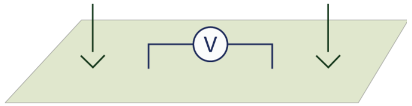
Assumptions:

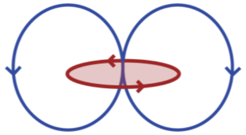
This course is for 3rd/4th year students in geoscience.

Basic background of physics are not preferred, but not required.

Apps developed by Jupyter Notebooks, Python and SimPEG are used, but strong software background is not required.

The Physics of Geophysical Prospecting Methods: Resistivity and Controlled Source vs Natural EM methods





Seogi Kang

Stanford University

Learning goals

Understand electrical resistivity and its linkage to geologic units.

Understand physical principles of Electromagnetic (EM) methods

- Charge build-up (or current channeling, galvanic currents)
- EM induction

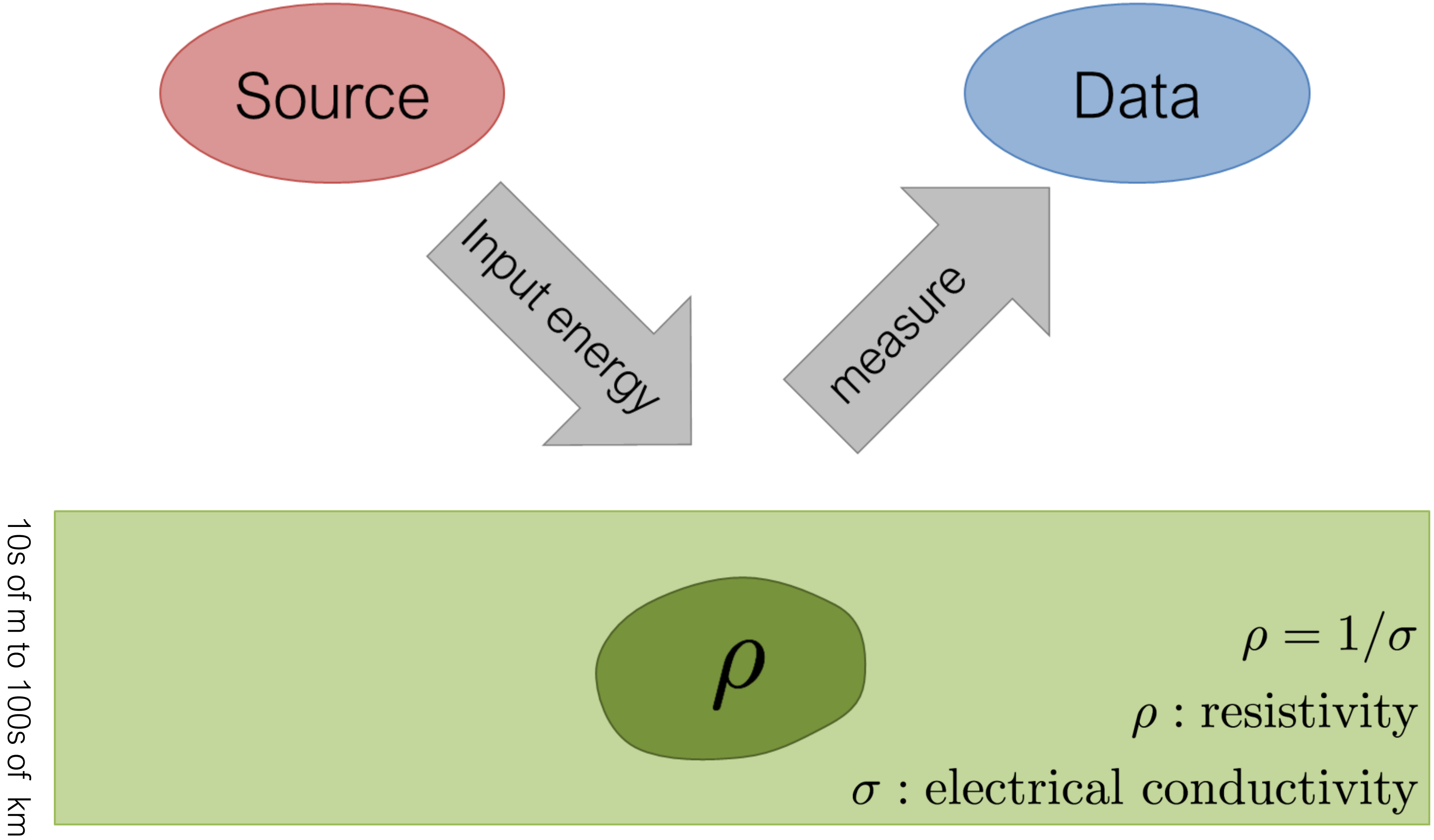
Identify difference between controlled-source EM and magnetotellurics (MT)

Identify all EM methods are governed by the same physical principles

Understand survey setup of EM methods and data

Understand sensitivity of EM methods to conductors & resistors

Electromagnetic (EM) survey

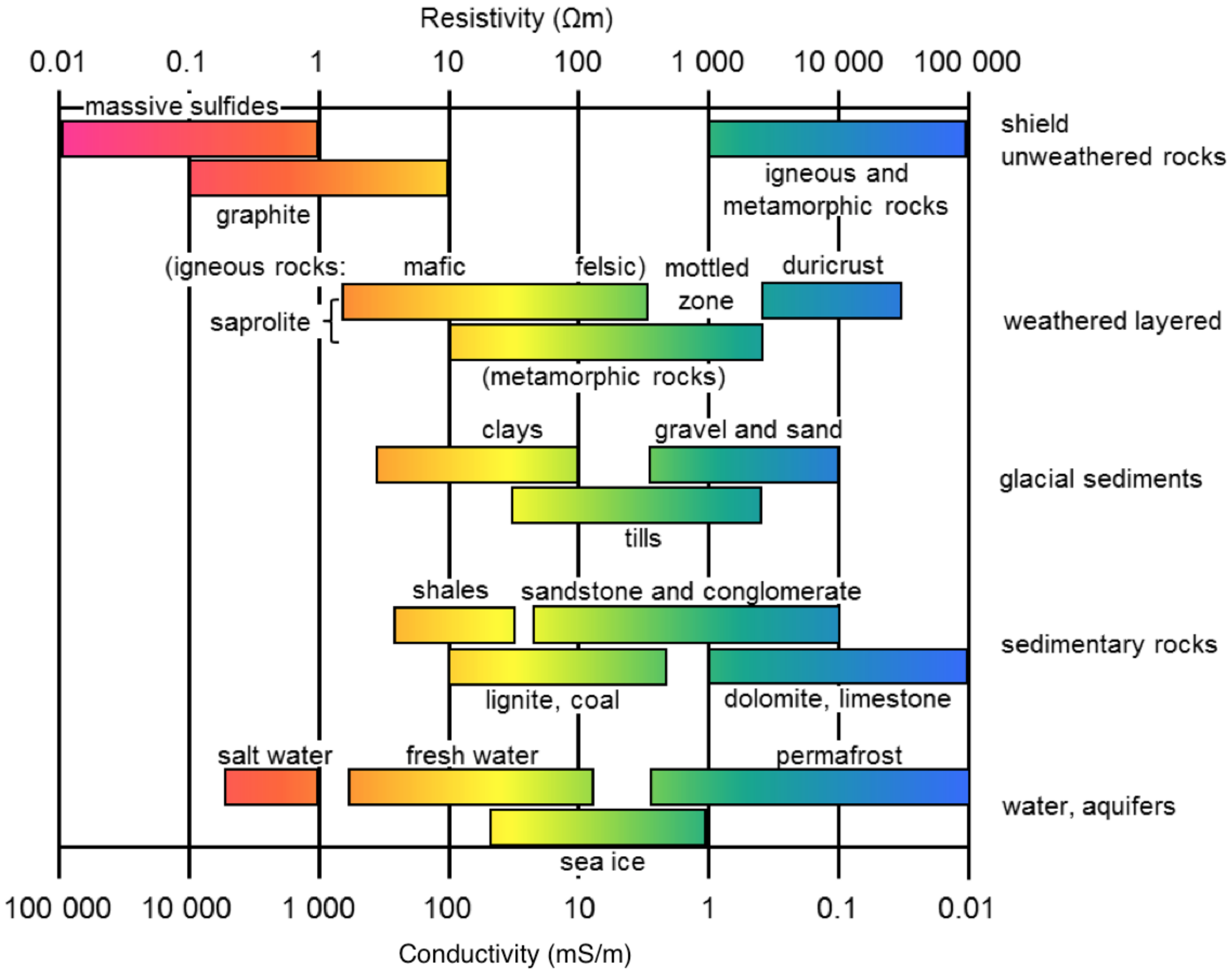


$$\rho = 1/\sigma$$

ρ : resistivity

σ : electrical conductivity

Electrical resistivity (or conductivity)



DC resistivity is sensitive to:

- σ: Conductivity [S/m]
- ρ: Resistivity [Ωm]
- $\sigma = 1/\rho$

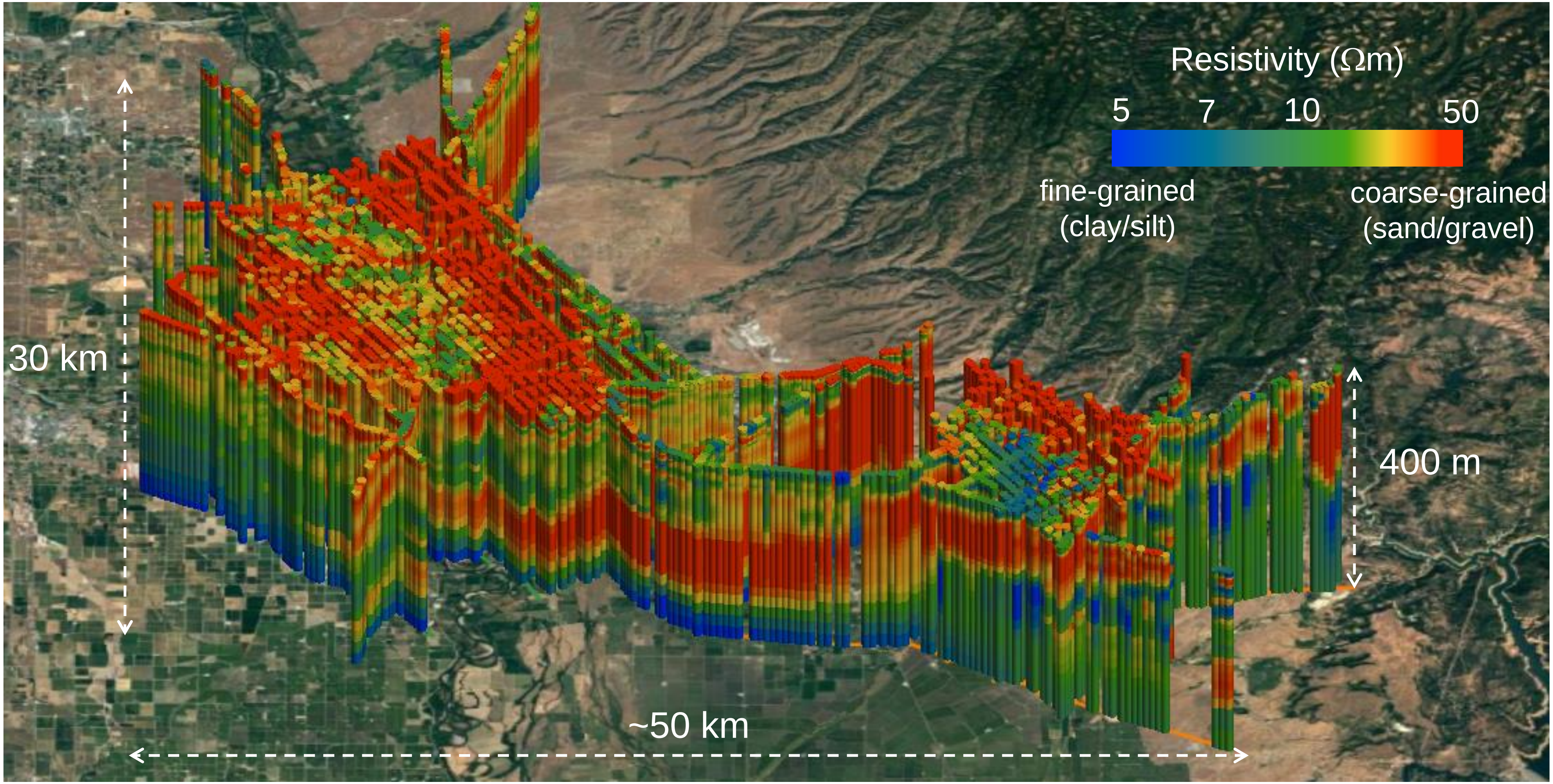
Varies over many orders of magnitude

Depends on many factors:

- Rock type
- Porosity
- Connectivity of pores
- Nature of the fluid
- Metallic content of the solid matrix

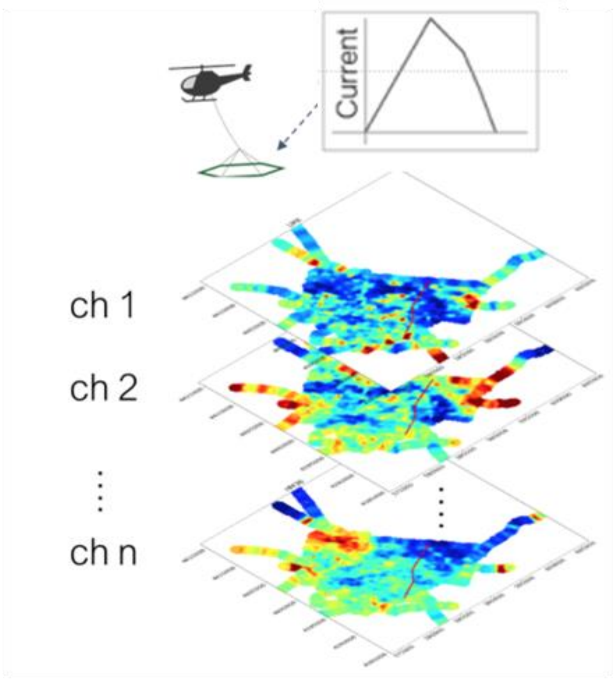
Electrical resistivity can be a diagnostic physical property for …

3D resistivity model from an airborne EM survey



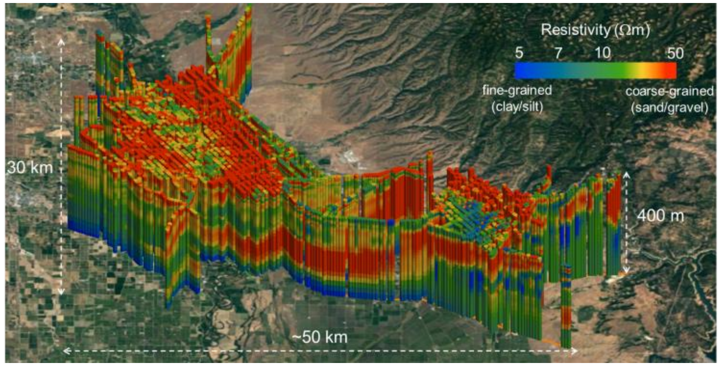
EM imaging – inverse problem

Data



Inversion

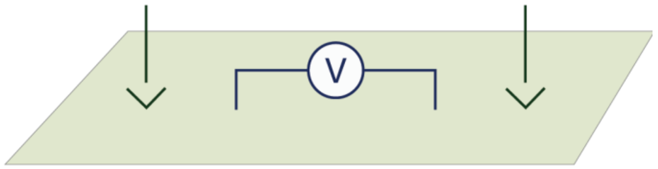
Resistivity



There are many models that can fit the data – non-uniqueness
CRITICAL to understand underlying physics

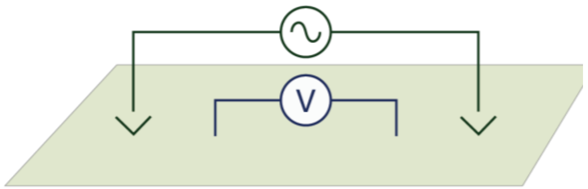
Resistivity, Controlled-source EM, and Magnetotellurics

Direct Current resistivity

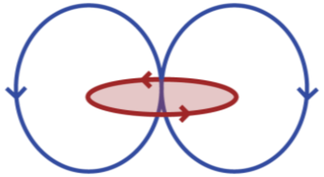


Controlled-source EM

Grounded source



Inductive source



Magnetotellurics

Depth range

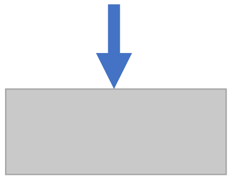
meters

hundreds of km

Electromagnetics: basic equations (quasi-static)

	Time	Frequency
Faraday's Law Ampere's Law No Magnetic Monopoles	$\nabla \times \vec{e} = -\frac{\partial \vec{b}}{\partial t}$ $\nabla \times \vec{h} = \vec{j} + \frac{\partial \vec{d}}{\partial t}$ $\nabla \cdot \vec{b} = 0$	$\nabla \times \vec{E} = -i\omega \vec{B}$ $\nabla \times \vec{H} = \vec{J} + i\omega \vec{D}$ $\nabla \cdot \vec{B} = 0$
Constitutive Relationships (non-dispersive)	$\vec{j} = \sigma \vec{e}$ $\vec{b} = \mu \vec{h}$ $\vec{d} = \varepsilon \vec{e}$	$\vec{J} = \sigma \vec{E}$ $\vec{B} = \mu \vec{H}$ $\vec{D} = \varepsilon \vec{E}$

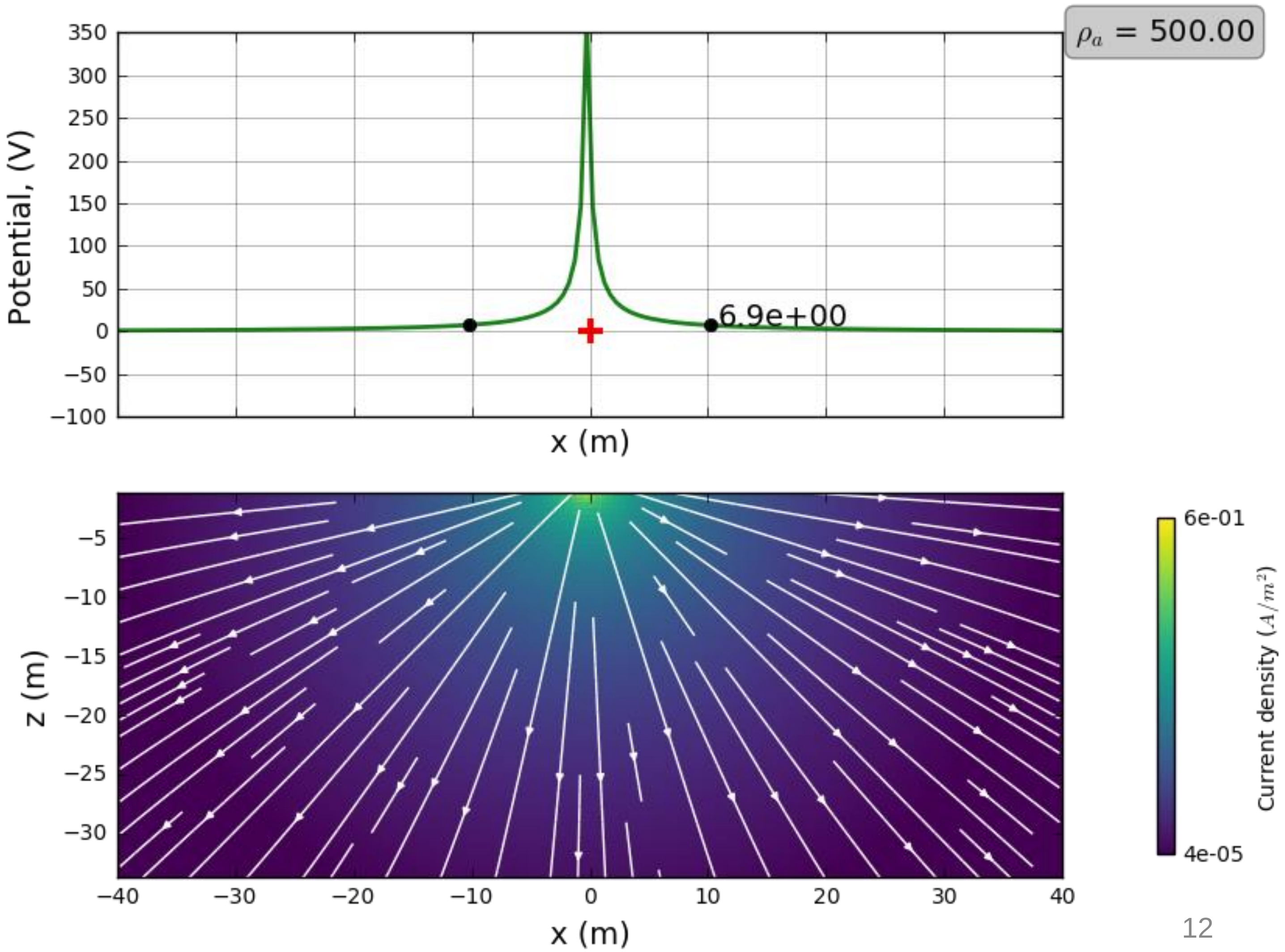
Steady-state Maxwell's equations – DC resistivity method

