Problem Chosen	2026 MCM Summary Sheet	Team Control Number
B		2613671

Optimizing Earth–Moon Logistics for Sustainable Lunar Settlement

Summary

Establishing a sustainable Earth–Moon logistics network is a fundamental prerequisite for large-scale lunar settlement. This paper develops an integrated mathematical modeling framework to determine an optimal transportation architecture that balances **economic cost, delivery efficiency, system resilience,** and **environmental sustainability** over a multi-decade horizon. We investigate three candidate logistics strategies: a rocket-based system, a space-elevator-based system, and an adaptive hybrid system that dynamically allocates cargo according to urgency, scale, and risk.

To quantify trade-offs among competing objectives, we formulate a **multi-objective optimization model** minimizing total generalized cost and delivery time subject to capacity and operational constraints. The model is solved using the **NSGA-II algorithm** with hybrid encoding and adaptive penalty functions, producing a Pareto-optimal frontier. Under ideal operating conditions, the optimal hybrid configuration assigns **34.5%** of cargo to rockets and **65.5%** to space elevators, achieving a minimum generalized cost of $\mathbf{2.38 \times 10^{13}}$USD with a projected completion horizon of **164 years**. These results establish the space elevator as the economic backbone of large-scale, routine Earth–Moon transportation.

Recognizing the inherent uncertainty of space infrastructure, we construct a **stochastic resilience assessment model** incorporating Markov state transitions, Weibull reliability functions, and Monte Carlo simulations to evaluate system performance under non-ideal conditions. When a 95% delivery reliability threshold is imposed, the optimal strategy adaptively increases rocket utilization to approximately **55%**, sacrificing cost efficiency in exchange for enhanced robustness. This demonstrates that no single transportation mode is globally optimal across all risk scenarios.

To support continuous lunar habitation, we further develop a **time-series-based sustainable water supply model** using an **ARMA(1,7)** process with feedback regulation for a 100,000-person lunar base. Simulation results reveal a stable functional division of labor: space elevators dominate steady-state supply, while rockets provide emergency and contingency support. Environmental externalities are internalized through a **Life Cycle Assessment (LCA)** framework combined with an entropy-weighted generalized cost function, confirming the long-term ecological viability of the hybrid strategy under strict carbon constraints.

In conclusion, our results demonstrate that an **adaptive hybrid Earth–Moon logistics architecture** consistently outperforms single-mode systems in terms of cost-effectiveness, resilience, and sustainability. The proposed framework is concise, extensible, and provides a robust quantitative foundation for future space logistics planning and infrastructure design.

Keywords: Earth-Moon Transportation ; Space Elevator System ; Multi-objective optimization ; NSGA-II ; ARMA Model

Contents

1 Introduction

1.1 Problem Background

In the deep space exploration era, sustainable lunar settlements are a core goal for global space agencies (NASA's Artemis Program, ESA's Moon Village)[1]. A large-scale, low-cost Earth-Moon cargo transport system is critical.

Current logistics rely on traditional heavy-lift rockets—direct and rapid but costly, low-frequency, limiting million-ton-scale transport. Emerging space elevators, enabled by materials/structural advances, offer low per-mass costs and near-zero pollution [2]. Yet rockets excel at oversized/emergency cargo, while space elevators face technical hurdles in materials, dynamic control, and orbital synchronization[3]. Lunar bases need massive construction material and continuous life support, requiring optimized hybrid architectures balancing cost, time, capacity, and reliability.

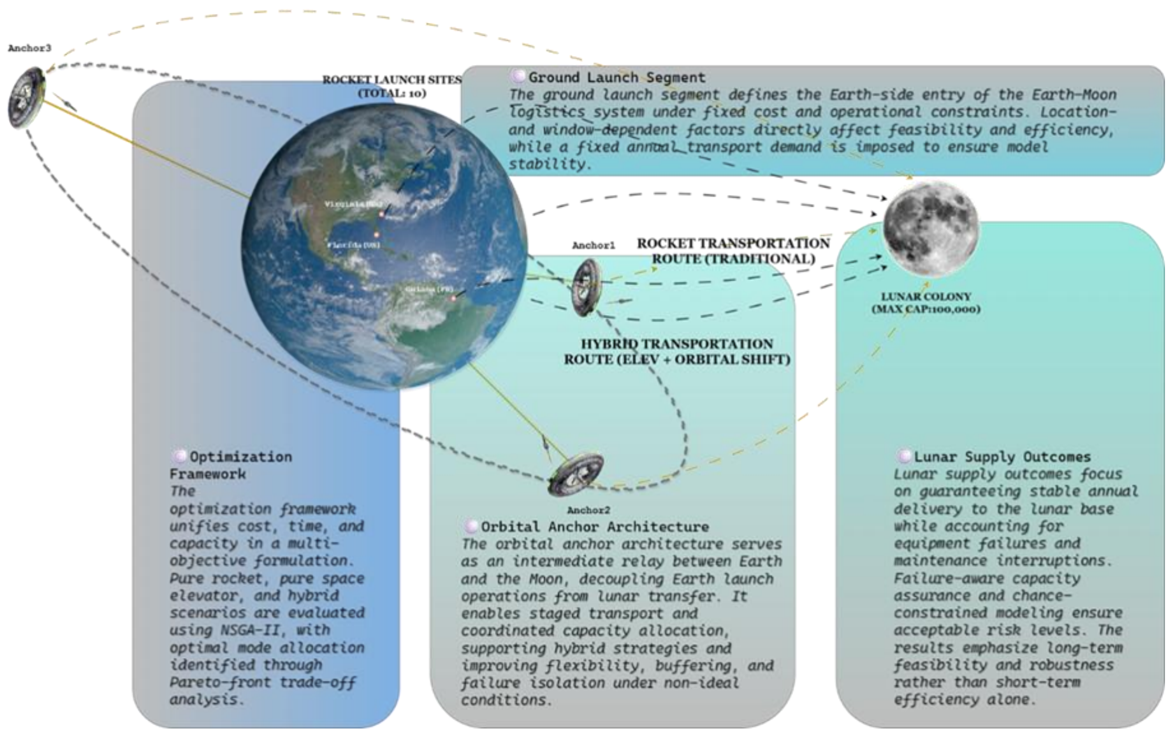

Fig.1 Earth-Moon Transport: Background Overview

1.2 Problem Restatement

Based on the given background, assumptions, and constraints, this paper evaluates alternative Earth–Moon transportation systems. The main objectives are to:

Task1: Model and compare the cost and delivery time of three transportation schemes: a space elevator system, a traditional rocket system with optimized launch site selection, and a hybrid system that allocates capacity by cargo type.

Task2: Analyze system resilience under non-ideal conditions, including capacity reduction and equipment failure, and propose adaptive strategies to mitigate disruptions.

Task3: Develop a continuous resupply plan for a lunar base by incorporating annual water demand and assessing long-term feasibility.

Task4: Evaluate and minimize environmental impacts by comparing rocket emissions and space elevator energy consumption.

Task5: Integrate all performance metrics to provide decision support and recommend an optimal transportation strategy with a phased implementation plan.

1.3 Our Work

For Problems 1, 2, and 4, we used multi-objective programming and solved them with NSGA-II. For Problem 3, to enhance system robustness and disturbance resistance, we reformulated it as an ARMA model and derived performance metrics from its coefficients. The workflow is illustrated below.

