \times xy + \left(5^2 \times y^2\right)$$

$$= (3x)^2 + 2 \times (3x) \times (5y) + (5y)^2$$

This expression is in the form of identity $a^2 + 2ab + b^2 = (a+b)^2$.

Hence, $(3x)^2 + 2 \times (3x) \times (5y) + (5y)^2 = (3x + 5y)^2$

$= (3x+5y)(3x+5y)$.

Therefore, the factors of $9x^2 + 30xy + 25y^2$ are $(3x+5y)$ and $(3x+5y)$.

Example 3.8 Factorise $4x^2 - 4xy + y^2$

Solution

$$4x^2 - 4xy + y^2 = 2^2 \times x^2 - (2 \times 2) \times xy + y^2$$

$$= (2x)^2 - 2 \times (2x) \times y + y^2$$

This expression is in the form of identity $a^2 - 2ab + b^2 = (a-b)^2$.

Say $2x = a$ and $y=b$, thus we have,

$$(2x)^2 - 2 \times (2x) \times y + y^2 = (2x - y)^2$$

$$= (2x - y)(2x - y)$$

Therefore, the factors of $4x^2 - 4xy + y^2$ are $(2x - y)$ and $(2x - y)$.

Think

Can we factorise the following expressions using any basic identities? Justify your answer.

(i) $x^2 + 5x + 4$

(ii) $x^2 - 5x + 4$

Example 3.9 Factorise: $x^2 - 2xy + y^2 - z^2$

Solution

$x^2 - 2xy + y^2 - z^2 = (x^2 - 2xy + y^2) - z^2 = (x - y)^2 - z^2$ [by identity-3]

Let, $(x - y) = a$ and $z = b$.

Therefore, $(x - y)^2 - z^2 = a^2 - b^2$

We know that, $a^2 - b^2 = (a + b)(a - b)$ [Identity-4]

Hence, $x^2 - 2xy + y^2 - z^2 = (x - y + z)(x - y - z)$.

Therefore, the factors of $x^2 - 2xy + y^2 - z^2$ are $(x - y + z)$ and $(x - y - z)$.

Activity

Here is an interesting number game to thrill your friends. Follow the steps.

1. Ask your friend to think a number in mind. Let the number be in between 1 to 10. (for example, let the number be 5).
2. Double the number and add to its square. (So, 10 + 25 = 35).
3. Add one to the result. (So, 35 + 1 = 36).
4. Ask the final resultant number.
5. The number will be a perfect square. Consider its base in the exponent form and reduce 1 from it. The result is the number in your friend's mind. ($36 = 6^2 \Rightarrow 6$ is the base; so, $6 - 1 = 5$).

   Remember the perfect square numbers and their corresponding base numbers $1 \to 1,\ 4 \to 2^2,\ 9 \to 3^2,\ 16 \to 4^2,\ 25 \to 5^2,\ 36 \to 6^2,\ 49 \to 7^2,\ 64 \to 8^2,\ 81 \to 9^2,$ $100 \to 10^2$

Exercise 3.1

1. Fill in the blanks.

   (i) $(p-q)^2$ = ---------------------------.

   (ii) The product of $(x + 5)$ and $(x - 5)$ is ---------------------.

   (iii) The factors of $x^2 - 4x + 4$ are ----------------------------.

   (iv) Express $24abc^2$ as product of its factors is ---------.

2. Say whether the following statements are True or False.

   (i) $(7x + 3)(7x - 4) = 49x^2 - 7x - 12$.

   (ii) $(a-1)^2 = a^2 - 1$.

   (iii) $(x^2 + y^2)(y^2 + x^2) = (x^2 + y^2)^2$.

   (iv) $2p$ is a factor of $8pq$.

3. Express the following as the product of its factors.

   (i) $24\ ab^2c^2$.

   (ii) $36\ x^3y^2z$.

   (iii) $56\ mn^2p^2$.

4. Using the identity $(x+a)(x+b) = x^2 + x(a+b) + ab$, find the following product.

   (i) $(x + 3)(x + 7)$ (ii) $(6a + 9)(6a - 5)$

   (iii) $(4x + 3y)(4x + 5y)$ (iv) $(8 + pq)(pq + 7)$

5. Expand the following squares, using suitable identities.

(i) $(2x+5)^2$ (ii) $(b-7)^2$ (iii) $(mn+3p)^2$ (iv) $(xyz-1)^2$

6. Using the identity $(a + b)(a - b) = a^2 - b^2$, find the following product.

(i) $(p + 2)(p - 2)$

(ii) $(1 + 3b)(3b - 1)$

(iii) $(4 - mn)(mn + 4)$

(iv) $(6x + 7y)(6x - 7y)$

7. Evaluate the following, using suitable identity.

(i) 51^2 (ii) 103^2 (iii) 998^2 (iv) 47^2

(v) 297×303 (vi) 990×1010 (vii) 51×52

8. Simplify: $(a+b)^2 - 4ab$

9. Show that $(m-n)^2 + (m+n)^2 = 2(m^2 + n^2)$

10. If $a + b = 10$ and $ab = 18$, find the value of $a^2 + b^2$

11. Factorise the following algebraic expressions by using the identity $a^2 - b^2 = (a+b)(a-b)$.

(i) $z^2 - 16$ (ii) $9 - 4y^2$ (iii) $25a^2 - 49b^2$ (iv) $x^4 - y^4$

12. Factorise the following using suitable identity.

(i) $x^2 - 8x + 16$

(ii) $y^2 + 20y + 100$

(iii) $36m^2 + 60m + 25$

(iv) $64x^2 - 112xy + 49y^2$

(v) $a^2 + 6ab + 9b^2 - c^2$

Objective Type Questions

13. If $a + b = 5$ and $a^2 + b^2 = 13$, then $ab = ?$

(i) 12 (ii) 6 (iii) 5 (iv) 13

14. $(5+20)(-20-5) = ?$

(i) -425 (ii) 375 (iii) -625 (iv) 0

15. The factors of $x^2 - 6x + 9$ are

(i) $(x - 3)(x - 3)$ (ii) $(x - 3)(x + 3)$ (iii) $(x + 3)(x + 3)$ (iv) $(x - 6)(x + 9)$

16. The common factors of the algebraic expressions ax^2y, bxy^2 and $cxyz$ is

(i) x^2y (ii) xy^2 (iii) xyz (iv) xy

3.5 Inequations

Earlier we have learnt to construct linear equations. Let us now study about 'Inequations'.

As per the norms the minimum age limit to own a driving licence is 18 years. Therefore, when Rajiv owns a driving licence, we can say that he is *at least* 18 years of age.

Now, if Rajiv's age is represented by x, then this statement can be written mathematically as $x \geq 18$, that is, we are not sure about his age, still we can say that his age is greater than or equal to 18 years.

Similarly, the statement *'This jug can hold up to 5 litres of water'* can be written mathematically as $x \leq 5$, where 'x' represents the volume of water in the jug.

We know that, the sum of the measures of any two sides of a triangle is greater than the measure of its third side. Thus, if the measure of the three sides are represented by a, b and c units, then we may write this fact as $a + b > c$, $a + c > b$ and $b + c > a$.

When $x \neq 10$, then either $x > 10$ or $x < 10$. That is, if the value of the variable 'x' is not 10, then the value of 'x' will either be greater than 10 or be less than 10.

An algebraic statement that shows two algebraic expressions being unequal is known as an algebraic inequation.

In general, when two expressions are compared, one might be; less than ($<$), less than or equal to ($\leq$), greater than ($>$), greater than or equal to ($\geq$) the other.

In an inequation, the algebraic expressions are connected by one out of the four signs of inequalities, namely, $>$, $\geq$, $<$ and $\leq$.

Try these

Construct inequations for the following statements:

1. Ramesh's salary is more than ₹25,000 per month.
2. This lift can carry maximum of 5 persons.
3. The exhibition will be there in town for at least 100 days.

3.5.1 Solving linear Inequations

A simple linear equation has atmost one solution, but a linear inequation may have many solutions.

To solve an inequation, it is necessary to know the set of values that the variable symbol can be substituted with. The collection of of all such values of an inequation is known as **solution** of the inequation.

For example, the solution of the equation $3x - 3 = 12$ is 5. (How?) Let us find the solution for the inequation $3x - 3 < 12$, where x is a natural number. Note that, the solutions of this inequation are 'natural numbers'. Now,

Add 3 on both sides, we get $3x - 3 + 3 < 12 + 3 \Rightarrow \quad 3x < 15$

Divide by 3 on both sides, we get $\frac{3x}{3} < \frac{15}{3} \Rightarrow x < 5$

Hence, x takes value which is less than 5 and x is a natural number. Thus, the solution for this inequation are 1, 2, 3 and 4.

Note

When 'x' is not restricted to a natural number, the solution includes all values less than 5.

Rules to solve inequation

While solving an inequation, the rules for transposition in case of inequalities are the same as for equations.

1. **Addition** of the same number on both sides of the inequation does not change the value of the inequation. Example: $10 > 5 \Rightarrow 10 + \mathbf{1} > 5 + \mathbf{1} \Rightarrow 11 > 6$.

   Extending this result, when adding any number 'x' instead of 1, the inequality $10 + x > 5 + x$ remains unchanged.

2. **Subtraction** of the same number from both sides of the inequation does not change the value of the inequation. Example: $10 > 5 \Rightarrow 10 - \mathbf{1} > 5 - \mathbf{1} \Rightarrow 9 > 4$.

   Extending this result, when subtracting any number x instead of 1, the inequatity $10 - x > 5 - x$ remains unchanged.

3. **Multiplication** by the same *positive number* on both sides of the inequation does not change the value of the inequation. Example: $10 > 5 \Rightarrow 10 \times \mathbf{2} > 5 \times \mathbf{2} \Rightarrow 20 > 10$.

   Similarly, when multiplying any positive number x instead of 2, the inequatity $10 \times x > 5 \times x$ remains unchanged.

4. **Division** by the same *non-zero positive number* on both sides of the inequation does not change the value of the inequation. Example: $10 > 5 \Rightarrow \frac{10}{5} > \frac{5}{5} \Rightarrow 2 > 1$.

   Similarly, when dividing any non zero positive number x instead of 5, the inequatity $\frac{10}{x} > \frac{5}{x}$ remains unchanged.

Note

When both sides of an inequation are multiplied or divided by the same *non-zero negative number*, the sign of inequality is reversed. For example, consider $3 < 12$.

Multiplying -1 on both sides, we get, $3 \times (-1) < 12 \times (-1)$

$$-3 < -12.$$

But, it is not true. Because, -3 is greater than -12.

So, $-3 > -12$. Note that the sign of inequality is reversed.

To generalize , when $x < y$ is multiplied by -1 on both sides , we get $-x > -y$.

DO YOU KNOW?

Interchanging the expressions on both sides of an equation, does not make any change in the equation. For example, $x+3=5$ and $5=x+3$ both are same.

But, if the expressions on both sides of an inequation are interchanged, the sign of inequality must be reversed.

For example, $30 > 20$ is the same as $20 < 30$ and $-18 < -9$ is the same as $-9 > -18$.

Example 3.10 Solve: $2x + 4 < 18$, where x is a natural number.

Solution

$$2x + 4 < 18$$

$$2x + 4 - 4 < 18 - 4 \quad \text{[Subtracting 4 from both sides]}$$

$$2x < 14 \quad \text{[Divide by 2 on both sides]}$$

$$x < 7$$

Since the solution belongs to natural numbers, that are less than 7, we take the values of the x as 1, 2, 3, 4, 5 and 6.

Therefore, the solutions are 1, 2, 3, 4, 5 and 6.

Example 3.11 Solve: $5 - 7x \geq 33$, where x is an integer.

Solution

$$5 - 7x \geq 33$$

$$5 - 5 - 7x \geq 33 - 5 \quad \text{[Subtracting 5 from both sides]}$$

$$-7x \geq 28$$

$$\frac{-7x}{-7} \geq \frac{28}{-7} \quad \text{[Dividing both sides by -7]}$$

$x \leq -4$ [since, it is divided by a negative number, the inequality is reversed]

Since, solution belongs to the set of integers, that are less than –4, we take the values of x as –4, –5, –6, ...

Therefore, the solutions are –4, –5, –6,... .

Think

Hameed saw a stranger in the street. He told his parent, “The stranger’s age is between 40 to 45 years, and his height is between 160 to 170 *cm*”.

Convert the above verbal statement into an algebraic inequations by using x and y as variables of age and height.

Example 3.12 If one worker earns ₹200 per day, how many workers can be employed within a monthly budget of ₹3 Lakh?

Solution

Let the number of workers be x.

Then, the wages that x workers will earn per day = ₹$200x$

The wages that x workers will earn per month= ₹$(200x \times 30)$= ₹$6000x$

Given that, this amount cannot exceed ₹300000.

Otherwise, it can be written as $6000x \leq 300000$

$$\frac{6000x}{6000} \leq \frac{300000}{6000} \quad \text{[Divide by 6000 on both sides]}$$

$$x \leq 50$$

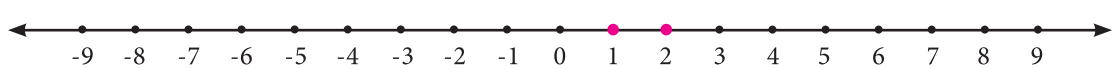
Thus, up to 50 workers can be employed on a monthly budget of ₹300000.

3.5.2 Graphical representation of Inequation

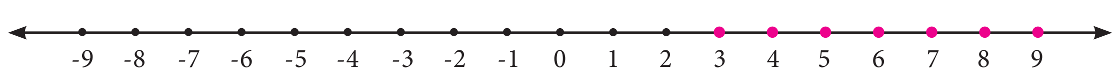
The solutions of an inequation can be represented on the number line by marking the true values of solutions with different colour on the number line.

Look into the following inequations and its graphical representation on number line. Here, we consider the solution belongs to natural numbers. That is, each and every value of the solution is a natural number.

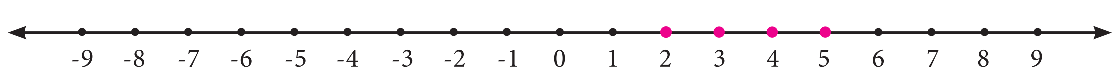
1. When $x < 3$, the solution in natural numbers are 1 and 2. Its graph on number line is shown below:



2. When $x \geq 3$, the solutions are natural numbers 3, 4, 5, ... and its graph is as shown below:



3. To mark the values represented by the inequation $2 \leq x \leq 5$, the solutions are set of natural numbers 2, 3, 4 and 5 and its graph is as given below:



Example 3.13 Represent the solutions $-8 < 2x < 10$ in a number line, where x is a natural number.

Solution

$-8 < 2x < 10$

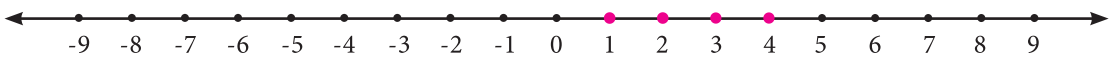
$$\frac{-8}{2} < \frac{2x}{2} < \frac{10}{2} \quad \text{[Dividing the inequation by 2]}$$

$-4 < x < 5$

Note

Since the solution is restricted to natural numbers, $-3, -2, -1$ and 0 have not been marked as solutions.

Since the solution belongs to the set of natural numbers, the solutions are 1, 2, 3 and 4. It's graph on the number line is shown below:

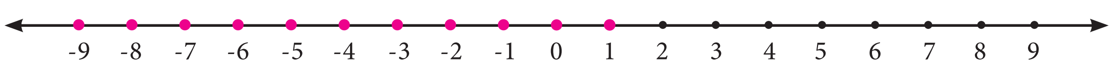


Example 3.14 Represent the solutions of $3x + 9 \leq 12$ in a number line, where x is an integer.

Solution

$$
\begin{aligned}
3x + 9 &\leq 12 \\
\frac{3x}{3} + \frac{9}{3} &\leq \frac{12}{3} && \text{[Dividing the inequation by 3 on both sides]} \\
x + 3 &\leq 4 \\
x + 3 - 3 &\leq 4 - 3 && \text{[Subtracting 3 from both sides]} \\
x &\leq 1
\end{aligned}
$$

Since the solution belongs to integers, the solutions are 1, 0, −1, −2, …. It's graph on the number line is shown below:

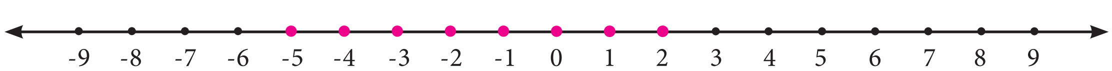


Example 3.15 Solve the inequation: $-2 \leq z + 3 \leq 5$, where z is an integer. Also, represent the solution, graphically.

Solution

$$
\begin{aligned}
-2 \leq\ & z + 3 \leq 5 \\
-2 - 3 \leq\ & z + 3 - 3 \leq 5 - 3 && \text{[Subtracting −3 from the inequation]} \\
-5 \leq\ & z \leq 2
\end{aligned}
$$

Since the solution belongs to integers, the solutions are −5, −4 , −3, −2, −1, 0, 1 and 2. It's graph on the number line is shown below:



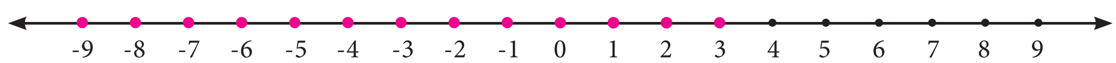
Example 3.16 Solve graphically: $6y - 5 \leq 2y + 7$, where y is an integer.

Solution

$$
\begin{aligned}
6y - 5 &\leq 2y + 7 \\
6y - 2y - 5 &\leq 2y - 2y + 7 && \text{[Subtracting } 2y \text{ from both sides]} \\
4y - 5 &\leq 7 \\
4y - 5 + 5 &\leq 7 + 5 && \text{[Adding 5 on both sides]}
\end{aligned}
$$

$$4y \leq 12$$
$$\frac{4y}{4} \leq \frac{12}{4} \quad \text{[Dividing by 4 on both sides]}$$
$$y \leq 3$$

Since the solution belongs to integers, the solutions are 3, 2, 1, 0, −1, −2,..... It's graph on the number line is shown below:



Exercise 3.2

1. Given that $x \geq y$. Fill in the blanks with suitable inequality signs.

   (i) $y \; \square \; x$

   (ii) $x + 6 \; \square \; y + 6$

   (iii) $x^2 \; \square \; xy$

   (iv) $-xy \; \square \; -y^2$

   (v) $x - y \; \square \; 0$

2. Say True or False.

   (i) Linear inequation has almost one solution.

   (ii) When x is an integer, the solutions for $x \leq 0$ are $-1, -2, \ldots$

   (iii) An inequation, $-3 < x < -1$, where x is an integer, cannot be represented in the number line.

   (iv) $x < -y$ can be rewritten as $-y < x$

3. Solve the following inequations.

   (i) $x \leq 7$, where x is a natural number.

   (ii) $x - 6 < 1$, where x is a natural number.

   (iii) $2a + 3 \leq 13$, where a is a whole number.

   (iv) $6x - 7 \geq 35$, where x is an integer.

   (v) $4x - 9 > -33$, where x is a negative integer.

4. Solve the following inequations and represent the solution on the number line:

   (i) $k > -5$, k is an integer.

   (ii) $-7 \leq y$, y is a negative integer.

   (iii) $-4 \leq x \leq 8$, x is a natural number

   (iv) $3m - 5 \leq 2m + 1$, m is an integer.

5. An artist can spend any amount between ₹ 80 to ₹ 200 on brushes. If cost of each brush is ₹ 5 and there are 6 brushes in each packet, then how many packets of brush can the artist buy?

Objective Type Questions

6. The solutions of the inequation $3 \le p \le 6$ are (where p is a natural number)

   (i) 4, 5 and 6 (ii) 3, 4 and 5 (iii) 4 and 5 (iv) 3, 4, 5 and 6

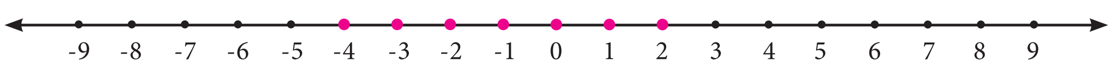
7. The solution of the inequation $5x + 5 \le 15$ are (where x is a natural number)

   (i) 1 and 2 (ii) 0, 1 and 2 (iii) 2, 1, 0, −1, −2.. (iv) 1, 2, 3..

8. The cost of one pen is ₹ 8 and it is available in a sealed pack of 10 pens. If Swetha has only ₹ 500, how many packs of pens can she buy at the maximum?

   (i) 10 (ii) 5 (iii) 6 (iv) 8

9. The inequation that is represented on the number line as shown below is ________



   (i) $-4 < x < 0$ (ii) $-4 \le x \le 0$ (iii) $-4 < x \le 0$ (iv) $-4 \le x < 0$

Exercise 3.3

Miscellaneous Practice problems

1. Using identity, find the value of

   (i) $(4.9)^2$

   (ii) $(100.1)^2$

   (iii) $(1.9)\times(2.1)$

2. Factorise : $4x^2 - 9y^2$

3. Simplify using identities (i) $(3p+q)(3p+r)$ (ii) $(3p+q)(3p-q)$

4. Show that $(x+2y)^2 - (x-2y)^2 = 8xy$

5. The pathway of a square paddy field has 5 m width and length of its side is 40 m. Find the total area of its pathway. (Note: Use suitable identity)

Challenge Problems

6. If $X = a^2 - 1$ and $Y = 1 - b^2$, then find $X+Y$ and factorize the same.

7. Find the value of $(x-y)(x+y)(x^2+y^2)$

8. Simplify $(5x-3y)^2-(5x+3y)^2$

9. Simplify: (i) $(a+b)^2-(a-b)^2$ (ii) $(a+b)^2+(a-b)^2$

10. A square lawn has a 2 m wide path surrounding it. If the area of the path is 136 m^2, find the area of lawn.

11. Solve the following inequalities.

(i) $4n + 7 \geq 3n + 10$, n is an integer.

(ii) $6(x + 6) \geq 5(x - 3)$, x is a whole number.

(iii) $-13 \leq 5x + 2 \leq 32$, x is an integer.

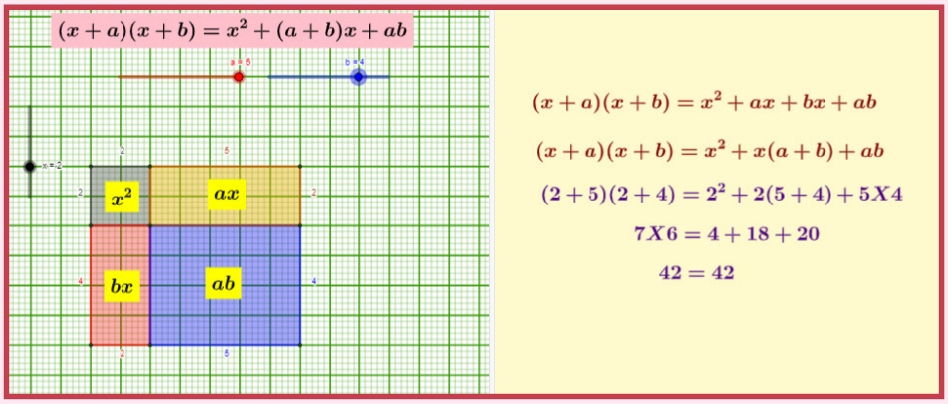
Summary

- The following identities are proved geometrically:
  - $(x + a)(x + b) = x^2 + x(a+b) + ab$
  - $(a+b)^2 = a^2 + 2ab + b^2$
  - $(a-b)^2 = a^2 - 2ab + b^2$ and
  - $(a + b)(a - b) = a^2 - b^2$.
- The factors of an algebraic expression is two or more expressions whose product is the given expression.
- The process of writing an algebraic expression as the product of its factors is called factorisation.
- An algebraic statement that shows two algebraic expression being unequal is known as an algebraic inequation.
- The algebraic expressions are connected with any one of the four signs of inequalities, namely, $>, \geq, <$ and $\leq$.
- When both sides of an inequation are added, subtracted, multiplied and divided by the same non-zero positive number, the inequality remains the same.
- When both sides of an inequation are multiplied or divided by the same non-zero negative number, the sign of inequality is reversed. For example, $x < y \Rightarrow -x > -y$.
- The solutions of an inequation can be represented on the number line by marking the true values of solutions with different colour on the number line.

