Application Note

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Lab 3: Making Small Things Big

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Contents

1. Abstract

This application note discusses the use and operation of an audio amplifier circuit using Texas Instruments OPA548T and TL072 operational amplifiers in a dual configuration to boost small signals.The choice of input and feedback resistor values for inverting opamps and their effect on gain and other factors are discussed. Additionally, problems that may occur if gain is too high or too low or if signal ranges are exceeded are also examined. Using mathematical analysis, simulation, and lab measurements, this application report discusses the fundamental aspects of a dual-opamp audio amplifier circuit.

2. Introduction

Audio amplifier circuits are used in many electrical devices and can be designed using a number of different methods. In this application note, a dual opamp configuration is the method of choice to create this type of circuit. When using this method, there are important aspects in the design of the circuit that must accounted for before its use and operation can be understood. These aspects include how the gain of each stage and the overall circuit gain is determined, how the range of values of feedback and input resistors are chosen, and how the output of the circuit is restricted by the opamps power supply and the input frequency.

3. Background

3.1 Inverting Opamp

An inverting opamp is an opamp configuration in which negative feedback is fed from the output of an opamp to its inverting input through a resistor called the “feedback resistor” . This results in a different signal on the inverting input terminal than the actual input voltage and therefore, the two signals must be separated using another resistor. This resistor is called the “input resistor”. The non-inverting terminal of the opamp is connected to common ground. This setup results in an amplifier with a closed-loop gain that can be controlled by adjusting the values of the feedback and input resistors. This gain can be calculated according to

$$A_V = -\frac{R_f}{R_{in}}$$

Different values for the feedback and input resistors of an inverting opamp can be used to produce the same gain, however, these values should be chosen according to a number of different factors which are discussed in this application note.

3.2 Potentiometer

A potentiometer is a three-terminal, “adjustable” resistor that can provide different resistances by sliding a wiper across a resistive strip. Adjusting a knob on the potentiometer causes a wiper to slide along the resistive strip which provides a “variable” resistance based on the position of the knob. Two terminals are connected across the resistor, creating a fixed maximum resistance. The third terminal is connected to the wiper which creates a break in resistance between the other two terminals. A linear potentiometer has a resistance between a terminal and the wiper that is proportional to the distance between them. This resistance is defined by

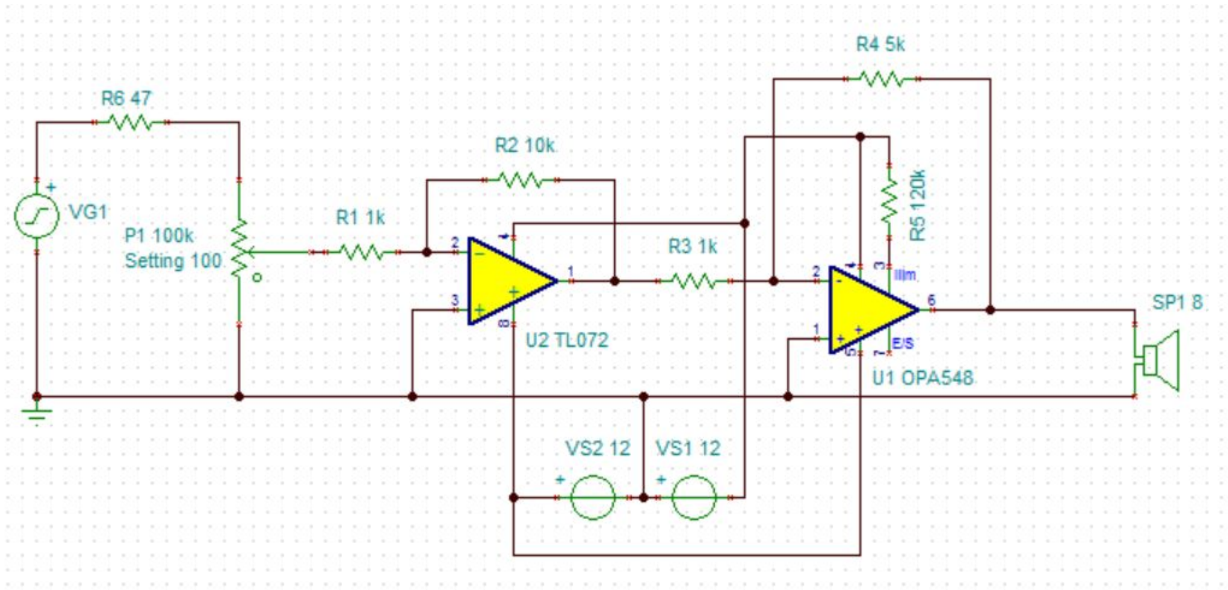
$$R(x) = x$$

3.3 Speaker

A loudspeaker, or speaker for short, is an electroacoustic transducer that is used to produce a sound based on an electrical audio signal. The speaker uses a diaphragm attached to a coil that moves back and forth to produce sound waves. This movement of this coil occurs when an alternating current electrical audio signal is applied to it. Speakers also have a resistance.

4. Discussions and Results

4.1 Description of each element



VG1: Function generator used to supply sinusoidal and square waveforms to circuit.

R6 47Ω: The purpose of the 47 ohm resistor in the circuit is to prevent a short circuit from occurring in the case that the potentiometer is set to 0%. For instance, if the potentiometer was set to 0% without this resistor present, there would be almost no electrical resistance in the circuit besides that of the wire. This would result in an unintendedly high current that could possibly damage the circuit.

P1 100k: The purpose of the potentiometer is to provide a “variable” resistance in order to control the input voltage to the opamp. This is desirable in order to change the volume output of the speaker. This resistance changes linearly or exponentially with the position (%) of the wiper based on whether the potentiometer is linear or logarithmic.

U2 TL072: Opamp used to amplify signal (stage 1).

R1 1kΩ & R2 10KΩ/50kΩ: Works in conjunction with TL072 in an inverting opamp format to produce a voltage gain of 10 or 50.

U1 OPA548: Opamp used to amplify signal (stage 2).

R3 1KΩ & R4 5KΩ: Works in conjunction with OPA548 in an inverting opamp format to produce a voltage gain of 5.

R5 120K: The R_{CL} (current-limiting) resistor is used to limit the current flowing into the opamp. This resistance is determined by the intended power usage of the opamp. 1.5W was chosen based on the thermal characteristics of the device and the decision not to use a heat sink.

SP1 8: The speaker was used to produce sound with a volume determined by the output generated by the circuit. The speaker has a resistance of 8 ohms.

VS1 12 & VS2 12: ±12V Power supply used to supply power to both opamps.

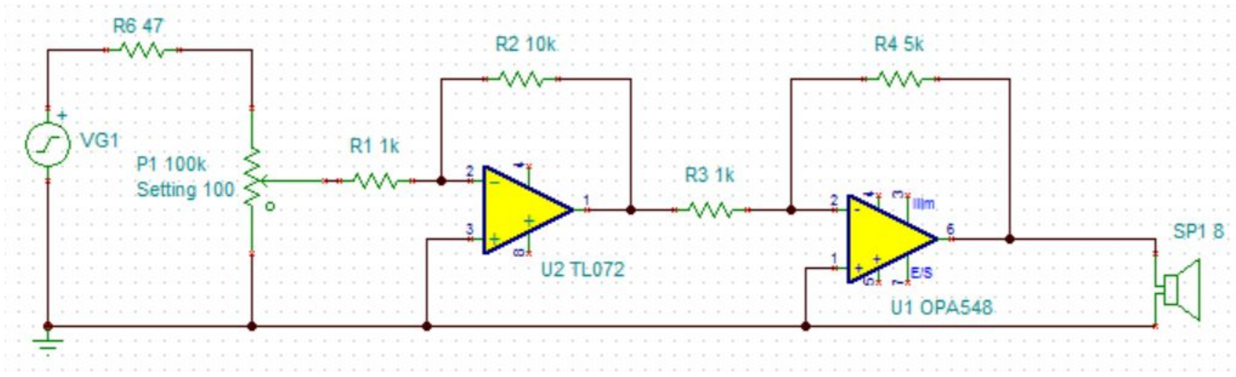
4.2 Picking resistor values

The closed-loop voltage gain (A_V) of an inverting opamp is given as:

$$A_V = -\frac{R_f}{R_{in}}$$

Where R_f is feedback resistance and R_{in} is input resistance.

Given the following circuit,



The gain of stage 1 (U2 TL072) is calculated as,

$$A_V = -\frac{R_2}{R_1}$$

$$A_V = -\frac{10k}{1k} = -10$$

The gain of stage 2 (U1 OPA548) is calculated as,

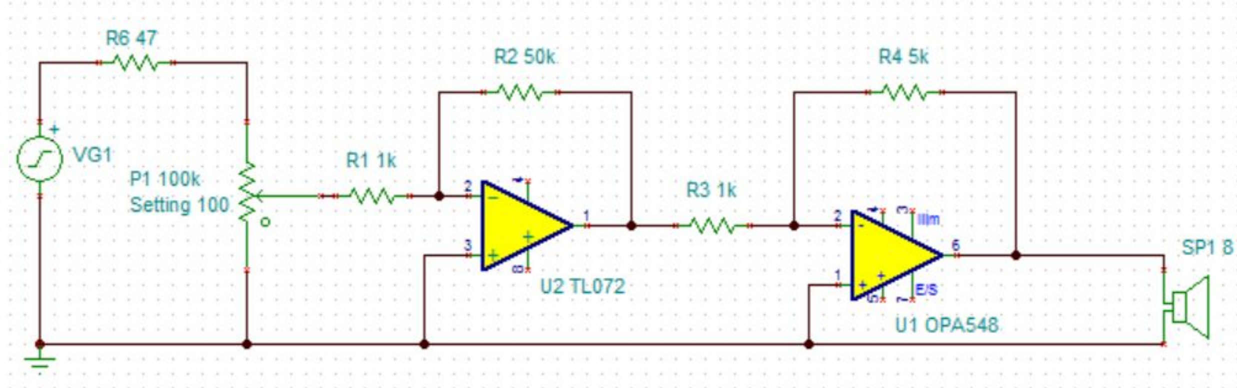
$$A_V = -\frac{R_4}{R_3}$$

$$A_V = -\frac{5k}{1k} = -5$$

The total gain of the circuit can then be calculated by the multiplying the gains of stage 1 and stage 2,

$$(-10) * (-5) = 50$$

Replacing the feedback resistance value of stage 1 (R2) with a 50k resistor results in a greater gain for stage 1 and a greater overall circuit gain.



The gain of stage 1 (U2 TL072) is now calculated as,

$$A_V = -\frac{50k}{1k} = -50$$

The total gain of the circuit can then be calculated by the multiplying the new gain of stage 1 and the previous gain of stage 2,

$$(-50) * (-5) = 250$$

* The simulation confirming these gain values can be found in section 4.4 of Discussion and Results.

The resistor values for an inverting opamp should be chosen according to a number of aspects. The benefits of using low resistance values include:

- improved performance in terms of DC accuracy
- lower noise
- reduced impact of parasitics(unwanted circuit elements of an electrical component), and hence better high-frequency behavior.
- Alternatively, the disadvantages of low resistance values include:

The disadvantages of using low resistance values include:

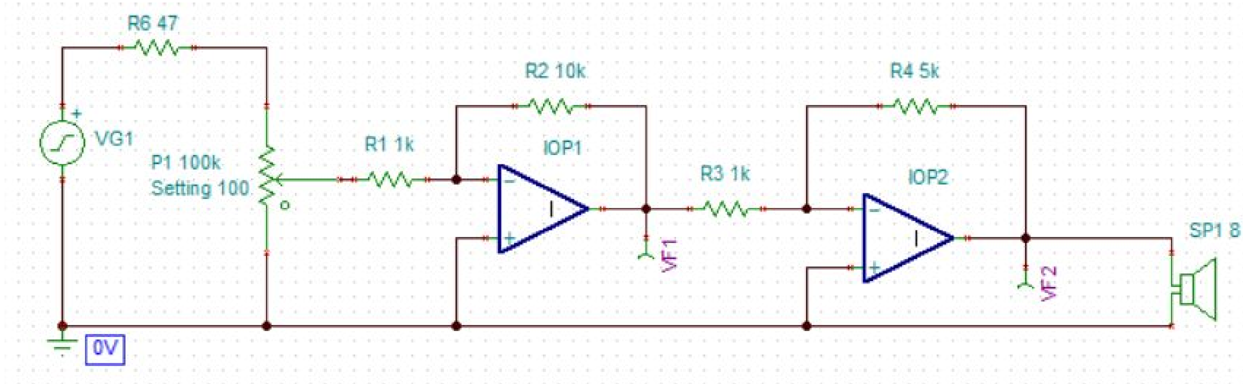
- Increased power consumption
- Additional loading effect on the output

Additionally, if the input resistor (R_I) is too large, it will cause the volume input control to be less accurate as the input resistor is technically connected in parallel with the bottom portion of the volume control potentiometer. This is discussed further in section 4.4.

