

# CSE321 - PS

# Problem 1

Construct an algorithm to find the missing element in a regularly increasing array.

## **Example:**

2 4 6 8 12 → missing element is 10

3 9 12 15 18 → missing element is 6

# Solution 1

**findMissingElement** (array):

*differences*  $\leftarrow$  [ ]

$n \leftarrow$  length of array

**for** i from 0 to  $n-1$ :

**append** (array[i+1]-array[i]) to *differences*

*difference*  $\leftarrow$  mostly seen element in *differences*

**for** i from 0 to  $n-1$ :

**if** (array[i+1]-array[i])  $\neq$  *difference*:

**return** array[i] + *difference*

# Solution 1

**findMissingElement** (array):

*differences*  $\leftarrow$  [ ]

$n \leftarrow$  length of array

**for** i from 0 to  $n-1$ :

**append** (array[i+1]-array[i]) to *differences*

*difference*  $\leftarrow$  mostly seen element in *differences*

**for** i from 0 to  $n-1$ :

**if** (array[i+1]-array[i])  $\neq$  *difference*:

**return** array[i] + *difference*

**Best Case:**  $\Omega(n)$

**Worst Case:**  $O(n)$

**Average Case:**  $\Theta(n)$

# Solution 1

A solution by using the *binary search*.

# Solution 1

3

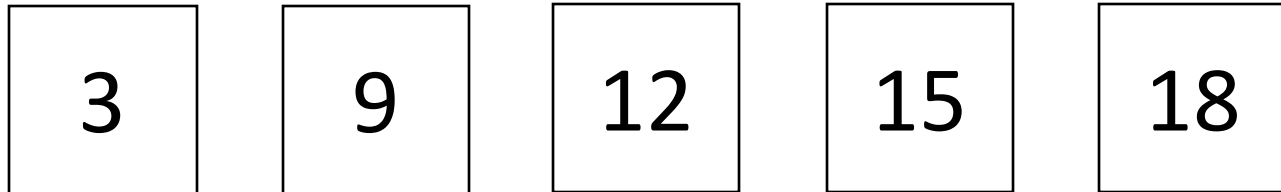
9

12

15

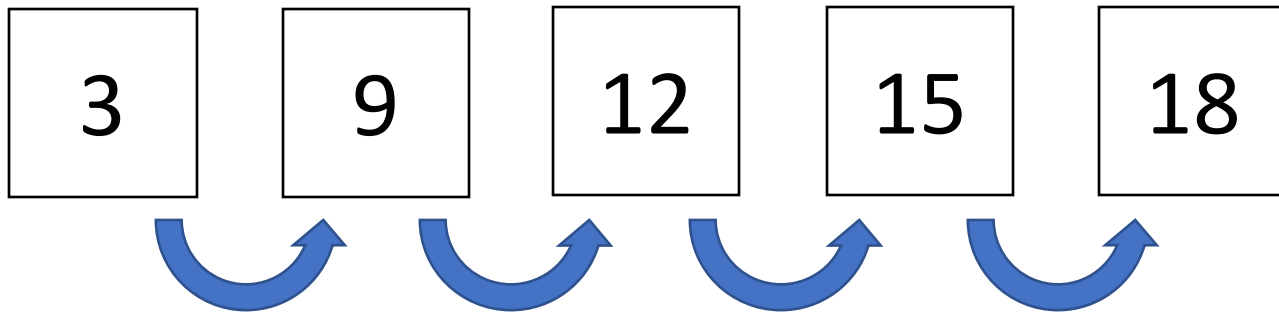
18

## Solution 1



difference = ?

## Solution 1

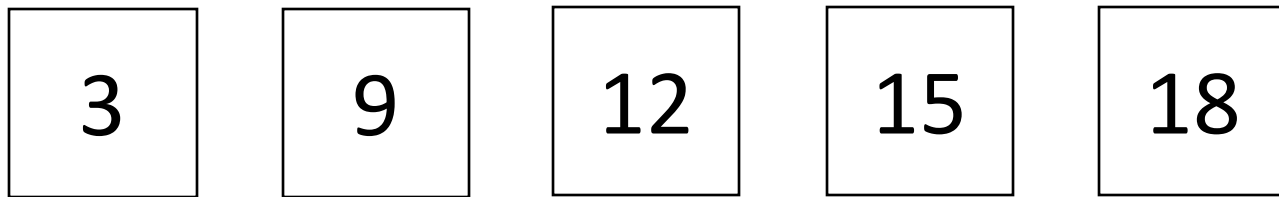


$n - 1 = 4$  additions

difference = ?



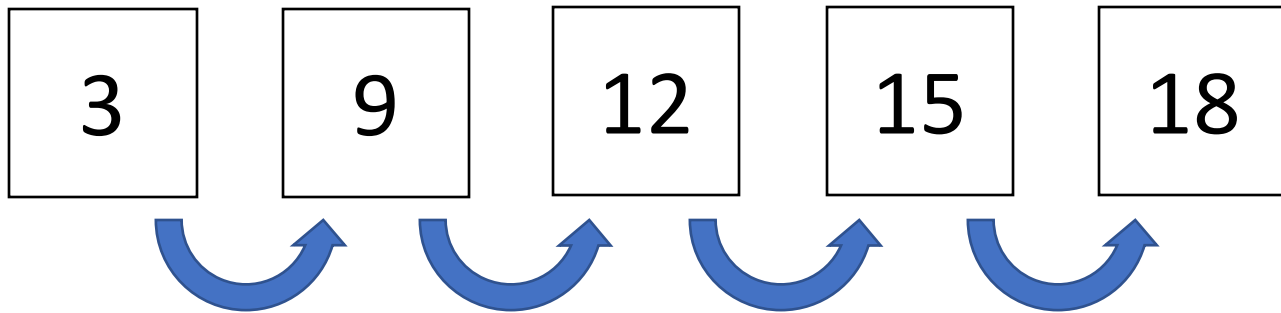
## Solution 1



$(n - 1) + 1 = n = 5$  additions

difference = ?

## Solution 1

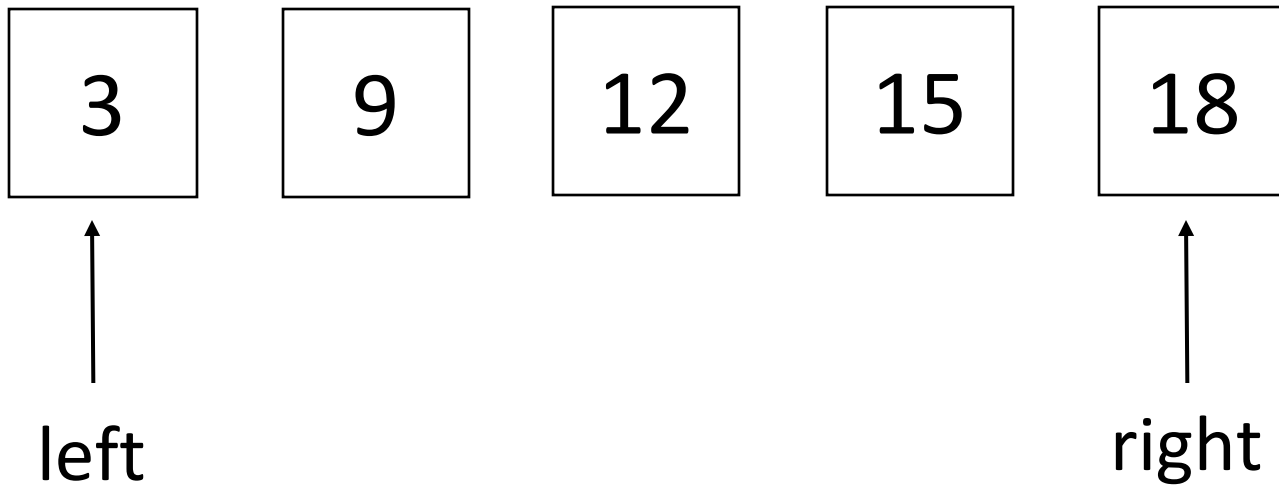


difference = ?