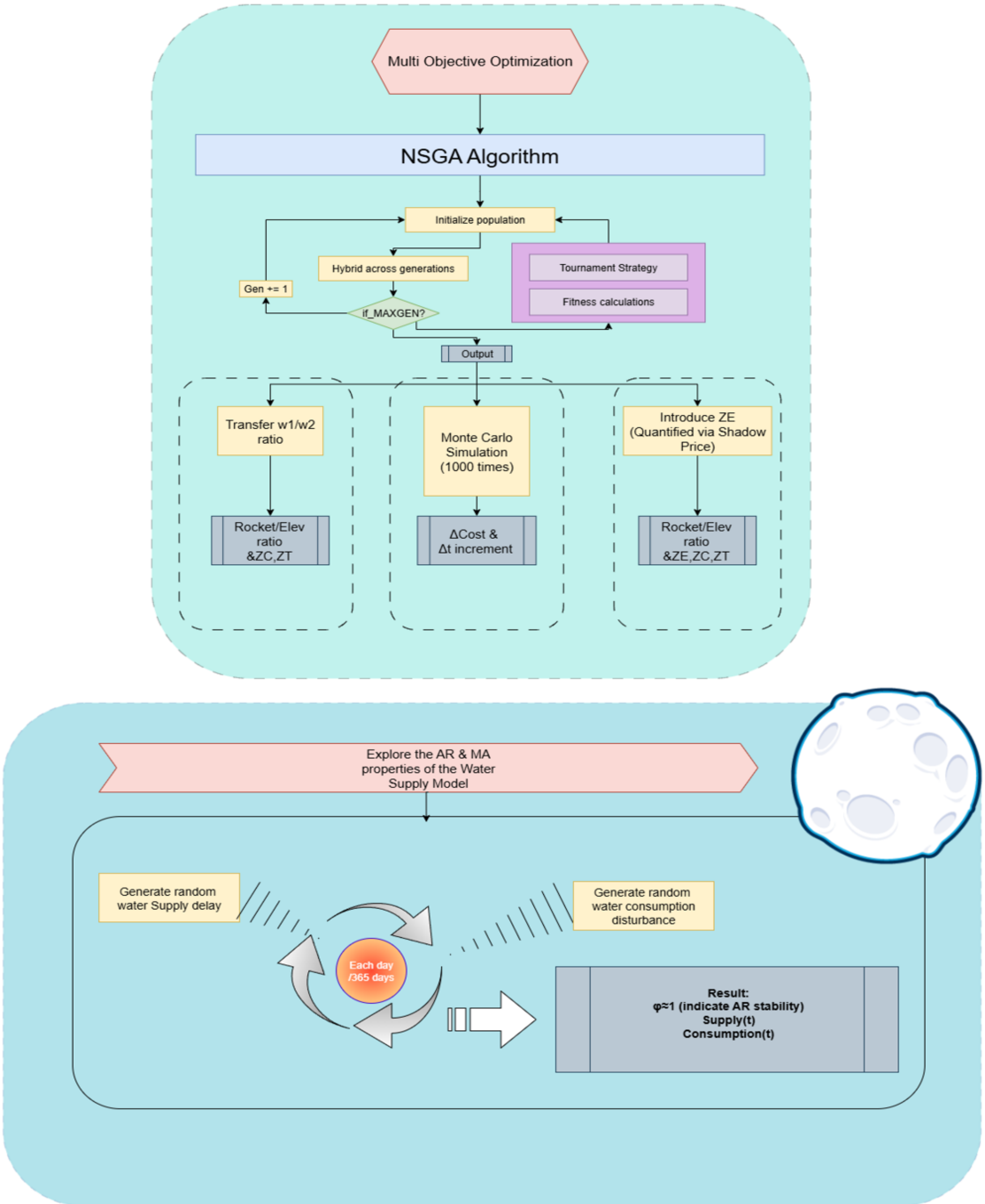

Fig.2 Conceptual Flowchart of our Models

2 Assumptions and Justifications

- **Assumption 1: Characteristics of "Effective Capacity" and "Marginal Cost" for Space Elevators**

Justification: According to the Tsiolkovsky rocket equation, rockets must carry enormous amounts of fuel to overcome Earth's gravitational well, with costs primarily tied to fuel and vehicle depreciation. In contrast, space elevators rely mainly on electricity, and gravitational potential energy can be partially recovered.

- **Assumption 2: Heavy-Lift Rocket Cost Model and Launch Window Constraints**

Justification: This constraint reflects considerations of real-world bottlenecks in the aerospace supply chain, based on global launch site data over the past 15 years.

- **Assumption 3: Discretization and Time-Lag Simplification of Transport Processes**

Justification: The core of the problem is a macro-logistics plan for 100 million tons of cargo (spanning decades), where hour/day-level loading/unloading errors have an impact of order ϵ on the overall timeline, which could be ignored.

- **Assumption 4: Neglect of Complex Real-World Social Conditions**

Content: We assume that all launch sites and Galactic Ports are centrally managed by the MCM organization, with no consideration of international political games, tariffs, or exclusive bidding between sites.

- **Assumption 5: Markov Property of Failures**

Content: We assume that the future state (normal/failed) of the system depends only on its current state, with failures occurring according to a Poisson distribution and a fixed failure repair time window.

- **Assumption 5: Risk and Cost-Free Apex-to-Moon Rocket Transportation**

Content: Leveraging the space elevator apex anchor's 7.6 km/s orbital velocity, we assumed to have negligible failure probability and marginal cost compared to Earth launch rocket transportation from the apex to the lunar base. [3]

3 Notations

The key mathematical notations used in this paper are listed in Table 1.

Table 1: Notations used in this paper

Symbol	Description	Unit
$R_{k,t}$	Number of heavy rocket launches conducted at the k-th launch site in year t ($k = 1,2,\ldots,10$)	launches/year
$E_{p,t}$	Volume of supplies transported via the p-th space elevator port in year t ($p = 1,2,3$)	metric tons
S_t	Binary indicator for whether the project is completed (all supplies delivered) in year t	N/A
D_{Total}	Total supplies required for lunar colony construction, with a value of 10^8	metric tons

K	Number of rocket launch sites	sites
P	Number of space elevator ports	ports
L_{max}	Maximum launches per site per year	launches/year
Cap_{rocket}	Payload per rocket launch	metric tons/launch
Cap_{lift}	Annual capacity per elevator port, 179,000	metric tons/year
$Cost_{rocket}$	Cost per metric ton for rocket transport	USD/metric ton
$Cost_{lift}$	Cost per metric ton for space elevator transport	USD/metric ton
Z_T	Total project timeline	Year
Z_C	Total project cost	USD
Z_E	Quantified environmental cost	N/A

4 Multi-Objective Optimization Model for Earth-Moon Logistics

4.1 Model Formulation

4.1.1 Problem Definition and Variable Setting

We address the Earth-Moon transportation optimization as a multi-objective linear programming problem. The model focuses on optimizing the allocation of transportation resources between rockets and space elevators over a multi-decade timeline.

4.1.2 Multi- Objective Optimization Framework

The model aims to minimize two core objectives: total cost (Z_C) and total timeline (Z_T). A weighted summation method is used for normalization (to eliminate unit differences), with weights w_1,w_2 (w_1+w_2=1, adjustable based on policy priorities).

Based on data from existing literature[4], we assume a maximum annual rocket launch rate of 8,000 missions post-2050.

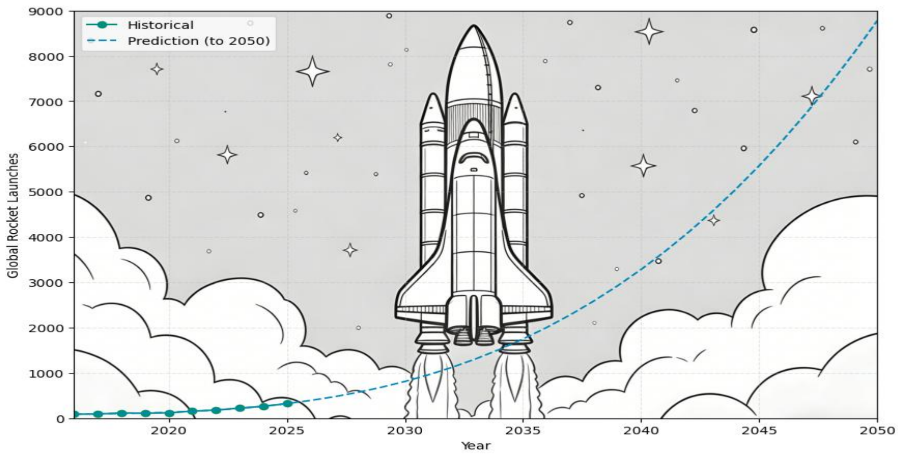

Fig.3 Rocket Launch Times Prediction Curve

Total Cost Z_C

Total cost includes cumulative rocket launch costs and elevator operational costs over the project period:

$$f_1 = Z_C = \sum_{t=1}^{T_{max}} \left(\sum_{k=1}^{10} (Cost_{rocket} \cdot R_{k,t}) + \sum_{p=1}^{3} (Cost_{lift} \cdot E_{p,t}) \right) \tag{1}$$

Total Timeline Z_T

Total timeline is the weighted sum of years using the completion indicator S_t (only the year of completion contributes to the timeline):

$$f_2 = Z_T = \sum_{t=1}^{T_{max}} t \cdot S_t \tag{2}$$

Normalized Dual-Objective Function

$$Min\ Z = w_1 \cdot \frac{Z_C}{C_{norm}} + w_2 \cdot \frac{Z_T}{T_{norm}} \tag{3}$$

Where C_{norm} is the normalized constant for cost (e.g., maximum possible cost if all materials use rockets), and T_{norm} is the normalized constant for timeline (e.g., T_{max}).

4.1.3 Constraint Conditions:

All constraints are summarized below to ensure the model complies with material demand and capacity limits:

$$\sum_{t=1}^{T_{max}} \left(\sum_{k=1}^{10} (Cap_{rocket} \cdot R_{k,t}) + \sum_{p=1}^{3} E_{p,t} \right) \geq D_{total} \tag{4}$$

4.1.4 NSGA-II Algorithm Implementation

Given the multiple variable nature of the problem and the requirement for multi-objective optimization, we employ the Elitist Non-dominated Sorting Genetic Algorithm (NSGA-II). For chromosome encoding, a hybrid coding strategy is adopted: real-number encoding is used to represent the space elevator transport volume $E_{p,t}$, integer encoding for the number of rocket launches $R_{k,t}$, and binary encoding for the project completion status S_t.Genetic operations include tournament selection, segment-based crossover, and Gaussian mutation (following an N(0,σ²) distribution), with safeguards to ensure non-negative costs after mutation. An elitist preservation strategy is implemented to retain the optimal solutions within the population. Constraint handling is achieved via an adaptive penalty function method based on violation severity, thereby guaranteeing the feasibility of the solutions.

