

What follows is a list of all implemented formulas in both rac.py and rac_utils.py:

$$\lambda^{[2/1]}(\kappa) = \lambda_0 \frac{\kappa^2 + 2\alpha^2\kappa + \alpha^4 + \beta^2}{\alpha^4 + \beta^2 + 2\alpha^2\kappa} = \lambda_0 g(\alpha, \beta; \kappa)$$

$$\lambda^{[3/1]}(\kappa) = \lambda_0 \frac{(\kappa^2 + 2\alpha^2\kappa + \alpha^4 + \beta^2)(1 + \delta^2)}{\alpha^4 + \beta^2 + \kappa(2\alpha^2 + \delta^2(\alpha^4 + \beta^2))}$$

$$\lambda^{[3/2]}(\kappa) = \lambda_0 \frac{(\kappa^2 + 2\alpha^2\kappa + \alpha^4 + \beta^2)(1 + \delta^2)}{\alpha^4 + \beta^2 + \kappa(2\alpha^2 + \delta^2(\alpha^4 + \beta^2) + \varepsilon^2\kappa^2)}$$

$$\lambda^{[4/2]}(\kappa) = \lambda_0 \frac{(\kappa^2 + 2\alpha^2\kappa + \alpha^4 + \beta^2)(\kappa^2 + 2\gamma^2\kappa + \gamma^4 + \delta^2)}{(\alpha^4 + \beta^2)(\gamma^4 + \delta^2) + \mu^2\kappa + \omega^2\kappa^2}$$

$$\text{where } \mu^2 = 2(\alpha^2(\gamma^4 + \delta^2) + \gamma^2(\alpha^4 + \beta^2))$$

$$\lambda^{[5/2]}(\kappa) = \lambda_0 \frac{(\kappa^2 + 2\alpha^2\kappa + \alpha^4 + \beta^2)(\kappa^2 + 2\gamma^2\kappa + \gamma^4 + \delta^2)(1 + \varepsilon^2\kappa)}{(\alpha^4 + \beta^2)(\gamma^4 + \delta^2) + \mu^2\kappa + \omega^2\kappa^2}$$

$$\lambda^{[5/3]}(\kappa) = \lambda_0 \frac{(\kappa^2 + 2\alpha^2\kappa + \alpha^4 + \beta^2)(\kappa^2 + 2\gamma^2\kappa + \gamma^4 + \delta^2)(1 + \varepsilon^2\kappa)}{(\alpha^4 + \beta^2)(\gamma^4 + \delta^2) + \mu^2\kappa + \zeta^2\kappa^2 + \omega^2\kappa^3}$$

$$\text{where } \mu^2 = 2(\alpha^2(\gamma^4 + \delta^2) + \gamma^2(\alpha^4 + \beta^2)) + \varepsilon^2(\alpha^4 + \beta^2)(\gamma^4 + \delta^2)$$

$$E_r = \beta^2 - \alpha^4$$

$$\Gamma = 4\alpha^2|\beta|$$

$$\chi^2 = \frac{1}{N} \sum_{i=1}^N |\lambda_0 g(\alpha, \beta; \kappa_i) - \lambda_i|^2$$

What follows is a list of essential references:

- 1) J. Horacek, I. Paidarova, and R. Curick. J. Chem. Phys. **143**, 184102 (2015);

<https://doi.org/10.1063/1.4935052>

- 2) J. Horacek, I. Paidarova, and R. Curick. Correction Paper (2016)

- 3) R. Curick. Private Communication (2019)

- 4) R. Curick. Private Communication (2020)