

1. Simplify the following function by using boolean algebra $F(x,y,z) = xy + x'z + yz$.

Solution:

$$\begin{aligned} xy + x'z + yz &= xy + x'z + yz(x + x') \\ &= xy + x'z + xyz + x'yz = xy(1+z) + x'z(1+y) \\ &= xy + x'z \end{aligned}$$

2. Derive that $(x + y)(x' + z)(y + z) = (x + y)(x' + z)$ by using boolean algebra.

Solution:

$$\begin{aligned} (x' + z)(y + z) &= x'y + x'z + zy + \underbrace{zz}_z \\ &= x'(y+z) + z\underbrace{(y+1)}_1 \\ &= x'z + x'y + z = z\underbrace{(1+x')}_1 + x'y \\ &= z + x'y \\ \text{So left side is } (x+y)(z + x'y) &= xz + \underbrace{xx'}_x y + yz + \underbrace{yy}_y x' \\ &= xz + x'y + yz + yx' \\ \text{so left side is } &= xz + yz + x'y \\ \text{And the right side is } &\underbrace{xx'}_0 + xz + yx' + yz = xz + x'y + yz \\ \text{So left side and right side equals each other} & \end{aligned}$$

3. (a) Express the following function in **sum of minterms** and **product of maxterms** by using truth table

$$F(A, B, C, D) = B'D + A'D + BD.$$

- (b) Simplify the standard expression

$$F(A, B, C, D) = B'D + A'D + BD.$$

Solution (a)

A	B	C	D	MINTERM(SOP)	MAXTERMS(POS)
0	0	0	0	$m_0 = A'B'C'D'$	$M_0 = A+B+C+D$
0	0	0	1	$m_1 = A'B'C'D$	$M_1 = A+B+C+D'$
0	0	1	0	$m_2 = A'B'CD'$	$M_2 = A+B+C'+D$
0	0	1	1	$m_3 = A'B'CD$	$M_3 = A+B+C'+D'$
0	1	0	0	$m_4 = A'BC'D'$	$M_4 = A+B'+C+D$
0	1	0	1	$m_5 = A'BC'D$	$M_5 = A+B'+C+D'$
0	1	1	0	$m_6 = A'BCD'$	$M_6 = A+B'+C'+D$
0	1	1	1	$m_7 = A'BCD$	$M_7 = A+B'+C'+D'$
1	0	0	0	$m_8 = AB'C'D'$	$M_8 = A'+B+C+D$
1	0	0	1	$m_9 = AB'C'D$	$M_9 = A'+B+C+D'$
1	0	1	0	$m_{10} = AB'CD'$	$M_{10} = A'+B+C'+D$
1	0	1	1	$m_{11} = AB'CD$	$M_{11} = A'+B+C'+D'$
1	1	0	0	$m_{12} = ABC'D'$	$M_{12} = A'+B'+C+D$
1	1	0	1	$m_{13} = ABC'D$	$M_{13} = A'+B'+C+D'$
1	1	1	0	$m_{14} = ABCD'$	$M_{14} = A'+B'+C'+D$
1	1	1	1	$m_{15} = ABCD$	$M_{15} = A'+B'+C'+D'$

Table 1: Minterm and Maxterm Table

Solution: **For sum of minterms**

$$\begin{aligned}
 B'D + A'D + BD &= D(B' + A' + B) = D(\underbrace{B' + B}_{1} + A') = D(\underbrace{A'}_{1} + \underbrace{1}_{1}) = D(\underbrace{A + A'}_{1}) \\
 &= DA + DA' \text{ and since all } (X+X')\text{s are 1} \\
 &= DA(B+B') + DA'(B+B') \\
 &= DAB + DAB' + DA'B + DA'B' \\
 &= DAB(C+C') + DAB'(C+C') + DA'B(C+C') + DA'B'(C+C') \\
 &= DABC + DABC' + DAB'C + DAB'C' + DA'BC + DA'BC' + DA'B'C + DA'B'C' \\
 &= m_{15} + m_{13} + m_{11} + m_9 + m_7 + m_5 + m_3 + m_1 \\
 &= \sum (15, 13, 11, 9, 7, 5, 3, 1)
 \end{aligned}$$

Solution: For product of maxterms

$$B'D + A'D + BD = D.(B' + A' + B) = D(\underbrace{B' + B}_{1} + A') = D(\underbrace{1 + A'}_{1}) = D$$

For $D = (D+0) = D + AA' = (D+A)(D+A')$ since $BB' = 0$

$$\begin{aligned} (D+A+BB')(D+A'+BB') &= (D+A+B)(D+A+B')(D+A'+B)(D+A'+B') \text{ since } CC' = 0 \\ &= (D+A+B+CC')(D+A+B'+CC')(D+A'+B+CC')(D+A'+B'+CC') \end{aligned}$$

$$= (D+A+B+C).(D+A+B+C').(D+A+B'+C).(D+A+B'+C').(D+A'+B+C).(D+A'+B+C').(D+A'+B'+C).(D+A'+B'+C')$$

$$= m_0 + m_2 + m_4 + m_6 + m_8 + m_{10} + m_{12} + m_{14}$$

$$= \prod (0,2,4,6,8,10,12,14)$$

(b)

$$D(B' + A' + B)$$

Because of $(B' + B) = 1$

$$= D(1 + A')$$

Because of $(1 + A') = 1$

$$= D$$