$(n - 1) + 1 = n = 5$  additions

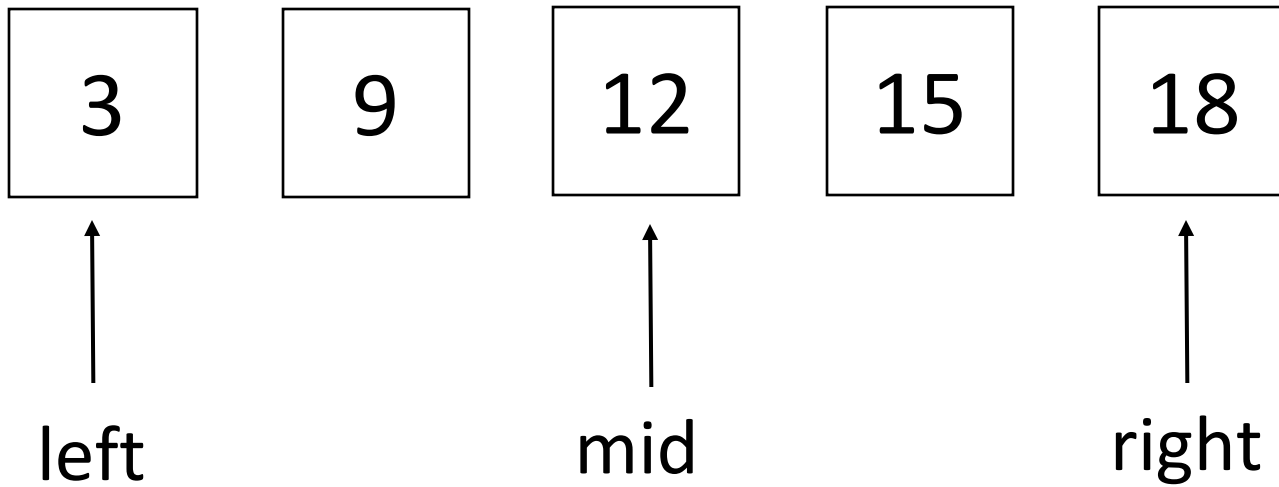
difference =  $(18 - 3) / 5 = 3$

## Solution 1



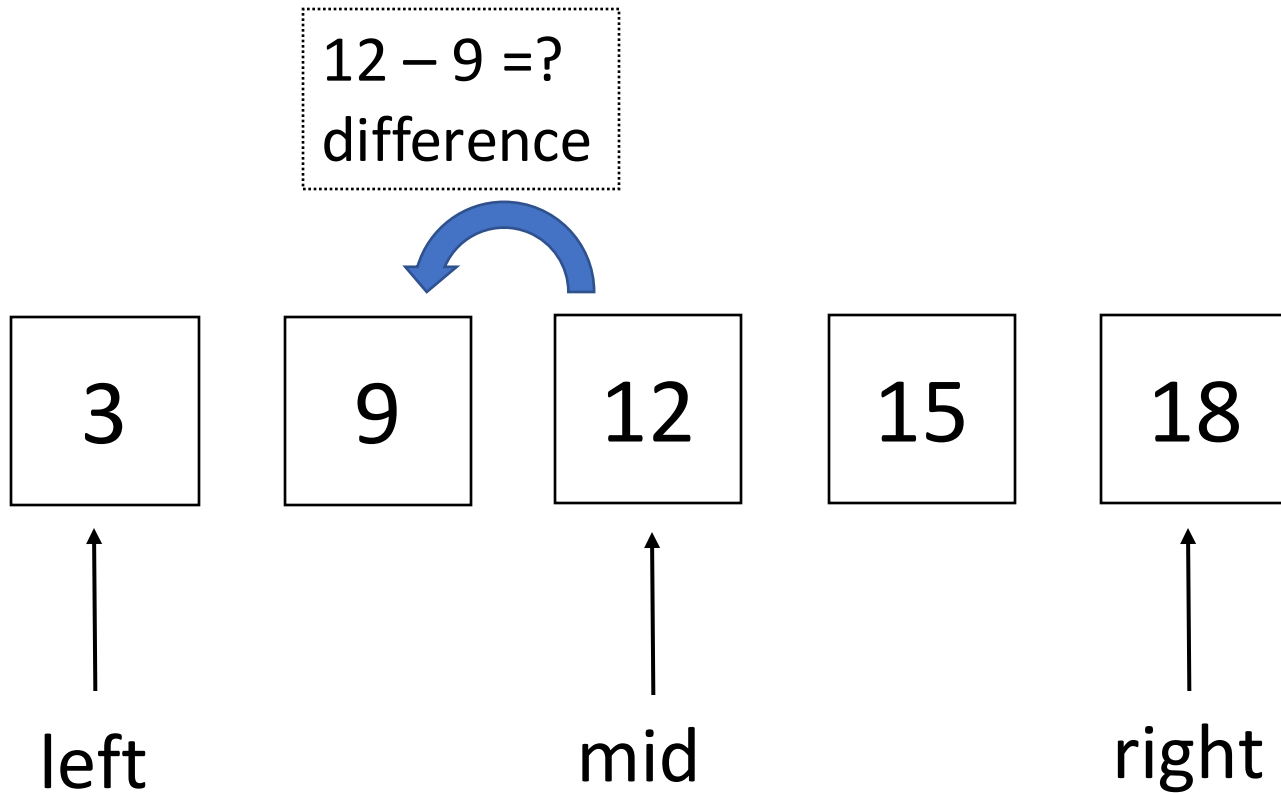
difference = 3

## Solution 1



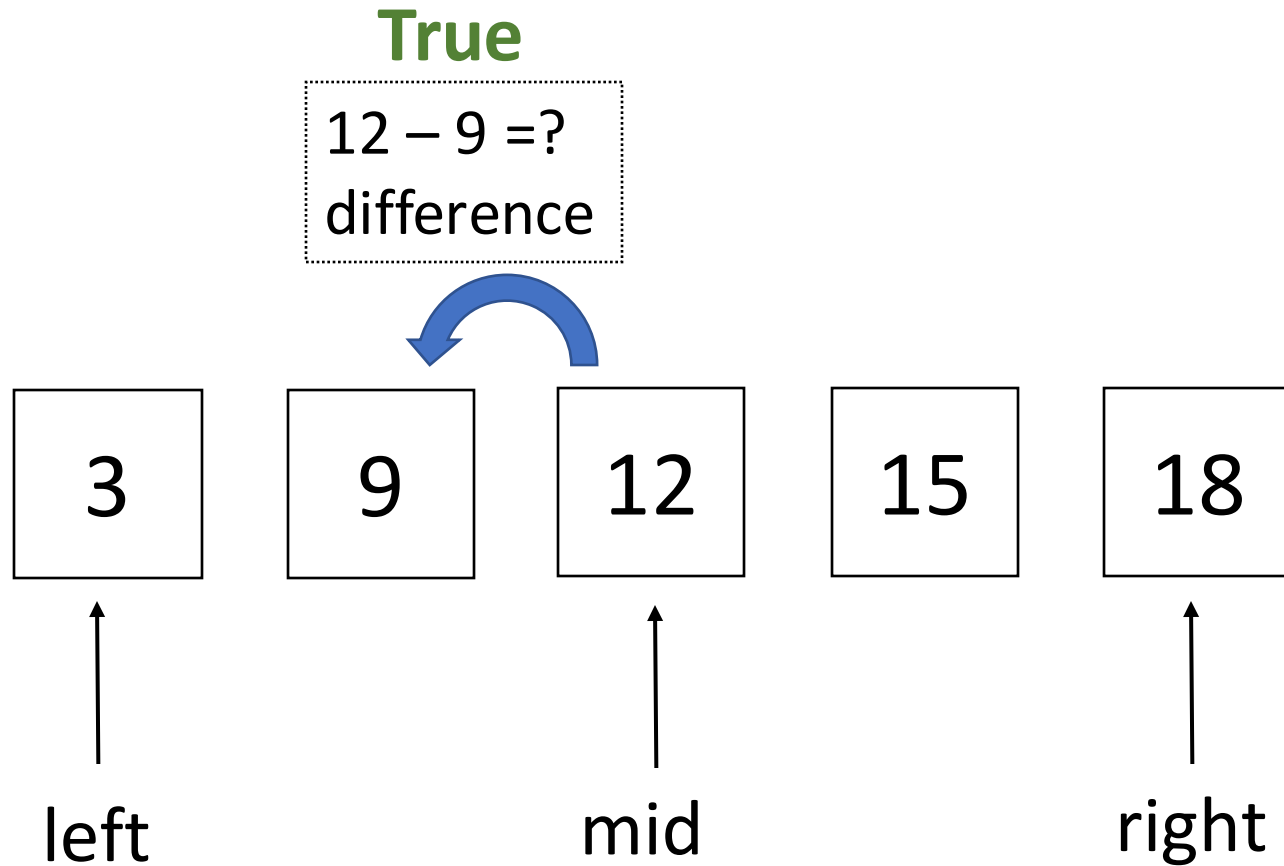
difference = 3

# Solution 1



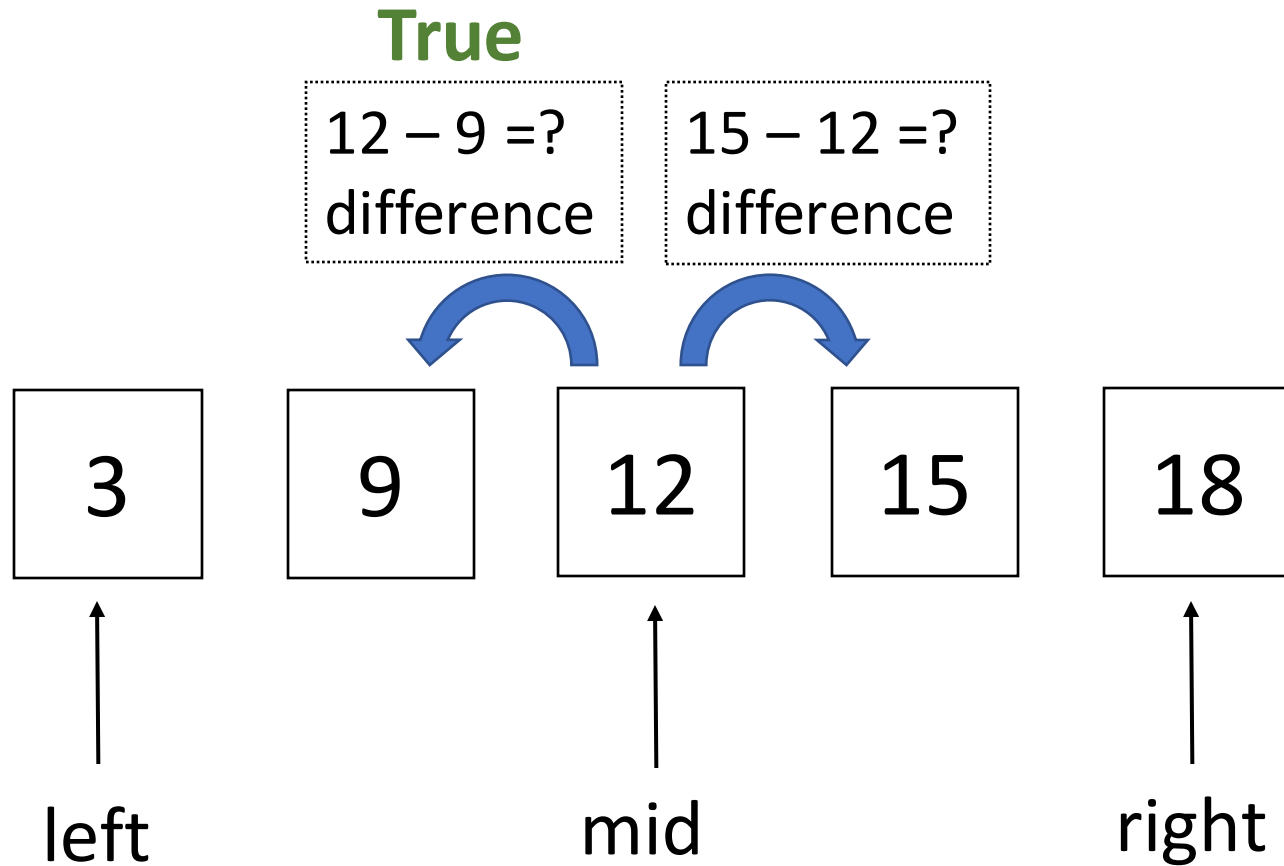
difference = 3

# Solution 1



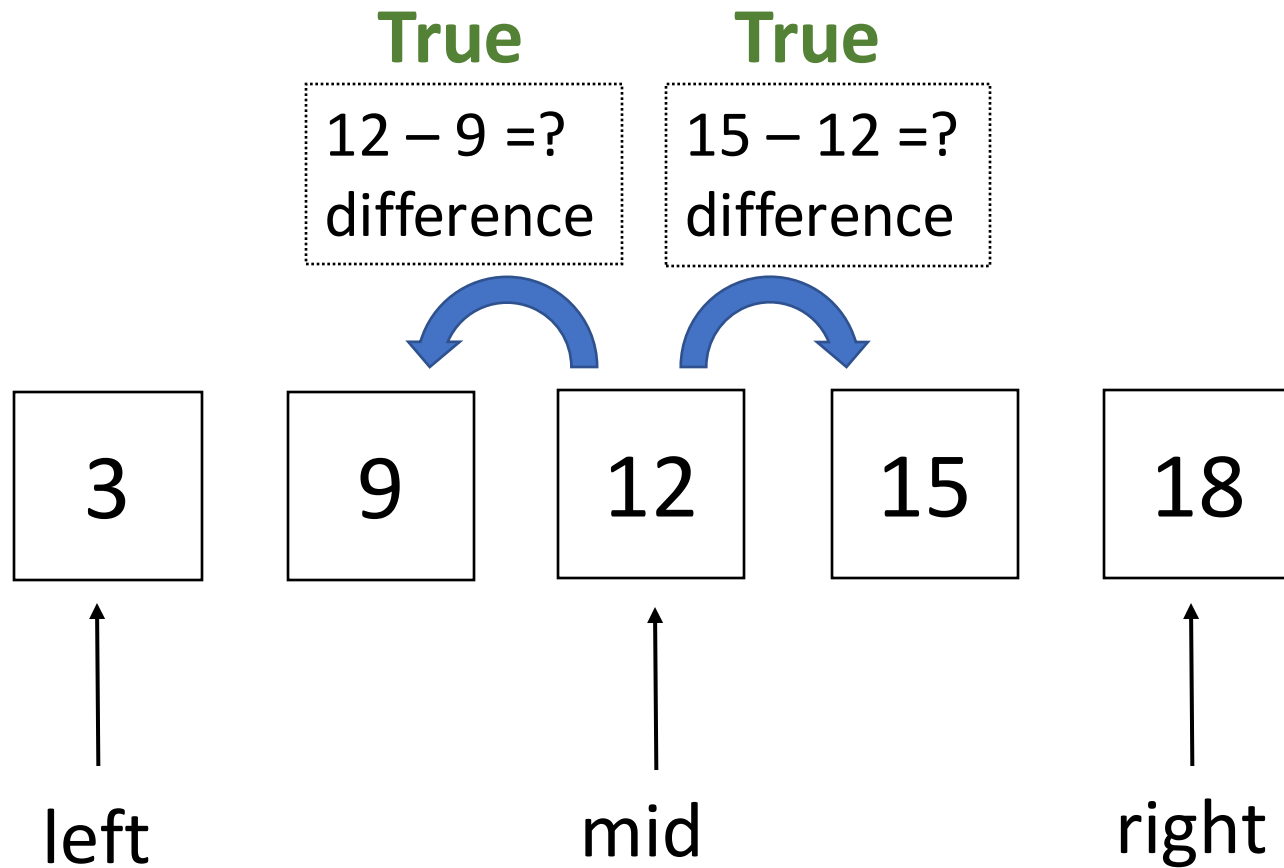
difference = 3

# Solution 1



difference = 3

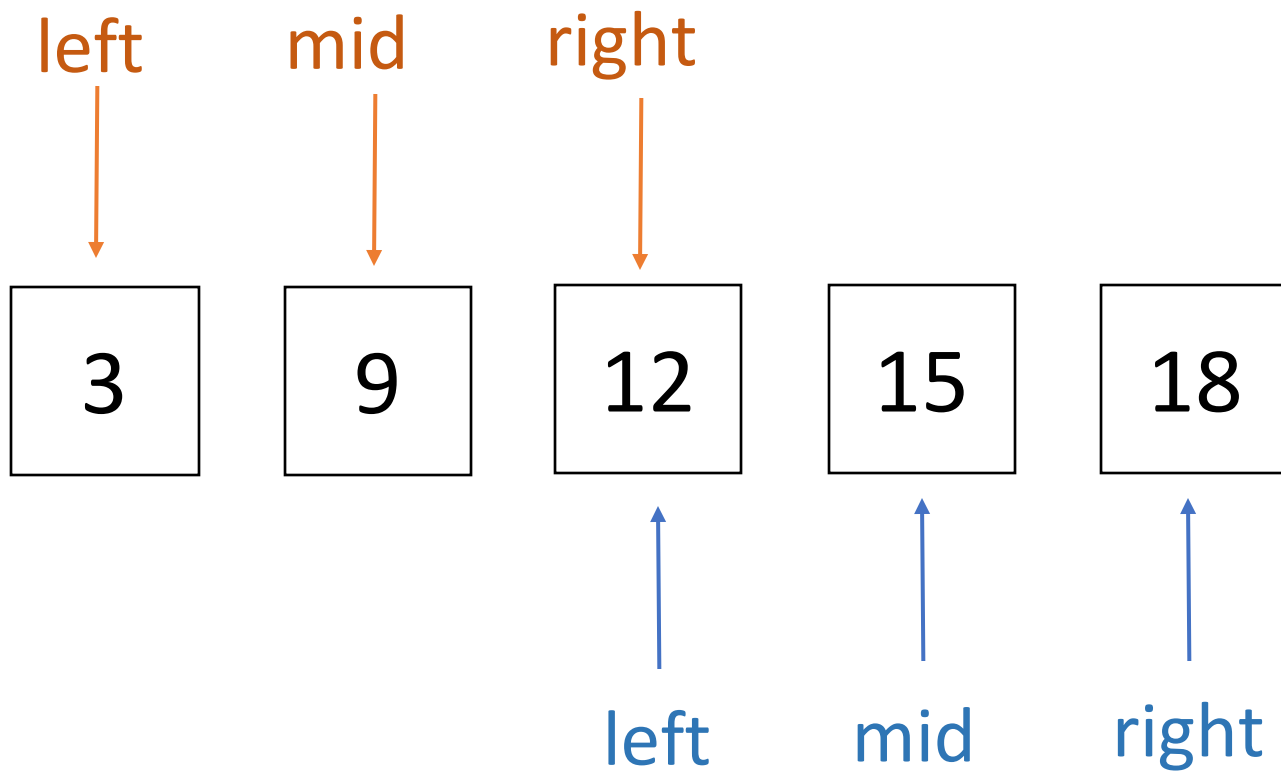