	Full	Steady State
Faraday	$\nabla \times \vec{e} = -\frac{\partial \vec{b}}{\partial t}$	$\nabla \times \vec{e} = 0 \qquad \vec{e} = -\nabla V$
Ampere	$\nabla \times \vec{h} = \vec{j} + \frac{\partial \vec{d}}{\partial t} + \vec{j}_s$	$\nabla \cdot \vec{j} = -\nabla \cdot \vec{j}_s$
Ohm's Law	$\vec{j} = \sigma \vec{e}$	
Put it together		$\nabla \cdot \sigma \nabla V = I\delta(r)$
Potential in a homogeneous halfspace		$V = \frac{I}{2\pi\sigma}\frac{1}{r} \qquad V = \frac{\rho I}{2\pi r}$

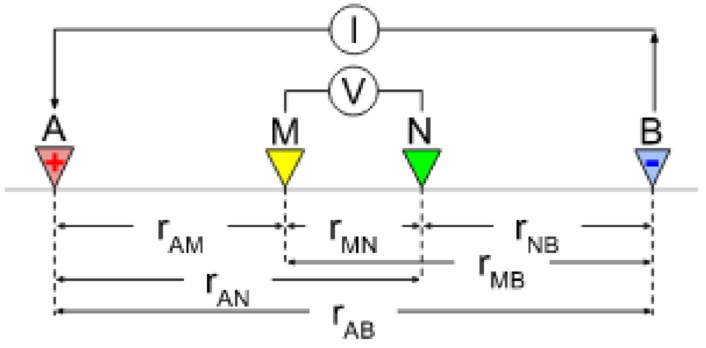
Currents and potentials: halfspace

$$V = \frac{\rho I}{2\pi r}$$

$$\rho = \frac{2\pi r V}{I}$$



Currents and potentials: 4-electrode array



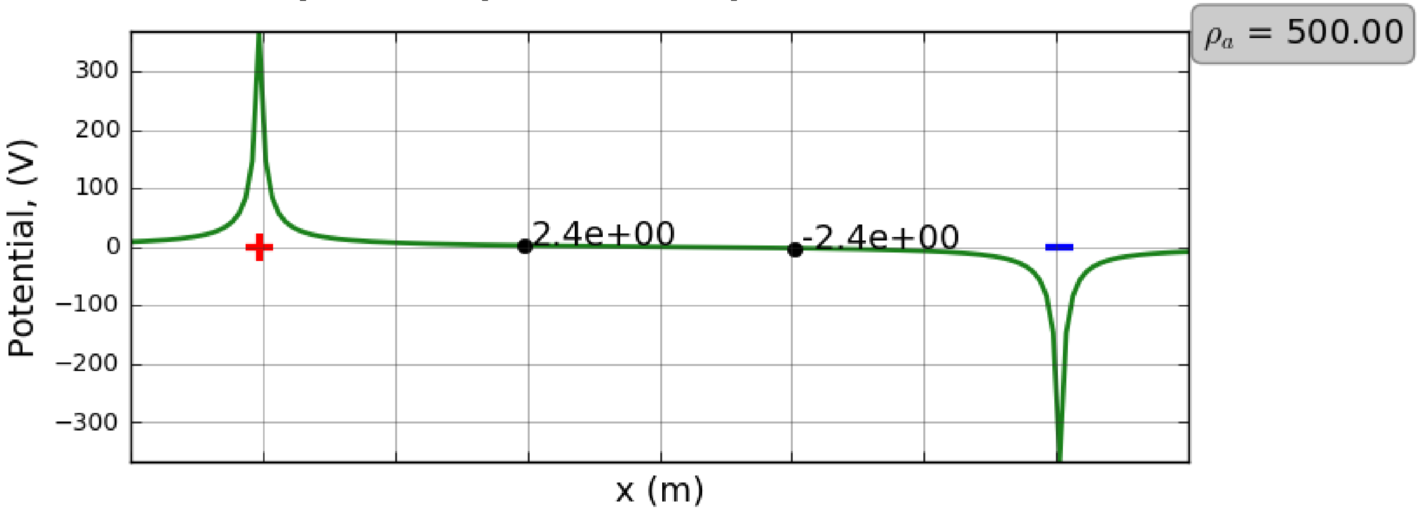
$$\Delta V_{MN} = \rho I \underbrace{\frac{1}{2\pi}\left[\frac{1}{AM} - \frac{1}{MB} - \frac{1}{AN} + \frac{1}{NB}\right]}_{G}$$

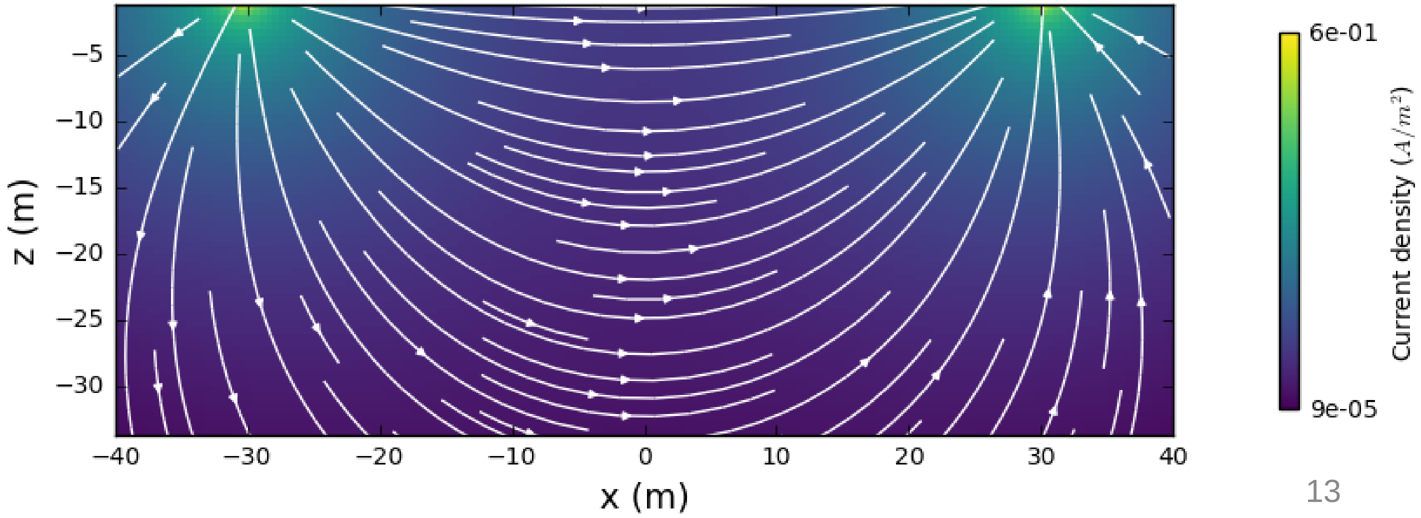
Resistivity

$$\rho = \frac{\Delta V_{MN}}{IG}$$

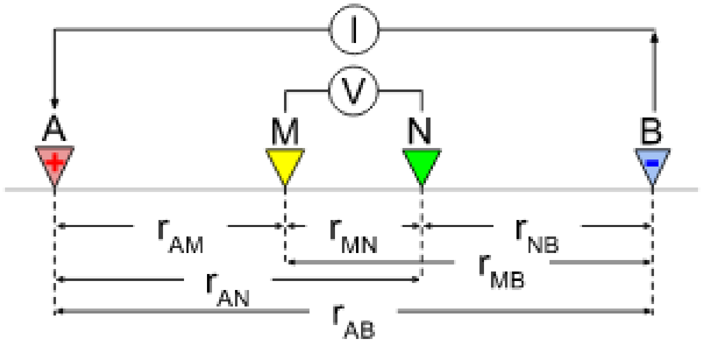
G: geometric factor

Halfspace (500 Ωm)





Currents and Apparent resistivity



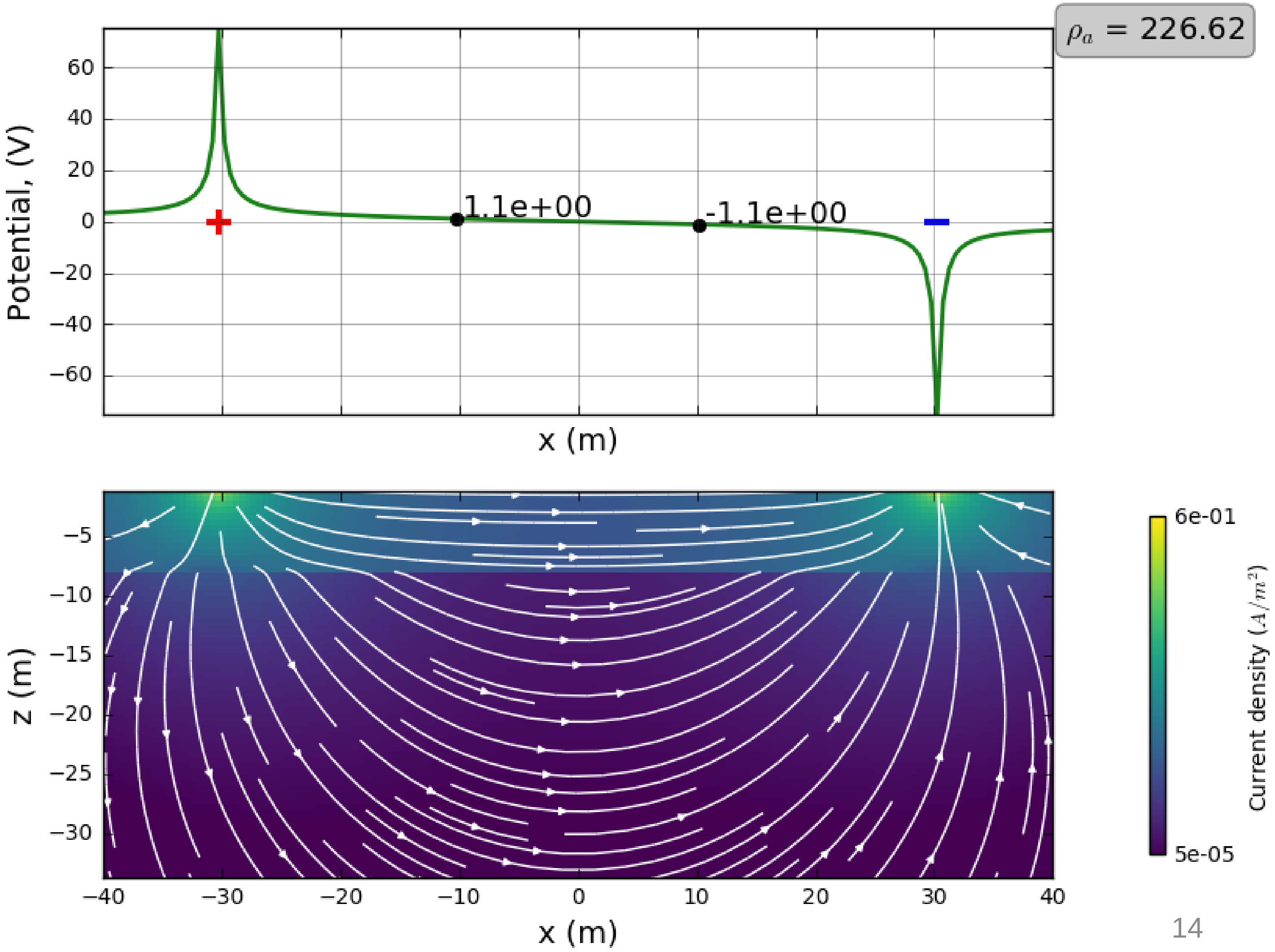
$$\Delta V_{MN} = \rho I \underbrace{\frac{1}{2\pi}\left[\frac{1}{AM} - \frac{1}{MB} - \frac{1}{AN} + \frac{1}{NB}\right]}_{G}$$

Apparent resistivity

$$\rho_a = \frac{\Delta V_{MN}}{IG}$$

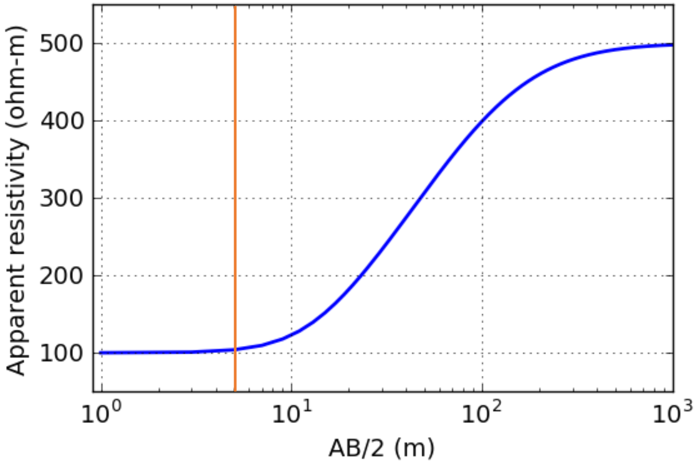
G: geometric factor

Conductive overburden (100 Ωm)

