Mobile Robotics: Assignment 3 Report

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Contents

1 Extended Kalman Filter

1.1 Brief Outline

The task is to estimate the state of the robot moving in a 2D map. The measurements available are the noisy odometer velocity readings as well as the laser readings with respect to 17 unique landmarks. The state that has to be estimated is (x, y, θ). The motion model is:

$$\begin{bmatrix} x_t \\ y_t \\ \theta_t \end{bmatrix} = \begin{bmatrix} x_{t-1} \\ y_{t-1} \\ \theta_{t-1} \end{bmatrix} + dt \begin{bmatrix} \cos(\theta_{t-1}) & 0 \\ \sin(\theta_{t-1}) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_t \\ \omega_t \end{bmatrix}$$

The observation model is:

$$\begin{bmatrix} r_t^l \\ \phi_t^l \end{bmatrix} = \begin{bmatrix} \sqrt{(x_l - x_t - d\cos(\theta_t))^2 + (y_l - y_t - d\sin(\theta))^2} \\ \operatorname{atan2}(y_l - y_t - d\sin(\theta_t), x_l - x_t - d\cos(\theta_t)) - \theta_t \end{bmatrix}$$

The observation model is non-linear, therefore we have to use the Extended Kalman Filter(EKF), where the observation model is the non-linear (but differentiable) function $h(r_t)$ while the motion model is the non linear function $g(r_t)$.

1.2 Predict Step

During, the predict step, we perform the following updates:

$$\bar{\mu_t} = g(\mu_t, \mu_{t-1})$$

$$\bar{\Sigma_t} = G_t \Sigma_{t-1} G_t^T$$

Here, G is the Jacobian matrix for the function g given as

$$G = \begin{bmatrix} 1 & 0 & -v_t \cdot dt \cdot \cos(\theta_{t-1}) \\ 0 & 1 & v_t \cdot dt \cdot \sin(\theta_{t-1}) \\ 0 & 0 & 1 \end{bmatrix}$$

1.3 Update Step

In the update step, we perform the correction using the laser readings. The update rules here are as follows:

$$K_t = \bar{\Sigma_t} H_t^T (H_t \bar{\Sigma_t} H_t^T)^{-1}$$

$$\mu_t = \bar{\mu_t} + K_t(z_t - h\bar{\mu_t})$$

$$\Sigma_t = (I - K_t H_t)\bar{\Sigma_t}$$

Here, K_t is the Kalman gain, H is the Jacobian matrix for the function h and z is the observed measurements. We obtain the corrected values for μ_t and Σ_t after this step. The Jacobian H $(2L \times 3)$ can be obtained as vertical stacking of following H_l (2×3):

$$H_l = \begin{bmatrix} \frac{-x_{\text{diff}}}{\alpha} & \frac{-y_{\text{diff}}}{\alpha} & \left(\frac{x_{\text{diff}} \cdot d \cdot \sin(\theta_k) - y_{\text{diff}} \cdot d \cdot \cos(\theta_k)}{\alpha}\right) \\ \frac{y_{\text{diff}}}{\alpha^2} & \frac{-x_{\text{diff}}}{\alpha^2} & \left[\left(\frac{-d \cdot \{x_{\text{diff}} \cos(\theta_k) + y_{\text{diff}} \sin(\theta_k)\}}{\alpha^2}\right) - 1\right] \end{bmatrix}$$

, where $x_{\text{diff}} = (x_l - x_i - d\cos(\theta_k))$, $y_{\text{diff}} = (y_l - y_i - d\cos(\theta_k))$ and $\alpha = \sqrt{(x_{\text{diff}})^2 + (y_{\text{diff}})^2}$.