ICT Corner

Algebra

Expected outcome



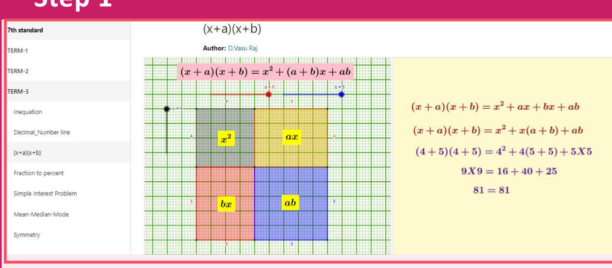
Step – 1

Open the Browser and type the URL Link given below (or) Scan the QR Code. GeoGebra work sheet named "(x+a)(x+b)" will open. Drag the sliders to change x, a, b values. Check the steps on right side.

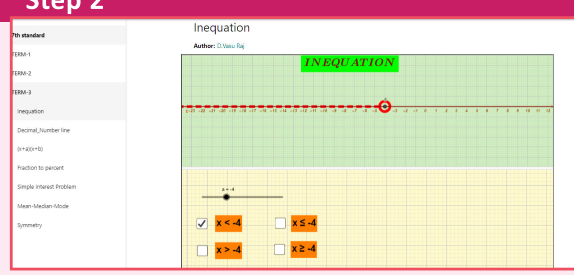
Step - 2

After completing Click on "Inequation" in the left side. Move the slider below to change "a" value.Click on the check boxes to see respective inequations on the number line.

Step 1



Step 2



Browse in the link

(x+a)(x+b): https://www.geogebra.org/m/f4w7csup#material/nguv3yey

or Scan the QR Code.

Chapter 4

GEOMETRY

Learning Objectives

- To recall the types of symmetry through diagrams.
- To learn symmetry through transformation (Translation, reflection and rotation).
- To construct circles and concentric circles.

Recap

In class VI we have learnt the concept of symmetry. Now we shall recall them.

Line of Symmetry

Look at the *Fig. 4.1* given on the right.

In each figure a line divides the figure into two identical halves. Such figures are symmetrical about the line. The line that divides any figure into two equal halves such that each half exactly coincides with the other is known as the line of symmetry or axis of symmetry.

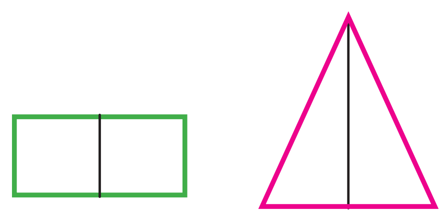


Fig. 4.1

A figure may have one, two, three or more lines of symmetry. Some figures which has lines of symmetry are shown in *Fig. 4.2* .

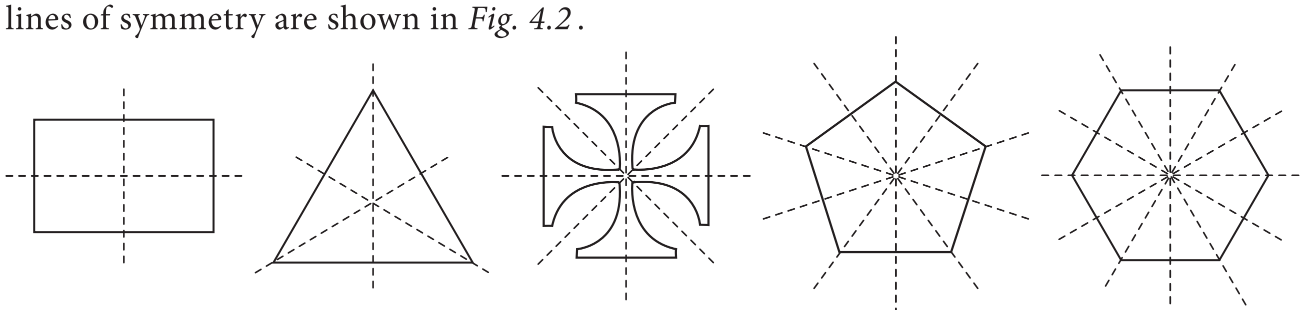
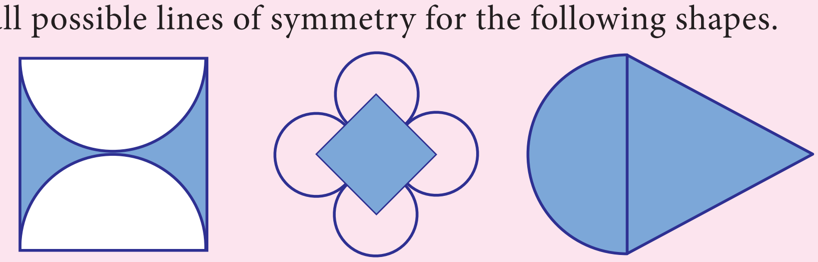


Fig. 4.2

Try these

1. Can you draw a shape which has no line of symmetry?
2. Draw all possible lines of symmetry for the following shapes.



Think

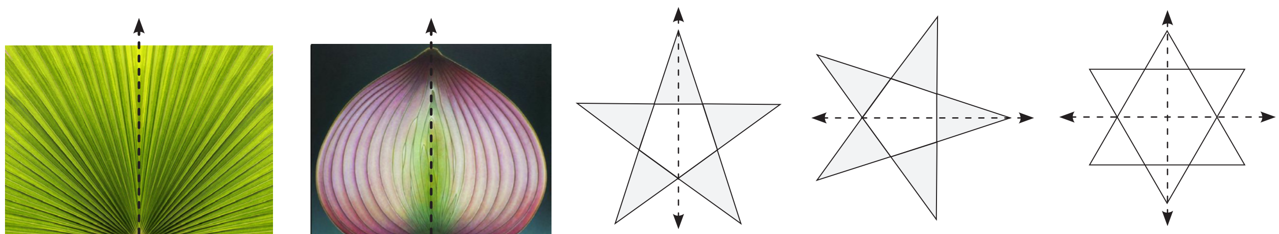
What can you say about the number of lines of symmetry of a circle?

Reflectional symmetry

When an object is seen in a mirror, the image obtained on the other side of the mirror is called its reflection.

An object and its mirror image are perfectly identical to each other. The left and right sides of an object appear inverted in the mirror. The object and its reflection image show mirror symmetry. The mirror line here is the line of symmetry. Mirror symmetry is called *reflectional symmetry*.

The following shapes are examples of reflectional symmetry.



Vertical Symmetry *Horizontal Symmetry* *Vertical and Horizontal Symmetry*

Fig. 4.3

The reflected shape will be exactly the same as the original, the same distance from the mirror line and the same size. While dealing with mirror reflection, care is needed to note down the left-right changes in the orientation.

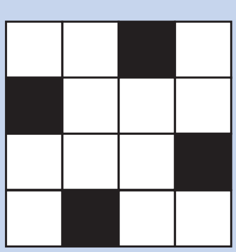
Try these

1. Reflect the words CHEEK, BIKE, BOX with horizontal line.
2. Reflect the following words with vertical line.

(i)	(ii)	(iii)
M	M	T
A	O	H
T	M	A
H		T

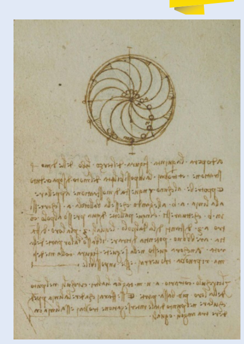
Think

Will the figure be symmetric about both the diagonals?



DO YOU KNOW?

The work of artist Leonardo da Vinci has an unusual characteristic. His hand writing is a mirror image of normal handwriting.



Rotational Symmetry

An object is said to have a *rotational symmetry* if it looks the same after being rotated about its centre through an angle less than 360°.

When an object rotates around a fixed axis if its appearance of size and shape does not change then the object is supposed to be rotationally symmetrical.

Rotational symmetry can be observed in the following *Fig. 4.4*

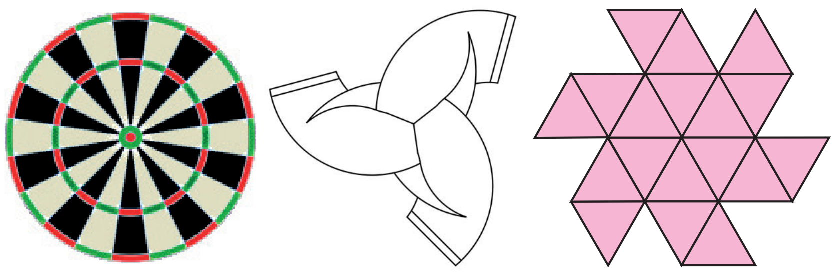


Fig. 4.4

The minimum angle of rotation of a figure to get exactly the same figure as original is called the angle of rotation.

The total number of times a figure coincides with itself in one complete rotation is called the order of rotational symmetry. We can only rotate the figure up to 360 degrees.

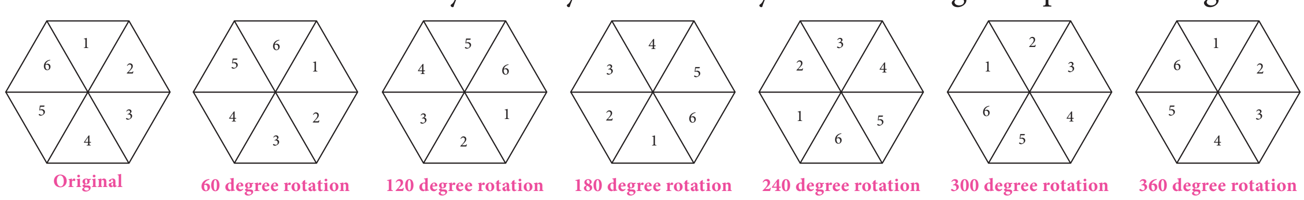
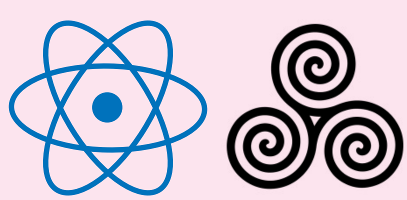


Fig. 4.5

It is important to understand that all figures have rotational symmetry of order 1, as can be rotated completely through 360° to come back to its original position. So we say that an object has rotational symmetry, only when the order of symmetry is more than 1. So, 2 is the smallest order of rotational symmetry.

Try these

1. Find the order of rotational symmetry of the following figures.



2. Find the order of rotational symmetry for an equilateral triangle.

Think

Can a parallelogram have a rotational symmetry?

Translational Symmetry

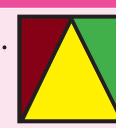
An image has translational Symmetry if it can be divided by straight lines into a sequence of identical figures. Translational symmetry results from moving a figure to a certain distance in a certain direction.

Thus, *translation symmetry* occurs when a pattern slides to a new position. The sliding movement involves neither rotation nor reflection.

Fig. 4.6

Try this

Using translational symmetry make new patterns with the given figure.



4.1 Introduction

Symmetry is a fundamental part of geometry, nature, and shapes. It creates patterns that help us to recognize the beauty of the nature. An object exhibits symmetry if it looks the same after a transformation, such as reflection or rotation.

Symmetry is the underlying mathematical principle behind all patterns. Symmetry plays a significant role in the field of Arts, Sciecne and architecture. We learnt the concept of symmetry in class VI.

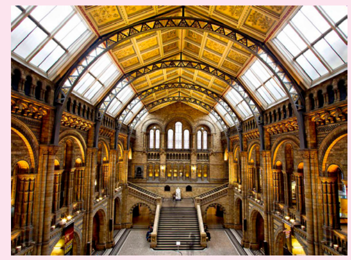
Now we are going to learn symmetry through transformations. Transformations describe how geometric figures of the same shape are related to one another.

One of the most important applications of mathematics in daily life is the concept of geometric transformation. Students need to learn this concept so as to understand the nature and environment they live in. Transformation concept is very important to learn since it helps students to understand their situations in daily life.

This concept is a necessary mental tool to be able to analyse mathematical situations. It enables students to make up rules and patterns, make explorations, be more motivated to do better works and gain rich experiences by doing maths.

Rotation, translation, reflection concepts within geometrical transformations are used in daily life, architectural designs, art and technology. Above all an aesthetic sense of beauty is observed in objects due to symmetry. Let us see the three types of transformation namely **translation, reflection** and **rotation** in this chapter.

MATHEMATICS ALIVE – Geometry in Real Life

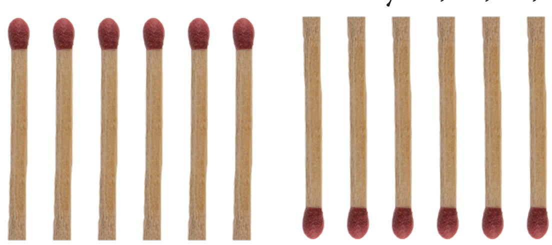
	
Symmetry in Plants	Symmetry in architecture

4.2 Symmetry through transformations

We learnt the types of symmetry. Now we are going to learn symmetry of figures through transformations.

Transformation describe how geometric figures of the same shape are related to one another. Figures or shapes in a plane can be translated, reflected or rotated to get new figures.

The original figure is called the **pre-image** and the new figure is called the **image**. Pre-images are denoted by A, B, C … etc., and the images are denoted by A', B', C', … etc. A' can be read as A prime.



Pre-image – A Image – A′

Fig. 4.7

DO YOU KNOW?

Leonardo da Vinci's Vitruvian Man (ca. 1487) is often used as a representation of symmetry in the human body and, by extension, the natural universe.

The operation that maps or moves the pre-image onto the image is called the transformation.

A transformation is a specific set of rules that change the pre-image onto the image.

In this chapter we are going to learn three types of transformation.

4.2.1. Translation:

A *translation* is a transformation that moves all points of a figure in the same distance in the same direction.

Look at the *Fig 4.8*.

From these examples we can observe that all points of a figure move in the same distance and in the same direction.

Using a grid paper, we can specify a translation by how far the shape is moved horizontally and then vertically.

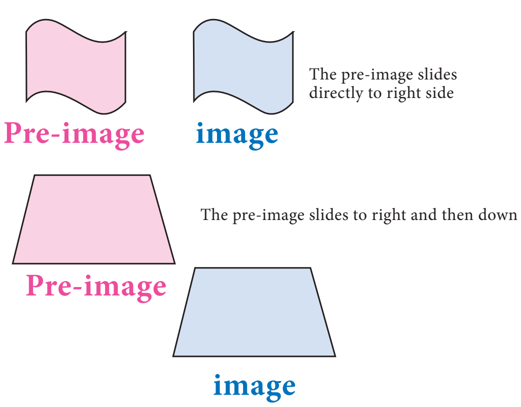
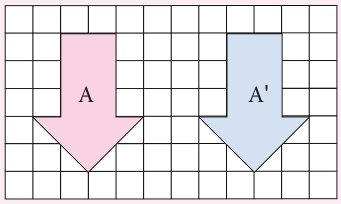
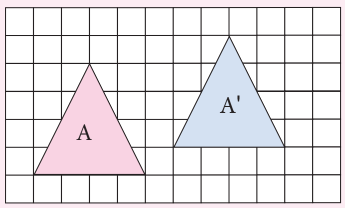
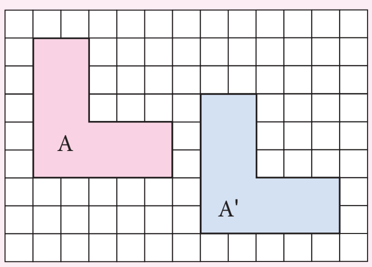
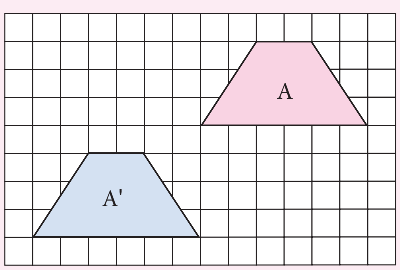


Fig. 4.8

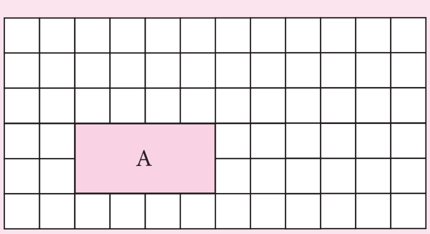
In horizontal, the right side movement is denoted by → and the left side movement is denoted ←.

In vertical, the upside movement is denoted ↑ and the downward movement is denoted ↓.

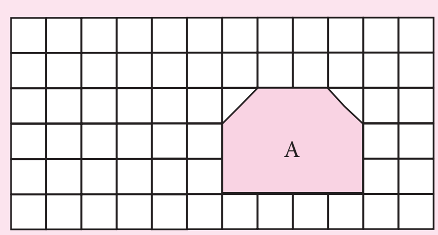
	The pre-image A is moved 6 units horizontally right side. No vertical movement. This translation is denoted by 6→
	Here the pre-image A is moved 5 units right side and then 1 unit upward. This is denoted by 5→,1↑
	Pre-image A moved 6 units to right and then 2 units downward. It is denoted by 6→,2↓ .
	Pre-image A moved 6 units to left and then 4 units downward. It is denoted by 6←,4↓

Try these

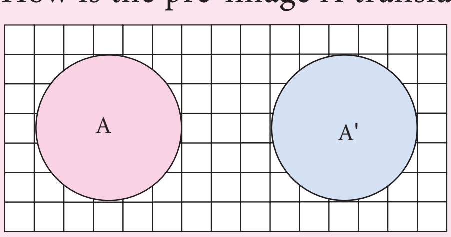
1. Translate this figure to 4 → 3 ↑

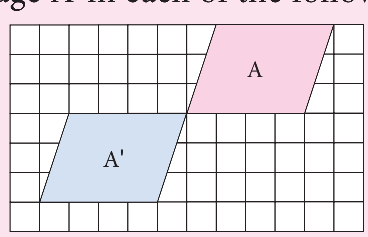


2. Translate this figure to 2 ↓ 1 ←



3. How is the pre-image A translated to image A' in each of the following figures?





Think

The pre-image and the image after a translation coincide. What can you say about the translation?

Activity

A	B	C	D	E	F
G	H	I	J	K	L
M	N	O	P	Q	R
S	T	U	V	W	X
Y	Z	*	*	*	*

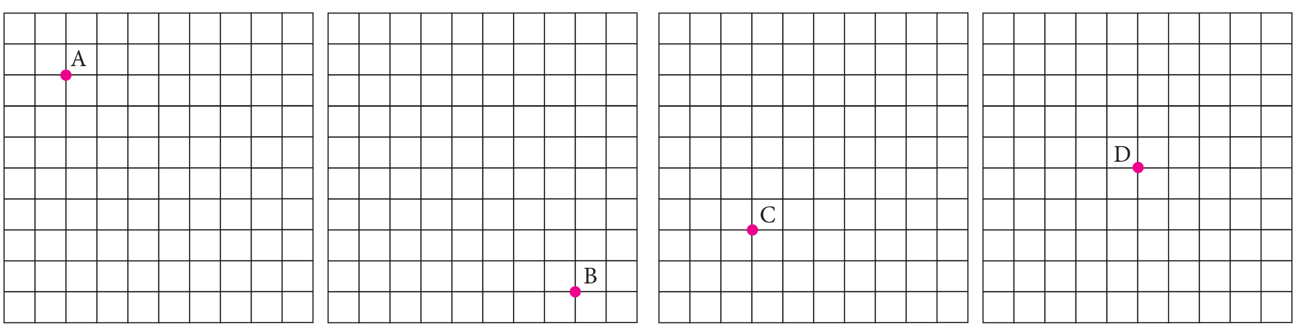
Here is a letter grid. Start with Square A. From A move 5 units right then 2 units down and stop. Then move 3 units left then 2 units up and stop. What mathematical word you get?

Start at square L. Move 3 units left and stop. Then move 1 unit left and 1 unit down and stop. Then move 3 units right then 2 unit up and stop. What mathematical word you get?

Give instruction to get (i) Right (ii) Angle (iii) Work (iv) Hard

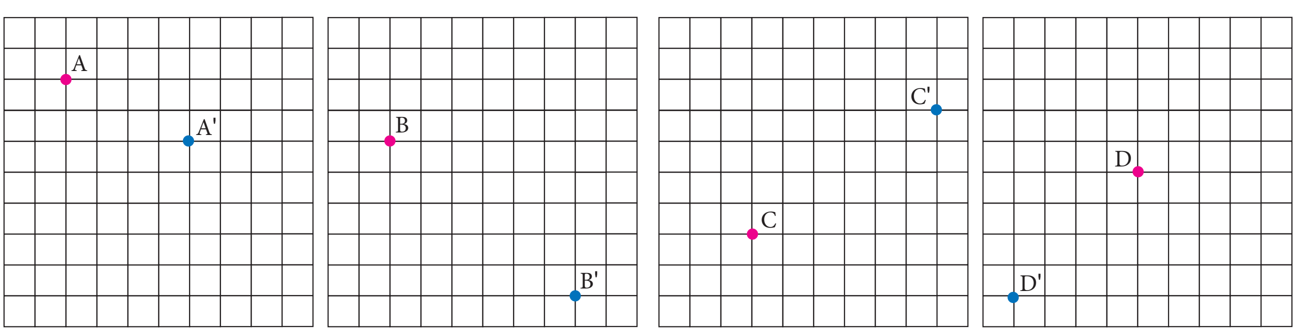
Example 4.1 Find the new position of each point using the translation given.

(i). 4 → 2 ↓ (ii). 6 ← 5 ↓ (iii). 6 → 4 ↑ (iv). 4 ← 4 ↓

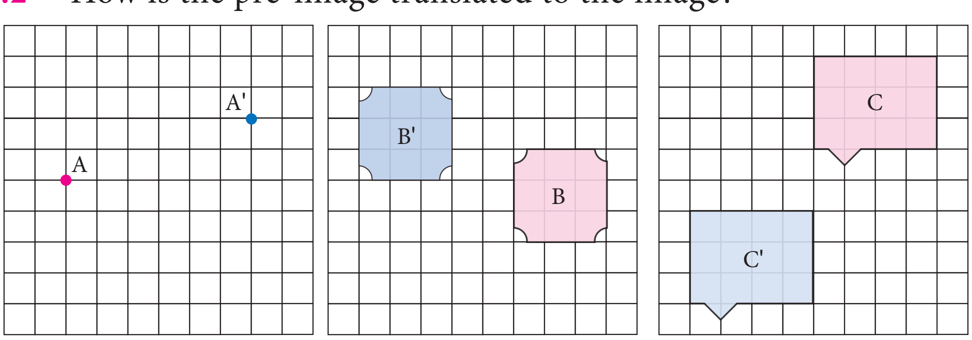


Solution:

(i) (ii) (iii) (iv)



Example 4.2 How is the pre-image translated to the image?



Solution:

(i) A is translated to A' by 6→,2↑

(ii) B is translated to B' by 5←,2↑

(iii) C is translated to C' by 4←,5↓

DO YOU KNOW?

The sights and pageantry of marching band performance can add to the excitement of a sporting event. Band members dedicate a lot of time and energy to learning the music as well as the movements required for a performance.

The movements of each band member as they progress throughout the show are examples of *translations*.

4.2.2. Reflection

A ***reflection*** is a transformation that "flips" or "reflects" a figure about a line.

After a figure is reflected, it looks like a mirror image of itself. The line that a figure is flipped over is called ***a line of reflection.***

We can observe reflection in water, a mirror or in a glossy surface as shown in *Fig. 4.9*.

Fig. 4.9

Observe the following pictures.

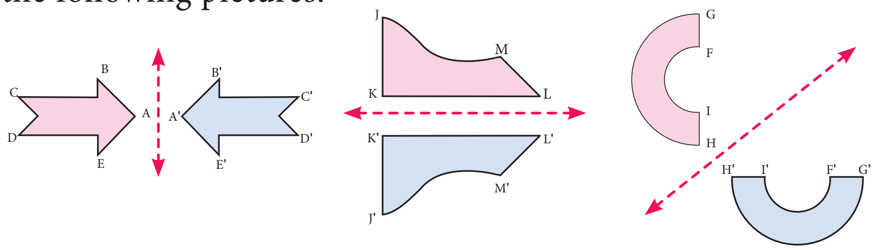


Fig. 4.10

In the above pictures (*Fig. 4.10*), the figures are reflected by a line. This line is called a line of reflection. Here the red line is the line of reflection.