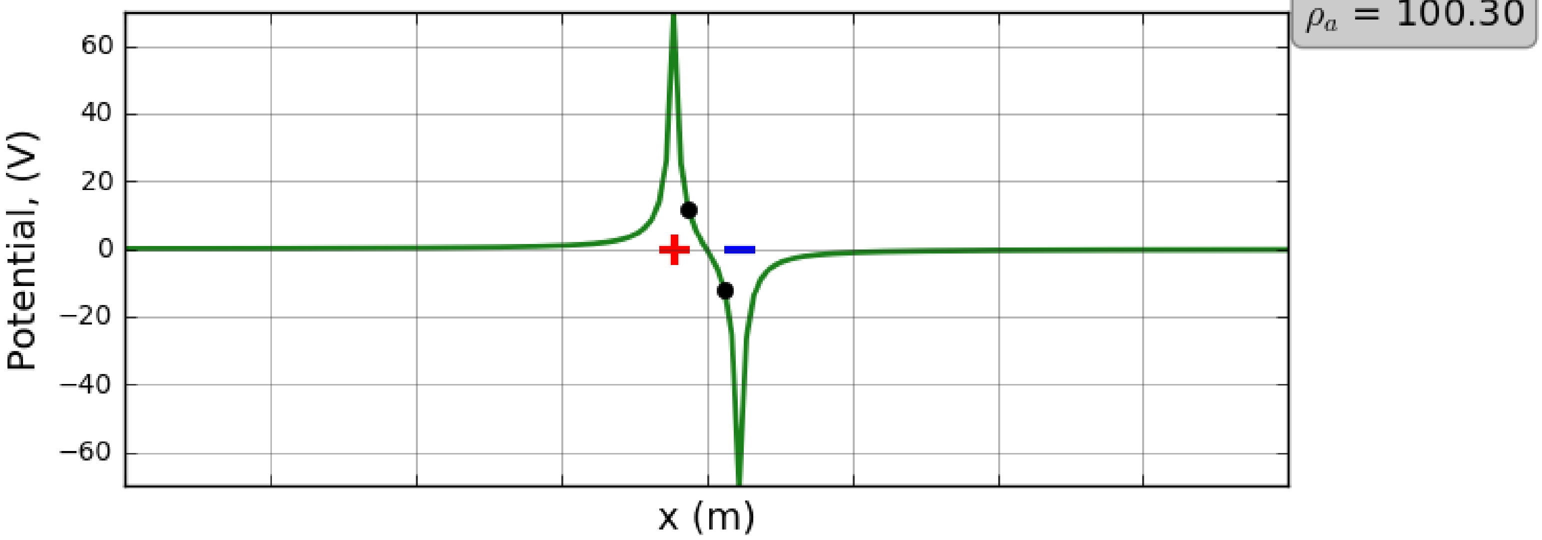


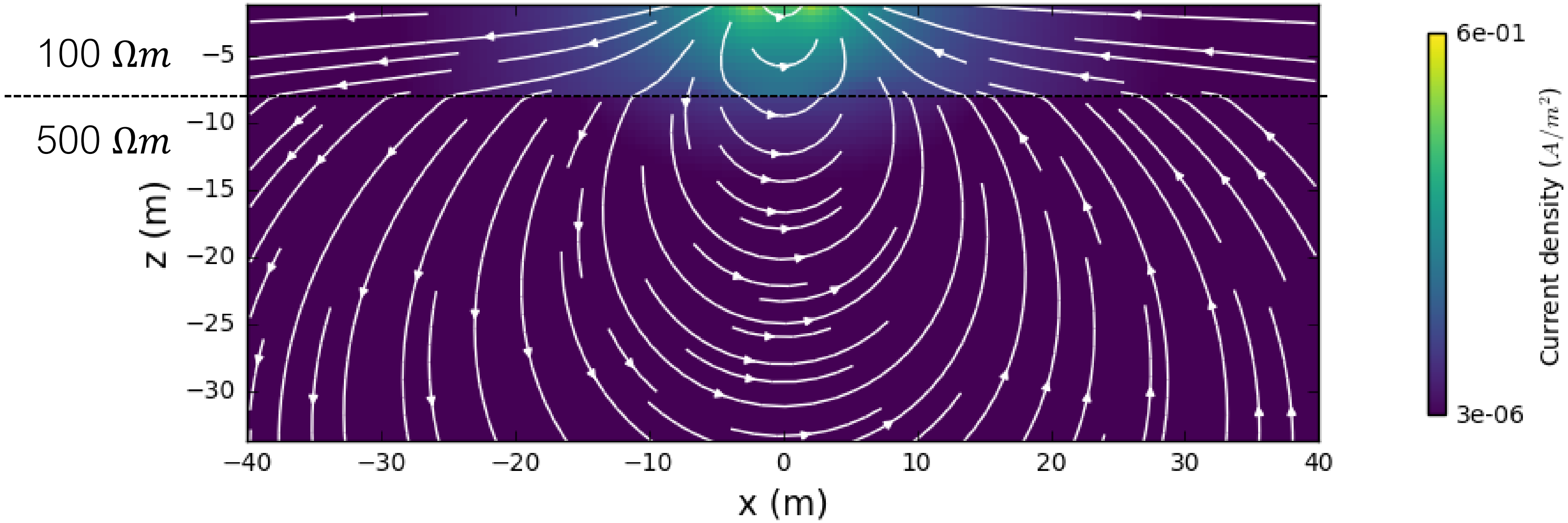
Geometric sounding

DC Sounding curve

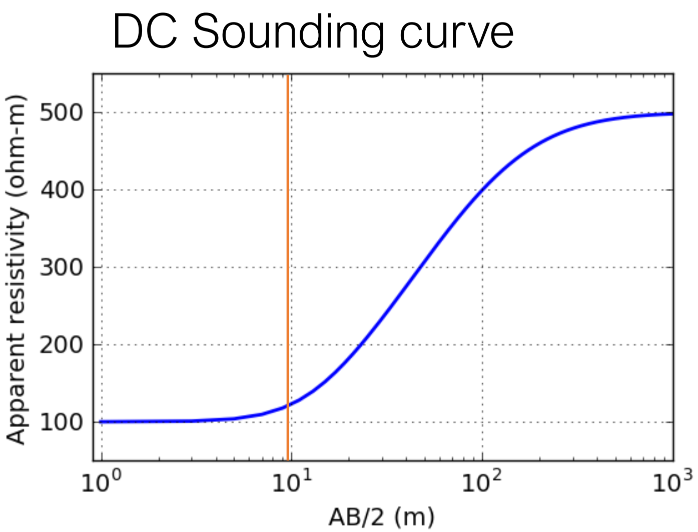


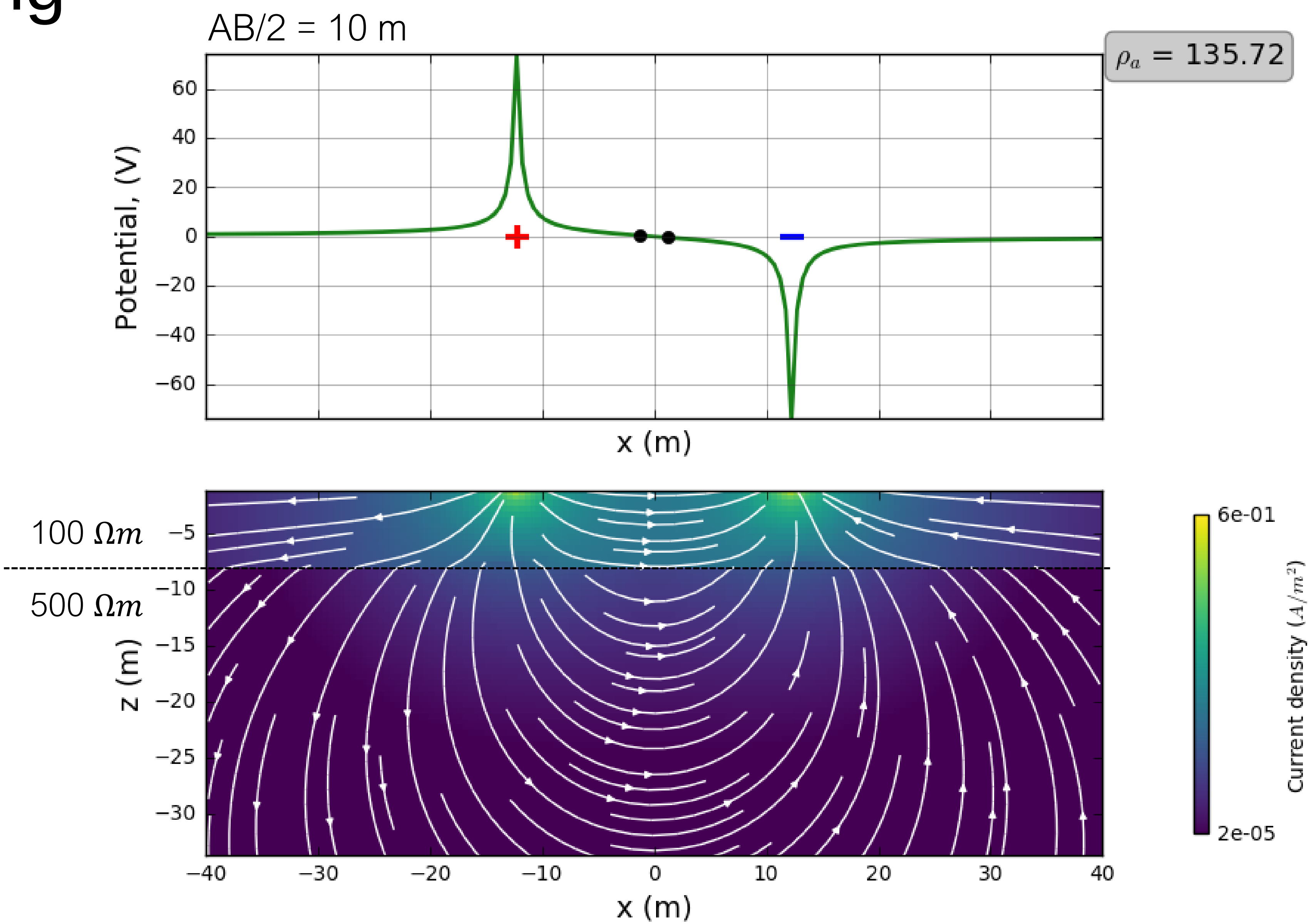
AB/2 = 5 m



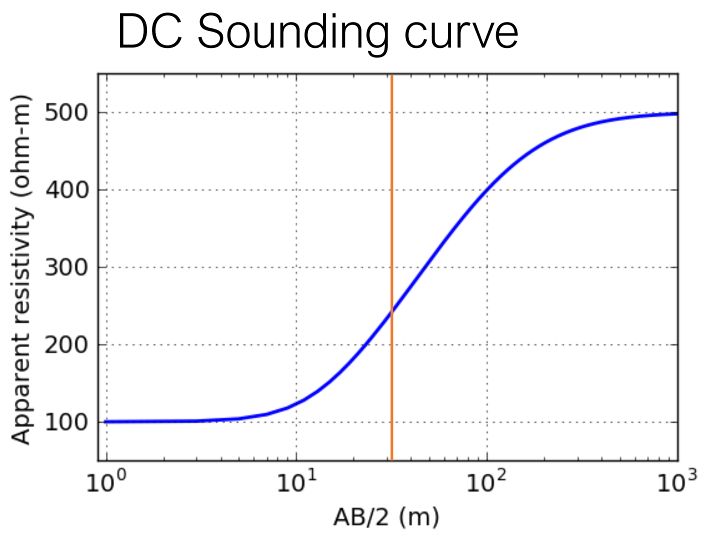


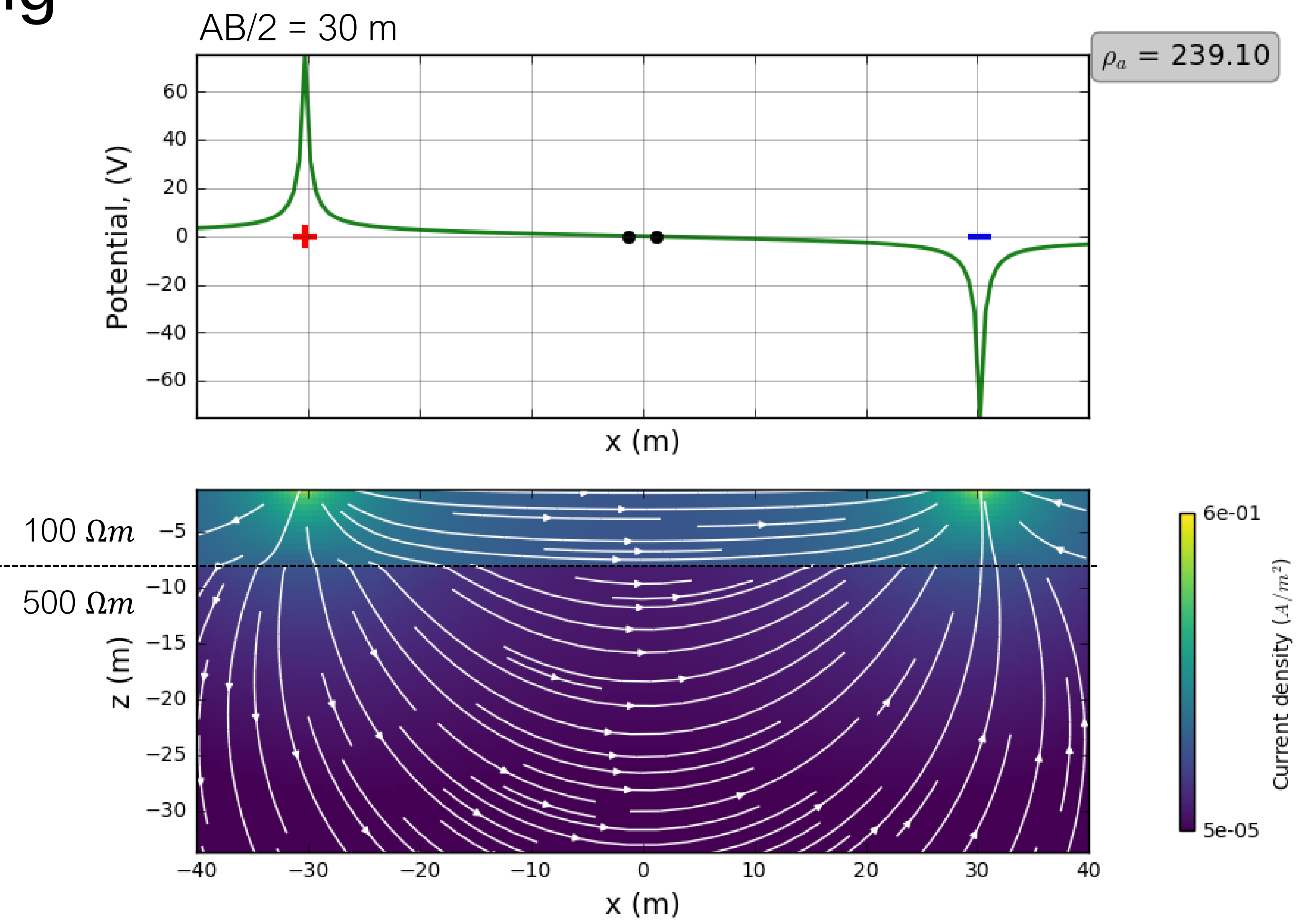
Geometric sounding





Geometric sounding





Concept of charges

Normal component of current density is continuous

$$J_{1n} = J_{2n}$$
$$\sigma_1 E_{1n} = \sigma_2 E_{2n}$$

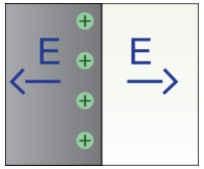
Conductivity contrast

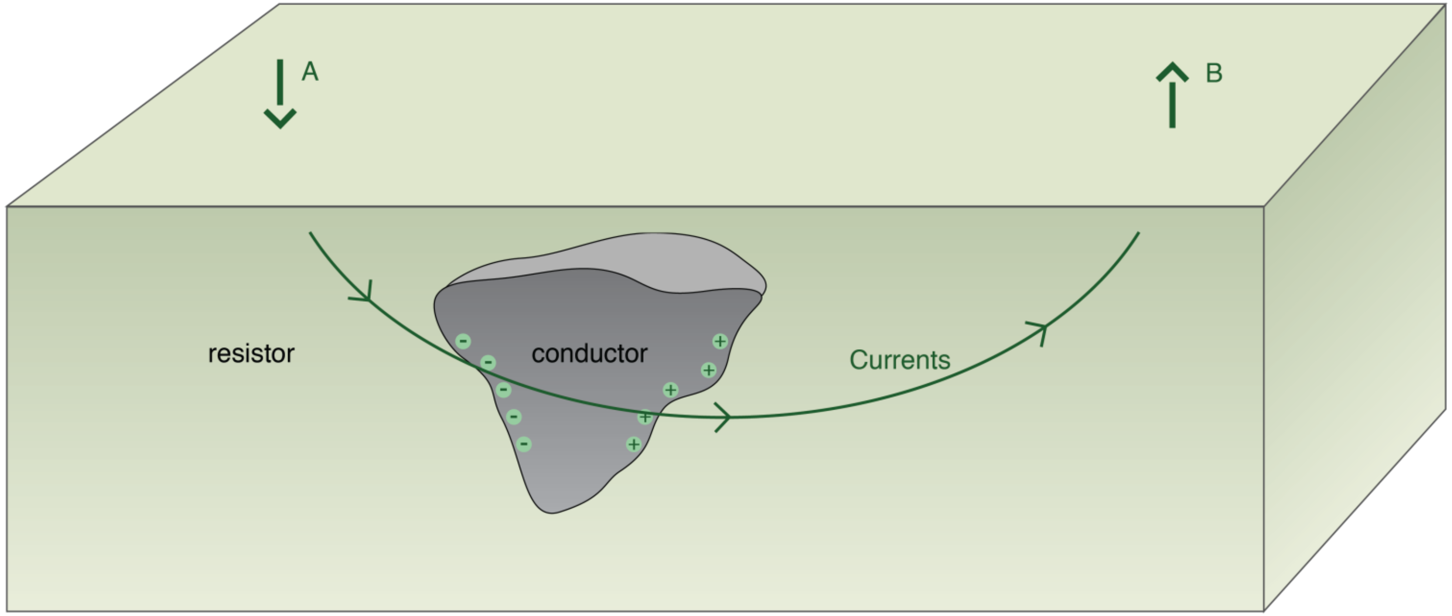
$$\sigma_1 \neq \sigma_2$$

Electric field discontinuous

Charge build-up

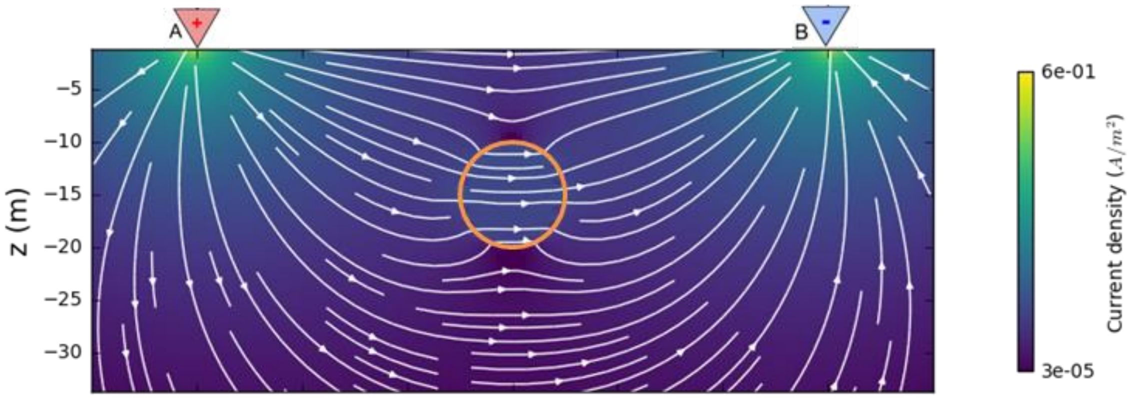
$$\mathbf{E} = \frac{Q}{4\pi\varepsilon_0|\mathbf{r}-\mathbf{r}'|^2}\hat{\mathbf{r}}$$



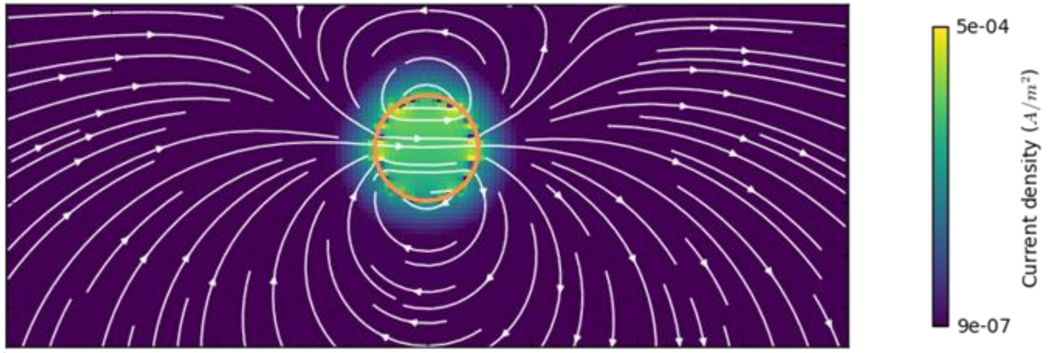


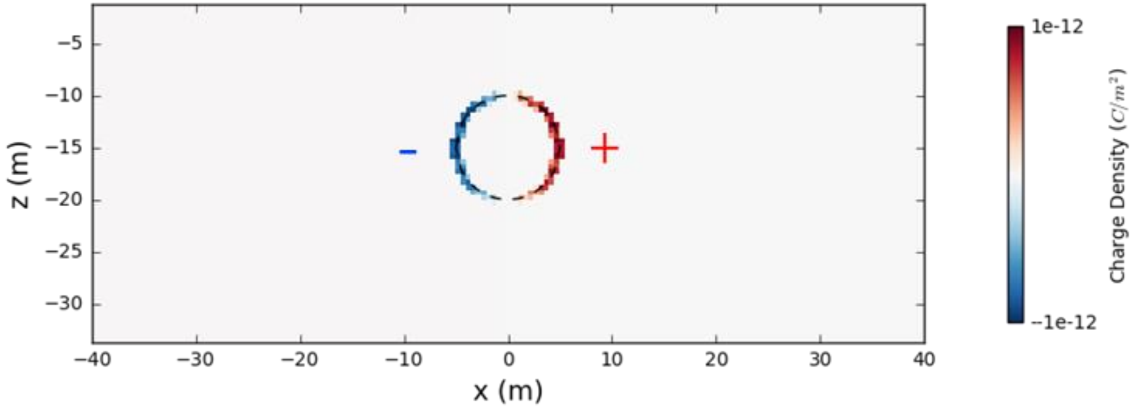
Currents, charges, and potentials

Total currents: J

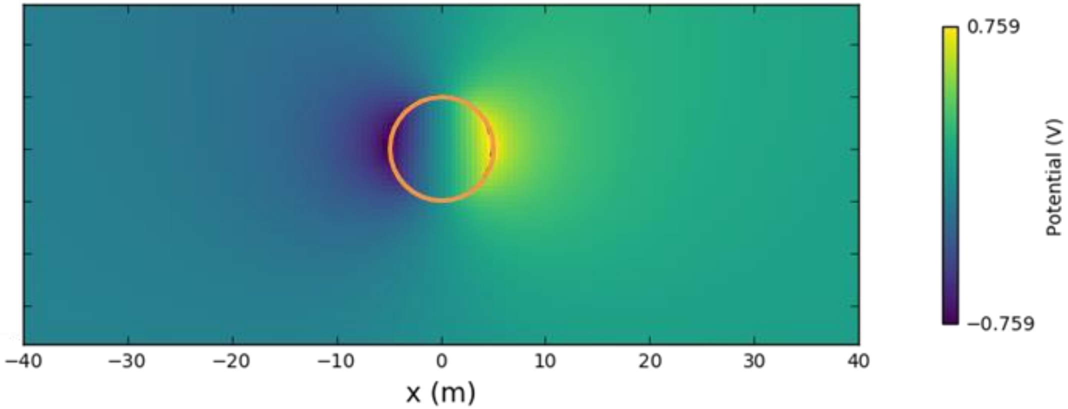


Secondary currents: J_s



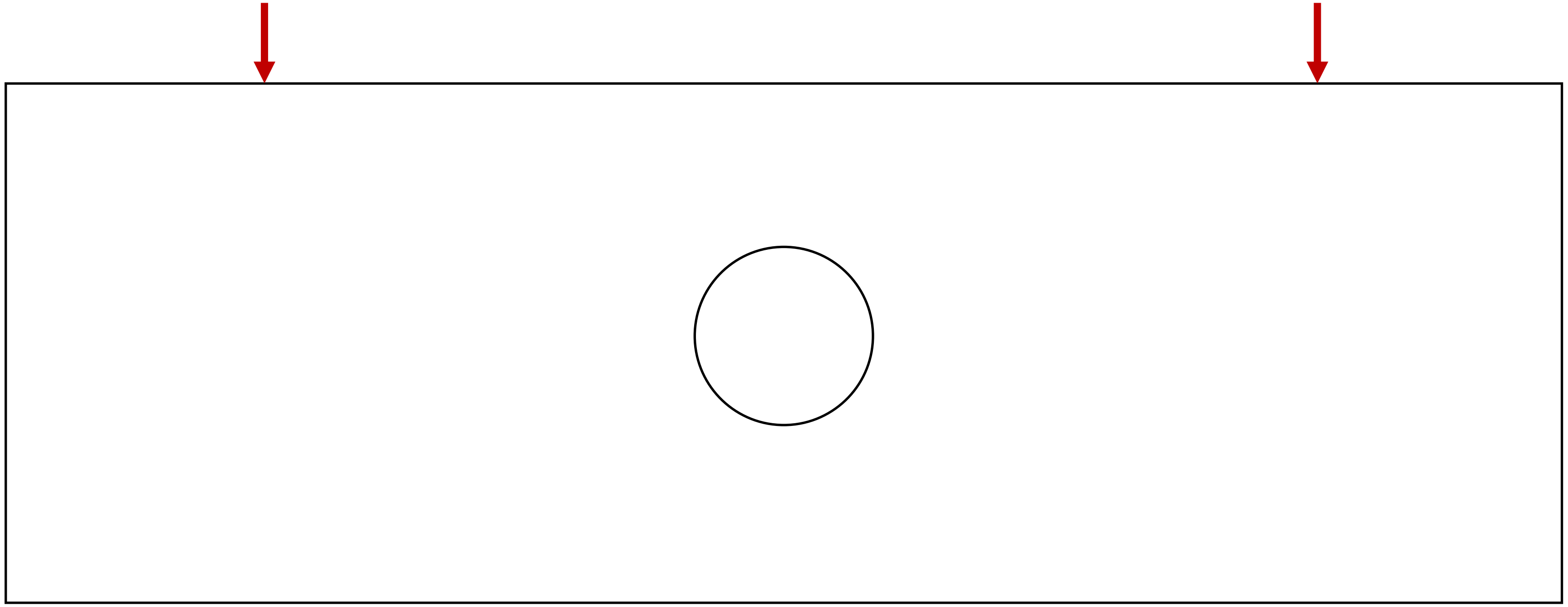


Secondary charges: Q_s



Secondary potential: ϕ_s

What would happen if there was a resistor



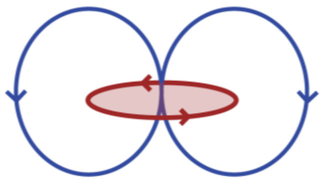
DC resistivity → EM; EM fields are varying with time or frequency

	Time	Frequency
Faraday's Law Ampere's Law No Magnetic Monopoles	$\nabla \times \vec{e} = -\frac{\partial \vec{b}}{\partial t}$ $\nabla \times \vec{h} = \vec{j} + \frac{\partial \vec{d}}{\partial t}$ $\nabla \cdot \vec{b} = 0$	$\nabla \times \vec{E} = -i\omega \vec{B}$ $\nabla \times \vec{H} = \vec{J} + i\omega \vec{D}$ $\nabla \cdot \vec{B} = 0$
Constitutive Relationships (non-dispersive)	$\vec{j} = \sigma \vec{e}$ $\vec{b} = \mu \vec{h}$ $\vec{d} = \varepsilon \vec{e}$	$\vec{J} = \sigma \vec{E}$ $\vec{B} = \mu \vec{H}$ $\vec{D} = \varepsilon \vec{E}$