1.4 Results

We apply the predict and update procedures for every time step. The error that is obtained can be quantified by the Root Mean Squared Error(RMSE). The RMSE we obtain is 1.65

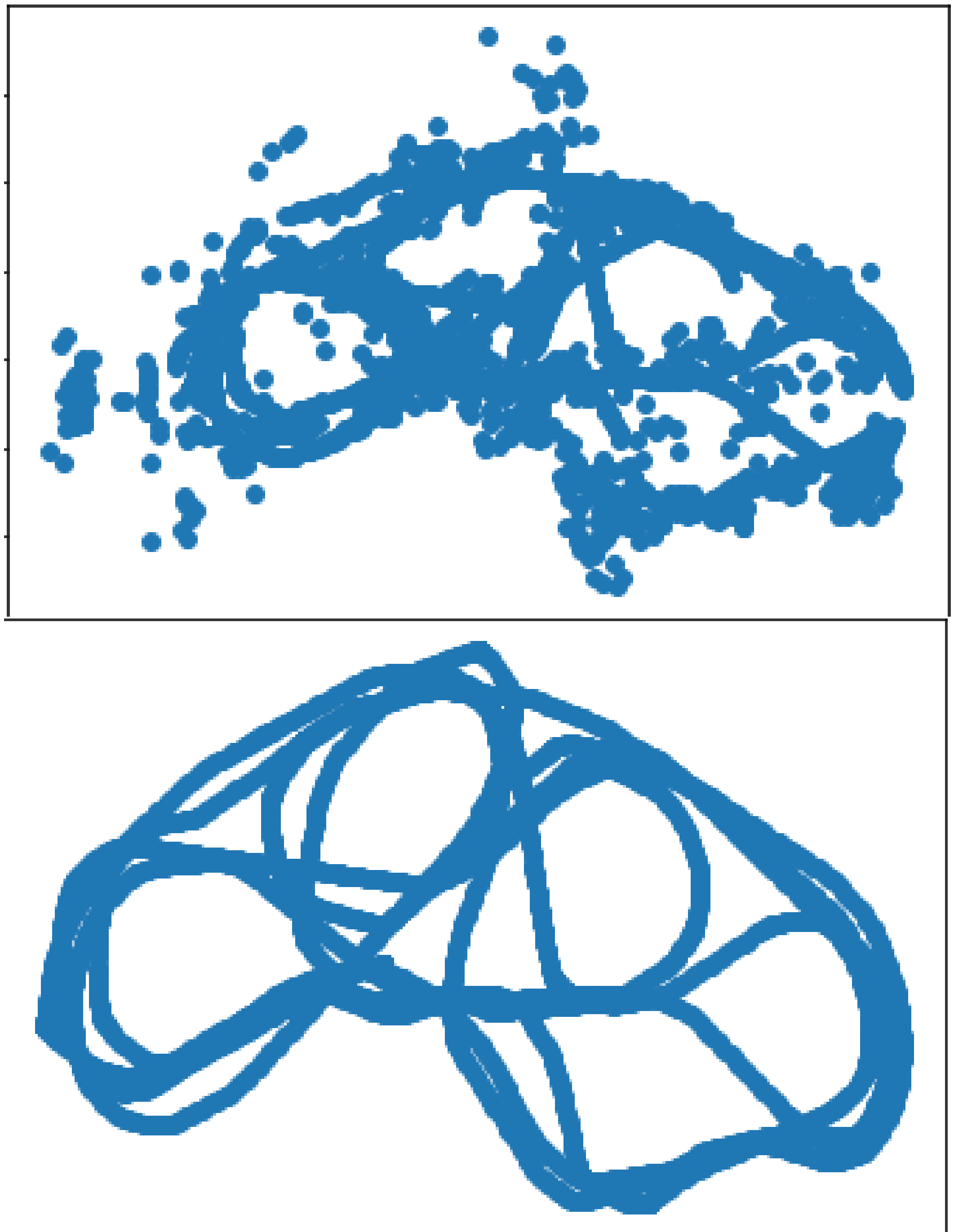

Figure 1: The scatter plot showing the paths of the ground truth as well as the predictions after 4000 time steps

