

1. Simplify the following function by using boolean algebra $F(x,y,z) = xy + x'z + yz$.

Solution:

$$\begin{aligned} xy + x'z + yz &= xy + x'z + yz(x + x') \\ &= xy + x'z + xyz + x'yz = xy(1+z) + x'z(1+y) \\ &= xy + x'z \end{aligned}$$

2. Derive that $(x + y)(x' + z)(y + z) = (x + y)(x' + z)$ by using boolean algebra.

Solution:

$$\begin{aligned} (x' + z)(y + z) &= x'y + x'z + zy + \underbrace{zz}_z \\ x'(y+z) + \underbrace{z(y+1)}_1 \\ x'z + x'y + z &= \underbrace{z(1+x')}_1 + x'y \\ &= z + x'y \\ \text{So left side is } (x+y)(z + x'y) &= xz + \underbrace{xx'}_x y + yz + \underbrace{yy}_y x' \\ &= xz + x'y + yz + yx' \\ \text{so left side is } &= xz + yz + x'y \\ \text{And the right side is } \underbrace{xx'}_0 &+ xz + yx' + yz = xz + x'y + yz \\ \text{So left side and right side equals each other} \end{aligned}$$

3. (a) Express the following function in **sum of minterms** and **product of maxterms** by using truth table

$$F(A, B, C, D) = B'D + A'D + BD.$$

- (b) Simplify the standard expression

$$F(A, B, C, D) = B'D + A'D + BD.$$

Solution (a)

A	B	C	D	MINTERM(SOP)	MAXTERMS(POS)
0	0	0	0	m0 = A'B'C'D'	M0 = A+B+C+D
0	0	0	1	m1 = A'B'C'D	M1 = A+B+C+D'
0	0	1	0	m2 = A'B'CD'	M2 = A+B+C'+D
0	0	1	1	m3 = A'B'CD	M3 = A+B+C'+D'
0	1	0	0	m4 = A'BC'D'	M4 = A+B'+C+D
0	1	0	1	m5 = A'BC'D	M5 = A+B'+C+D'
0	1	1	0	m6 = A'BCD'	M6 = A+B'+C'+D
0	1	1	1	m7 = A'BCD	M7 = A+B'+C'+D'
1	0	0	0	m8 = AB'C'D'	M8 = A'+B+C+D
1	0	0	1	m9 = AB'C'D	M9 = A'+B+C+D'
1	0	1	0	m10 = AB'CD'	M10 = A'+B+C'+D
1	0	1	1	m11 = AB'CD	M11 = A'+B+C'+D'
1	1	0	0	m12 = ABC'D'	M12 = A'+B'+C+D
1	1	0	1	m13 = ABC'D	M13 = A'+B'+C+D'
1	1	1	0	m14 = ABCD'	M14 = A'+B'+C'+D
1	1	1	1	m15 = ABCD	M15 = A'+B'+C'+D'

Table 1: Minterm and Maxterm Table

Solution: For sum of minterms

$$\begin{aligned}
 B'D + A'D + BD &= D(B' + A' + B) = D(\underbrace{B' + B}_{1} + A') = D(\underbrace{A' + 1}_{1}) = D(\underbrace{A + A'}_{1}) \\
 &= DA + DA' \text{ and since all } (X+X')\text{s are } 1 \\
 &= DA(B+B') + DA'(B+B') \\
 &= DAB + DAB' + DA'B + DA'B' \\
 &= DAB(C+C') + DAB'(C+C') + DA'B(C+C') + DA'B'(C+C') \\
 &= DABC + DABC' + DAB'C + DAB'C' + DA'BC + DA'BC' + DA'B'C + DA'B'C' \\
 &= m_{15} + m_{13} + m_{11} + m_9 + m_7 + m_5 + m_3 + m_1 \\
 &= \sum (15, 13, 11, 9, 7, 5, 3, 1)
 \end{aligned}$$

Solution: For product of maxterms

$$B'D + A'D + BD = D.(B' + A' + B) = D(\underbrace{B' + B}_1 + A') = D(\underbrace{1 + A'}_1) = D$$

$$\text{For } D = (D+0) = D + AA' = (D+A)(D+A') \text{ since } BB' = 0$$

$$(D+A+BB')(D+A'+BB') = (D+A+B)(D+A+B')(D+A'+B)(D+A'+B') \text{ since } CC' = 0$$

$$= (D+A+B+CC')(D+A+B'+CC')(D+A'+B+CC')(D+A'+B'+CC')$$

$$= (D+A+B+C).(D+A+B+C').(D+A+B'+C).(D+A+B'+C').(D+A'+B+C).(D+A'+B+C').(D+A'+B'+C).(D+A'+B'+C')$$

$$= m_0 + m_2 + m_4 + m_6 + m_8 + m_{10} + m_{12} + m_{14}$$

$$= \prod (0,2,4,6,8,10,12,14)$$

(b)

$$D(B' + A' + B)$$

$$\text{Because of } (B' + B) = 1$$

$$= D(1 + A')$$

$$\text{Because of } (1 + A') = 1$$

$$= D$$