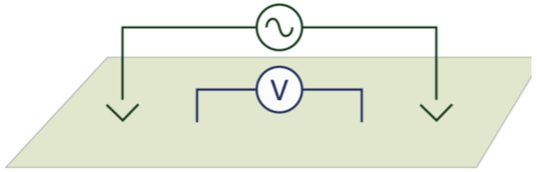
Controlled-source EM methods

Types of EM source

Inductive source

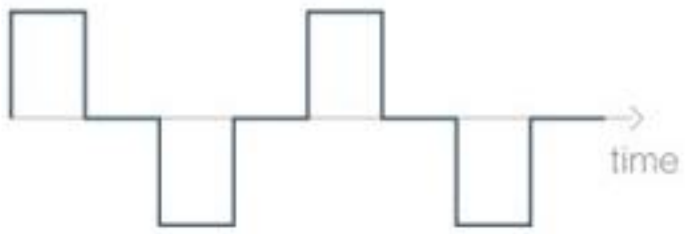


Grounded source

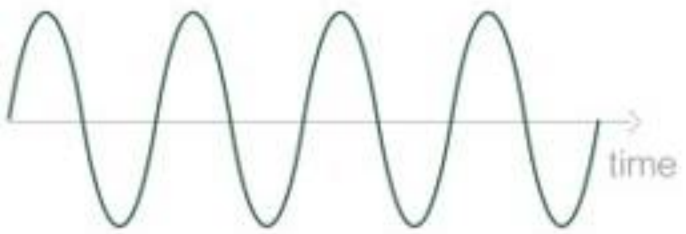


Types of current waveform

Time-domain; Transient

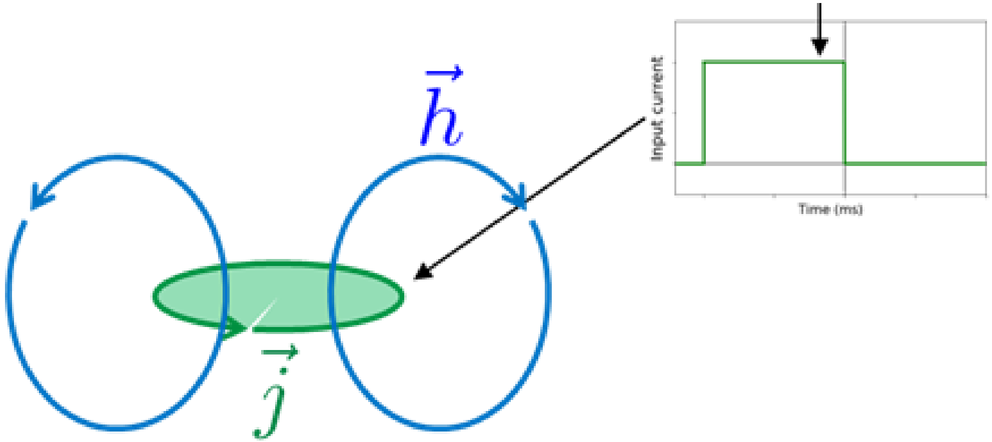


Frequency-domain; Harmonic



EM induction

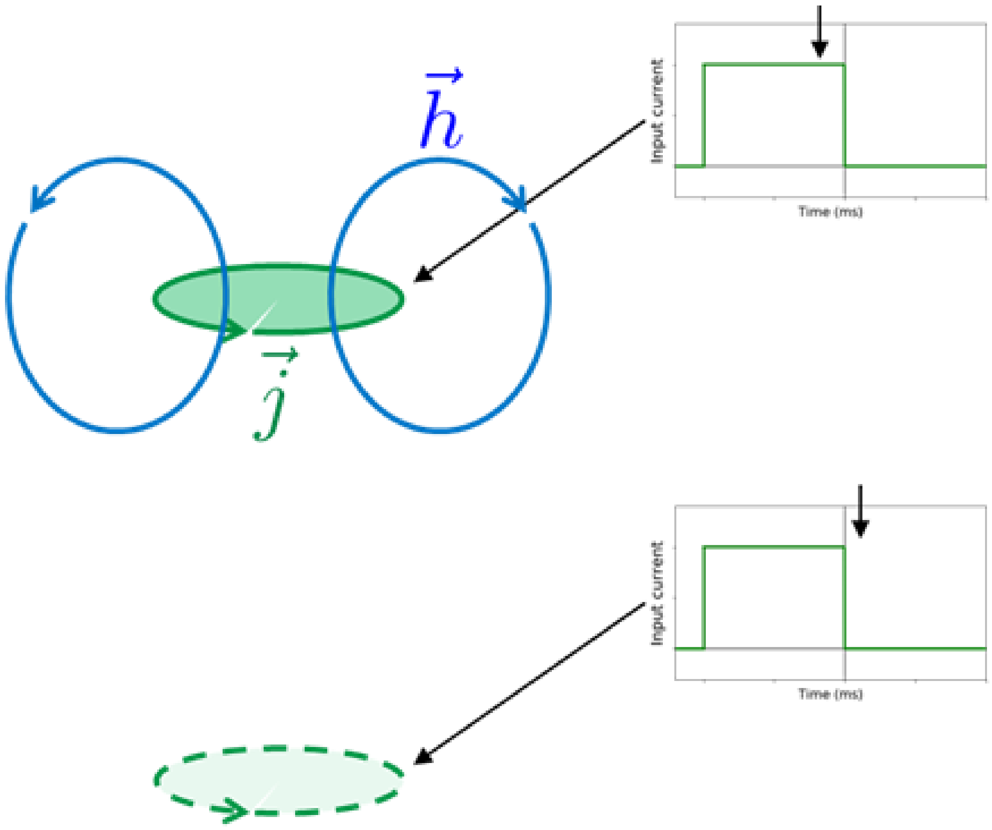
Ampere's Law: $\vec{\nabla} \times \vec{h} = \vec{j}$



EM induction

Ampere's Law: $\vec{\nabla} \times \vec{h} = \vec{j}$

Faraday's Law: $\vec{\nabla} \times \vec{e} = -\mu_0 \dfrac{\partial \vec{h}}{\partial t}$

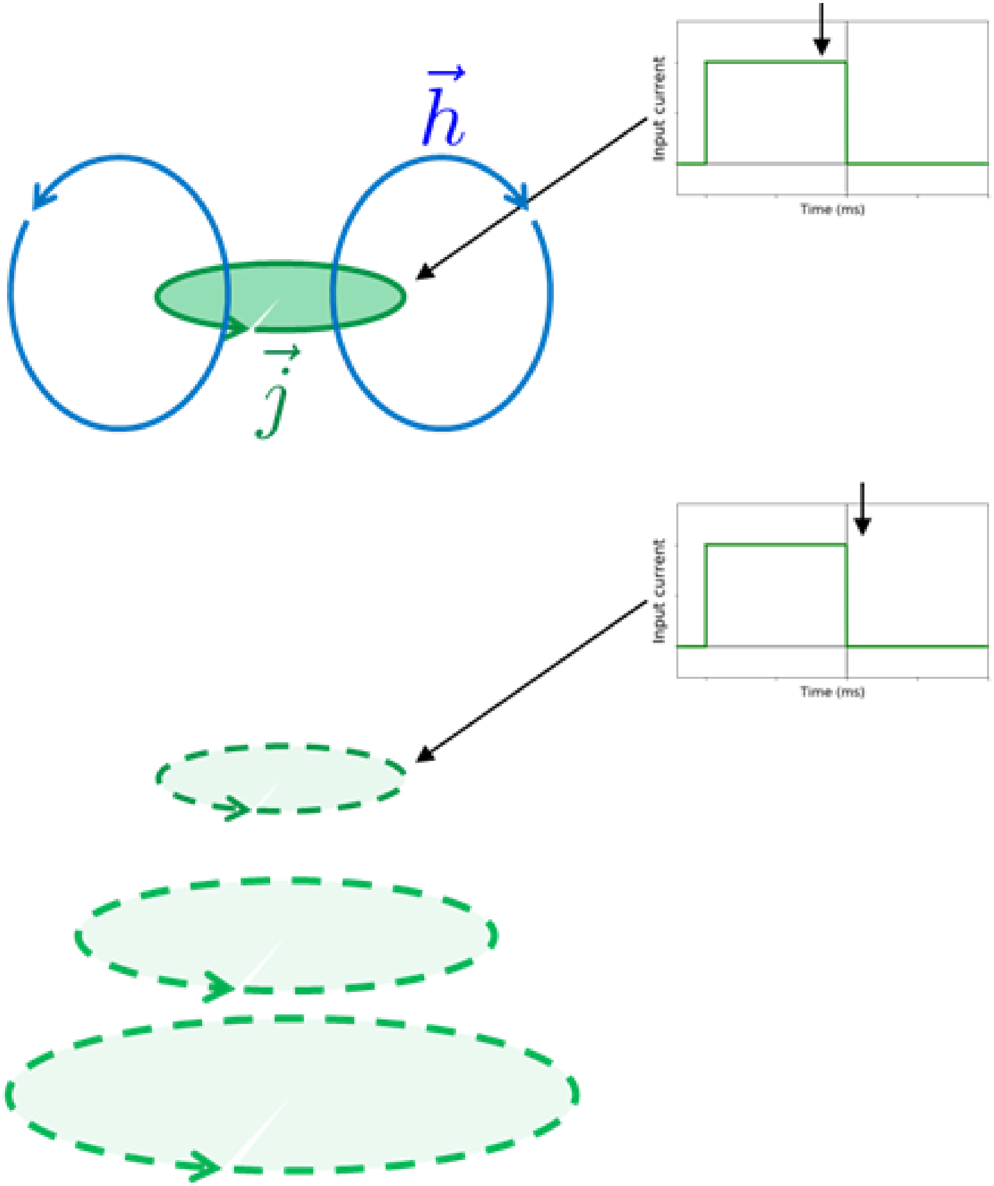


EM induction

Ampere's Law: $\vec{\nabla} \times \vec{h} = \vec{j}$

Faraday's Law: $\vec{\nabla} \times \vec{e} = -\mu_0 \dfrac{\partial \vec{h}}{\partial t}$

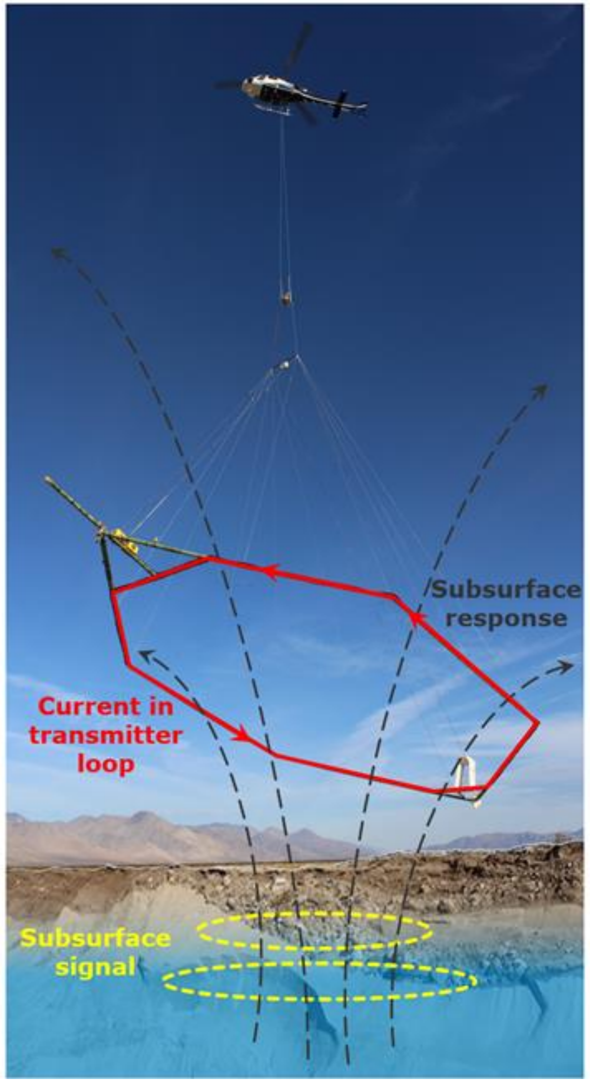
Ohm's Law: $\vec{j} = \sigma \vec{e}$



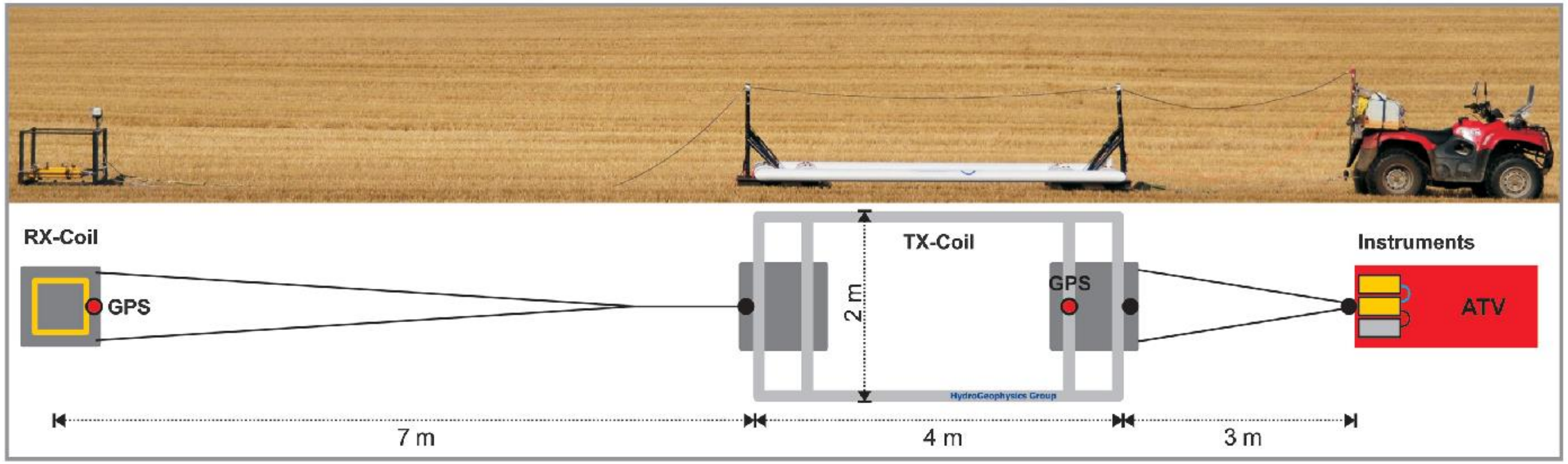
Currents diffuse away (EM induction)

EM induction methods (no need to “contact”)