In short, resistor values for an inverting opamp should be picked in the moderate range according to the intended use and configuration of the opamp.

4.3 Non-inverting and inverting inputs at each opamp stage

Consider the following circuit with two ideal opamp stages,



In an ideal opamp circuit the inverting pin will equal the voltage non-inverting pin.
$V_- = V_+$

In the circuit above, the non-inverting pin is connected to ground which is 0V.
$V_- = V_+ = 0V$

In an ideal opamp, the difference between the inverting and non-inverting input voltages should be zero.

Simulations

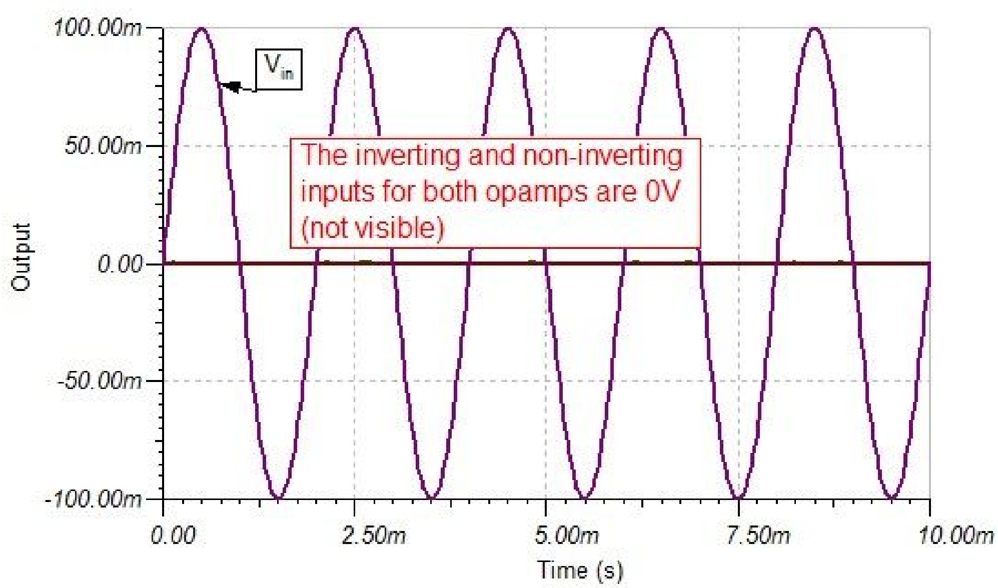


Figure 1: Simulation of inverting and non-inverting inputs of ideal opamp

The simulation shows that for ideal opamps the non-inverting pin will equal the inverting pin. In the circuit used in this example (the non-inverting pin is tied to ground), both pins will equal 0V as simulated.

Measured Data:

Opamp Stage @ Potentiometer Position %	Non-inverting Voltage (mV DC)	Inverting Voltage (mV DC)	Δ Voltage (mV DC)
TL072 @ 0%	0.015	0.114	0.099
OPA548T @ 0%	0.015	1.760	1.745
TL072 @ 50%	0.005	0.102	0.097
OPA548T @ 50%	0.005	1.830	1.825
TL072 @ 100%	0.005	0.106	0.100
OPA548T @ 100%	0.005	1.835	1.830

Generally speaking, an ideal opamp has the following characteristics:

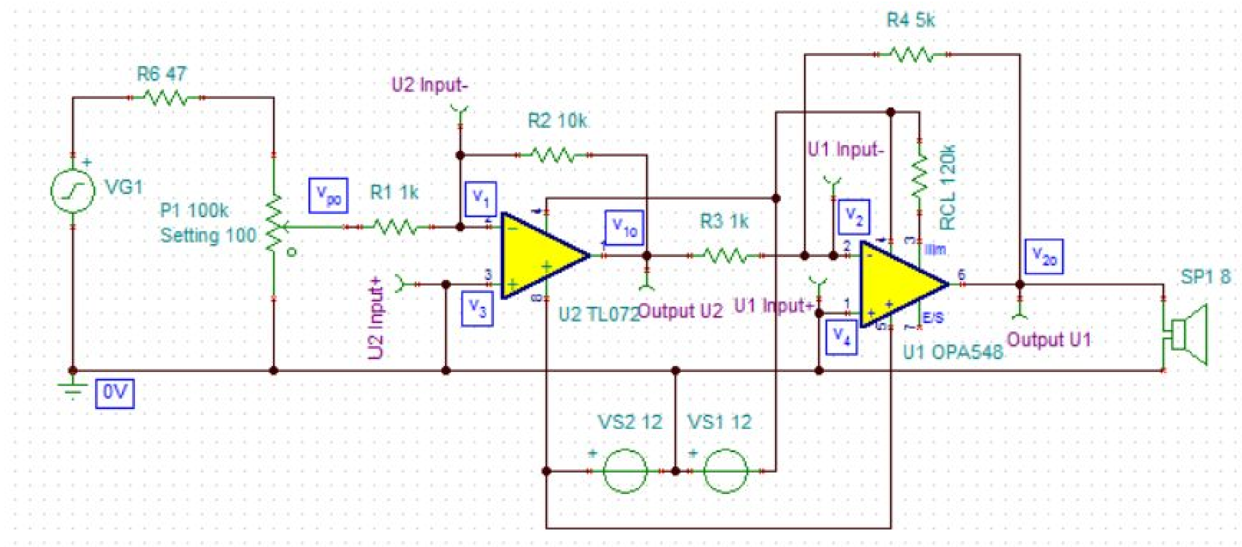
- Infinite voltage gain
- Infinite input impedance
- Zero output impedance
- Infinite bandwidth
- Zero input offset voltage (zero in, zero out)

The opamps used in this application report (the OPA548T and TL072) are not completely ideal in real-world application. For example, the average difference between the non-inverting pins on the TL072 is 0.99mV. The average difference between the non-inverting pins on the OPA548T is 1.80mV. While the opamps used in this application report are not completely ideal, the difference of voltages between the pins are within acceptable range.

Although it is known these opamps are not ideal in real-world applications, for this application report and for all mathematical analysis proceeding this section, the opamps used in this circuit will be assumed to be ideal. Therefore, in all mathematical analysis proceeding this section the inverting pin of the opamps will assumed to be equal to the non-inverting pin.

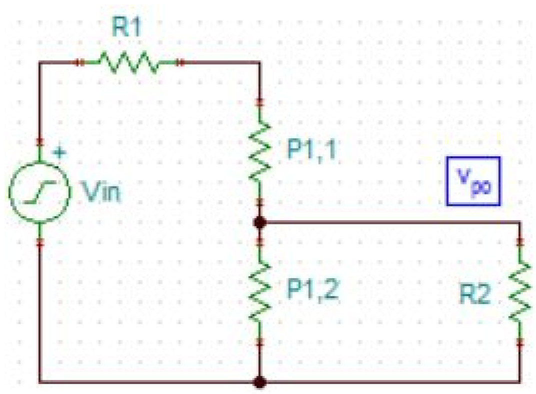
4.4.1 Circuit with overall gain of 50

The following circuit has an overall gain of 50



Given the following values,
$V_{in} = 100mV, P1 = 100k\Omega, R1 = 1k\Omega, R2 = 10k\Omega$
$R3 = 1k\Omega, R4 = 5k\Omega$
$R6 = 47\Omega, R_{CL} = 120k\Omega$
$SP1 = 8\Omega$
The input to the TL072 and OPA548T are ± 12V.

To calculate the voltages v_1, v_{1o}, v_2, v_{2o} the voltage at the output of the potentiometer (v_{po}) must first be calculated.
The volume control portion of the circuit can be drawn as:



When the knob of the potentiometer is set at 100%, the values for the components above are:

$V_{in} = 0.1V$
$R1 = 47\Omega$
$P1,1 = 0\Omega, P1,2 = 100k\Omega$
$R2 = 1k\Omega$

As seen in the diagram, resistors R1 and P1,1 can be combined in series. Additionally, the resistors P1,2 and R2 can be combined in parallel.
The full equation for solving for v_{po} is written by:

$$v_{po} = \frac{(V_{in})(\frac{P1,2*R2}{P1,2+R2})}{(P1,1+R1)+(\frac{P1,2*R2}{P1,2+R2})} = \frac{(V_{in})(P1,2||R2)}{(P1,1+R1)+(P1,2||R2)}$$

$$v_{po} = \frac{(0.1V)(100k\Omega||1k\Omega)}{(0\Omega+47\Omega)+(100k\Omega||1k\Omega)} = 0.095468V = 95.47mV$$

$$v_{po} = 95.47mV$$

Solving for v_1, v_{1o}, v_2, v_{2o} when the potentiometer knob is at 100% position and $R2 = 10k\Omega$:

Because the opamps are ideal, the non-inverting pin will equal the inverting pin.
The non-inverting pin of the TL072 and OPA548T are directly connected to 0V.
Therefore,
$v_3 = v_1 = 0V$
$v_2 = v_4 = 0V$

Nodal analysis can be performed on v_1 to receive the value v_{1o}

$$\frac{v_1 - v_{po}}{R1} + \frac{v_1 - v_{1o}}{R2} = 0$$

$$\frac{0V - 0.09546V}{1k\Omega} + \frac{0V - v_{1o}}{10k\Omega} = 0$$

Solving for v_{1o}:
$v_{1o} = -0.9547V$
Comparing $v_{po}(0.09547V)$ to $v_{1o}(-0.9547V)$ it can be seen that the first opamp stage has a gain of -10. ($0.09547V * (-10) = -0.9547V$)

Knowing v_{1o}, v_{2o} can be solved for using nodal analysis at v_2

$$\frac{v_2 - v_{1o}}{R3} + \frac{v_2 - v_{2o}}{R4} = 0$$

$$\frac{0V - (-0.9547V)}{1k\Omega} + \frac{0V - v_{2o}}{5k\Omega} = 0$$

Solving for v_{2o}:

$v_{2o} = 4.77V$

Comparing $v_{1o}(-0.9547V)$ to $v_{2o}(4.77V)$ it can be seen that the second opamp stage has a gain of -5. ($(-0.9547V) * (-5) = 4.77V$).

