Resilient Networks (1.208) Project Report

Modelling Strategic Patient Behaviour in the US Deceased-Donor Organ Allocation System

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1. Introduction

The number of patients in the United States alone who were registered on a waitlist to receive an organ transplant was over 110 000 at the start of April 2020. But organs used for transplantation are a scarce resource – only 40 000 transplants were completed in 2019 [1].

We consider deceased-donor organs in this project, which are organs that have become available through the death of a patient listed as a donor. Procurement and transplantation of these organs both take place within the Organ Procurement and Transplantation Network (OPTN), which is operated by the United Network for Organ Sharing (UNOS) according to an extensive set of policies that have been developed in accordance with federal law [2].

Within the OPTN, regional units called Organ Procurement Organisations (OPOs) each maintain waitlists of patients who require a transplant. When an organ becomes available, a process to match the organ with a patient on the waitlist is initiated. The main steps in this process are:

- The local OPO selects a medically compatible patient on their waitlist to offer the organ.
- The patient chooses to accept or reject the organ. If they accept, the transplant takes place.
- If the patient rejects the organ, the OPO may offer the organ to another patient, and the process repeats.

One source of inefficiency in the system stems from the fact that organs must be transplanted within a limited time (called the *cold ischemia* time) after they have been procured [3]. Organs that are not transplanted within their cold ischemia time must simply be discarded.

Unfortunately, offering an organ to a patient and waiting for their decision takes valuable time – and organs can therefore only survive a finite number of rejections before they must be discarded. We can see how a game arises between the local OPO (whose objective is to maximize social welfare) and the patients (whose objective is to maximize their personal welfare). While an offer of an organ may be beneficial for the system as a whole, a patient acting in their own interests may be best to reject it.

It is this strategic nature of the patient behaviour in the presence of deteriorating organs that this project intends to explore. In particular, it aims to:

(1) Modify existing allocation models to allow for strategic patient behaviour.

(2) Compute or characterize strategic equilibria that may arise in the resulting game, and compare the allocation outcomes under these equilibria.

1.1. Related Work

There have been many specific models introduced to study the deceased-donor organ allocation system, but this area of research also overlaps considerably with the literature on general models for matching resources and consumers. This section highlights some related work from both areas. The main point to observe is that models which focus on the strategic behaviour of patients are not so frequently encountered – and when they are, they include assumptions which do not account for organ deterioration and wastage.

In [4], the authors model the problem faced by a patient when choosing to accept or reject an offer as an optimal stopping problem. They do not consider concurrently the strategic behaviour of other patients on the waitlist.

[5] studies a first-come, first-served (FCFS) queueing model for the deceased-donor organ waitlist and aims to characterize an optimal acceptance policy for a patient on the waitlist. Patients are assumed to be homogeneous, and there are Poisson arrival processes for both new patients joining the queue and new organs arriving to be matched. Each arriving organ has a random *quality* associated with it, say $x \in [0, 1]$, and a patient must choose whether to accept or reject the arriving organ by setting acceptance quality thresholds, $(x_1, \ldots, x_n, \ldots)$, for each position in the queue, n. However, the authors do not consider the effect of organ deterioration with rejection and note that their analysis is likely to understate the resulting welfare loss caused by patient choice.

The model in the literature that is closest to the model used in this report is an online version of the edge-weighted bipartite matching problem – termed the *Display Ads* problem and introduced in [6]. In this problem, we start with an underlying bipartite graph with two sets of vertices U and V, and edges E. The vertices $u \in U$ are known in advance and vertices $v \in V$ arrive sequentially. Each arriving v must be immediately matched to some u with $(u, v) \in E$, and matches cannot later be revoked. Each vertex $u \in U$ has capacity c_u giving the maximum number of arriving vertices that it can be matched with, and each edge has a weight w_{uv}.

The objective is to maximise the sum of the weights of edges in the matching under various input models. The authors describe an algorithm that achieves a competitive ratio of $1 - 1/e$ under the IID input model. We note that when each $c_u = 1$, we recover a problem which is very close to the model of the organ allocation system that we consider in this paper. However, the Display Ads problem includes no notion of strategic choice for the nodes in U and once a match is proposed, it must take place.

2. Model

In this section, we introduce a model which is a modification of the Display Ads problem described in Section 1.1 and [6]. It allows one set of vertices (the patients) in the bipartite graph to reject a proposed matching, and it also includes the notion of organ deterioration.

