



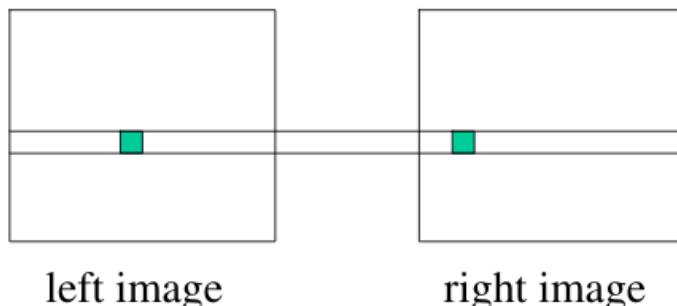
CSE463 - Vision

In stereo vision, the two images, captured by the two cameras separated by a distance, can be used to get the 3d location (x, y, z) of the points of images in real world i.e. the depth -- z location in addition to 2d -- x and y location.

The **disparity** is the difference in image location of the same 3D point when projected under perspective to two different cameras.

Any point in the scene that is visible in both cameras will be projected to a pair of image points in the two images, called a conjugate pair. The displacement between the positions of the two points is called the disparity.

● 3D point



The disparity map/image is simply the image that given where each pixel gives the disparity of that 3d point.

The **depth** (the actual z location of 3d point) can be calculated by using the disparity of the corresponding point e.g. in simple cases, as follows:

```
depth = (baseline * focal length) / disparity
```

where `baseline` is the distance b/w the cameras.

By getting the depth of every pixel, you get the depth map/image.

STEREO MATCHING EXAMPLES

<https://github.com/eborboihuc/stereo-matching>

<https://github.com/erget/StereoVision/blob/master/stereovision/blockmatchers.py>

Rotation matrix in stereo → 3×3 $x, y, z / x, y, z$

Correspondence için aynı doğru üzerinde + fundamental matrix

<https://youtu.be/GXysgyXgRSY?t=9420> fundamental matrix correspondence

fundamental matrix diğer noktayı arayacağımız doğruya bize söyler.

denklik = correspondence

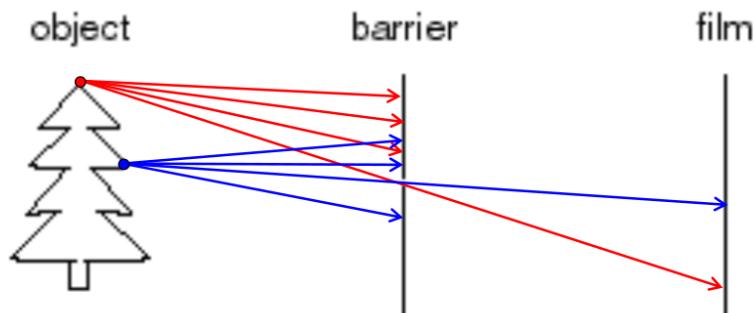
Keypointteki pikselleri kendi aralarında karşılaştırıp ordan depth map ve disparity çıkar.