4.2 The Solution of Model 1

Based on the parameter settings provided in Section 4.2, we conducted numerical simulations to solve the spatiotemporal network flow model. The key parameters are set as follows:

4.2.1 Scenario a: Pure Space Elevator System

In this scenario, all materials are transported via the space elevators of the three Galactic Harbours. The annual lifting capacity per Galactic Harbour is $\boldsymbol{Cap_{lift} = 179000}$ ***metric tons/year***.

Although the pure space elevator scenario ($\boldsymbol{T_a \approx 186.2\ years,\ C_a = 1.0\times10^{13}\ USD}$) boasts the lowest cost and zero atmospheric emissions, its extended timeline makes it less suitable for this project.

4.2.2 Scenario b: Pure Rocket Launch System

This scenario relies solely on the 10 global rocket launch sites. Given the maximum launch rate of 800 per site per year and a payload capacity of 150 tons per launch, the total annual delivery capacity is $\boldsymbol{1{,}200{,}000\ metric\ tons/year}$.

The exclusive use of rockets results in a prohibitive timeline exceeding $\boldsymbol{T_a \approx 83.3}$ ***years*** and an astronomical cost of approximately $\boldsymbol{C_b = 5.0\times10^{13}\ USD}$. This clearly demonstrates the infeasibility of relying solely on current rocket technology for large-scale lunar colonization logistics.

4.2.3 Scenario c: Hybrid Transportation System (Optimized Solution)

To achieve an optimal cost-time trade-off, this study establishes an NSGA-II-based multi-objective optimization model. Parameters: population size 100, iterations 700, crossover probability 0.7, mutation probability 0.2. Constraints are handled via tournament selection and an adaptive penalty function. With a 1:1 cost-time weight ratio, the Pareto optimal set is derived iteratively, with key parameters as follows:

Optimization results show the earliest completion year of the hybrid transportation system is:

$$Z_T = 164 \text{ years}$$

In terms of material allocation, the transport volume distribution is determined as:

$$D_{rocket} = 34521300\ tons(34.52\%)$$
$$D_{lift} = 65478279.421\ tons(65.48\%)$$

Total optimized cost:

$$Z_C = 2.381\times10^{13}\ \ USD$$

The optimized hybrid solution represents the **Pareto-optimal solution** for the equal weighting of cost and time, adopting a capacity allocation scheme where rockets undertake **34.52%** of the transport volume and space elevators assume **65.48%**. This result verifies the space elevator's economic superiority as the backbone of large-scale Earth-Moon logistics, with rockets supplementing critical and specialized cargo. The solution outperforms standalone systems in balancing timeline and cost, underscores the hybrid architecture's advantages, and provides a scientifically feasible optimization pathway for lunar base logistics.

4.2.4 Algorithm Convergence and Result Visualization

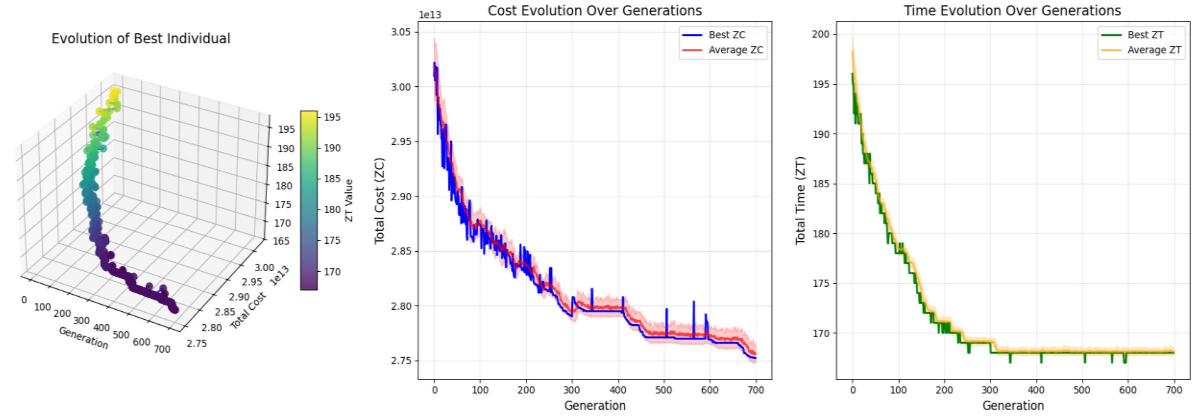

Fig. 4 Evolution of Cost and Timeline During NSGA-II Iteration

This figure illustrates the dynamic convergence characteristics of the optimal and average values of total cost and project timeline with the iteration process after 1000 generations of the NSGA-II algorithm, intuitively verifying the stability and effectiveness of the algorithm in solving the multi-objective optimization problem of Earth-Moon logistics.

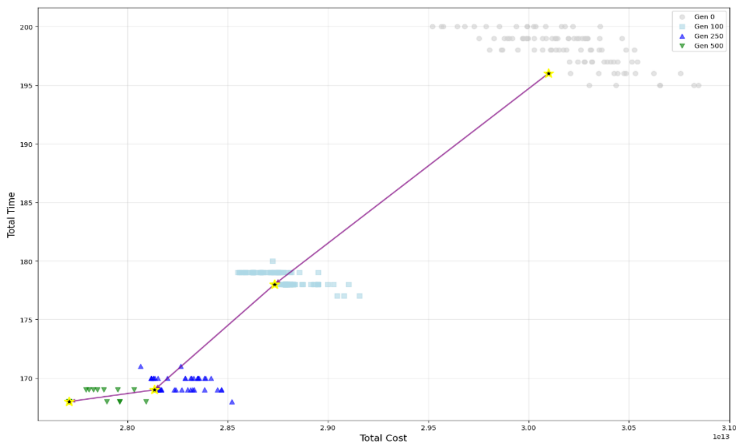

Fig. 5 Evolution of Pareto Front Across Different Iteration Stages

This figure presents the evolutionary process of the Pareto front from the initial iteration to the final generation (Gen999), including gradual compaction, homogenization, and convergence towards the ideal optimal region, reflecting the algorithm's efficient exploration capability of the solution space and the screening effect of high-quality non-dominated solutions.

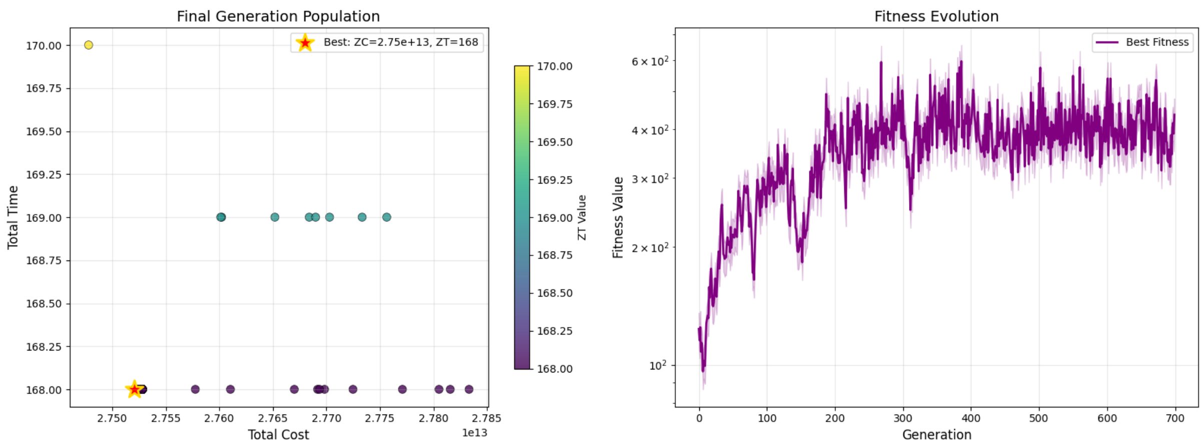

Fig. 6 Final Generation Population Distribution and Fitness Evolution

Characteristics of the Gen999 population in the two-dimensional cost-timeline space and the marked raw optimal individual, while the right part presents the stable evolutionary trend of fitness during the iteration process, laying a solid foundation for deriving the constrained optimized solution that meets the total material demand.

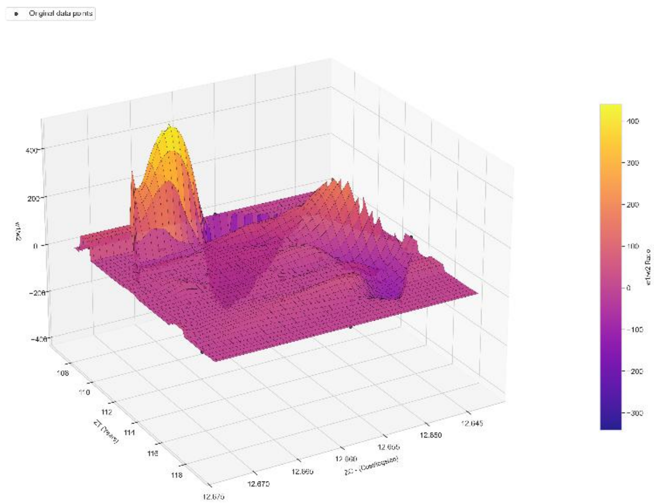

Fig. 7 The Distribution Characteristics of Pareto Optimal Solutions

The Pareto optimal solution distribution for the first problem is shown in Figure .

