Application Note

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Lab 4 Add-on: Frequency Space is the Place

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Table of Contents

1. Abstract

In this addon application note, we will mathematically calculate, simulate, demonstrate, and discuss the process of designing a 3-band audio equalizer using [improved] second-order filters from *Lab 4 Application Note*. We will analyze each audio band, improve audio bands where applicable, then combine to design a 3-band audio equalizer. The equalizer circuit will be analyzed, simulated, and discussed and the phase components will be verified to ensure all components are in phase.

2. Introduction

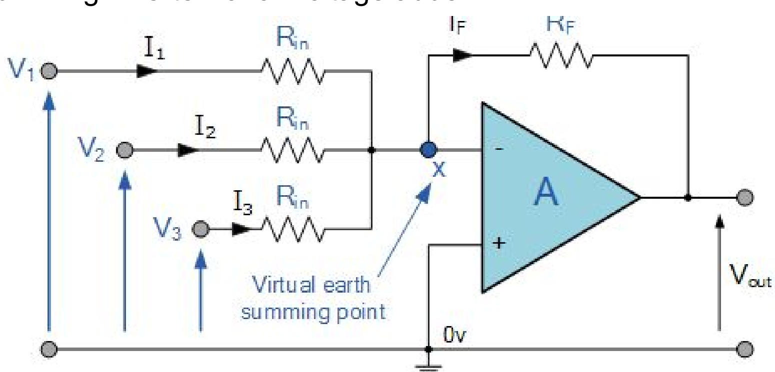
Audio filters are used to control frequencies which pass through a system. Many forms of audio equipment (e.g: equalisers, synthesizers) use audio filters to alter sound. There are many different types of filters available. High pass filters will allow only high frequencies to pass through. Inversely, low pass filters will allow only low frequencies to pass through. These filters and its components are frequency dependent - as in changing values of components and frequency will change the gain and cutoff frequencies of the circuit. Circuit analysis can be completed in the frequency domain to show how each circuit component affects the overall transfer function of the circuit.

Equalization is the process of adjusting the balance between frequency components with an electronic signal. Equalizers are used in sound recording and music reproduction. Audio equalizer devices can either strengthen (boost) or weaken (cut) the energy of specific frequency bands or “frequency ranges.” A 3-band audio equalizer (the one in this application note) will have three adjustable frequency ranges - low frequency (bass), mid frequency (mid), and high frequency (treble).

3. Background

3.1 Summing Operational Amplifier

A summing amplifier is a type of operational amplifier circuit configuration that is used to combine the voltages from two or more inputs into a single output voltage. A summing amplifier is also called a “summing inverter” or a “voltage adder”.

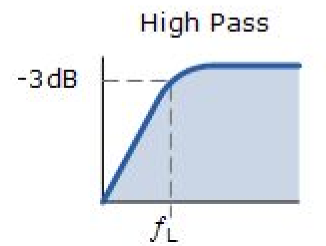


3.2 Potentiometer

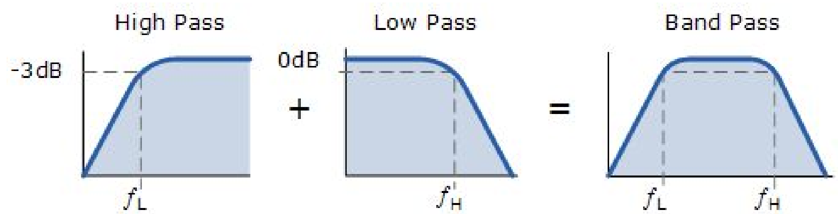
A potentiometer is a three-terminal component that provides different resistances by sliding a wiper across a resistive strip. Adjusting a knob on the potentiometer causes a wiper to slide along the resistive strip which provides a “variable” resistance based on the position of the knob. Two terminals are connected across the resistor, creating a fixed maximum resistance. The third terminal is connected to the wiper which creates a break in resistance between the other two terminals. Therefore, a potentiometer can be seen as a ratio of two resistors for mathematical analysis. A linear potentiometer has a resistance between a terminal and the wiper that is proportional to the distance between them.

3.3 Low Pass Filter
A low pass filter only allows low frequency signals from 0Hz to its cutoff frequency point to pass while blocking those any higher. Using an operational amplifier, we can create an active low pass filter which provides amplification and gain control. The frequency response of a passive low pass filter can be seen on the right.

3.4 High Pass Filter
A high pass filter only allows high frequency signals from its cutoff frequency point and higher to infinity to pass through while blocking those any lower. Using an operational amplifier, we can create an active low pass filter which provides amplification and gain control. The frequency response of a passive pass filter can be seen on the right.



3.5 Band Pass Filter
A band pass filter allows signals falling within a certain frequency band setup between two points to pass through while blocking both the lower and higher frequencies on either side of this frequency band. The frequency response of a band pass filter is found by combining the response of a highpass filter and a low pass filter as seen below:



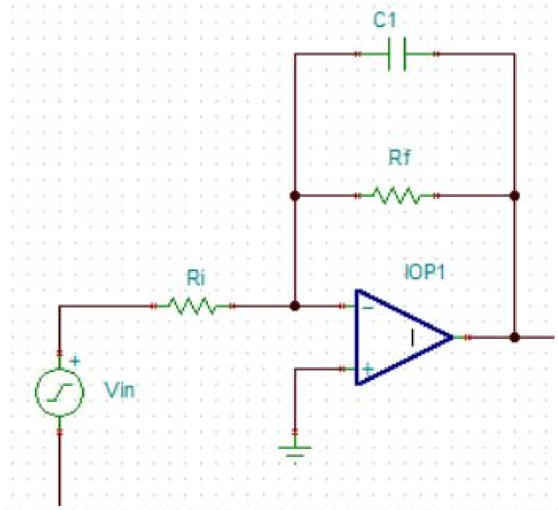
3.6 Audio Frequency
Audio frequency is characterized as a periodic vibration whose frequency is audible to the average human. The unit of audio frequency is in the unit Hertz (Hz). Frequency is the property of sound that mostly determines pitch. The standard range of audible frequencies is 20Hz to 20kHz. The seven standard frequency bands are:

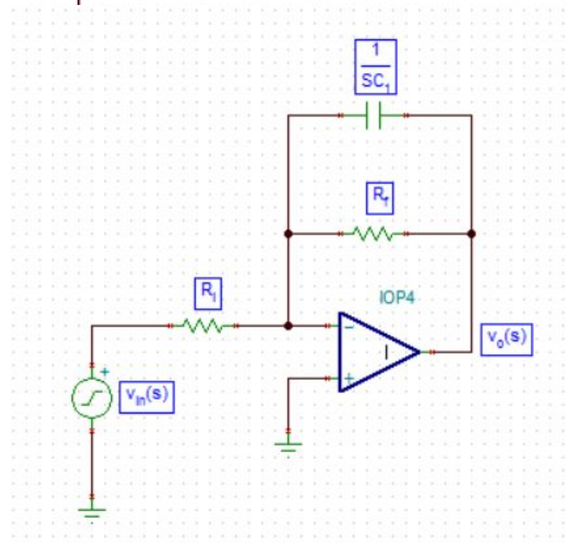
1. Sub-bass (20Hz - 60Hz)
2. Bass (60Hz - 250Hz)
3. Low midrange (250Hz - 500Hz)
4. Midrange (500Hz - 2kHz)
5. Upper midrange (2kHz - 4kHz)
6. Presence (4kHz - 6kHz)
7. Brilliance (6kHz - 20kHz)

4. Discussions and Results

4.1 Low EQ Band

Consider the following first-order active low pass filter (right).





The active low-pass filter stage should provide a DC gain of +3.75dB and a cutoff frequency of 500Hz. We need to find the transfer function for this circuit. The circuit in the s-domain can be seen on the right.

The impedance of the capacitor and the feedback resistor can be combined in parallel,

$$Z_f = \frac{1}{j\omega C_1}||R_f$$

$$Z_f = \frac{\frac{1}{j\omega C_1} \cdot R_f}{R_f + \frac{1}{j\omega C_1}}$$

$$Z_f = \frac{R_f}{j\omega C_1 R_f + 1}$$

For an inverting operational amplifier circuit in the s-domain,

$$H(j\omega) = \frac{v_{out}}{v_{in}} = -\frac{Z_f}{Z_i}$$

$$H(j\omega) = -\frac{\frac{R_f}{j\omega C_1 R_f + 1}}{R_i}$$

Therefore, the transfer function for the circuit is

$$H(j\omega) = -\frac{R_f}{R_i(1 + j\omega C_1 R_f)}$$

We will set the gain for the first stage to be +3.75dB. The gain for +3.75dB can be calculated by,

$$+3.75dB = 20log(\frac{R_f}{R_i})$$

Let $R_f = 10k\Omega$

$$+3.75dB = 20log(\frac{10k\Omega}{R_i})$$

$$R_i = 6493\Omega \approx 6500\Omega$$

After rounding the value of the input resistor, the resistor values for our circuit are,

$$R_i = 6500\Omega$$
$$R_f = 10k\Omega$$

The gain of the circuit can be recalculated such as,

$$A_v = 20log(\frac{10k\Omega}{6.5k\Omega}) = +3.74$$

To obtain the value of the capacitor in the circuit we will set the transfer function equal to the output gain and solve for capacitance **when the cutoff frequency for this stage is 500 Hz**.

Note, $\omega = 2\pi f$

$$f = 500Hz$$

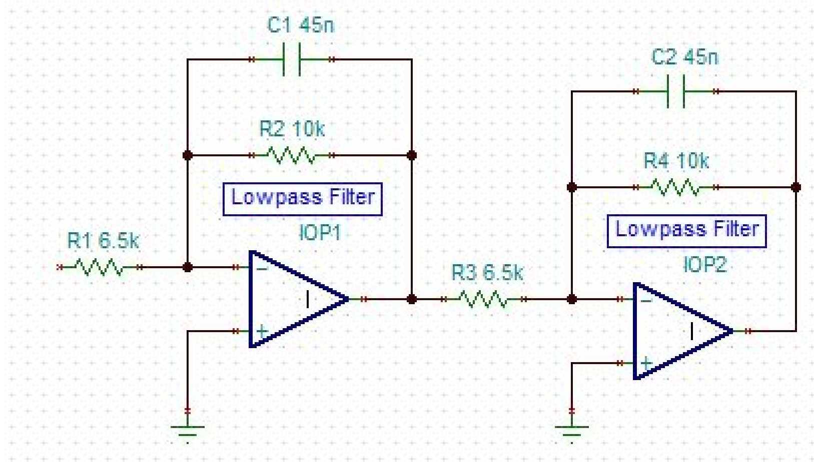
$$+3.74 = -\frac{R_f}{R_i(1 + j\omega C_f R_f)} = -\frac{10k\Omega}{6.5k\Omega(1 + (2\pi 500Hz) \cdot 10k\Omega \cdot C_f)}$$

$$C_{f2} = 4.49 \cdot 10^{-8}F = 0.049\mu F$$

The component values of this circuit are,

$$R_f = 10k\Omega$$
$$R_i = 6.5k\Omega$$
$$C_f = 0.049\mu F$$

By cascading the same low pass filter circuit we can create a second-order active low pass filter,



To find the overall gain (transfer function) for the second-order filter we can multiply the first and second stage of the circuit,

$$H(j\omega) = \frac{v_{o2}}{v_{o1}} \cdot \frac{v_{o1}}{v_i}$$

$$H(j\omega) = \frac{(R_{f1}/R_{i1})(R_{f2}/R_{i2})}{(1 + j\omega C_{f1}R_{f1})(1 + j\omega C_{f2}R_{f2})}$$

Analyzing the poles of the transfer function above, the circuit can be seen to be second-order. Recall from the Lab 4 Application Note (section 4.5.3 Cascading filters effect on gain) how to calculate the overall gain of a second-order circuit. We can solve for the overall gain in decibels by adding each gain stage (decibels),

$$A_v = 20\log_{10}(A_{v1}) + 20\log_{10}(A_{v2})$$
$$A_v = 20\log_{10}(10k\Omega/6.5k\Omega) + 20\log_{10}(10k\Omega/6.5k\Omega)$$
$$A_v = 3.74dB + 3.74dB$$
$$A_v = 7.48dB$$

The overall magnitude of the gain for the cascaded second-order filter will be 7.48dB prior to any filtering.

The slope of the cascaded second-order filter will also have a decreased slope (increased magnitude). The total roll-off by a higher order circuit is given by,

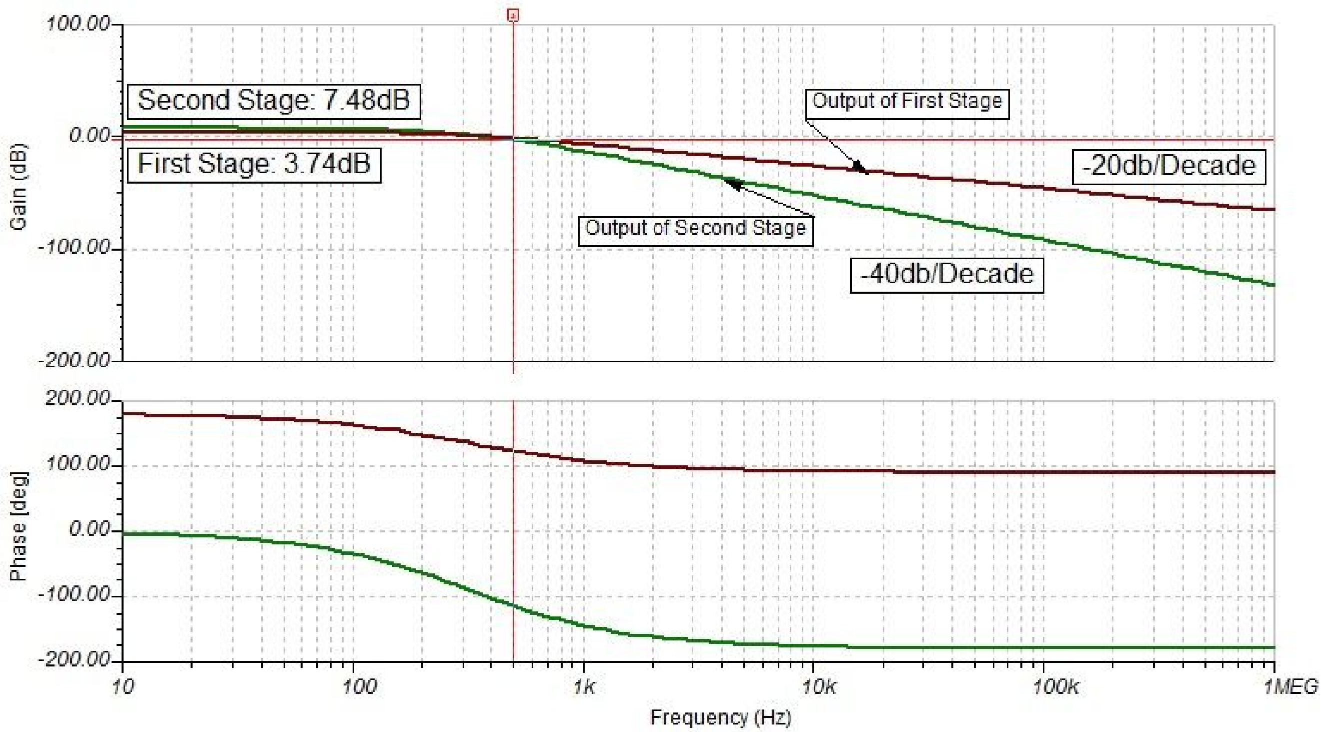
$$\Delta L_T = n\Delta L = 20ndB/Decade$$

The total roll-off for a first-order filter can be calculated by,
$\Delta L_T = (1)\Delta L = 20(1)dB/Decade = 20dB/Decade$

The total roll-off for a second-order filter can be calculated by,
$\Delta L_T = (2)\Delta L = 20(2)dB/Decade = 40dB/Decade$

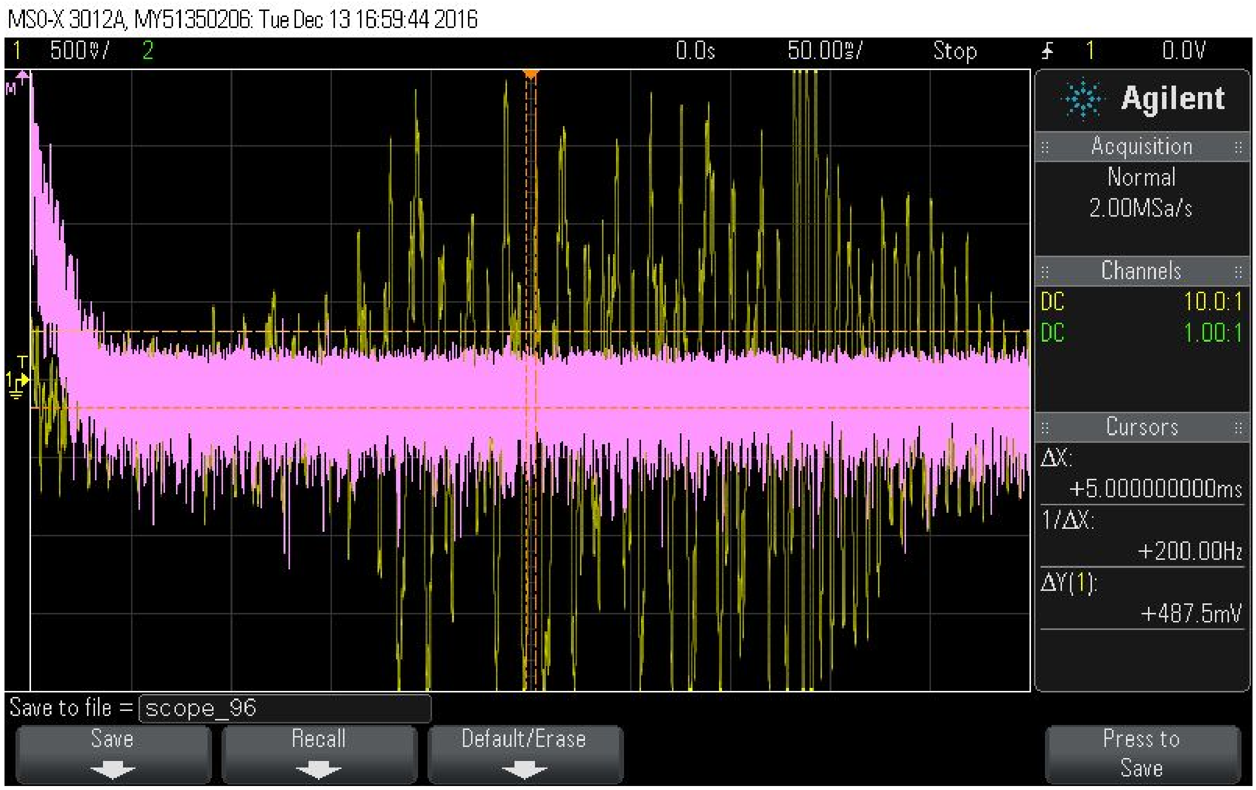
Therefore, the roll-off slope of the cascaded second-order will be 40dB/Decade per mathematical calculations.