It is hoped that this model will capture the idea that patients sometimes have an incentive to reject organs if their future payoff will bring them a better reward – and that this behaviour is a threat to socially optimal outcomes.

2.1. Model Definition

We start with the definition of an *underlying* bipartite graph $\bar{G} = (\bar{U}, \bar{V}, \bar{E})$, which has a left set of nodes $\bar{U}$, a right set of nodes $\bar{V}$, and a set of edges $\bar{E} \subseteq \bar{U} \times \bar{V}$. It also has a value $\bar{r}_{ij} \in \mathbb{R}_+$, associated with each edge $(i, j) \in \bar{E}$.

The nodes in $\bar{U}$ represent organ types: practically, we can think of these as discrete clusters of organs which have similar medical characteristics. The nodes in $\bar{V}$ similarly represent patient types. The presence of an edge $(i, j) \in \bar{E}$ indicates that it is medically feasible for an organ of type i to be transplanted into a patient of type j, and the value $\bar{r}_{ij}$ associated with this edge gives the benefit obtained by the patient. Figure 1 provides an example of what this graph may look like.

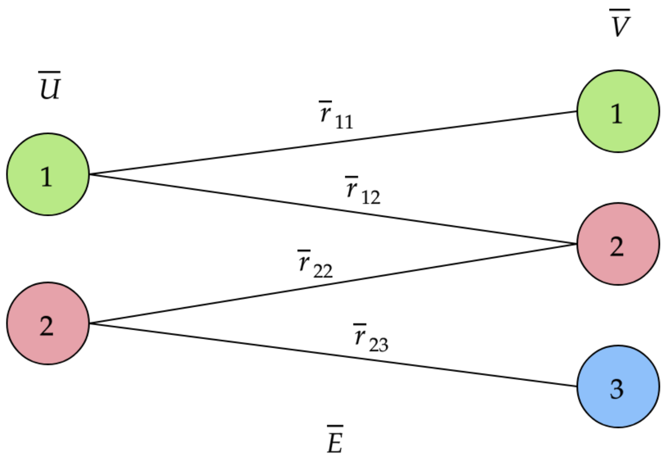

Figure 1: An example of an underlying graph $\bar{G} = (\bar{U}, \bar{V}, \bar{E})$. This graph defines the organ ($\bar{U}$) and patient ($\bar{V}$) types in the problem, and whether they are medically compatible with each other ($\bar{E}$).

An instance of the game is defined by an *instance graph* which represents the patients on the waitlist and organs that become available for transplant. Once again, this is a bipartite graph $G = (U, V, E)$. We let $M = |U|$ and $N = |V|$. Each node in U has an organ type associated with it, and each node in V a patient type; for $m \in U$, let $\sigma(m)$ be the type of organ m, and for $n \in V$, let $\sigma(n)$ be the type of patient n. There is an edge $(m, n) \in E$ if $(\sigma(m), \sigma(n)) \in \bar{E}$ in the underlying graph. Similarly, we let $r_{mn} = \bar{r}_{\sigma(m)\sigma(n)}$. Figure 2 shows an example of an instance graph arising from the underlying graph in Figure 1.

In an instance graph, it is important to note that U is actually a *sequence* rather than just a set. The order of the organs in this sequence has a direct impact on how the game will play out – because it defines the order in which they are offered.

For a fixed instance, it will be useful to understand the flow of the game when discussing the strategy sets. The game proceeds as follows:

(i) Organs arrive sequentially according to the order in U. Suppose organ m has just arrived.

(ii) A player who we will call the central decision maker (CDM) selects a patient, $n \in V$, such that $(m, n) \in E$ and n has not previously accepted an organ, and offers them organ m.

(iii) Patient n chooses whether to accept or reject the organ. If they accept, the organ is matched and patient n receives the reward r_{mn}.