Airborne EM



Towed time-domain EM

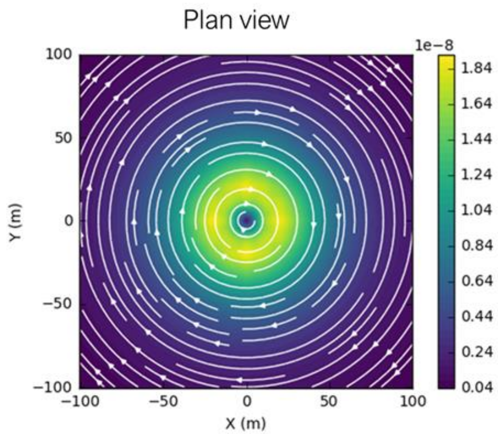


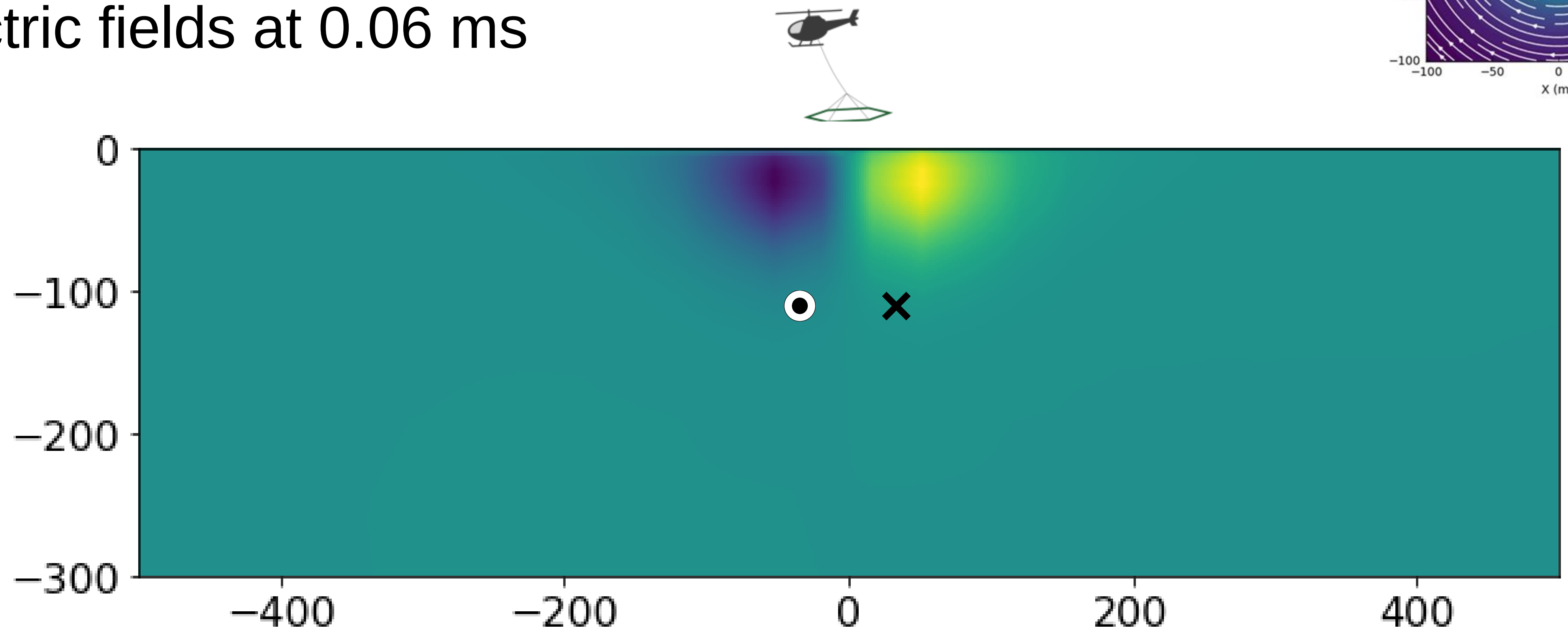
EM-31

DualEM

Diffusive EM wave

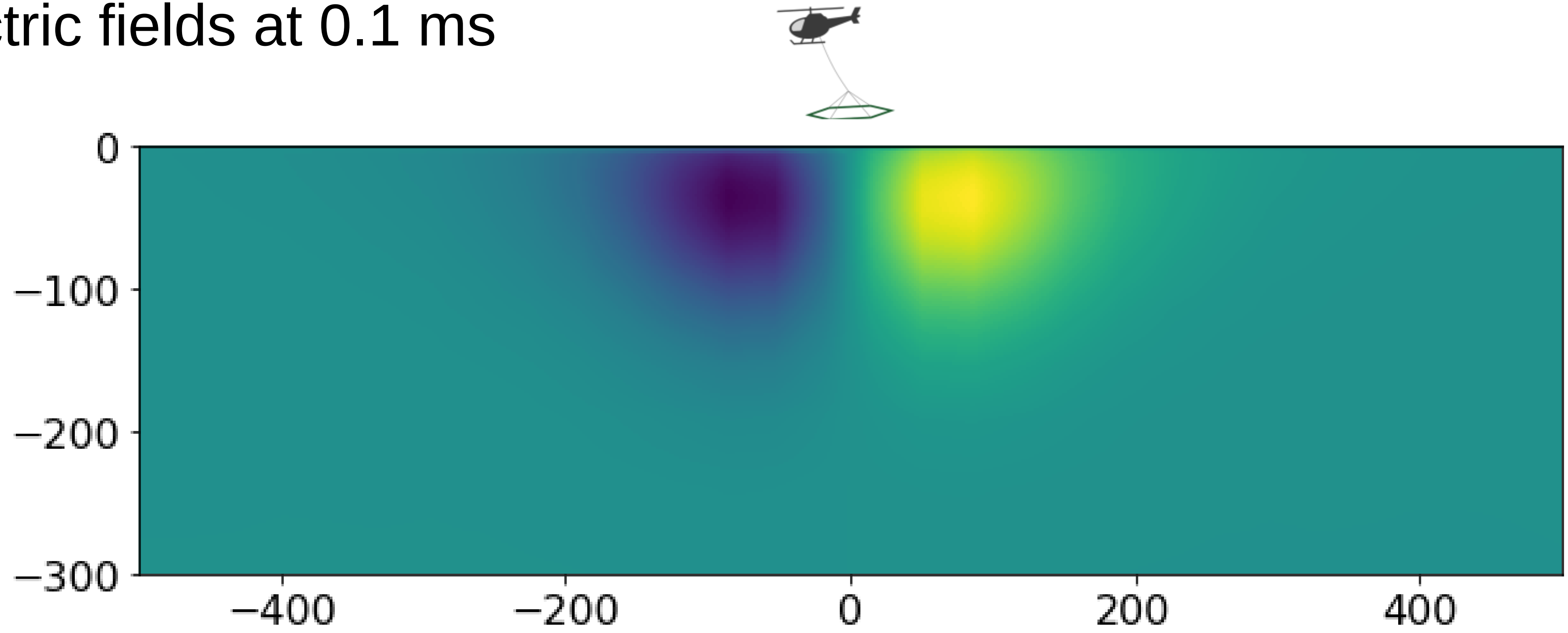
Electric fields at 0.06 ms





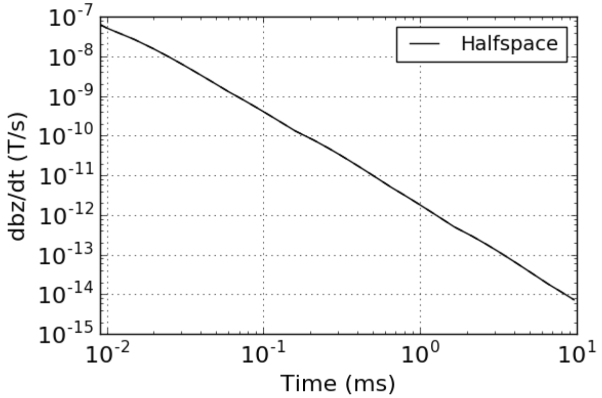
Diffusive EM wave

Electric fields at 0.1 ms

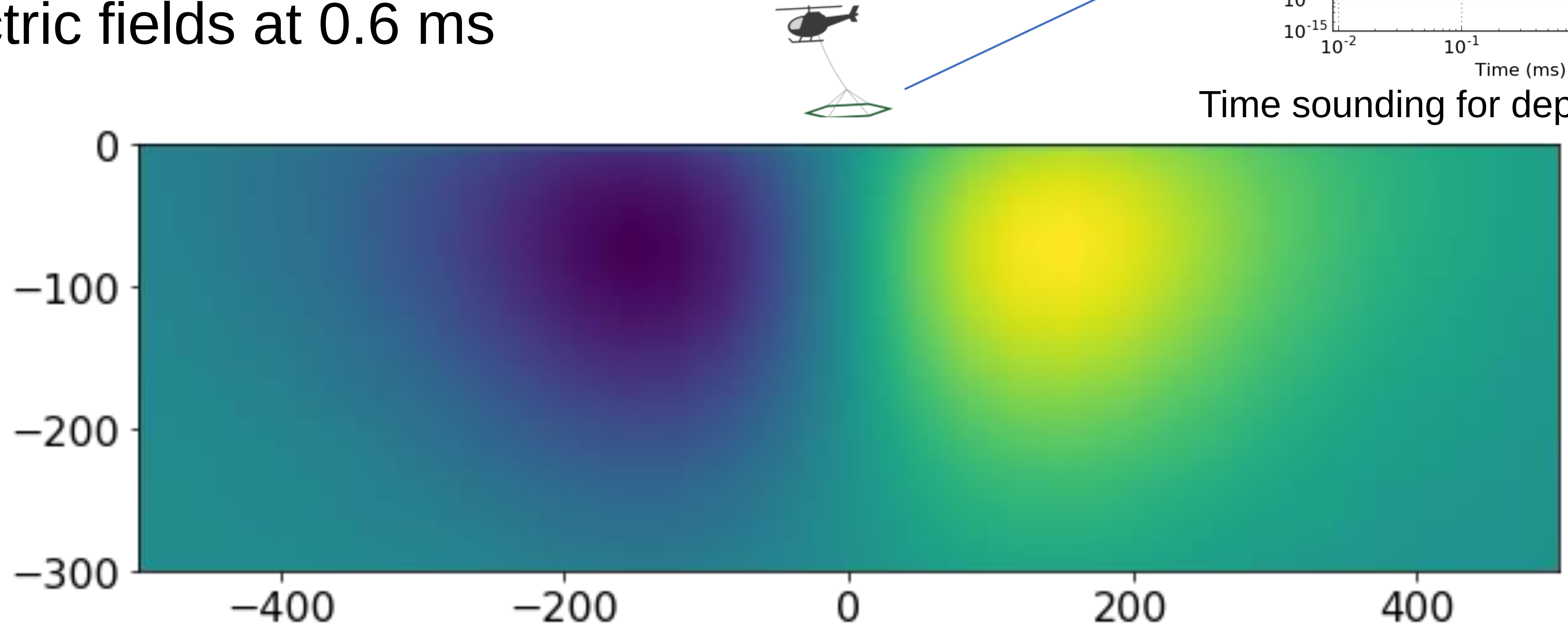


Diffusive EM wave

Electric fields at 0.6 ms

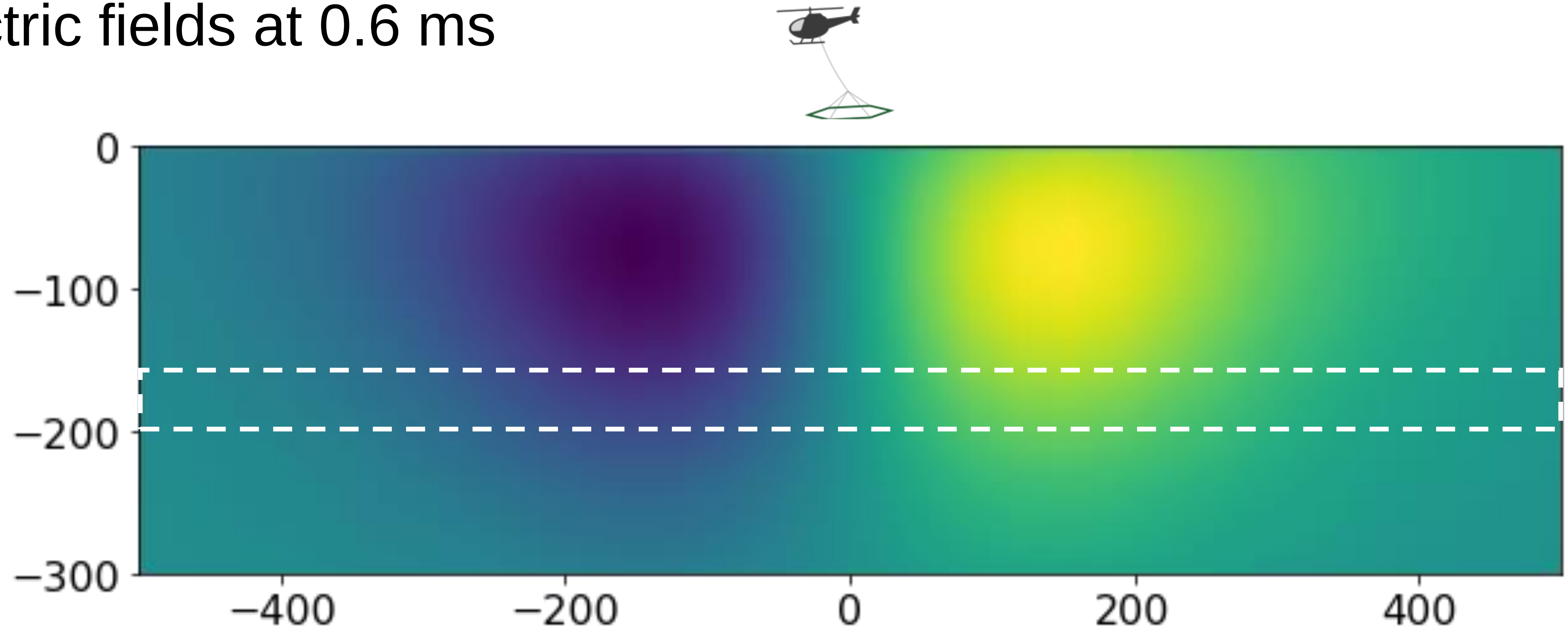


Time sounding for depth resolution

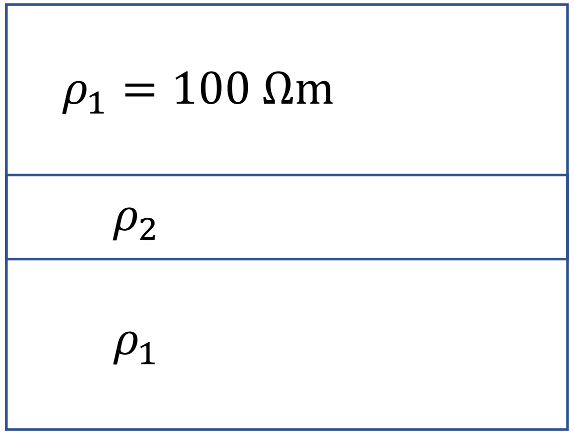


Electrical fields are parallel to the layered structure

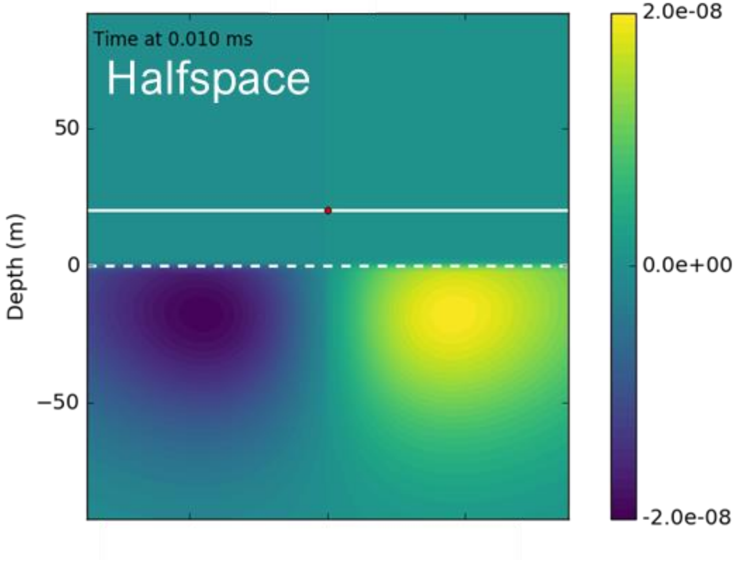
Electric fields at 0.6 ms



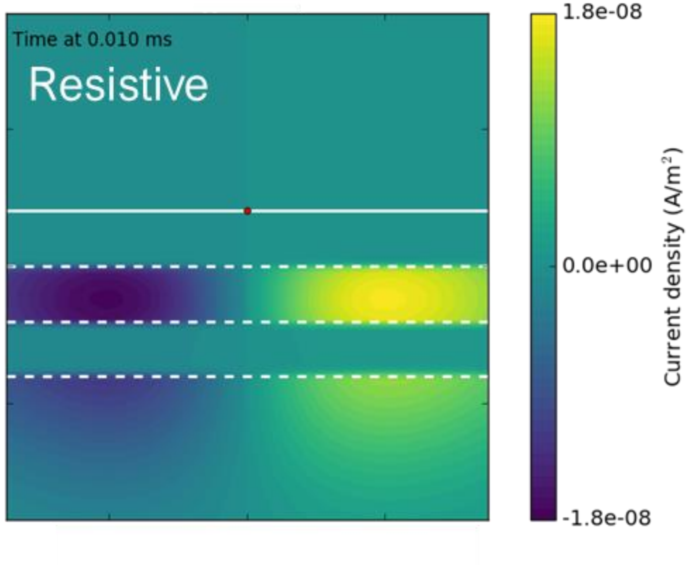
Layered structure



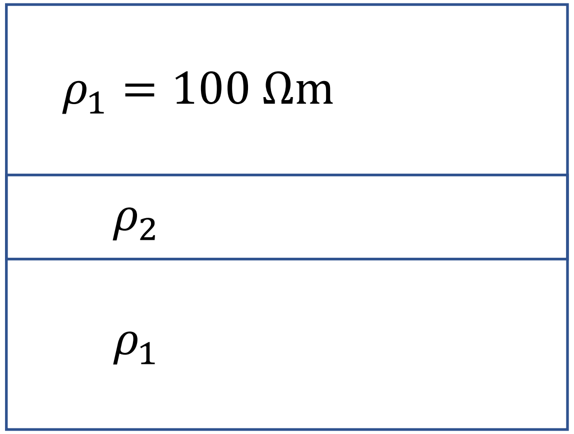
$\rho_2 = 100\ \Omega\text{m}$



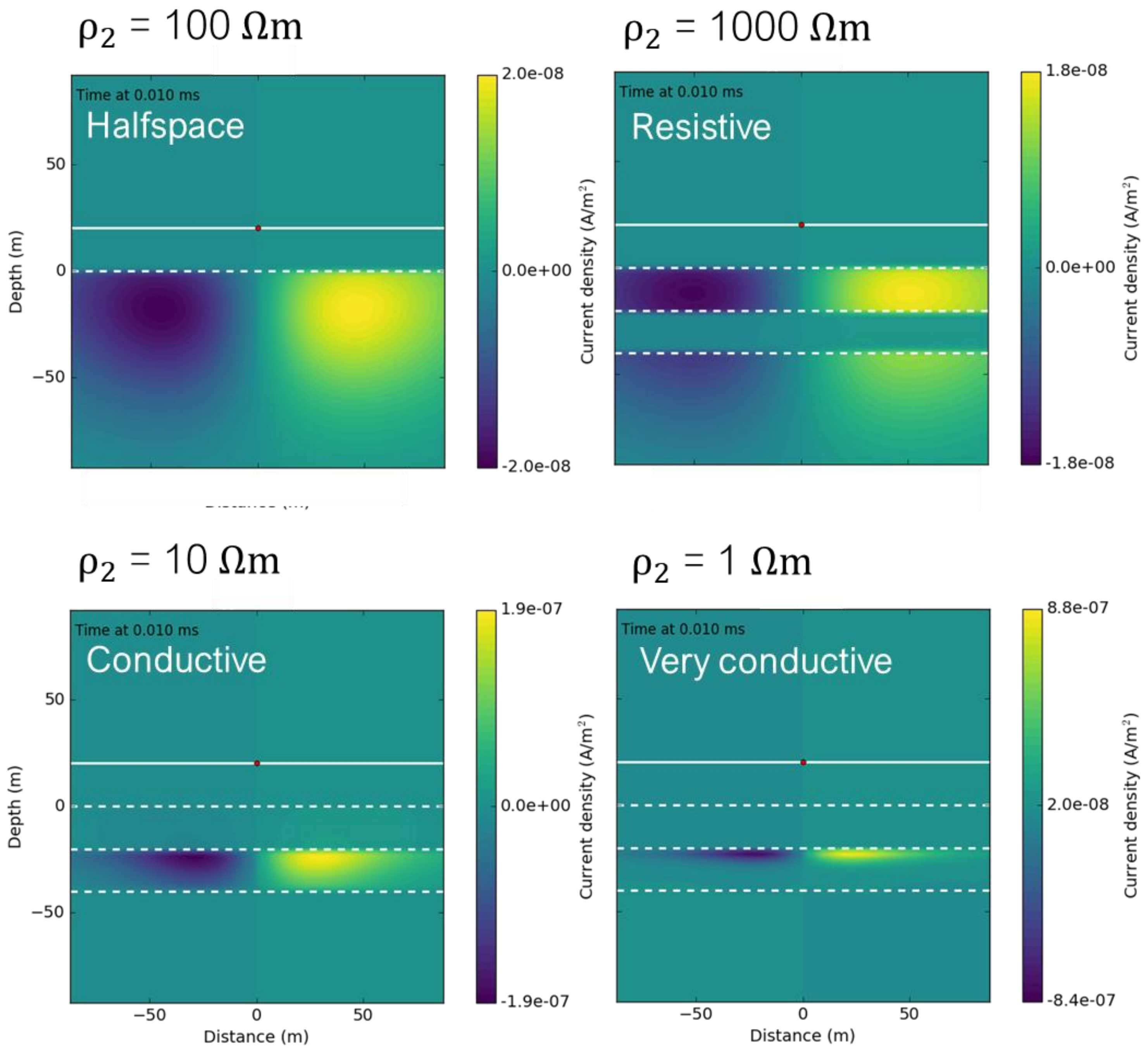
$\rho_2 = 1000\ \Omega\text{m}$



Layered structure

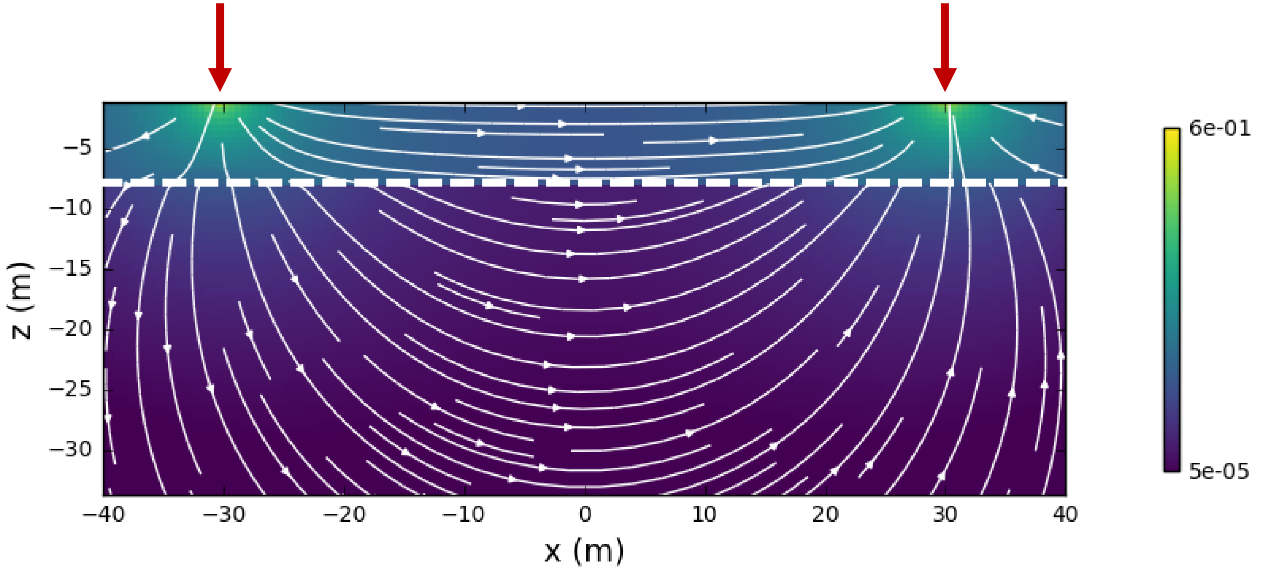


Biased sensitivity towards a conductor

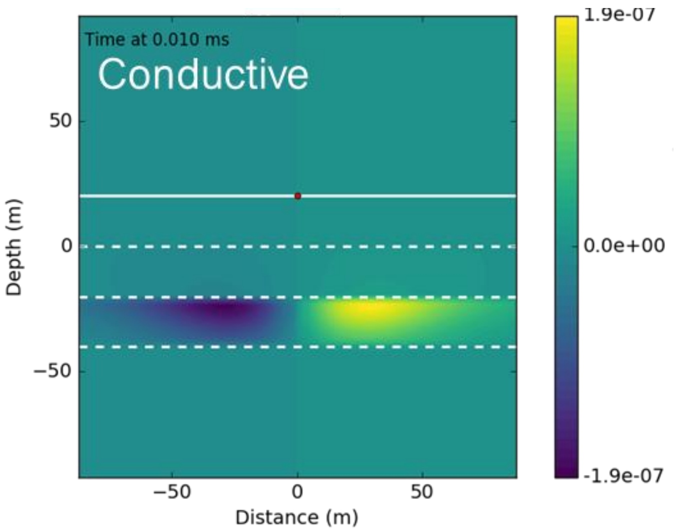


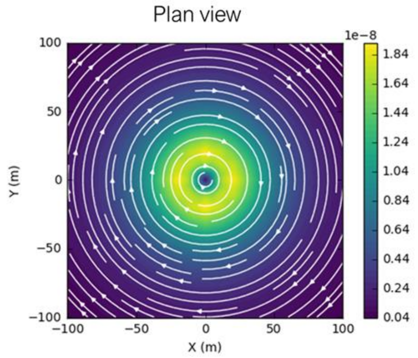
DC resistivity vs. Inductive source EM

E-fields crosses the layer boundary
(charge build-up; galvanic currents; current channeling)



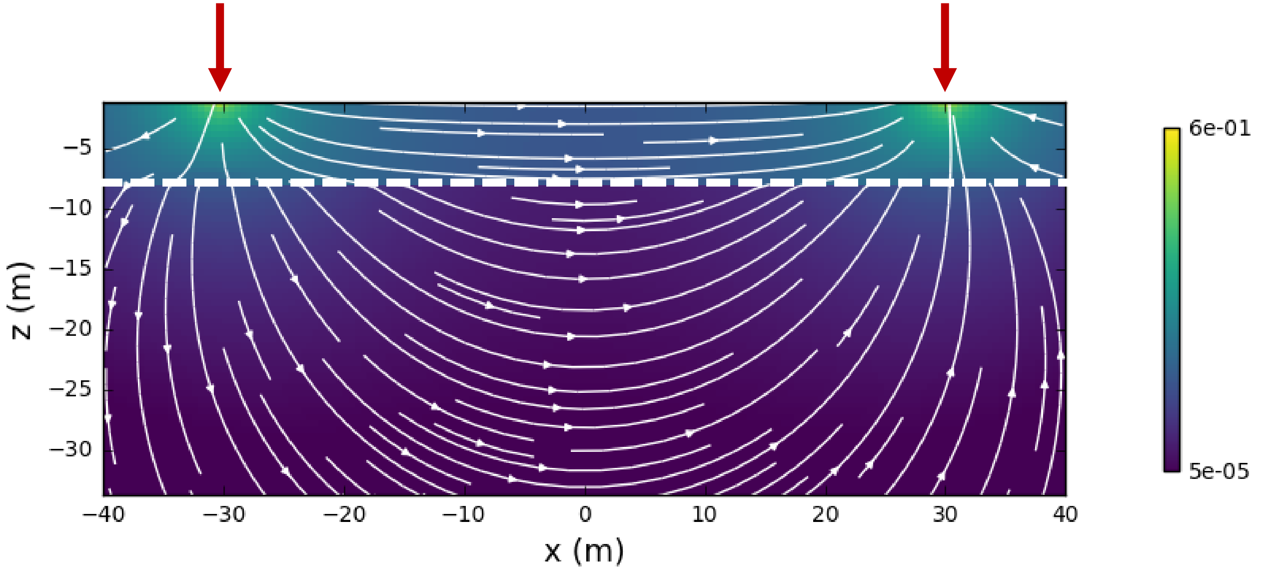
E-fields parallel to the layer boundary
(EM induction)