Simulation (bode plot) of the output of the first and second stage of the second-order low pass filter circuit.



The second stage of the second-order low pass filter has a decreased slope of -40dB/Decade compared to the first stage with a slope of -20dB/Decade. The second stage has a gain magnitude of 7.48dB compared to the 3.74dB magnitude of the first stage. All calculations correspond with the simulations.

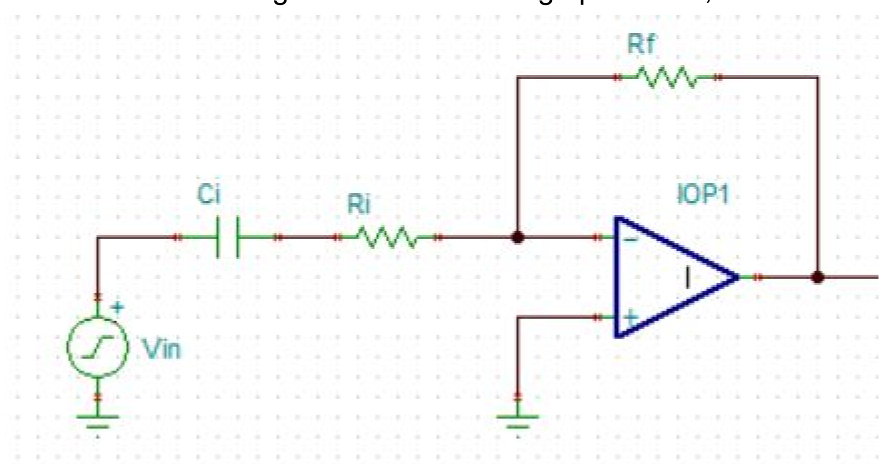
The following is a measured frequency domain approximation using the fast-fourier transform function (FFT) on the oscilloscope. The input signal for the circuit in this test measurement was electronic music. The mid-band and high-band on the circuit were set to 0% with the low-band set to 100%.



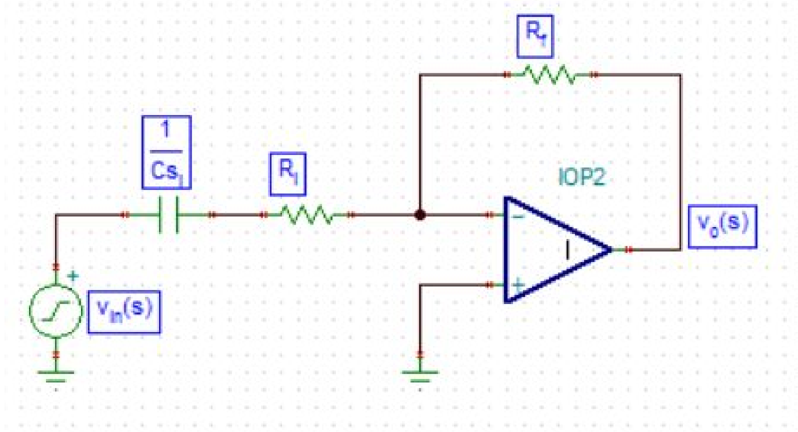
The measurements correspond with mathematical calculations and simulations. In the measured result, the gain of the output decreases as the frequency increases (second-order low pass filter).

4.2 High EQ Band

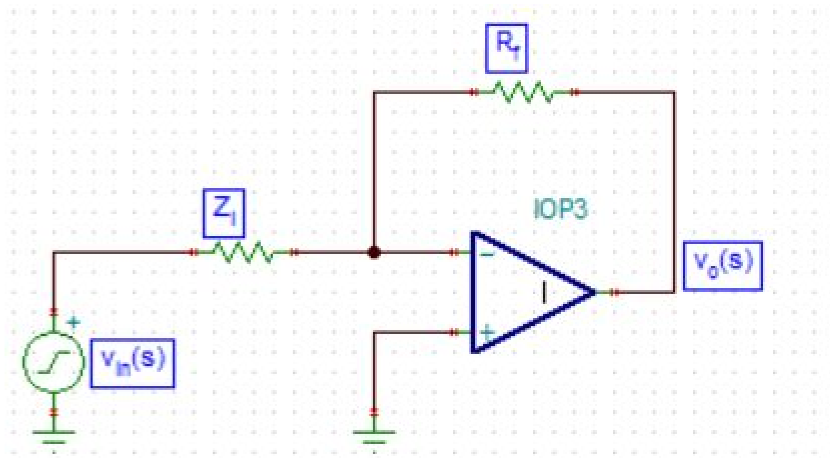
Consider the following first-order active high pass filter,



The active low-pass filter stage should provide a DC gain of +3.75dB and a cutoff frequency of 2kHz. We need to find the transfer function for this circuit. The circuit re-written in the s-domain,



The input capacitor and input resistor impedance can be combined in series. The circuit can be re-written such as,



Where Z_i equals the impedance of the input resistor and capacitor in series.
The impedance of the input resistor and capacitor in series,

$$Z_i = R_i + \frac{1}{j\omega C_i}$$

The impedance of the feedback portion is simply the feedback resistor,

$$Z_f = R_f$$

For an inverting opamp circuit in the s-domain,

$$H(j\omega) = -\frac{Z_f}{Z_i}$$

Therefore, the transfer function for the circuit,

$$H(j\omega) = \frac{-R_f}{R_i + \frac{1}{j\omega C_i}} = \frac{-j\omega C_i R_f}{1 + j\omega RiC_i}$$

We need to find values for the resistor and capacitor components such that we can get a gain of +3.75dB for the first stage and a cutoff frequency of 2kHz. Setting the gain to +3.75dB and solving for resistor values,

$$+3.75dB = 20log(\frac{R_f}{R_i})$$

Let $R_f = 10k\Omega$

$$+3.75dB = 20log(\frac{10k\Omega}{R_i})$$

$$R_i = 6493\Omega \approx 6500\Omega$$

After rounding the value of R_i, the values for our resistors are,

$$R_i = 6500\Omega$$

$$R_f = 10k\Omega$$

The gain of our circuit can be recalculated such as,

$$A_v = 20log(\frac{10k\Omega}{6.5k\Omega}) = +3.74$$

To obtain the value of the capacitor in the circuit we will set the transfer function equal to the output gain and solve for capacitance when the **cutoff frequency for this stage is 2kHz.**

Note, $\omega = 2\pi f$

$$f = 2000Hz$$

$$+3.74 = \frac{-j\omega C_i R_f}{1 + j\omega Ri C_i} = \frac{-(2\pi \cdot 2000Hz)(10k\Omega)(C_i)}{1 + (2\pi \cdot 2000Hz)(6.5k\Omega)(C_i)}$$

$$C_i = 8.67 \cdot 10^{-9} F = 0.00867\mu F$$

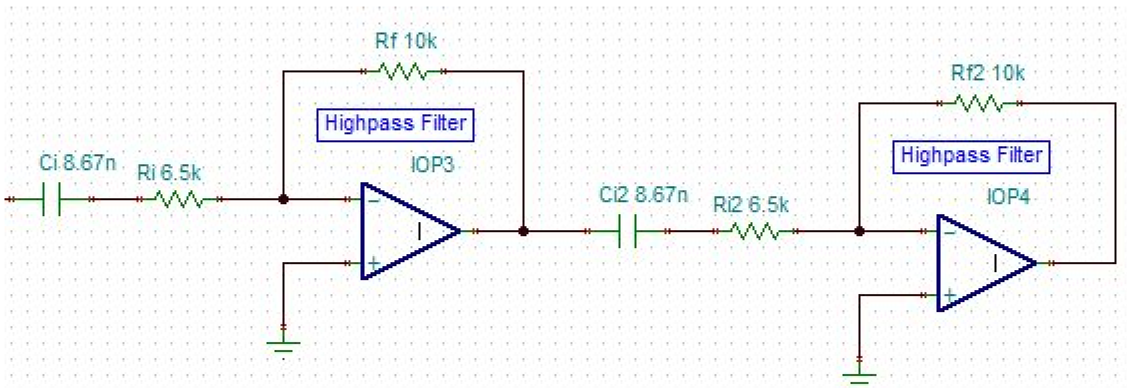
The component values of this circuit are,

$$R_f = 10k\Omega$$

$$R_i = 6.5k\Omega$$

$$C_i = 8.67nF$$

By cascading the same high-pass filter circuit we can create a second-order active high pass filter,



To find the overall gain (transfer function) for the second-order filter we can multiply the first and second stage of the circuit,

$$H(j\omega) = \frac{v_{o2}}{v_{o1}} \cdot \frac{v_{o1}}{v_i}$$

$$H(j\omega) = \frac{j\omega^2 C_{i1} C_{i2} R_{f1} R_{f2}}{(1 + j\omega R_{i1} C_{i1})(1 + j\omega R_{i2} C_{i2})}$$

Analyzing the poles of the transfer function above, the circuit can be seen to be second-order. Recall from the Lab 4 Application Note (section 4.5.3 Cascading filters effect on gain) how to calculate the overall gain of a second-order circuit. We can solve for the overall gain in decibels by adding each gain stage (decibels),

$$A_v = 20\log_{10}(A_{v1}) + 20\log_{10}(A_{v2})$$
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The overall magnitude of the gain for the cascaded second-order filter will be 7.48dB prior to any filtering.

The slope of the cascaded second-order filter will also have a decreased slope (increased magnitude). The total roll-off by a higher order circuit is given by,

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The total roll-off for a first-order filter can be calculated by,

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The total roll-off for a second-order filter can be calculated by,

$$\Delta L_T = (2)\Delta L = 20(2)dB/Decade = 40dB/Decade$$

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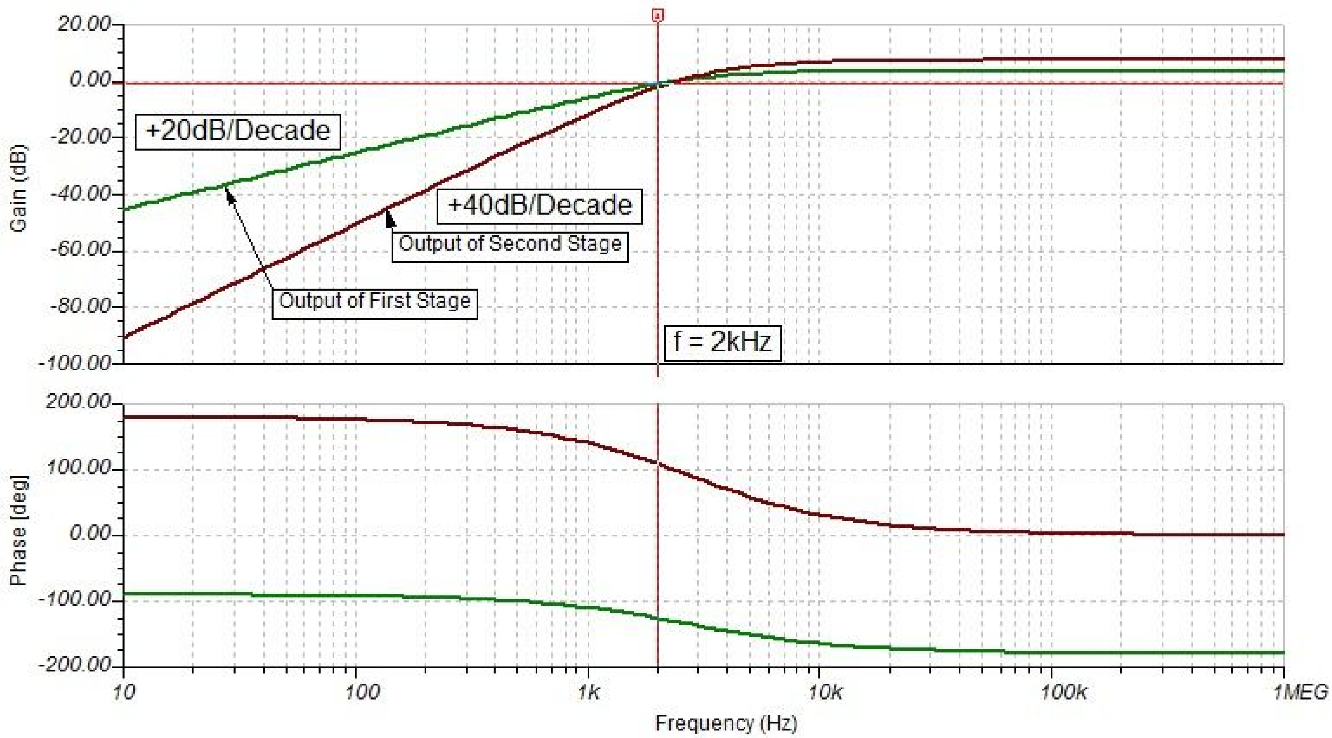
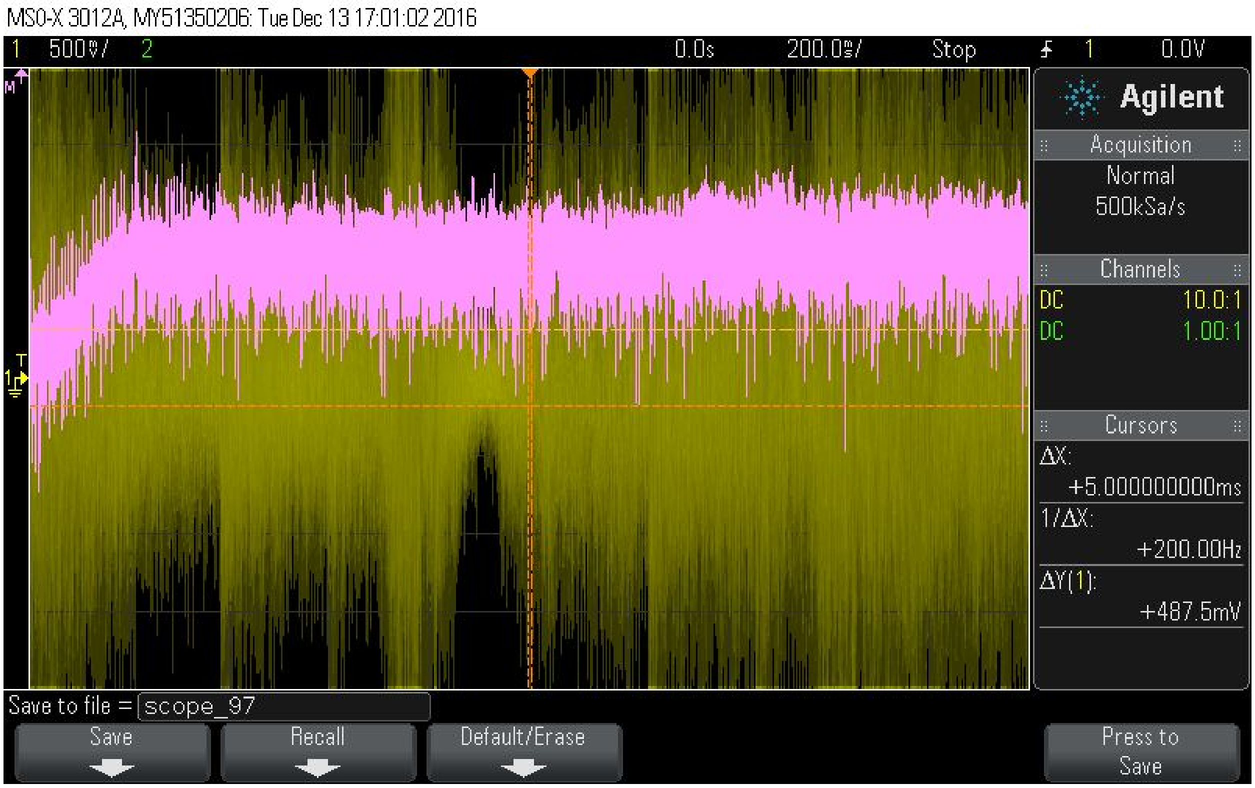
Simulation (bode plot) of the output of the first and second stage of the second-order high pass filter circuit.