We can observe that the figures and its reflections are exactly the same distance from the line of reflection on both sides.

The line of reflection may be horizontal or vertical or slanting and also it may be on the shape or outside the shape.

Note

The line of reflection is the perpendicular bisector of the line joining at any point and its image

How to reflect a shape about a line?

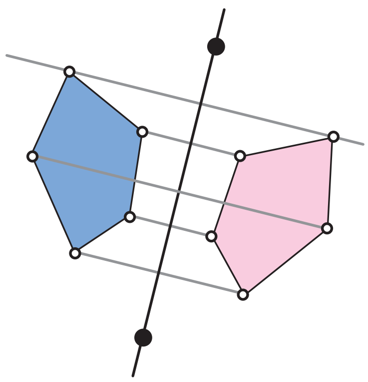


Fig. 4.11

To reflect the shape about the line of reflection, we have to reflect every vertex individually and then connect them again.

First, choose one of the vertices and draw the line through this vertex so that it is perpendicular to the line of reflection.

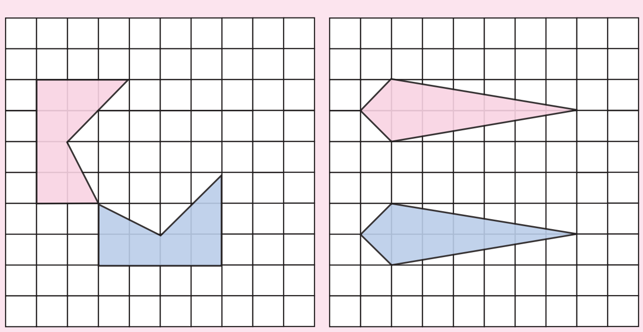
Now measure the distance from the vertex to the line of the reflection, and mark a point that has the same distance on the other side. It can be done by using either a ruler or a compass.

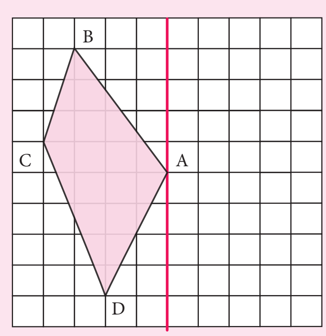
Repeat the process for all the other vertices of the shape.

Finally connect all the reflected vertices in the correct order to get the reflection of the shape.

Try these

1. Draw the line of reflection in the following pictures.
2. Reflect the shape with given line of reflection.

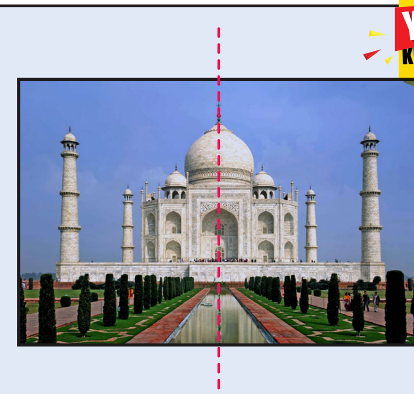




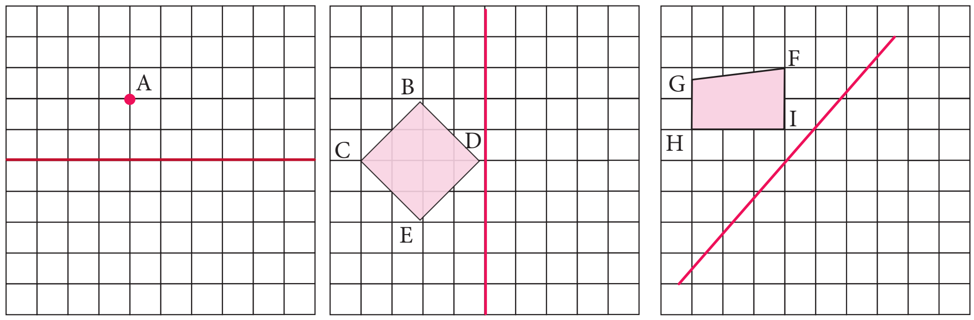
DO YOU KNOW?

Taj Mahal at Agra is planned by following the axis with Bilateral symmetrical design in plan and overall campus as the mirror image as shown in figure.

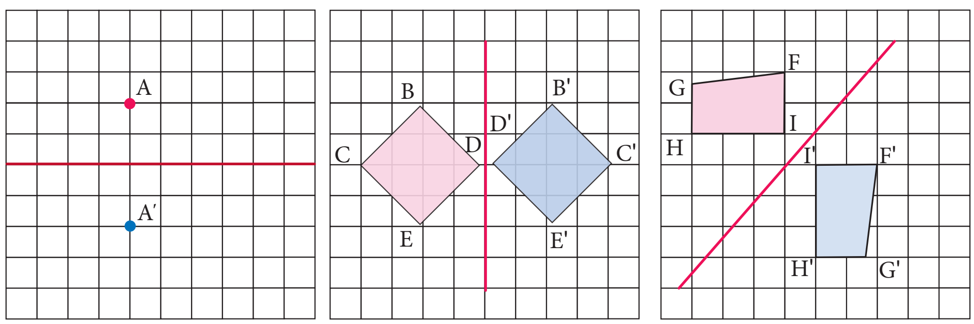
The symmetry in architecture is implied by its axiality or centrality in the form of the building. The monumental architecture often uses symmetrical design i.e. mirrored, which show stability, balance and control.



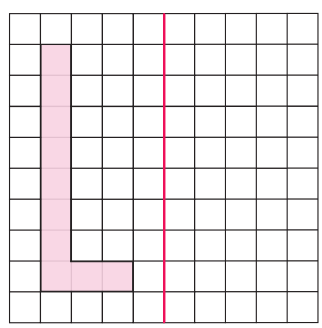
Example 4.3 Reflect the shape in each of the following pictures with given line of reflection.



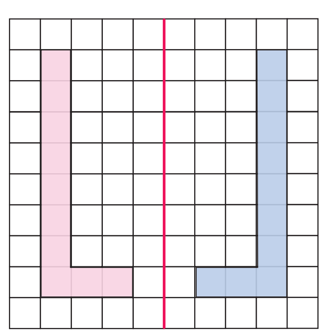
Solution:



Example 4.4 Reflect the letter about the red line.



Solution:



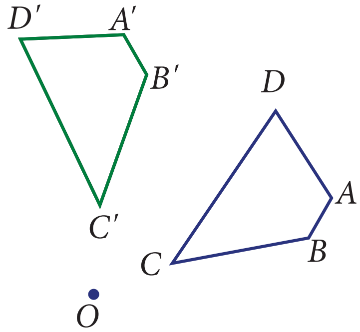
4.2.3. Rotation

A **rotation** is a transformation that turns every point of the pre-image through a specified angle and direction about a point.

The fixed point is called the centre of rotation. The angle is called the angle of rotation. A rotation is also called a turn.

The default direction of a rotation is the anti-clockwise direction. The angle of rotation can be any value between 0 and 360 degrees, both are included.

Rotation of 360° is called a full turn, rotation of 180° is called a half turn, rotation of 90° is called a quarter turn.



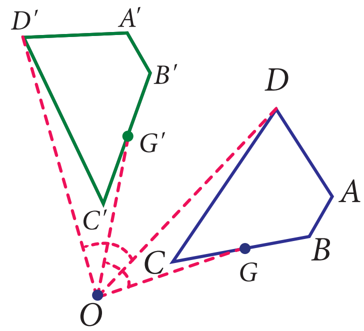
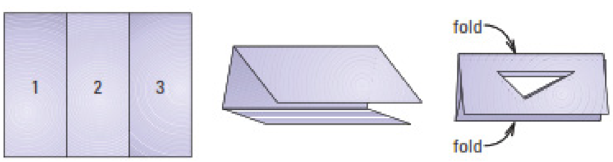


Fig. 4.12

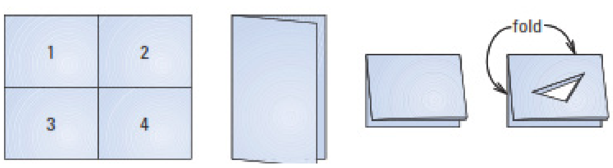
In *Fig. 4.12* the preimage ABCD is rotated about the point O to get the image A'B'C'D'. Here the angles ∠AOA', ∠BOB', ∠COC', ∠DOD' are equal. Any point G on the preimage ABCD will have a corresponding image G' on A'B'C'D' such that ∠GOG' = ∠AOA' = ∠BOB' = ∠COC' = ∠DOD' which is the angle of rotation.

Activity

- Fold a piece of paper and lable it as shown. Cut scalene triangle out of the folded paper and unfold the paper. You can see triangles in all three parts How are the triangles in parts 2 and 3 are related to triangle in part 1?



- Fold a piece of paper and lable it as shown. Cut scalene triangle out of the folded paper and unfold the paper. You can see triangles in all four parts. How are the triangles in part 2, part 3 and part 4 are related to triangle in part 1?



How to rotate a shape about a point?

To rotate a shape about a point with the given angle, we have to rotate every vertex individually and connect them again.

Here ΔABC is rotated about O with angle of 100°.

Step1. Draw CO. Make angle of 100° with vertex C and side CO using a protractor.

Step2. Use a compass to construct C'O = CO

Step3. Locate A' and B' in the similar way.

Step4. Join A', B', C' to form ΔA'B'C'

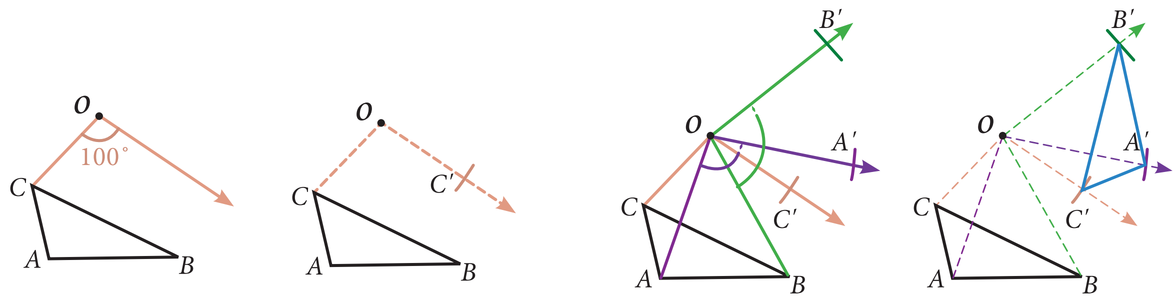


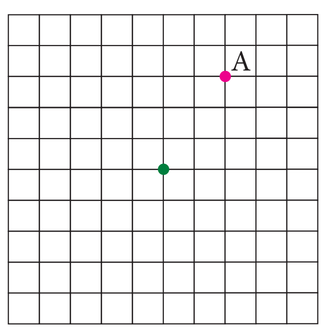
Fig. 4.13

Note

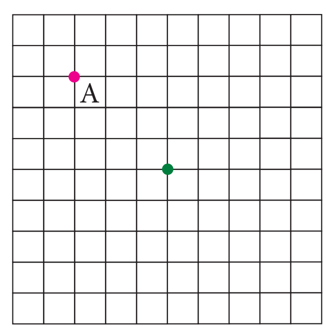
A 180° clockwise rotation and a 180° counter clockwise rotation have the same image. So, you do not need to specify direction when rotating a figure 180°.

Example 4.5 Rotate the pink point about the green point by given angle of rotation and direction

(i) 90° counter clockwise

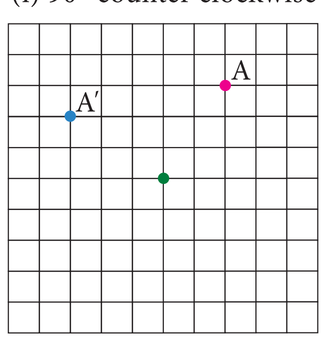


(ii) 180°

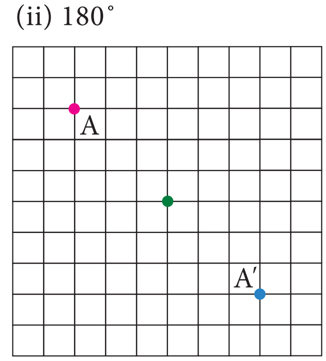


Solution:

(i) 90° counter clockwise

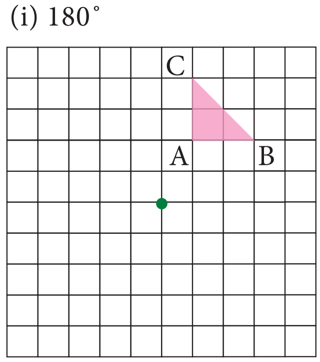


(ii) 180°

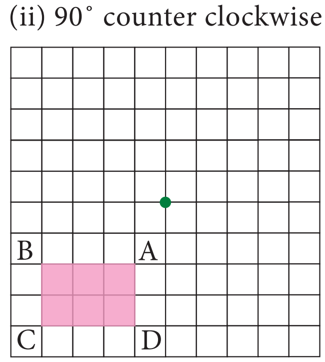


Example 4.6 Rotate the pink shape about the green point by given angle of rotation and direction

(i) 180°

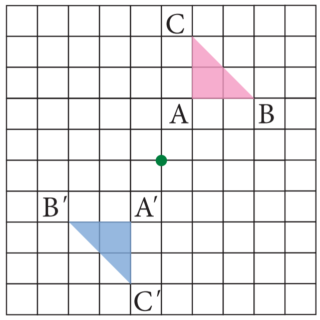


(ii) 90° counter clockwise

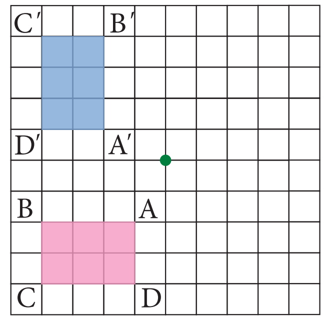


Solution:

(i) 180°

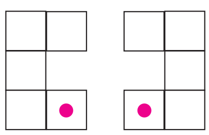


(ii) 90° counter clockwise

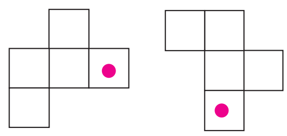


Example 4.7 Describe the transformation involved in the following pair of figures. Write translation, reflection or rotation.

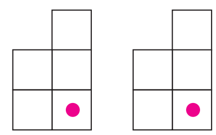
(i)



(ii)



(iii)



Solution:

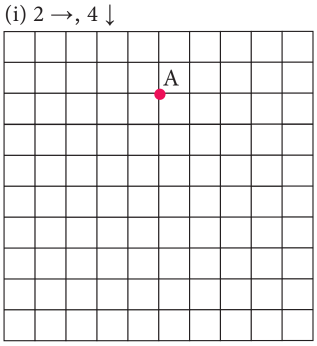
(i) Reflection (ii) Rotation (iii) Translation

> DO YOU KNOW?
>
> A *glide reflection* is a combination of two transformations: a reflection about a line and a translation. Here the translation is parallel to the line of reflection. Reversing the order of the composition will not affect the outcome. We can translate first and then reflect, or reflect first and then translate.

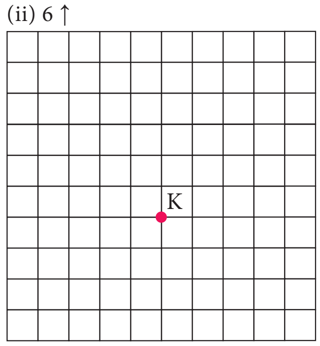
Exercise 4.1

1. Find the new position of each point using the translation given.

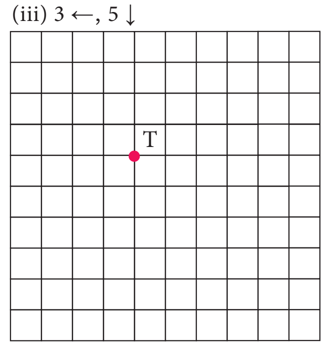
(i) 2 →, 4 ↓



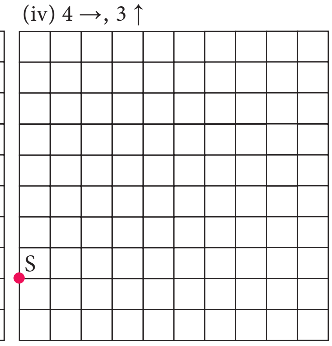
(ii) 6 ↑



(iii) 3 ←, 5 ↓

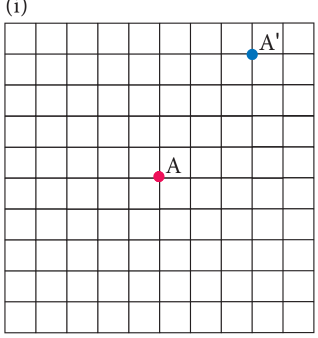


(iv) 4 →, 3 ↑

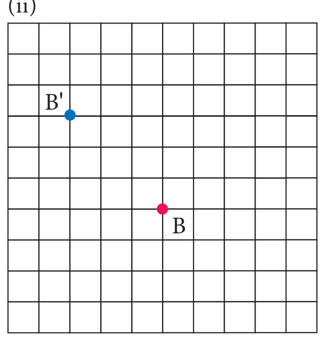


2. How is the pre-image translated to the image?

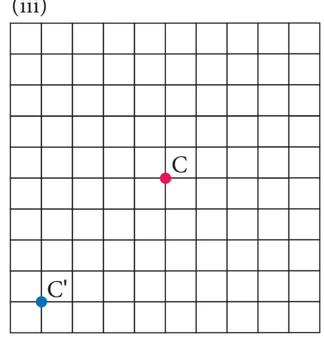
(i)



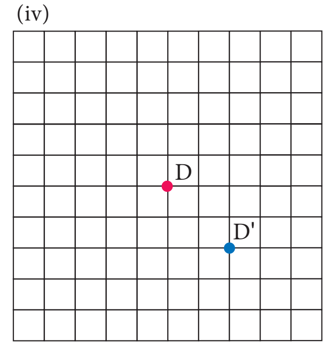
(ii)



(iii)

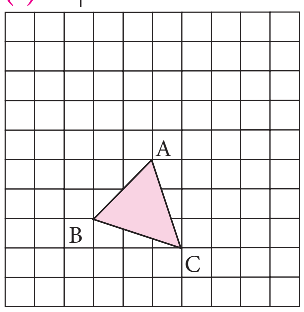


(iv)

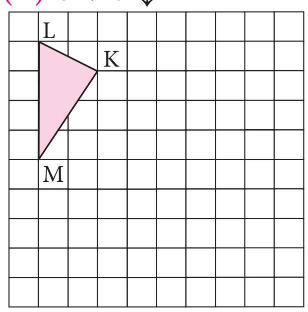


3. Find the image of the given triangle with given translation.

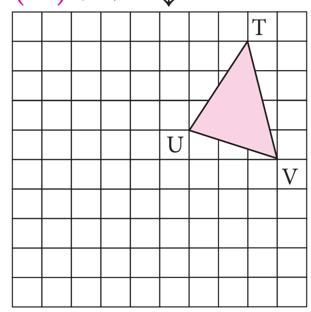
(i) 4 ↑



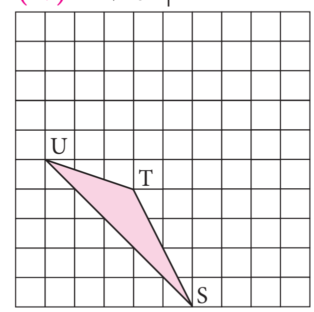
(ii) 6 → 3 ↓



(iii) 5 ← 4 ↓

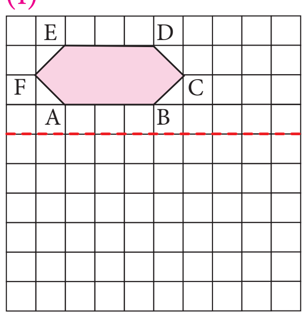


(iv) 4 → 3 ↑

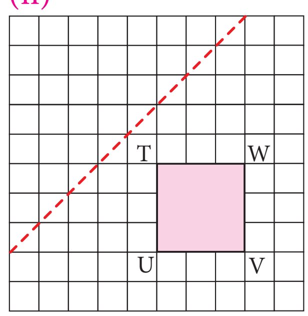


4. Reflect the shape with given line of reflection.

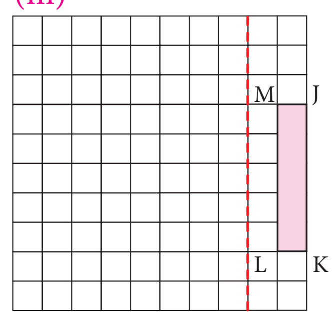
(i)



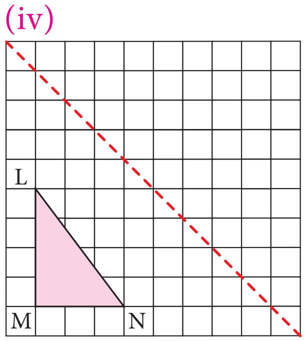
(ii)



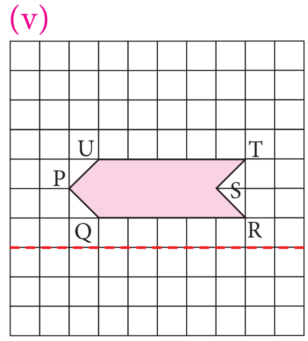
(iii)



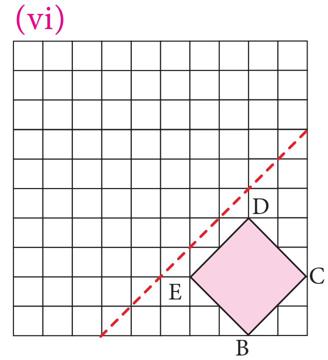
(iv)



(v)

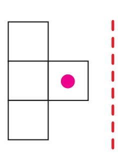


(vi)

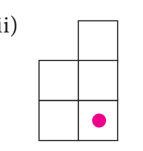


5. Reflect the shape in each of the following pictures with given line of reflection.

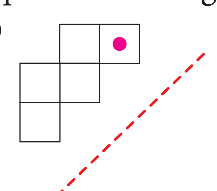
(i)



(ii)

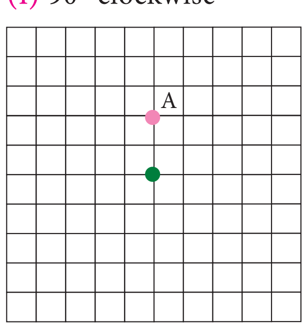


(iii)

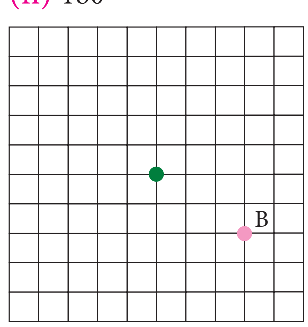


6. Rotate the preimages in each case as directed about the green point.

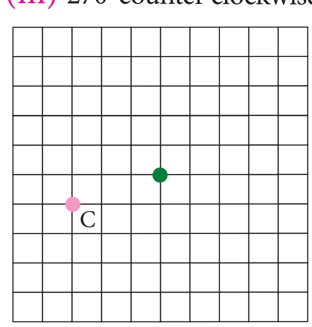
(i) 90° clockwise



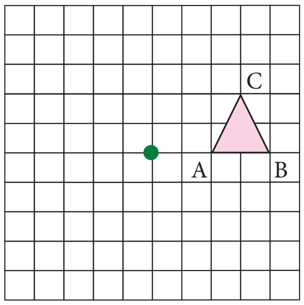
(ii) 180°



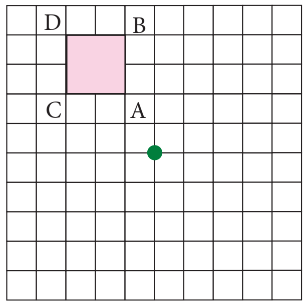
(iii) 270° counter clockwise



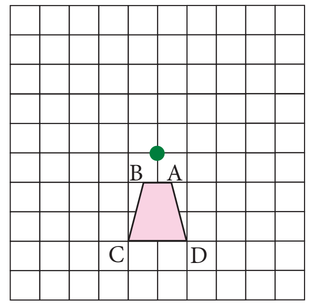
(iv) 90° counter clockwise



(v) 90° clockwise

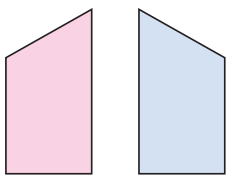


(vi) 180°

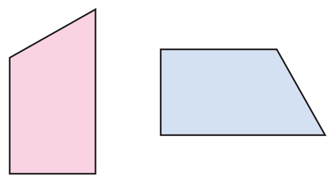


Identify the transformation

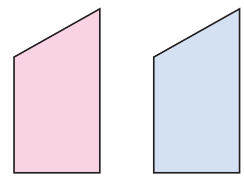
7.