5 Non-Ideal Operating Conditions Resilience Assessment Model

5.1 Model Establishment

5.1.1 Weibull Distribution + Nonlinear Dynamics Integrated Modeling Principle

This spatiotemporal stochastic network flow model assesses the resilience of the Earth-Moon transportation system under non-ideal conditions. It uses the Weibull distribution to model space elevator failure probability and characterize random equipment failures, and literature-derived high-dimensional nonlinear dynamics to simulate tether oscillation and its nonlinear chaotic characteristics. Integrated with the two core modules, Monte Carlo simulation generates time-series failure samples, which are input into a heuristic optimization module for adaptive spatiotemporal capacity allocation to realize rational fault-state resource scheduling. This method balances scenario characterization accuracy and computational efficiency, providing a reliable tool for the system's resilience assessment.

5.2 Mathematical Formulation

5.2.1 Weibull Distribution-Based Failure Probability Modeling

Since space elevators currently only exist in theoretical concepts, the Weibull distribution is selected to characterize the randomness of their failures over time:

$$P_{fail}(t) = 1 - e^{-(t/\eta_E)^{\beta_E}} \tag{5}$$

where η_E (scale parameter) and β_E (shape parameter) are typically estimated by expert experience or historical data. For simplified calculation, we assume the annual failure probability of the space elevator is 5‰ with a single maintenance duration of 2 months, resulting in an annual available time ratio δ_b=0.8‰.

5.2.2 Emergency Risk Quantification via Statistical Analysis

For unforeseen emergencies in the Apex-Moon segment, based on engineering experience and space mission risk analysis, this study assumes an emergency occurrence probability of 1% with a corresponding 30% capacity loss, and defines the comprehensive capacity loss coefficient under this scenario as $\alpha_{k,t}$=3‰.

5.2.3 Rocket Launch Success Probability Based on Empirical Data

The rocket launch success probability is a key parameter for calculating the effective delivery volume. Taking the Cape Canaveral/KSC launch site as a reference and based on open data[5], the launch success probability is assumed to be PR=98% and subject to the binomial distribution.

5.2.4 Total Transport Capacity Guarantee Under Chance Constraints

Incorporating the impact coefficients of faults and oscillations, the actual delivery volume is calculated in two parts as follows:

Space elevator delivery volume:

$$D_{t.lift} = Cost_{lift} \cdot E_{p,t} = \sum_{p} E_{p,t}\, \alpha_{p,t}(1 - \delta_b) \tag{6}$$

Where $E_{p,t}$=125denotes the annual baseline transport capacity of a single space elevator port, determined by truncating the normal distribution N(μ =125, σ^2=15^2) to the interval [100, 150]i.e.,$E_{p,t}$= $max\,(\,min\,(N(\mu$=125,σ^2=15^2),150),100)=125.

Rocket delivery volume:

$$D_{t,rocket} = Cost_{rocket} \cdot R_{k,t} \sum_{k=1}^{10} \sum_{i=1}^{R_{k,t}} P_{R,i}\, Cap_{rocket} \tag{7}$$

5.2.5 Total Transport Capacity Assurance Under Chance Constraints

the chance constraint for the stochastic spatiotemporal network flow is formulated as follows:

$$P(\sum_{t=2050}^{T} \mathrm{D}_{\mathrm{t,rocket}} + \mathrm{D}_{\mathrm{t.lift}} \geq 10^{8}) \geq 1-\varepsilon \tag{8}$$

Where ε denotes the allowable violation probability, set to 5% with reference to general engineering risk management criteria.

5.2.6 Construction of Resilience Evaluation Index System

To comprehensively evaluate system resilience, three categories of core indicators are defined to reflect the impacts of non-ideal operating conditions:

Time increment $\Delta T = T_{imperfect} - T_{perfect}$ quantifies transportation delays caused by non-ideal operating conditions.

Cost increment $\Delta Cost = Cost_{imperfect} - Cost_{perfect}$ measures the additional costs arising from faults and subsequent adjustments.

Transport capacity structure change：Characterizes the adjustment range of the volume share between space elevator and rocket transportation, reflecting the adaptability of resource allocation.

5.3 Adaptation Strategy Design

5.3.1 Redundant Capacity Replacement Strategy

The 10% capacity replacement principle is a heuristic parameter derived from engineering empirical values, system redundancy and cost optimization strategies. To compensate for the capacity loss caused by space elevator faults, rocket transportation is adopted for supplementary delivery, with a unit replacement cost：$\Delta Cost \approx 40{,}000\ USD/\ metric\ tons$.

5.3.2 Dynamic Capacity Allocation Based on Real-Time Status

Based on real-time operational status, the task ratio between the space elevator and rockets is adjusted. When one mode is affected by non-ideal conditions, the other mode’s proportion is increased to ensure overall efficiency.

This adaptive strategy mitigates non-ideal condition impacts via capacity allocation and maintenance scheduling adjustments; its effectiveness is verified by the following quantitative results.

5.4 Model Result

Based on 1,000 random failure scenarios generated via Monte Carlo simulation, the core performance indicators of the Earth-Moon transportation system under non-ideal conditions are as follows: total durationZ_T =169 years, total cost Z_C= 4.714×10^{13}USD, rocket payload 54,954,000 tons, space elevator payload 45,045,315 tons, with capacity shares of 54.95% and 45.05%, respectively.

Combined with the Pareto-optimal solution under ideal conditions (Problem 1), the impact of non-ideal conditions is quantified by resilience metrics: time increment ΔT=5years, delay rate ~3.04%; cost increment *Δ Cost* =2.333×1013 USD, an increase of 97.9%.

Under ideal operating conditions, the space elevator system dominates the logistics network due to its significantly lower unit cost and stable continuous capacity, resulting in an optimal allocation of approximately 65% elevator transport and 35% rocket transport.

However, once non-ideal operational factors are introduced—such as tether oscillations, system failures, maintenance downtime, and stochastic capacity reductions—the role of the space elevator fundamentally changes. Although its expected annual capacity remains high, the increased variance in available capacity introduces substantial schedule risk under a hard delivery deadline.

In contrast, traditional rockets, despite higher unit costs, provide highly reliable, modular, and rapidly recoverable transport capacity. As a result, the optimization shifts toward a risk-hedged strategy, increasing rocket utilization to 55% in order to maintain delivery reliability and satisfy probabilistic completion constraints.

This transition reflects not a failure of the space elevator concept, but a rational rebalancing of cost efficiency and system resilience in a stochastic logistics environment.

The distribution of core system indicators and stability of capacity structure under non-ideal conditions can be further illustrated and verified via statistical charts:

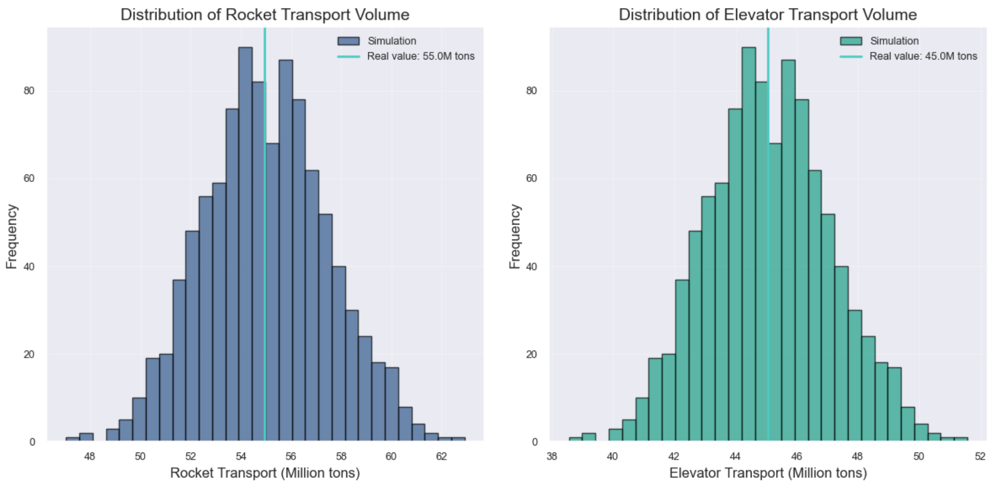

Fig. 8 Bar Chart of Core Indicators Comparison Under Non-Ideal Conditions

Comparison of the two bar datasets in the figure intuitively presents quantitative results of core system indicators under non-ideal conditions and clarifies their deviation from ideal conditions.