The second stage of the second-order high pass filter has an increased slope of +40dB/Decade compared to the first stage with a slope of +20dB/Decade. The second stage has a gain magnitude of 7.48dB compared to the 3.74dB magnitude of the first stage. The gain increases towards it reaches the cutoff frequency of 2kHz. All calculations correspond with the simulations.

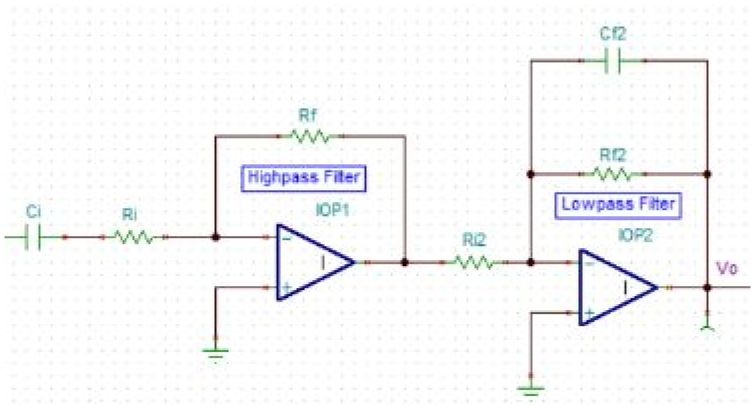
The following is a measured frequency domain approximation using the fast-fourier transform function (FFT) on the oscilloscope. The input signal for the circuit in this test measurement was electronic music. The mid-band and low-band on the circuit were set to 0% with the high-band set to 100%.



The measurements correspond with mathematical calculations and simulations. In the measured result, the gain of the output increases as the frequency increases toward the cutoff frequency (second order high-pass filter).

4.3 Mid EQ Band

As discussed in the Lab 4 Application Note, combining a active low-pass filter and active high-pass filter will result in a bandpass filter (In section 4.4 we will discuss an improved method).



Recall the transfer function for the first-order high-pass filter,

$$H(j\omega) = \frac{-R_f}{R_i + \frac{1}{j\omega C_i}} = \frac{-j\omega C_i R_f}{1 + j\omega RiC_i}$$

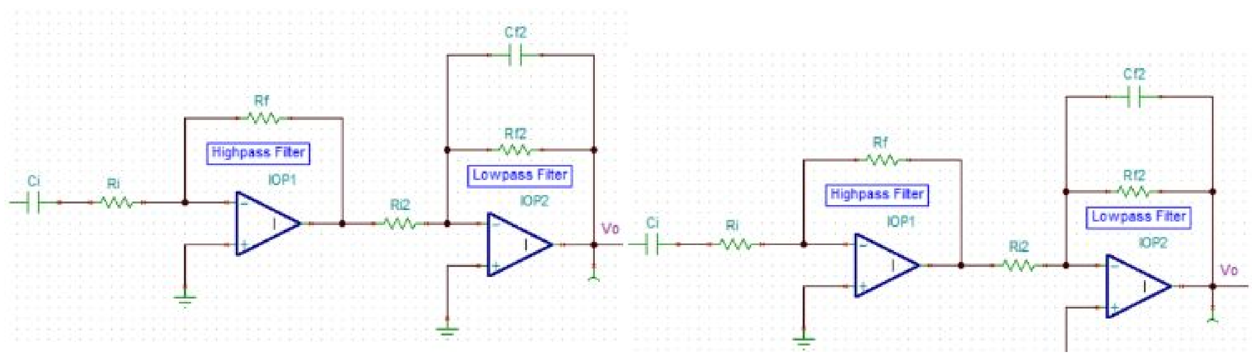
Recall the transfer function for the first-order low-pass filter,

$$H(j\omega) = -\frac{R_f}{R_i(1 + j\omega C_1 R_f)}$$

Multiplying the two transfer functions together for the transfer function for the first-order bandpass filter,

$$H(j\omega) = \frac{j\omega C_{i1} R_{f1} R_{f2}}{R_{i2}(1 + j\omega C_{i1} R_{i1})(1 + j\omega C_{f2} R_{f2})}$$

By cascading the same bandpass filter circuit we can create a second-order active bandpass filter,



To find the second-order variant of the transfer function, multiply the first-order transfer function by itself,

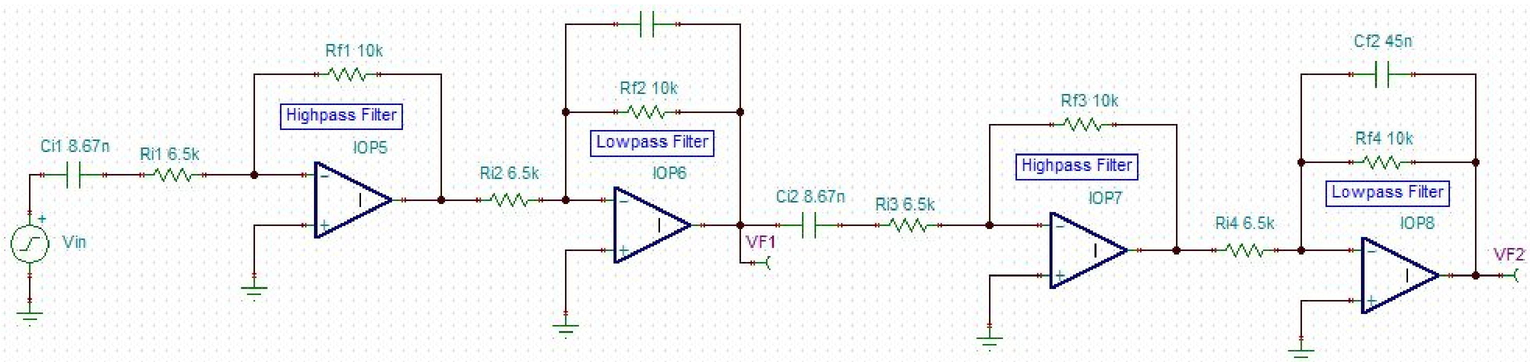
$$H(j\omega) = \frac{j\omega^2 C_{i1} C_{i3} R_{f1} R_{f2} R_{f3} R_{f4}}{R_{i2} R_{i4}(1 + j\omega C_{i3} R_{i3})(1 + j\omega C_{i1} R_{i1})(1 + j\omega C_{f4} R_{f4})(1 + j\omega C_{f2} R_{f2})}$$

The values of components for both high-pass stages will be the same as the second-order highpass filter in section 4.2,

$$R_{f1,3} = 10k\Omega$$
$$R_{i1,3} = 6.5k\Omega$$
$$C_{i1,3} = 8.67nF$$

The values of component for both low-pass stages will be the same as the second-order lowpass filter in section 4.1,

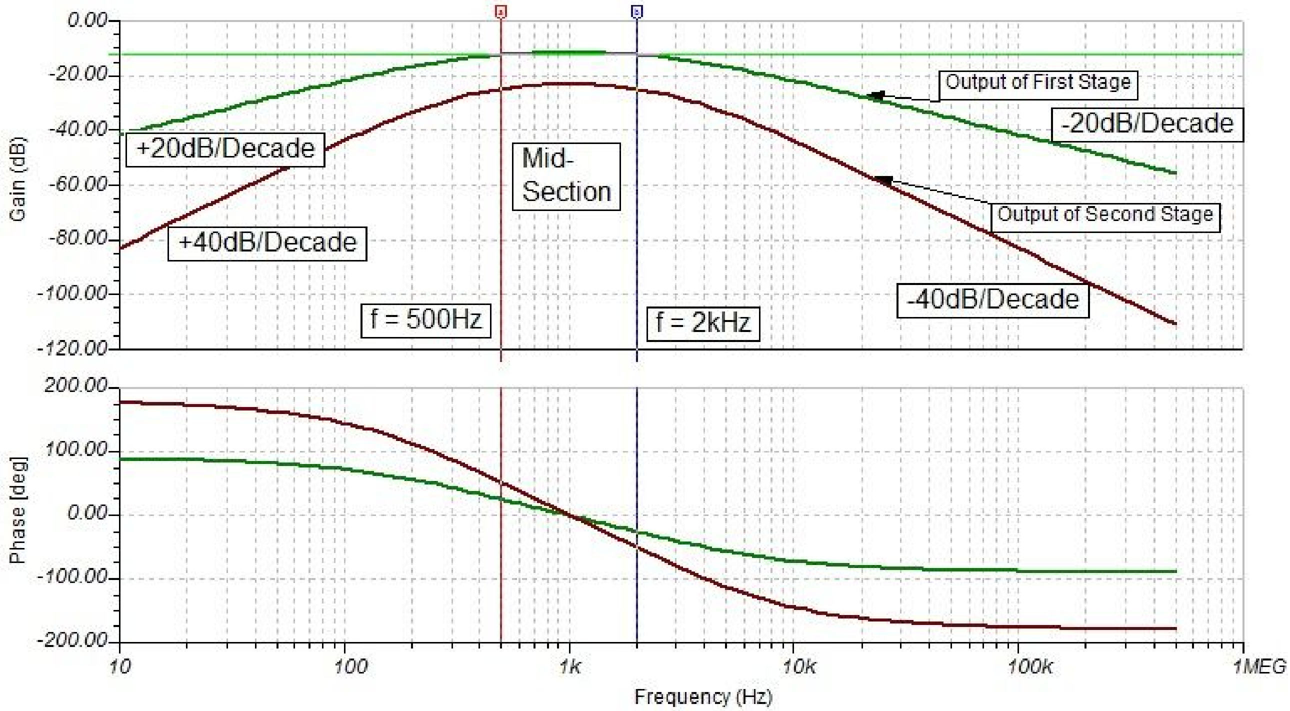
$$R_{f2,4} = 10k\Omega$$
$$R_{i2,4} = 6.5k\Omega$$
$$C_{f2,4} = 0.049\mu F$$



The total roll-off for a second-order filter can be calculated by,

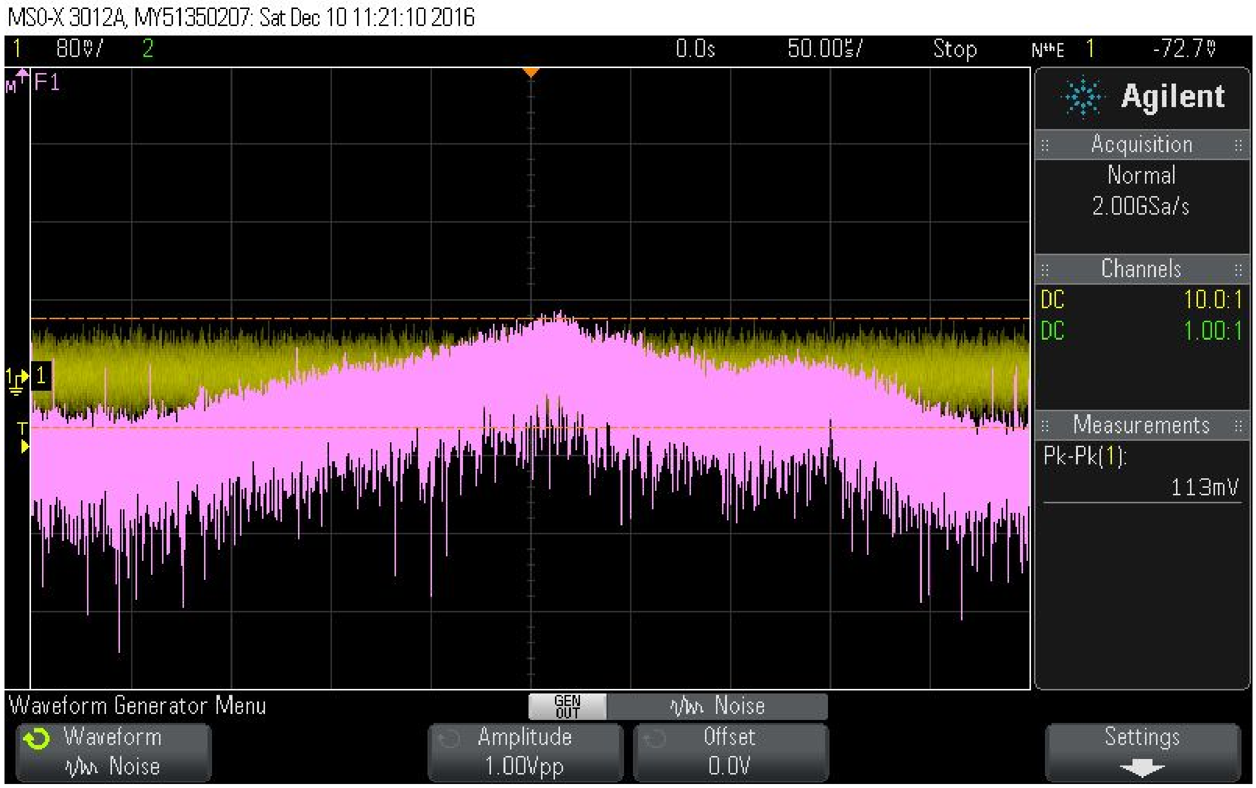
$$\Delta L_T = (2)\Delta L = 20(2)dB/Decade = 40dB/Decade$$

Simulation (bode plot) of the first and second stage of the second-order bandpass filter circuit,



The second stage of the second-order bandpass filter has an increased slope of ±40dB/Decade compared to the first stage with a slope of ±20dB/Decade. The filter increases in gain towards the highpass cutoff frequency of 500Hz. The filter then decreases in gain passed the lowpass cutoff frequency of 2kHz. All calculations correspond with the simulations.

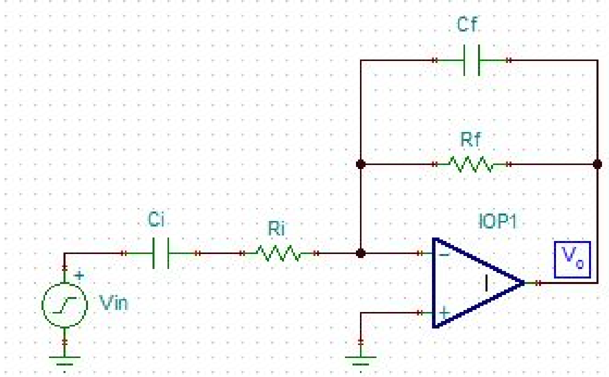
The following is a measured frequency domain approximation using the fast-fourier transform function (FFT) on the oscilloscope. The input signal for the circuit in this test measurement was white noise. The low-band and high-band were set to 0% with the mid-band set to 100%.

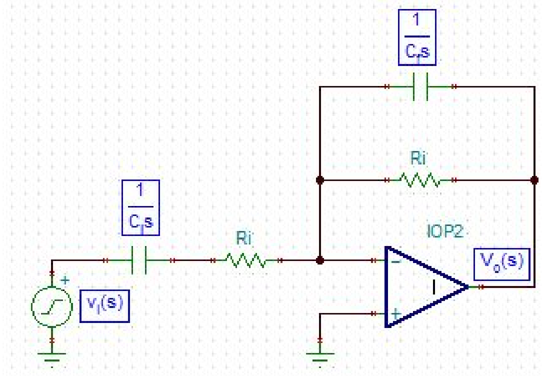


The measurements correspond with mathematical calculations and simulations. In the measured result, the gain increases toward the first cutoff frequency (500Hz) then decreases gain after the second cutoff frequency (2kHz).