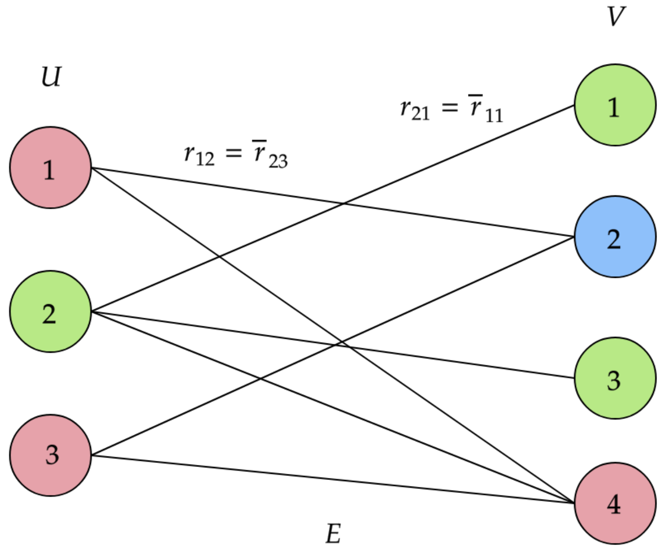

Figure 2: An example of an instance graph $G = (U, V, E)$, for the underlying graph in Figure 1. Organ and patient types are denoted by the colours of the nodes.

(iv) If they reject the organ, then the CDM picks another valid patient $m' \in V$, and the process repeats. If an organ is rejected K times, then it is discarded and not matched at all.

(v) The game concludes when all organs have been matched or discarded. Any patient n who received an organ m gains reward r_{mn}, and the CDM gains a reward that is the sum across all of these.

2.2. CDM Strategies

We consider only pure strategies for the CDM, which correspond to be a sequence of permutations (one for each organ) over the patients. We will call such a strategy $s^{\text{CDM}} = (\phi^1, \ldots, \phi^M)$. Here, $\phi_k^m = n$ indicates that patient n is the k^{th} most preferred patient for organ m from the perspective of the CDM. As rejections occur and patients are matched to previous organs, the CDM must make offers that fall further down their preferences in the permutations.

Note that each ϕ^m is not a true permutation, but rather a *sub-permutation* of V which may not contain all patients. Indeed, for a strategy to be feasible we must have that $(m, \phi_k^m) \in E$ for every m and k (meaning that the permutations only contain patients that are medically compatible with the organ being offered).

Because we are more interested in observing how strategic patient behaviours contribute to allocation outcomes, we will only look at games where the strategy of the CDM is fixed. The computational experiments in Section 3 use two different fixed strategies:

1. **FCFS.** This queueing system is often used in the analysis of deceased-donor waitlist systems. If we consider V to be a sequence that describes the order of patients in the queue (instead of just a set) then we may define:

$$\phi_k^m = n, \text{ where } n \text{ is the } k^{\text{th}} \text{ highest patient in } V \text{ such that } (m, n) \in E$$

That is: all compatible patients are taken in the same order for organs of the same type.

2. **Maximum weighted matching (MWM).** This is a heuristic strategy that the CDM may naturally adopt when faced with the game we have described, given that their objective is to achieve the socially optimal allocation. The strategy is constructed as follows:
   - Initialize ϕ_m to be empty for each $m \in U$.
   - Solve for a maximum weighted matching $X^* \subseteq E$, in the bipartite graph G and for each match $(m, n) \in X^*$, append n to the end of ϕ_m.
   - Remove all edges in X^* from E, solve for another maximum weighted matching, and repeat the process above until all edges have been removed from the graph.

   If patients accept all offers, then the outcome of the game when the CDM uses this strategy is the social optimum corresponding to the maximum weighted matching obtained from the full graph G. The iterative procedure above finds backup offers for the CDM that are contingency plans given that patients may reject organs.

2.3. Patient Strategies

The strategy set we consider for a patient n is much simpler than for the CDM. It is just, for each organ type $i \in \bar{U}$, a decision $x_{in} \in \{0, 1\}$ corresponding to whether the patient would accept an organ of type i if it were offered to them.

2.4. Discussion of Assumptions

Some assumptions have been implicitly made in the previous definition of the model and the strategy sets. Here, we include some comments on these assumptions:

- We are assuming that perfect information is available to all the patients. This deviates from the online setting in the Display Ads problem and means that information on all organ types (and therefore rewards) is available at the start of the game. It is clearly an unrealistic assumption for organ allocation, but it is a simplifying one that allows us to begin understanding the model.
- To restrict the strategy set of each patient, we have assumed that their rejections are consistent across time – that is, if they reject an organ of type $i \in \bar{U}$ at some time, they will reject any future organ of that type too. This decision is one taken for tractability, but we hope that it still allows us to observe the effects that we aim to in the model.

3. Computational Experiments

Though the original goal was to obtain some analytical insight for the model, it is not a trivial game to analyze (it involves multiple stages and multiple players with their own objectives). The techniques used to obtain analytical results in traditional bipartite matching papers could not be obviously extended to the setting where patients are strategic agents.