The gain of the first opamp stage was found to be -10. The gain of the second opamp stage was found to be -5. The overall gain of the circuit is $(-10) * (-5) = 50$

The voltage at the output of the TL072 (first stage): $v_{1o} = -0.9547V$

The voltage at the output of the OPA548T (second stage): $v_{2o} = 4.77V$

Simulations

Using TINA-TI AC analysis, the calculated results can be verified:

v_{po}: Amplitude 95.47mV

v_1: Amplitude 161.43uV

v_{1o}: Amplitude 954.62mV

v_2: Amplitude 1.93mV

v_{2o}: Amplitude 4.77V

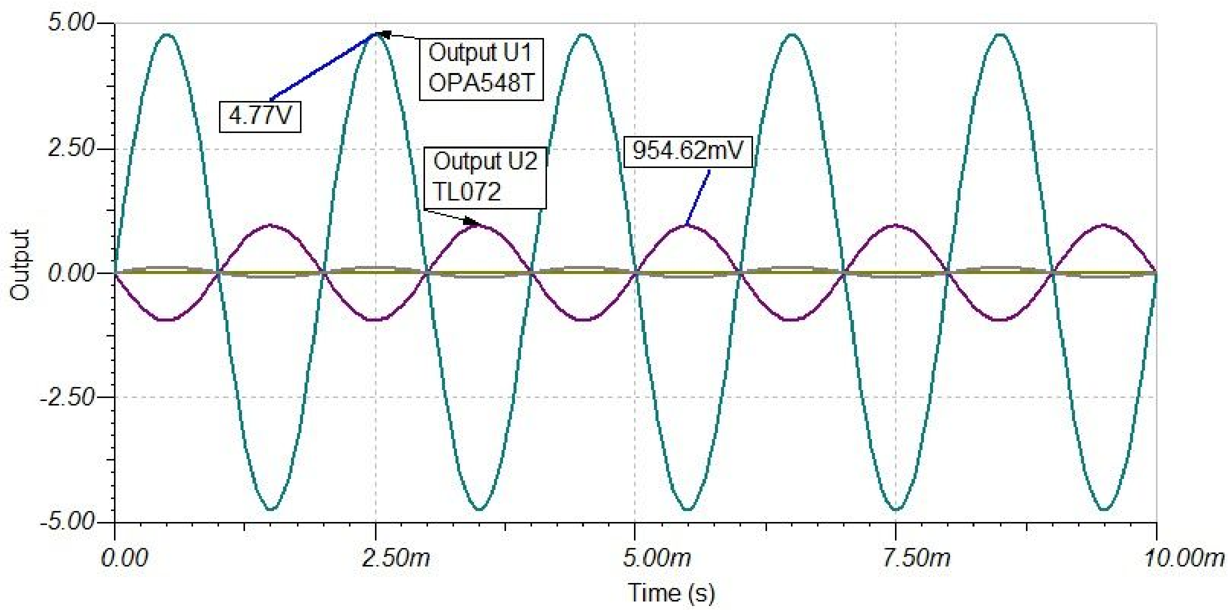


Figure 2: Transient analysis with speaker disconnected

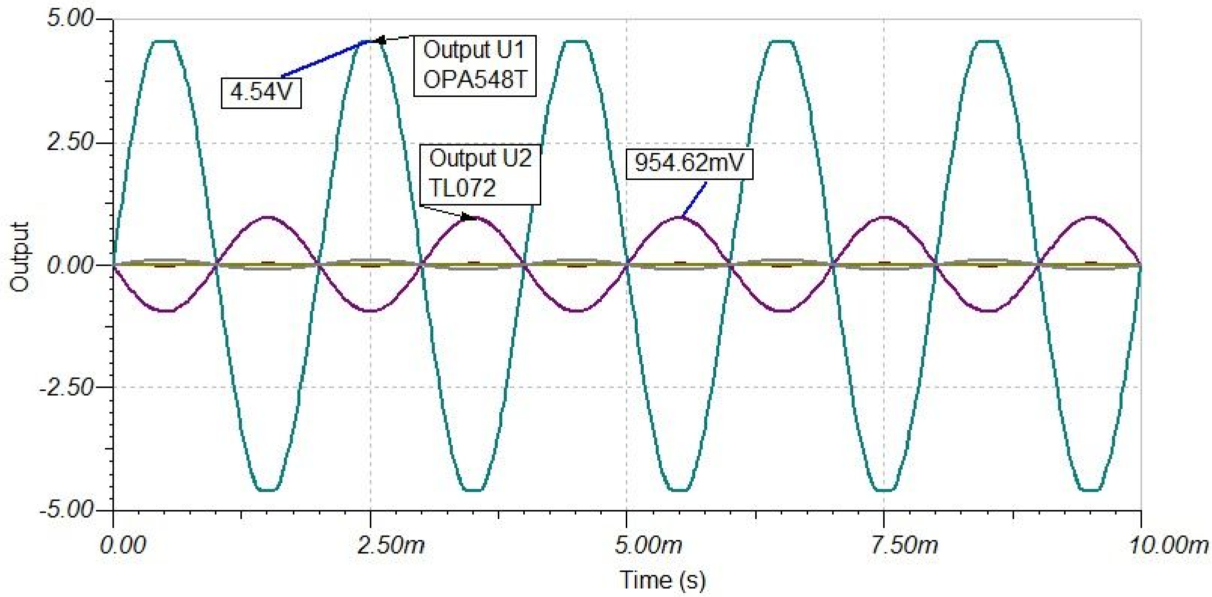


Figure 3: Transient analysis with speaker connected

When the speaker is connected, audio clipping occurs. Discussed in the discussions section.