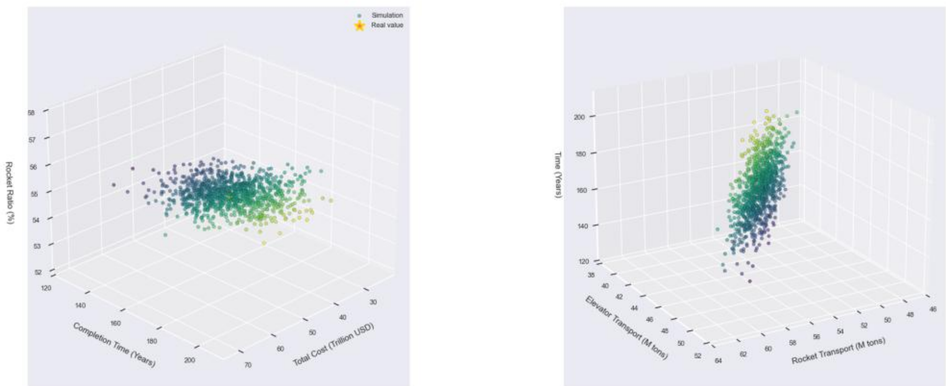

Fig. 9 Scatter Plot of Capacity Structure Ratio Under Non-Ideal Conditions

The figure reflects the discrete characteristics of capacity proportion via scatter distribution, verifying the stability and anti-disturbance capability of the system capacity structure under non-ideal conditions.

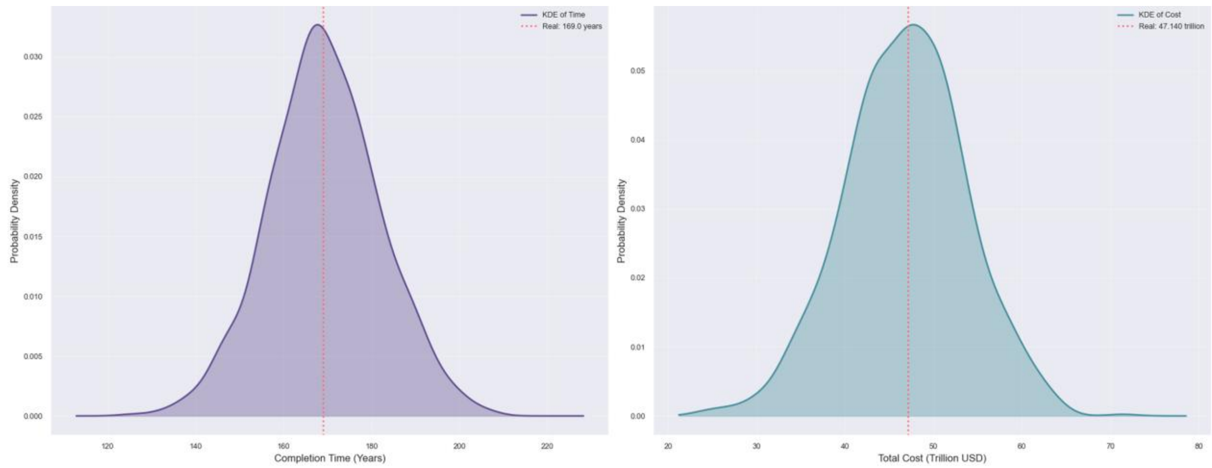

Fig. 10 Comparative Probability Density Distributions of Time and Cost

This figure compares the distributions of two key variables via side-by-side kernel density estimation (KDE) curves.

6 Sustainable Water Supply Planning for Lunar Base

6.1 Model Establishment

To address continuous water supply for a 100,000-person lunar base, this study develops an ARMA(1,7) time-domain model integrating autoregressive and moving average components. With a daily time step, the model simulates one-year water dynamics to accurately compute the additional transportation cost for maintaining water system steady state and evaluate the Earth-Moon hybrid transportation system's adaptability to this routine supply mission.

The fundamental dynamics of the water system follow the stock balance principle, where temporal stock changes arise from the dynamic equilibrium of supply, consumption, and recovery, expressed in differential form as:

$$\frac{dW}{dt} = Supply(t) - Consumption(t) + Recycle(t) \tag{9}$$

Assuming an efficient water treatment and recycling system, the water recovery efficiency is set as η= 0.98. [6]Substituting this relation and discretizing with a one-day time step yields the basic difference equation:

$$W_{t+1} = W_t + \Delta\mathrm{t} \cdot [Supply_t - (1-\eta) \cdot Consumption_t] \tag{10}$$

It is apparent that $W_{t+1} - W_t$ adheres to the AR(1) model structure. However, approximating $Supply_t$ and $Consumption_t$ as constant values is overly simplistic, failing to reflect practical complexities and exhibiting insufficient robustness. Accordingly, an MA model is incorporated to decompose these variables into mean and fluctuation components, thereby accurately capturing the dynamic characteristics of water demand variability, transportation uncertainties, and inventory feedback regulation.

For the 100,000-person base, the daily average total water consumption is set at 30,000 tons (300 liters per capita per day)[7], falling within the typical urban comprehensive water consumption range of 250-350 liters per capita per day, covering research, daily life and potential losses. The daily actual water consumption is $Consumption_t = \overline{C} + \varepsilon_t$,with random fluctuation $\varepsilon_t \sim N(0, 1500^2)$, weekend consumption is 20% lower than weekdays. The steady-state supply demand is $\bar{S} = (1-\eta) \cdot \bar{C} = 600$tons/day, and the daily actual total supply is $Supply_t = \bar{S} + \mu_t + \gamma_t$,where μ_t is the dynamic adjustment command based on inventory feedback, and $\gamma_t \sim N(0, \sigma_\gamma^2)$ denotes transportation uncertainty.

The model incorporates practical transportation constraints, with a mean transportation delay of *d*=7days for supply commands. Based on the deviation between the current inventory level W_t and the target safety inventory $W_{target} = 100000$ tons (approximately 1.7 days of water buffer), the system dynamically adjusts supply commands via a feedback function:

$$\mu_t=\beta \cdot (\bar{S}+\gamma_t) \cdot f(\frac{W_t}{W_{target}}) \tag{11}$$

The feedback function is defined as:：

$$f(r) = \begin{cases} 0 & if \quad r > 1.2 \\ 0.5 & if\ 0.8 < r \le 1.2 \\ 1.0 & if\ 0.5 < r \le 0.8 \\ 1.5 & if \quad r \le 0.5 \end{cases} \tag{12}$$

The logic of adjusting resupply intensity based on inventory deviation is as follows: When inventory is excessively high, resupply is halted to avoid storage cost waste; when inventory is normal, resupply is conducted at 0.5 times the intensity; when inventory is low, resupply proceeds at 1.0 times the intensity; and when inventory is critically insufficient, emergency resupply is activated at 1.5 times the intensity to ensure water security for the base.

Integrating all mechanisms, the complete ARMA (1,7) model expression is:

$$W_{t+1} = \varphi \cdot W_t + \Delta t \left[\beta \cdot (\bar{S} + \gamma_t) \cdot f\left(\frac{W_t}{W_{target}}\right) + \sum_{i=1}^{d} \alpha_i u_{t-1} \cdot I - (1-\eta)(\bar{C} + \varepsilon_t) \right] \tag{13}$$

Here I denotes the resource preemption coefficient, set to 0.8, representing the occupation degree of transportation corridors by water supply missions. As water supply is critical to crew safety, it prioritizes 80% of transportation corridors. u_{t-1} is the supply command fluctuation at time t-1, and the weights α_i satisfy $\sum_{i=1}^{d} \alpha_i = 1$.

In the model, the autocorrelation function (ACF) is calculated based on covariance, and the partial autocorrelation function (PACF) is solved via the Yule-Walker equation to ensure model stationarity and fitting performance. Transportation cost accounting adopts the existing hybrid transportation weight, with 35% borne by rocket transportation and 65% by space elevator, which is consistent with the expected composition of the short-to-medium-term Earth-Moon logistics system.

6.2 Model Solution and Result Analysis

6.2.1 Solution Method

Based on the ARMA(1,7) model with a 365-day simulation period, the solution process is as follows: Generate the water consumption fluctuation sequence$\varepsilon_t$with 20% weekend attenuation and the transportation uncertainty sequence $\gamma_t \sim N(0,73.4^2)$to iteratively derive the annual operation trajectory of the water resource management system; verify model stationarity by computing the partial autocorrelation function via the Yule-Walker equation and the autocorrelation function based on covariance; substitute basic parameters and the feedback function to iteratively calculate the daily water inventory W_t and supply $Supply_t$; allocate the daily supply according to the probability weights of 35% for rocket transportation and 65% for space elevator transportation; summarize the annual additional transportation cost by using the unit transportation cost.

In the coefficients determined by the final ARMA model, the calculated value of φ≈1(AR ≈1). This precisely demonstrates that when the daily resupply requirement is met, the daily water inventory level remains approximately equal to that of the previous day, while maintaining robustness against sudden fluctuations in water consumption and supply delays.