4.4 Improved Mid EQ Band

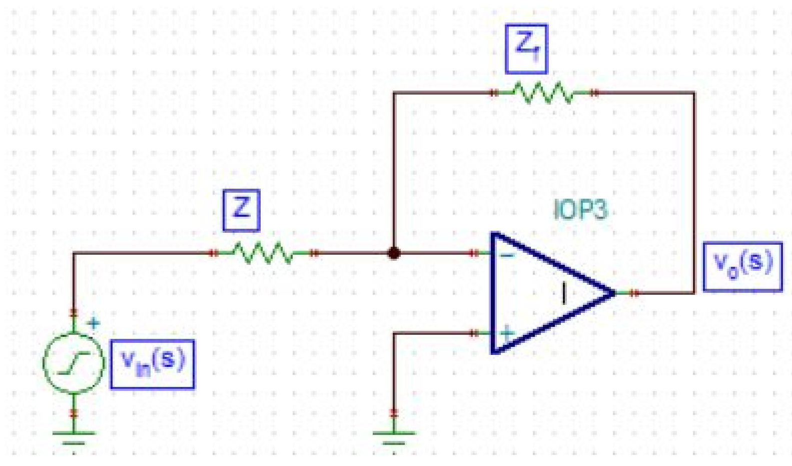
In the previous section (4.3) we discussed the design of a second-order bandpass filter in the most simplistic and brute force method. In this section we will discuss how to design a more efficient bandpass filter (using less opamps and total components). Recall the circuit for a first-order high pass filter. If we add a capacitor in parallel with the feedback resistor, it becomes a bandpass filter. Likewise, if we add a capacitor in series with the input resistor in a low pass filter it becomes a bandpass filter (same circuit).





We can calculate for a new transfer function for this circuit. The circuit in the s-domain can be rewritten as shown above (right).

The input capacitor and input resistor impedance can be combined in series. The feedback capacitor and feedback resistor impedance can be combined in parallel. The circuit can be rewritten such as,



Where Z_i equals the impedance of the input capacitor and input resistor; Z_f equals the impedance of the feedback capacitor and the feedback resistor.

The impedance of the feedback capacitor and feedback resistor in parallel,

$$Z_f = \frac{R_f}{1 + j\omega R_f C_f}$$

The impedance of the input capacitor and input resistor in series,

$$Z_i = R_i(1 + \frac{1}{j\omega R_i C_i})$$

For an inverting opamp circuit in the s-domain,

$$H(j\omega) = -\frac{Z_f}{Z_i}$$

$$H(j\omega) = \frac{-R_f}{(1 + j\omega R_f C_f)(1 + \frac{1}{j\omega R_i C_i})R_i}$$

$$H(j\omega) = \frac{\frac{-R_f}{R_i}}{(1 + j\omega R_f C_f)(1 + \frac{1}{j\omega R_i C_i})}$$

Therefore, the transfer function for this circuit is calculated such as,

$$H(j\omega) = \frac{-j\omega C_i R_f}{(j\omega C_i R_i + 1)(j\omega C_f R_f + 1)}$$

By analyzing the denominator of the transfer function we can see the circuit has two cutoff frequencies,

Note: $\omega = 2\pi f$

$$f_{c1} = \frac{1}{2\pi R_i C_i} \quad f_{c2} = \frac{1}{2\pi R_f C_f}$$

The circuit has a DC gain of,

$$A_v = -\frac{R_f}{R_i}$$

For a DC gain of +3.75 we know from previous calculations,

$$R_f = 10k\Omega$$
$$R_i = 6.5k\Omega$$

Solving for the input capacitor in regards to the high pass cutoff frequency (2kHz),

$$f_{c1} = \frac{1}{2\pi R_i C_i}$$

$$2000Hz = \frac{1}{2\pi \cdot 6.5k\Omega \cdot C_i}$$

$$C_i = 1.224 \cdot 10^{-8}F = 0.012\mu F$$

Solving for the feedback capacitor in regards to the low pass cutoff frequency (500Hz),

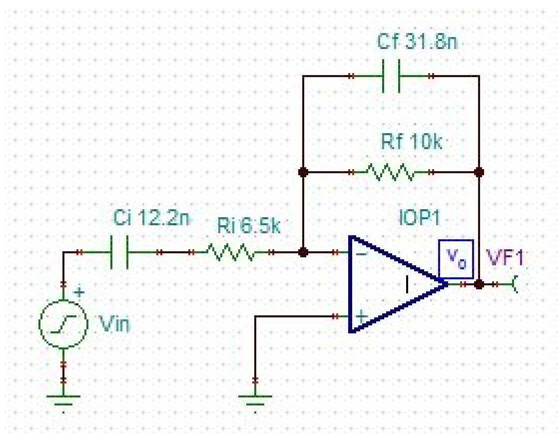
$$f_{c2} = \frac{1}{2\pi R_f C_f}$$

$$500Hz = \frac{1}{2\pi \cdot 10k\Omega \cdot C_f}$$

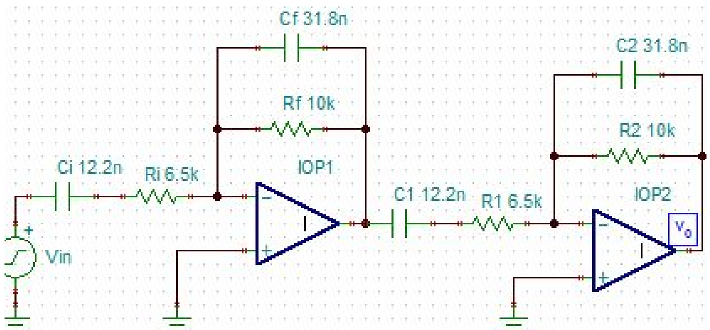
$$C_f = 3.18 \cdot 10^{-8}F = 0.032\mu F$$

Therefore, the component values of a first-order bandpass filter using this methodology are,

$$R_f = 10k\Omega$$
$$R_i = 6.5k\Omega$$
$$C_i = 0.012\mu F$$
$$C_f = 0.032\mu F$$



We can make the first-order bandpass filter a second-order bandpass filter by cascading the same stage in series,



To find the overall gain (transfer function) for the second-order filter we can multiply the first and second stage of the circuit,

$$H(j\omega) = \frac{v_{o2}}{v_{o1}} \cdot \frac{v_{o1}}{v_i}$$

$$H(j\omega) = \frac{j\omega^2 C_{i1} C_{i2} R_{f1} R_{f2}}{(1 + j\omega C_{i2} R_{i2})(1 + j\omega C_{i1} R_{i1})(1 + j\omega C_{f2} R_{f2})(1 + j\omega C_{f1} R_{f1})}$$

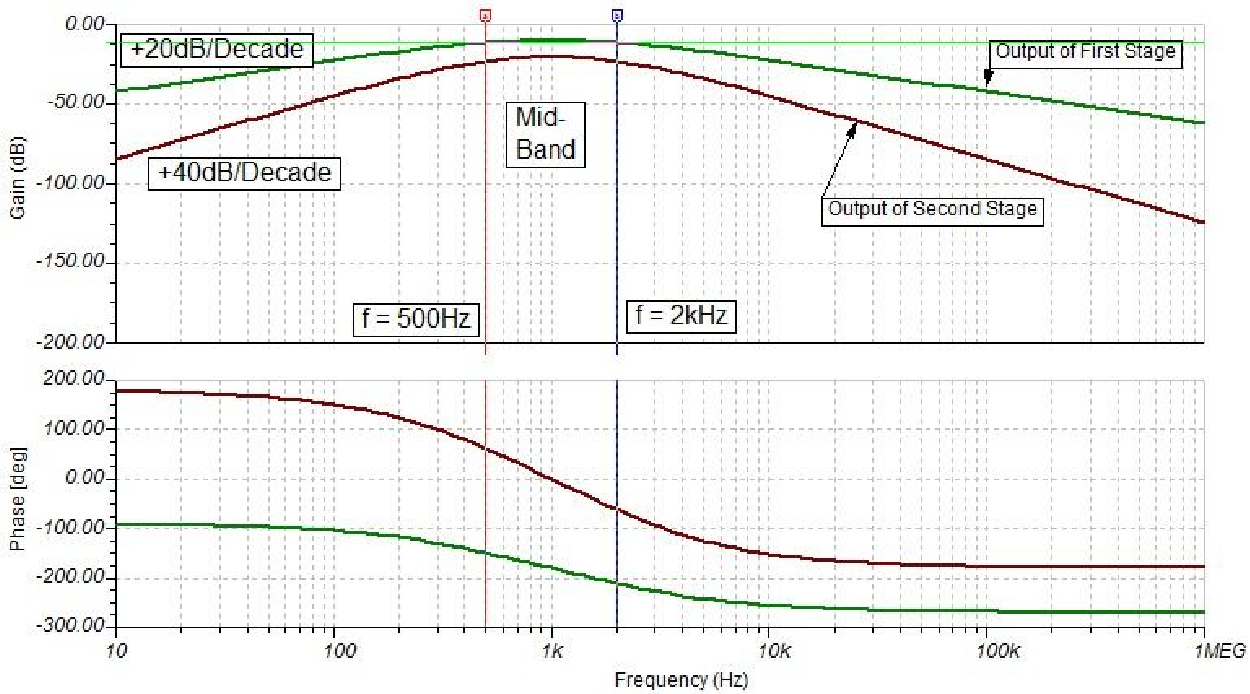
The slope of the cascaded second-order filter will also have an increased slope magnitude. The total roll-off by a higher order circuit is given by,

$$\Delta L_T = n\Delta L = 20ndB/Decade$$

The total roll-off for a second-order filter can be calculated by,

$$\Delta L_T = (2)\Delta L = 20(2)dB/Decade = 40dB/Decade$$

Simulation (bode plot) of the first and second stage of the improved second-order bandpass filter,

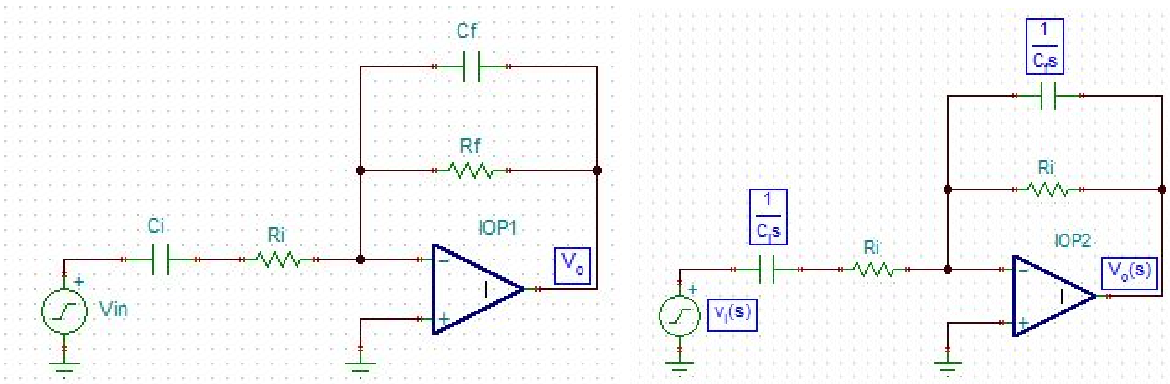


The improved second-order filter corresponds with the mathematical calculations. The simulation is identical to the non-improved filter in the previous section (4.4) as anticipated.

4.5 Improved High EQ Band

A problem with the non-improved second-order high pass filter from section 4.2 is that it will have a large DC gain with a large frequency input. We can transform the highpass filter into a bandpass filter by placing a capacitor in parallel with the feedback resistor. By making a highpass filter a bandpass filter we can limit the DC gain of the filter at very large frequencies.

Recall the circuit for the improved bandpass filter,



Recall the transfer function for the first-order circuit as calculated in previous sections,

$$H(j\omega) = \frac{-j\omega C_i R_f}{(j\omega C_i R_i + 1)(j\omega C_f R_f + 1)}$$

Recall the equations to find the cutoff frequencies,

Note: $\omega = 2\pi f$

$$f_{c1} = \frac{1}{2\pi R_i C_i} \quad f_{c2} = \frac{1}{2\pi R_f C_f}$$

We are still using a DC gain of +3.75 in the calculations. First solving for the first cutoff frequency of 2kHz (highpass frequency),

$$f_{c1} = \frac{1}{2\pi R_i C_i}$$

$$2kHz = \frac{1}{2\pi \cdot 6.5k\Omega \cdot C_i}$$

$$C_i = 12.2nF$$

To limit the DC gain we will set the second cutoff frequency to 20kHz as human ears cannot hear frequencies greater than 20kHz.

Solving for the second cutoff frequency of 20kHz (limiting frequency),

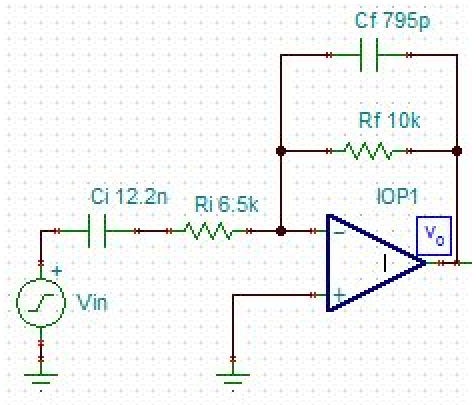
$$f_{c2} = \frac{1}{2\pi R_f C_f}$$

$$20kHz = \frac{1}{2\pi \cdot 10k\Omega \cdot C_f}$$

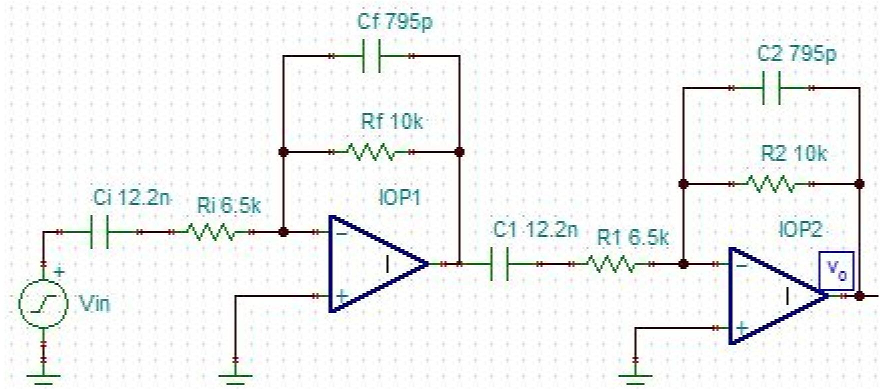
$$C_f = 7.95 \cdot 10^{-10} F = 795pF$$

Therefore, the component values of our first-order improved high pass (bandpass) filter are,

$$R_f = 10k\Omega$$
$$R_i = 6.5k\Omega$$
$$C_i = 12.2nF$$
$$C_f = 795pF$$



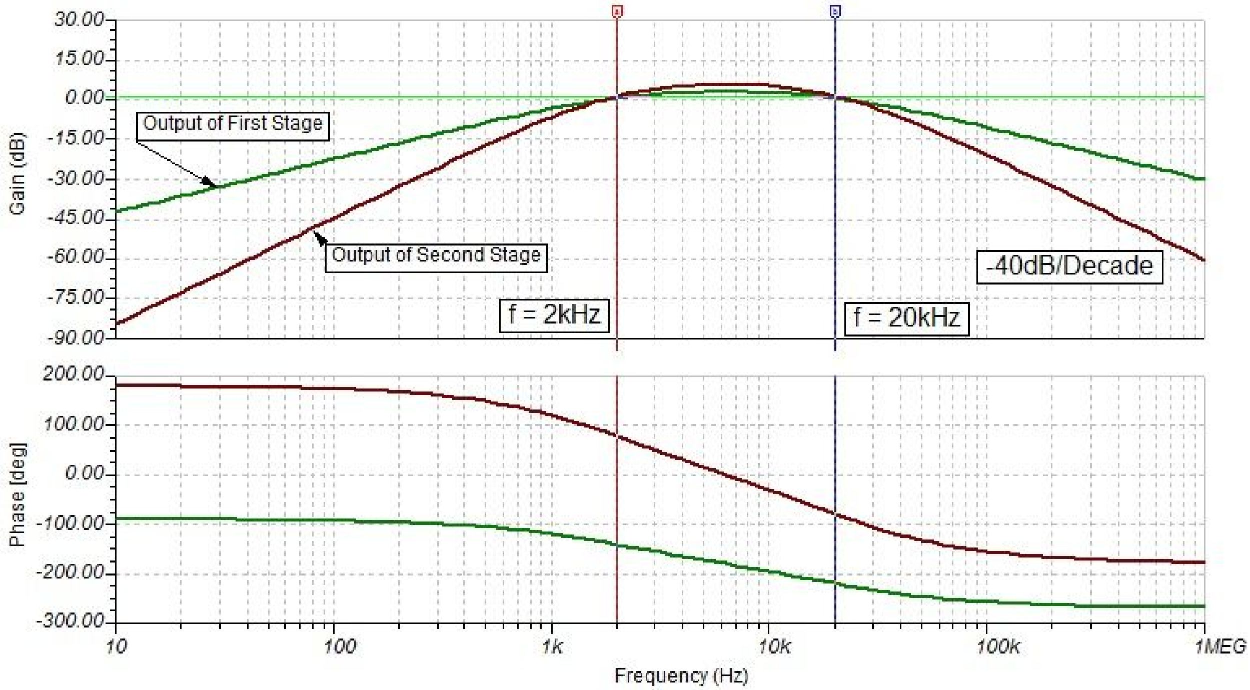
We can create an improved second-order high pass filter by simply cascading the same stage in series,