However, for an arbitrary instance and fixed CDM strategy it was possible to model the outcome of the game (the organ allocations) with binary variables representing the patient strategies. If we let $x_{in} \in \{0, 1\}$ be the acceptance decisions of the patients, $y_{mn} \in \{0, 1\}$ indicate whether patient n received organ m, and S be the linearly-constrained feasible set of the formulation (explained in Appendix A), the integer program (IP) formulation allowed us to:

- Compute the socially optimal outcome when all players cooperate. That is, solve the model for the objective function:

$$\max_{(\mathbf{x},\mathbf{y})\in S} \quad \sum_{m,n} r_{mn} y_{mn}$$

- Run an iterated best response procedure to optimize for each patient n in turn and hope for convergence to a Nash equilibrium, by solving for the objective function:

$$\max_{(\mathbf{x},\mathbf{y})\in S} \quad \sum_{m} r_{mn} y_{mn}$$

So that the results in this section were not tied to the characteristics of specific instances of the game, we performed each experiment on organs and patients taken from 10 randomized underlying graphs with 5 organ and patient types and 40 patients and organs with $K = 1$. The focus of these experiments were:

- For both the FCFS and MWM strategies of the CDM, use the IP model to perform iterated best response and check whether this converges towards a set of strategies that are a Nash equilibrium.
- For both CDM strategies, compute the socially optimal outcome (if all patients work together to optimize aggregate welfare) and compare it with the outcomes in the Nash equilibrium (if it was obtained). We also compare these outcomes with a baseline where all patients accept all offers.
- Compute the socially optimal solution on different problem sizes and observe how well the IP formulation scales. For this experiment, we varied the number of organs and patients.

3.1. Socially Optimal and Best Response Experiments

We first look at the FCFS strategy for the CDM. In every instance we observed that the iterated best response procedure converged to a Nash equilibrium after just one round of play for each of the patients.

Then, we computed the objective of the socially optimal allocation. When we compared this against the social objective value at the Nash equilibria (summed across all patients), we observed the gap that we were hoping the model would capture. Figure 3 shows this comparison.

We can see that the socially optimal allocations lead to higher reward per organ than both the equilibrium allocations and the baseline allocations where all patients accept all offers. This illustrates the effect that we hoped – where strategic play leads to an equilibrium that is less efficient than the social optimum. However, the degree of efficiency is not significant compared with the baseline allocation procedure, which needs further investigation.

For the MWM strategy used by the CDM we observed some different behaviour. In this setting, the baseline allocation procedure where all patients accept all offers is now equivalent to the socially optimal one (as it corresponds to the maximum matching in the instance graph).

The iterated best response procedure only converged to a Nash equilibrium in 89% of instances. Out of the convergent instances, 70% attained a Nash equilibrium with a total objective (summed across all patients) that matched the socially optimal allocation. This was interesting to observe – and suggests (as is intuitive) that changing the strategy employed by the CDM can have the effect of aligning the incentives of the patients with the CDM.

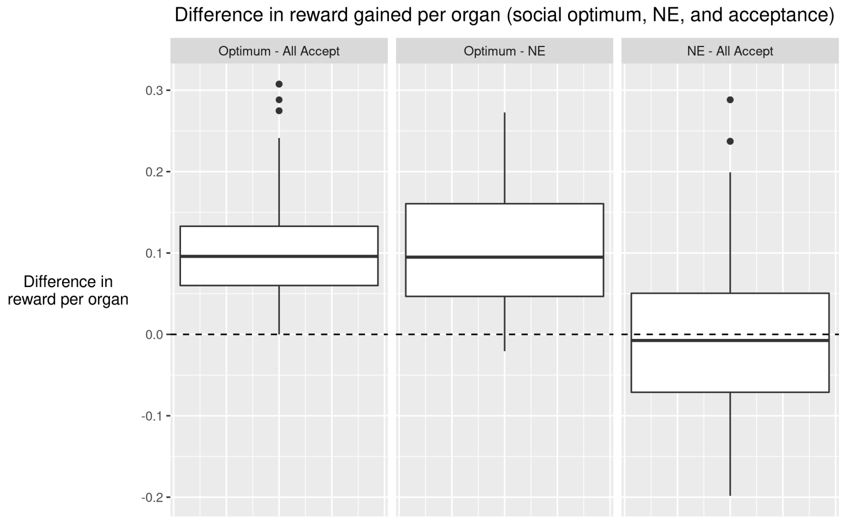

Figure 3: Difference in reward per organ obtained by the socially optimal allocation, compared to the equilibrium allocations and the baseline allocations. The measure is in reward per organ and the reward for any organ is in $[0, 1]$. Column labels indicate the difference that was taken.