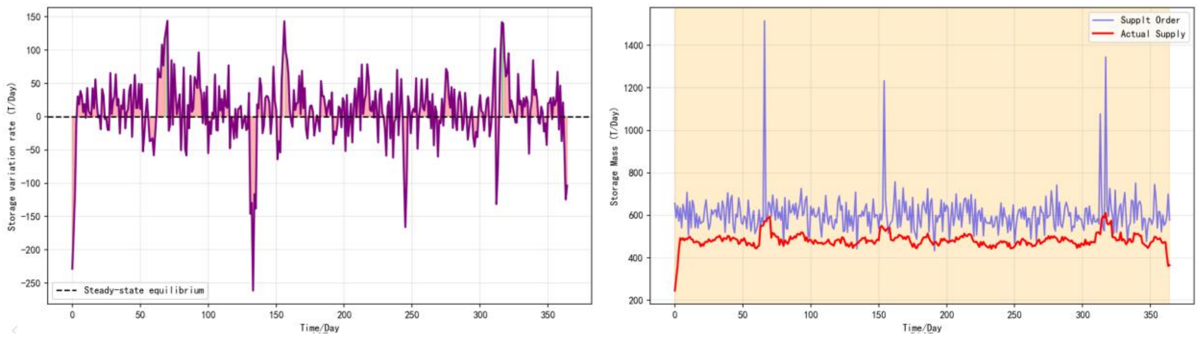

Fig. 11 Storage Variation & Supply Delay in a Year

6.2.2 Quantification of Core Results

The ratio of the annual total supply volume to the annual total water consumption is:

$$\frac{\sum_{t=1}^{365} Supply_t}{\sum_{t=1}^{365} Consumption_t} = 2.02\%$$

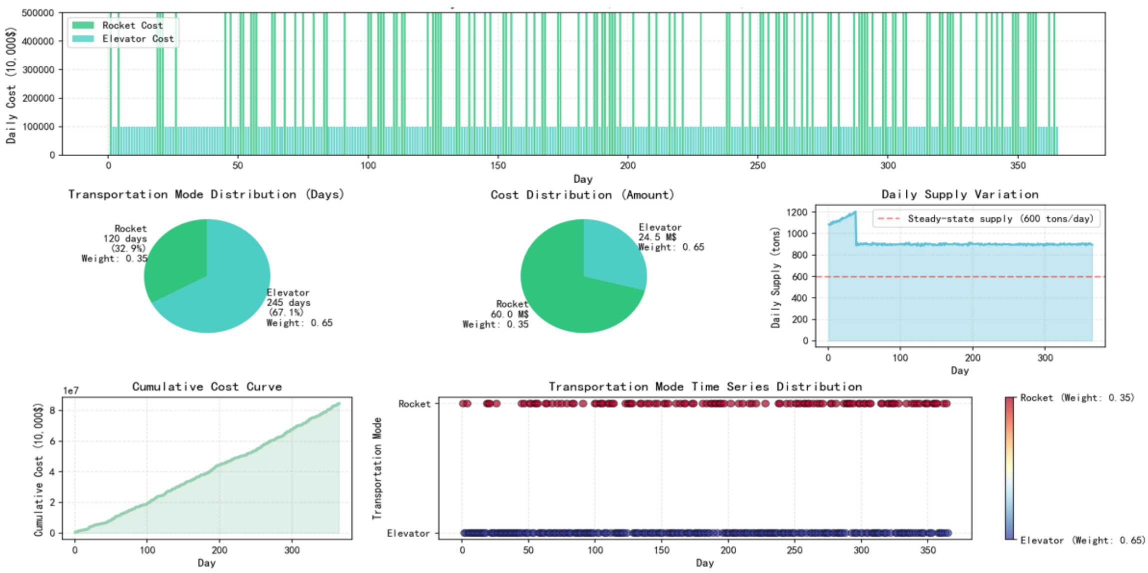

Fig. 12 Earth-Moon Base Water Supply and Transportation Cost

The daily supply follows a dynamic decomposition: the steady-state component $\bar{S}$ accounts for approximately 90%, and the command fluctuation μ_t and transportation uncertainty γ_t together account for about 10%, consistent with the dynamic characteristics of regular supply. The transportation mode allocation of daily supply is intuitively shown in the scheduling timeline:

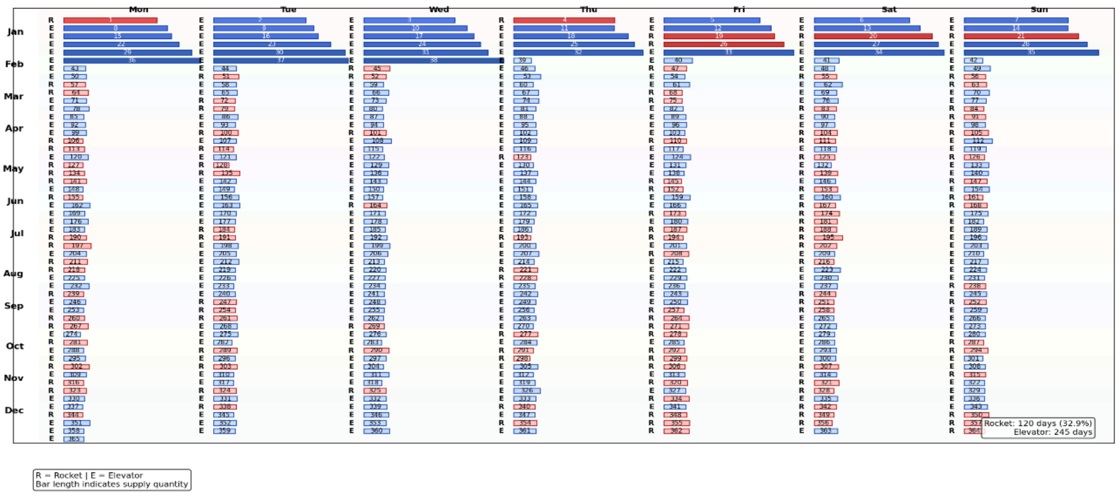

Fig. 13 Water Supply Calendar of a Year

Rockets account for 32.9% of supply days, mainly for emergency supply under low inventory; elevators account for 67.1%, primarily responsible for regular steady-state supply. Bar lengths in the table correspond to daily supply volume, further verifying the feedback regulation logic of "increased supply under low inventory". Annual additional transportation demand is:

$$D_{water} = \sum_{t=1}^{365} Supply_t \approx 337{,}479\ tons$$

Total annual supply cost is:

$$C_{water} = 8.45 \times 10^7\ USD$$

Cost structure analysis shows:

$$Cost_{rocket} = 6.00 \times 10^7 \text{ USD.}$$
$$Cost_{lift} = 2.45 \times 10^7 \text{ USD}$$

6.2.3 Interpretation of Result Implications

The AR coefficient is close to 1, indicating slight inventory decay and strong inertia in the system. The MA (7) structure reflects weekly operational and scheduling cycles in consumption and transportation. The consistency between supply proportion and theoretical net loss rate verifies the rationality of model parameterization and mechanism design. Dynamic decomposition of daily supply reflects the model's accurate characterization of real-world randomness; incorporating command fluctuation and transportation uncertainty enhances the scheme's practicality. The model's dynamic characteristics explain the observations: The impact of water consumption fluctuation is significantly attenuated by the coefficient $(1-\eta) = 0.02$. $\Delta W_\varepsilon = -(1-\eta)\varepsilon_t = -0.02\varepsilon_t$

This attenuation effect prevents drastic inventory fluctuations caused by short-term water consumption variations, further verifying the stability design of the model.

The transportation delay effect is reflected by the moving average term: $\Delta W_u =$

$\sum_{i=1}^{d} \alpha_i u_{t-1}$, and recent transportation delays carry higher weight in affecting the current inventory.

Cost results reflect the economic efficiency of the hybrid transportation mode: the space elevator reduces long-term supply burdens through its low-cost advantage, while rockets ensure emergency demands via high priority. Their combination achieves both cost control and supply reliability, providing a scientifically feasible solution for regular water supply in lunar bases. The model results offer a quantitative basis for scheduling regular supply in the Earth-Moon transportation system. Future work may integrate in-situ water resource utilization technologies to further reduce dependence on Earth supply.

7 Environmental Impact Optimization of Transportation Systems

7.1 Model Framework and Quantitative Methodology

To achieve synergistic economic and environmental optimization of the Earth-Moon transportation system, this study develops a comprehensive model framework integrating Life Cycle Assessment (LCA) and generalized cost analysis. The core innovation of this framework is the precise monetization of abstract environmental impacts, providing a quantitative basis for decision-making under the 2050 carbon quota regime.

7.1.1 Core Decision-Making Mechanism: Generalized Cost Function

By introducing the carbon shadow price, the model internalizes environmental externalities and establishes the core decision-making function:

$$Generalized\ Cost = Financial\ Cost + \lambda \times Environmental\ Cost \tag{14}$$

The Entropy Weight Method is adopted for the objective allocation of weights to each environmental indicator, which avoids subjective biases and ensures the neutrality of the model results.

1.Data Standardization: Standardize cost-type indicators x_{ij}using $r_{ij} = \frac{max(x_j) - x_{ij}}{max(x_j) - min(x_j)}$

2. Calculate Proportion Matrix: $p_{ij} = \frac{r_{ij}}{\sum_{i=1}^{m} r_{ij}}$

3. Calculate Information Entropy: $e_j = -\frac{1}{ln\,(m)} \sum_{i=1}^{m} p_{ij}\, ln\,(p_{ij})$

4. Calculate Variation Coefficient: $d_j = 1 - e_j$

5. Determine Weights: $w_j = \frac{d_j}{\sum_{j=1}^{n} d_j}$

Where:i is the solution number, j is the indicator number, m is the total number of solutions, n is the total number of indicators.