NOTES

Digimask → put your face on 3d avatar

Range image sana yakın olan daha koyu renk

Pinhole camera

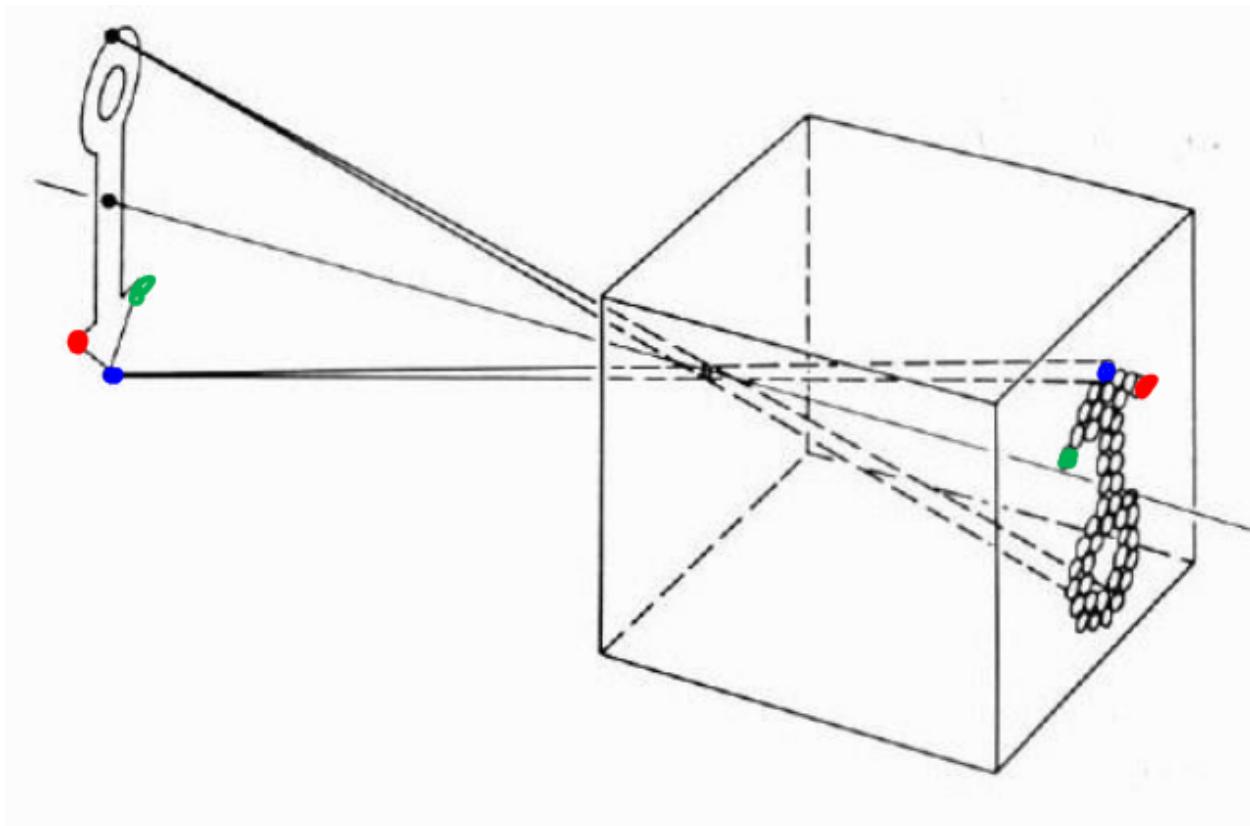


Add a barrier to block off most of the rays

- This reduces blurring
- The opening known as the **aperture**

aperture = açıklık

pupil = gözbebeği



$$1/\text{focus} = 1/\text{distO} + 1/\text{distI}$$

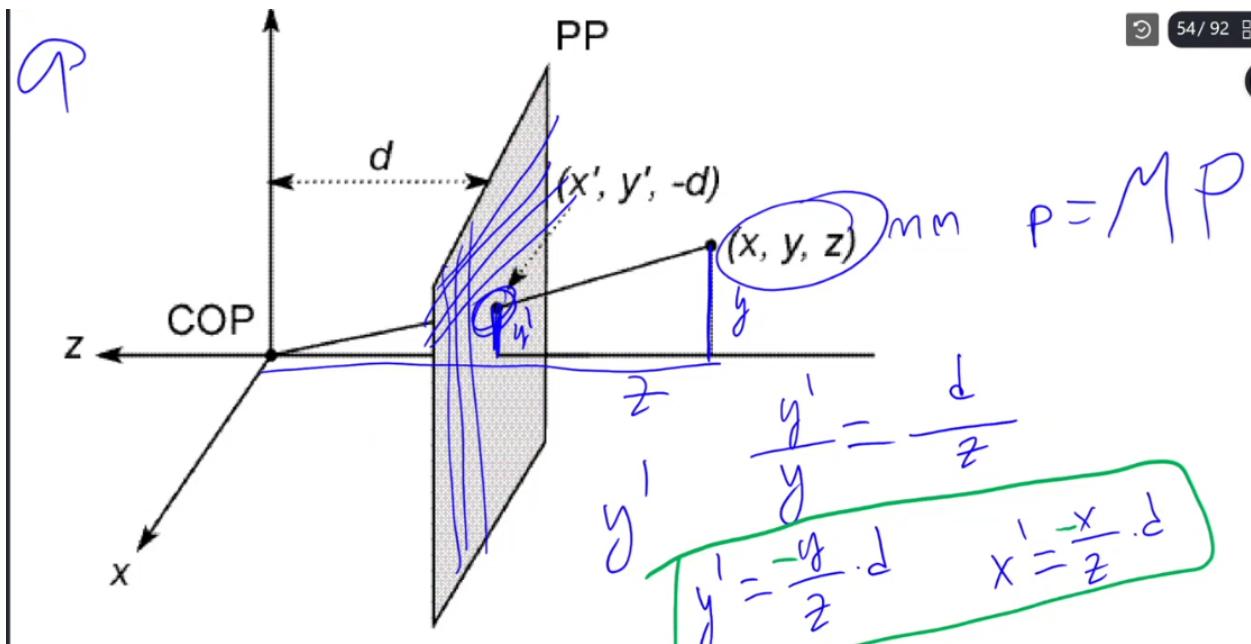
focal = odak

When intensity of lights reflected from objects is high \rightarrow cone vision (else rod vision)

Put optical center at the origin. Image plane in front of center of projection.

$r = \text{non homogen} \mid h = \text{homogen}$

telecentric lens = make distance same



$p = MP$. We will multiply projection matrix M with P , and will get p in terms of image coordinates.

◆ `projectPoints()`

```
void cv::projectPoints
InputArray rvec,
InputArray tvec,
InputArray cameraMatrix,
InputArray distCoeffs,
InputArray imagePoints,
InputArray jacobian = noArray(),
double aspectRatio = 0
)
```

InputArray
Camera
Calibration



process of recovering these parameters is called → camera calibration.

$$p_{1 \dots 10} = M \cdot P_{1 \dots 10}$$

M contains 12 elements. P=3D, p=2d

barrel and cushion(yastık) distortion

Extrinsic parameters → depend on real world objects