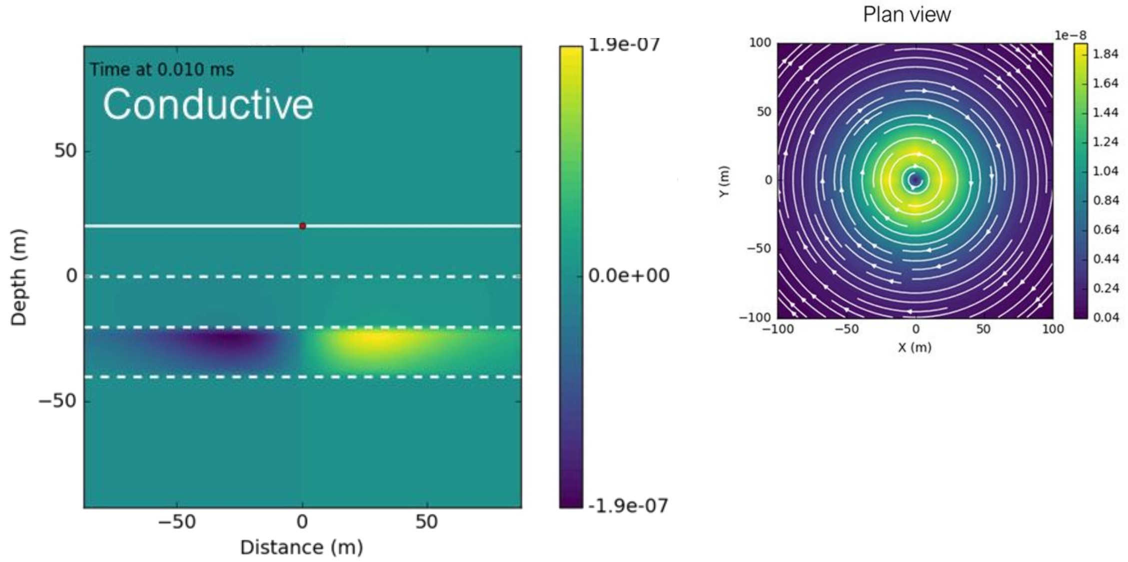


DC resistivity vs. Inductive source EM

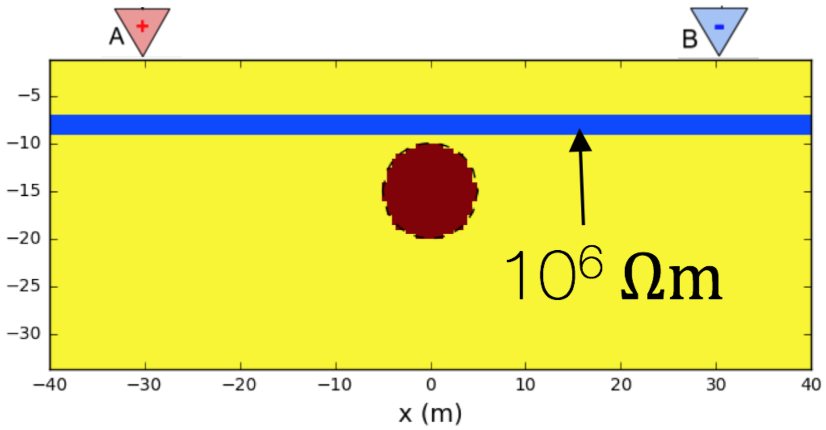
E-fields crosses the layer boundary
(charge build-up; galvanic currents; current channeling)



E-fields parallel to the layer boundary
(EM induction)

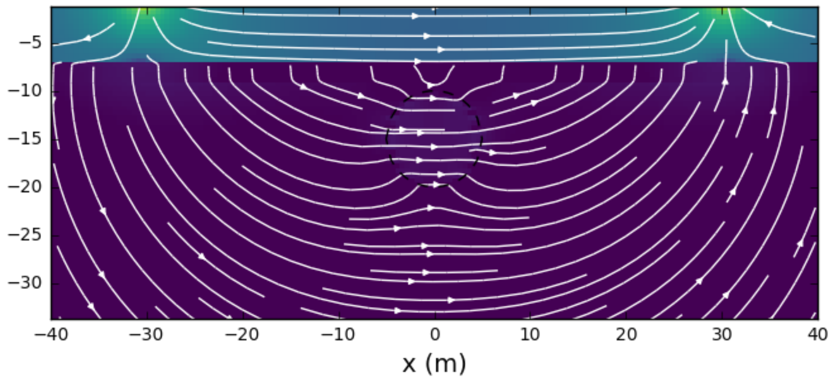


Shielding problem



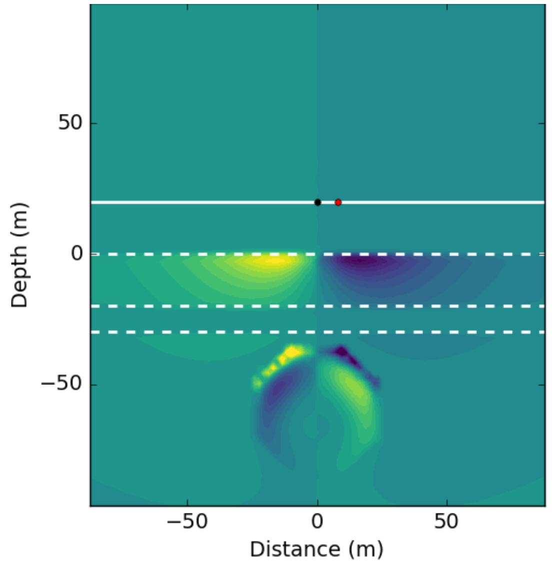
DC resistivity

Hard to see below the insulator



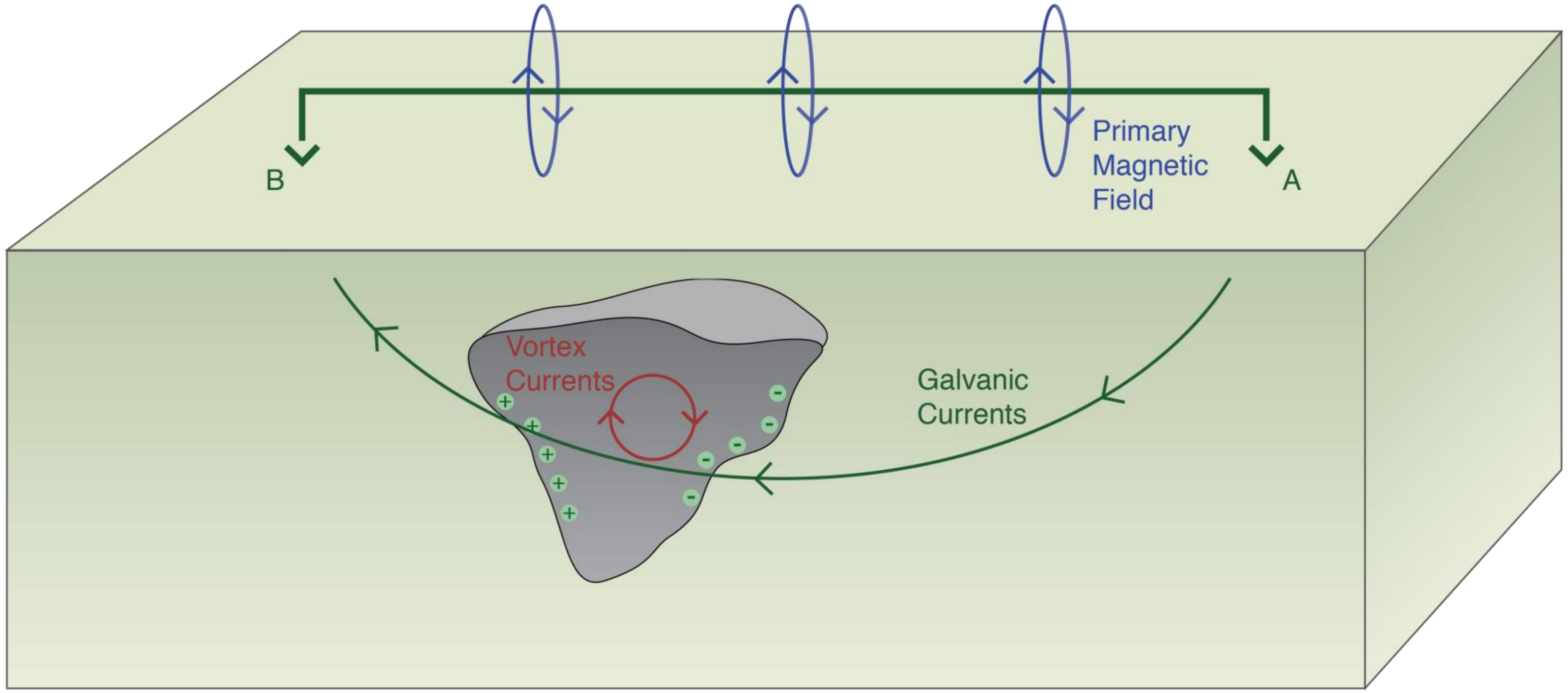
Inductive source EM

Easy to see below the insulator



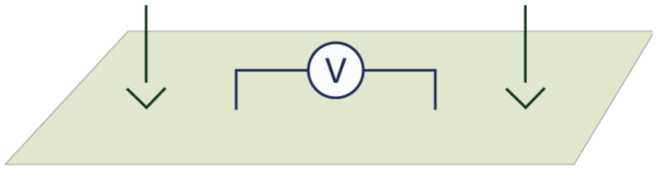
Grounded source EM

Current channeling + EM induction



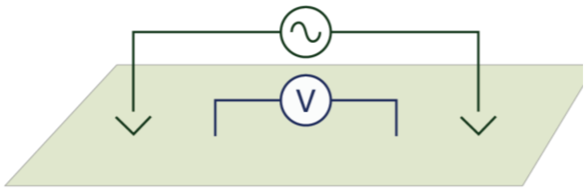
For deeper imaging (a few km to 100s of km)

Direct Current resistivity

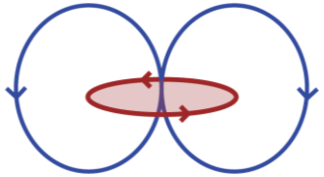


Controlled-source EM

Grounded source



Inductive source



Magnetotellurics

Depth range

meters

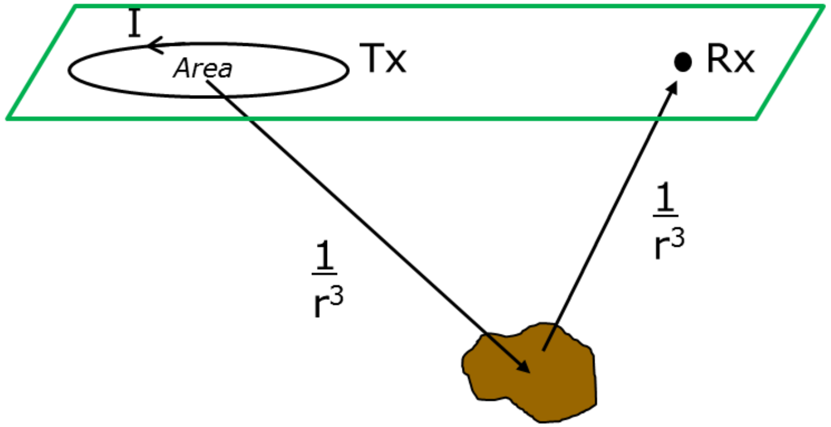
hundreds of km

What is required to see deeper?

Penetration depth depends upon system power

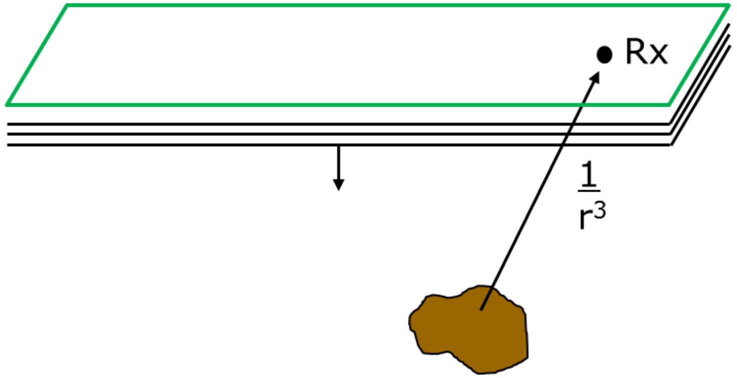
Controlled source:

- Using a small loop
- Magnetic moment
$$m = IA$$
- Total geometric decay
$$\sim \frac{1}{r^6}$$



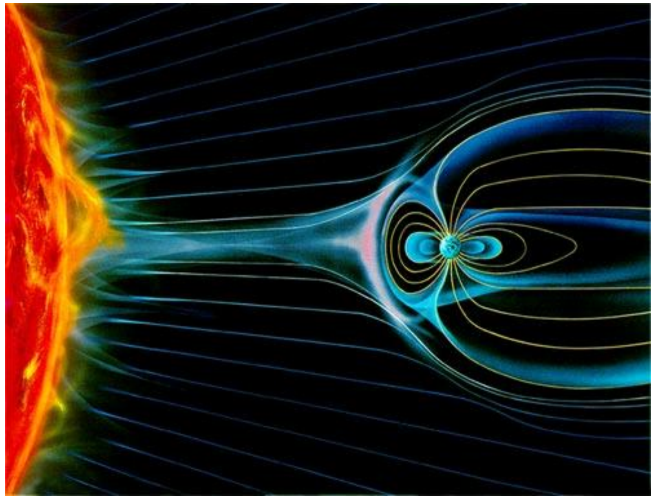
Infinitely large loop source

- Sheet currents generate plane waves
- Total geometric decay
$$\sim \frac{1}{r^3}$$



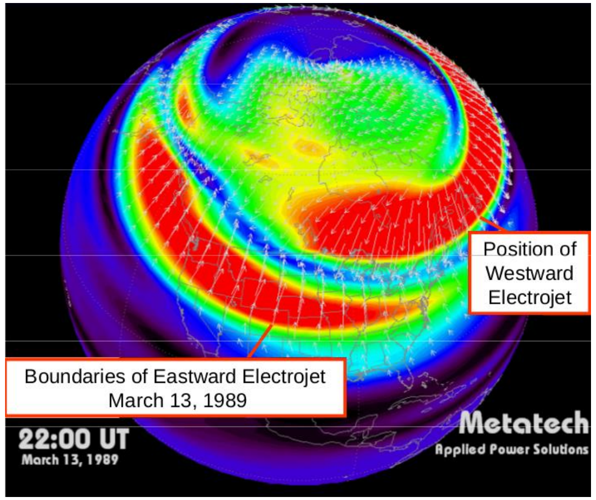
Natural EM sources

Sun and magnetosphere, solar storms



Lightning

Auroral electrojet



Aurora

Refraction of waves

Snell's law

$$k_i \sin\theta_i = k_t \sin\theta_t$$

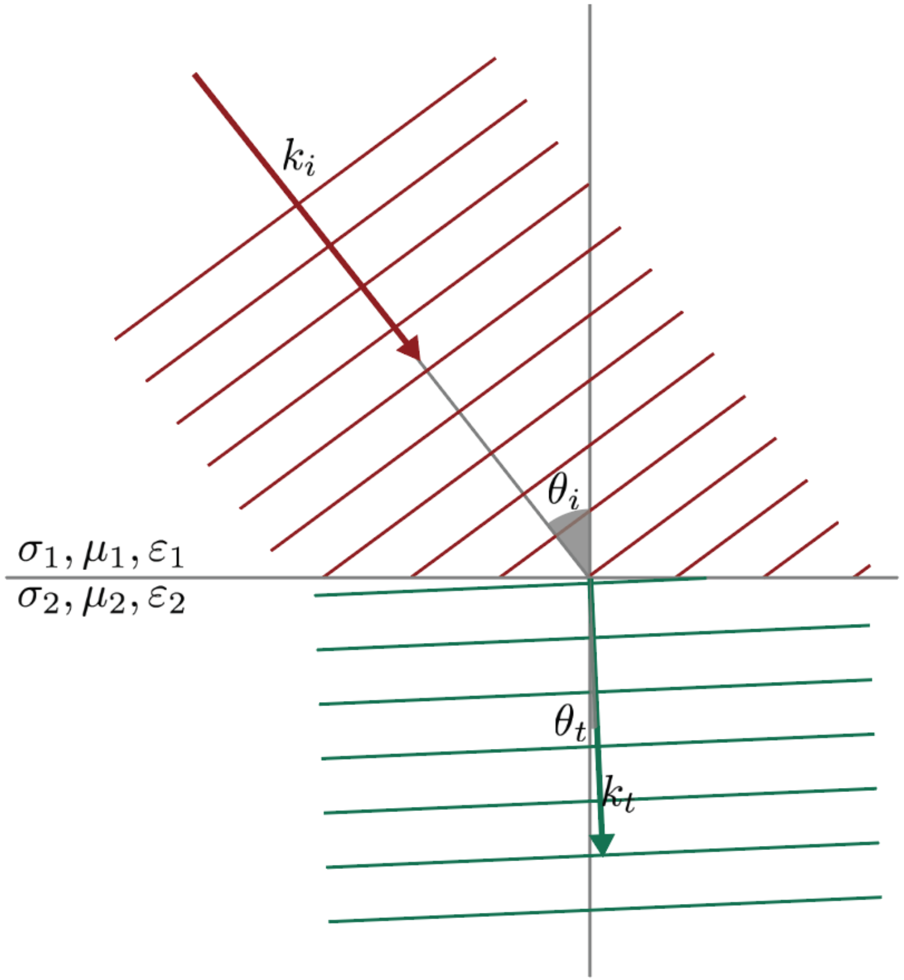
k is complex wave number

$$k^2 = \omega^2 \mu\varepsilon - i\omega\mu\sigma$$

Quasi-static: $\frac{\omega\varepsilon_0}{\sigma} \ll 1$

$$\sin\theta_t = \sqrt{\frac{2\omega\varepsilon_0}{\sigma}} \sin\theta_i$$

Angle of refraction is $\theta_t = 0°$ in almost every instance



Example for 10,000 Hz

$\sigma = 10^{-3}$ S/m

$\theta_i = 89°$

Then $\theta_t = 1.35°$

Plane waves and skin depth

Skin depth (meters)

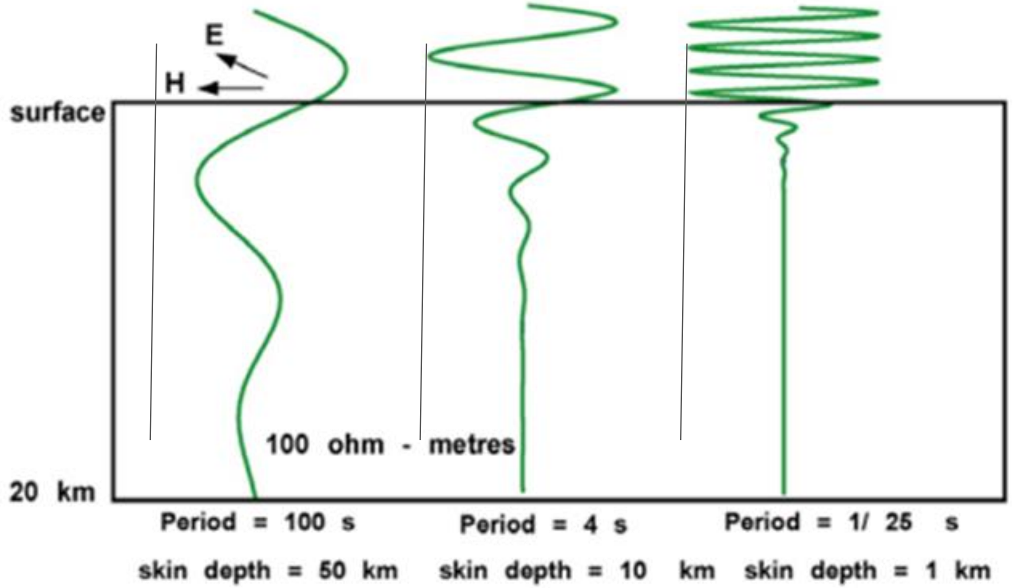
$$\delta = \sqrt{\frac{2}{\omega\mu\sigma}} = 503\sqrt{\frac{1}{\sigma f}}$$

Low frequency waves propagate further

Frequency sounding

Depth of propagation

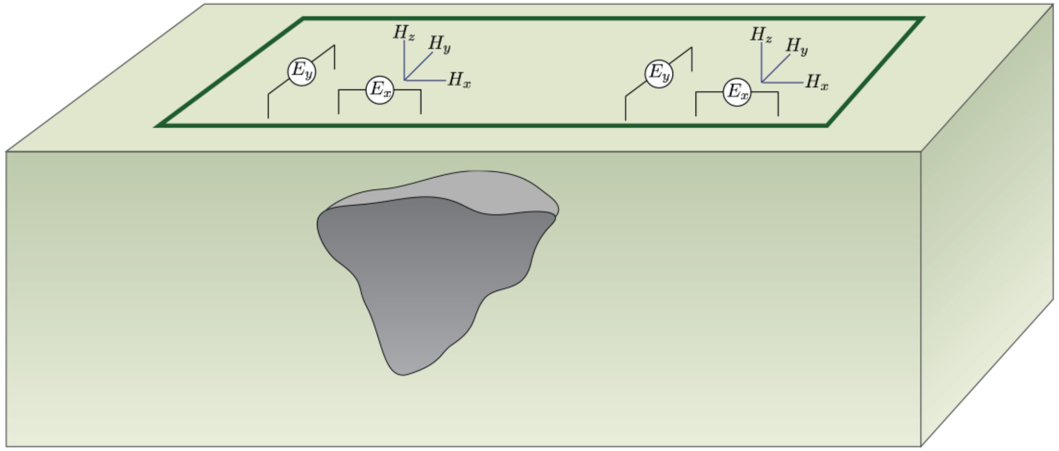
- A few skin depths
- Only a portion of a wavelength