7.1.2 Quantification of Multi-dimensional Environmental Impacts

The model breaks through the limitation of a single carbon emission indicator, proposes four key environmental impact parameters, and accurately depicts the unique environmental impacts of rocket launches:

Ozone Layer Depletion: $E_{ozone} = N \cdot M_{Cl} \cdot \alpha$ Focusing on the amplification effect of stratospheric injection.α denotes the stratospheric amplification factor (solid rockets > liquid rockets), which enables us to effectively distinguish the differences between technical pathways.

Climate Warming: $Eclimate = \beta \cdot N \cdot M_{BC} \cdot Tres$ Repeated rocket launches emit a large amount of black carbon into the stratosphere, which has become a non-negligible factor contributing to the greenhouse effect.

Aerosol Disturbance:$E_{aerosol} = N \cdot M_{Al_2O_3} \cdot \gamma$ Quantifying the disturbances to cloud formation and radiation.

Cumulative Greenhouse Gas Emissions: The comprehensive emissions throughout the entire life cycle of transportation are accounted for and quantified by the total volume method.

7.1.3 Environmental Impact Target Function

Based on the aforementioned parameters, the model establishes the environmental impact target function:

$$f_3 = Min\ Z_E = \sum_{t=1}^{T} (E_{rocket} \cdot R_{k,t} + E_{lift} \cdot E_{p,t}) \tag{15}$$

Where E_{rocket} and E_{lift} denote the unit environmental costs accurately quantified by sub-models. The model integrates multi-dimensional environmental impacts into a unified target function to realize comprehensive optimization:

$$minZ = w_1 \cdot \frac{f_1}{f_{1,ref}} + w_2 \cdot \frac{f_2}{f_{2,ref}} + w_3 \cdot \frac{f_3}{f_{3,ref}} \tag{16}$$

Where $f_{i,ref}(i = 1,2,3)$denotes the reference benchmark value for each objective, and the weight w_i can be set via the Entropy Weight Method or policy preferences.

7.1.4 Environmental Impact Function and Critical Mechanism

The model defines the comprehensive environmental impact judgment function:

$$I_{env} = w_1 \frac{P_e}{\Phi_{\odot}} + w_2 \frac{S}{S_{bio}} + w_3 \frac{\overline{E} - E_0}{\sigma_{env}}, w_1 + w_2 + w_3 = 1 \tag{17}$$

When $I_{env} < 1$, we consider the environmental impact during the operational phase to be negligible. Among them, P_e denotes the system's effective emission power, and $\Phi_{\odot}$is the solar constant, which together form a relative scale for energy input.Srepresents the total spatial occupation area of the mission chain, and $S_{bio}$is the biosphere carrying capacity of the reference area, which is used to evaluate the eco−logical pressure of spatial occupation.$\overline{E} - E_0$quantifies the deviation degree of the additional radiation flux induced by human activities from the natural background value E_0, and is normalized by the natural standard deviation of environmental disturbanceσ_{env}.

Table 2 Data and Database Websites

Database Names	Description
S_{bio}、S	https://www.footprintnetwork.org/
P_e、$\Phi_{\odot}$	https://science.nasa.gov/earth/earth-observatory/

$\bar{E}$、E^0	https://www.iaea.org/data
$\sigma_e nv$	https://www.ncdc.noaa.gov

7.1.5 Scenario Setting and System Constraints

To ensure the credibility of simulation results, practical constraint conditions have been incorporated into the model:

$$N \leq N_{crit} = \frac{E_{crit}}{w_1 M_{Cl}\alpha + w_2 M_{BC} T_{res}\beta + w_3 M_{Al_2O_3}\gamma + w_4 E_{other}} \tag{18}$$

Among them, N_{crit}denotes the critical annual launch frequency.$E_{crit}$is the annual upper limit of acceptable environmental impacts derived by back−solving the condition $I_{env} = 1$ $M_j$represents the mass of specific substances emitted per single launch, andα, β, γ为 are the environmental impact amplification factors or residence times of the corresponding substances in the stratosphere.

7.2 Multi-scenario Simulation and Analysis of Optimization Strategies

7.2.1 Environmental Impact and Multi-objective Value Comparison of Three Transportation Scenarios

Based on the multi-objective optimization solution of the genetic algorithm (NSGA), combined with constraints including a maximum annual rocket launch frequency of 8000, an annual transportation capacity of 537,000 tons for the space elevator, and a launch success rate of 98.5%, the environmental impacts and multi-objective Z_E optimization paths for the three types of scenarios are as follows:

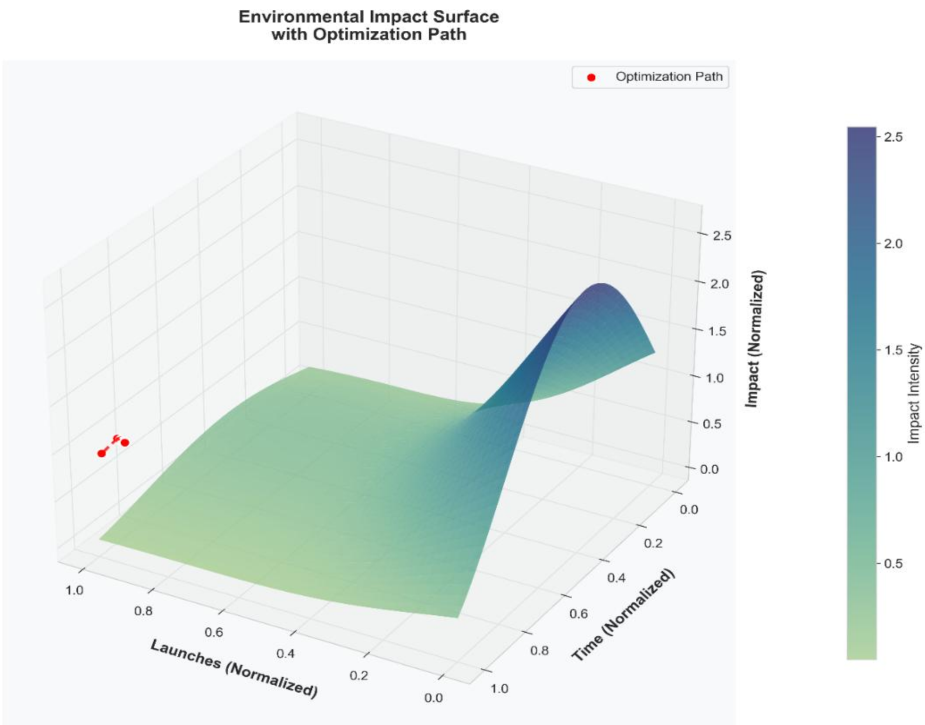

Fig. 14 ZE Impact Surface Optimization

The Pareto frontier solutions of the final generation show that the optimal value of Z_E is achieved when the proportion of rocket transportation reaches 32.9% and that of space elevator transportation stands at 67.1%. At this point, the cost is $Z_C = 6.9\times10^{12}$USD, the time consumption is $Z_T = 160$ years, and the shadow price of environmental cost is $Environmental\ Cost = 1.09 \times 10^{13}$. The generalized cost is $Generalized\ Cost = 1.78 \times 10^{13}$. In addition, the final results indicate that the average annual launch frequency $N = 1899.4 < N_{crit} \approx 9638$, which is consistent with the expected results.