Intrinsic parameters → They don't depend on where my camera is in real world

x'e göre kısmı türev [partial derivative] f_x X dışındakileri sabit kabul et

lecture 2 Filtering- spring 2020

$\nabla I \rightarrow$ Gradient of image

Derivatives in 2 Dimensions

Given function $f(x, y)$

Gradient vector $\nabla f(x, y) = \begin{bmatrix} \frac{\partial f(x, y)}{\partial x} \\ \frac{\partial f(x, y)}{\partial y} \end{bmatrix} = \begin{bmatrix} f_x \\ f_y \end{bmatrix}$ At every point actually we have 2 derivatives

Gradient magnitude $|\nabla f(x, y)| = \sqrt{f_x^2 + f_y^2}$

Gradient direction $\theta = \tan^{-1} \frac{f_x}{f_y}$

Needed for edge detection

1 dimensional gaussian → $g(x) = e^{\frac{-x^2}{2\sigma^2}}$

2d gaussian $g(x) = e^{\frac{-(x^2+y^2)}{2\sigma^2}}$

when σ is increases, image is more blurred and noise is more suppressed.

türev sayesinde kenar bulma

EDGE → Sudden changes in an image intensity [not because object borders]

For edge detection you need to find sudden changes and we need to use DERIVATIVE for this

Filters are linear operations applied on images

when derivative makes a peak, we know that there is an edge

2.türevin x ekseninden geçtiği yer [0 crossings] tepe noktasını verir (peak positions)

To find edges, look for peaks in $\frac{d(f*g)}{dx} = f * \frac{d}{dx} g$.