# Solution 1



difference = 3

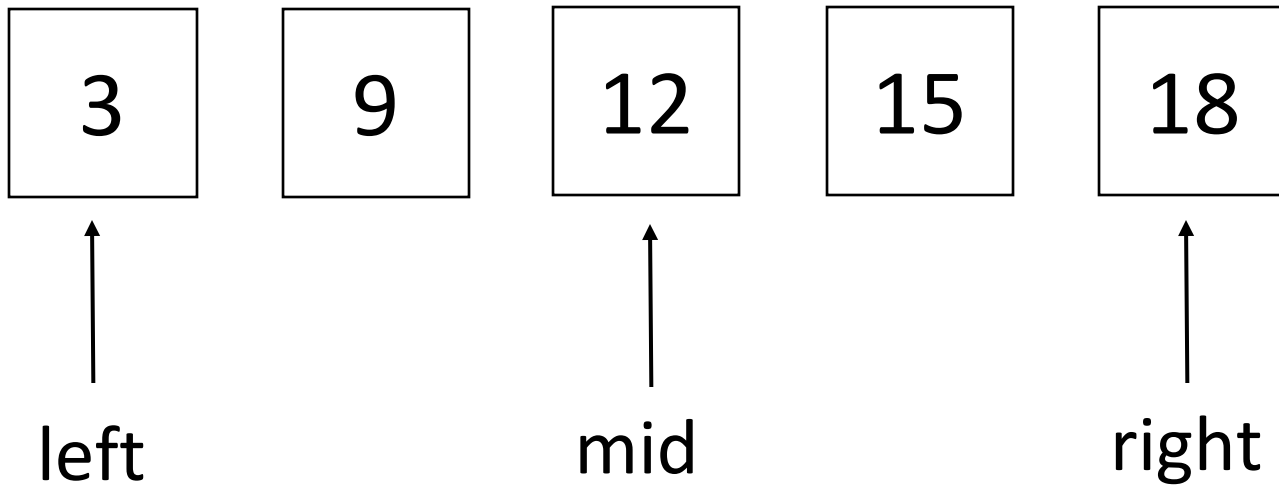


# Solution 1



difference = 3

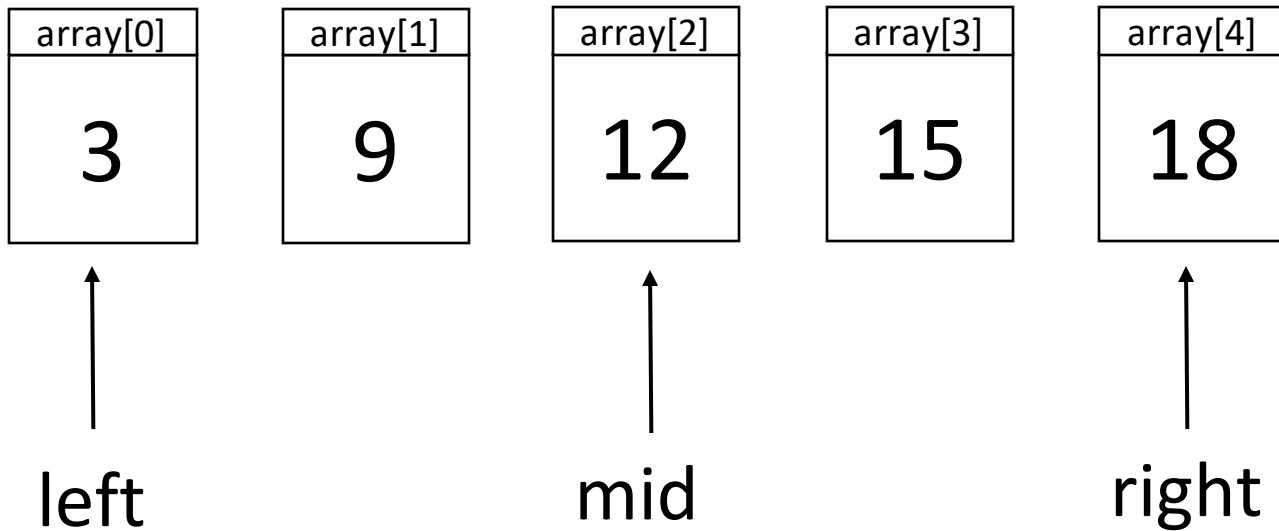
## Solution 1



difference = 3

## Solution 1

$$a = \text{right} - \text{mid} = 4 - 2 = 2 \text{ (number of additions)}$$

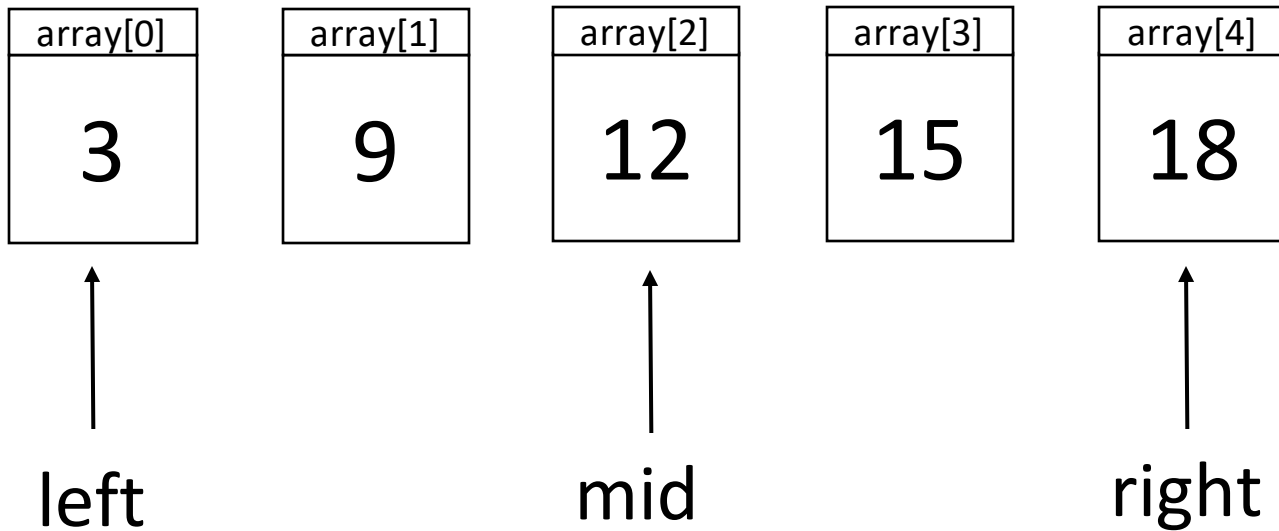


difference = 3

## Solution 1

$a = \text{right} - \text{mid} = 4 - 2 = 2$  (number of additions)

$\text{array}[\text{mid}] + a * \text{difference} = ? \text{array}[\text{right}]$