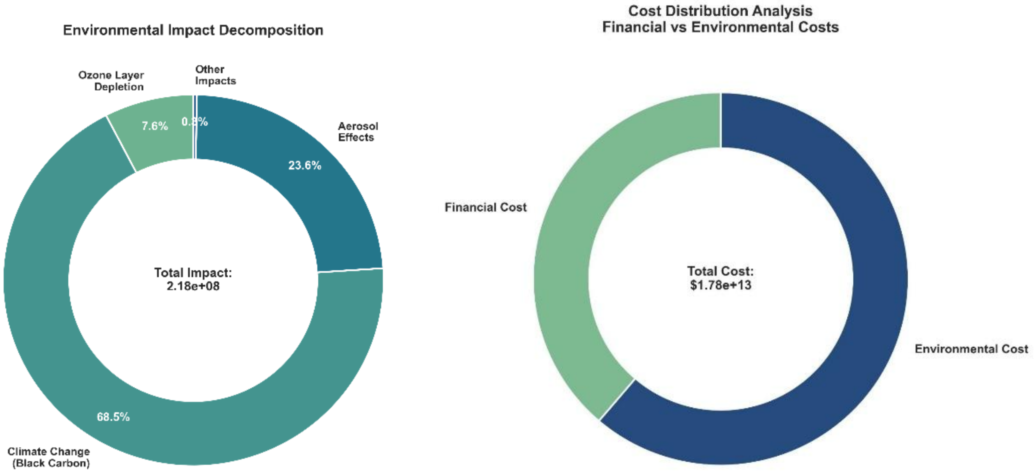

Fig. 15 Pollutant Component & ZC/ZE Ratio

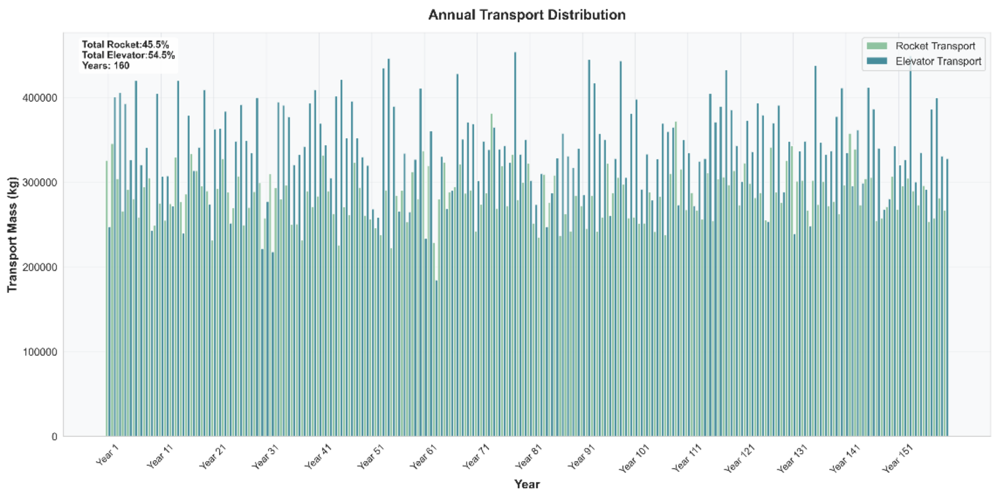

Fig. 16 Timetable and Rocket/Lift Ratio of the Solution

7.3 Model Result

Based on the multi-objective optimization incorporating environmental externalities through the generalized cost framework, the NSGA-II-derived Pareto-optimal solution identifies a hybrid transport structure—comprising 32.9% rocket and 67.1% space elevator delivery—as the most effective strategy for balancing economic and environmental objectives. This configuration achieves a total financial cost of $Z_C = 6.9 \times 10^{12}$ USD, a project timeline of $Z_T = 160$ years, and an environmental shadow cost of 1.09×10^{13}, resulting in a minimized generalized cost of 1.78×10^{13}. The solution not only stays well within the critical annual launch limit ($N < N_{\text{crit}}$), but also demonstrates that the space elevator serves as the low-emission backbone for bulk logistics while rockets provide essential redundancy and responsiveness, collectively ensuring system sustainability under mid-21st-century carbon constraints.

8 Conclusion

This work consistently applies goal programming to Problems 1, 2, and 4, utilizing NSGA-II to solve the resulting multi-objective optimizations. Solutions align with our assumptions and realistic engineering constraints, demonstrating strong internal consistency and interpretability.

8.1 Sensitivity Analysis

Sensitivity

Tests on core parameters (costs, capacities, failure rates) confirm the robustness of the ideal hybrid allocation (35% rocket, 65% elevator). Timeline and cost respond predictably to parameter shifts. Resilience and environmental outcomes are more sensitive to failure probabilities and environmental weighting, underscoring the influence of policy choices.

Convergence Analysis

The evolutionary optimization exhibits rapid convergence: total cost Z_C and timeline Z_T stabilize by generation 600, with Z_C converging to 2.75×10^{13} and Z_T to 168. Tight alignment between best and average values confirms consistent convergence, while the trajectory of optimal individuals demonstrates no premature stagnation, validating the algorithm's robust convergence to the optimal hybrid allocation. Parameter values were selected based on convergence stability tests, and further variation showed no significant change in the Pareto frontier structure.

8.2 Model Evaluation

Strengths:

1.High Coherence: Integrates theory with practical constraints, yielding quantifiable, self-consistent solutions.

2.Modular Design: Parameterized framework allows easy adaptation and scenario testing.

3.Methodological Rigor: Appropriate use of NSGA-II, Monte Carlo, and ARMA modeling addresses the problem's complexity.

Weaknesses:

1.Computational Limits: Monte Carlo sample size may affect precision in resilience confidence intervals.

2.Data Limitations: Pioneering subject matter necessitates assumptions that future data could revise.

8.3 Further Discussion

Improvements: Incorporate more advanced stochastic processes (e.g., MDPs) or high-fidelity simulators for deeper validation. Enable dynamic, endogenous parameter updates.

Extensions: Integrate Lunar ISRU (water extraction) to transform supply logistics. Model phased infrastructure buildup. Expand to a full system-of-systems optimization including base design and energy infrastructure.

References

[1] Krawciw, A., Olmedo, N., Rehmatullah, F., Desjardins-Goulet, M., Toupin, P., & Barfoot, T. D. (2026). Lunar Rover Cargo Transport: mission concept and field test. *IEEE Transactions on Field Robotics.*, 1. https://doi.org/10.1109/tfr.2026.3652156

[2] Woods, A., Vogler, A., & Collins, P. (2023). The Lunar Space Elevator: a key technology for realising the Greater Earth Lunar Power Station (GEO-LPS). *Journal of the British Interplanetary Society*, *76*(7), 252–262. https://doi.org/10.59332/jbis-076-07-0252

[3] Kapman, J. M. (2023). Innovation and research for space elevators. *Journal of the British Interplanetary Society*, *76*(7), 238–246. https://doi.org/10.59332/jbis-076-07-0238

[4] Revell, L. E., Bannister, M. T., Brown, T. F. M., Sukhodolov, T., Vattioni, S., Dykema, J., Frame, D. J., Cater, J., Chiodo, G., & Rozanov, E. (2025). Near-future rocket launches could slow ozone recovery. Npj Climate and Atmospheric Science, 8(1), 212.

[5] https://ntrs.nasa.gov/citations/20180006947

[6] Williamson, J., Wilson, J. P., & Luong, H. (2024). Status of ISS water management and recovery. Proceedings of the 53rd International Conference on Environmental Systems, Louisville, KY, USA.

[7] Bloetscher, F., & Eustice, R. J. (2016). Do per Capita Water Use Regulations Penalize Growing Urban Economies. *Current Urban Studies*, *04*(03), 376–391.

Letter to the MCM Agency

Dear Members of the Board,

In response to the 2026 MCM Problem B, we developed a comprehensive modeling framework to assess the cost, timeline, reliability, and environmental impacts of building and sustaining a 100,000-person lunar colony. Using multi-objective optimization and scenario-based simulation, we present the following recommendation.

Executive Recommendation

We strongly recommend adopting a phased hybrid Earth–Moon transportation architecture that integrates space elevator systems with traditional heavy lift rockets. This approach consistently outperforms single-method strategies across all key performance metrics.

Strategic Rationale

Space elevators at three Galactic Harbours form the primary logistics backbone, driven by low marginal cost, high throughput, and near-zero emissions. Rockets, launched from ten globally distributed sites, complement the system by providing redundancy, handling time-critical or oversized cargo, and maintaining operational continuity. Under optimal conditions, a roughly **65% elevator / 35% rocket** allocation achieves the best balance between cost efficiency and delivery speed.

Implementation Plan

Beginning in 2050, we propose simultaneous construction of the space elevator and expansion of rocket manufacturing. Using the optimized hybrid model, 10^8 metric tons of materials are delivered in about 164 years at a cost of 2.38×10^{13} USD, reducing costs by over 50% versus rockets alone and outperforming an elevator-only timeline.

Resilience and Sustainment

To address non-ideal conditions, we propose a dynamic capacity reallocation strategy. During space-elevator maintenance or failures, rocket utilization rises to about 55%, extending the timeline by only 5 years while preserving uninterrupted supply. Once operational, the colony's annual water demand of roughly 337,500 metric tons can be met within the same hybrid framework at an estimated cost of 8.45×10^7 USD per year.

Environmental Considerations

By internalizing environmental impacts via a shadow carbon price, our model shows that the hybrid strategy substantially reduces rocket launches, thereby lowering stratospheric emissions and ozone-depletion risks relative to a rockets-only scenario.

Conclusion

This hybrid transportation architecture is a quantitatively justified, resilient, and environmentally responsible solution. We strongly recommend its adoption as the foundational strategy for humanity's first large-scale lunar settlement.

Respectfully submitted,

Team Control Number: 2613671
2026 Mathematical Contest in Modeling

Report on use of AI

1.To enhance the rigor and normativity of non-native academic expressions, the research team integrates AI into paper writing. Before incorporating AI-assisted content into designated paper modules, a multi-tier manual verification mechanism is applied for compliance and academic validation, ensuring alignment with the expressive norms and logical demands of mathematical modeling papers. For standardized and controllable AI application, a global semantic framework is first built as a foundational constraint, with elaborately designed prompts then providing directional guidance for the orientation, dimensions and logical reasoning of AI-generated content. The whole technical application strictly follows scientific research ethical guidelines, and institutional mechanisms are established to prevent inappropriate AI use.

2.Google Gemini(Gemini 3 Flash)

Query: <Can water resources be recycled instantaneously on the Moon?>

Output:

On the Moon, water resources cannot be recycled "instantaneously" in a literal sense, but they can achieve highly efficient, near real-time closed-loop circulation.Current aerospace technology (taking the International Space Station, or ISS, as a blueprint for future lunar bases) can achieve a water recovery rate of 98% from urine, sweat, and breath condensate. However, the process of turning "waste" back into "potable water" involves a series of complex physical and chemical treatments that typically take several hours or longer.

On the Moon, water recycling is a **"continuous flow closed-loop"** rather than an "instant transformation." Current processing speeds are sufficient to maintain the dynamic balance of a base, but due to safety protocols and physical reaction times, there will always be a processing delay.