lecture 3 EdgeDetection

Derivative in Two-Dimensions

- Definition

$$\frac{\partial f(x, y)}{\partial x} = \lim_{\varepsilon \rightarrow 0} \left(\frac{f(x + \varepsilon, y) - f(x, y)}{\varepsilon} \right) \quad \frac{\partial f(x, y)}{\partial y} = \lim_{\varepsilon \rightarrow 0} \left(\frac{f(x, y + \varepsilon) - f(x, y)}{\varepsilon} \right)$$

- Approximation

Derivative with respect to x

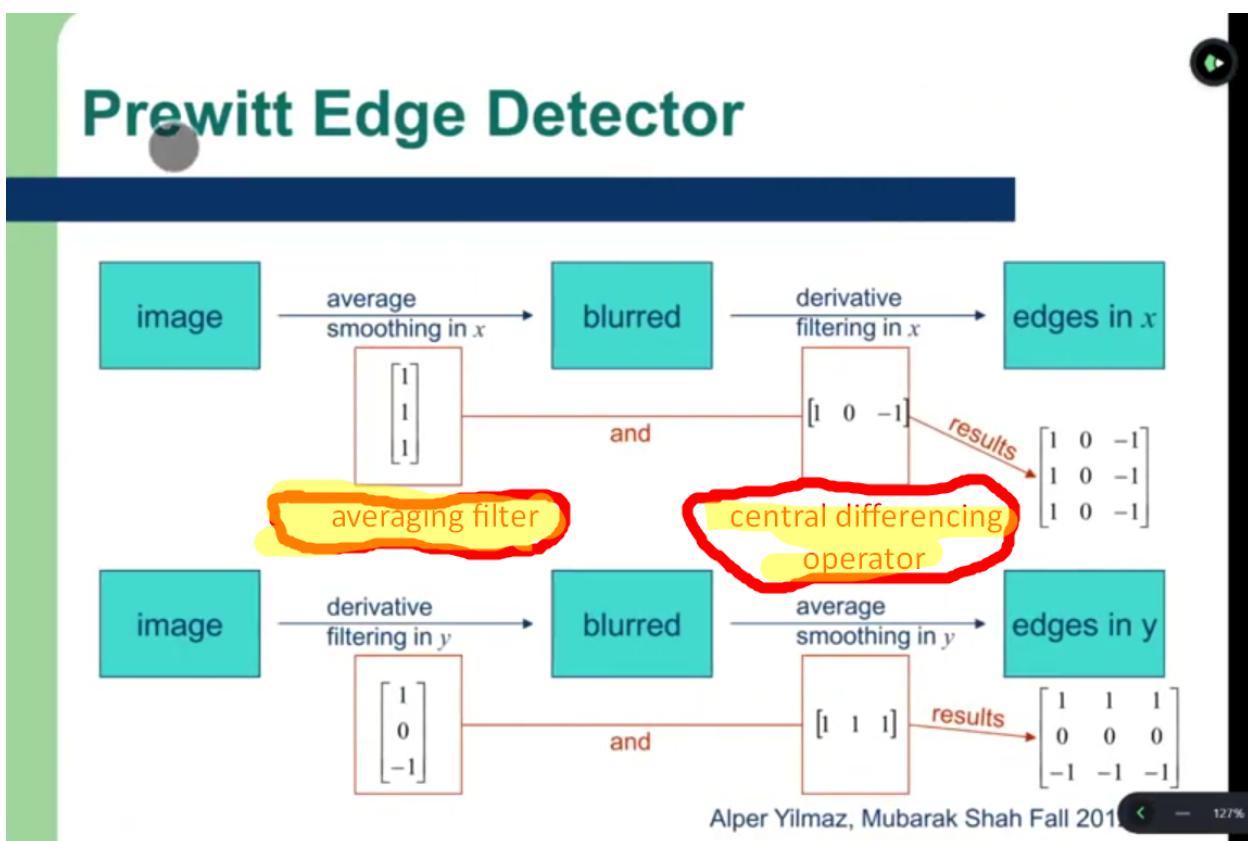
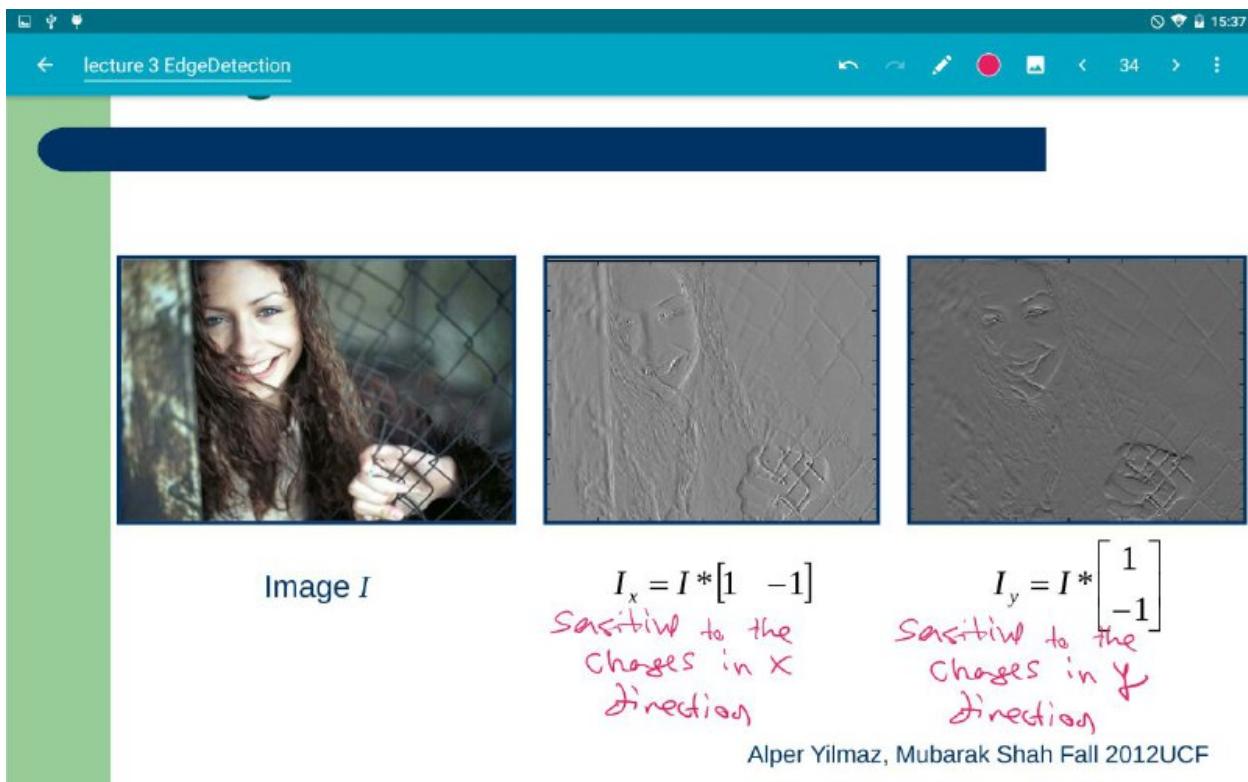
$$\frac{\partial f(x, y)}{\partial x} \approx \frac{f(x_{n+1}, y_m) - f(x_n, y_m)}{1} \rightarrow \text{we take finite value.}$$

Derivative with respect to y

$$\frac{\partial f(x, y)}{\partial y} \approx \frac{f(x_n, y_{m+1}) - f(x_n, y_m)}{1}$$

- Convolution kernels

$$f_x = [1 \quad -1]$$
$$f_y = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