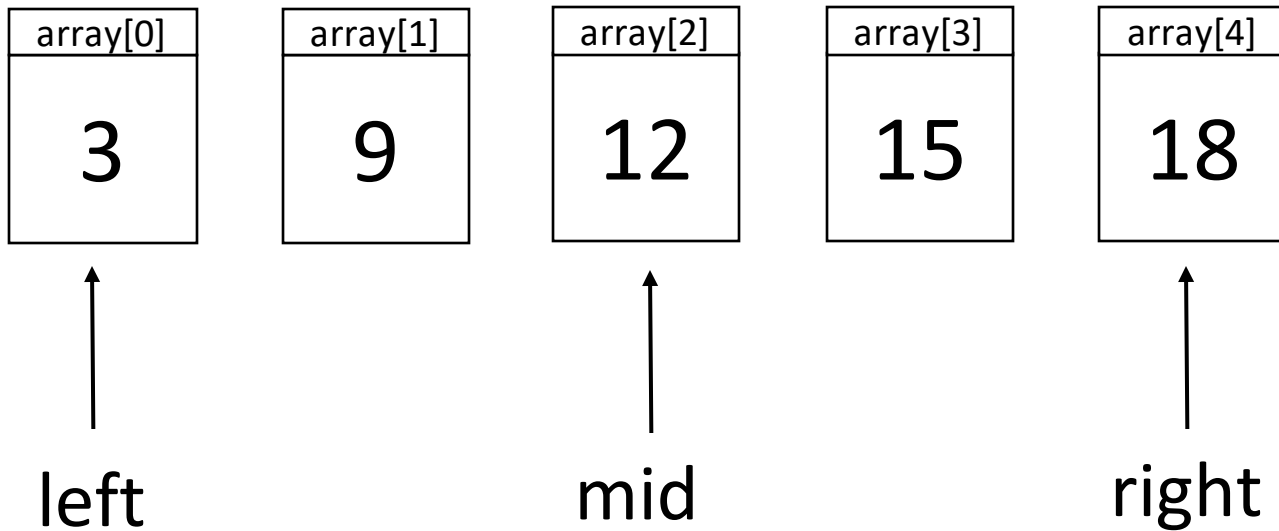
difference = 3

## Solution 1

$a = \text{right} - \text{mid} = 4 - 2 = 2$  (number of additions)

$\text{array}[\text{mid}] + a * \text{difference} = ? \text{array}[\text{right}]$

$12 + 2 * 3 = ? 18$



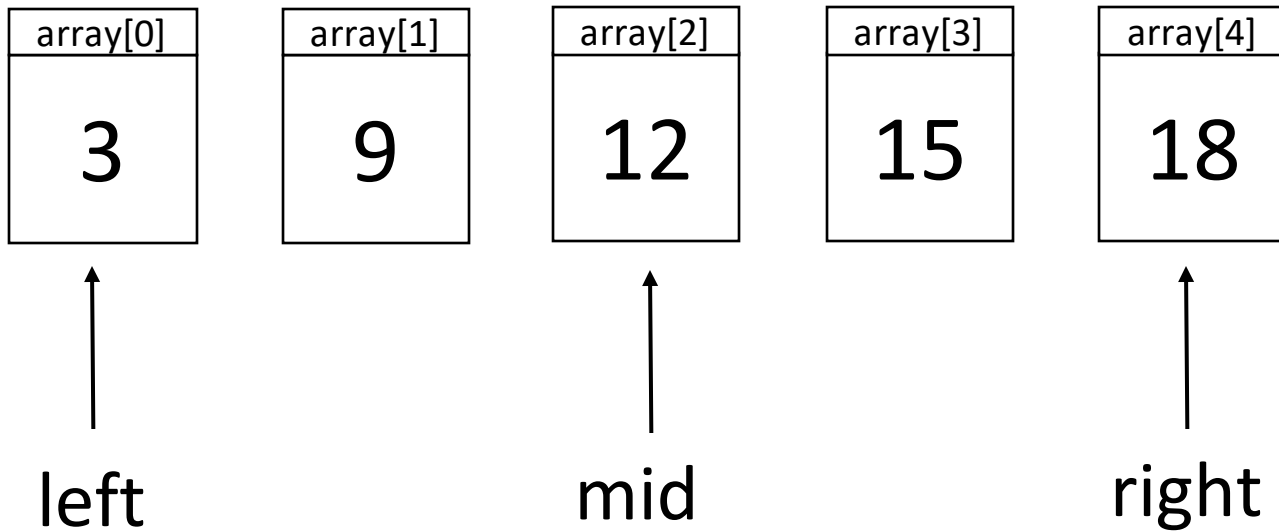
difference = 3

## Solution 1

$a = \text{right} - \text{mid} = 4 - 2 = 2$  (number of additions)

$\text{array}[\text{mid}] + a * \text{difference} = ? \text{array}[\text{right}]$