And the transfer function for the second-order filter becomes (same as the transfer function in section 4.4),

$$H(j\omega) = \frac{j\omega^2 C_{i1} C_{i2} R_{f1} R_{f2}}{(1 + j\omega C_{i2} R_{i2})(1 + j\omega C_{i1} R_{i1})(1 + j\omega C_{f2} R_{f2})(1 + j\omega C_{f1} R_{f1})}$$

Simulation (bode plot) of the first and second stage of the improved second-order high pass filter,

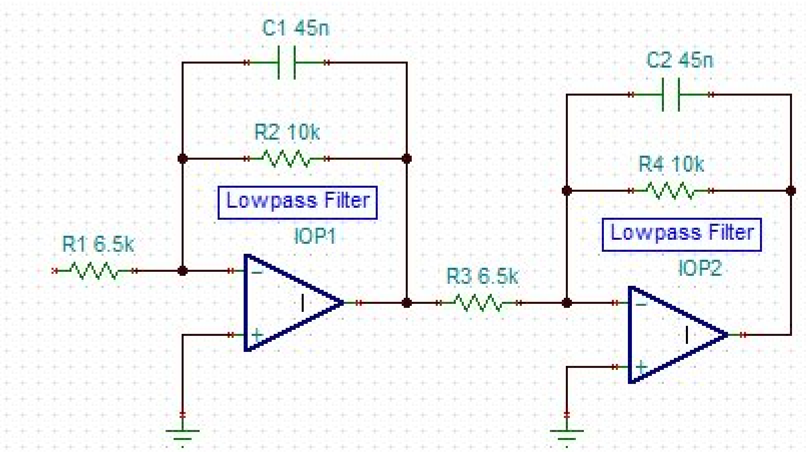


The improved second-order high pass filter corresponds with the mathematical calculations. The gain of the filter increases as it increases towards the first cutoff frequency. After the 20kHz cutoff frequency the gain begins to decrease. Therefore, this improved filter will limit DC gain at high frequencies.

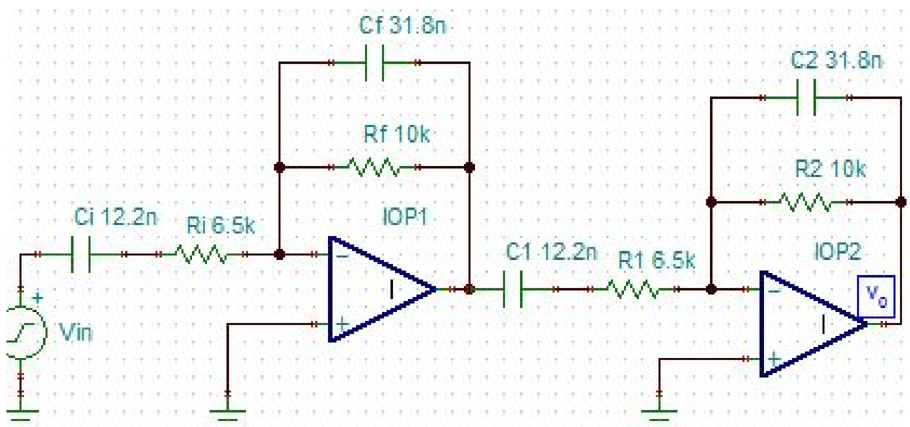
4.6 3-band audio equalizer

We can combine low pass band (4.1), improved mid pass band (4.4), and improved high pass band (4.5) to create a simple adjustable 3-band audio equalizer. The output of each band will have a potentiometer where the output leads into an inverting summing amplifier.

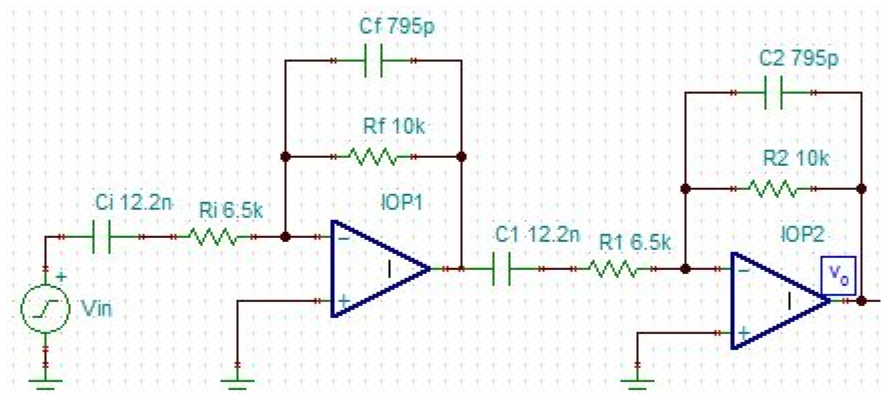
Recall the circuit for the lowpass band,



Recall the circuit for the improved mid pass band,

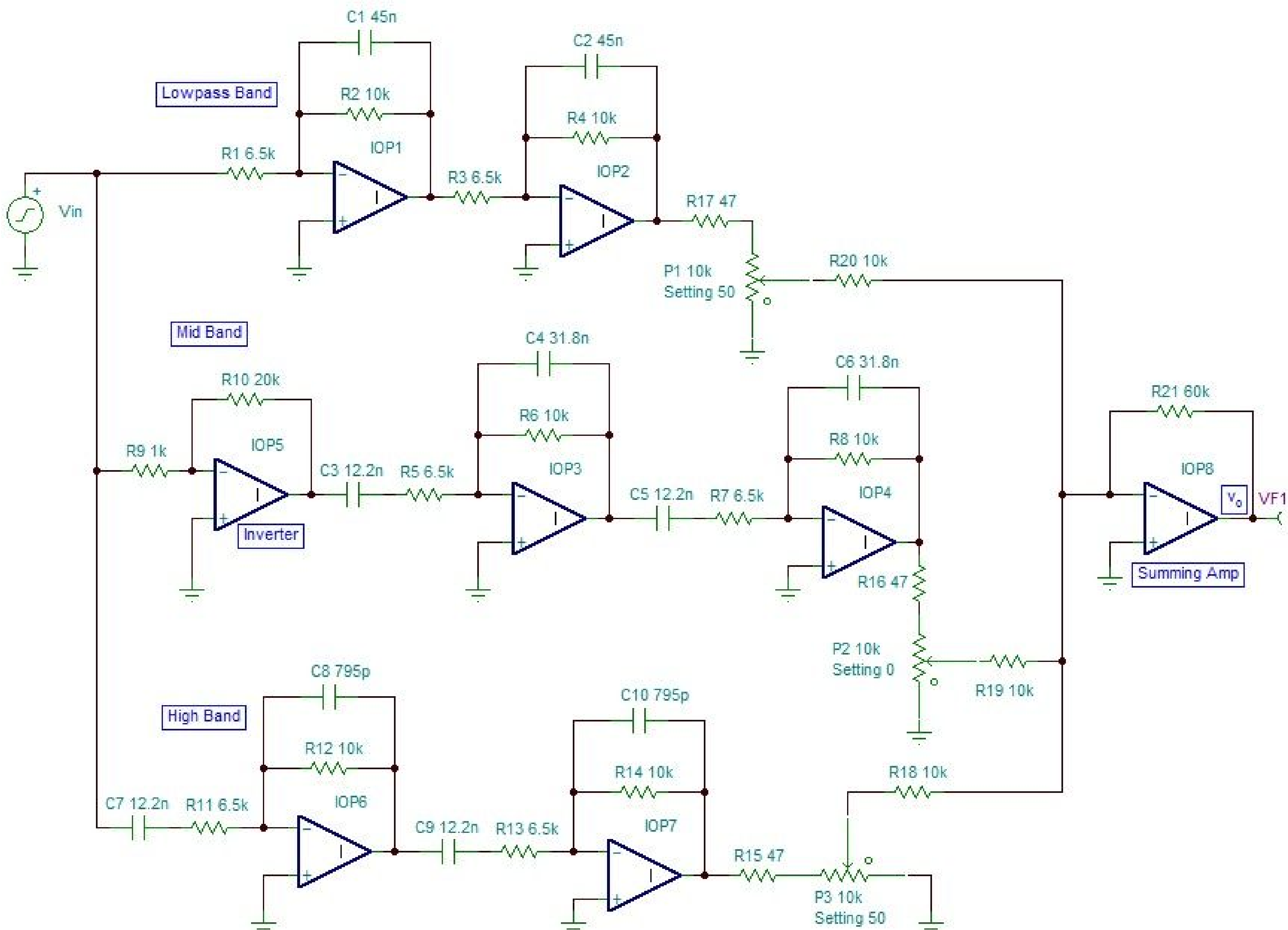


Recall the circuit for the improved high pass band,



To combine for a 3-band audio equalizer:

1. Connect the input of each individual band to a common source.
2. Add an additional inverting amplifier [with gain] for the mid band to correct phase shifts (explained with more detailed in section 4.7).



3. Add a potentiometer to the output of each band with a 47 Ohm resistor.
4. Connect the output of each potentiometer to the input of an inverting summing amplifier.

The following is the completed circuit for a second-order 3-band audio equalizer,

Items to note:

- The additional inverting amplifier to correct phase shifts for the mid band provides a gain of +26dB to compensate for what would normally have low gain.

$$A_v = 20\log_{10}(20k\Omega/1k\Omega) = +26dB$$

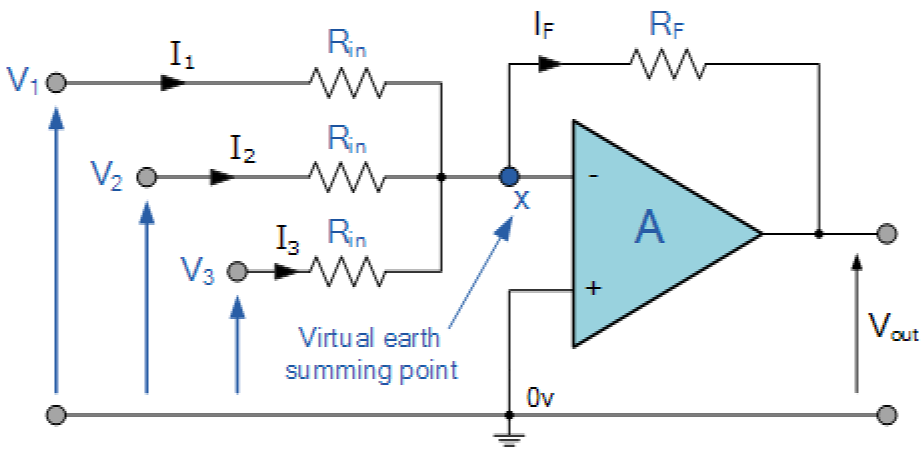
- The bottom leg of the output potentiometer is in parallel with the input resistor for the summing amplifier. E.g at 50% position (10kOhm potentiometer):

$$V_{po} = V_{in} \cdot \frac{(P_2||R_i)}{47\Omega + P_2 + (P_1||R_i)}$$

$$V_{po} = V_{in} \cdot \frac{(5k\Omega||10k\Omega)}{47\Omega + 5k\Omega + (5k\Omega||10k\Omega)}$$

$V_{po} = V_{in} \cdot 0.3977$ (Equation for solving output of the potentiometer [50% position] in circuit)

The following is a general diagram for an inverting summing amplifier,



Source: http://www.electronics-tutorials.ws/opamp/opamp_4.html

Solving for V_{out} in the diagram is calculated such by,

$$-V_o = [V_1\frac{R_f}{R_i} + V_2\frac{R_f}{R_i} + V_3\frac{R_f}{R_i}]$$

We can solve for the output of the inverting summing amp using the equation to find the voltage at the end of the potentiometer and the equation of the output of an inverting summing amplifier.

Low Pass Band Output Calculations

The amplitude of the output for the low pass band at 1kHz is 263.15mV:

Amplitude	263.15mV

Calculating for the output of the potentiometer at 50% position,

$V_{po} = V_{in} \cdot 0.3977$

$V_{po} = 0.26315V \cdot 0.3977$

$V_{po} = 0.104655V$

Mid Band Output Calculations

The amplitude of the output for the mid pass band at 1kHz is 1.89V:

Amplitude	1.89V

Calculating for the output of the potentiometer at 50% position,

$V_{po} = V_{in} \cdot 0.3977$

$V_{po} = 1.89V \cdot 0.3977$

$V_{po} = 0.75012V$

High Band Output Calculations

The amplitude of the output for the high pass band at 1kHz is 469.56mV:

Amplitude	469.56mV

Calculating for the output of the potentiometer at 50% position,

$V_{po} = V_{in} \cdot 0.3977$

$V_{po} = 0.46956V \cdot 0.3977$

$V_{po} = 0.186744V$

We can now substitute the calculated outputs into the equation for finding the output of the inverting summing amplifier. Recall the equation for finding the output of the inverting summing amplifier,

$$-V_o = [V_1 \frac{R_f}{R_i} + V_2 \frac{R_f}{R_i} + V_3 \frac{R_f}{R_i}]$$

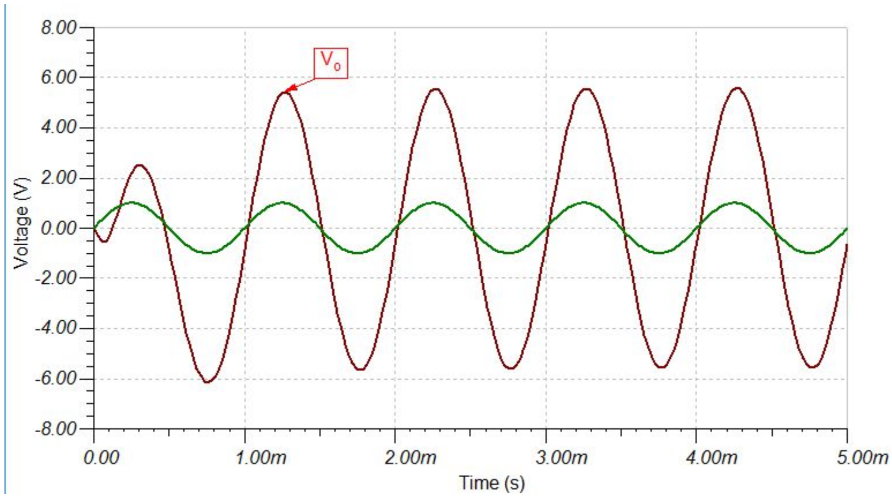
Substituting for the calculated values,

$$-V_o = [0.104655V \frac{60k\Omega}{10k\Omega} + 0.751653V \frac{60k\Omega}{10k\Omega} + 0.186744V \frac{60k\Omega}{10k\Omega}]$$

$$-V_o = [0.62793V + 4.50992V + 1.12046V]$$

$$-V_o = 6.25V$$

The following is a time domain simulation of the output of the inverting summing amplifier,



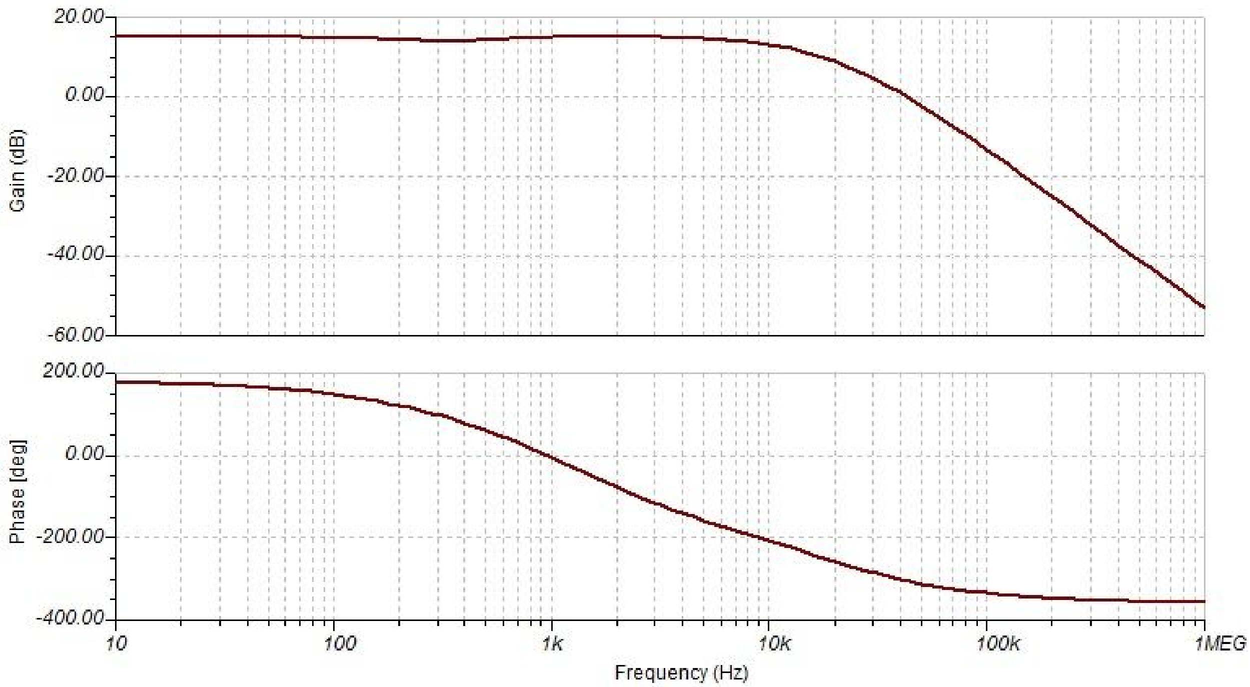
The output of the inverting summing amplifier correspond with the calculations.