Measurements

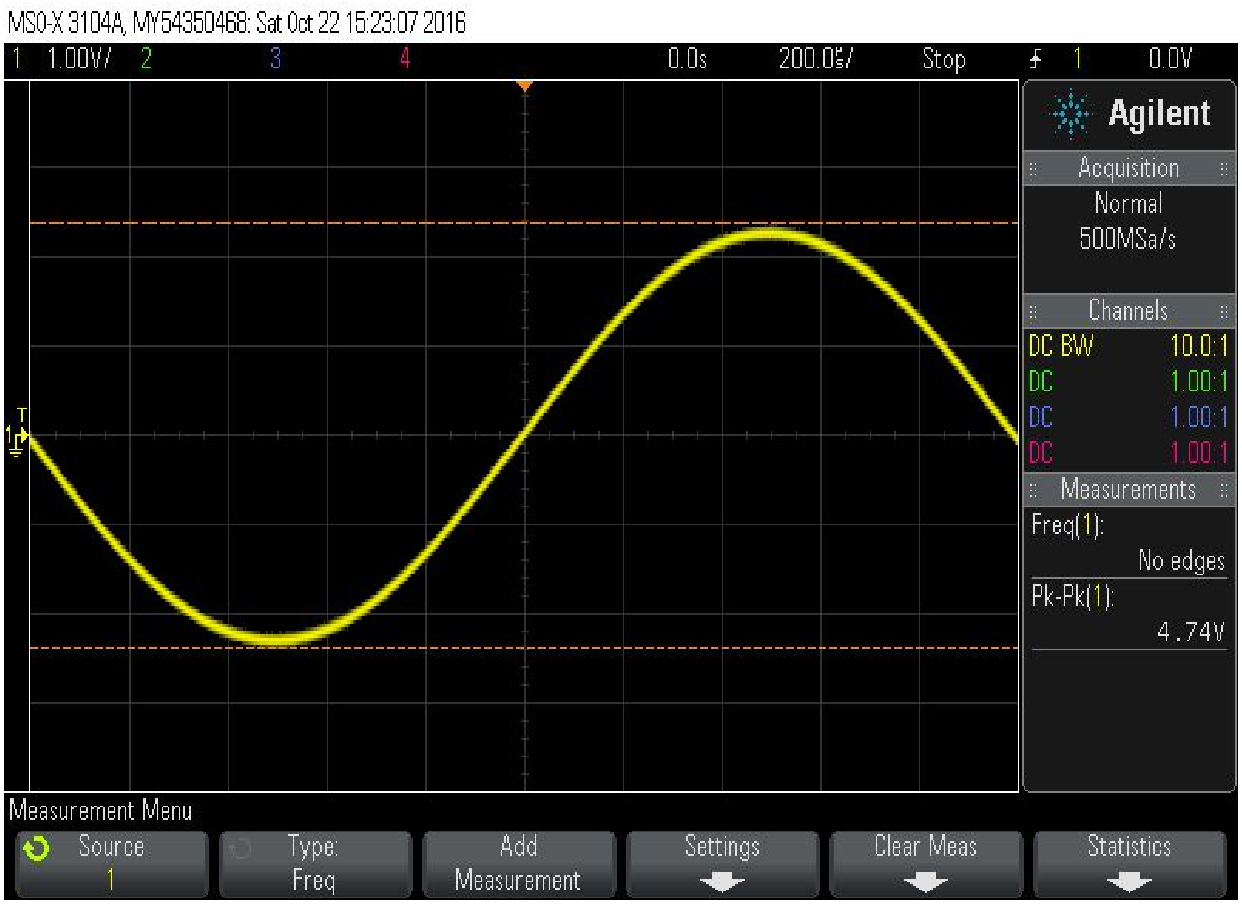


Figure 4: Output OPA548T, pot @ 100% position, speaker disconnected

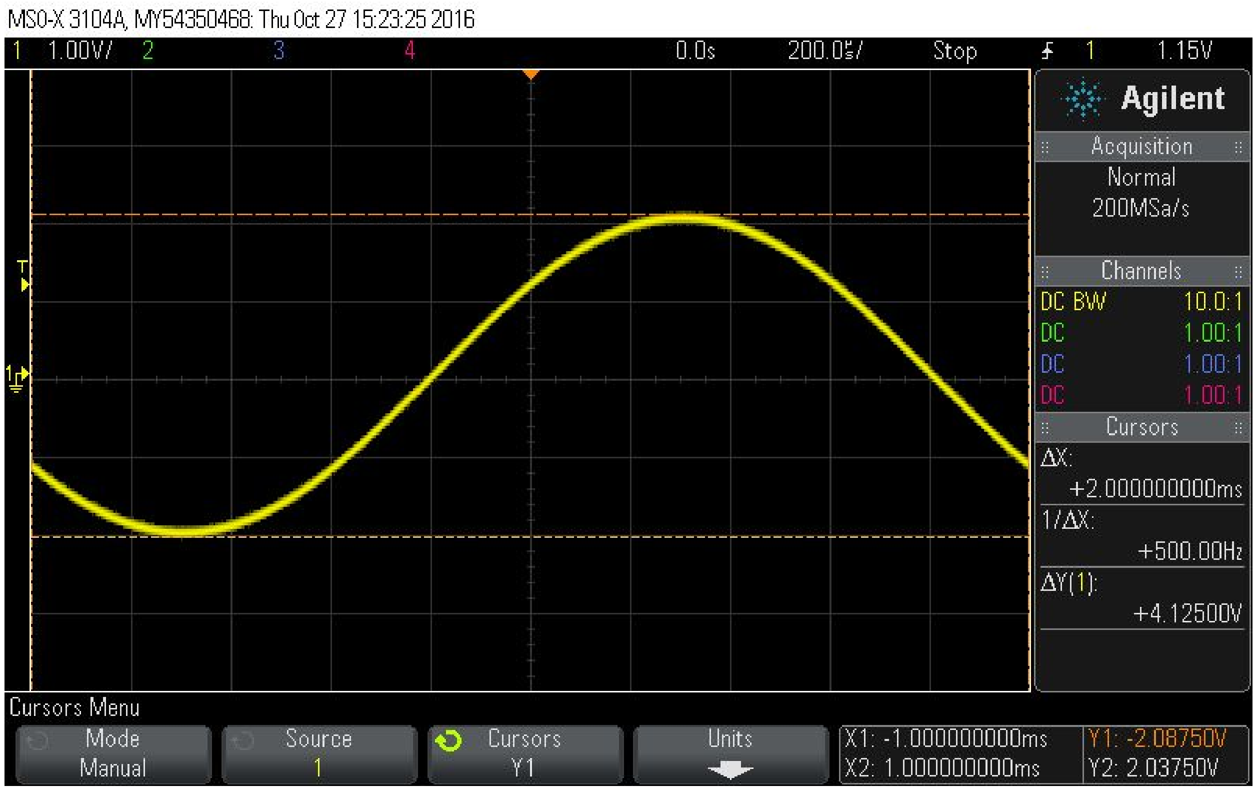


Figure 5: Output OPA548T, pot @ 100% position, speaker connected

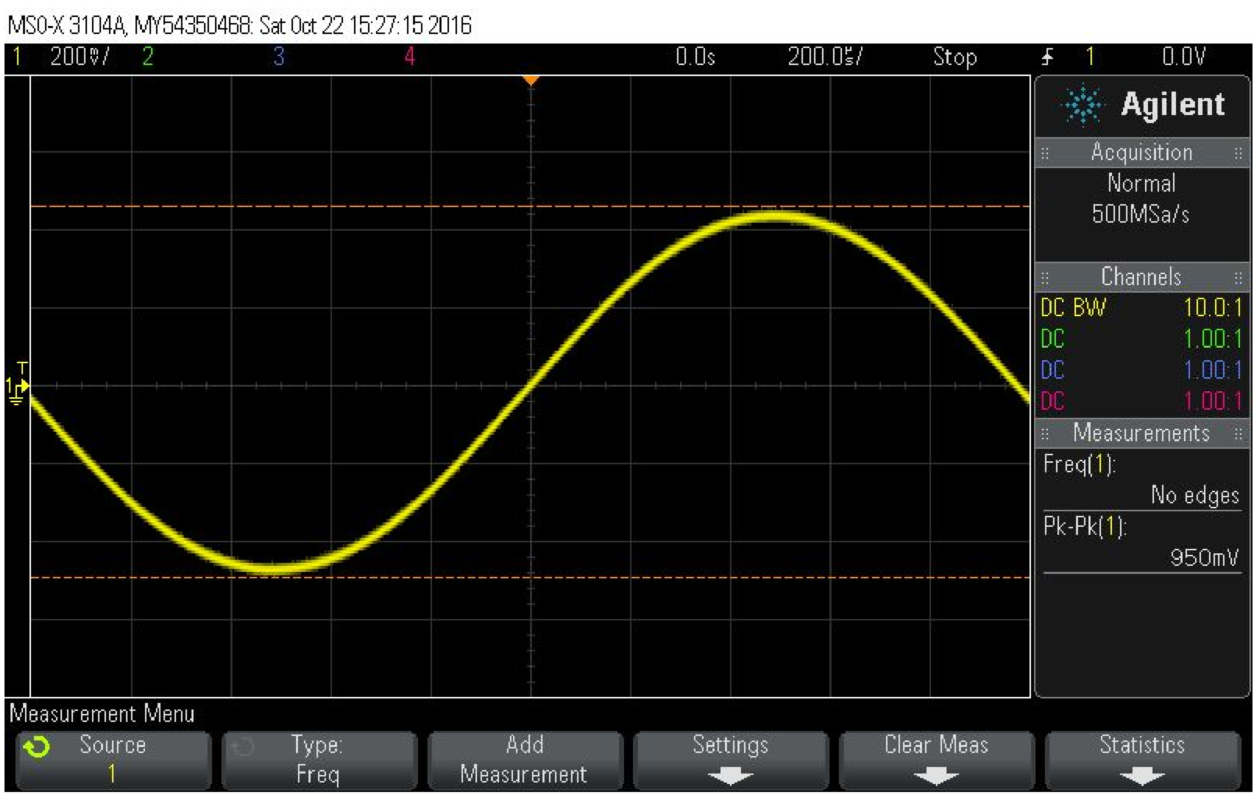
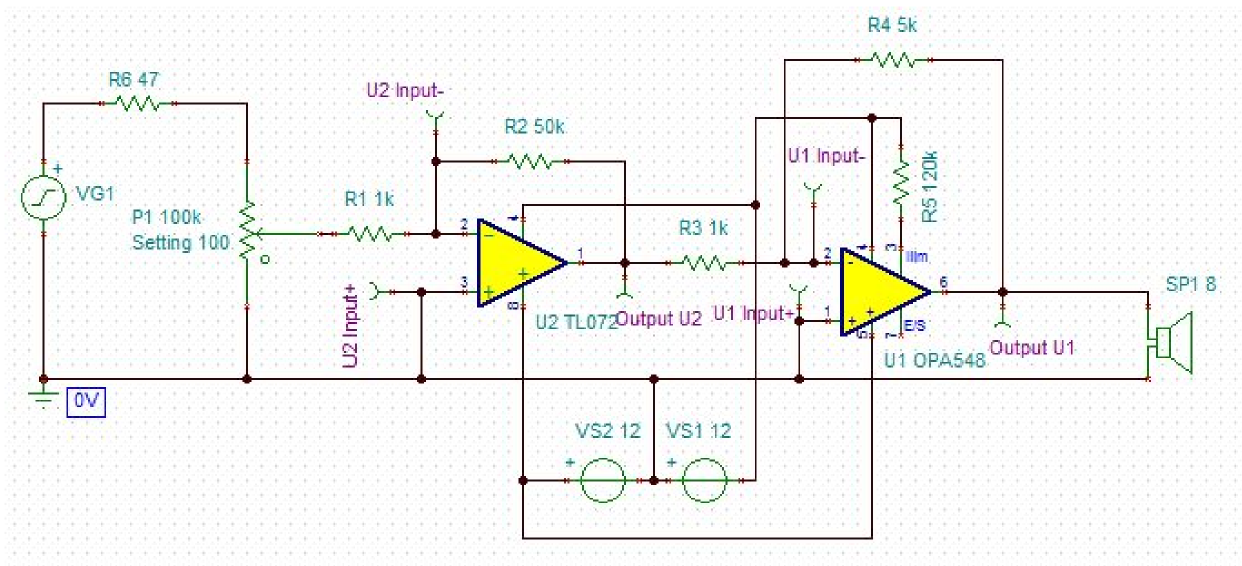


Figure 6: Output TL072, pot @ 100% position, speaker connected/disconnected

4.4.2 Circuit with overall gain of 250

The following circuit has an overall gain of 250,



Given the following values,

$V_{in} = 100mV, P1 = 100k\Omega, R1 = 1k\Omega, R2 = 50k\Omega$

$R3 = 1k\Omega, R4 = 5k\Omega$

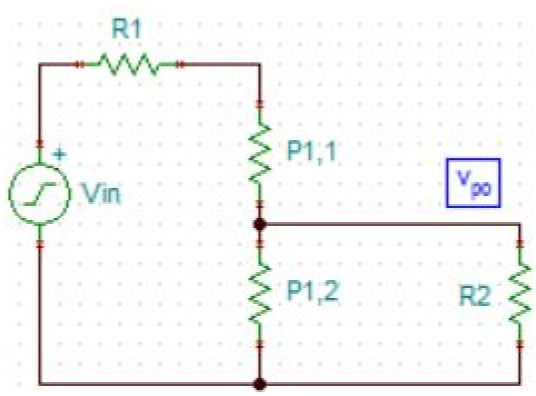
$R6 = 47\Omega, R_{CL} = 120k\Omega$

$SP1 = 8\Omega$

The input to the TL072 and OPA548T are ± 12V.

To calculate the voltages v_1, v_{1o}, v_2, v_{2o} the voltage at the output of the potentiometer (v_{po}) must first be calculated.

The volume control portion of the circuit can be drawn as:



When the knob of the potentiometer is set at 100%, the values for the components above are:

$V_{in} = 0.1V$
$R1 = 47\Omega$
$P1,1 = 0\Omega, P1,2 = 100k\Omega$
$R2 = 1k\Omega$

As seen in the diagram, resistors R1 and P1,1 can be combined in series. Additionally, the resistors P1,2 and R2 can be combined in parallel.
The full equation for solving for v_{po} is written by:

$$v_{po} = \frac{(V_{in})(\frac{P1,2*R2}{P1,2+R2})}{(P1,1+R1)+(\frac{P1,2*R2}{P1,2+R2})} = \frac{(V_{in})(P1,2||R2)}{(P1,1+R1)+(P1,2||R2)}$$

$$v_{po} = \frac{(0.1V)(100k\Omega||1k\Omega)}{(0\Omega+47\Omega)+(100k\Omega||1k\Omega)} = 0.095468V = 95.47mV$$

$$v_{po} = 95.47mV$$

Solving for v_1, v_{1o}, v_2, v_{2o} when the potentiometer knob is at 100% position and $R2 = 10k\Omega$:

Because the opamps are ideal, the non-inverting pin will equal the inverting pin.
The non-inverting pin of the TL072 and OPA548T are directly connected to 0V.
Therefore,
$v_3 = v_1 = 0V$
$v_2 = v_4 = 0V$

Nodal analysis can be performed on v_1 to receive the value v_{1o}

$$\frac{v_1 - v_{po}}{R1} + \frac{v_1 - v_{1o}}{R2} = 0$$

$$\frac{0V - 0.09546V}{1k\Omega} + \frac{0V - v_{1o}}{50k\Omega} = 0$$

Solving for v_{1o}:
$v_{1o} = -4.77V$
Comparing $v_{1o}(-4.77V)$ to $v_{po}(0.09547V)$ it can be seen that the first opamp stage has a gain of -50. ($0.09547V*(-10) = -4.77V$)

Knowing v_{1o}, v_{2o} can be solved for using nodal analysis at v_2

$$\frac{v_2 - v_{1o}}{R3} + \frac{v_2 - v_{2o}}{R4} = 0$$

$$\frac{0V - (-4.77V)}{1k\Omega} + \frac{0V - v_{2o}}{5k\Omega} = 0$$

Solving for v_{2o}:
$v_{2o} = 23.84V$
Comparing $v_{1o}(-4.77V)$ to $v_{2o}(23.84V)$ it can be seen that the second opamp stage has a gain of -5. ($(-4.77V) * (-5) = 23.84V$).

The gain of the first opamp stage was found to be -50. The gain of the second opamp stage was found to be -5. The overall gain of the circuit is $(-50) * (-5) = 250$
The voltage at the output of the TL072 (first stage): $v_{1o} = -4.77V$
The voltage at the output of the OPA548T (second stage): $v_{2o} = 23.84V$

Simulations
Using TINA-TI AC, the calculated results can be verified:

v_{po}: Amplitude 95.47mV

v_1: Amplitude 798.56uV

v_{1o}: Amplitude 4.77V

v_2: Amplitude 19.39mV

v_{2o}: Amplitude 23.84V

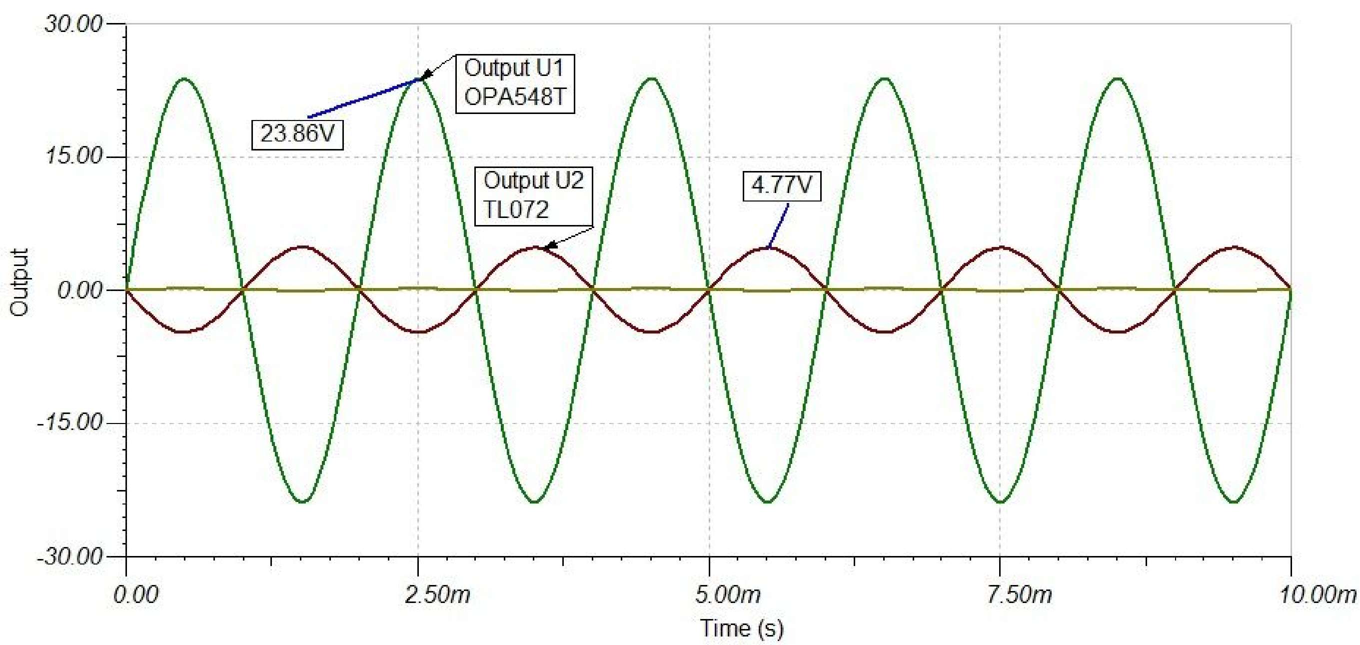


Figure 7: Transient analysis with speaker disconnected

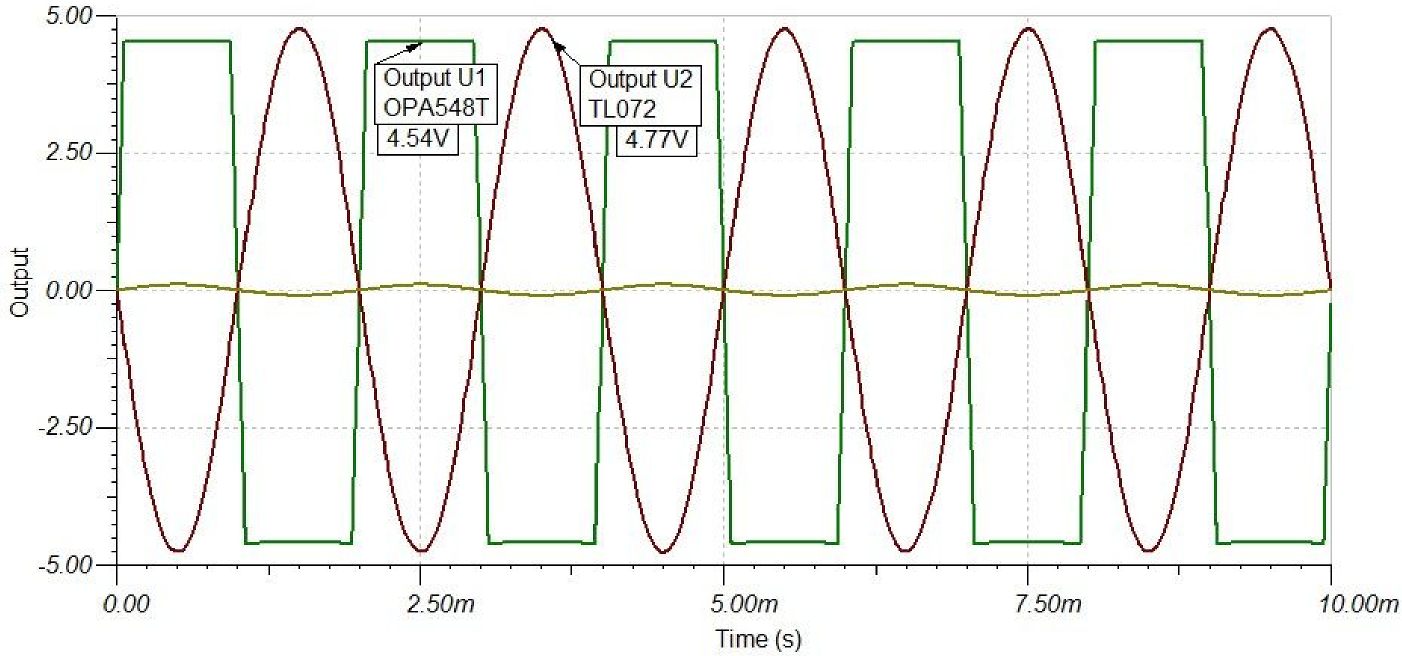


Figure 8: Transient analysis with speaker connected

Measurements

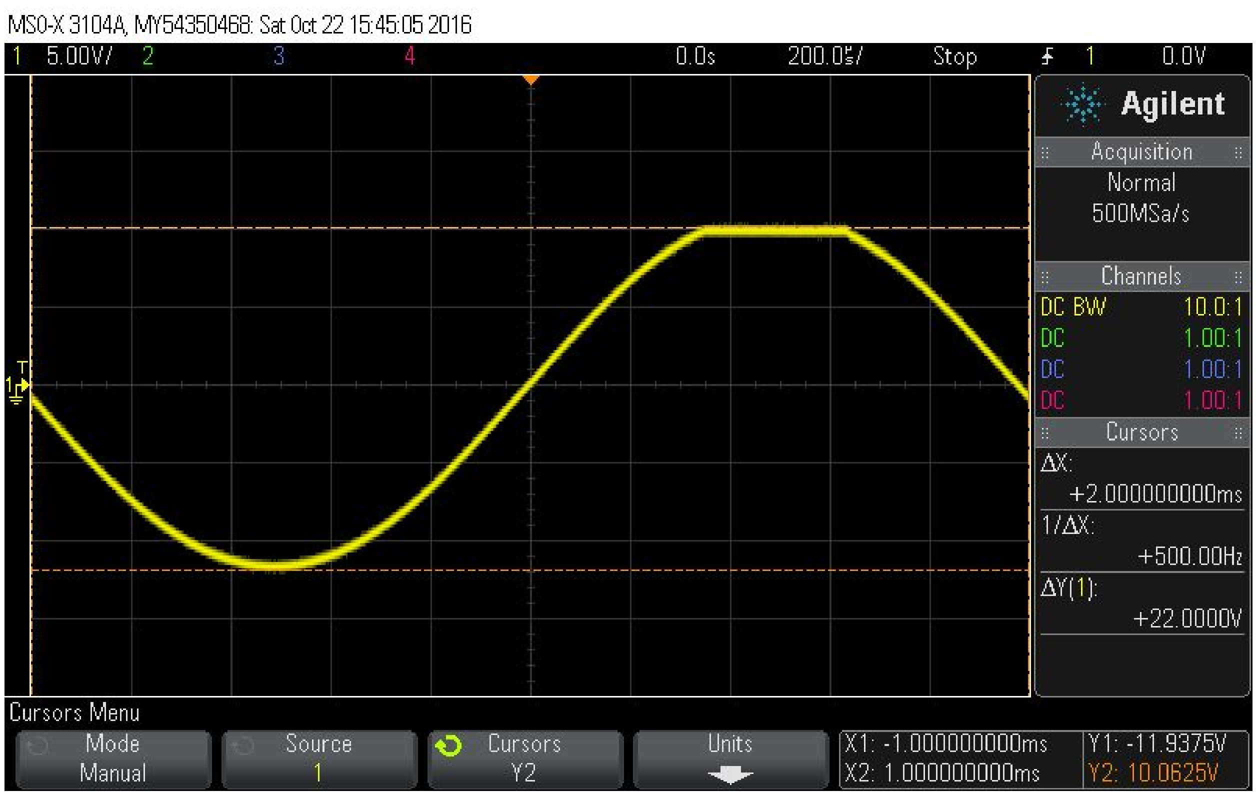


Figure 9: Output of OPA548T, pot @ 100% position, speaker disconnected

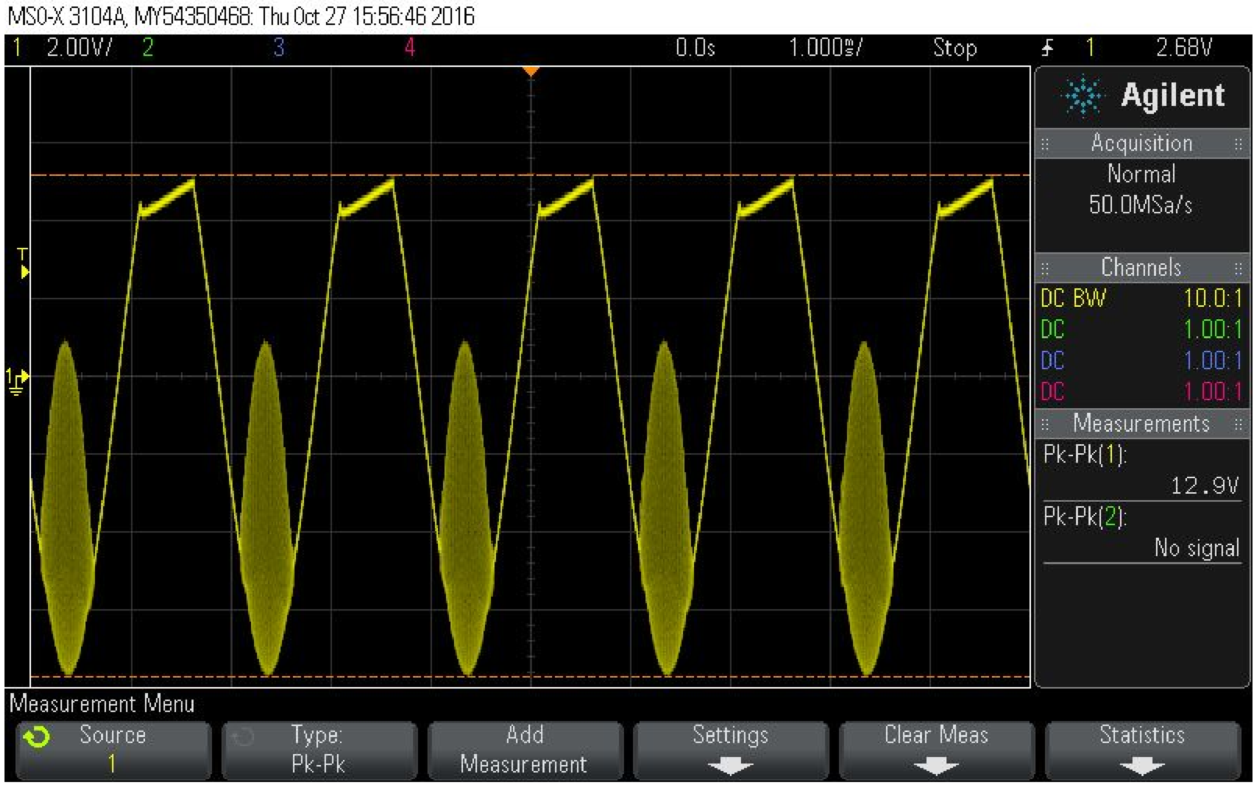


Figure 10: Output of OPA548T, pot @ 100% position, speaker connected

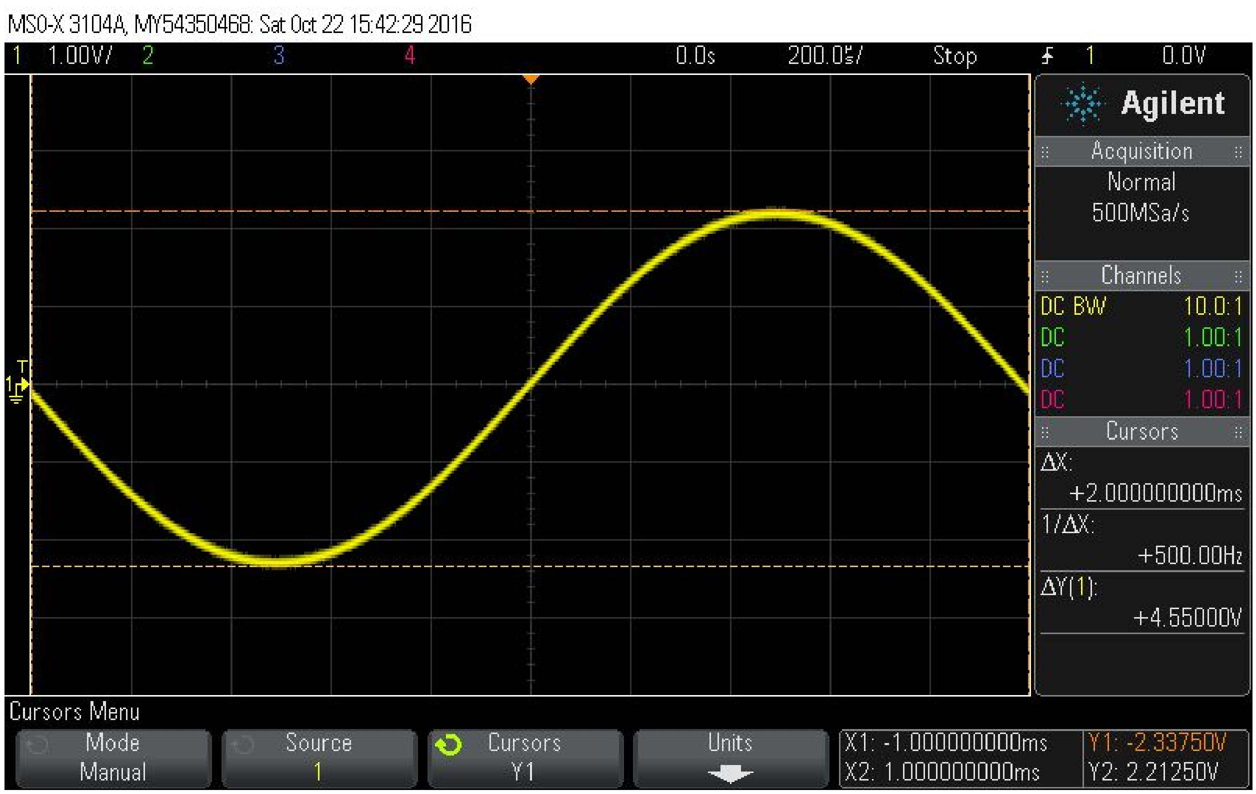


Figure 11: Output of TL072, pot @ 100% position, speaker connected/disconnected

Discussions

Audio clipping is a form of waveform distortion that occurs when an amplifier attempts to deliver an output voltage or current beyond its maximum capability. This maximum capability is determined by the amplifier's power supply. When an amplifier attempts to output a signal with more power than its power supply can produce, the signal is amplified only up to the power supply's max capacity and the rest of the signal is said to be "clipped". This results in a sine wave becoming a distorted square wave.