$12 + 2 * 3 = ? 18$  **True**



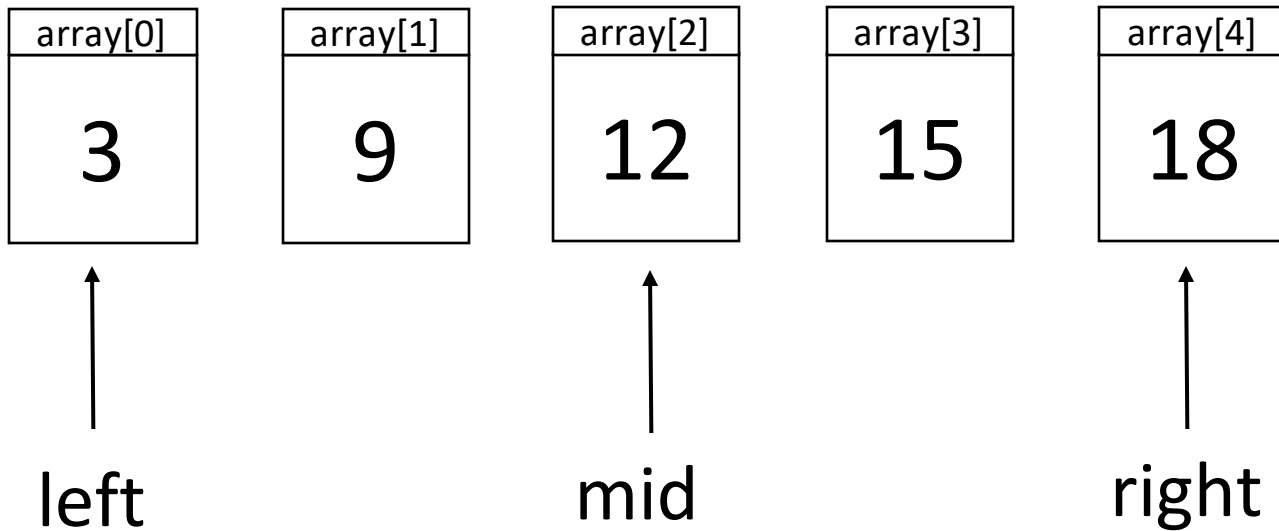
difference = 3

## Solution 1

$a = \text{mid} - \text{left} = 2 - 0 = 2$  (number of additions)

$\text{array}[\text{mid}] - a * \text{difference} = ? \text{array}[\text{left}]$

$3 = ? 12 - 2 * 3$



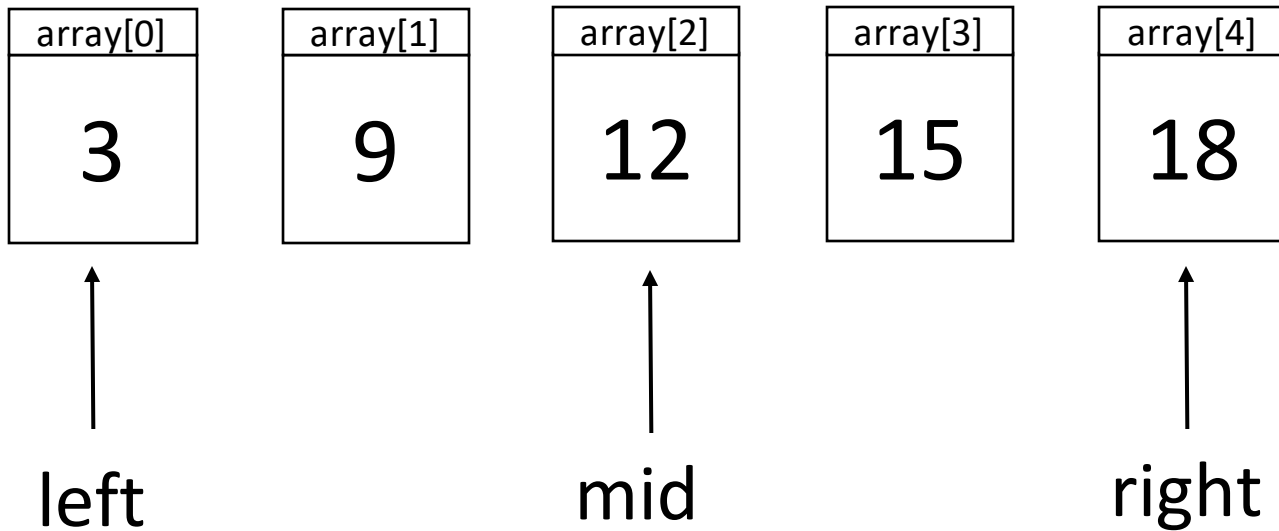
difference = 3

## Solution 1

$a = \text{mid} - \text{left} = 2 - 0 = 2$  (number of additions)