The transfer function of the 3-band audio equalizer in its entirety can be written as,

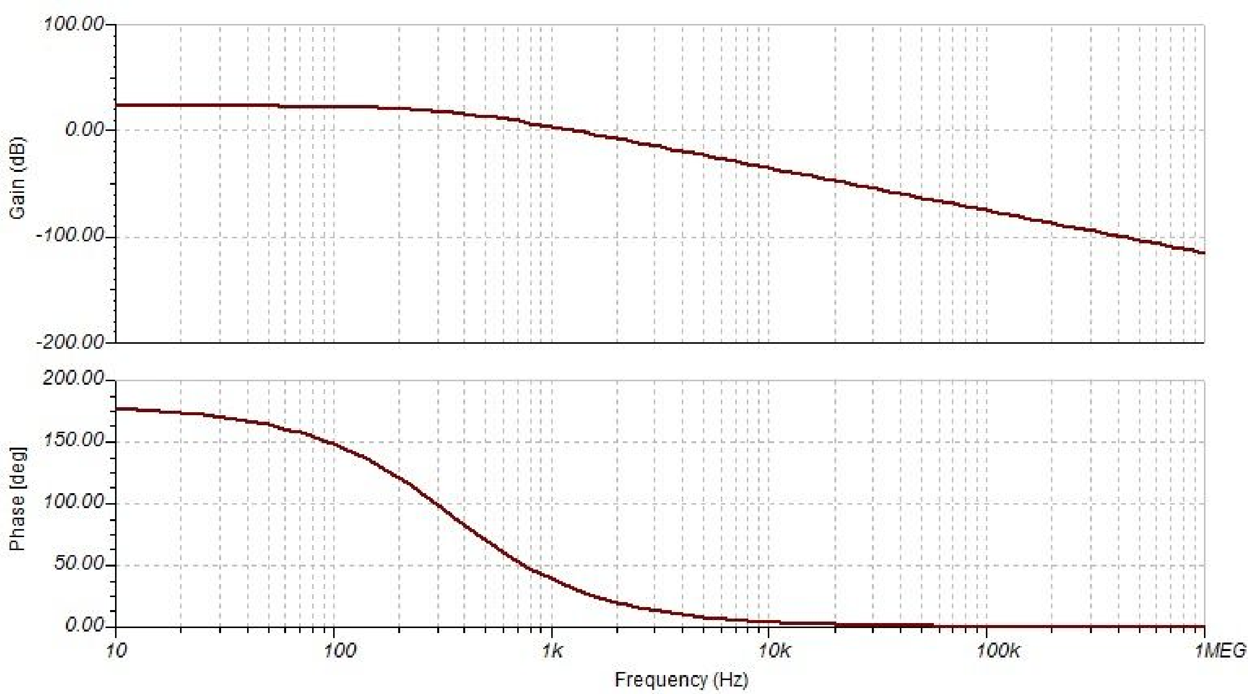
$$H(j\omega) = -\frac{R_{fs}}{R_{is}}\left[\frac{(R_{f1}/R_{i1})(R_{f2}/R_{i2})}{(1+j\omega C_{f1}R_{f1})(1+j\omega C_{f2}R_{f2})} + \frac{j\omega^2 C_{i1}C_{i2}R_{f1}R_{f2}}{(1+j\omega C_{i2}R_{i2})(1+j\omega C_{i1}R_{i1})(1+j\omega C_{f2}R_{f2})(1+j\omega C_{f1}R_{f1})} \frac{j\omega^2 C_{i1}C_{i2}R_{f1}R_{f2}}{(1+j\omega C_{i2}R_{i2})(1+j\omega C_{i1}R_{i1})(1+j\omega C_{f2}R_{f2})(1+j\omega C_{f1}R_{f1})}\right]$$

Where R_{fs} is the feedback resistor of the summing amplifier, R_{is} is the input resistor for the summing amplifier. Each term in the function represents an individual band of the 3-band audio equalizer (e.g low,mid,highs) and therefore each component subscript is peritant to that band only.

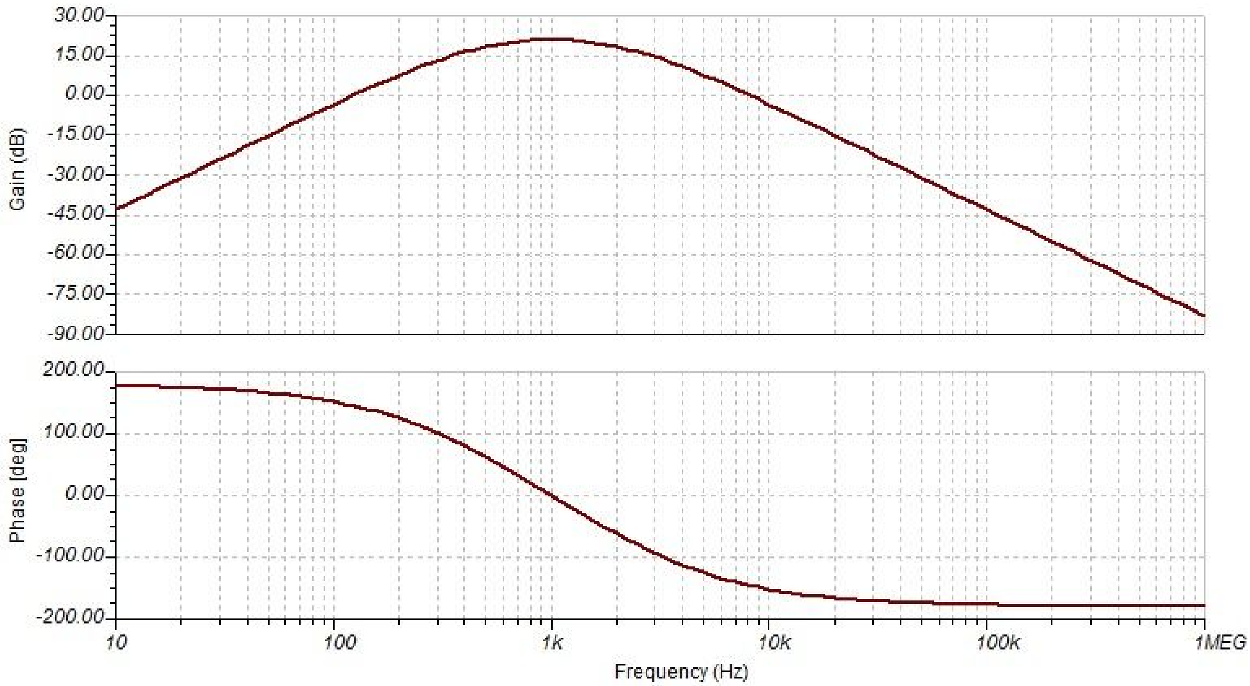
The following is a frequency domain (bode) simulation of the 3-band audio equalizer when the low,mid,high bands are set to the 50% position,



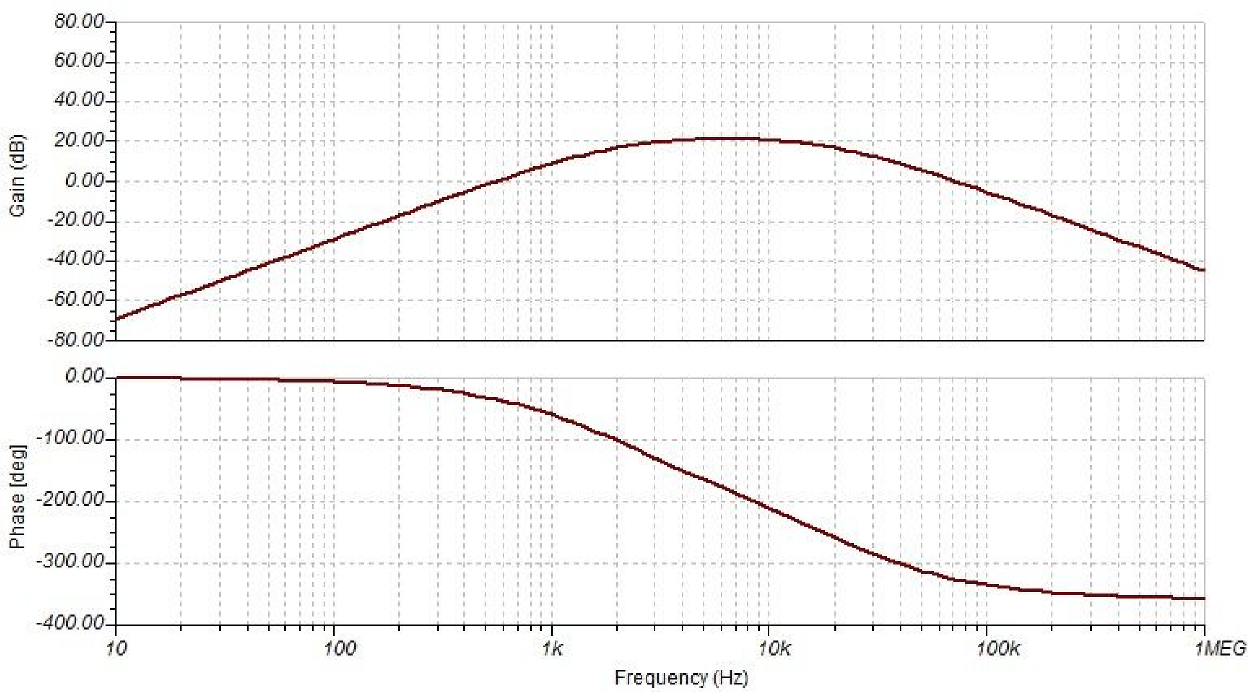
The following is a frequency domain (bode) simulation of the 3-band audio equalizer when the low band is at the 100% position; mid and high bands are at the 0% position,



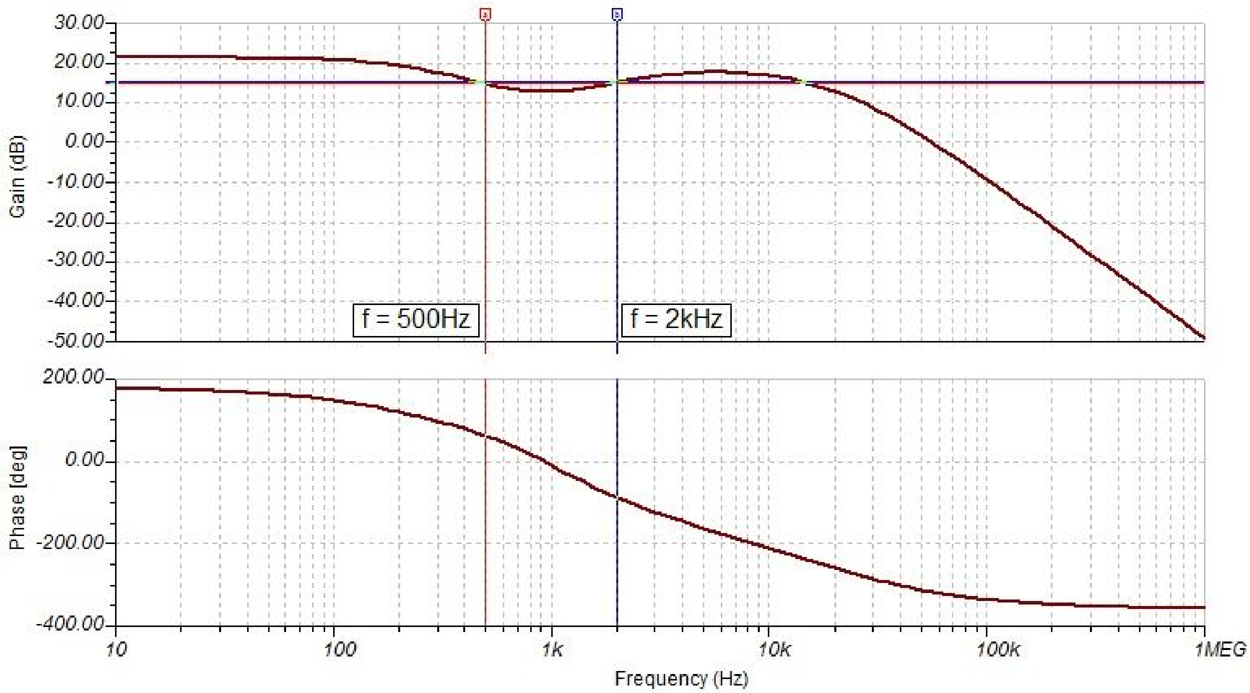
The following is a frequency domain (bode) simulation of the 3-band audio equalizer when the mid band is at the 100% position; low and high bands are at the 0% position,



The following is a frequency domain (bode) simulation of the 3-band audio equalizer when the high band is at the 100% position; low and mid bands are at the 0% position,

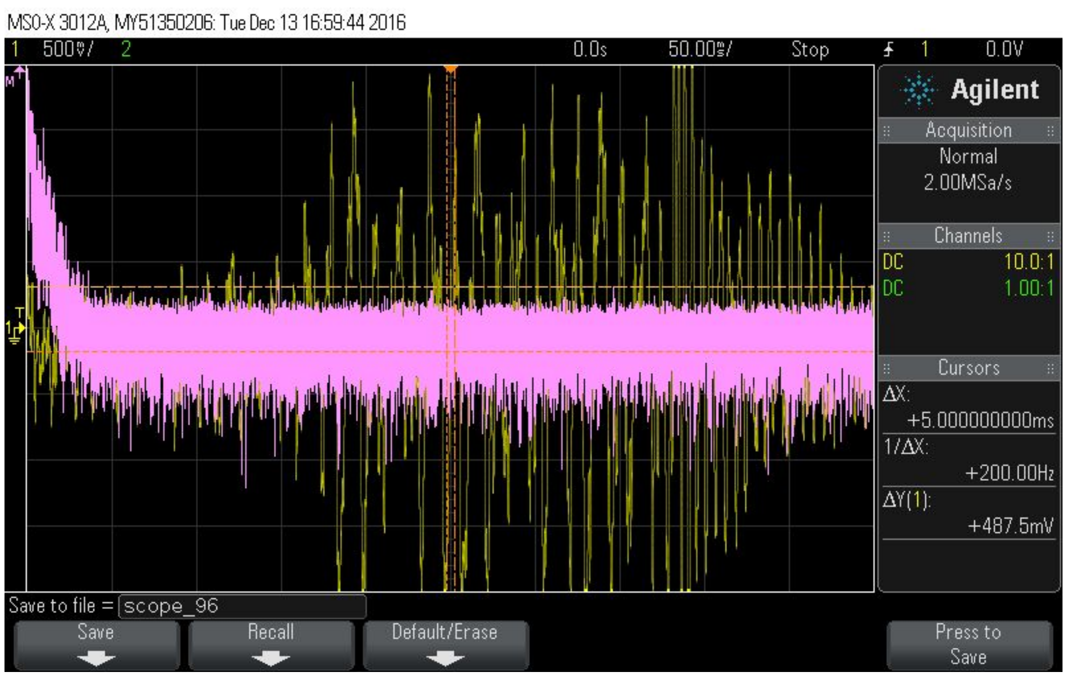


The following is a frequency domain (bode) simulation of the 3-band audio equalizer when the low band is at 90% position, mid band is at 25% position, and high band is at 75% position,

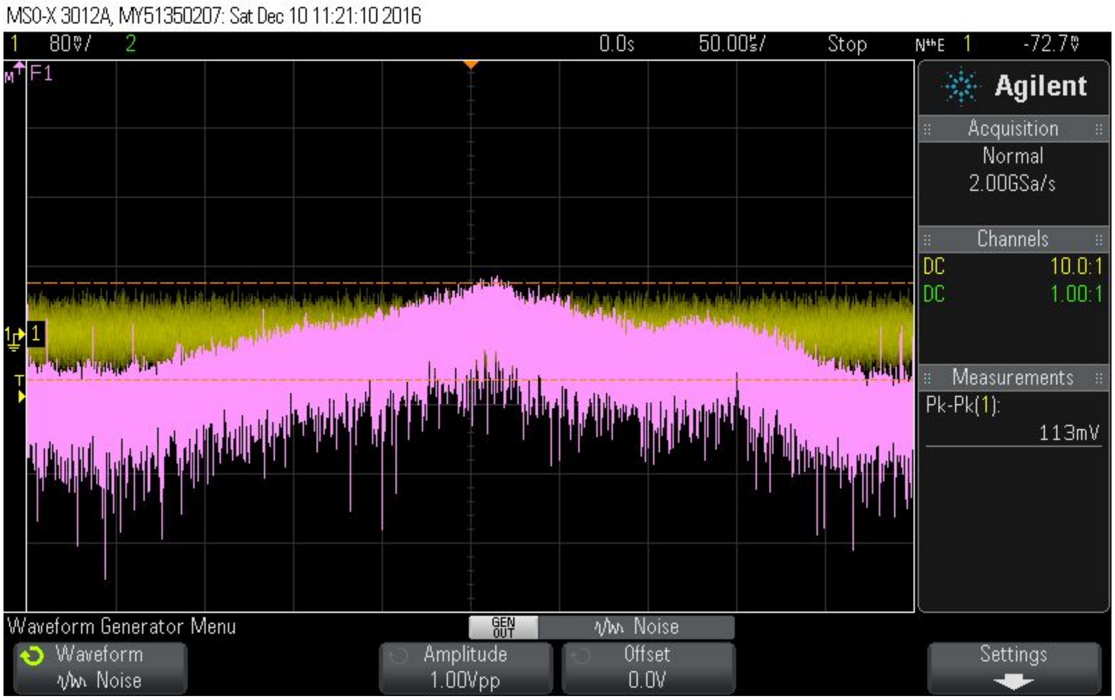


The simulations correspond with the mathematical calculations for each band. The gain in the frequency domain simulations adjust depending on the position of each potentiometer for each band.