Clipping is displayed in the circuit at the output of the OPA548T when the potentiometer is set to 100% and the speaker is disconnected, as seen in the figure above. When the overall circuit gain is 250, I_{lim} is exceeded, resulting in the positive voltage output to be clipped by 2V as described by the voltage output when I_O is 0.6A on data sheet. I_{lim} is calculated using an equation on the datasheet and set by using the appropriate value for R_{CL}.

As seen in the second figure on the previous page *Output OPA548T @ 100% Position, Speaker Connected*, the output of the waveform is unconventional. The circuit configuration used to acquire this waveform was an overall circuit gain of 250. Maximum input voltage was also applied by having the input volume control potentiometer set to the 100% position. The waveform looks *strange* due to the very high gain amount and the large amount of amplification done by the opamp which exceeds its power ratings.

The output voltages and gains of the opamps can be determined using mathematical analysis. When solving for the input voltage of the circuit, the bottom portion of the potentiometer is connected in parallel with the resistor R1 (R_{IN}). The parallel equivalent resistance is calculated as a voltage divider with the top portion of the potentiometer combined in series with the 47Ohm resistor. The mathematical analysis for computing v_{po} (input voltage) can be seen in this section.

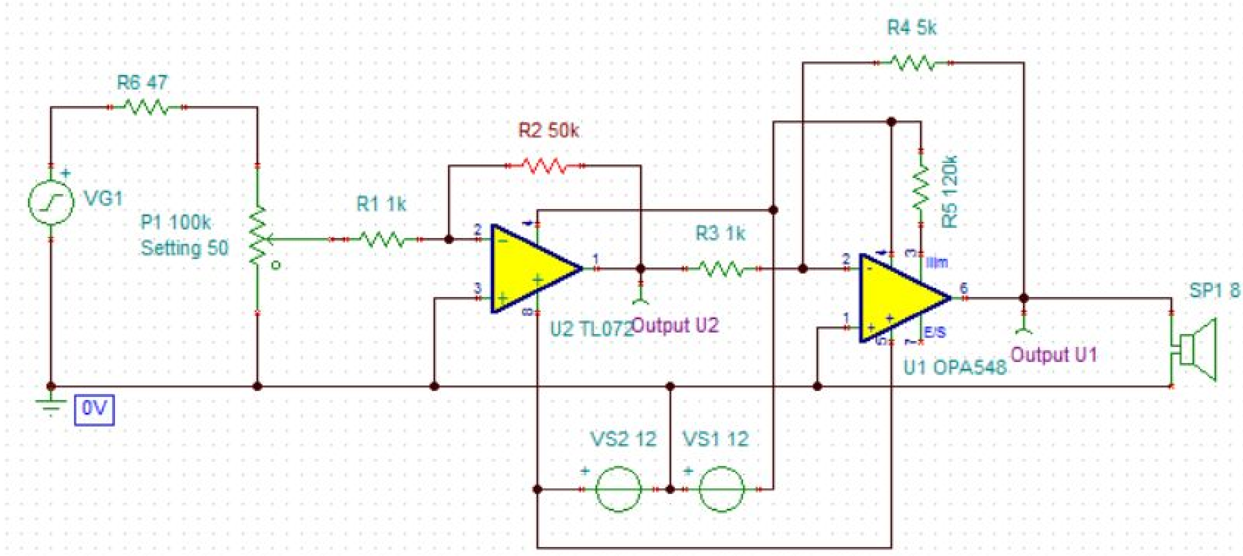
Once the input voltage v_{po} is known nodal analysis can be used to calculate the output of the opamp. Because it is assumed the opamps are ideal, the non-inverting pin is equal to the inverting pin which in this case is 0V. The output of the first opamp stage can be used as the input voltage for the nodal analysis for the second opamp stage.

When the feedback resistor for the first opamp stage R_f (R2) is changed from 10kOhm to 50kOhm the gain of the first opamp stage (TL072) increases from 10 to 50. When the feedback resistor is at 10kOhm the gain for the individual opamp stage is 10. When the feedback resistor is at 50kOhm the gain for the individual opamp stage is 50. When the gain of the opamp stage increases the output voltage of the opamp increases.

The values of the feedback and input resistor were purposely chosen to be within the range of 1kOhm to 500kOhm. The reasons for this are outlined in section 4. The values of these resistors theoretically could be any number as long as the ratio between the feedback and input give us the desired gain. However, in this circuit, the input resistor is connected in parallel with the bottom portion of the potentiometer. If an outrageously high resistance value was chosen for the input resistor, it would cause the volume control to be altered (become not linear anymore). By choosing a resistance value close to the resistance value of the potentiometer, it reduces the risk of alteration because the resulting parallel equivalence would still be within similar range.

4.5 Output of opamp stages as input frequency increases

Consider the following circuit,



The overall gain of this circuit is 250 (as calculated in previous sections). The position of the volume control potentiometer is set to 50% to ensure a reasonable volume with an output waveform that looks nominal.

The voltage input (v_{in}) of the circuit is 0.1V. The function generator connected to the input of the circuit is set to generate square waves.

Consider the function generator frequency is set to **50KHz**,
The opamps in use have finite frequency responses, i.e at high frequencies the gain of the amplifiers are expected to drop.

The transfer function of the opamp is assumed to be,

$$|v_{out}| = |v_{in}|\frac{A}{|1+j\frac{w}{wc}|}$$

$$|v_{out}| = |v_{in}|\frac{250}{\sqrt{1+(\frac{w}{wc})^2}}$$ for a gain of A = 250

Assume a scenario where $\omega = 2\pi x 50 x 10^3 rad/s (50KHz)$ and $v_{in} = 0.1V$

$$|v_{out}| = 0.1V \frac{250}{\sqrt{1 + (\frac{2\pi x 50 x 10^3}{wc})^2}}$$ **(Equation 1)**

Now, assume a scenario when $\omega = 2\pi x 500 x 10^3 rad/s (500KHz)$ and $v_{in} = 0.1V$

$$|v_{out}| = 0.1V \frac{250}{\sqrt{1 + (\frac{2\pi x 500 x 10^3}{wc})^2}}$$ **(Equation 2)**

Compare the two scenarios shown in Equation 1 and Equation 2:

- Equation 2 contains a larger value for frequency than Equation 1

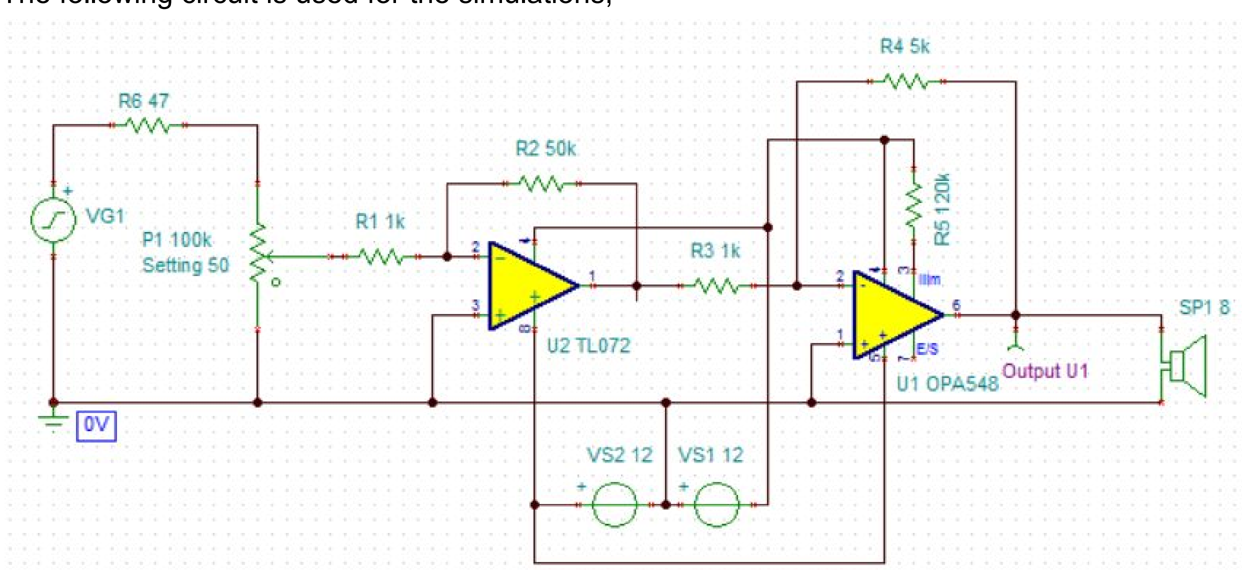
By simple mathematical analysis, we can make the following conclusions in the for the above transfer equations:

- As frequency increases greatly the denominator of the equation increases
- As the denominator of the equation increases the value for v_{out} decreases

Therefore, by mathematical analysis we can conclude that increasing frequency decreases the voltage output of the opamp.

Simulations

The following circuit is used for the simulations,



The input is a 0.1V square wave.
When the frequency of the square wave is 50KHz,

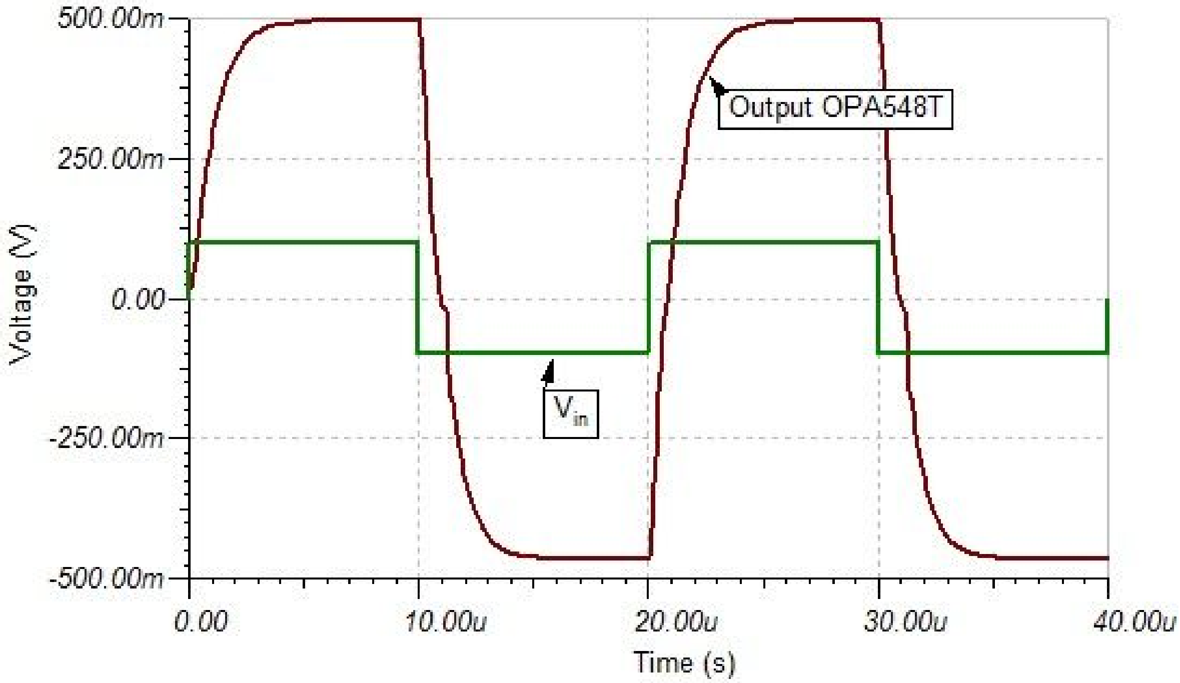


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Figure 12: Output of OPA548T with 50KHz square wave

The output voltage in this simulation is about 500mV.