$\text{array}[\text{mid}] - a * \text{difference} = ? \text{array}[\text{left}]$

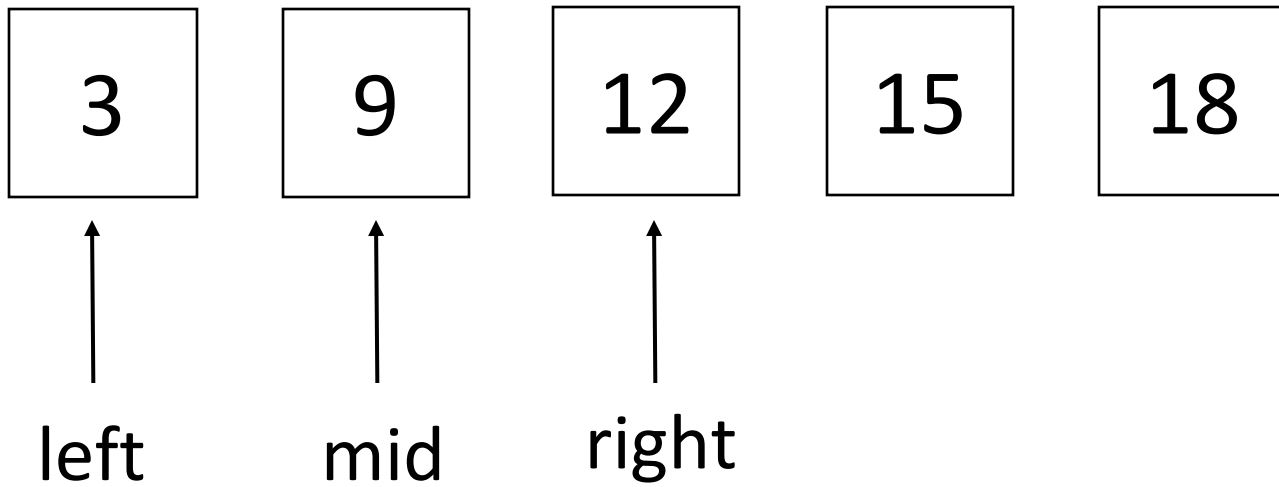
$3 = ? 12 - 2 * 3$  **False**



difference = 3

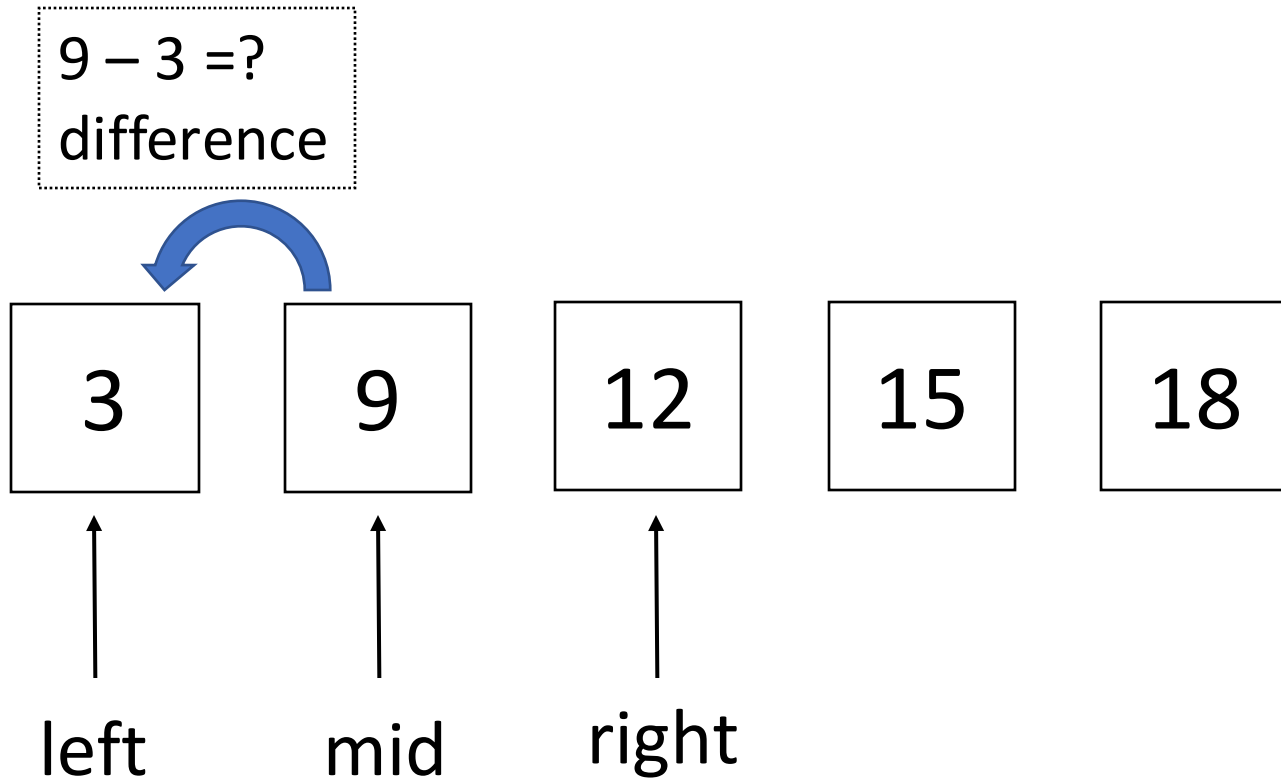


## Solution 1



difference = 3

# Solution 1

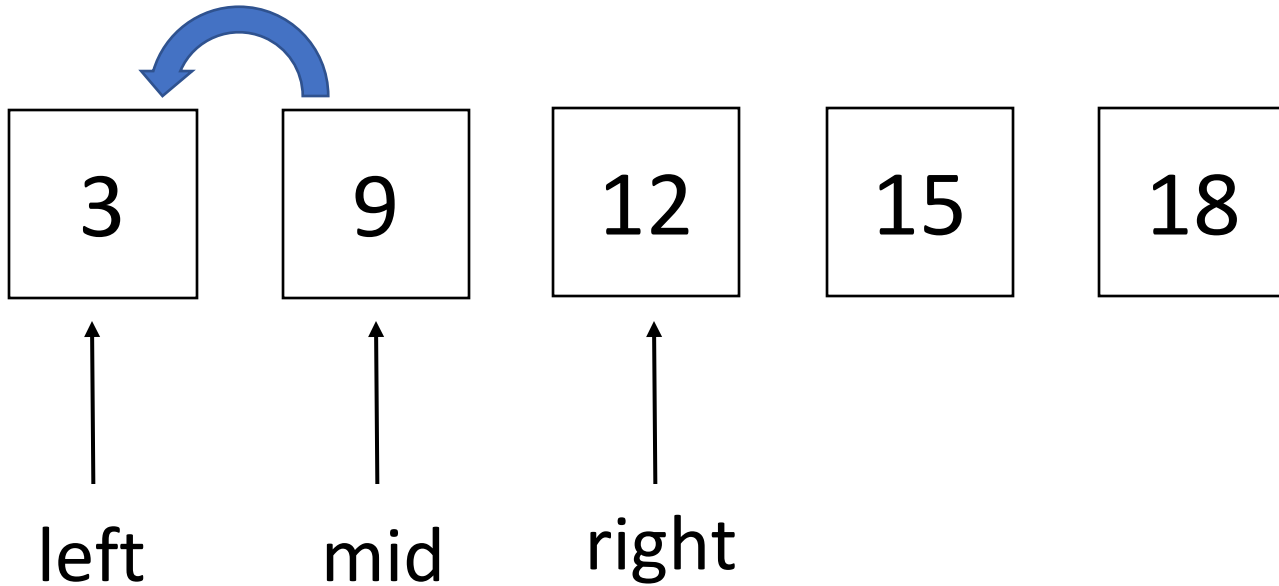


difference = 3

# Solution 1

**False**

9 - 3 = ?  
difference



difference = 3

# Solution 1

**False**

$9 - 3 = ?$   
difference

array[mid] - difference =

3

9

12

15

18

difference = 3

↑  
left

↑  
mid

↑  
right

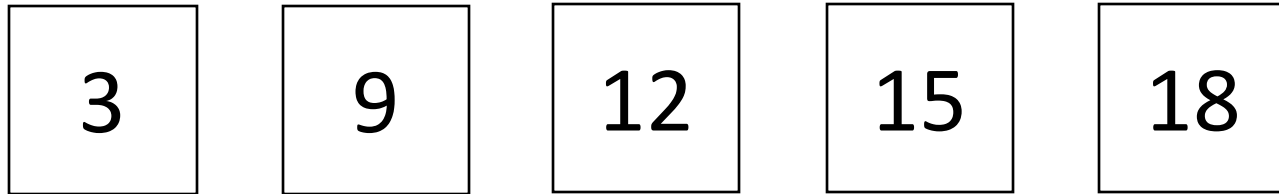
# Solution 1

**False**

$9 - 3 = ?$   
difference

$\text{array}[\text{mid}] - \text{difference} = 9 - 3 = 6$

The missing element is 6



difference = 3

# Solution 1

A solution by using the *binary search*

*Complexity?*

# Solution 1

A solution by using the *binary search*

<b><i>Best Case:</i></b>	<b><math>\Omega(1)</math></b>
<b><i>Worst Case:</i></b>	<b><math>O(\log n)</math></b>
<b><i>Average Case:</i></b>	<b><math>\Theta(\log n)</math></b>

# Solution 1

**findMissingElement** (array):

*differences*  $\leftarrow$  [ ]

$n \leftarrow$  length of array

**for** i from 0 to  $n-1$ :

**append** (array[i+1]-array[i]) to *differences*

*difference*  $\leftarrow$  mostly seen element in *differences*

**for** i from 0 to  $n-1$ :

**if** (array[i+1]-array[i])  $\neq$  *difference*:

**return** array[i] + *difference*



# Solution 1

**findMissingElement** (array):

*difference*  $\leftarrow$  (last element – first element) / (length of the array)

**for** i from 0 to n-1:

**if** (array[i+1]-array[i])  $\neq$  *difference*:

**return** array[i] + *difference*

# Solution 1

**findMissingElement** (array):

*difference*  $\leftarrow$  (last element – first element) / (length of the array)

**for** i from 0 to n-1:

**if** (array[i+1]-array[i])  $\neq$  *difference*:

**return** array[i] + *difference*

***Complexity?***

# Solution 1

**findMissingElement** (array):

*difference*  $\leftarrow$  (last element – first element) / (length of the array)

**for** i from 0 to n-1:

**if** (array[i+1]-array[i])  $\neq$  *difference*:

**return** array[i] + *difference*

**Best Case:**  $\Omega(1)$

**Worst Case:**  $O(n)$

**Average Case:**  $\Theta(n)$



## Problem 2

Construct an algorithm to find the maximum difference between two elements such that the smaller element appears before the larger one.

**Example:**

6 10 **1** **7** 5  $\rightarrow$  maximum difference is  $7 - 1 = 6$

15 6 **2** 3 **5**  $\rightarrow$  maximum difference is  $5 - 2 = 3$

## Solution 2

A solution with  $O(n^2)$  complexity:

```
findMaxDiff (array)
    difference  $\leftarrow 0$ 
     $n \leftarrow$  length of array
    for  $i$  from 0 to  $n-1$ :
        for  $j$  from  $i+1$  to  $n$ :
            if  $\text{array}[j] > \text{array}[i]$ :
                 $\text{difference} = \max(\text{difference}, A[j] - A[i])$ 
    return difference
```

## Solution 2

A solution with  $O(n)$  complexity:

findMaxDiff (array)

$max\_difference \leftarrow 0$

$max\_element \leftarrow array[-1]$

**for** i from n-2 to 0:

**if**  $array[i] < max\_element$

$max\_element = array[i]$

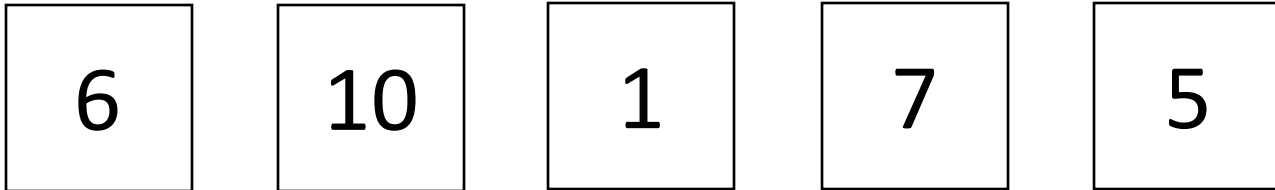
**else**

$max\_difference = \max ( difference, array[i+1] - array[i] )$

**return**  $max\_difference$

## Solution 2

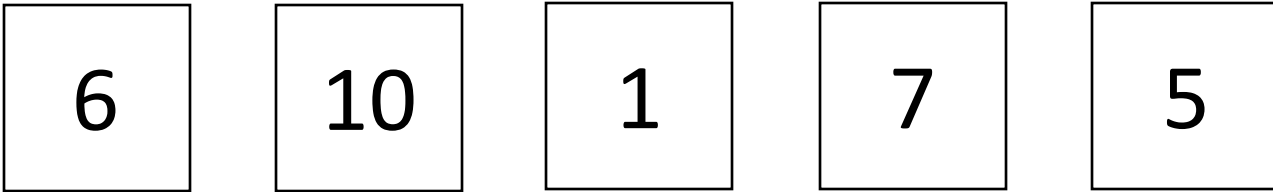
A solution with  $O(n)$  complexity:





## Solution 2

A solution with  $O(n)$  complexity:

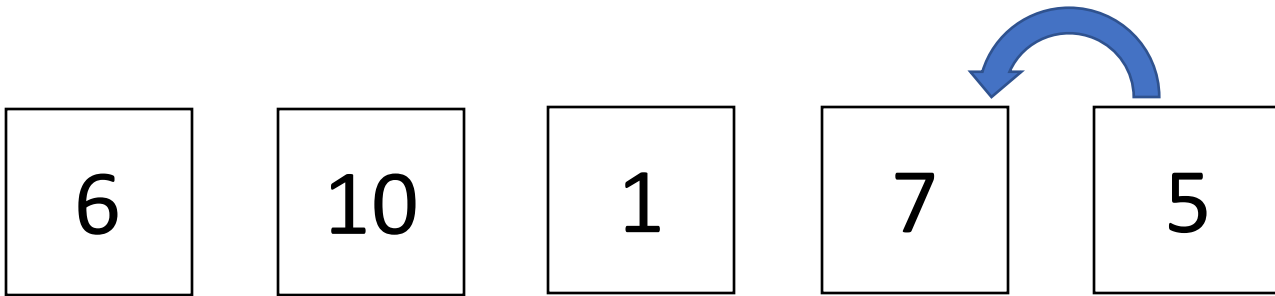


`max_element = array[n-1] = 5`

`max_difference = 0`

## Solution 2

A solution with  $O(n)$  complexity:

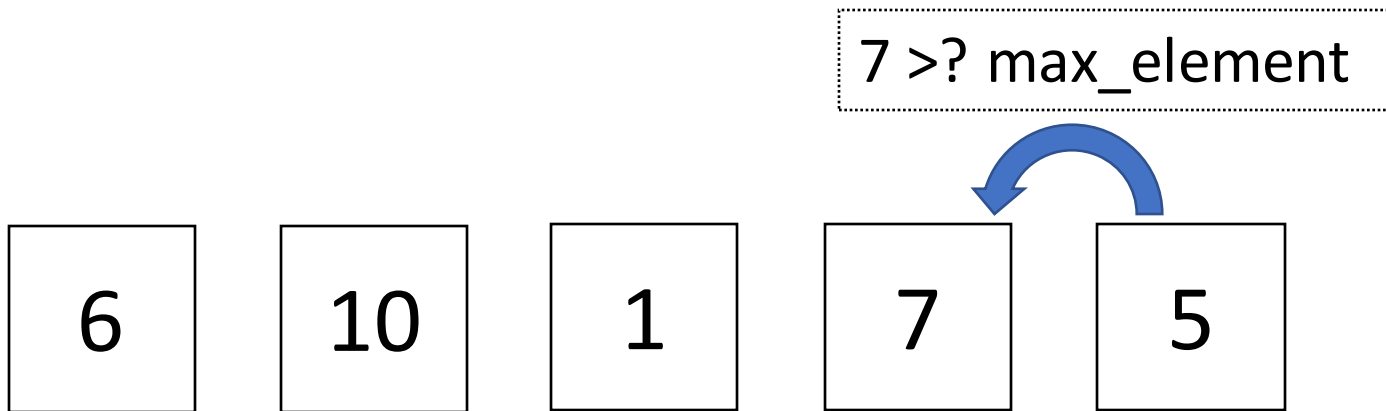


max\_element = 5

max\_difference = 0

## Solution 2

A solution with  $O(n)$  complexity:

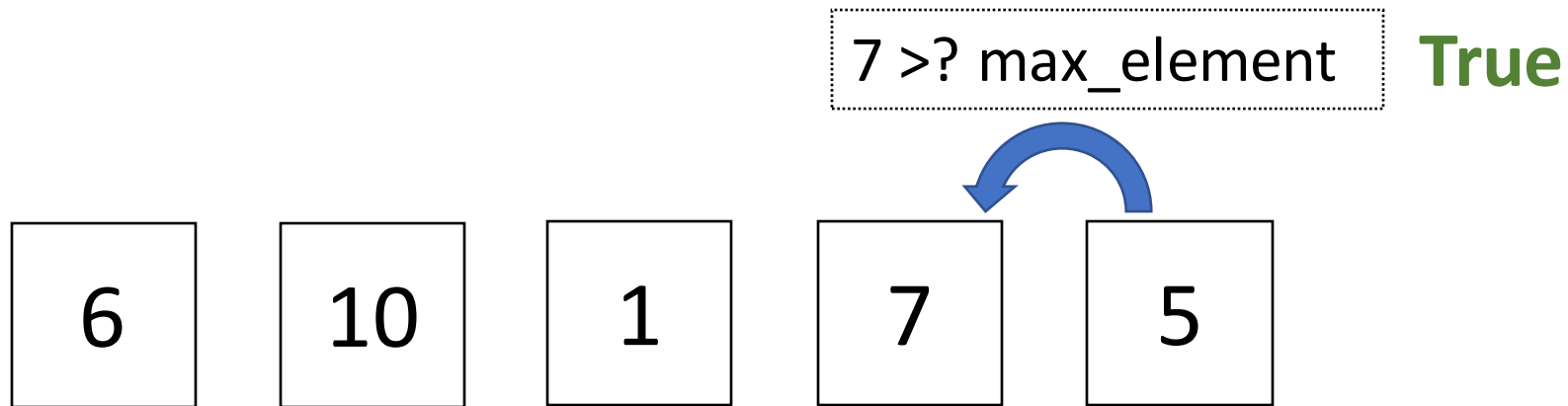


max\_element = 5

max\_difference = 0

## Solution 2

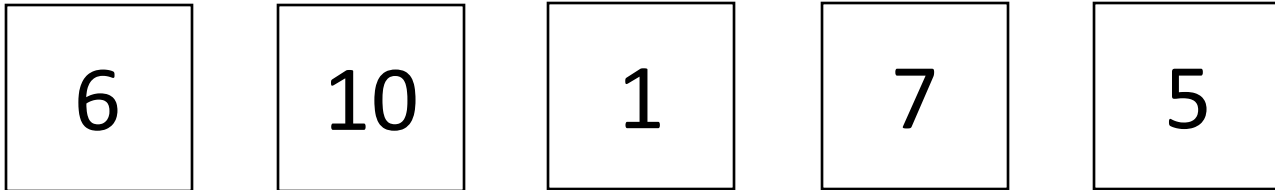
A solution with  $O(n)$  complexity:



max\_element = current\_element = 7  
max\_difference = 0

## Solution 2

A solution with  $O(n)$  complexity:



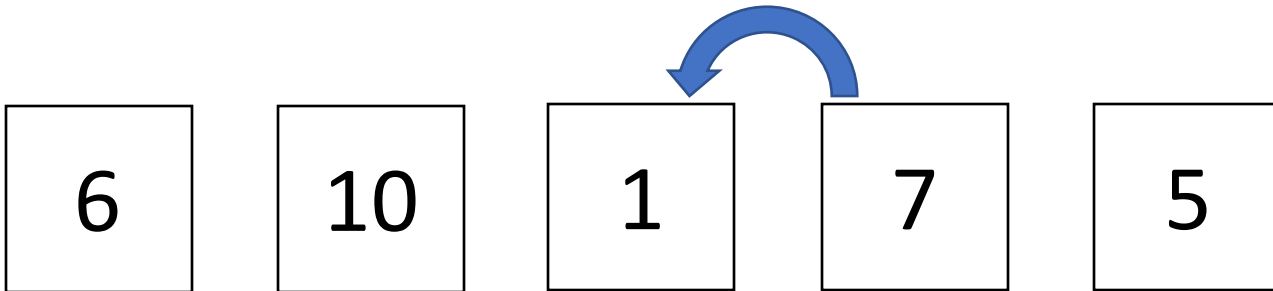
max\_element = 7

max\_difference = 0

## Solution 2

A solution with  $O(n)$  complexity:

1 >? max\_element



max\_element = 7

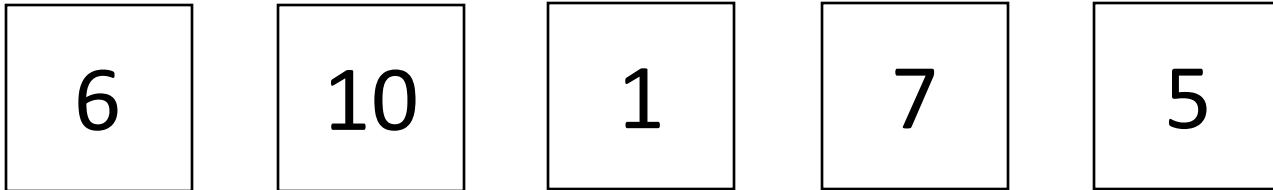
max\_difference = 0

## Solution 2

A solution with  $O(n)$  complexity:

1 >? max\_element

**False**



max\_element = 7

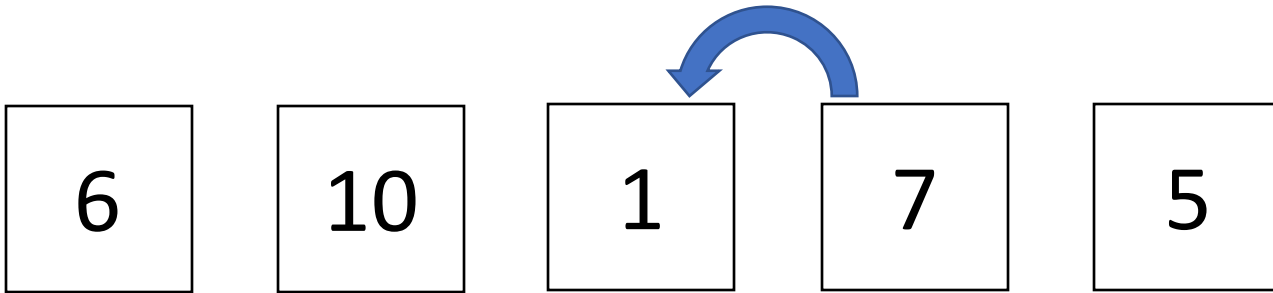
max\_difference = 0

## Solution 2

A solution with  $O(n)$  complexity:

$1 >? \text{max\_element}$

**False**



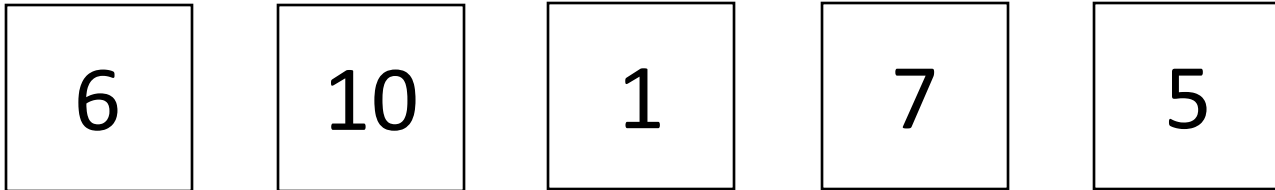
$\text{max\_element} = 7$

$\text{max\_difference} = \max(\text{max\_difference}, \text{max\_element} - \text{current\_element})$   
 $= \max(0, 7 - 1) = \max(0, 6) = 6$