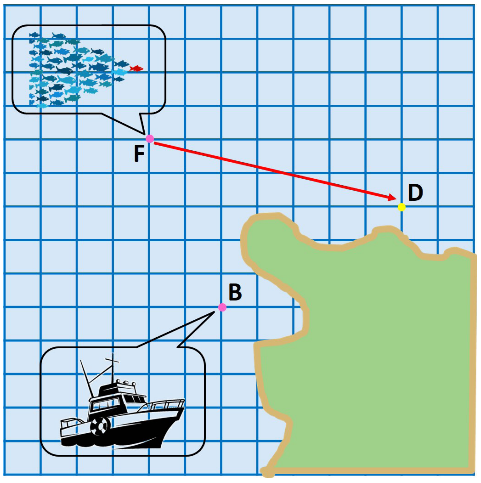
8.



9.



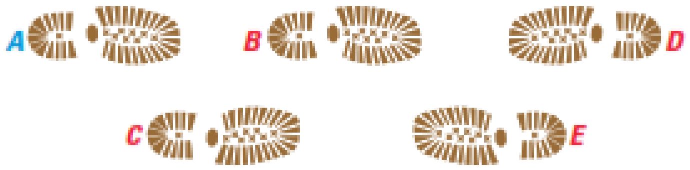
10. A pool of fish translates from point F to point D.
    a. Describe the translation of the pool of fish.
    b. Can the fishing boat make the same translation? Explain.
    c. Describe a translation the fishing boat could make to get to point D.



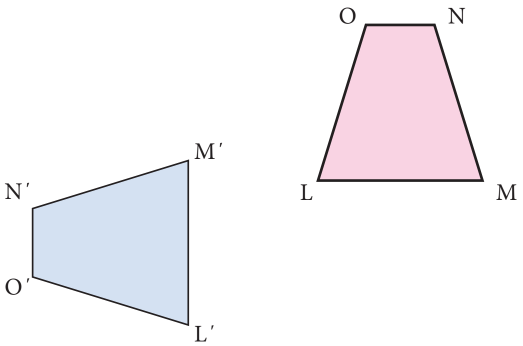
11. Name the transformation that will map footprint A onto the indicated footprint.

    (i) Footprint B (ii) Footprint C

    (iii) Footprint D (iv) Footprint E



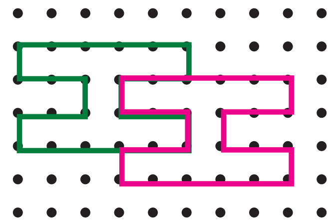
12. In given diagram, the blue figure is an image of the pink figure.



(i) Choose an angle or a vertex from the preimage and name its image.

(ii) List all pairs of corresponding sides.

13. In the diagram at the right, the green figure is a translation image of the pink figure. Write a coordinate rule that describes the translation.



Objective type questions.

14. A _____ is a turn about a point.

(i) Translation (ii) Rotation (iii) Reflection (iv) Glide Reflection

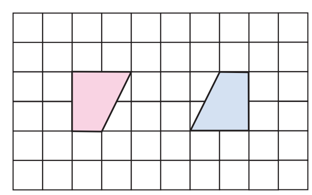
15. A _____ is a flip over a line.

(i) Translation (ii) Rotation (iii) Reflection (iv) Glide Reflection

16. A _____ is a slide; move without turning or flipping the shape.

(i) Translation (ii) Rotation (iii) Reflection (iv) Glide Reflection

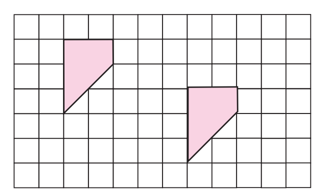
17. The transformation used in the picture is



(i) Translation (ii) Rotation

(iii) Reflection (iv) Glide Reflection

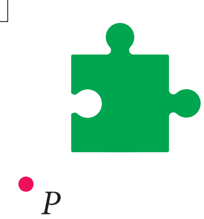
18. The transformation used in the picture is



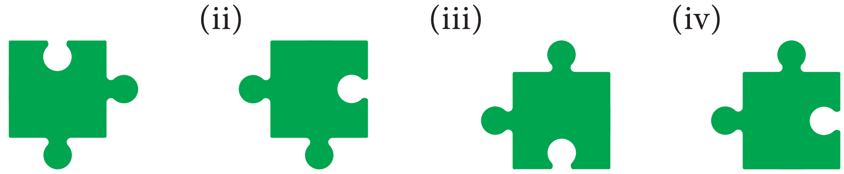
(i) Translation (ii) Rotation

(iii) Reflection (iv) Glide Reflection

19. You must rotate the puzzle piece 270° clockwise about the point P to fit it into a puzzle. Which piece fits in the puzzle as shown?



(i) (ii) (iii) (iv)



4.3 Construction of circles and concentric circles

In previous term we have learnt to find the area and the circumference of a circle. Now we can learn more about circles.

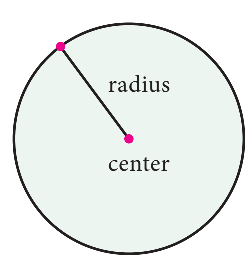


Fig. 4.14

4.3.1 circles

The collection of all the points in a plane, which are at a fixed distance from a fixed point in the plane, is called a circle.

The fixed point is called the centre of the circle and the fixed distance is called the radius of the circle. The word radius is used in two senses – in the sense of a line segment which joins the centre of the circle and a point on the circle and in the sense of length of the line segment.

A circle groups all points in the plane on which it lies into three categories. They are: (i) the points which are inside the circle, which is also called the interior of the circle; (ii) the points on the circle and (iii) the points outside the circle, which is also called the exterior of the circle.

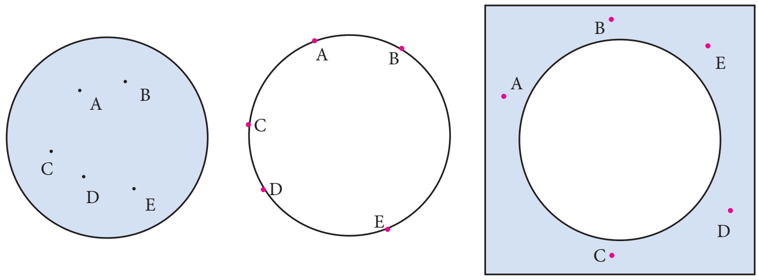


Fig. 4.15

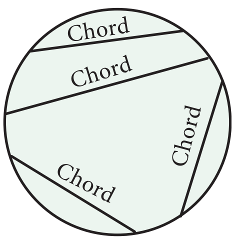


Fig. 4.16

If two points on a circle are joined by a line segment, then the line segment is called a chord of the circle. Since there are many points on the circles, any number of chords can be drawn in a circle.

The chord, which passes through the centre of the circle, is called a diameter of the circle.

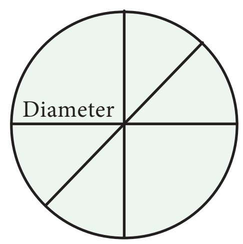


Fig. 4.17

As in the case of radius, the word 'diameter' is also used in two senses, that is, as a line segment and also as its length.

It can be easily verified that the diameter is the longest chord and all diameters have the same length. The diameter is equal to two times the radius.

4.3.2 Construction of circles

Now let us learn to construct circle with given radius and diameter.

Example Construct a circle of radius 5 cm with centre O.

Step 1: Mark a point O on the paper.

Step 2: Extend the compass distance equal to the radius 5 cm

Step 3: At center O, Hold the compass firmly and place the pointed end of the compass.

Step 4: Slowly rotate the compass around to get the circle.

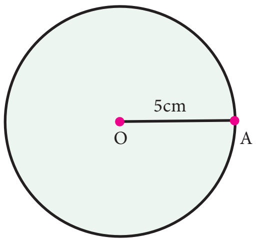


Fig. 4.18

4.3.3 The Concentric Circles

Circles drawn in a plane with a common centre and different radii are called ***concentric circles*** (*Fig. 4.19*).

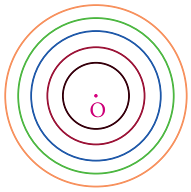


Fig. 4.19

The area between the two concentric circles is known as ***circular ring*** (*Fig. 4.20*).

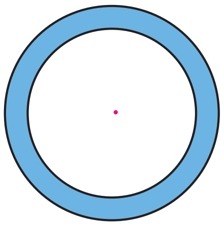


Fig. 4.20

Width of the circular ring (see *Fig. 4.21*)

= OB – OA= r_2 - r_1.

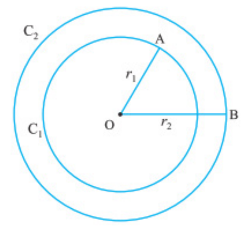


Fig. 4.21

4.3.2 Construction of Concentric Circles

Example Draw concentric circles with radii 4 cm and 6 cm and shade the circular ring. Find its width.

Step 1: Draw a rough diagram and mark the given measurements.

Step 2: Take any point O and mark it as the centre.

Step 3: With O as centre and draw a circle of radius OA = 4 cm

Step 4: With O as centre and draw a circle of radius OB = 6 cm.

Thus the concentric circles C_1 and C_2 are drawn.

Width of the circular ring = OB – OA = 6 – 4 = 2 cm.

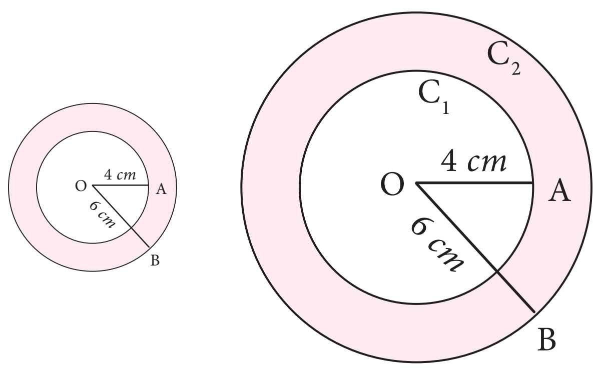


Fig. 4.22

Exercise 4.2

1. Draw circles for the following measurements of radius (r)/ diameters(d).

   (i) r = 4 cm (ii) d = 12 cm. (iii) r = 3.5 cm (iv) r = 6.5 cm.

   (v) d = 6 cm

2. Draw concentric circles for the following measurements of radii / diameters. Find out the width of each circular ring.

   (i) r = 3 cm and r = 5 cm.

   (ii) r = 3.5 cm and r = 6.5 cm.

   (iii) d = 6.4 cm and d = 11.6 cm.

   (iv) r = 5 cm and r = 7.5 cm.

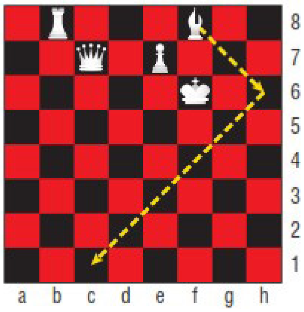
   (v) d = 6.2 cm and r = 6.2 cm.

   (vi) r = 7.1 cm and d = 12 cm.

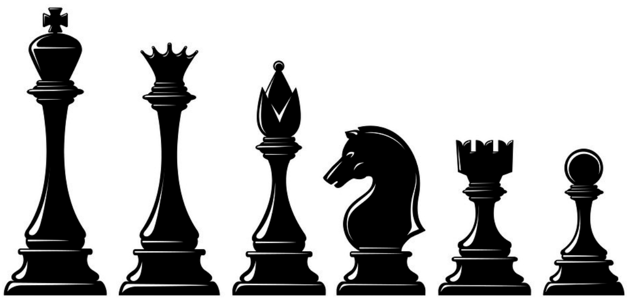
Exercise 4.3

Miscellaneous Practice problems

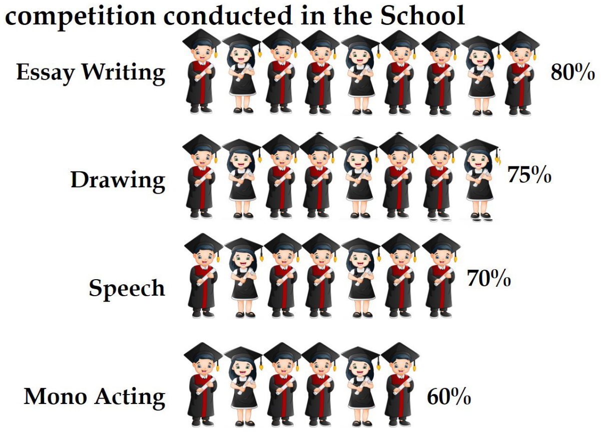
1. The bishop, in given picture of chess board, can move diagonally along dark squares. Describe the translations of the bishop after two moves as shown in the figure.



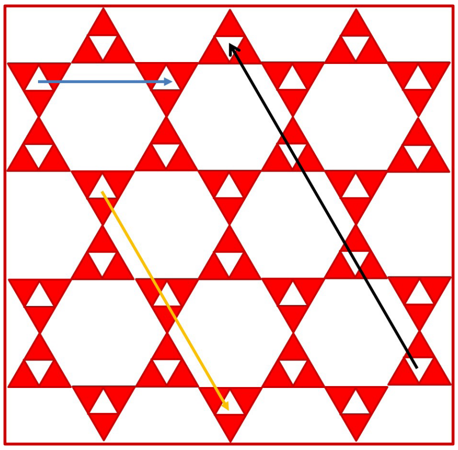
2. Write a possible translation for each of chess piece for a single move.



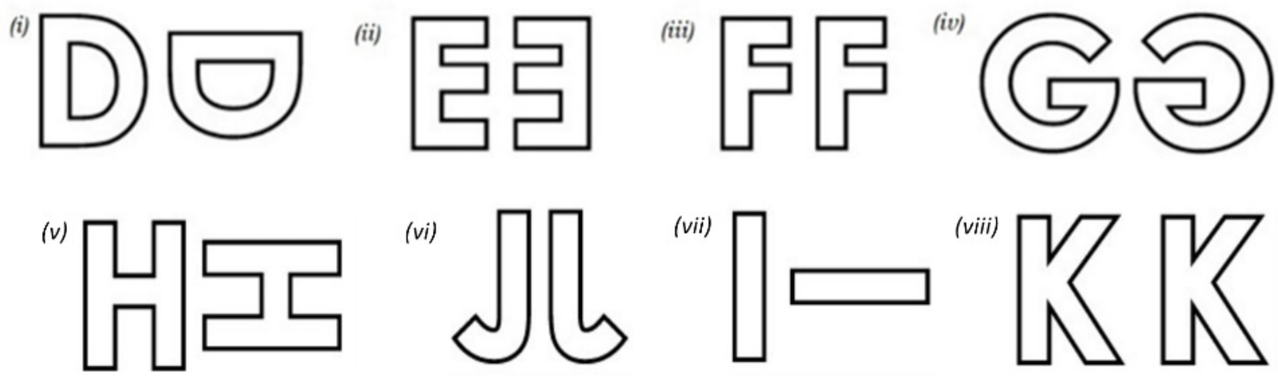
3. Referring the graphic given, answer the following questions. Each bar of the category is made up of boy-girl-boy unit. (i) Which categories show a boy-girl-boy unit that is translation within the bar? (ii) Which categories show a boy-girl-boy unit that is reflected within the bar?



4. Given figure is a floor design in which the length of the small red equilateral triangle is 30 *cm*. All the triangles and hexagons are regular. Describe the translations in *cm,* represented by the (i) yellow line (ii) black line (iii) blue line.



5. Describe the transformation involved in the following pair of figures (letters). Write translation, reflection or rotation.



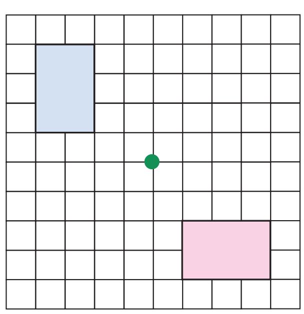
Challenge Problems

6. In chess, a knight can move only in an L-shaped pattern:
   - two vertical squares, then one horizontal square;
   - two horizontal squares, then one vertical square;
   - one vertical square, then two horizontal squares; or
   - one horizontal square, then two vertical squares.

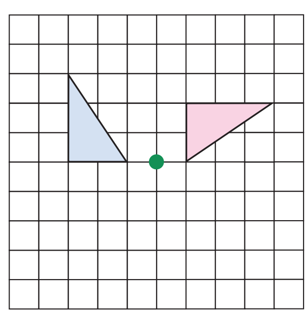
   Write a series of translations to move the knight from g8 to g5 [at most two moves].

7. The pink shape is congruent to blue shape. Describe a sequence of transformations in which the blue shape is the image of pink shape.

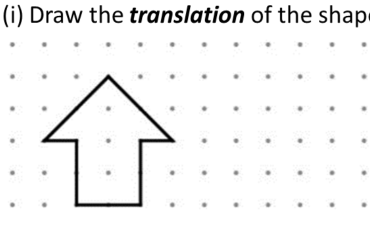
(i)



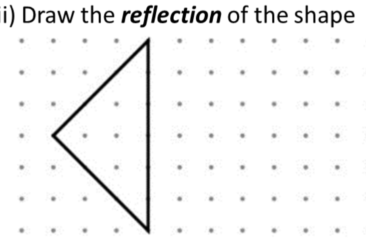
(ii)



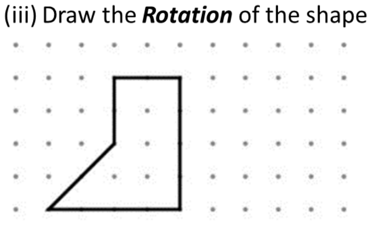
8 (i) Draw the ***translation*** of the shape



(ii) Draw the ***reflection*** of the shape



(iii) Draw the ***Rotation*** of the shape



9. Draw concentric circles given that radius of inner circle is 4.5 cm and width of circular ring is 2.5 *cm*.

10. Draw concentric circles given that radius of outer circle is 5.3 cm and width of circular ring is 1.8 *cm*.

Summary

- A transformation is a specific set of rules that change the preimage onto the image.
- A ***translation*** is a transformation that moves all points of a figure in the same distance in the same direction.
- In horizontal, the right-side movement is denoted by → and the left side movement is denoted by ←.
- In vertical, the upside movement is denoted by ↑ and the downward movement is denoted by ↓.
- A ***reflection*** is a transformation that "flips" or "reflects" a figure about a line.
- A ***rotation*** is a transformation that turns every point of the pre-image through a specified angle and direction about a point. The fixed point is called the centre of rotation. The angle is called the angle of rotation.
- A rotation is also called a turn.
- The default direction of a rotation is the anti-clockwise direction.
- Rotation of 360° is called a full turn, rotation of 180° is called a half turn, rotation of 90° is called a quarter turn.
- The collection of all the points in a plane, which are at a fixed distance from a fixed point in the plane, is called a circle. The fixed point is called the centre of the circle and the fixed distance is called the radius of the circle.
- If two points on a circle are joined by a line segment, then the line segment is called a chord of the circle.
- The chord, which passes through the centre of the circle, is called a diameter of the circle.
- Circles drawn in a plane with a common centre and different radii are called concentric circles.
- The area between the two concentric circles is known as circular ring
- Width of the circular ring(w) = $r_2 - r_1$

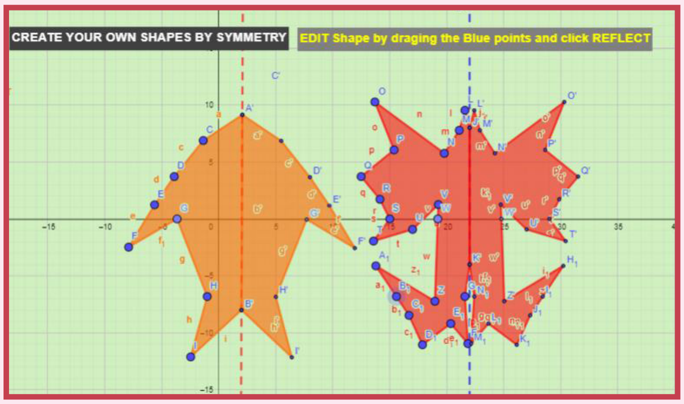
ICT Corner

Geometry

Step – 1

Open the Browser and type the URL Link given below (or) Scan the QR Code. GeoGebra work sheet named "Symmetry" will open. Drag the points on left side of the line to change the shape. Click "REFLECT" check box to see the symmetricity.

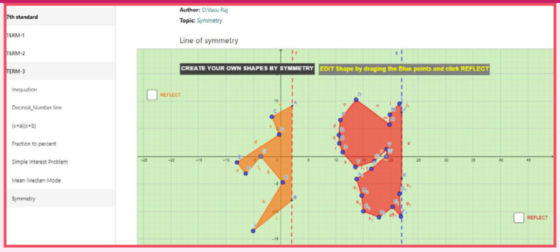
Expected outcome



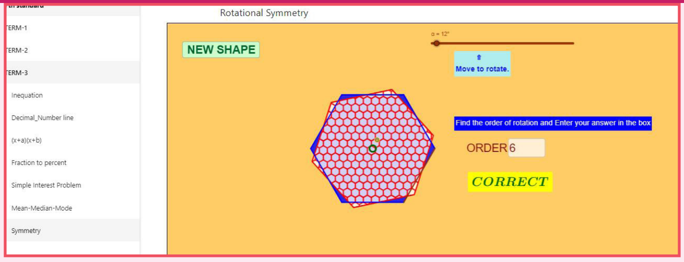
Step - 2

Scroll down to see another worksheet called "Rotational symmetry". Move the slider to rotate the shape and find the order of rotation and enter in the box given. By pressing enter, you can check whether your answer is correct. Click on "NEW SHAPE" and repeat the process.

Step 1



Step 2



Browse in the link

Symmetry: https://www.geogebra.org/m/f4w7csup#material/udcrmzyr

or Scan the QR Code.

Chapter 5

STATISTICS

Learning Objectives

- To recall collection and organisation of discrete data.
- To recognise and distinguish among the different types of averages such as the Arithmetic Mean, Median and Mode of discrete data.
- To learn to compute different types of averages.

5.1 Introduction

Recap

We have studied about data, classification and pictorial representation of data in class VI. Let us recall them now.

Observe the following tables and answer the questions given below.

Table 1

The marks obtained by 30 students in a mathematics test (out of 50 marks) is given below.

33	22	50	28	20	21	25	23	25	34	28	26	28	22	28
28	37	35	21	42	24	2	49	29	35	25	26	32	33	29

(i) What is the maximum mark scored?

(ii) How many students scored above 40 marks?

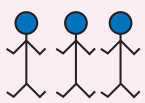
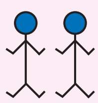
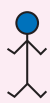
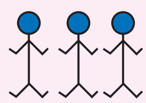
(iii) Find the minimum mark scored?

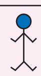
(iv) How many students scored between 21 and 23 (both inclusive)?

(v) Find the difference between maximum and minimum scores?

Table 2

Data shows the children who are absent in class VII during a particular week.

Monday	Tuesday	Wednesday	Thursday	Friday
		–		

 - represents five children.

(i) Find the day with minimum number of absentees?

(ii) Mention the day with ten absentees.

(iii) Which day of the week has no absentees ?

(iv) Find the day on which maximum number of children were absent?

5.2 Collection of data

We need to collect data having the specific information in our mind. Suppose the specific information needed by us is to know the height of class VII students, then we should collect specific data related to their heights and ages rather than the data related to health record of students.

From the above discussion, we conclude that the purpose for which a data is to be collected has to be kept in mind before starting the process of data collection. Then only we can get the desired information, which is appropriate to the purpose. Let us look into a few situations that are given below.

Data can be generated in many situations around us. For example,

- The height of class VII students.
- The performance of class VII students in the term II examinations.
- The number of trees planted in your locality.
- The highest temperatures recorded in all the major cities of India during the year 2018.
- The least amount of rainfall recorded in all the districts of Tamil Nadu during the year 2018.