3.2. Performance Experiments

To observe how well the IP formulation scaled, it was paired with the objective function measuring social welfare and used to solve instances of different sizes (in terms of the number of organs and patients). We tested systems with between 10 and 50 organs and patients, and where the CDM used the FCFS strategy.

All tests were implemented in Julia using the Gurobi solver with a 30 minute time-limit, and conducted on the Engaging MIT cluster using nodes with 16GB RAM. Figure 4 shows the results.

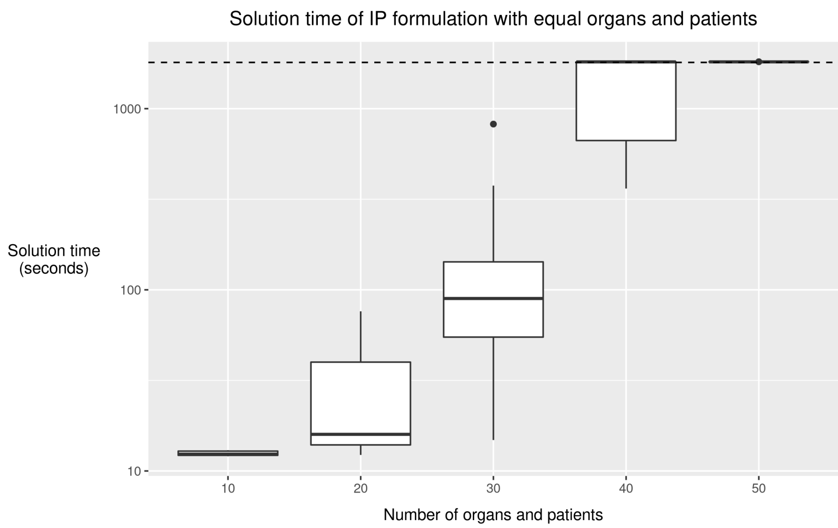

Figure 4: Solution times for the IP model with increasing problem size (note the log scale on the y axis). The 30 minute time-limit is indicated by the dashed line.

We can see that once the number of organs and patients reached a value of 40, the majority of problem instances took longer than 30 minutes to solve. All instances with 50 patients and organs took longer than 30 minutes to solve. Though the IP formulation clearly does not scale to extremely large numbers of patients and organs (the likes of which we might encounter in the real-life deceased-donor queuing system), we can still hope that the solvable problems shed some light on the behaviour of the model being analyzed. It is also possible that the IP formulation can be significantly tightened.

4. Summary and Conclusions

The aim of this study was to introduce a model of the deceased-donor organ allocation system in the US which took into account strategic patient behaviour. We did this by modifying the online maximum weighted bipartite matching problem to give patients the power to reject a proposed match, and also introduced the notion of organ deterioration.

Though no analytic insight was obtained for the model, the outcome of the game could be represented using linearly constrained binary variables. This allowed us to generate random instances and compute the socially optimal outcomes, as well as run an iterated best response procedure on random instances and obtain Nash equilibria.

The conclusions we were able to draw from the computational analysis were as follows:

- The IP formulation could scale to solve games with 5 organ and patient types, and around 40 patients and organs. Though this is far smaller than the size of the games that would arise in real-life, it may be large enough that we can hope the results provide some insight into how strategic patient behaviour works in this context.
- When the CDM used the FCFS strategy, there was a significant gap between the socially optimal outcomes and the social welfare in the equilibrium outcomes – suggesting that the model did capture this tradeoff to some degree.
- When the CDM used the MWM strategy, the gap between the socially optimal outcomes and the social welfare in the equilibrium outcomes reduced (often shrinking to zero) – suggesting that the strategy of the CDM can be changed so that the objectives of the patients more closely align with those of the CDM.

A. IP Formulation

A.1. Notation

Let $i \in [I]$ index the organ types and $j \in [J]$ index the patient types. Let $m \in [M]$ index the individual organs (which are ordered) and $n \in [N]$ index the individual patients. As before, we let $\sigma(m)$ be a function which maps organ m onto its type. Recall that a strategy for the CDM is $s^{\text{CDM}} = (\phi^1, \ldots, \phi^M)$, which is a sequence of partial permutations of compatible patients.