When the frequency of the square wave is 500KHz (increased),

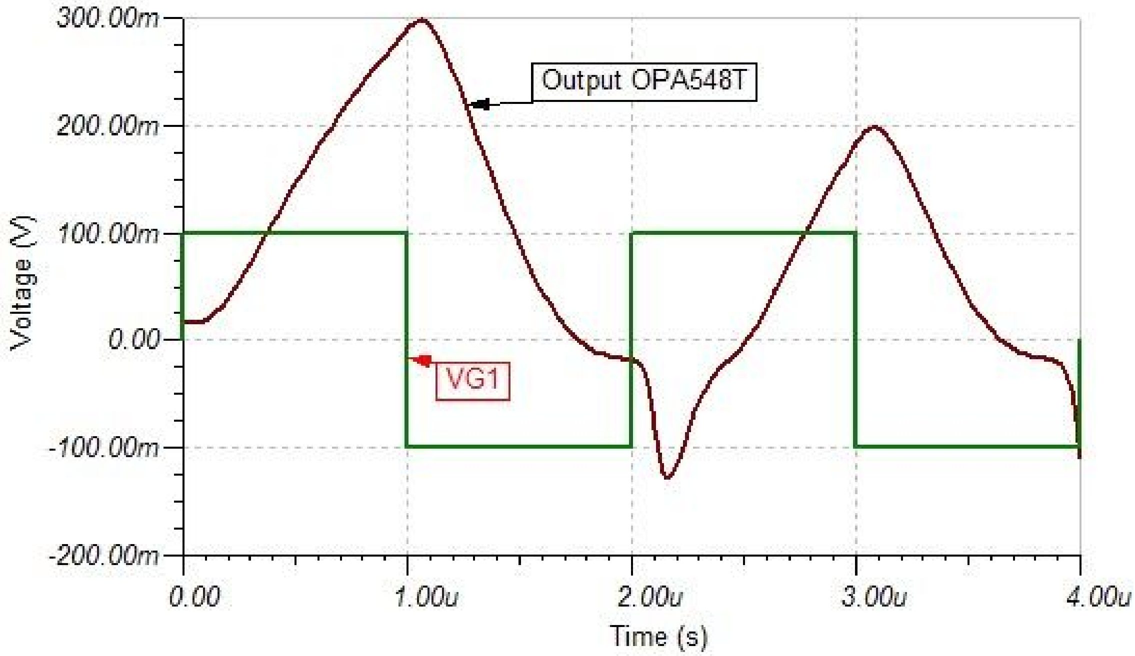


Figure 13: Output of OPA548T with 500KHz square wave

The output voltage in this simulation okis about 300mV.

As seen from the above simulations, the amplitude of the voltage output decreased when the frequency of the input square waves increased.

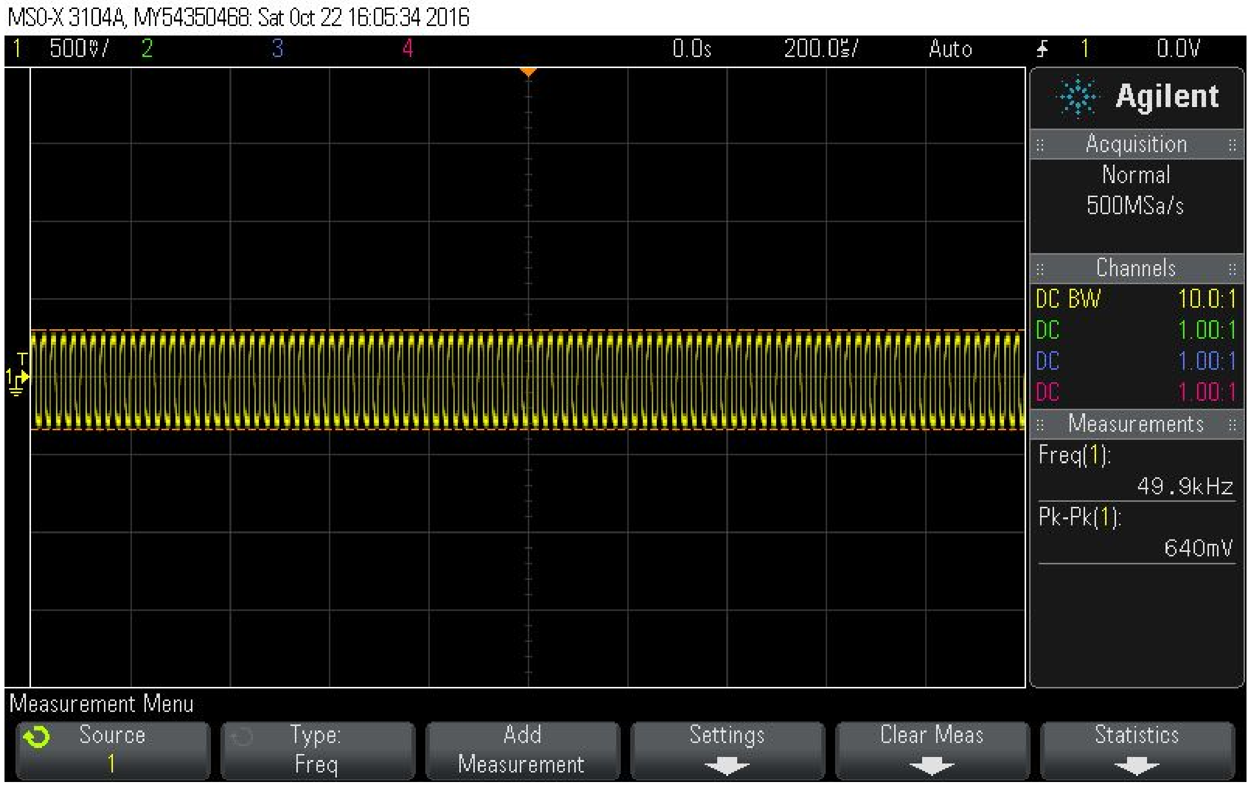


Figure 14: Amplitude of square wave at frequency of 50KHz

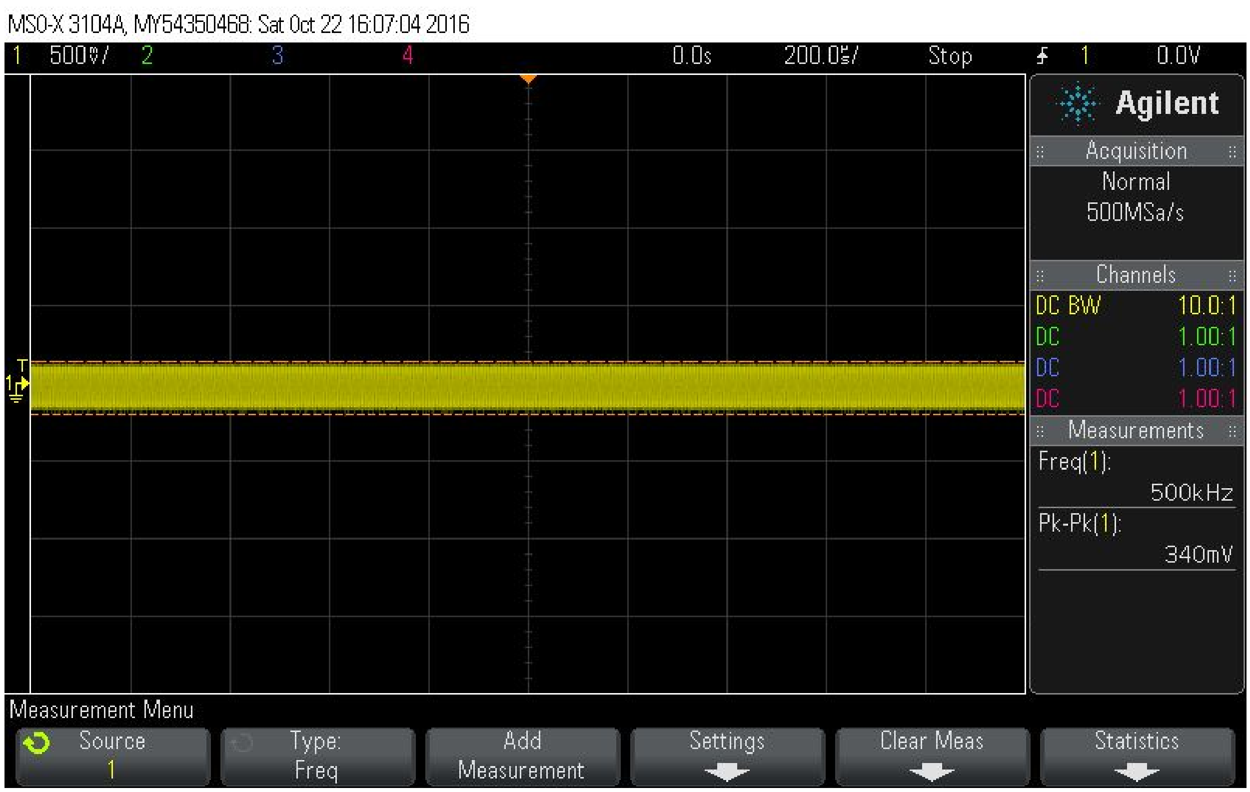


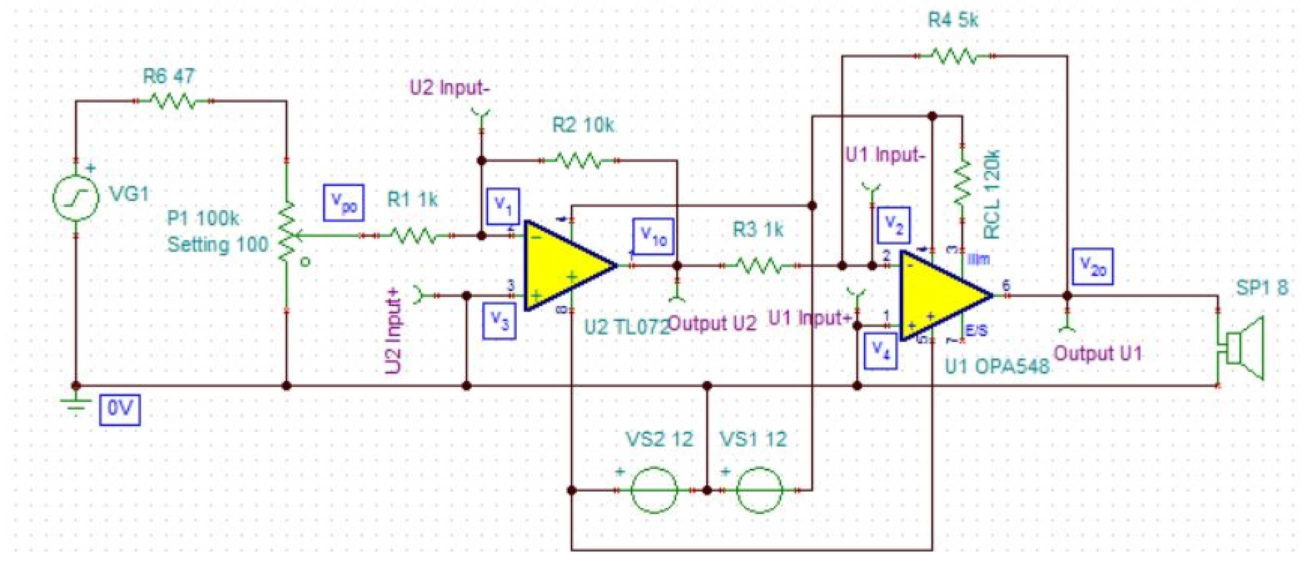
Figure 15: Amplitude of square wave at frequency of 500KHz

In the mathematical analysis, simulations, and lab measurements the voltage output decreases as the frequency increases.

