Thapar Institute of Engineering and Technology, Patiala
SCHOOL OF MATHEMATICS
END SEMESTER EXAMINATION

Course No. UCS410 | *Course Name: PROBABILITY AND STATISTICS*
Time: 3 hours | *MM: 100*

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There are a total of 5 Questions each having two parts. Attempt any 4 questions. In case of over attempt first 4 questions will be considered. Non programmable calculators are allowed. A table for CDF of Z is given at the end.

1 (a) The breaking strength X of a certain rivet used in a machine engine has a mean 5000 units and standard deviation 400 units. A random sample of 36 rivets is taken. Consider the distribution of $\bar{X}$, the sample mean breaking strength and answer the following
(i) What is the probability that the sample mean falls between 4800 and 5200?
(ii) What sample size n would be necessary in order to have $P(4900 < \bar{X} < 5100) = 0.99$?

[12 marks]

(b) (i)A machinist is making engine parts with axale diameters of 0.700 inch. A random sample of 10 parts shows a mean diameter of 0.742 inch with a standard deviation of 0.040 inch. Compute the statistic you would use to test whether the work is meeting specifications?
(ii) A sample of 10 is drawn randomly from a certain population. The sum of the squared deviations from the mean of the given sample is 50. The population standard deviation is 5. Calculate the test statistics needed to test the hypothesis that the population variance is 5.

[13 marks]

2 (a) Let X be a continuous random variable with the PDF given by

$$f_X(x) = \begin{cases} k\exp\left(-\frac{x}{3}\right); & \text{for } |x| \le 1, \\ 0 & \text{otherwise} \end{cases}$$

Find (i) the value of k, (ii) the CDF of X, (iii) $P(1 < X < 4)$. [12 marks]

(b) Concentrations of pollutants produced by chemical plants historically are known to exhibit behaviour that resembles a lognormal distribution. Suppose it is assumed that the concentration of a certain pollutant, in parts per million, has a lognormal distribution with parameters $\mu = 3.2$ and $\sigma = 1$. Then find
(i) the probability that the concentration exceeds 8 parts per million.
(ii) Find the fifth percentile.

[13 marks]

3 (a) If X is uniformly distributed over the interval (0, 1), i.e., $X \sim U(0,1)$. Show that the random variable $Y = -2\ln(X)$ has a chi-squared distribution with 2 degrees of freedom.

[12 marks]

(b) Let X be continuous random variable having probability density function

$$f(x) = \begin{cases} \frac{1}{2\sqrt{3}}, & -\sqrt{3} < x < \sqrt{3}; \\ 0, & \text{elsewhere.} \end{cases}$$

Find the actual probability $P\left(|X - \mu| \le \frac{3}{2}\sigma\right)$ and compare it with the upper bound obtained by Chebyshev's inequality.

[13 marks]

4 (a) Suppose that $\{X_1, X_2, \ldots, X_n\}$ be a random sample taken from a population with mean μ and variance σ^2. Prove that sample mean, $\bar{X} = \frac{1}{n}(\sum_{i=1}^{n} X_i)$ and sample variance, $S^2 = \frac{1}{(n-1)}(\sum_{i=1}^{n}(X_i - \bar{X})^2)$, are the unbiased estimators for μ and σ^2, respectively. [12 marks]

(b) Consider a Poisson distribution with probability mass function $f(x) = \frac{e^{-\mu}\mu^x}{x!}, x = 0, 1, 2, \cdots$. Suppose that n independent observations $x_1, x_2, \cdots, x_n$ are taken from the distribution. What is the maximum likelihood estimator of μ.

[13 marks]

5 (a) A study was conducted to assess the effects that occur when children are exposed to cocaine before birth. Children were tested for object assembly skills. The 191 children born to cocaine users had a mean score of 7.5. The 181 children not exposed to cocaine had a mean score of 8.3. The population standard deviations are 3.0 of 2.9 respectively. Assuming that the two samples are independent, use a 0.05 significance level to test the claim that prenatal cocaine exposure is associated with lower score.

[12 marks]

(b) At a certain college, it is estimated that 25% of the students ride bicycles to class. Does this seem to be a valid estimate if in a random sample of 90 college students, 30 are found to ride bicycles to class. Use (i) $\alpha = 0.05$ and (ii) $\alpha = 0.1$ level of significance for hypothesis testing.

[13 marks]

STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
0.1	.53983	.54380	.54776	.55172	.55567	.55962	.56356	.56749	.57142	.57535
0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
0.4	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793
0.5	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
0.6	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
0.7	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
0.9	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891
1.0	.84134	.84375	.84614	.84849	.85083	.85314	.85543	.85769	.85993	.86214
1.1	.86433	.86650	.86864	.87076	.87286	.87493	.87698	.87900	.88100	.88298
1.2	.88493	.88686	.88877	.89065	.89251	.89435	.89617	.89796	.89973	.90147
1.3	.90320	.90490	.90658	.90824	.90988	.91149	.91309	.91466	.91621	.91774
1.4	.91924	.92073	.92220	.92364	.92507	.92647	.92785	.92922	.93056	.93189
1.5	.93319	.93448	.93574	.93699	.93822	.93943	.94062	.94179	.94295	.94408
1.6	.94520	.94630	.94738	.94845	.94950	.95053	.95154	.95254	.95352	.95449
1.7	.95543	.95637	.95728	.95818	.95907	.95994	.96080	.96164	.96246	.96327
1.8	.96407	.96485	.96562	.96638	.96712	.96784	.96856	.96926	.96995	.97062
1.9	.97128	.97193	.97257	.97320	.97381	.97441	.97500	.97558	.97615	.97670
2.0	.97725	.97778	.97831	.97882	.97932	.97982	.98030	.98077	.98124	.98169
2.1	.98214	.98257	.98300	.98341	.98382	.98422	.98461	.98500	.98537	.98574
2.2	.98610	.98645	.98679	.98713	.98745	.98778	.98809	.98840	.98870	.98899
2.3	.98928	.98956	.98983	.99010	.99036	.99061	.99086	.99111	.99134	.99158
2.4	.99180	.99202	.99224	.99245	.99266	.99286	.99305	.99324	.99343	.99361
2.5	.99379	.99396	.99413	.99430	.99446	.99461	.99477	.99492	.99506	.99520
2.6	.99534	.99547	.99560	.99573	.99585	.99598	.99609	.99621	.99632	.99643
2.7	.99653	.99664	.99674	.99683	.99693	.99702	.99711	.99720	.99728	.99736
2.8	.99744	.99752	.99760	.99767	.99774	.99781	.99788	.99795	.99801	.99807
2.9	.99813	.99819	.99825	.99831	.99836	.99841	.99846	.99851	.99856	.99861
3.0	.99865	.99869	.99874	.99878	.99882	.99886	.99889	.99893	.99896	.99900
3.1	.99903	.99906	.99910	.99913	.99916	.99918	.99921	.99924	.99926	.99929
3.2	.99931	.99934	.99936	.99938	.99940	.99942	.99944	.99946	.99948	.99950
3.3	.99952	.99953	.99955	.99957	.99958	.99960	.99961	.99962	.99964	.99965
3.4	.99966	.99968	.99969	.99970	.99971	.99972	.99973	.99974	.99975	.99976

Note that copies will be shown on 26th December, 2023.
(Check LMS for more details.)

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