5.3 Organisation of Data

We first collect data, record it and organise them. To understand this, consider an example which deals with the weights of 10 students of class VII. These are collected to know whether the weights of students are appropriate to their heights. The data is given below.

- Anbu-20 *kg*; Nambi-19 *kg*; Nanthitha- 20 *kg*; Arul- 24 *kg*;
- Mari-25 *kg*; Mathu-22 *kg*; Pavithra – 23 *kg*; Beeman- 26 *kg*;
- Arthi-21 *kg*; Kumanan-25 *kg*.

Let us try to answer the following questions.

(i) Who is the least weight of all?

(ii) How many students weigh between 22 *kg* to 24 *kg*?

(iii) Who is the heaviest of all?

(iv) How many children are above 23 *kg* and how many are below 23 *kg*?

The data mentioned above is not easy to understand.

If the data is arranged according to the order of weights, it will be easy for answering the questions. Observe the following table.

S. No	Name	Weight (kg)
1	Nambi	19
2	Anbu	20
3	Nanthitha	20
4	Arthi	21
5	Mathu	22

S. No	Name	Weight (kg)
6	Pavithra	23
7	Arul	24
8	Kumanan	25
9	Mari	25
10	Beeman	26

Now we can answer the above questions easily. Hence it is essential to organise the data to obtain any kind of inferences from the data.

Organisation of data is helpful to understand quickly and get an overall view of data. It becomes easy to understand and interpret which in turn also helps to take decision accordingly.

5.4 Representative values

We have come across situtations where we use the term 'average' in our day-to-day life. Consider the following statements.

- The average temperature at Chennai in the month of May is 40° c.
- The average marks in mathematics unit test of class VI is 74.
- Mala's average study time is 4 hours.
- Mathan's average pocket money per week is ₹100.

We come across many more statements of such kind in our daily life. Let us take the example, *"the average marks scored by class* VI *students in maths test is 74"*. Does it mean that every student has scored 74? No, certainly not. Some students would have got more than 74 and some students would have got less than 74. Average is the value that represents the general performance of class VI students in maths test.

Try these

Collect the height of students of your class. Organise the data in ascending order.

Similarly, 40° c is the representative temperature of Chennai in the month of May which does not mean that everyday temperature is 40° c in the month of May. Since the average lies between the highest and the lowest value of the given data, we say average is a measure of central tendency of the group of data. Different forms of data need different forms of representative or central value to describe it. We study three types of central values of data namely Arithmetic Mean, Mode and Median in this chapter.

5.5 Arithmetic Mean

Now, let us see one of the measures of central tendency, that is the Arithmetic Mean. Consider this situation.

Mani and Ravi started collecting shells in the sea shore with an agreement to share the shells equally after collection. Finally, Mani collected 50 shells and Ravi collected 30 shells. Now, if both of them share equally, find the number of shells each one gets?

We find it using arithmetic mean or average. To find the average, add the numbers and divide by 2. Hence,

$$\text{Average} = \frac{50+30}{2} = \frac{80}{2} = 40$$

Average lies between 30 and 50.

Hence, each of them will get 40.

Try these

Find the Arithmetic Mean or average of the following data.

(i) The study time spent by Kathir in a week is 3 hrs, 4 hrs, 5 hrs, 3 hrs, 4 hrs, 3:45 hrs and 4:15 hrs.

(ii) The marks scored by Muhil in five subjects are 75, 91, 48, 63 and 51.

(iii) Money spent on vegetables for five days is ₹ 120, ₹ 80, ₹ 75, ₹ 95 and ₹ 86.

Thus to find the arithmetic mean (average), we have to add all the observations and divide the sum of all observations by the number of observations.

$$\text{Hence, Arithmetic Mean} = \frac{\text{Sum of all observations}}{\text{Number of observations}}.$$

Example 5.1 The daily wages of a worker for 10 days is as follows. Find the average income of the worker.

Day	1	2	3	4	5	6	7	8	9	10
Wages (₹)	250	350	100	400	150	270	450	320	610	750

Solution

$$\text{Arithmetic Mean} = \frac{\text{Sum of all observations}}{\text{Number of observations}}$$

$$= \frac{250+350+100+400+150+270+450+320+610+750}{10}$$

$$= \frac{3650}{10} = 365$$

Hence, the average income of the worker is ₹365.

Example 5.2 The mean of 9 observations is 24. Find the sum of the 9 observations.

Solution

$$\text{Arithmetic Mean} = \frac{\text{Sum of all observations}}{\text{Number of observations}}$$

$$\text{Thus, } 24 = \frac{\text{Sum of all observations}}{9}$$

$$\text{Sum of all observations} = 9 \times 24 = 216.$$

Example 5.3 The mean age of 15 teachers in a school is 42. The ages of the teachers are 35, 42, 48, x, x+8, 40, 43, 50, 46, 50, 37, 32, 38, 41, 40 (in years). Find the value of X and unknown ages of the two teachers?

Solution

$$\text{Mean} = \frac{\text{Total ages of teachers}}{\text{Number of teachers}}$$

$$42 = \frac{35+42+48+x+(x+8)+40+43+50+46+50+37+32+38+41+40}{15}$$

$$\frac{550+2x}{15} = 42$$

$$550+2x = 42 \times 15$$

$$= 630$$

$$2x = 630-550$$

$$2x = 80$$

$$x = \frac{80}{2}$$

$$x = 40$$

Therefore, the age of the teacher (x) is 40 and the age of the another teacher (x+8) is 40+8=48.

Example 5.4 If the mean of the following numbers is 38, find the value of x.
48, x, 37, 38, 36, 27, 35, 34, 38, 49, 33.

Solution

$$\text{Mean} = \frac{\text{Sum of all observations}}{\text{Number of observations}}$$

$$38 = \frac{48+x+37+38+36+27+35+34+38+49+33}{11}$$

$$38 = \frac{375+x}{11}$$

$$38 \times 11 = 375+x$$

$418 = 375+x$

$x = 418 - 375$

$x = 43$

Hence, the value of x is 43.

Think

Check the properties of arithmetic mean for the example given below:

X	3	6	9	12	15

(i) If the mean is increased by 2, then what happens to the individual observations.

(ii) If first two items are increased by 3 and last two items are reduced by 3, then what will be the new mean?

DO YOU KNOW?

Here are few interesting averages.

1. On an average, you blink your eyes 17 times per minute. That is 5.2 million times a year.
2. The average person has about 1460 dreams a year. That is about 4 per night.
3. Based on the average life of a G2 star the present age of the sun is estimated to be 4.5 billion years, halfway through its lifetime.

Exercise 5.1

1. Fill in the blanks.
   (i) The mean of first ten natural numbers is ________.
   (ii) If the average selling price of 15 books is ₹ 235, then the total selling price is ________.
   (iii) The average of the marks 2, 9, 5, 4, 4, 8, 10 is _________.
   (iv) The average of integers between -10 to 10 is ________.
2. Ages of 15 students in 8th standard is 13, 12, 13, 14, 12, 13, 13, 14, 12, 13, 13, 14, 13, 12, 14. Find the mean age of the students.
3. The marks of 14 students in a science test out of 50 are given below. 34, 23, 10, 45, 44, 47, 35, 37, 41, 30, 28, 32, 45, 39 Find
   (i) the mean mark.
   (ii) the maximum mark obtained.
   (iii) the minimum mark obtained.

4. The mean height of 11 students in a group is 150 cm. The heights of the students are 154 *cm*, 145 *cm*, Y *cm*, Y + 4 *cm*, 160 *cm*, 151 *cm*, 149 *cm*, 149 *cm*, 150 *cm*, 144 *cm* and 140 *cm*. Find the value of Y and the heights of two students?

5. The mean of runs scored by a cricket team in the last 10 innings is 276. If the scores are 235, 400, 351, x, 100, 315, 410, 165, 260, 284, then find the runs scored in the fourth innings.

6. Find the mean of the following data.

   5.1, 4.8, 4.3, 4.5, 5.1, 4.7, 4.5, 5.2, 5.4, 5.8, 4.3, 5.6, 5.2, 5.5

7. Arithmetic mean of 10 observations was found to be 22. If one more observation 44 was to be added to the data, what would be the new mean?

Objective type questions

8. _________ is a representative value of the entire data.

   (i) Mean (ii) range (iii) minimum value (iv) maximum value

9. The mean of first fifteen even numbers is ________

   (i) 4 (ii) 16 (iii) 5 (iv) 10

10. The average of two numbers are 20. One number is 24, another number is _________

    (i) 16 (ii) 26 (iii) 20 (iv) 40

11. The mean of the data 12, x, 28 is 18. Find the value of x.

    (i) 18 (ii) 16 (iii) 14 (iv) 22

5.6 Mode

As we have discussed earlier that the arithmetic mean is one of the form of representative value or measures of central tendency of a group of data. Depending upon the data and its purpose, other measures of central tendency may be used.

Consider the example of sale details of different sizes of footwear in a shop for a week.

Size of the footwear	5"	6"	7"	8"	9"	10"	11"
Number of footwear sets sold	20	35	16	65	32	25	10

The shopkeeper has to replenish his stock at the end of the week. Suppose we find the arithmetic mean of the footwear sold,

$$\text{Mean} = \frac{20+35+16+65+32+25+10}{7} = \frac{203}{7} = 29.$$

Average number of footwear is 29. This means that the shopkeeper has to get 29 pairs of footwear in each size. Will it be wise to decide like this?

It has to be observed that the maximum purchase falls on the footwear of size 8 inches. So the shopkeeper has to get more number of footwear of size 8 inches. Hence arithmetic mean does not suit for this purpose. Here we need another type of representative value of data called 'Mode'.

Mode is the value of the data which occurs maximum number of times.

Consider another example.

A shopkeeper analyses his sales data of readymade shirts to plan for the stock according to the demand. The sale details of shirts are given below.

Size	28"	30"	32"	34"	36"
Number of shirts sold	20	35	22	35	15

Here he observes that there is a equal demand for shirts of sizes 30" and 34". Now this data has two modes as there are two maximum occurrences namely 30" and 34". He stocks more shirts of these 2 sizes. Note that, this data has two mode and it is known as **bimodal data**.

Try these

(1) Find the mode of the following data.

2, 6, 5, 3, 0, 3, 4, 3, 2, 4, 5, 2

(2) Find the mode of the following data set.

3, 12, 15, 3, 4, 12, 11, 3, 12, 9, 19

(3) Find the mode of even numbers within 20.

Example 5.5 Find the mode of the given set of numbers. 5, 7, 10, 12, 4, 5, 3, 10, 3, 4, 5, 7, 9, 10, 5, 12, 16, 20, 5

Solution

Arranging the numbers in ascending order without leaving any value, we get,

3, 3, 4, 4, 5, 5, 5, 5, 5, 7, 7, 9, 10, 10, 10, 12, 12, 16, 20

Mode of this data is 5, because it occurs more number of times than the other values.

Note

To find mode, arranging the raw data in ascending order is not mandatory. It helps us to ensure that each and every value is taken into account for the calculation of mode and helps in identifying the mode value easily.

Example 5.6 The marks obtained by 11 students of a class in a test are 23, 2, 15, 38, 21, 19, 23, 23, 26, 34, 23. Find the mode of the marks.

Solutionv

Arranging the given marks in ascending order, we get,

2, 15, 19, 21, 23, 23, 23, 23, 26, 34, 38.

Clearly, 23 occurs maximum number of times. Hence mode of marks=23.

Example 5.7 Find the mode of the following data 123, 132, 145, 176, 180, 120

Solution

From the above data, we can see that there is no repetition of values in the given data. Each observation occurs only once, so there is no mode.

Note

When each of the observations have occurred only once, then there is no mode for the data.

Think

1. A toy factory making variety of toys for kids, wants to know the most popular toy liked by all the kids. Which average will be the most appropriate for it?
2. Is there a mode exists between the odd numbers from 20 to 40? Discuss.

5.6.1 Mode of large data

When size of the data is large, it is not easy to identify the value which occurs maximum number of times. In that case, we can group the data by using tally marks and then find the mode.

Consider the example to find the mode of the number of goals scored by a football team in 25 matches. The goal scored are 1, 3, 2, 5, 4, 6, 2, 2, 2, 4, 6, 4, 3, 2, 1, 1, 4, 5, 3, 2, 2, 4, 3, 0, 1.

To find the mode of this data, the number of goals score starting from 0 and ending with a maximum of 6 is represented in the form of a table.

Number of goals	Tally marks	Frequency
0	|	1
1	||||	4
2	~~||||~~ ||	7
3	||||	4
4	~~||||~~	5
5	||	2
6	||	2
		25

From the table we observe that the highest frequency is 7, which corresponds to number of goals, that is 2. Hence, the mode is 2.

Example 5.8 Find the mode of the following data. 14, 15, 12, 14, 16, 15, 17, 13, 16, 16, 15, 12, 16, 15, 13, 14, 15, 13, 15, 17, 15, 14, 18, 19, 12, 14, 15, 16, 15, 16, 13, 12.

Solution

We tabulate the data as follows.

The whole data ranges from 12 to 19.

Observation	Tally marks	Frequency
12	||||	4
13	||||	4
14	卌	5
15	卌 ||||	9
16	卌 |	6
17	||	2
18	|	1
19	|	1
	Total	32

The highest frequency is 9 which corresponds to the value 15. Hence the mode of this data is 15.

Example 5.9 The following data shows that the number of hours spent by the students for study.

Number of study hours	1	2	3	4	5	6
Number of students	4	2	1	2	1	0

Find the mode.

Solution

Since the one hour study time is spent by maximum number of students, the mode of the data is 1 hour.

Think

Which average will be most aprropriate for the companies producing the following goods? why?

(i) Diaries and notebooks. (ii) School bags. (iii) Jeans and T-shirts.

Exercise 5.2

1. Find the mode of the following data.
   2, 4, 5, 2, 6, 7, 2, 7, 5, 4, 8, 6, 10, 3, 2, 4, 2

2. The number of points scored by a Kabaddi team in 20 matches are 36, 35, 27, 28, 29, 31, 32, 31, 35, 38, 38, 31, 28, 31, 34, 33, 34, 31, 30, 29. Find the mode of the goals scored by the team.

3. The ages (in years) of 11 cricket players are given below. 25, 36, 39,38 40, 36, 25, 25, 38, 26, 36. Find the mode of their ages.

4. Find the mode of the following data.
   12, 14, 12, 16, 15, 13, 14, 18, 19, 12, 14, 15, 16, 15, 16, 16, 15, 17, 13, 16, 16, 15, 15, 13, 15, 17, 15, 14, 15, 13, 15, 14

Objective type questions

5. The colours used by the six students for drawing is blue, orange, yellow, white, green and blue then the mode is ________.

   (i) blue (ii) green (iii) white (iv) yellow

6. Find the mode of data 3, 6, 9, 12, 15

   (i) 1 (ii) 2 (iii) 3 (iv) No mode

7. Find the modes of the data 2, 1, 1, 3, 4, 5, 2

   (i)1 and 5 (ii) 2 and 3 (iii) 2 and 1 (iv) 1 and 4

5.7 Median

We have discussed the situations where arithmetic mean and mode are the representative values of the given data. Let us think of any other alternative representative value or measures of central tendency. For this let us consider the following situation.

Rajam an old student of the school wanted to provide financial support to a group of 15 students, who are selected for track events. She wanted to support them on the basis of their family income. The monthly income of those 15 families are given below.

₹3300, ₹5000, ₹4000, ₹4200, ₹3500, ₹4500, ₹3200, ₹3200, ₹4100, ₹4000, ₹4300, ₹3000, ₹3200, ₹4500, ₹4100.

Rajam would like to give them an amount to their family.

If we find the mean, we get

Arithmetic mean, A.M $= \dfrac{\text{sum of all values}}{15}$

$$= \frac{\begin{Bmatrix} 3300+5000+4000+4200+3500+4500+3200 \\ +3200+4100+4000+4300+3000+3200+4500+4100 \end{Bmatrix}}{15}$$

$$= \frac{58100}{15} = 3873.3$$

Can the amount of ₹3873.3 be given to all of them irrespective of their salary? Is ₹3873.3 is the suitable representative here? No, this is not suitable here because a student with family income ₹3000 and a student with family income ₹5000 will receive the same amount. Because the representative measure used here is not sutable for the above data, let us find the mode for this data.

3000		1
3200		3
3300		1
3500		1
4000		2

4100		2
4200		1
4300		1
4500		2
5000		1

Here mode is 3200 which means there are more number of students with a family income of ₹3200. But this does not suite our purpose.

Hence, mode is also not suitable. Is there any other representative measures that can be used here? Yes.

Let us look at another representative value which divides the data into two halves exactly. First, let us arrange the data in ascending order.

That is, ₹3000, ₹3200, ₹3200, ₹3200, ₹3300, ₹3500, ₹4000, ₹4000, ₹4100, ₹4100, ₹4200, ₹4300, ₹4500, ₹4500, ₹5000.

After arranging the income in ascending order, Rajam finds 8th value (₹4000) which divides the data into two halves. It helps her to decide the amount of financial support that can be given to each of the students. Note that ₹4000 is the middle most value.

This kind of representative value which is obtained by choosing the middle item is known as Median.

Thus in a given data, arranged in ascending or descending order, the median gives us the middle value.

Consider another example, where the data contains even number of terms 13, 14, 15, 16, 17 and 18. How to find the middle term for this example? Here the number of terms is 6, that is an even number. So we get, two middle terms namely 3rd and 4th term. Then, we take the average of the two terms (3rd and 4th term) and the value we get is the median.

That is, Median $= \frac{1}{2}\left\{3^{rd} \text{ term} + 4^{th} \text{ term}\right\}$

$= \frac{1}{2}\left\{15 + 16\right\}$

$$= \frac{15+16}{2} = \frac{31}{2} = 15.5$$

Here, to find median we arrange the values of the given data either in ascending or descending order, then find the average of the two middle values.

So we conclude that, to find median,

(i) arrange the data in ascending or descending order.

(ii) If the number of terms (n) is odd, then $\left(\frac{n+1}{2}\right)^{th}$ term is the median.

(iii) If the number of terms (n) is even, then average of $\left(\frac{n}{2}\right)^{th}$ and $\left(\frac{n}{2}+1\right)^{th}$ terms is the median.

Try these

1. Find the median of 3, 8, 7, 8, 4, 5, 6.
2. Find the median of 11, 14, 10, 9, 14, 11, 12, 6, 7, 7.

Activity

Create a group of 6 to 7 students and collect the data of weight of the students in your class. In each group find mean, median and mode. Also, compare the averages among groups. Are they same for all the groups?

Also find all the three averages for entire class. Now, compare the results with the average of each of the groups.

Example 5.10 Find the median of the following golf scores.

68, 79, 78, 65, 75, 70, 73.

Solution

Arranging the golf scores in ascending order, we have,

65, 68, 70, 73, 75, 78, 79

Here $n = 7$, which is odd.

Therefore, Median $= \left(\frac{n+1}{2}\right)^{th}$ term

$= \left(\frac{7+1}{2}\right)^{th}$ term.

$= \left(\frac{8}{2}\right)^{th}$ term

$= 4^{th}$ term $= 73$

Hence, the Median is 73.

Example 5.11 The weights of 10 students (in *kg*) are 35, 42, 40, 38, 25, 32, 29, 45, 20, 24 Find the median of their weight?

Solution

Arranging the weights in ascending order, we have,

20, 24, 25, 29, 32, 35, 38, 40, 42, 45

Here, $n = 10$, which is even.

$$\text{Therefore, median weight} = \frac{1}{2}\left\{\left(\frac{n}{2}\right)^{th} term + \left(\frac{n}{2}+1\right)^{th} term\right\}$$

$$= \frac{1}{2}\left\{\left(\frac{10}{2}\right)^{th} term + \left(\frac{10}{2}+1\right)^{th} term\right\}$$

$$= \frac{1}{2}\left\{5^{\text{th}} \text{ term} + 6^{\text{th}} \text{ term}\right\}$$

$$= \frac{1}{2}\{32 + 35\}\ kg = \frac{67}{2} = 33.5\ kg$$

Hence, Median is 33.5 *kg*.

Example 5.12 Create a collection of 12 observations with median 16.

Solution

As the number of observations is even, there are two middle values.

The average of those values must be 16.

We will now find any pair of numbers whose average is 16. Say 14 and 18.

Now an example of data with median 16 can be 2, 4, 7, 9, 12, 14, 18, 24, 28, 30, 45, 62.

Note

We can find more than one answer for this question

Example 5.13 The lifetime (in days) of 11 types of LED bulbs is given in days. 365, 547, 730, 1095, 547, 912, 365, 1460, 1825, 1500, 2000. Find the median life time of the LED bulbs.

Solution

Arranging the data in ascending order, we have,

365, 365, 547, 547, 730, 912, 1095, 1460, 1500, 1825, 2000.

The number of observations are 11, which is odd.

$$\text{Median} = \left(\frac{n+1}{2}\right)^{th} \text{term}$$

$$= \left(\frac{11+1}{2}\right)^{th} \text{ term}$$

$$= 6^{th} \text{ term} = 912$$

Therefore, the median is 912.

Hence, the median lifetime of the LED bulb is 912 days.

Example 5.14 Find the Median of the following data.

12, 7, 23, 14, 19, 10, 5, 26

Solution

Arranging the data in ascending order, we have

5, 7, 10, 12, 14, 19, 23, 26.

Here, $n = 8$, which is even.

Therefore, Median $= \frac{1}{2}\left\{\left(\frac{8}{2}\right)^{th} term + \left(\frac{8}{2}+1\right)^{th} term\right\}$

$$= \frac{1}{2}\left\{4^{\text{th}}\ term + 5^{\text{th}}\ term\right\}$$

$$= \frac{1}{2}\{12+14\}$$

$$= \frac{26}{2} = 13$$

Therefore, the Median is 13.

Think

Complete the table given below and observe it to answer the following questions.

Series	Values	Mean	Median
A	99,100,101		
B	90,100,110		
C	50,100,150		
D	99,100,200		

(i) Which are all the series having common mean and median?

(ii) Why median is same for all the 4 series?

(iii) How mean is unchanged in the series A, B and C.

(iv) What change is to be made in the data, so that mean and median of 'D' series is equal to other series?

Exercise 5.3

1. Fill in the blanks.

   (i) The median of the data 12, 14, 23, 25, 34, 11, 42, 45, 32, 22, 44 is __________.

   (ii) The median of first ten even natural numbers is _________.

2. Find the median of the given data: 35, 25, 34, 36, 45, 18, 28.

3. The weekly sale of motor bikes in a showroom for the past 14 weeks given below. 10, 6, 8, 3, 5, 6, 4, 7, 12, 13, 16, 10, 4, 7. Find the median of the data.

4. Find the median of the 10 observations 36, 33, 45, 28, 39, 45, 54, 23, 56, 25. If another observation 35 is added to the above data, what would be the new median?

Objective type questions

5. If the median of *a, 2a, 4a, 6a, 9a* is 8, then find the value of *a* is

   (i) 8 (ii) 6 (iii) 2 (iv) 10

6. The median of the data 24, 29, 34, 38, 35 and 30, is______________

   (i) 29 (ii) 30 (iii) 34 (iv) 32

7. The median first 6 odd natural numbers is _________

   (i) 6 (ii) 7 (iii) 8 (iv) 14

Exercise 5.4

Miscellaneous Practice problems

1. Arithmetic mean of 15 observations was calculated as 85. In doing so an observation was wrongly taken as 73 for 28. What would be correct mean?

2. Find the median of 25, 16, 15, 10, 8, 30.

3. Find the mode of 2, 5, 5, 1, 3, 2, 2, 1, 3, 5, 3.

4. The marks scored by the students in social test out of 20 marks are as follows: 12, 10, 8, 18, 14, 16. Find the mean and median?

5. The number of goals scored by a football team is given below. Find the mode and median for the data of 2, 3, 2, 4, 6, 1, 3, 2, 4, 1, 6.

6. Find the mean and mode of 6, 11, 13, 12, 4, 2.

Challenge Problems

7. The average marks of six students is 8. One more student mark is added and the mean is still 8. Find the student mark that has been added.