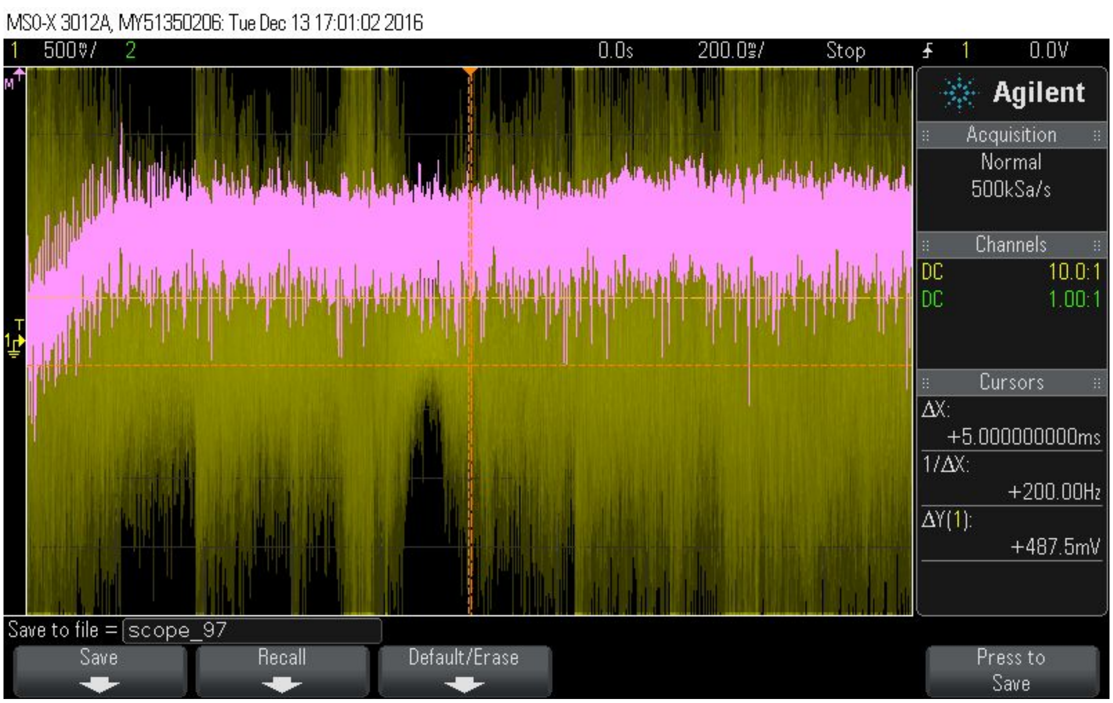
The following is the measured frequency domain plot of the 3-band audio equalizer using the FFT function on the oscilloscope when the low band is at 100% position and the mid and high bands are at the 100% position. The input for the circuit is electronic music,



The following is the measured frequency domain plot of the 3-band audio equalizer using the FFT function on the oscilloscope when the mid band is at 100% position and the low and high bands are at the 0% position. The input for the circuit is electronic music,



The following is the measured frequency domain plot of the 3-band audio equalizer using the FFT function on the oscilloscope when the high band is at 100% position and the mid and high bands are at the 0% position. The input for the circuit is electronic music,



The following is the measured frequency domain plot of the 3-band audio equalizer using the FFT function on the oscilloscope when all bands are at the 50% position. The input for the circuit is electronic music,

All measurements correspond with mathematical calculations and simulations.

Video Demonstration

https://www.youtube.com/watch?v=ydMIfWoZ1mg

4.7 Cutoff frequency selections
Why did we choose frequencies of 500Hz and 2kHz for bandpass filter?

We chose the values of 500Hz and 2kHz because the spectrum of the midrange frequencies of audio are frequencies from 500Hz to 2kHz. The audio spectrum is the audible frequency range at which humans can hear. The audio spectrum range spans from 20Hz to 20kHz and can be broken down into different frequency bands, with each having a different impact on the total sound.

The seven frequency bands are:

8. Sub-bass (20Hz - 60Hz)
9. Bass (60Hz - 250Hz)
10. Low midrange (250Hz - 500Hz)
11. **Midrange (500Hz - 2kHz)**
12. Upper midrange (2kHz - 4kHz)
13. Presence (4kHz - 6kHz)
14. Brilliance (6kHz - 20kHz)

For the mid-band we wanted to affect the gain on the midrange frequencies (500Hz to 2kHz). The high pass filter will cut-off at 500Hz and begin to increase gain. The low pass will cut-off at 2kHz and begin to decrease gain.

Why did we choose 20kHz cutoff frequency for high pass filter?

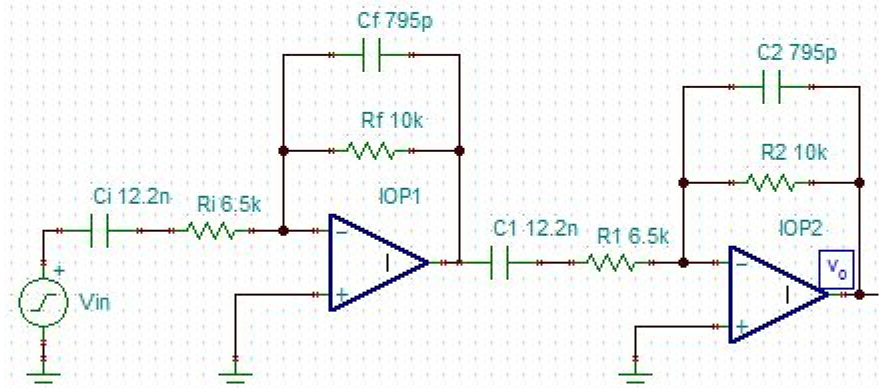
The cutoff frequency of 20kHz was chosen for the high pass filter to limit DC gain. Human ears cannot hear any sound from frequencies greater than 20kHz.

4.8.1 Phase shift analysis

In order to verify all the phase angles of each band are not out of phase, we will separate each band individually and perform analysis. We don't want any band to be shifted out of phase from any other.

4.8.2 High-band phase analysis

Recall the high pass band portion of the audio equalizer,



Before calculating for the phase angle we need to find the center frequency of the filter. Recall the lower cutoff frequency is 2kHz and the upper cutoff frequency is 20kHz. The center frequency can be calculated by,

$$f_c = \sqrt{f_L \cdot f_U}$$

$$f_c = \sqrt{2kHz \cdot 20kHz}$$

$$f_c = 6324Hz$$

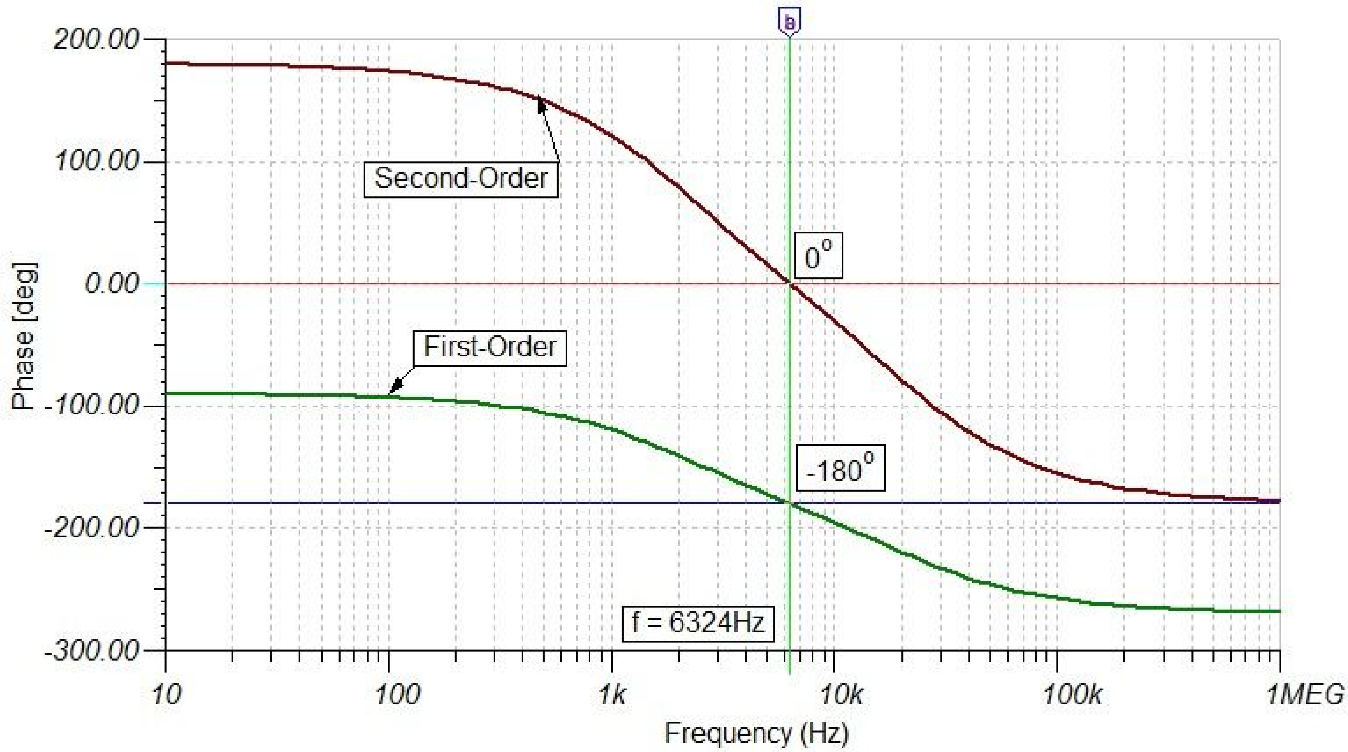
The phase angle for the first stage of the filter (first-order) can be calculated by,

$$\phi = 180° - \tan^{-1}(2\pi \cdot f_c C_i R_i C_f R_f)$$
$$\phi = 180° - \tan^{-1}(2\pi \cdot 6.3kHz \cdot 12.2 \cdot 10^{-9}F \cdot 6.5k\Omega \cdot 795 \cdot 10^{-12}F \cdot 10k\Omega)$$
$$\phi = -180°$$

To find the phase angle of the second stage (second order), subtract the same calculated phase angle from the first stage (another 180° shift),

$$\phi = 180° - 180°$$
$$\phi = 0°$$

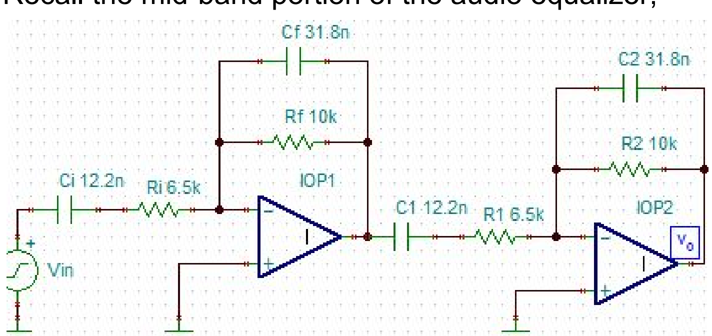
The following is a frequency domain phase simulation (phase bode) of the high pass stage from the 3-band audio equalizer,



The simulation corresponds with the mathematical analysis. The first-order stage is at a -180^0 phase angle and the second-order stage is at a 0° phase angle at the center cutoff frequency.

4.8.3 Mid-band phase analysis

Recall the mid-band portion of the audio equalizer,



Before calculating for the phase angle we need to find the center frequency of the filter. Recall the lower cutoff frequency is 500Hz and the upper cutoff frequency is 2kHz. The center frequency can be calculated by,

$$f_c = \sqrt{f_L \cdot f_U}$$

$$f_c = \sqrt{500Hz \cdot 2kHz}$$

$f_c = 1kHz$

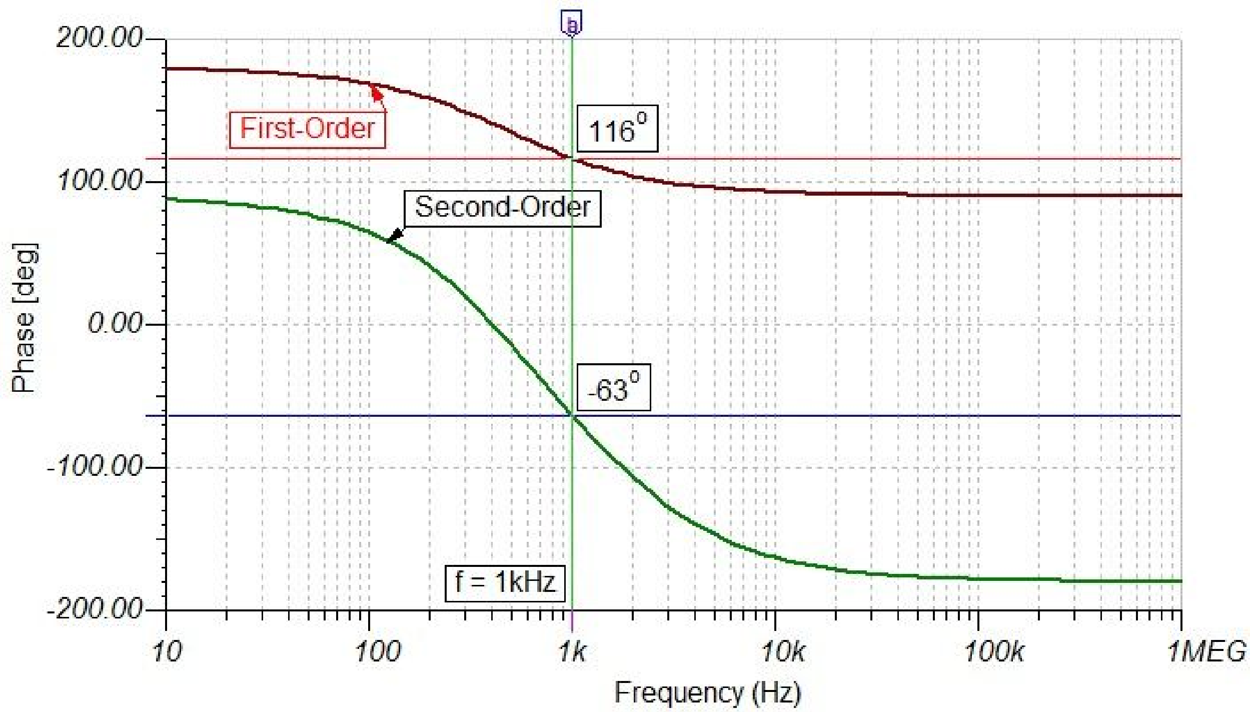
The phase angle for the first stage of the filter (first-order) can be calculated by,

$$\phi = 180^\circ - \tan^{-1}(2\pi \cdot f_c C_i R_i C_f R_f)$$
$$\phi = 180^\circ - \tan^{-1}(2\pi \cdot 1kHz \cdot 12.2 \cdot 10^{-9}F \cdot 6.5k\Omega \cdot 31.8 \cdot 10^{-9}F \cdot 10k\Omega)$$
$$\phi = -180^\circ$$

