1 Problem

Show that for composition of finitely many invertible functions is invertible.

2 Solution

This proof can be performed with induction. The base case consists of a single invertible function. By definition, such a function is invertible.

Suppose that this holds true for a function composed of k invertible functions is itself invertible.
$g(x) = (f_k \circ \ldots \circ f_1)(x) \Rightarrow \exists h(x) : (h \circ g)(x) = 1$
Then in order to complete the proof by induction, we must demonstrate that a function composed of $k+1$ invertible functions is invertible.

Consider $i(x) = (f_{k+1} \circ g)(x)$. We know that for $\forall f_{k+1}(x) \exists f_{k+1}^{-1}(x) : (f_{k+1}^{-1} \circ f_{k+1})(x) = 1$. Let $j(x) = (h \circ f_{k+1}^{-1})(x)$. Then $(j \circ i)(x) = ((h \circ f_{k+1}^{-1}) \circ (f_{k+1} \circ g))(x) = (h \circ (f_{k+1}^{-1} \circ f_{k+1}) \circ g)(x) = (h \circ g)(x) = 1$. This demonstrates that a function composed of $k+1$ invertible functions is invertible. Consequently, we complete our proof by induction.