8. Calculate the mean, mode and median for the following data:
   22, 15, 10, 10, 24, 21.

9. Find the median of the given data: 14, −3, 0, −2, −8, 13, −1, 7.

10. Find the mean of first 10 prime numbers and first 10 composite numbers.

Summary

- Based on the purpose, appropriate data has to be collected and organised to find the representative of data.
- Representative of data are also known as measures of central tendency.
- Arithmetic mean is the most commonly used representative of data and is calculated by the formula.

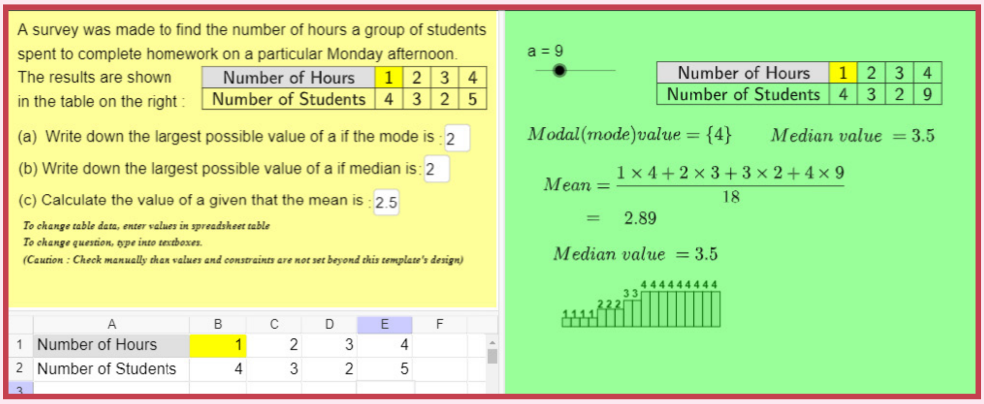
$$\text{Arithmetic Mean} = \frac{\text{Sum of all observations}}{\text{Number of observations}}$$

- Mode is the value of the data which occurs maximum number of times.
- A data may have more than one mode as well as no mode.
- A data is of large size, mode can be found out after grouping.
- Median is the middle most value of the given data.
- To find the median for the given data,
  - (i) arrange the data in ascending or descending order.
  - (ii) If the number of terms (n) is odd, then $\left(\frac{n+1}{2}\right)^{th}$ term is the median.
  - (iii) If the number of terms (n) is even, then average of $\left(\frac{n}{2}\right)^{th}$ and $\left(\frac{n}{2}+1\right)^{th}$ terms is the median.

ICT Corner

Statistics

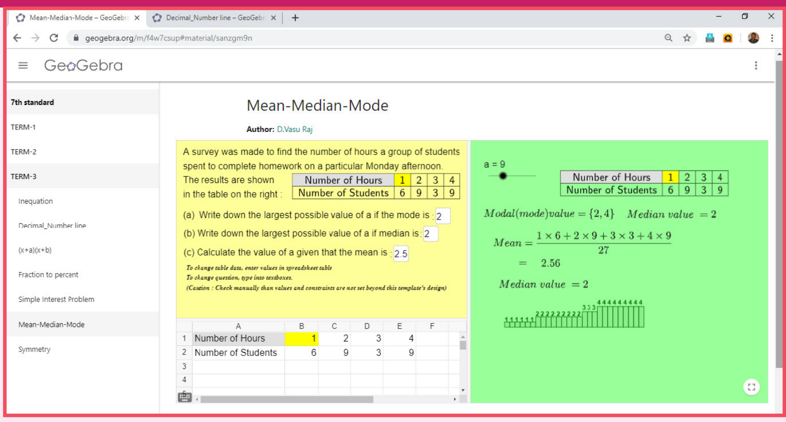
Expected outcome



Step – 1

Open the Browser and type the URL Link given below (or) Scan the QR Code. GeoGebra work sheet named “Mean-Median-Mode” will open. Change the number of students by typing your number. Right side you can see the Mean, Median and Mode.

Step 1



Browse in the link

Mean-Median-Mode: https://www.geogebra.org/m/f4w7csup#material/sanzgm9n

or Scan the QR Code.

Chapter 6

INFORMATION PROCESSING

Learning Objectives

- Scheduling of tasks under given set of constraints.
- Able to create and use of flowcharts.

6.1 Scheduling

The Process that involves deciding of the ordering of tasks and allocating of appropriate resources among the different type of possible tasks is called scheduling. Schedules can usually be short term periodical plan such as a daily or weekly schedule, and long – term periodical plan which extends for several months or years.

A schedule consists of a list with a timeline with which possible tasks, events or activities are intended to take place or a sequence of is intended to take place.

Situation 1

Here is a programme list of two days Zonal level sports meet held on 15th Nov 2018 and 16th Nov 2018 at Bala's school. To submit the annual report, prepare a schedule and make a table using the column headings given below.

S.No.	Date of the events	Time of the events	Name of the Events	Collections of photography

Programmes
15-11-2018

1. Preparation of play ground – 8.00 AM
2. Inauguration – 9.00 AM
3. Instructions given by Teacher – 9.15 AM
4. Registration for events - 9.30 AM
5. Allotment report for team games – 9.45 AM
6. Inauguration of sports events – 10.00AM
7. Indoor games – 11.00 AM
8. Individual Events – 02.30 PM

16-11-2018

9. Team Events – 10.30 AM
10. Prize Distribution – 4.30 PM
11. Closing ceremony – 5.15 PM
12. National Anthem – 06.00 PM












Zonal Level Sports Meet 15.11.2018 & 16.11.2018				
S.No.	**Date of the events**	**Time of the events**	**Name of the Events**	**Collections of photography**
1	15.11.2018	8.00 AM	Preparation of play ground	
2	15.11.2018	9.00 AM	Inauguration	
3	15.11.2018	9.15 AM	Instructions given by Teacher	

4	15.11.2018	9.30 AM	Registration for events	
5	15.11.2018	9.45 AM	Allotment report for team games	
6	15.112018	10.00 AM	Inauguration of sports events	
7	15.11.2018	11.00 AM	Indoor games	
8	15.11.2018	2.30 PM	Individual Events	
9	16.11.2018	10.30 AM	Team Events	
10	16.11.2018	4.30 AM	Prize Distribution	
11	16.11.2018	5.15 PM	Closing ceremony	
11	16.11.2018	6.00 PM	National Anthem	

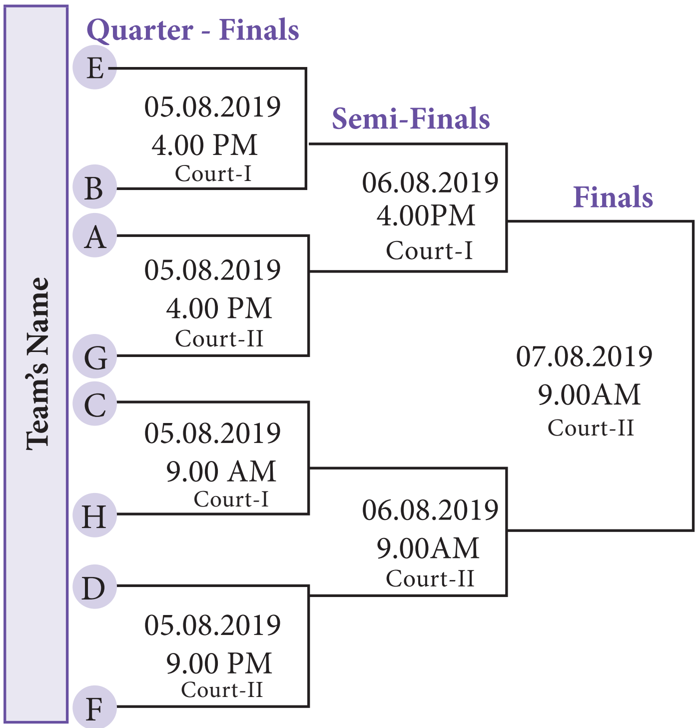
Activity

Using the time table given below, fill in the trip schedule, date and time according to your school.

Schedule for going an excursion and parental permit letter			
S.No.	**Name of the Events**	**Date**	**Time**
1.	Instruction given by a class teacher about an excursion		
2.	Receiving parents permission Letter		
3.	Preparation of the name list by the class teacher who are going for an excursion		
4.	Instructions given by HM		
5.	Time to start an excursion		
6.	Time to reach the First destination		
7.	Lunch Break		
8.	Time to reach second destination		
9.	Tea Break		
10.	Time of return to school		
I am willing to send my child for an excursion **Class Teacher signature** **Parent's signature**			

Activity

A fixture given below illustrates the team allocation for the game of kabaddi.



Instruction

- **8** school teams has been selected for quarter finals.
- **8** teams are named as **A** to **H**
- Matches will be held in **2** courts
- Here team **A** and **E** meet in the first semi final match, team **D** and **H** meet in the second semi final match.
- **H** wins the final match against **E**.

Using the fixtures, complete the table with the instruction given above:

S.No.	Match Name	Date	Time	Participating Teams	Venue of the match
1.	Quarter Finals – 1		4.00 PM		
2.		05-08-2019		**A** and **G**	
3.			9.00 PM		Court-I
4.	Quarter Finals – IV			**D** and **F**	
5.	Semi Finals – I	06-08-2019			
6.					Court-II
7.	Final				

I. Semi Finalist team Name 1) ______________ Vs _________________

2) ______________ Vs _________________

II. Winner of the Match ______________________

Activity

Complete the partially filled 7th class model timetable using the instruction given below.

Periods Allotment		
Tamil	-	7
English	-	7
Maths	-	7
Science	-	7
Social Science	-	7
PET	-	2
Drawing	-	1
Music	-	1
Library	-	1
Total	-	40 Periods

Instruction

- 2 Periods can be combined for all subjects.
- Drawing and music periods can be alloted only in the afternoon.
- 1 PET period can be alloted in the morning session and one in the afternoon session.
- Library period should be alloted in the second period only

VII – Time Table

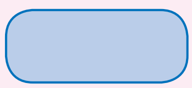
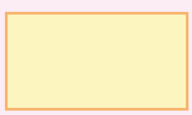
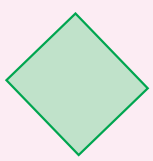
Days	Periods							
	1	2	3	4	5	6	7	8
Monday	Tamil		English		Science	Social Science		English
Tuesday	Tamil		English			Social Science	Social Science	Maths
Wednesday	Tamil	Tamil	Maths	Maths	Science	Social Science	PET	
Thursday	Tamil	Tamil	Maths	Maths				
Friday	English	English	Maths				Science	Science

6.2 Flowchart

Can you think of a day in our life which goes without problem solving? No. In our life we are bound to solve problems. In our day to day activity such as browsing some web pages, purchasing something from a general store and making payments, depositing fee in school, or withdrawing money from an ATM involves some kind of problem solving. It can be said that whatever activity a human being or machine does for achieving a specified objective comes under problem solving. During the process of solving any problem, one tries to find the necessary steps to be under taken in a sequence.

Often the best way to understand a problem is to draw pictures. Pictures often provide us with a more complete idea of the situation than a series of short words or phrases can. However, pictures combined with text provide an extremely powerful tool for communication and problem solving.

A Flowchart is a pictorial representation and it gives an idea of instructions to perform a task or calculation. They are widely used in multiple fields to prepare a document, study, plan, improve and communicate often complex process clearly as it is easy to understand a diagram. In the flowchart, each shape has a particular meaning. They are given in the following table.

Shape	Name	Meaning
→	Flowline	Used to connect shapes and indicates the flow of instructions.
	Terminal	Used to represent the Start and End of the task.
	Input / Output	Used to the instruction to be read or displayed are described inside.
	Processing	Used for calculating or indicating normal process of flow step.
	Decision	Used for any logic or comparison of an operation. The flow direction chosen depends on whether answer to the question is "yes" or "no".

To understand better, let us see, how we can apply flowchart practically.

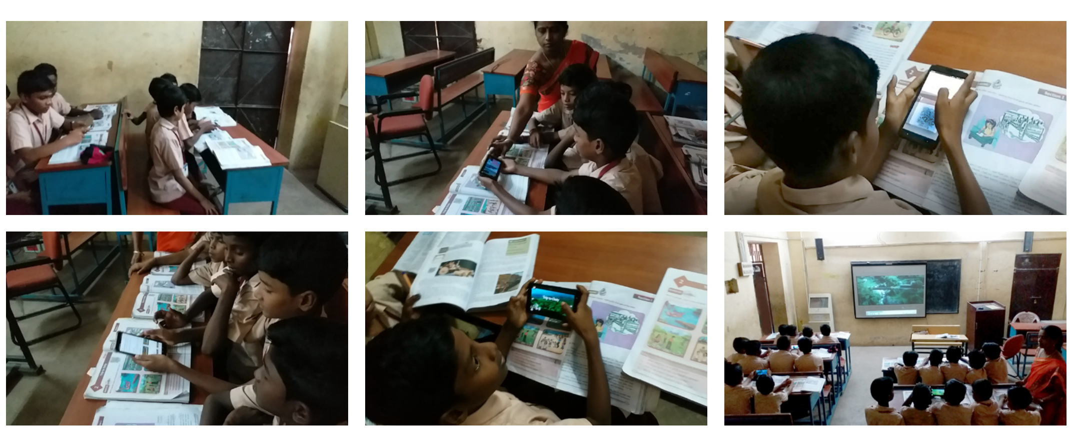
6.2.1 Types of Flowchart

Sequential flowchart

A flowchart describes the operations and in what sequence the problem is to be solved.

Situation 1

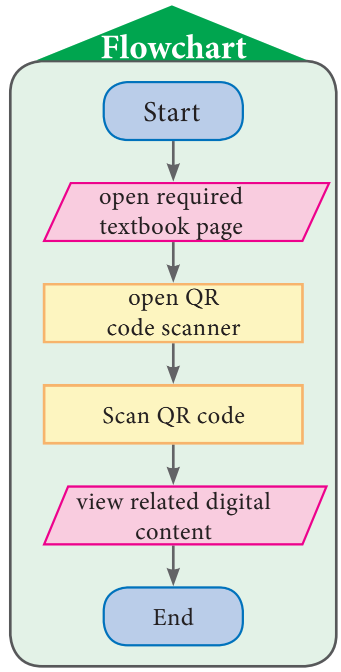
Scan QR Code from your textbook to view related digital content. as shown in the pictures below.



First, let us write an step by step process to scan QR code:

Step by Step process
First, let us write an step by step process to scan QR code: 1. Take correct textbook page on which required QR code were situated. 2. Open QR code scanner on your mobile or tab. 3. Scan QR code to view related digital content. 4. View digital content.

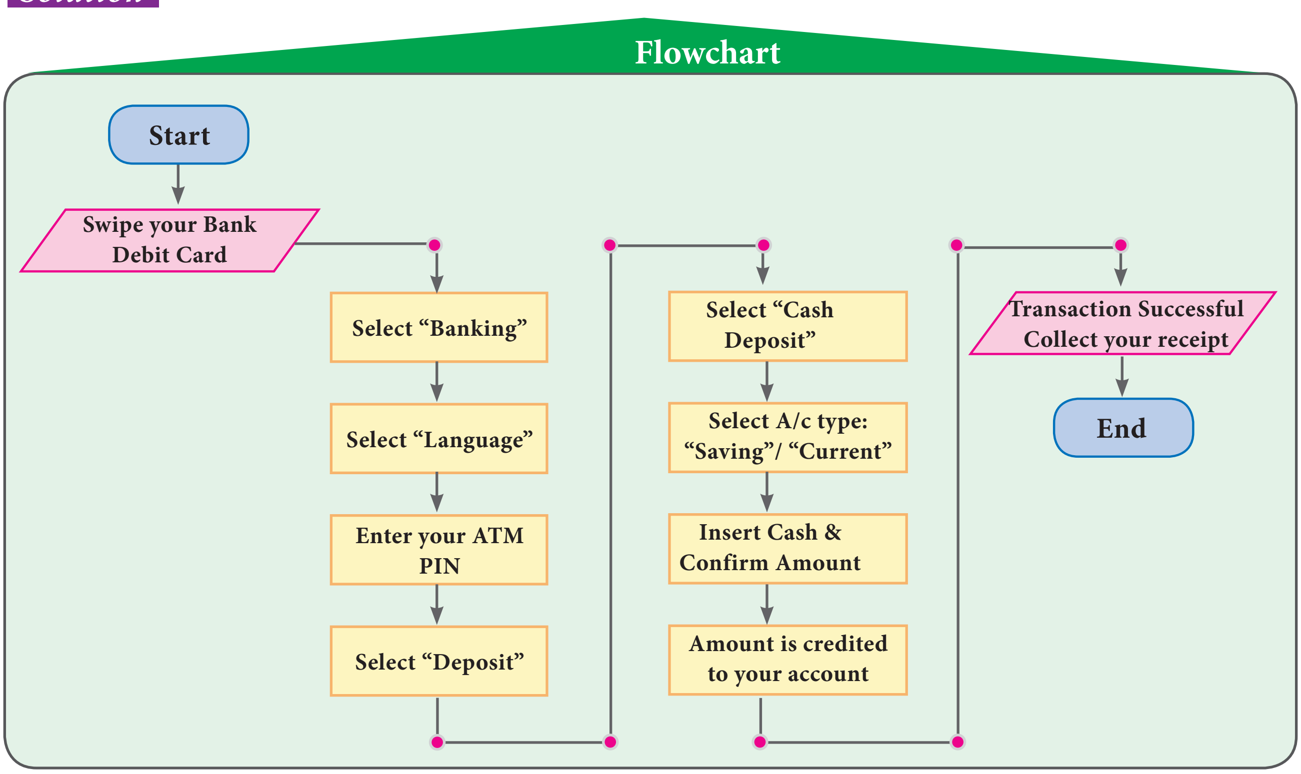
Now, let us write a sequence flowchart for this given in figure beside.



Example 6.1 Construct an appropriate flowchart for depositing a sum of money in an ATM using the instruction given below.

Input	Process		Output
1 Swipe your Bank Debit Card	2 Select "Banking"	6 Select "Cash Deposit"	10 Transaction Successful Collect your receipt
	3 Select Language	7 Select A/c type: "Saving"/ "Current"	
	4 Enter your ATM PIN	8 Insert Cash & Confirm Amount	
	5 Select "Deposit"	9 Amount is credited to your account	

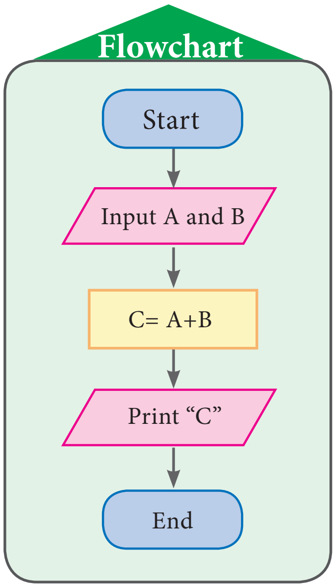
Solution



Conditional flowchart

Situation 2

Construct the flowchart to find the sum of given two numbers. (To achieve this, first the two numbers have to be received and kept in two places, under two names. Then the sum of them is to be found and printed. The flow chart for this is given in figure beside.)



In the above flow chart, there is a special meaning in writing C = A + B. Here, the value of C indicates the sum of A and B values. For example, if we input **A = 325** and **B = 486** then the value of C will be printed as **811**. Let us learn more from following examples.

Example 6.2

Construct the flow chart to find whether the given number is a positive integer or negative integer or zero and write step by step process.

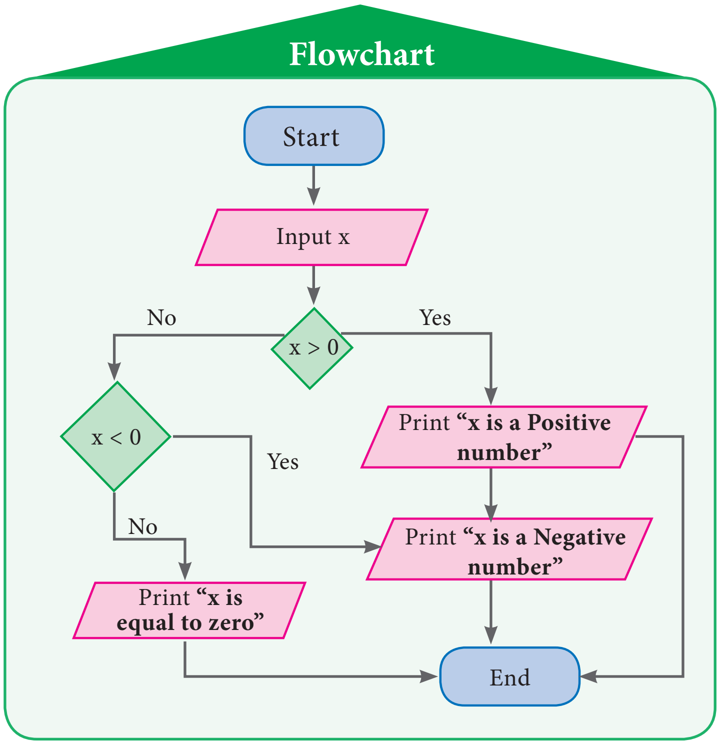
Solution

In this question we are asked to find the given integer **x** is a positive number or a negative number or a zero. For that, first we have to assign the value of **x** and check whether **x** is greater than 0. If '**yes**', we will print "**x is a Positive number**". If '**no**' then we will check again if the value of **x** is less than **0**. If 'yes', we will print "**x is a Negative number**". If 'no', then we will print "**x = zero**".

Step by Step process:

Step by Step process

- assign the value of **x**
- check **x > 0**
- if 'yes' print
- "**x is a Positive number**"
- check again **x < 0**
- if 'yes' print
- "**x is a Negative number**"
- if 'no' print "**x is equal to zero**".