Controlled-source vs. Natural source

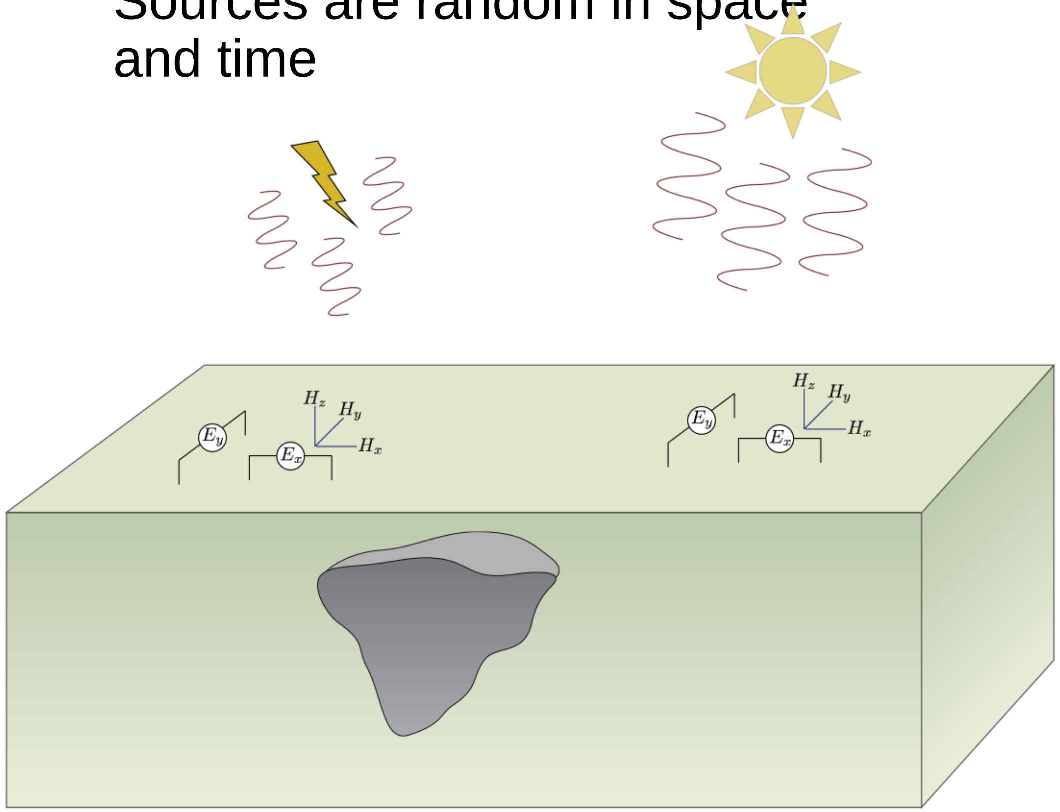
Controlled-source

Well-defined location, geometry, and amplitude

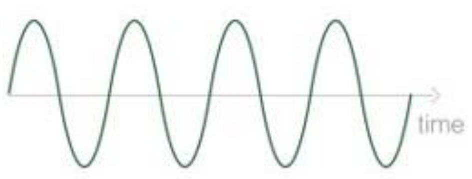


Natural sources

Sources are random in space and time



MT Station



Frequency-domain

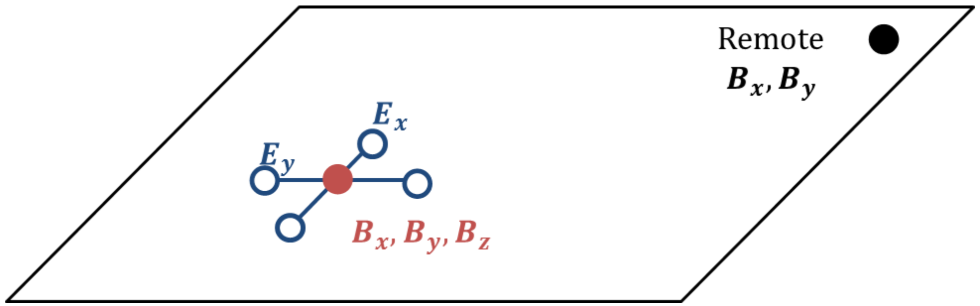
Maxwell's equations:

- Linear in J_s
- E and H affected in the same way

$$\nabla \times \mathbf{E} + i\omega\mu\mathbf{H} = 0$$

$$\nabla \times \mathbf{H} - \sigma\mathbf{E} = \mathbf{J_s}$$

Effects of unknown source removed by taking ratio



Transfer function $\mathbf{E} = \mathbf{Z}\mathbf{H}$

impedance (matrix)

$$\begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{pmatrix} \begin{pmatrix} H_x \\ H_y \end{pmatrix}$$

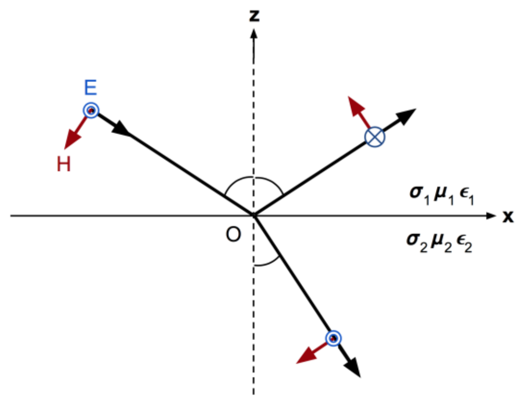
Impedance and resistivity

Plane wave in homogenous media:

- E and H fields are perpendicular

E-field parallel to the boundary
(EM induction)

TE mode



Homogeneous half space

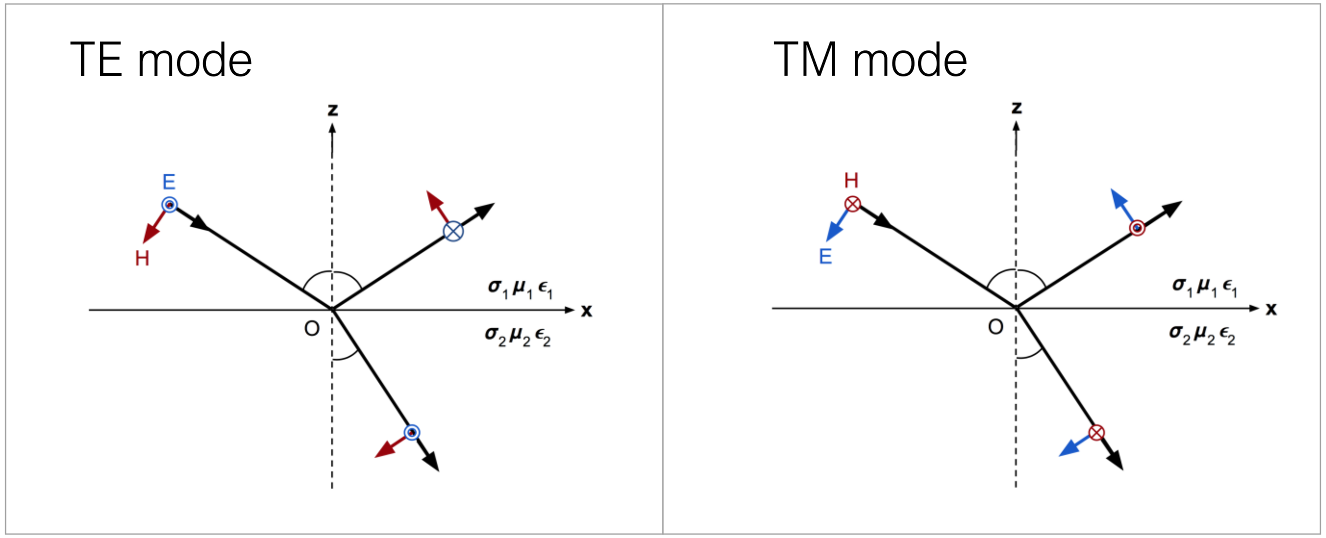
Impedance	Resistivity	Phase
$Z_{xy} = \frac{E_x}{H_y}$	$\rho = \frac{1}{\omega\mu}\left\|Z_{xy}\right\|^2$	$\Phi = \tan^{-1}\left(\frac{Im(Z_{xy})}{Re(Z_{xy})}\right) = \frac{\pi}{4}$

Impedance and resistivity

Plane wave in homogenous media:

- E and H fields are perpendicular

E-field parallel to the boundary
(EM induction)



E-field crosses the boundary
(Current channeling)

Homogeneous half space

Impedance	Resistivity	Phase
$Z_{xy} = \dfrac{E_x}{H_y}$	$\rho = \dfrac{1}{\omega\mu}\left\lvert Z_{xy}\right\rvert^2$	$\Phi = \tan^{-1}\left(\dfrac{Im(Z_{xy})}{Re(Z_{xy})}\right) = \dfrac{\pi}{4}$

MT soundings in 1D

- In general:

$$Z = \begin{pmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{pmatrix}$$

- Apparent resistivity:

$$\rho_a = \frac{1}{\omega\mu_0}\left|Z_{xy}\right|^2$$

- Phase:

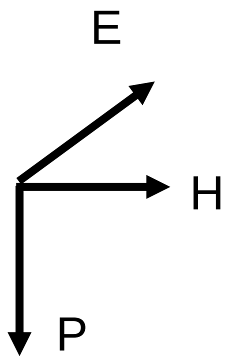
$$\Phi = \tan^{-1}\left(\frac{Im(Z_{xy})}{Re(Z_{xy})}\right)$$

- In 1D:

$$Z = \begin{pmatrix} 0 & Z_{xy} \\ Z_{yx} & 0 \end{pmatrix}$$

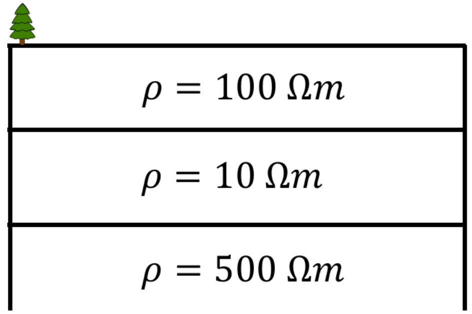
$$Z_{xy} = \frac{E_x}{H_y}$$

$$Z_{xy} = -Z_{yx}$$

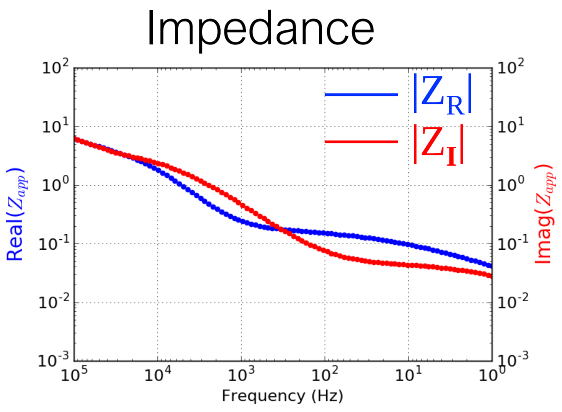


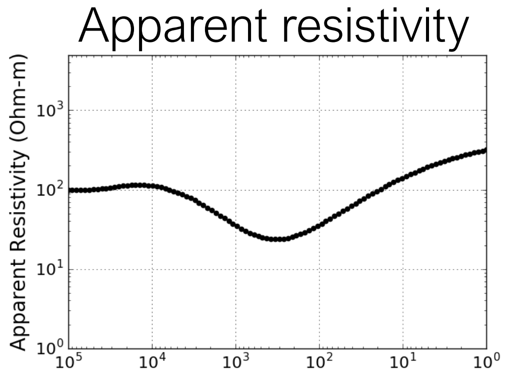
E x H = P
P: ponyting vector

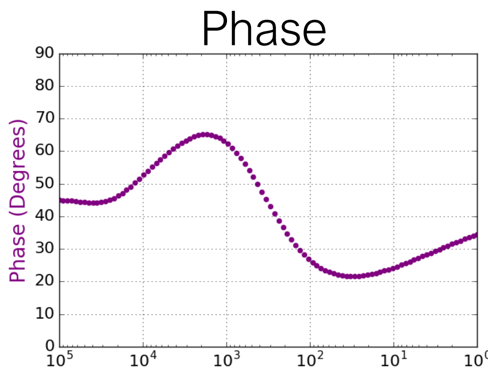
Very similar to inductive source EM
(E-fields parallel to the boundary)



Frequency sounding
for depth resolution







MT soundings in 2D

- In general:

$$Z = \begin{pmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{pmatrix}$$

- In 2D:

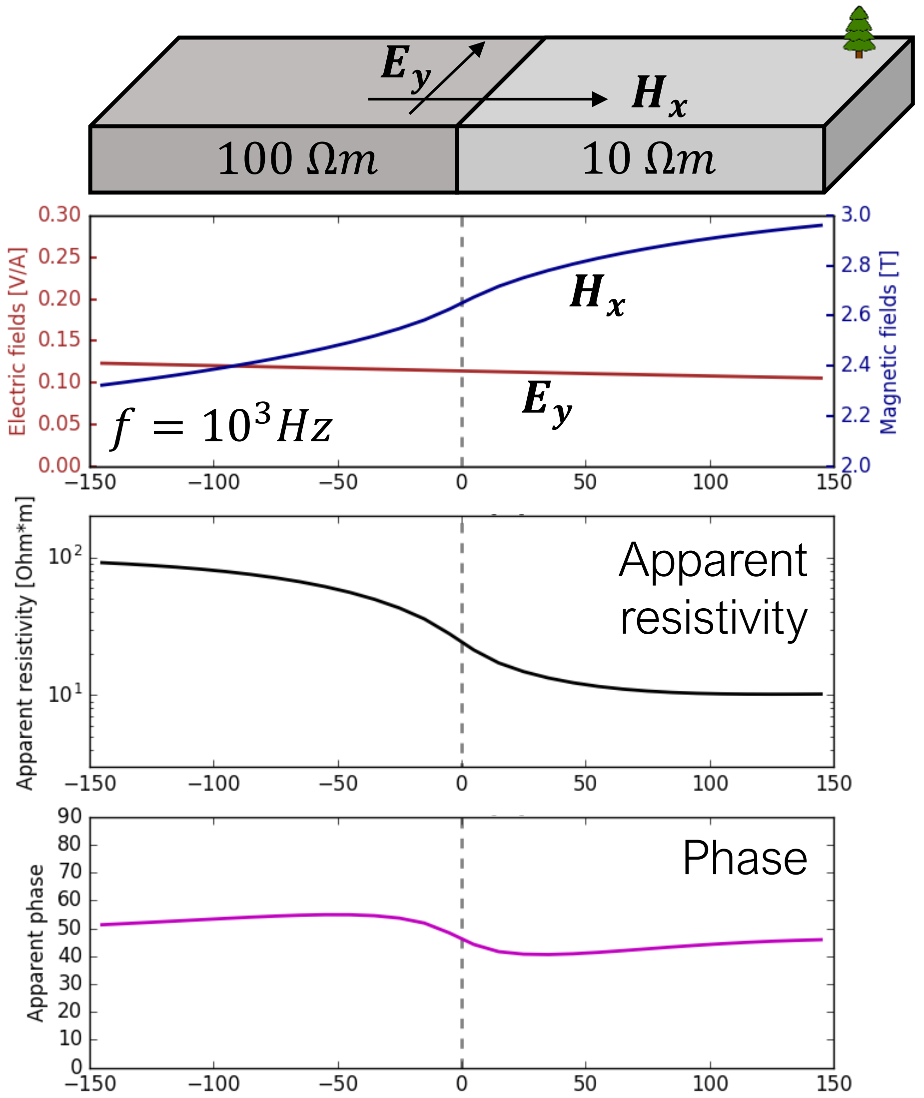
$$Z = \begin{pmatrix} 0 & Z_{xy} \\ Z_{yx} & 0 \end{pmatrix}$$

$$Z_{xy} \neq Z_{yx}$$

- TE mode
  - E-field parallel to structure

$$Z_{yx} = \frac{E_y}{H_x}$$

E-field parallel to the boundary
(EM induction; biased sensitivity towards a conductor)



MT soundings in 2D

- In general:

$$Z = \begin{pmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{pmatrix}$$

- In 2D:

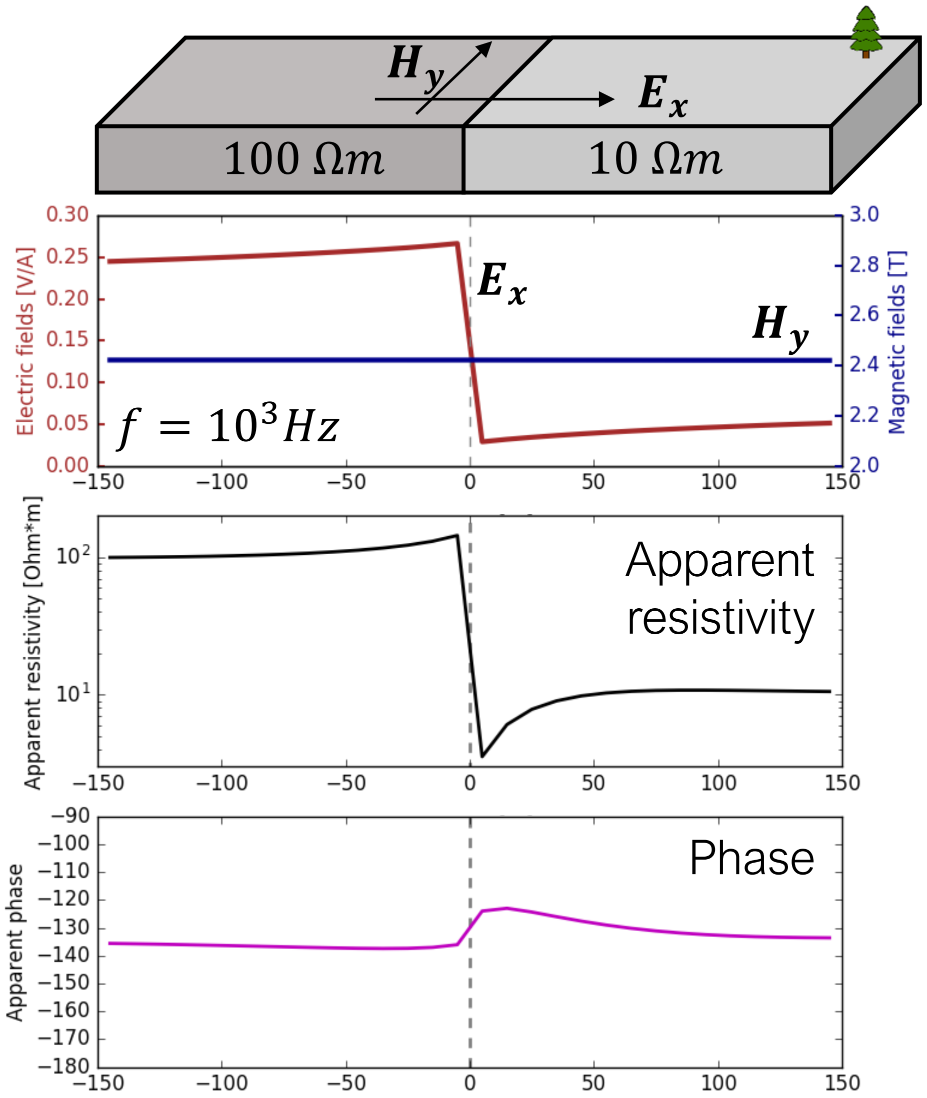
$$Z = \begin{pmatrix} 0 & Z_{xy} \\ Z_{yx} & 0 \end{pmatrix}$$

$$Z_{xy} \neq Z_{yx}$$

- TM mode
  - H-field parallel to structure
  - E_x discontinuous

$$Z_{xy} = \frac{E_x}{H_y}$$

E-field crosses the boundary
(Current channeling; sensitive to both conductor & resistor)