## Solution 2

A solution with  $O(n)$  complexity:



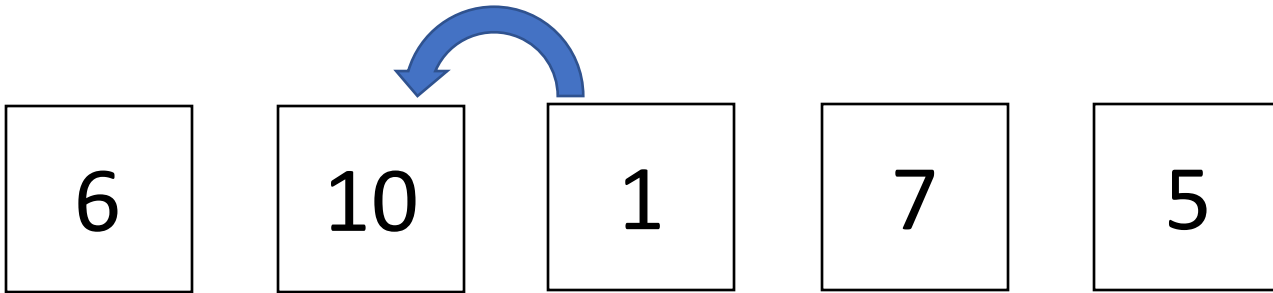
max\_element = 7

max\_difference = 6

## Solution 2

A solution with  $O(n)$  complexity:

10 >? max\_element



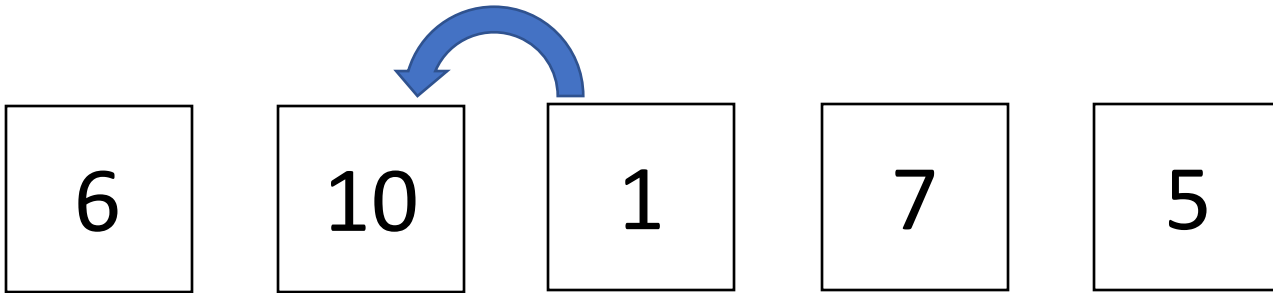
max\_element = 7

max\_difference = 6

## Solution 2

A solution with  $O(n)$  complexity:

10 >? max\_element **True**



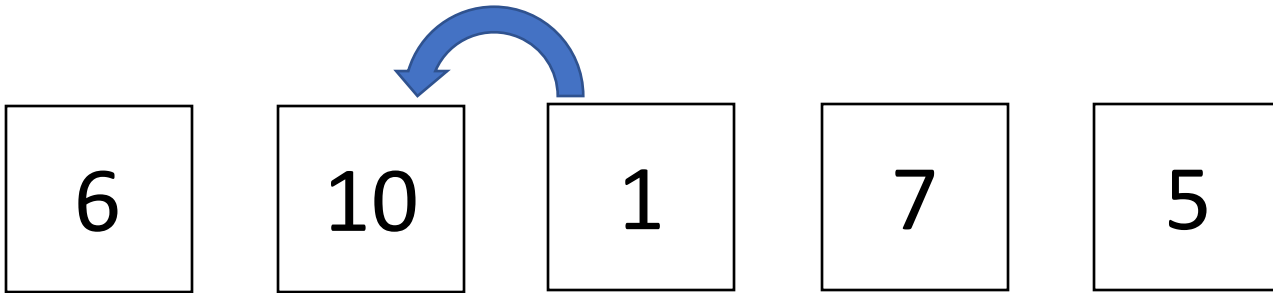
max\_element = 7

max\_difference = 6

## Solution 2

A solution with  $O(n)$  complexity:

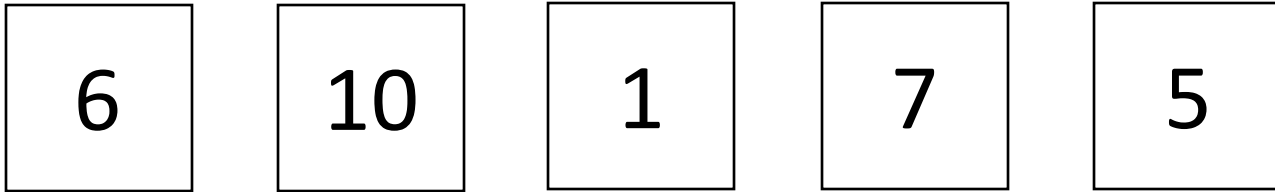
10 >? max\_element **True**



max\_element = current\_element = 10  
max\_difference = 6

## Solution 2

A solution with  $O(n)$  complexity:



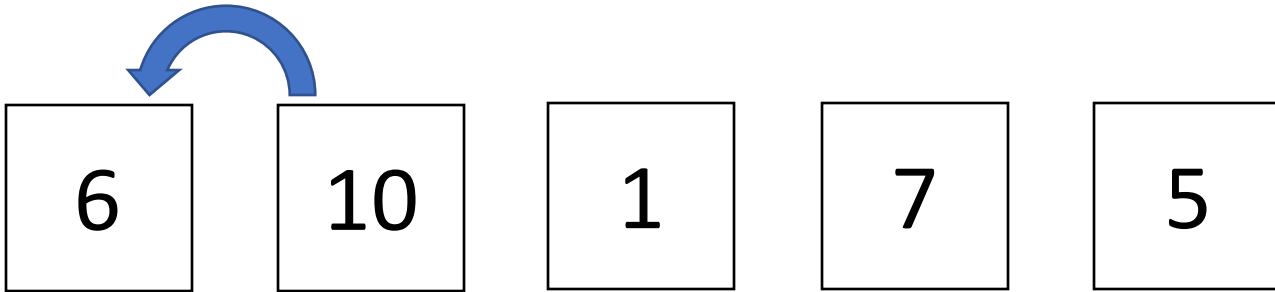
max\_element = 10

max\_difference = 6

## Solution 2

A solution with  $O(n)$  complexity:

6 >? max\_element



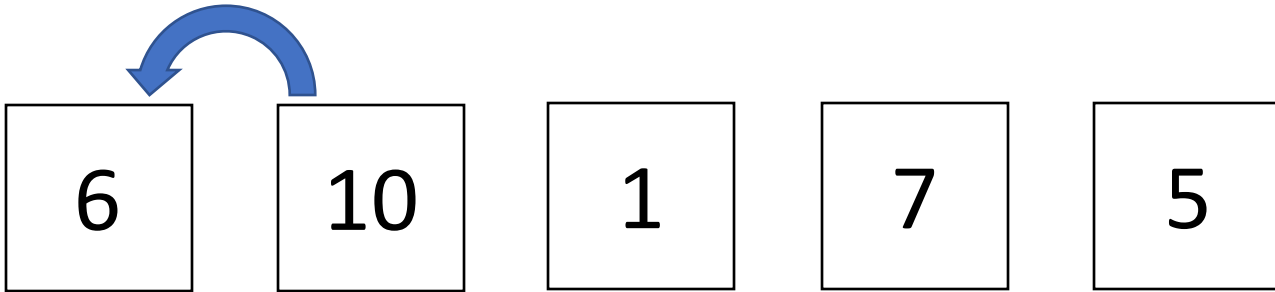
max\_element = 10

max\_difference = 6

## Solution 2

A solution with  $O(n)$  complexity:

6 >? max\_element **False**



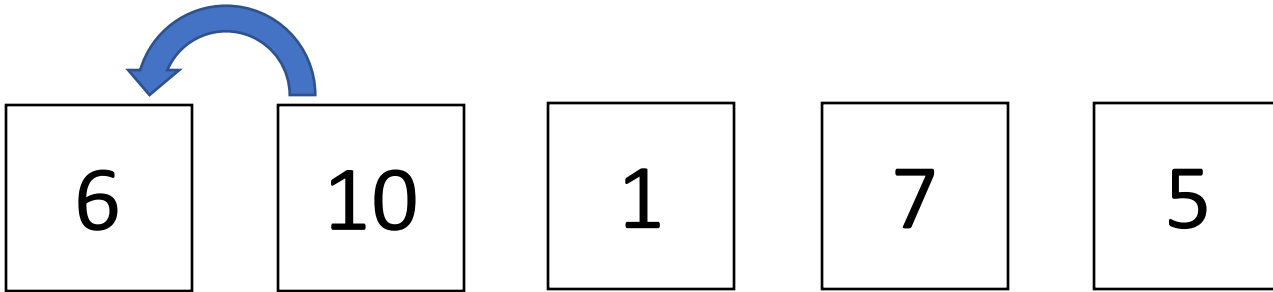
max\_element = 10

max\_difference = 6

## Solution 2

A solution with  $O(n)$  complexity:

6 >? max\_element **False**



max\_element = 10

max\_difference =  $\max(\text{max\_difference}, \text{max\_element} - \text{current\_element})$   
=  $\max(6, 10 - 6) = \max(6, 4) = 6$