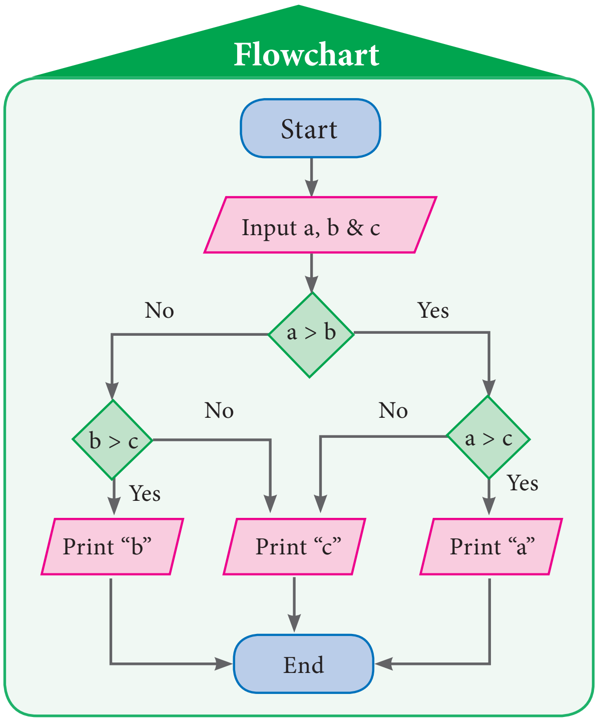
Verifying the condition x > 0	Verifying the condition x < 0	Verifying the condition x = 0
For a particular value x = 7926 7926 > 0 Yes Print **x is a Positive number**	For a particular value **x = -2589** -2589 > 0 No -2589 < 0 Yes Print **x is a Negative number**	For a particular value **x = 0** 0 > 0 No 0 < 0 No Print **x is equal to zero**

Example 6.3 Construct the flow chart to explain the process of finding the greatest number among the given 3 natural numbers and write step by step process

Solution

In this question, we are asked to find the greatest of the given 3 numbers. For that, first we have to assign the values of **a , b and c** and then check whether **a** is greater than **b**. If 'yes', we will check again if the value of **a** is greater than **c.** If '**yes**', we will print the value of **a.** If '**no**', then we will check again if the value of **b** is greater than **c.** If '**yes**', we will print the value of **b.** If '**no**', then we will print the value of **c.**

Step by Step process
• assign the values of **a, b & c**
• check **a** > **b.**
• if 'yes', check again **a** > **c**
• if 'yes', print the value of **a**
• if 'no', check again **b** > **c**
• if 'yes', print the value of **b**
• if 'no', print the value of **c**



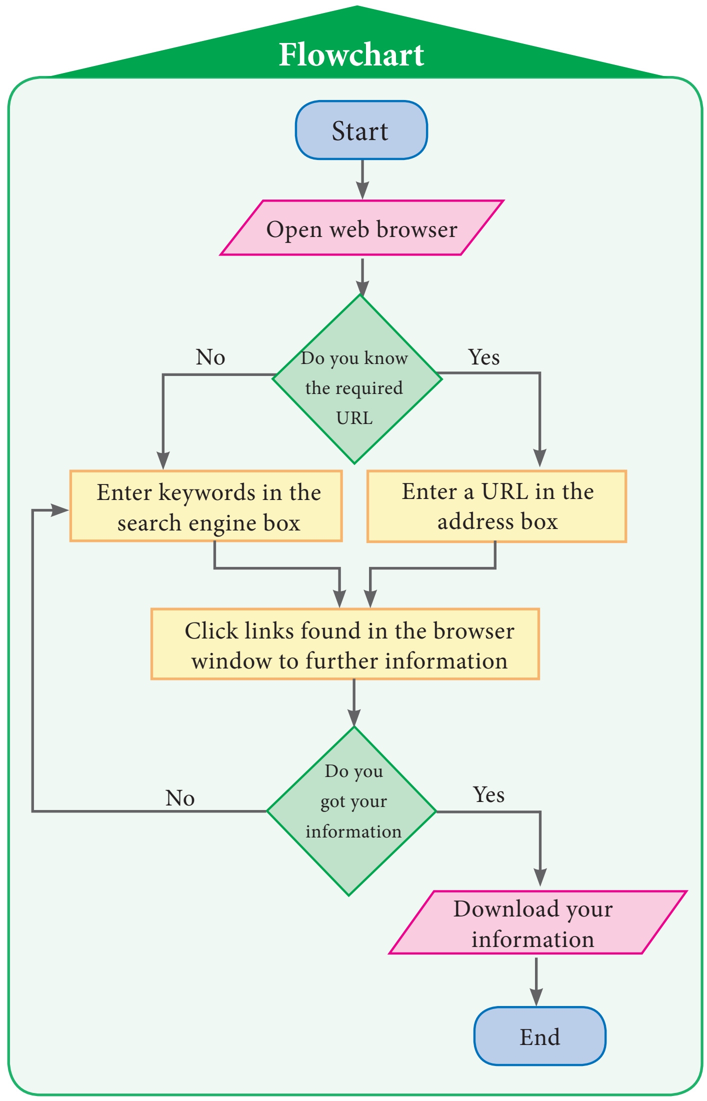
Verifying the condition a > b	Verifying the condition b > c	Verifying the condition a > c
For a particular value	For a particular value	For a particular value
a = 75, b = 56 & c = 20	**a = 68, b = 82 & c = 45**	**a = 185, b = 393 & c = 852**
check is **75 > 56**	check is **68 > 82**	check is **185 >393**
'yes' check again **75 > 20** 'yes'	'no' check again **82 > 45**	'no' check again **393 > 852**
print **75**	'yes' print **82**	'no' print **852**

Situation 3

Are you familiar with the Internet? If you want to know how to search in the Internet, then you have to find the right search engine, type in your search keywords as accurately as possible, and browse through the results to find the one you want. Let us learn more about this from the following algorithm and flowchart.

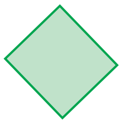
Step by Step process

- Open your web browser
- If you know the required **URL** then enter an URL in the address box or
- Enter **keywords** in the search engine box
- Click the **link** found in the browser window to further information
- If you get the information you need, **download** your materials (text or image or audio or video or web links). Otherwise continue search.

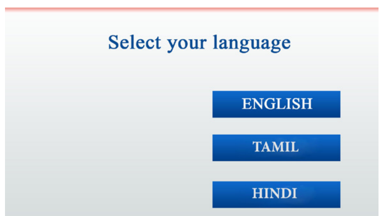
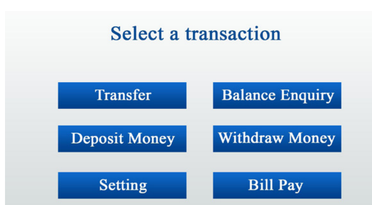
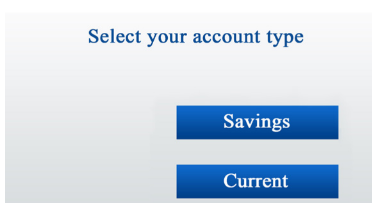
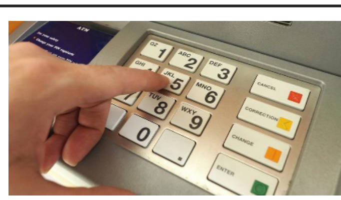
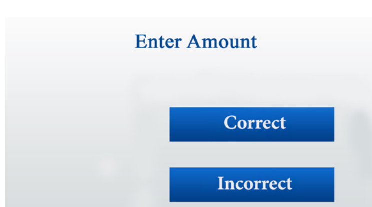


Exercise 6.1

1. Match the following:

	Symbols		**Uses**
(i)		(a)	Input / Output
(ii)		(b)	c = a + b
(iii)		(c)	Start / End
(iv)		(d)	a >= 0
(v)		(e)	It shows direction of flow

2. The steps of withdrawing cash from your saving bank account using ATM card are explained in the figures given below. Construct an appropriated flow chart.

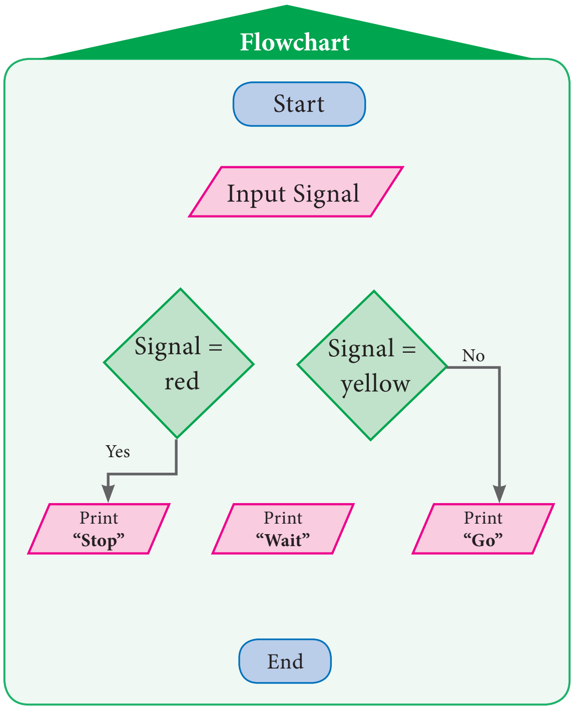
(i)		Insert a ATM debit /credit card
(ii)		Select your language
(iii)		Select your transaction
(iv)		Select your account type
(v)		Enter the pin number
(vi)		Enter the amount do you want to withdraw
(vii)		Collect your money

3. Using given step by step process to recharge mobile phone , draw a sequence flowchart.

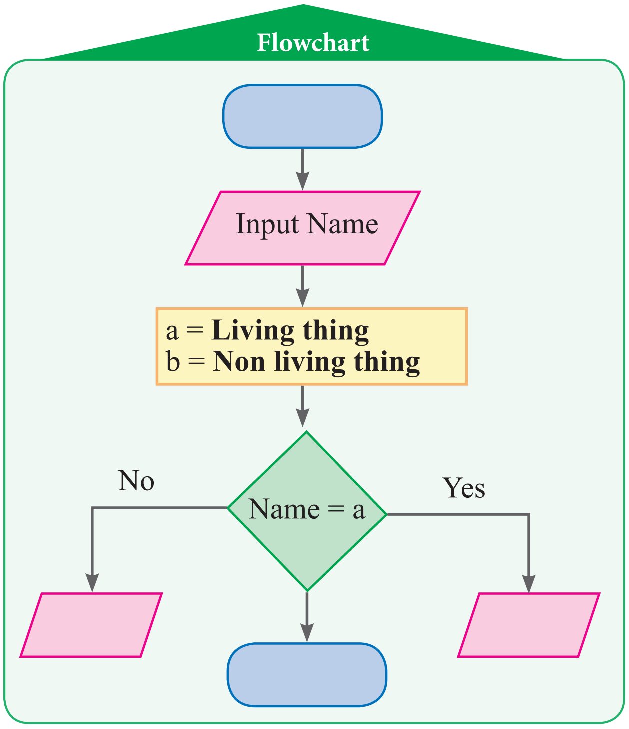
Step by Step process

- Login the mobile recharge web browser
- Select prepaid or postpaid
- Enter mobile number
- Select operator and browse plans to choose your recharge plan
- Enter amount to recharge
- Proceed to recharge

4. Complete the direction of the flowchart using arrows for the flow chart explaining the traffic rule given below.

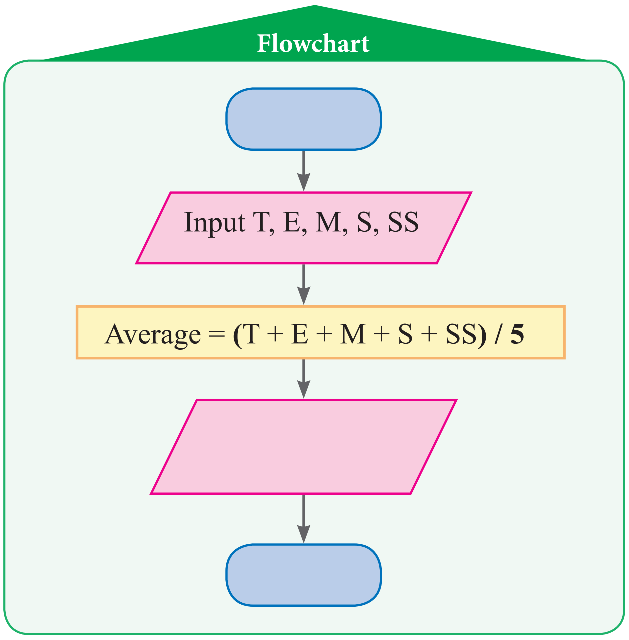


5. Complete the given flowchart, input names of things and check whether it is living or non - living.

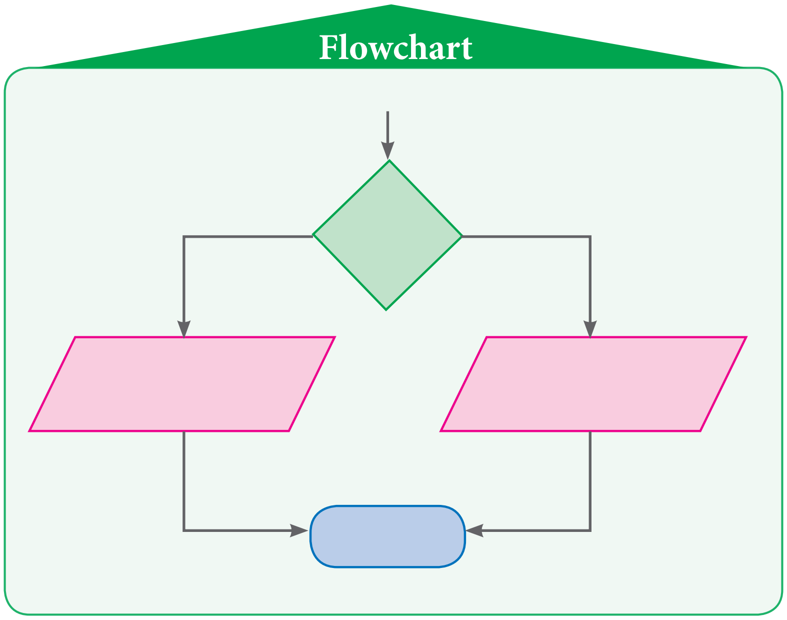


6. Complete the given flowchart.

   (i) Fill in the flow chart to print the average mark by giving your I term or II term marks as inputs.



   (ii) Construct the flow chart to print teachers comment as "very good" if your average mark is above 75 out of 100 or else, as "still try more" can be inserted in the flow chart with earlier one.



7. A merchant calculates the cost price (**CP**) and the selling price (**SP**) of the product bought by him. Construct the flow chart to print 'PROFIT' if the selling price (**SP**) is more than the cost price (**CP**) or else 'LOSS'.

ANSWERS

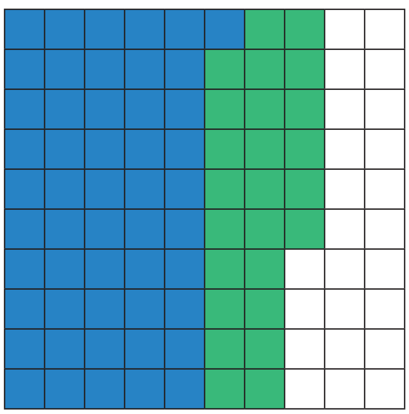
1. Number system

Exercise 1.1

1. (i) 9 (ii) 26 (iii) 69 (iv) 104
   (v) 50 (vi) 101 (vii) 40 (viii) 1
2. (i) 6.0 (ii) 21.81 (iii) 35.001
3. (i) 123.4 (ii) 20.0 (iii) 910.6
4. (i) 87.76 (ii) 301.51 (iii) 80.00
5. (i) 24.400 (ii)1251.235 (iii) 61.002

Exercise 1.2

1.



Ans 0.76

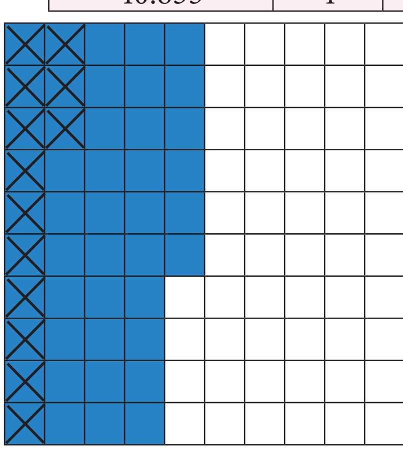
2. (i)

Decimal No	Tens	Ones	Tenths	Hundredths
25.80	2	5	8	0
18.53	1	8	5	3
44.33	4	4	3	3

(ii)

Decimal No	Tens	Ones	Tenths	Hundredths	Thousands
17.400	1	7	4	0	0
23.435	2	3	4	3	5
40.835	4	0	8	3	5

3.



Ans 0.33

4. (i)

Decimal No	Ones	Tenths	Hundredths	Thousandths
9.231	9	2	3	1
6.567	6	5	6	7
2.664	2	6	6	4

(ii)

Decimal No	Ones	Tenths	Hundredths	Thousandths
7.000	7	0	0	0
3.235	3	2	3	5
3.765	3	7	6	5

5. 14.01 6. 4.650 *kg* 7. 6.387 8. 18.1 9. ₹ 2.60 10. 11.4 *cm*

Objective type questions

11. (iii) 1.83 12. (ii) 4.17 13. (iv) 2.16 14. (i) 1.57 15. (i) 128.89

Exercise 1.3

1. (i) 1.5 (ii) 22.50 (iii) 200.8
   (iv) 0.27 (v)3171.21 (vi) 2.8

2. 23.80 *sq.cm* 3. 360.24 *sq.cm*

4. (i) 25.7 (ii) 5.1 (iii) 12536.7 (iv) 3451 (v) 6273.5
   (vi) 7.0 (vii) 3 (viii) 400

5. 497*cm* 6. ₹ 30

7. (i) 1.08 (ii) 5.23 (iii) 107.48 (iv) 0.036 (v) 14.306
   (vi) 0.051 (vii) 10.5525 (viii) 1.0101 (ix) 110.011

Objective type questions

8. (ii) 0.107 9. (i) 20.8 10. (iii) 53.0 *cm*

Exercise 1.4

1. (i) 0.2 (ii) 0.18 (iii) 1.02 (iv) 3.08
   (v) 0.094 (vi) 10.34 (vii) 99.4
2. (i) 0.57 (ii) 9.37 (iii) 0.09 (iv) 30.1301
   (v) 0.083 (vi) 0.0062
3. (i) 0.007 (ii) 0.038 (iii) 0.493 (iv) 4.6385
   (v) 0.003 (vi) 0.274
4. (i) 0.0189 (ii) 0.00087 (iii) 0.0493 (iv) 0.0003
   (v) 0.3824 (vi) 0.0938
5. (i) 8 (ii) 9.9 (iii) 14.7 (iv) 0.19
   (v) 9 (vi) 1.69

6. 1.91 *kg* 7. 64 *km* 8. 650 *sq.ft* 9. ₹ 53.80 10. 2.8

Objective type questions

11. (iv) 11.2 12. (ii) 67.0 13. (ii) 0.1

Exercise 1.5

1. 36.06 *m* 2. 1.976 *kg* 3. 20.24 *l* 4. 9 5. 0.3924 *kg*

6.(i)120.198 (ii) 33.258

7. 15 8. 2376.1 *m* 9. 0.0523 10. 69 pages

Challange Problems

11 0.267 *km* 12. Mala travelled 0.5 *km* more than Anbu.

13. ₹ 3421.25 14. 463.53 *km/hour* 15. 325.08 *km*

2. Percentage and Simple Interest

Exercise 2.1

1. (i) $\frac{58}{100}$, 0.58, 58% (ii) $\frac{53}{100}$, 0.53, 53%

(iii) $\frac{25}{50}$, 0.50, 50% (iv) $\frac{17}{25}$, 0.68, 68%

(v) $\frac{15}{30}$, 0.50, 50%

2. (i) 50% (ii) 50% (iii) $31\frac{4}{16}$% (iv) $68\frac{12}{16}$%

3. White colour– 50% ; Black colour – 50 %

4. (i) 72% (ii) 270% (iii) 75% (iv) 225%

(v) 160%

5. 87.2 %

6. (i) $\frac{21}{100}$ (ii) $\frac{931}{1000}$ (iii) $\frac{151}{100}$ (iv) $\frac{13}{20}$ (v) $\frac{4}{625}$

7. 83.33% 8. $\frac{12}{25}$ 9. ₹ 1, 875

Objective type questions

10. (iii) 25% 11. (i) 60% 12. (iv) $\frac{7}{10,000}$

Exercise 2.2

1. (i) 0.21 (ii) 0.931 (iii) 1.51 (iv) 0.65 (v) 0.0064

2. (i) 28.2% (ii) 151 % (iii) 109 % (iv) 71 % (v) 85.8 %

3. 0.75 4. 0.705 5. 0.86 6. 675%

7. 156%   8. 25%

Objective type questions

9. 1.425   10. 0.5 %   11. 470 %

Exercise 2.3

1. 20%   2. 10 liters   3. ₹1334   4. ₹240   5. 75%

6. 30%   7. Mathametics 85 %   8. 600   9. 8%

10. Education - ₹6,000 and 33.33%; Savings – ₹3,000 and 16.66%; other expenses – ₹9,000 and 50%

Exercise 2.4

1. ₹6,300   2. I= ₹1,120 ; A = ₹9,120   3. ₹56,000

4. 10%   5. 3 years   6. 2 years   7. 7%   8. ₹12,500

Objective type questions

9. (i) ₹500   10. (iii) ₹100   11. (i) 10%

Exercise 2.5

1. 10%   2. 60%   3. 58.33%   4. 70 %

5. 74% cam swim; 26% canot swim   6. 20%

7. ₹75   8. ₹1,770   9. 4%   10. 800%

11. ₹800   12. 6 years   13. 8 years

Challange Problems

14. 20% ; 80%   15. 77.77%   16. 58.33%   17. 32%

18. 16   19. 7.225 *kg*   20. 47.82%   21. 70 *l*

22. $16\frac{2}{3}$ 23. 1,31,220   24. 12 %   25. 7%

26. 10 %   27. 25 %

3. Algebra

Exercise 3.1

1. (i) $p^2-2pq+q^2$ (ii) x^2-25

(iii) $(x-2)$ and $(x-2)$ (iv) $2 \times 2 \times 2 \times 3 \times a \times b \times c \times c$

2. (i) True   (ii) False   (iii) True   (iv) True.

3. (i) $2 \times 2 \times 2 \times 3 \times a \times b \times b \times c \times c$

(ii) $2 \times 2 \times 3 \times 3 \times x \times x \times x \times y \times y \times z$

(iii) $2 \times 2 \times 2 \times 7 \times m \times n \times n \times p \times p$

4. (i) $x^2 + 10x + 21$ (ii) $36a^2 + 24a - 45$

(iii) $16x^2 + 32xy + 15y^2$ (iv) $p^2q^2 + 15pq + 56$

5. (i) $4x^2 + 20x + 25$ (ii) $b^2 - 14b + 49$

(iii) $m^2n^2 + 6\,mnp + 9p^2$ (iv) $x^2y^2y^2 - 2\,xyz + 1$

6. (i) $p^2 - 4$ (ii) $9b^2 - 1$ (iii) $16 - m^2n^2$ (iv) $36x^2 - 49y^2$

7. (i) 2, 601 (ii) 10, 609 (iii) 9,96,004 (iv) 2, 209

(v) 89,991 (vi) 9,99,900 (vii) 2,652

8. $(a-b)^2$ 10. 64

11. (i) $(z + 4)(z - 4)$ (ii) $(3 + 2y)(3 - 2y)$

(iii) $(5a + 7b)(5a - 7b)$ (iv) $(x^2 + y^2)(x + y)(x - y)$

12. (i) $(x-4)(x-4)$ (ii) $(y + 10)(y + 10)$

(iii) $(6m + 5)(6m + 5)$ (iv) $(8x - 7y)(8x - 7y)$

(v) $(a + 3b + c)(a + 3b - c)$

Objective type questions

13. (ii) 6 14. (iii) – 625 15. (i) $(x - 3)(x-3)$

16. (iv) xy

Exercise 3.2

1. (i) $y \leq x$ (ii) $x + 6 \geq y + 6$ (iii) $x^2 \geq xy$

(iv) $-xy \leq -y^2$ (v) $x - y \geq 0$

2. (i) False (ii) False (iii) True (iv) False

3. (i) $x = 1, 2, 3, 4, 5, 6$ and 7

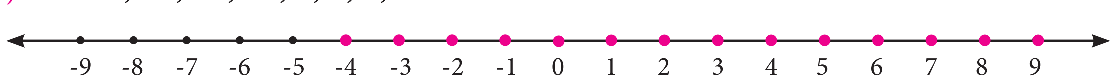
(ii) $x = 1, 2, 3, 4, 5$ and 6

(iii) $a = 0, 1, 2, 3, 4$ and 5

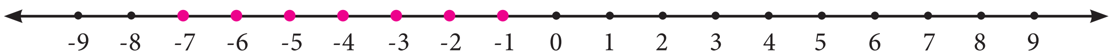
(iv) $x = 7, 8, 9, 10, \ldots.$

(v) $x = -5, -4, -3, -2$ and -1

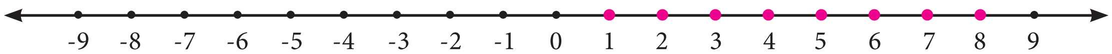
4. (i) $k = -4, -3, -2, -1, 0, 1, 2, \ldots$



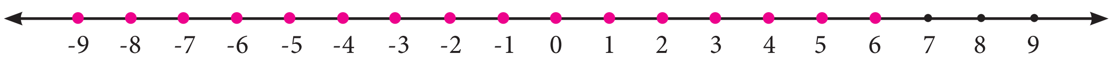
(ii) $y = -7, -6, -5, -4, -3, -2$ and -1



(iii) $x = 1, 2, 3, 4, 5, 6, 7$ and 8



(iv) $m = \ldots -3, -2, -1, 0, 1, 2, 3, 4, 5, 6$



5. The artist can buy $3 \leq x \leq 6$ brushes or $x = 3, 4, 5$ and 6 brushes.

Objective type questions

6. (iv) 3, 4, 5 and 6 7. (i) 1 and 2 8. (iii) 6 9. (ii) $-4 \leq x \leq 0$

Exercise 3.3

1. (i) 24.01 (ii) 10020.01 (iii) 3.99

2. $(2x + 3y)(2x - 3y)$

3. (i) $9p^2 + 3p(q + r) + qr$ (ii) $9p^2 - q^2$

4. 900 *sq.m*

Challenge problems

6. $(a - b)(a + b)$ 7. $x^4 - y^4$ 8. $-60xy$

9. (i) $4ab$ (ii) $2(a^2 + b^2)$

10. 225 *sq.m*

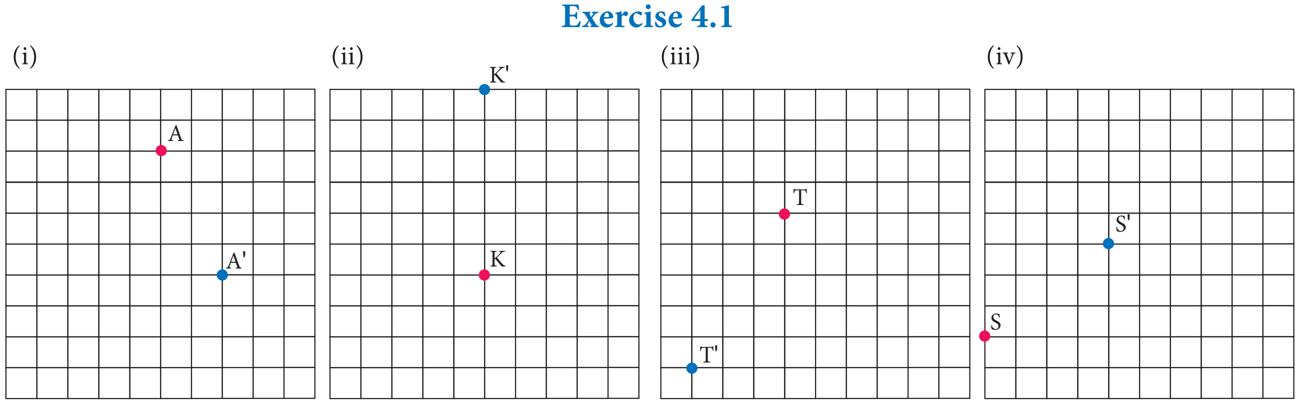
11. (i) $n = 3, 4, 5, 6, \ldots$

(ii) $x = 0, 1, 2, 3, \ldots$

(iii) $x = -3, -2, -1, 0, 1, 2, 3, 4, 5$ and 6

4. Geometry

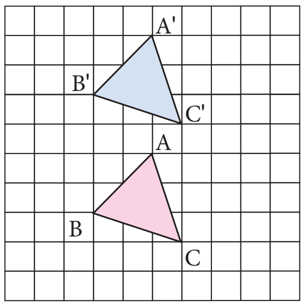
Exercise 4.1



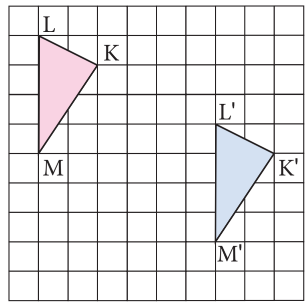
2. (i) $3\rightarrow, 4\uparrow$ (ii) $3\leftarrow, 3\uparrow$ (iii) $4\leftarrow, 4\downarrow$ (iv) $2\rightarrow, 2\downarrow$

3.