To find the phase angle of the second stage (second order), subtract the same calculated phase angle from the first stage (another 180° shift),

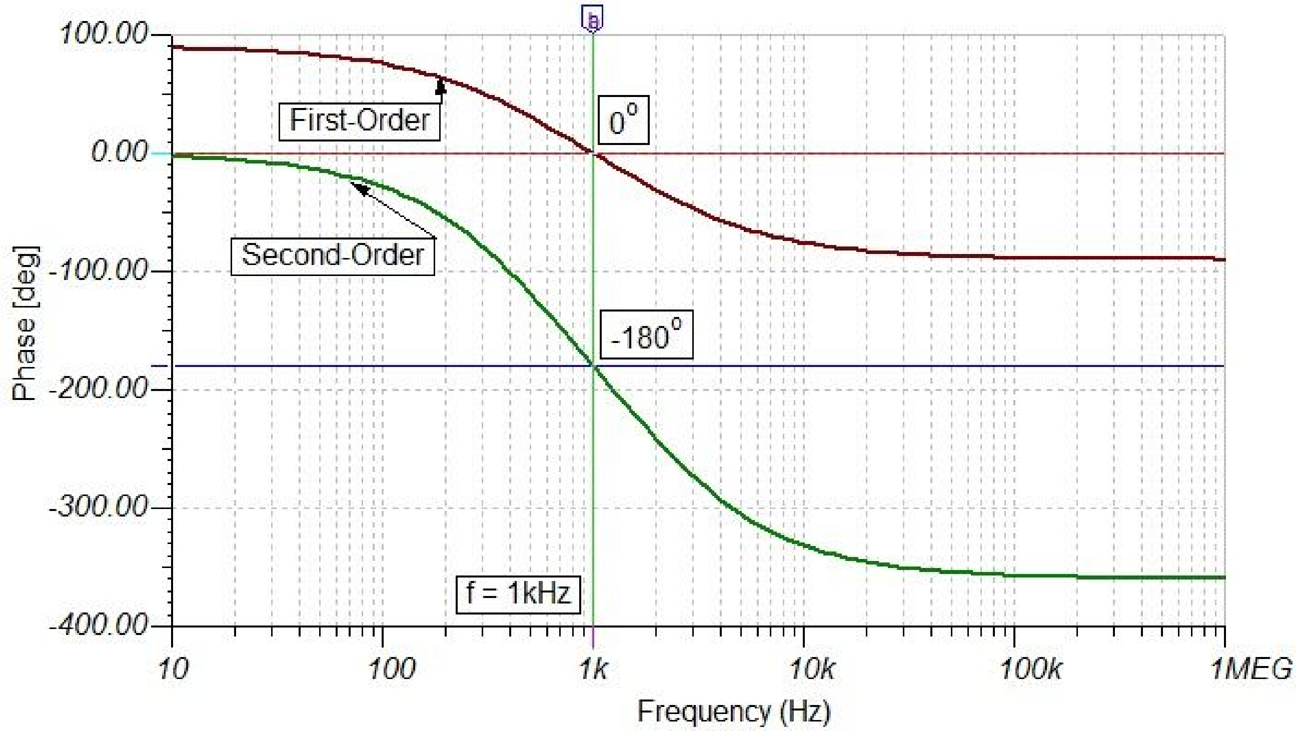
$$\phi = 180^\circ - 180^\circ$$
$$\phi = 0^\circ$$

The following is a frequency domain phase simulation (phase bode) of the mid-band stage from the 3-band audio equalizer (without being fixed),

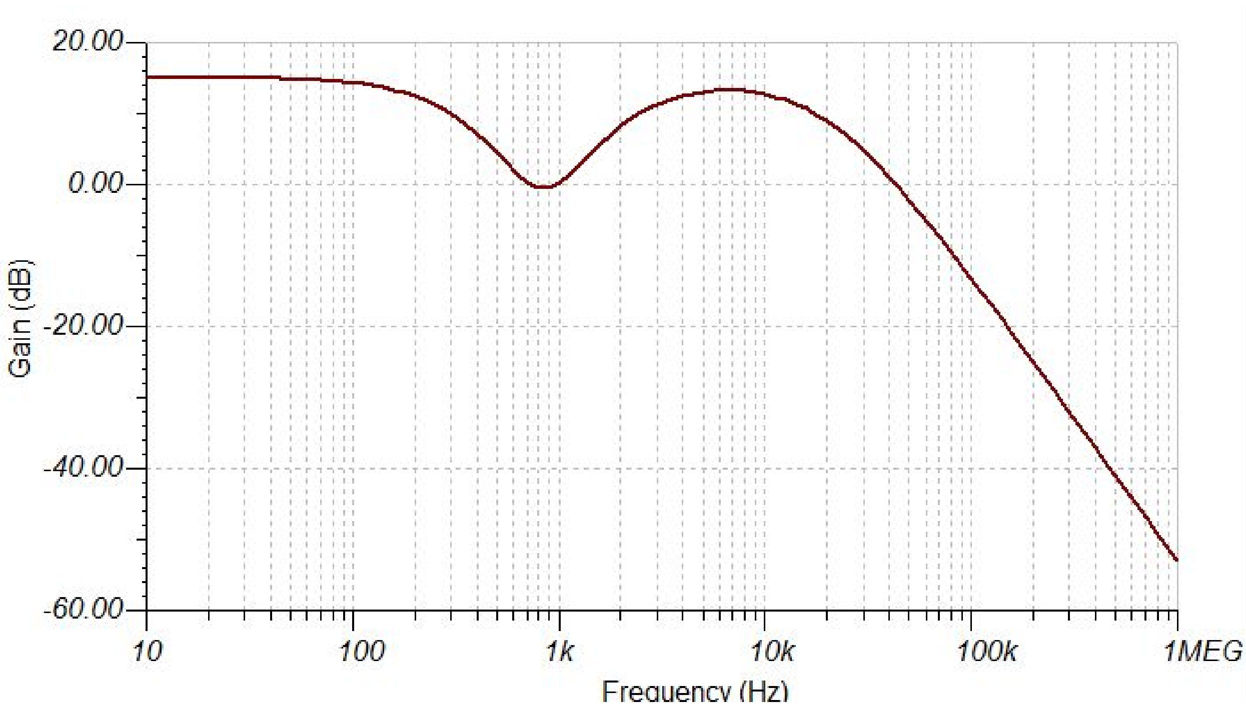


The band pass stage creates an extra 180° degree shift.

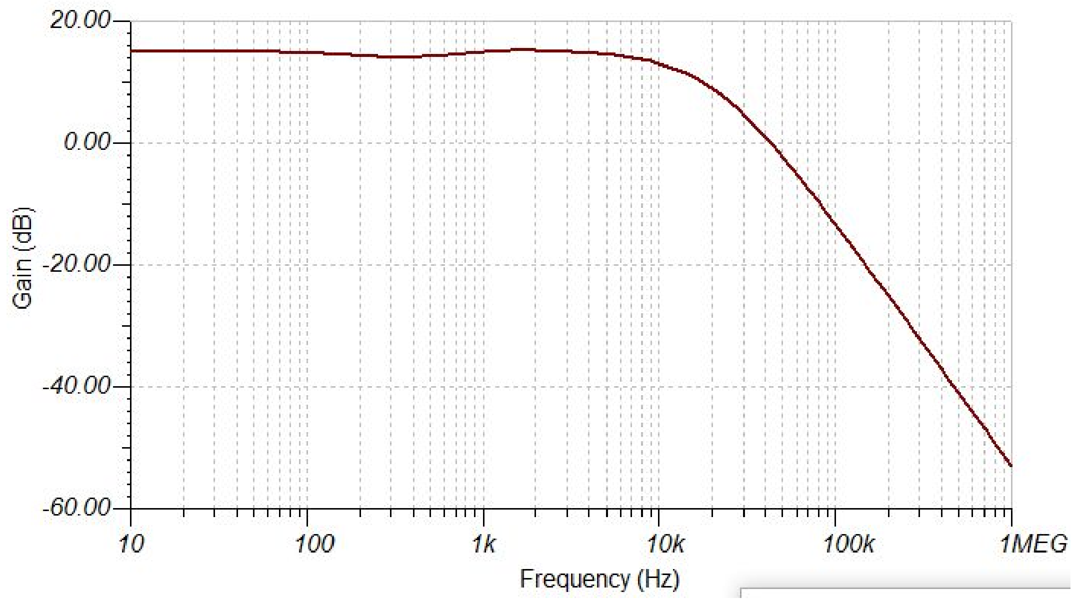
The following is a frequency domain phase simulation (phase bode) of the mid-band stage from the 3-band audio equalizer (with the inverting opamp with gain of 20 added beforehand),



The following is a frequency domain simulation (amplitude bode) of the 3-band equalizer **without** the correction for the phase shift (all knobs at 50%),



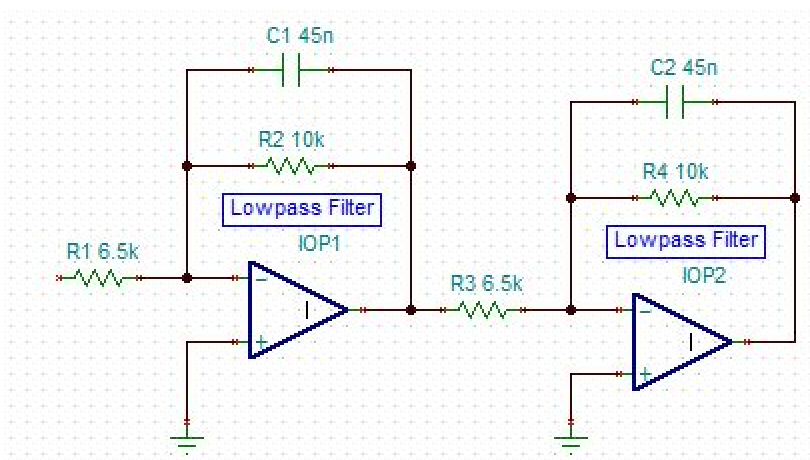
As the knob for the mid section increases towards 100% (more mids) the band pass curve actually goes downwards instead of upwards which is not expected. The following is a frequency domain The following is a simulation (amplitude bode) of the 3-band equalizer **with** the correction for the phase shift (all knobs at 50%),



The circuit with this correction correctly adjusts the mid-section upwards and downwards depending on the potentiometer position for the mid-band.

4.8.4 Low-band phase analysis

Recall the low pass band portion of the audio equalizer,



The cutoff frequency for this filter is 500Hz. The phase angle for the first stage of the filter (first-order) can be calculated by,

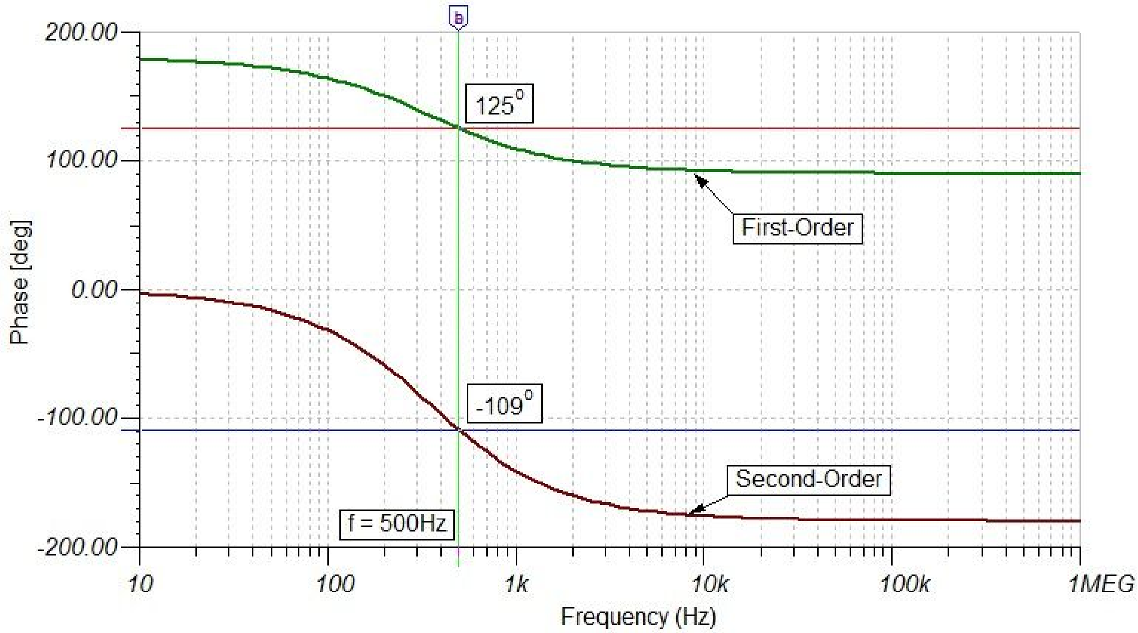
$\phi = 180° - \tan^{-1}(2\pi f_c C_f R_f)$

$\phi = 180° - \tan^{-1}(2\pi \cdot 500Hz \cdot 44.8 \cdot 10^{-9} \cdot 10k\Omega)$

$\phi = 125.394°$

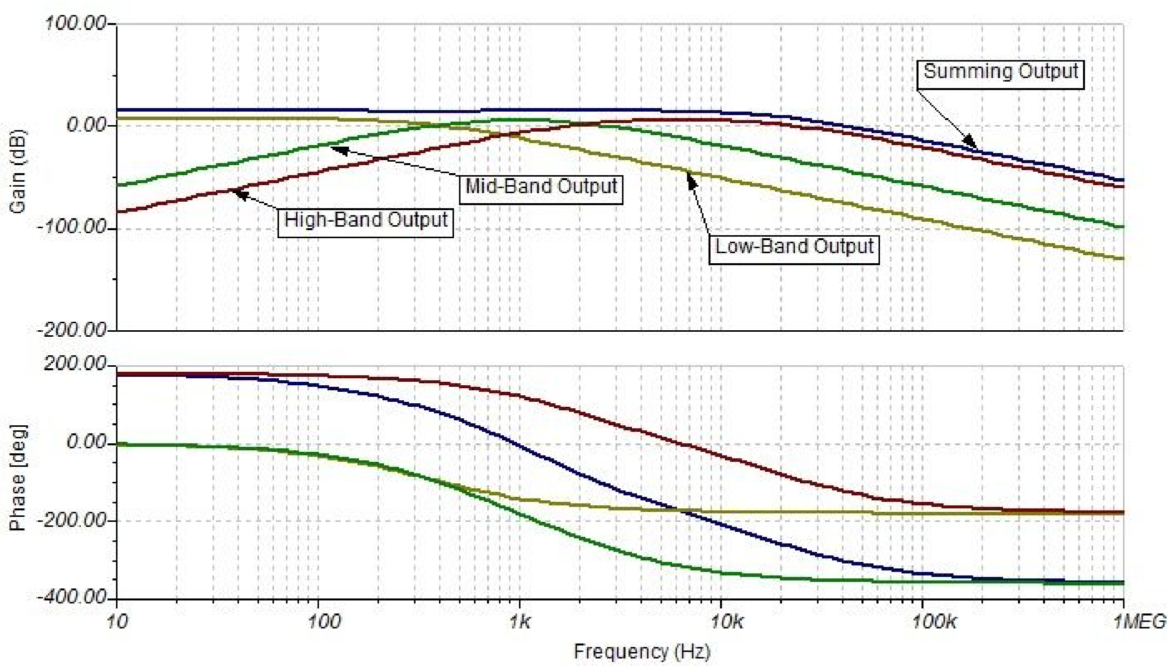
Calculating for the output of the second-order stage,

$$\phi = -\tan^{-1}(2\pi \cdot 500Hz \cdot 44.8 \cdot 10^{-9} \cdot 10k\Omega) - \tan^{-1}(2\pi \cdot 500Hz \cdot 44.8 \cdot 10^{-9} \cdot 10k\Omega) = -109^{\circ}$$



The simulation corresponds with the mathematical analysis. The first-order stage is at a 125° phase angle and the second-order stage is at a -109° phase angle.

The following is a frequency domain simulation of the 3-band audio equalizer circuit with outputs at each stage and final output,



None of the outputs are out of phase and the circuit performs as expected.