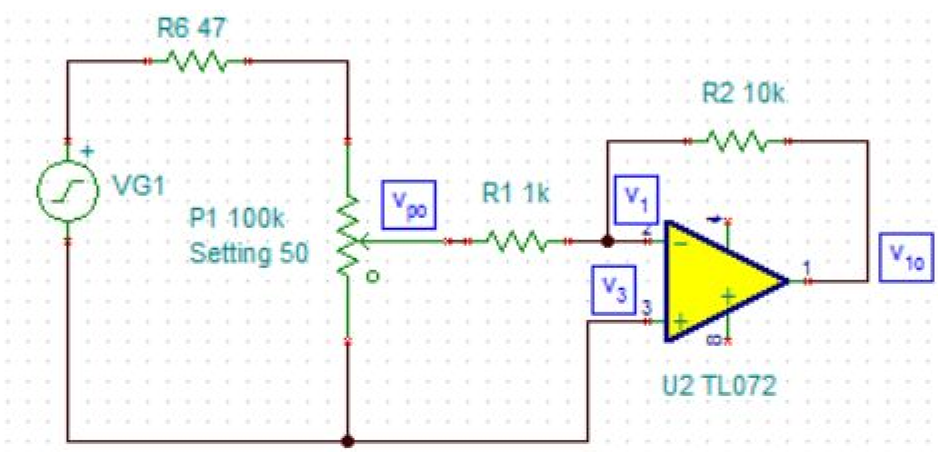
The AC transfer characteristics equation shows that the voltage output will decrease as the frequency increases. Another factor that affects amplitude due to frequency is the slew rate of the opamp. The slew rate of an opamp or any amplifier circuit is the rate of change in the output voltage caused by a step change on the input. The slew rate of the OPA548T is 50Vp-pV/µs at a gain of 1. When the gain increases the slew rate will also change. The voltage will begin to decrease if the opamp input exceeds the slew rate.

4.6 Reasoning for volume control potentiometer at the input

Consider the circuit with an overall gain of 50,



The volume control potentiometer in the above circuit is located at the input of the circuit. Consider the following portion of the above circuit (the volume input control and the first opamp gain stage),



As discussed in section 4.4.1, the calculations for the output after the first gain stage (opamp TL072) is as follows (when the potentiometer is set at 50%):

$$v_{po} = \frac{(V_{in})(\frac{P1,2*R2}{P1,2+R2})}{(P1,1+R1)+(\frac{P1,2*R2}{P1,2+R2})} = \frac{(V_{in})(P1,2||R2)}{(P1,1+R1)+(P1,2||R2)}$$

$$v_{po} = \frac{(0.1V)(50k\Omega||1k\Omega)}{(50k\Omega+47\Omega)+(50k\Omega||1k\Omega)} = 0.001921 = 1.92mV$$

$$v_{po} = 1.92mV$$

$$v_3 = v_1 = 0V$$

$$v_2 = v_4 = 0V$$

Nodal analysis can be performed on v_1 to receive the value v_{1o}

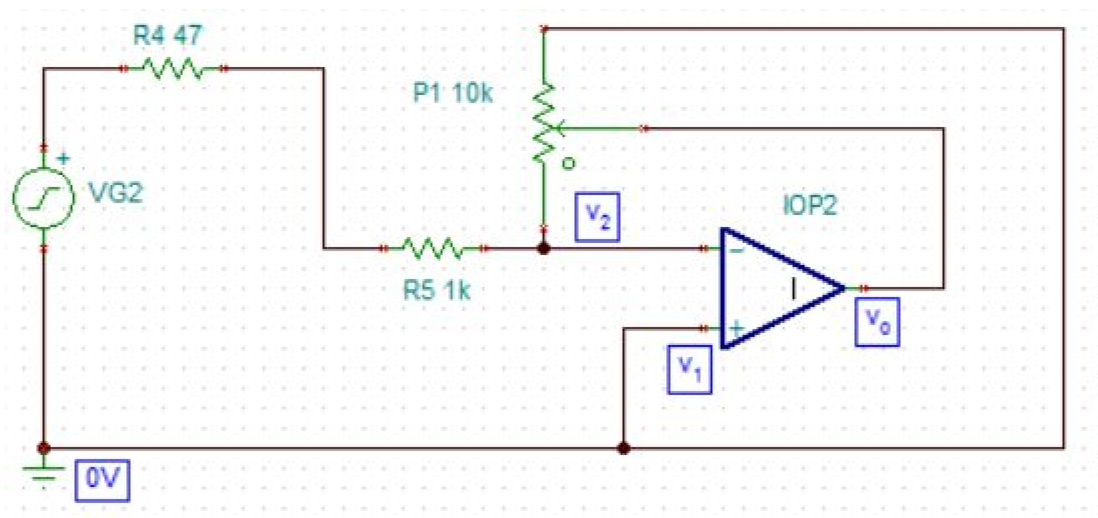
$$\frac{v_1 - v_{po}}{R1} + \frac{v_1 - v_{1o}}{R2} = 0$$

$$\frac{0V - 0.001921V}{1k\Omega} + \frac{0V - v_{1o}}{10k\Omega} = 0$$

Solving for v_{1o}:

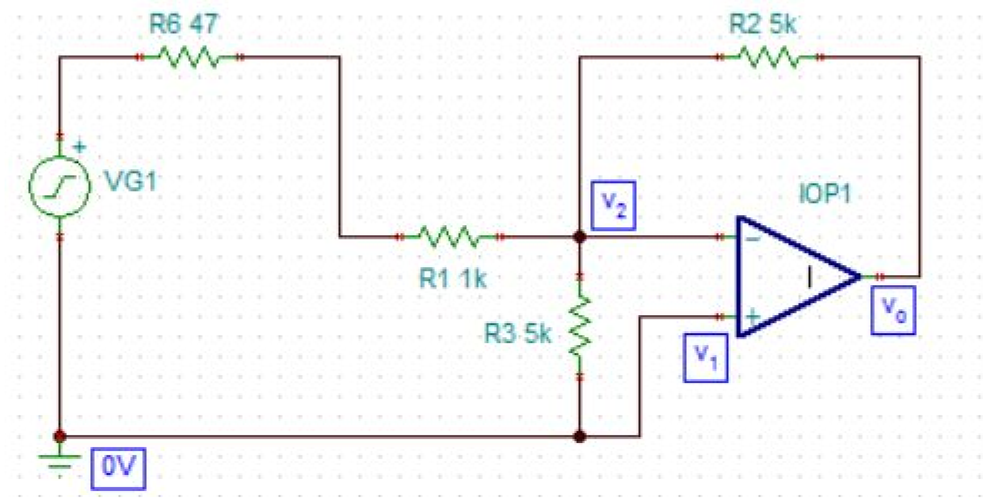
$$v_{1o} = -19.21mV$$

Consider the following circuit where the potentiometer is used to adjust the gain of the opamp directly (instead of at the circuit input),



Which can be redrawn and simplified to the following circuit,

For this circuit, the 10kOhm potentiometer is set at 50% position.

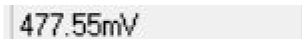


Using nodal analysis to solve for the output of the opamp,
$v_1 = v_2 = 0V$
$v_{in} = 0.1V$

$$\frac{v_2 - v_{in}}{R6 + R1} + \frac{v_2 - v_o}{R2} = 0$$
$$\frac{0V - 0.1V}{1047\Omega} + \frac{0V - v_o}{5k\Omega} = 0$$

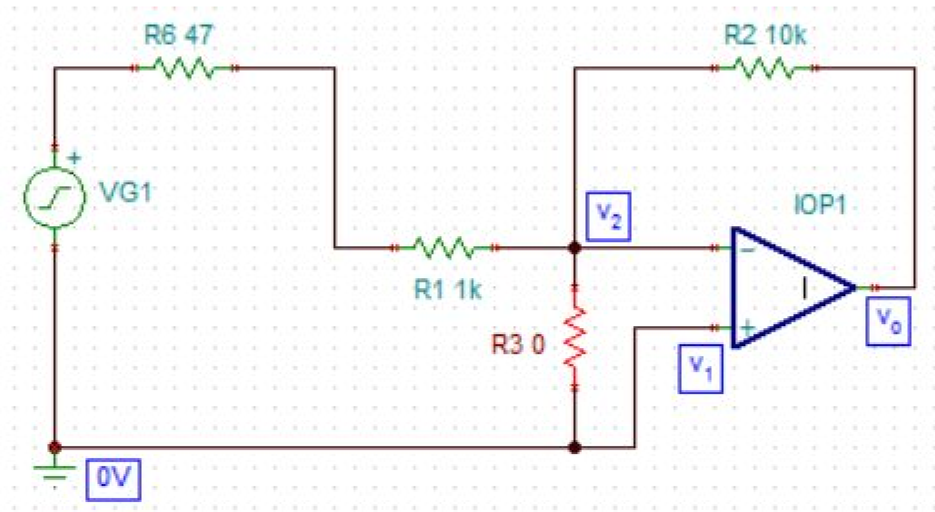
$$v_o = -0.47755V = 477.55mV$$
$$v_o = -0.477mV$$

Simulation

v_o when a potentiometer at 50% position is used as a replacement for R2 (to adjust the gain of the opamp stage directly): 

Mathematical Analysis, continued

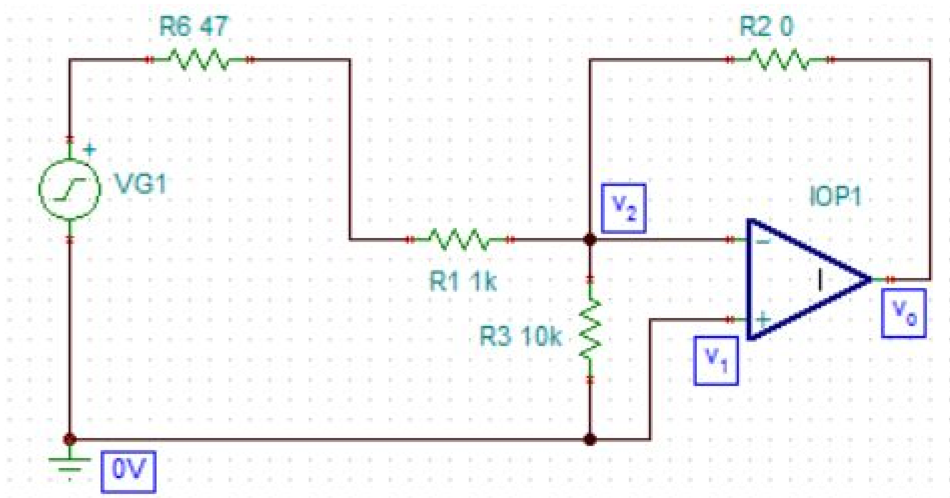
Consider the identical circuit from above,
The potentiometer is set to the 100% position in this example,



When the potentiometer is set to the 100% position, the bottom portion of the potentiometer is connected to ground and therefore shorts out the circuit. In this situation, the circuit is irregular and no longer functional.

Simulation of the above circuit
Simulation error is provided due to irregular circuitry (this was expected).

Consider the identical circuit from above,
The potentiometer is set to the 0% position in this example,



The non-inverting terminal will equal the inverting terminal in an ideal opamp.
$v_1 = v_2 = 0V$

The output v_o is connected directly to the node v_2 (resistor of 0Ω can be considered a wire).
Therefore, when the potentiometer of the circuit configuration is set to 0% position,
$v_1 = v_2 = v_o = 0V$

Simulation of the above circuit

v_o: 0V

Discussion

Having the volume control right at the input allows the ability to increase or decrease the **overall** gain of the circuit. If a potentiometer is used to change the resistance of the opamp gain stage directly, it would necessarily affect the opamp stage proceeding it. Remember that the output of the first opamp will be multiplied by the gain of the additional opamp gain stages. This method does not equate to a 1:1 volume control system and may even lead to overloading the proceeding opamp.

Additionally, there are also some problems that may occur by using a potentiometer to adjust the opamp gain directly. As observed from mathematical analysis and simulations, if the potentiometer reaches the 100% (maximum) or 0% (minimum) position, the circuit will short out. This is an unintentional and unwanted result that will occur when this volume control method is used.