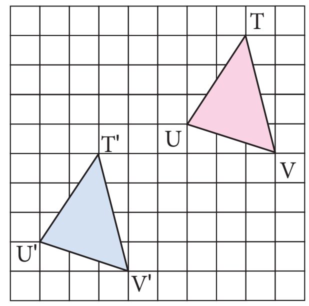
(i)



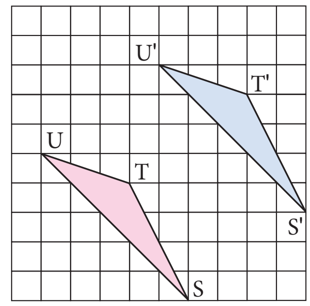
(ii)



(iii)

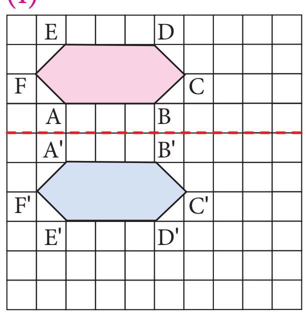


(iv)

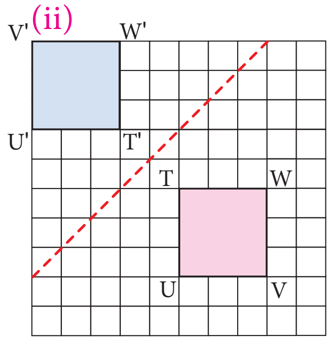


4.

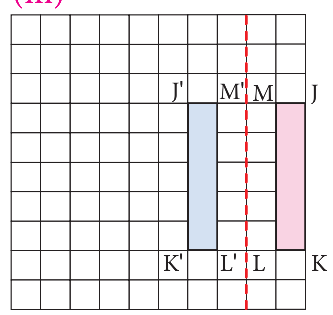
(i)



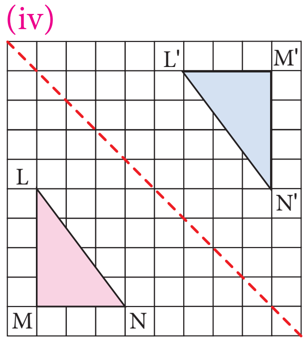
(ii)



(iii)



(iv)



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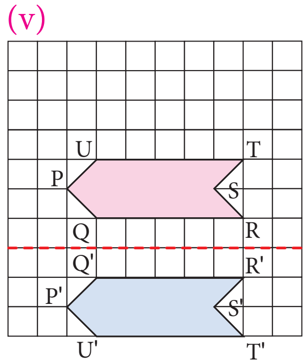

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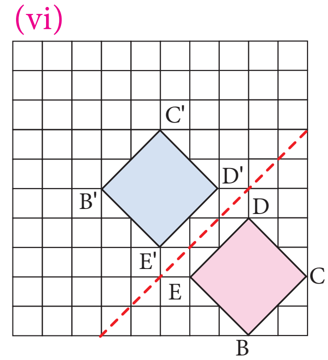
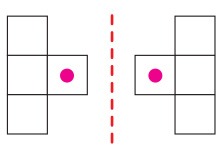


5. (i)



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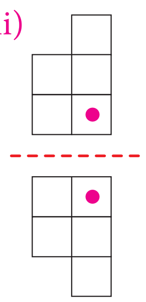

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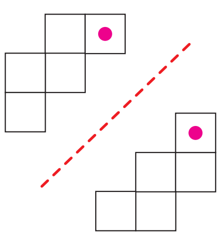
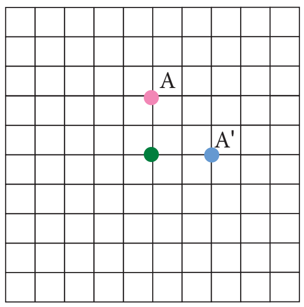
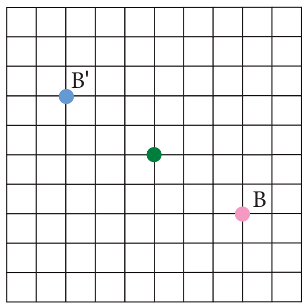


6. (i)



(ii)



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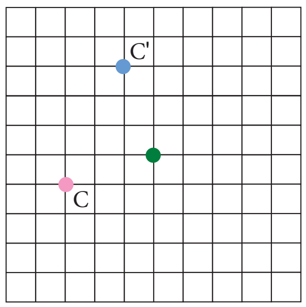

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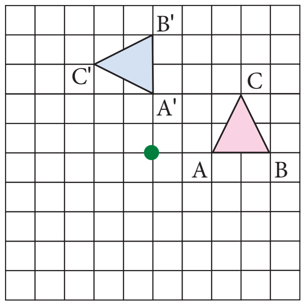

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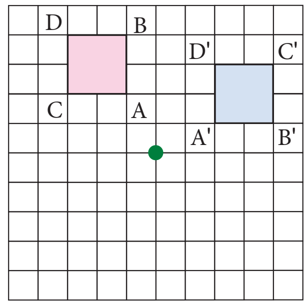

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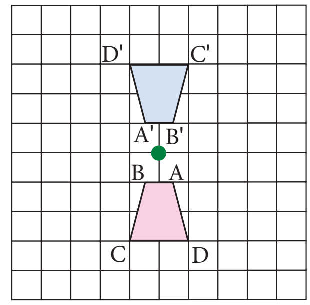


7. Reflection

8. Rotation

9. Translation

10. a. 7→,2↓

  b. No. It will be landed on the island.

  c. 5→,3↓

11. (i) Translation (ii) Reflection about horizontal line

  (iii) Reflection about vertical line (iv) Rotation about the heel

12. (i) Image of ∠L is ∠L', Image of ∠M is ∠M',

  Image of ∠N is ∠N', Image of ∠O is ∠O'

  Image of vertex L is L', Image of vertex M is ∠M'

  Image of vertex N is ∠N', Image of vertex O is O'

  (ii) Corresponding sides are LM and L'M', MN and M'N', NO and N'O' and OL and O'L'

13. 3→ 1↓

Objective type questions.

14. (ii) Rotation 15. (iii) Reflection 16. (i) Translation

17. (ii) Rotation 18.(i) Translation 19.(iii)

Exercise 4.3

Miscellaneous Problems

1. For first move: 2→,2↓; For second move: 5←,5↓

2. Pawn – 1↑ or 2↑

  Rook – 1 to 8 ↑

  Knight – 2⟶,1↑ or 2⟵,1↑ or1⟶,2↑ or 1⟵,2↑

  Bishop – 1⟶,1↑ or 2⟶,2↑or 3⟶,3↑or 4⟶,4↑or 5⟶5↑1⟵,1↑ or 2⟵,2↑ or 3⟵,3↑ or 4⟵,4↑ or 5⟵5↑

Queen – 1 to 8 ↑, 1→, 1↑ or 2→,2↑ or 3→,3↑ or 4→,4↑ or 5→,5↑ or 1←,1↑ or 2←,2↑ or 3←,3↑ or 4←,4↑ or 5←5↑

King – 1→ or ← or ↑

3. (i) More brains category shows translation

(ii) More brains category shows reflection

4. (i) 120cm→,210cm↓ (ii) 270cm←,330cm↑ (iii) 150cm→

5. (i) rotation (ii) reflection (iii) translation (iv) reflection

(v) rotation (vi) reflection (vii) rotation. (viii) translation

challenge Problems

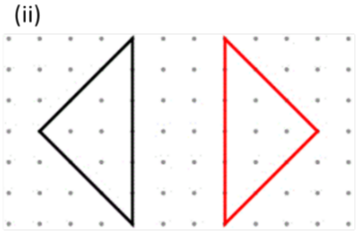
6. 2←,1↓ and then 1←, 2↓ (or) 2←, 1↓ and then 1←,2↓

7. (i) Translation 3←,5↑ and 90° counter clockwise rotation about the green point and translates 5 ←, 2↓

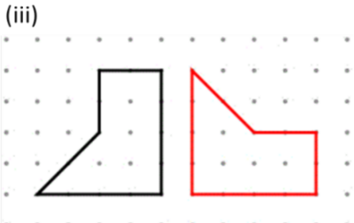
(ii) Translation 2←90° counter clockwise rotation about the green point and translates 2 ←, 2↓

8. (i)

(ii)



(iii)



5. Statistics

Exercise 5.1

1. (i) 5.5 (ii) 3,525 (iii) 6 (iv) 0

2. 13 3. (i) 35 (ii) 47 (iii) 10

4. Y=152 ; Height of two students are 152 *cm* and 156 *cm*

5. 240 6. 5 7. 24

Objective type questions:

8. (i) Mean 9. (ii) 16 10. (i) 16 11. (iii) 14

Exercise 5.2

1. 2 2. 31 3. 25 and 36 4. 15

Objective type questions:

5. (i) blue 6. (iv) No mode 7. (iii) 2 and 1

Exercise 5.3

1. (i) 25 (ii). 11

2. 34 3. 7 4. 37.5 ; 36

Objective type questions:

5. (iii) 2 6. (iv) 32 7. (i) 6

Exercise 5.4

1. 82 2. 15.5 3. 2, 3 and 5 4. 13;13

5. 2;3 6. 8; No mode.

Challenge Problems

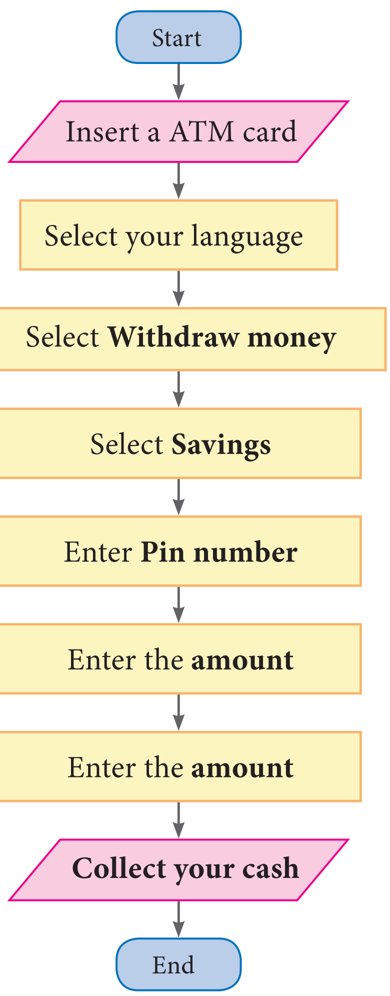
7. 8 8. 17; 10 ; 18 9. –0.5 10. 12.9 ; 11.2

6. Information Processing

Exercise 6.1

1. (i) (e) (ii) (c) (iii) (a) (iv) (b) (v) (d)

2.



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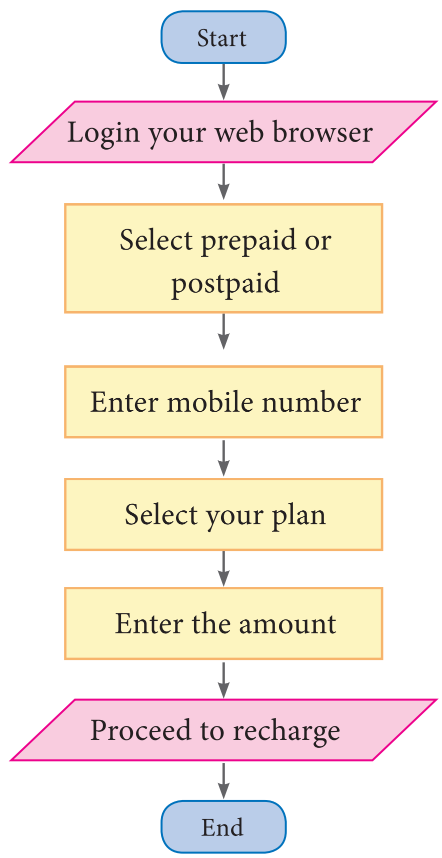

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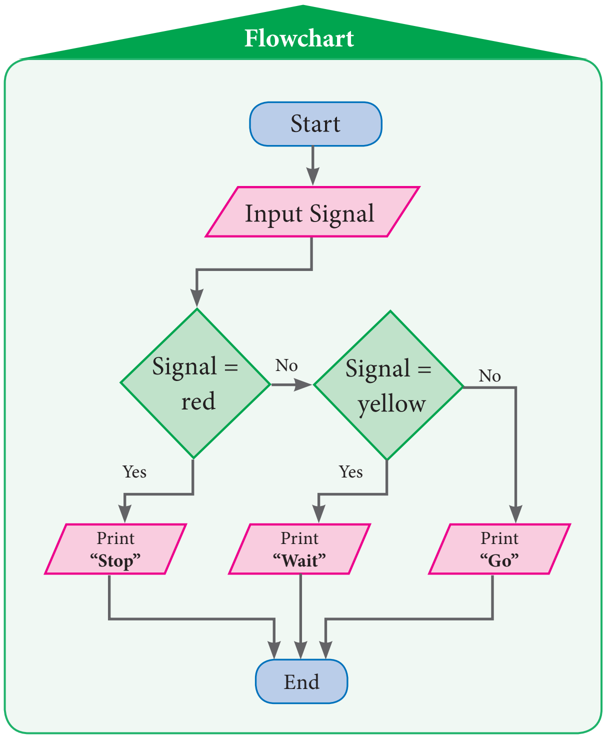
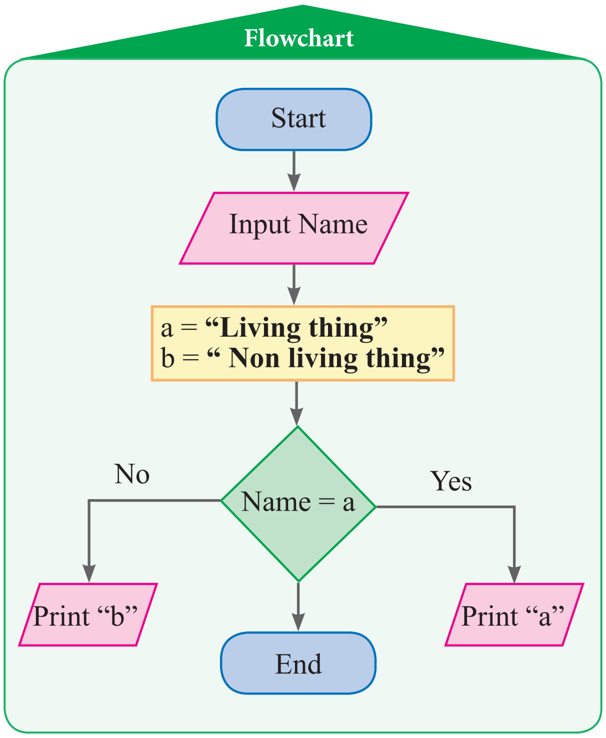
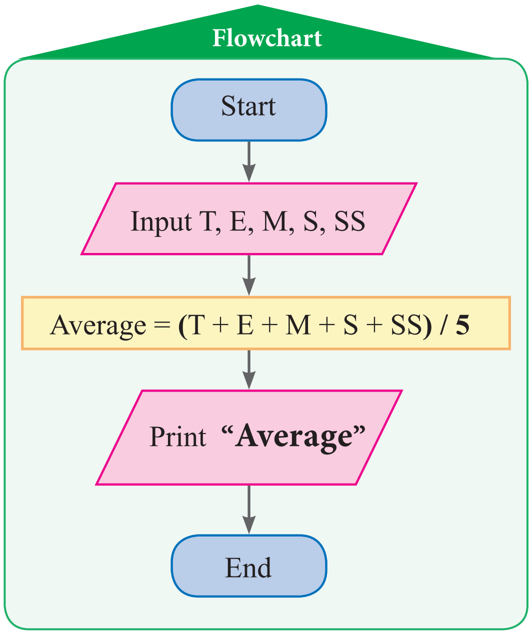

Flowchart
Start
Input Signal
Signal = red
No
Signal = yellow
No
Yes
Yes
Print "Stop"
Print "Wait"
Print "Go"
End

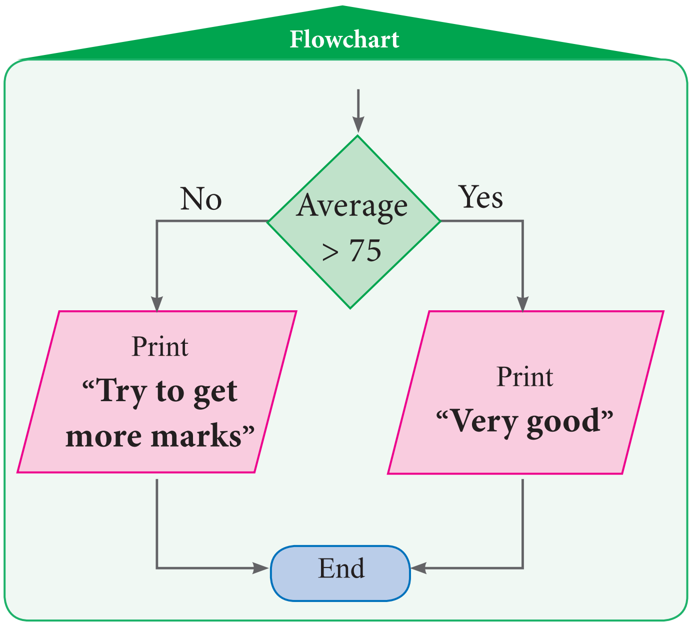
5.


Flowchart
Start
Input Name
a = "Living thing"
b = " Non living thing"
No
Name = a
Yes
Print "b"
End
Print "a"

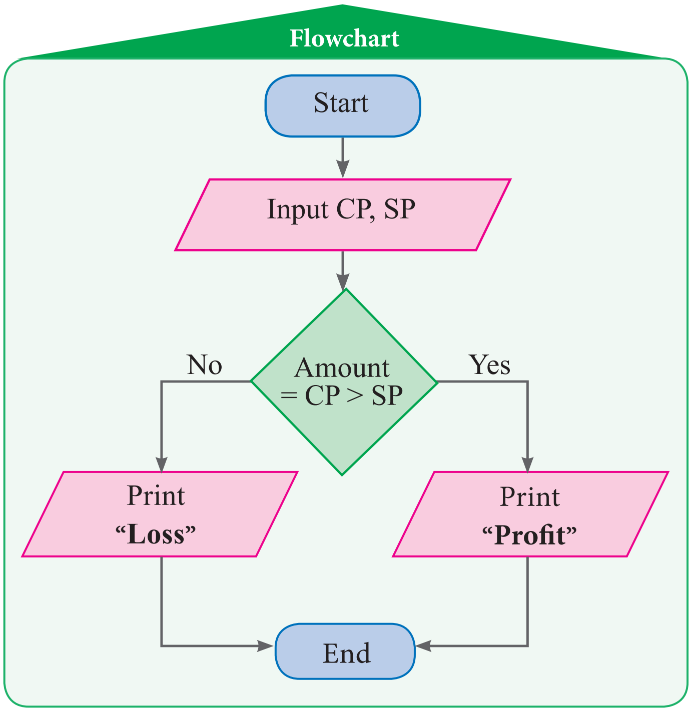
6. (i)


Flowchart
Start
Input T, E, M, S, SS
Average = (T + E + M + S + SS) / 5
Print "Average"
End

(ii)


Flowchart
No
Average > 75
Yes
Print "Try to get more marks"
Print "Very good"
End

7.


Flowchart
Start
Input CP, SP
No
Amount = CP > SP
Yes
Print "Loss"
Print "Profit"
End

GLOSSARY

Amount	கூட்டுத் தொகை
Angle of rotation	சுழற்சிக்கோணம்
Arithmetic Mean	எண்கணித சராசரி கூட்டுச் சராசரி)
Average	சராசரி
Beads	மணிகள்
Bimodal	இருமுகடுகள்
Borrower	கடன் வாங்குபவர்
Central tendency	மையநிலை அளவுகள் (மையப்போக்கு)
Chord	நாண்
Circular ring	வட்டவலயம்
Cutouts	வெட்டுத் துண்டுகள்
Data	தரவு
Decimal	பத்தின் கூரான
Decimal point	தசம புள்ளி
Dimension	பரிமாணம்
Discrete data	வெவ்வேறான தரவுகள் (தொடர்ச்சியற்ற)
Dividend	வகுபடும் எண்
Divisor	வகுக்கும் எண்
Estimation	மதிப்பிடுதல்
Factorisation	காரணிப்படுத்துதல்
Factors	காரணிகள்
Flip Turn	திருப்பம்
Frequency	நிகழ்வெண்
Graph	வரைபடம்
Hundredths	நூறில் ஒரு பங்கு

Identity	முற்றொருமை
Inequality	அசமத்தன்மை
Inequations	அசமன்பாடுகள்
Lender	கடன் அளிப்பவர்
Line of reflection center of rotation	சுழறசிமையம்
Median	இடைநிலை
Mode	முகடு
Monomials	ஒருறுப்புக் கோவை
Number line	எண்கோடு
Per annum	ஓர் ஆண்டுக்கு
Percentage	சதவீதம்
Place Value	இட மதிப்பு
Principal	அசல்
Range	வீச்சு
Rate of Interest	வட்டிவிகிதம்
Replacement set	பிரதியிடும் கணம்
Representative value	தேர்ந்தெடுக்கப்பட்ட மதிப்பு (பிரதிநிதி)
Simple interest	தனிவட்டி
Solution set	தீர்வு கணம்
Tally Marks	நேர்க்கோட்டு குறிகள்
Tenths	பத்தாவதாக
Thousandths	ஆயிரத்தில் ஒரு பங்கு
Transformation	உருமாற்றம்

Upper Primary Mathematics - Class 7, Term-III
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This book has been printed on 80 GSM
Elegant Maplitho paper

Printed by offset at :

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