MT soundings in 3D

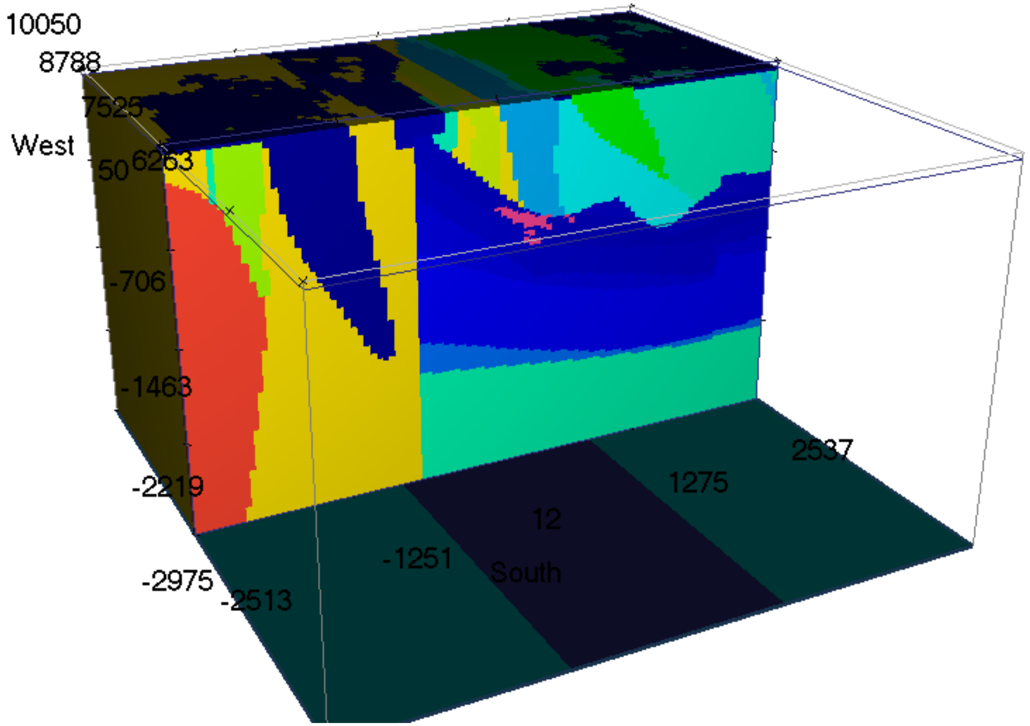
- In general:

$$Z = \begin{pmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{pmatrix}$$

- In 3D:

$$Z = \begin{pmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{pmatrix}$$

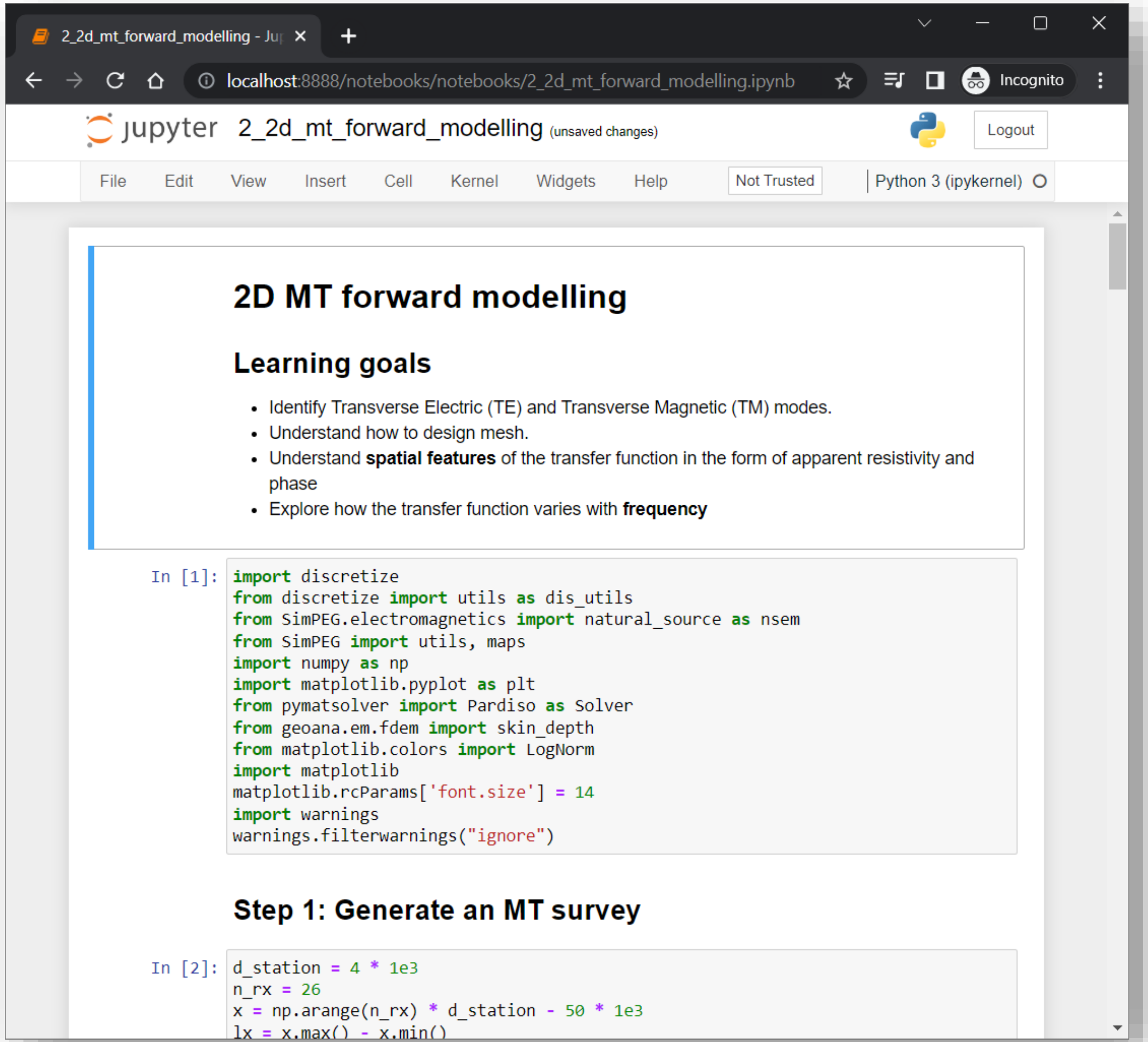
- No symmetry or special conditions



Demo: 2D MT forward modelling app

Why interactive apps?

- Visualization aids understanding
- Learn through interaction
  ask questions and investigate
- Open source:
  Free to use
  Welcome contributions!



Learning goals

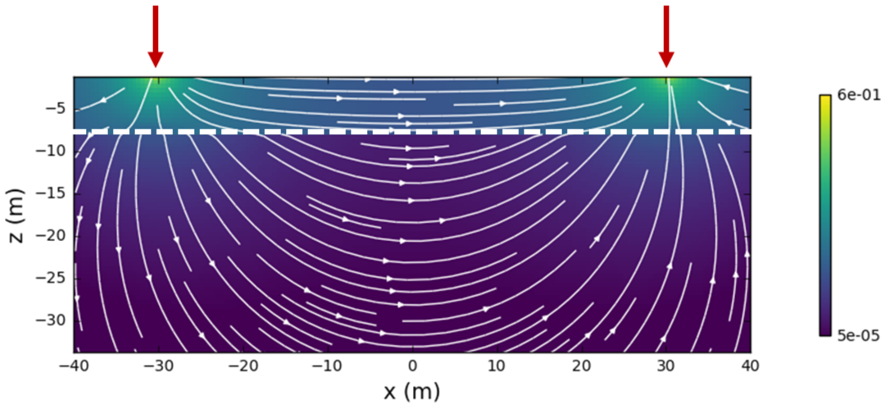
Understand electrical resistivity and its linkage to geologic units.

Understand physical principles of Electromagnetic (EM) methods

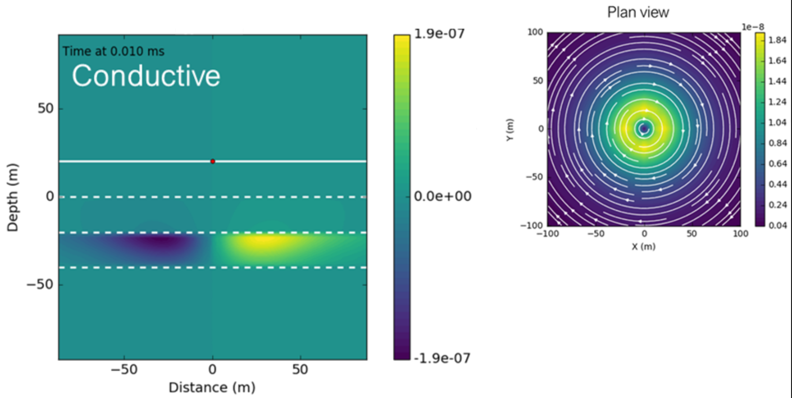
- Charge build-up (or current channeling, galvanic currents)
- EM induction

DC resistivity vs. Inductive source EM

E-fields crosses the layer boundary
(charge build-up; galvanic currents; current channeling)

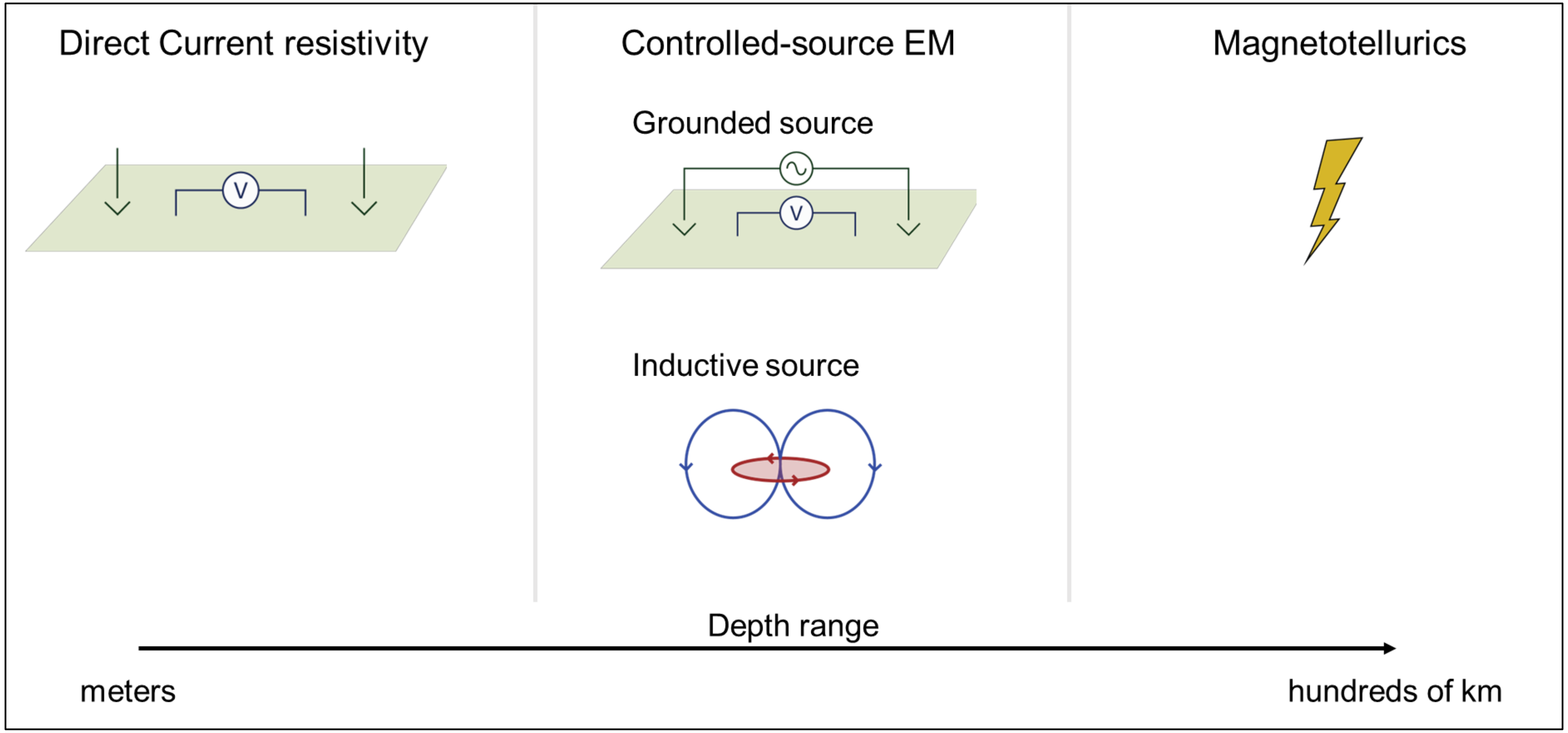


E-fields parallel to the layer boundary
(EM induction)



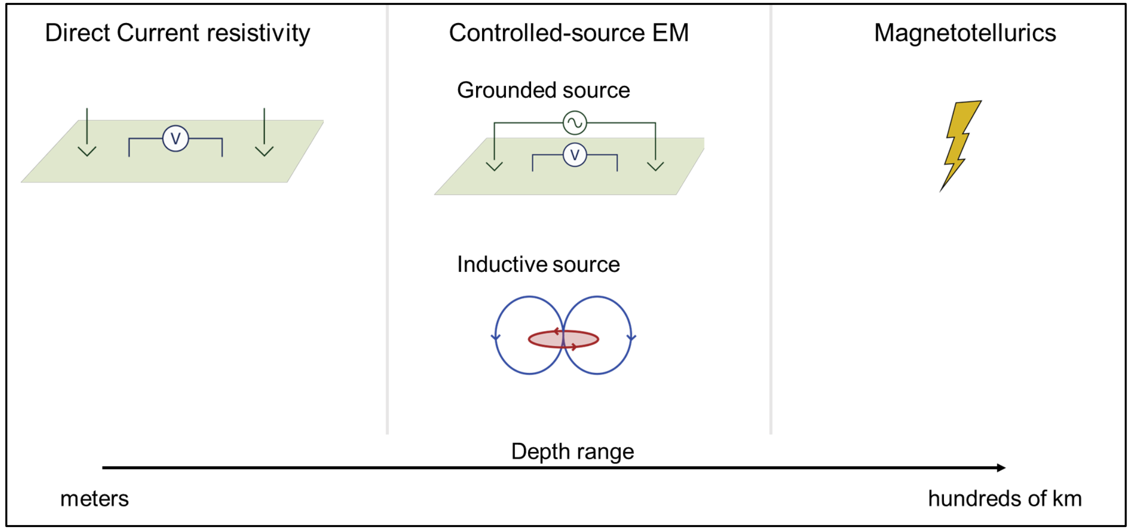
Learning goals

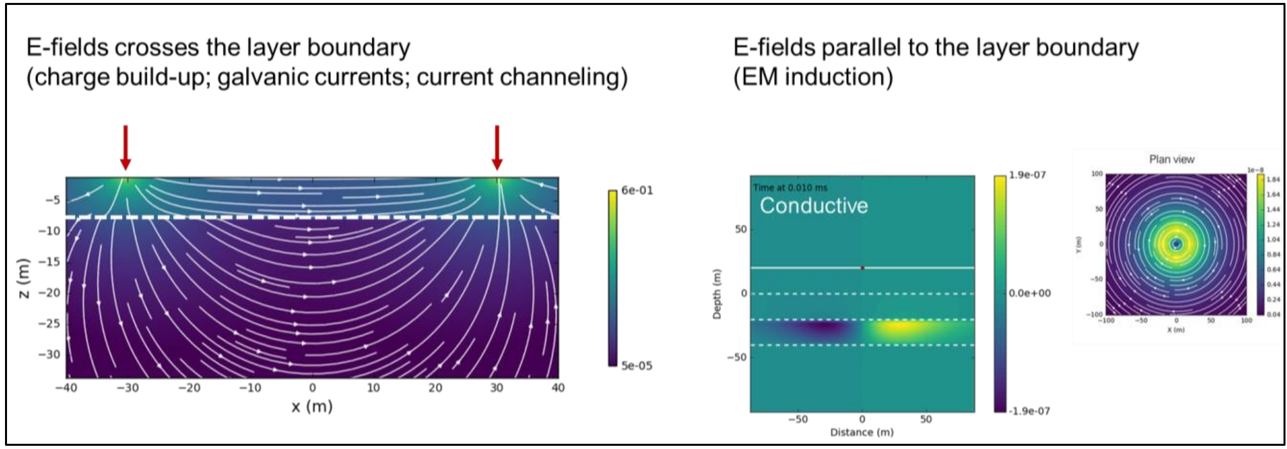
Identify difference between controlled-source EM and magnetotellurics (MT)



Learning goals

Identify all EM methods are governed by the same physical principles





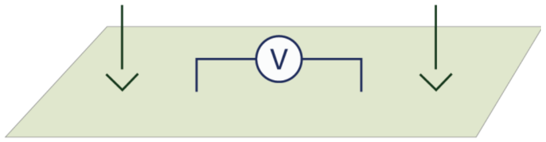
Current channeling
EM induction

Learning goals

Understand survey setup of EM methods and data

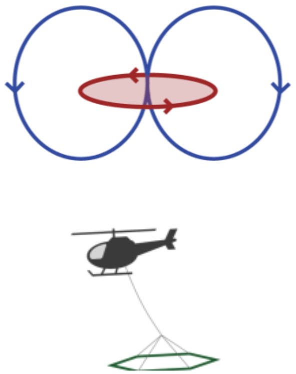
Understand sensitivity of EM methods to conductors & resistors

DC resistivity



Current channeling
Sensitive to both conductors and resistors

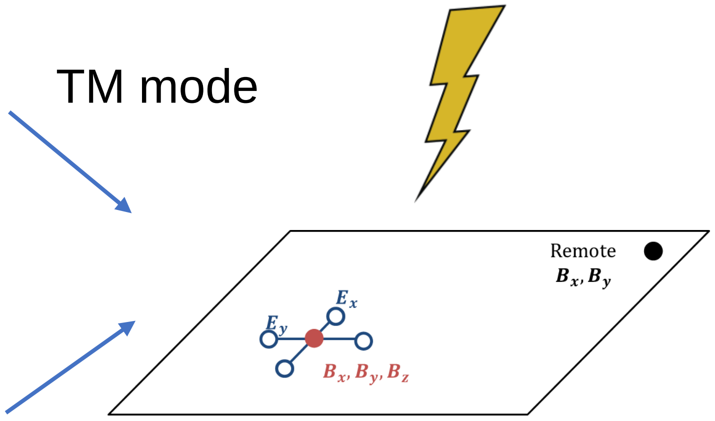
Inductive source EM



EM induction
Biased sensitivity towards conductors

Magnetotellurics

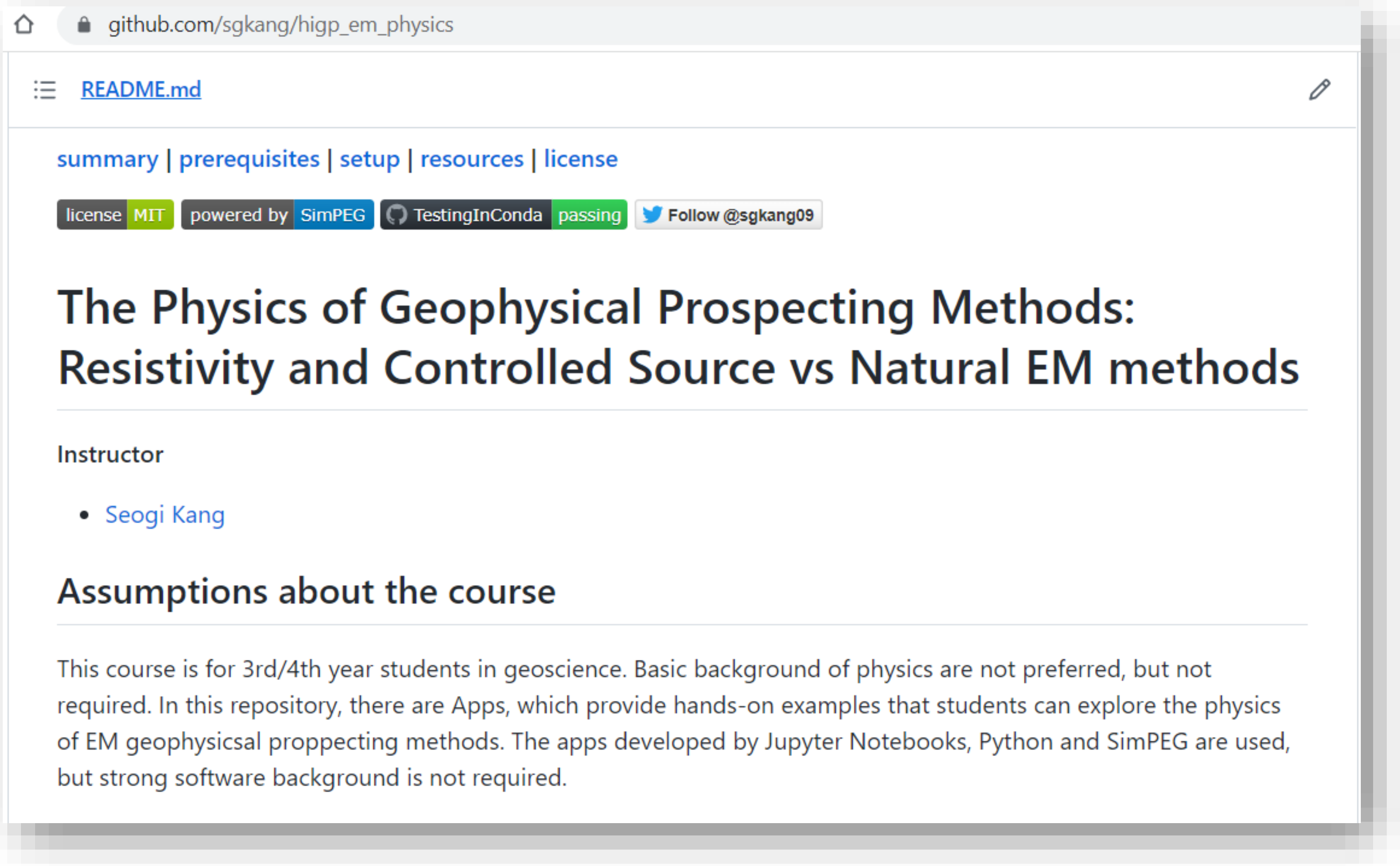
TM mode



TE mode

Resources:

https://github.com/sgkang/higp_em_physics


github.com/sgkang/higp_em_physics

README.md

summary | prerequisites | setup | resources | license

license MIT | powered by SimPEG | TestingInConda passing | Follow @sgkang09

The Physics of Geophysical Prospecting Methods: Resistivity and Controlled Source vs Natural EM methods

Instructor

- Seogi Kang

Assumptions about the course

This course is for 3rd/4th year students in geoscience. Basic background of physics are not preferred, but not required. In this repository, there are Apps, which provide hands-on examples that students can explore the physics of EM geophysicsal proppecting methods. The apps developed by Jupyter Notebooks, Python and SimPEG are used, but strong software background is not required.

The end