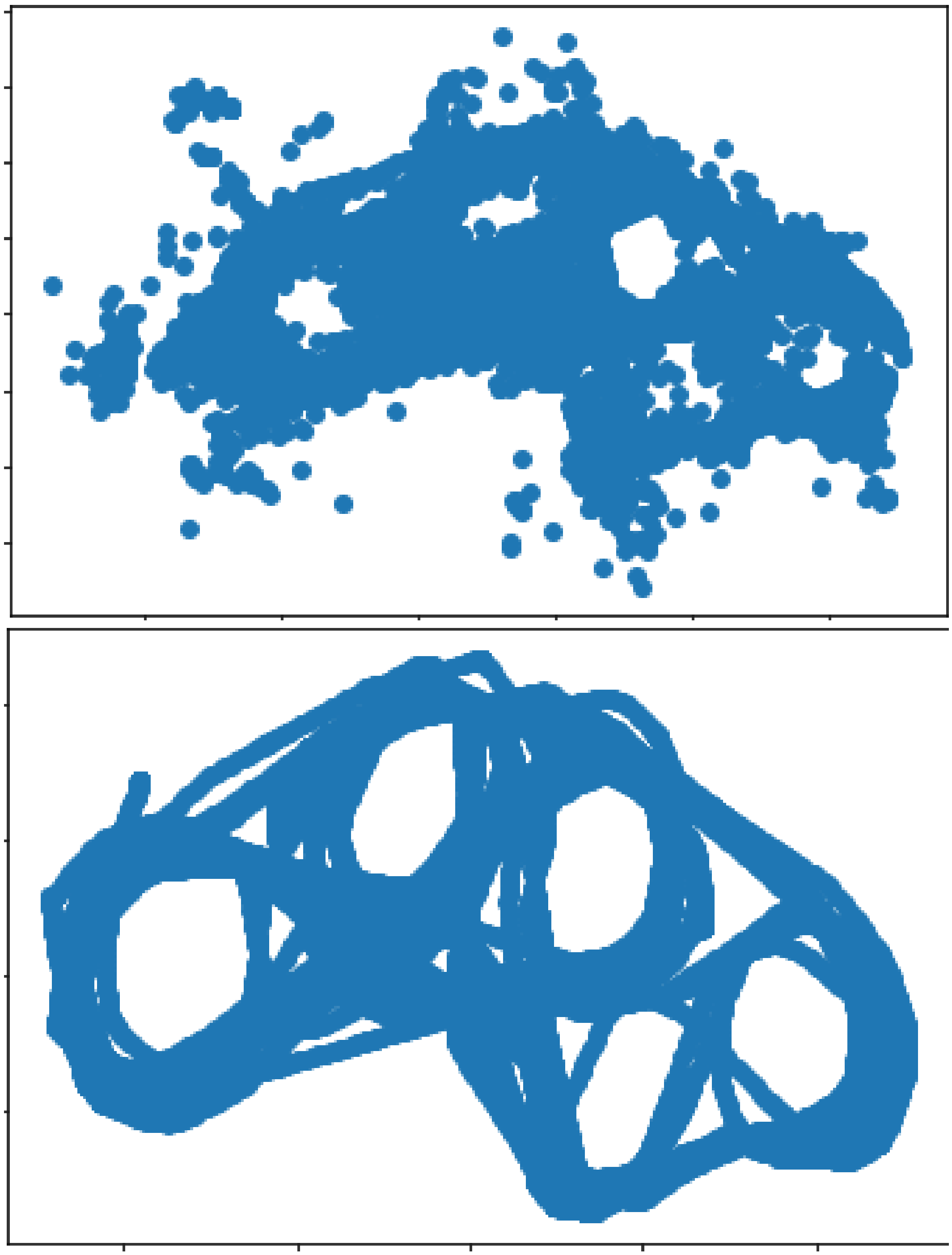

Figure 2: The scatter plot showing the final paths of the ground truth as well as the predictions