CSE 211: Discrete Mathematics **(Due: 17/01/21)**

Homework #4

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Course Policy: Read all the instructions below carefully before you start working on the assignment, and before you make a submission.

- It is not a group homework. Do not share your answers to anyone in any circumstance. Any cheating means at least -100 for both sides.
- Do not take any information from Internet.
- No late homework will be accepted.
- For any questions about the homework, send an email to gizemsungu@gtu.edu.tr
- The homeworks (both latex and pdf files in a zip file) will be submitted into the course page of Moodle.
- The latex, pdf and zip files of the homeworks should be saved as "Name_Surname_StudentId".{tex, pdf, zip}.
- If the answers of the homeworks have only calculations without any formula or any explanation -when needed- will get zero.
- Writing the homeworks on Latex is strongly suggested. However, hand-written paper is still accepted **IFF** hand writing of the student is clear and understandable to read, and the paper is well-organized. Otherwise, the assistant cannot grade the student's homework.

Problem 1 (15+15=30 points)

Consider the nonhomogeneous linear recurrence relation $a_n = 3a_{n-1} + 2^n$.

(a) Show that whether $a_n = -2^{n+1}$ is a solution of the given recurrence relation or not. Show your work step by step.
(Solution)

1) If $a_n = -2^{n+1}$ then $a_{n-1} = -2^n$

2) Put a_n and a_{n-1} in the recurrence relation.

3) $-2^{n+1} = 3*(-2^n) + 2^n$

4) $-2^{n+1} = -2*2^n$ so $2^{n+1} = 2*2^n$

5) 2 = 2 so that means it is a solution of given recurrence relation.

(b) Find the solution with $a_0 = 1$.
(Solution)

0) $a_n = a_n^{(h)} + a_n^{(p)}$

1) Firstly $a_n - 3a_{n-1} = 0$ for $a_n^{(h)}$

2) r - 3 = 0 so r = 3

3) $a_n^{(h)} = \alpha 3^n$

4) Secondly $a_n - 3a_{n-1} = 2^n$ for $a_n^{(p)}$

5) Guessing $A2^n$ form

6) $A2^n - 3A2^{n-1} = 2^n$ (Let's divide by 2^n)

7) $A - \frac{3}{2}A = 1$ so A = -2

8) Rewriting (5) again $a_n^{(p)} = -2.2^n = -2^{n+1}$

9) Rewriting (0) again $a_n = \alpha 3^n - 2^{n+1}$ (for n⩾1)

10) Put $a_0 = 1$ in relation in question so $a_1 = 3*1 + 2 = 5$

11) Put $a_1 = 5$ in (9) so $a_1 = 5 = \alpha 3^1 - 2^2$ so $\alpha = 3$

12) Put $\alpha = 3$ in (9) So $a_n = 3*3^n - 2^{n+1}$

13) So the solution for $a_0 = 1$ is $a_n = 3^{n+1} - 2^{n+1}$ (for n ⩾1)

Problem 2 (35 points)

Solve the recurrence relation f(n) = 4f(n-1) - 4f(n-2) + n^2 for f(0) = 2 and f(1) = 5.
(Solution)

1) f(n) - 4f(n-1) + 4f(n-2) = n^2

2) $f(n) = f^{(h)}(n) + f^{(p)}(n)$

3) (For $a_n^{(h)}$) Characteristic equation is r^2 - 4r + 4 = 0

4) So r=2 (coincident roots) so $a_n^{(h)} = \alpha 2^n + \beta$n$2^n$

5) (For $a_n^{(p)}$) Guessing $Kn^2 + Ln + M$ form.

6) So f(n) = $Kn^2 + Ln + M$ and f(n-1) = $K(n-1)^2 + L(n-1) + M$ and f(n-2) = $K(n-2)^2 + L(n-2) + M$

7) Put in (1) $Kn^2 + Ln + M$ -4[$K(n-1)^2 + L(n-1) + M$ $-K(n-2)^2 - L(n-2) - M$]

8) K($n^2 - 4(n-1)^2 + (n-2)^2) + L(n - 4n + 4 + n - 2) + 3M = n^2$

9) K$n^2 + (-8K + L)n + (12K - 4L + M) = n^2$ so equalize to the coefficients

10) K=1 then solve (-8K+L=0) so L=8. Then 12K-4L+M = 12-32+M=0 so M=20

11) $a_n^{(p)} = n^2 + 8n + 20$ and $a_n^{(h)} = \alpha 2^n + \beta$n$2^n$

12) Put everything in (2) so f(n) = $n^2 + 8n + 20 + 2^n(\alpha + n\beta)$

13) f(0)=2 so 20 + α = 2 so α=-18 and f(1)=5 so 1+8+20+2(-18+β)=5 so $\beta = 6$

14) So f(n) = $n^2 + 8n + 20 + 2^n(6n - 18)$

Problem 3 (20+15 = 35 points)

Consider the linear homogeneous recurrence relation $a_n = 2a_{n-1}$ - $2a_{n-2}$.
(a) Find the characteristic roots of the recurrence relation.
(Solution)

1) a_n - $2a_{n-1}$ + $2a_{n-2}$.= 0

2) $2a_{n-2}$ will be showed as 2 (like a constant) to find the characteristic roots.

3) So it will be like $->$ r^2 - 2r + 2 = 0

4) It is same with $(r-1)^2 + 1 = 0$

5) $(r-1) = i$ or $-(r-1) = i$ because i = $\sqrt{-1}$

6) So r = 1 - i $\bigvee$ r = 1 + i

(b) Find the solution of the recurrence relation with $a_0 = 1$ and $a_1 = 2$.
(Solution)

1) $\alpha(1-i)^n + \beta(1+i)^n = a_n$

2) $a_0 = \alpha + \beta = 1$ and $a_1 = \alpha + \beta + i(\beta - \alpha) = 2$

3) If $\alpha + \beta = 1$ then, $\beta - \alpha$ should be = -i

4) When sum those, $2\beta = 1 - i$ so $\beta = \frac{1-i}{2}$ and $\alpha = 1 - \frac{1-i}{2} = \frac{1+i}{2}$

5) So $a_n = \frac{1+i}{2}(1-i)^n + \frac{1-i}{2}(1+i)^n$