We also need to define, for each organ $m \in [M]$ and patient $n \in \phi^m$, the set H_n^m. This set will represent the patients higher in the permutation (meaning that they are more highly preferred by the CDM) than patient n for organ m.

A.2. System Description

The dynamics of the system can be modelled with linear constraints and binary variables. This becomes clear when we break down the rules of the system into smaller pieces that reveal its logical structure:

(i) If organ m has $\sigma(m) = i$, and patient n would not accept an organ of type i, then m cannot be assigned to n.

(ii) Consider an organ m, and patient n. If there is some patient $h \in H_n^m$ which has not been assigned an organ by the time m is offered and h accepts organ type $\sigma(m)$, then n should not be offered to m.

(iii) Again, consider an organ m, and patient n. If out of the patients in H_n^m, there are more than K who were not already assigned an organ and rejected organ type $\sigma(m)$, then organ m should not be offered to m (since it should be discarded).

(iv) If patient n has already received some other organ $m' \leq m - 1$, then this patient cannot be assigned organ m.

These four observations govern the behaviour of the system, and it is now our goal to model them with linear constraints and binary variables.

A.3. Variables and Constraints

We will define the variables $x_{in} \in \{0, 1\}$ and $y_{mn} \in \{0, 1\}$ to represent the following:

$$x_{in} = \begin{cases} 1 & \text{if patient } n \text{ accepts organ type } i \\ 0 & \text{otherwise} \end{cases}$$

$$y_{mn} = \begin{cases} 1 & \text{if patient } n \text{ receives organ } m \\ 0 & \text{otherwise} \end{cases}$$

Auxiliary variables $z_{mn} \in \{0, 1\}$ will also help make the constraints in the formulation clearer:

$$z_{mn} = \sum_{m'=1}^{m-1} y_{m'n} = \begin{cases} 1 & \text{if patient } n \text{ has received an organ by the time organ } m \text{ is available} \\ 0 & \text{otherwise} \end{cases}$$

Adding some more auxiliary logical variables will help further with the formulation. These are modelled using standard IP techniques for logical constraints:

$$\begin{aligned}\alpha_{mn} &= \neg z_{mn} \wedge x_{\sigma(m)n} \\ &= \begin{cases} 1 & \text{if patient } n \text{ not assigned an organ when } m \text{ available, and accepts type } \sigma(m) \\ 0 & \text{otherwise} \end{cases}\end{aligned}$$

$$\begin{aligned}\beta_{mn} &= \neg z_{mn} \wedge \neg x_{\sigma(m)n} \\ &= \begin{cases} 1 & \text{if patient } n \text{ not assigned an organ when } m \text{ available, and rejects type } \sigma(m) \\ 0 & \text{otherwise} \end{cases}\end{aligned}$$

$$\begin{aligned}\gamma_{mn} &= \mathbb{1}\{\sum_{h \in H_n^m} \beta_{mh} \leq K\} \\ &= \begin{cases} 1 & \text{if } \leq K \text{ patients ranked higher than } n \text{ are unassigned and would reject type } \sigma(m) \\ 0 & \text{otherwise} \end{cases}\end{aligned}$$

With all of these variables defined, we can model the logical behaviour of the system that has previously been described with the following constraints:

$$y_{mn} \leq x_{\sigma(m)n} \qquad \forall m \in [m],\ \forall n \in [N] \tag{1}$$

$$y_{mn} \leq \alpha_{mh} \qquad \forall m \in [m],\ \forall n \in [N],\ \forall h \in H_n^m \tag{2}$$

$$y_{mn} \leq \gamma_{mn} \qquad \forall m \in [m],\ \forall n \in [N] \tag{3}$$

$$y_{mn} \geq 1 - \sum_{h \in H_{mn}} \alpha_{mh} - \left(1 - x_{\sigma(m)n}\right) - (1 - \gamma_{mn}) - z_{mn} \quad \forall m \in [m],\ \forall n \in [N] \tag{4}$$

An interpretation of each constraint is:

- (1) ensures that patient n does not receive an organ if they reject its type.
- (2) ensures that patient n does not receive an organ if there is some other patient $h \in H_n^m$ who is is unassigned when organ m is available, and would accept it.
- (3) ensures that patient n does not receive an organ when it should be discarded.
- (4) ensures that patient n receives an organ if none of the previous scenarios apply, and n has not previously received an organ.

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