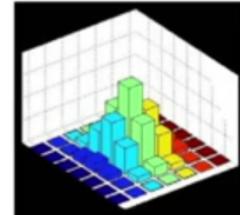


For sobel edge detector, instead [1,1,1] it's [1,2,1]

laplacian is second derivative of the gauss

• Gaussian smoothing

$$\text{smoothed image } \hat{S} = \text{Gaussian filter } g * \text{image } I \quad g = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2+y^2}{2\sigma^2}}$$



• Find Laplacian

$$\Delta^2 S = \underbrace{\frac{\partial^2}{\partial x^2} S}_{\text{second order derivative in } x} + \underbrace{\frac{\partial^2}{\partial y^2} S}_{\text{second order derivative in } y}$$

- ∇ is used for gradient (first derivative)
- Δ^2 is used for Laplacian (Second derivative)

Alper Yilmaz, Mubarak Shah Fall 2012

taking second derivative creates more noise in the image

Taking second derivative may not be the best option. So CANNY says take first derivative do rest yourself

canny → using 2 thresholds for ex. 50 and 100. above 100 is strong but also take above 50

consider neighbors iteratively and if it's connected declare it as edge pixel.

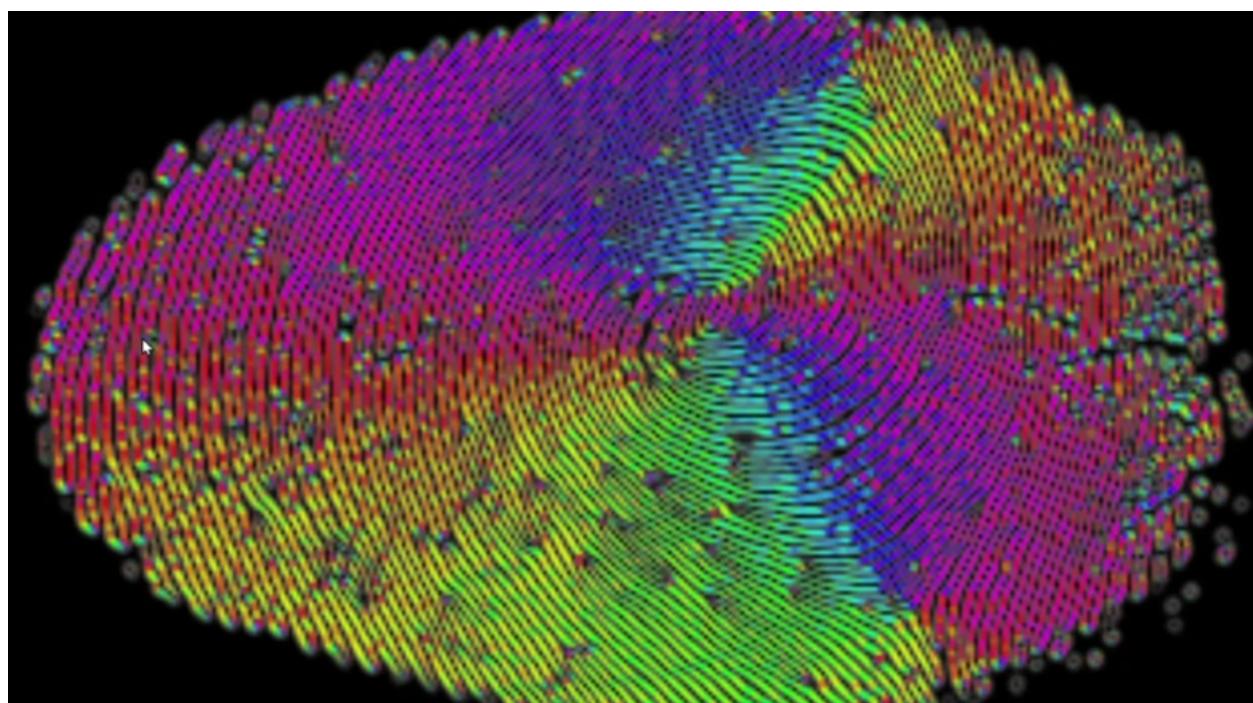
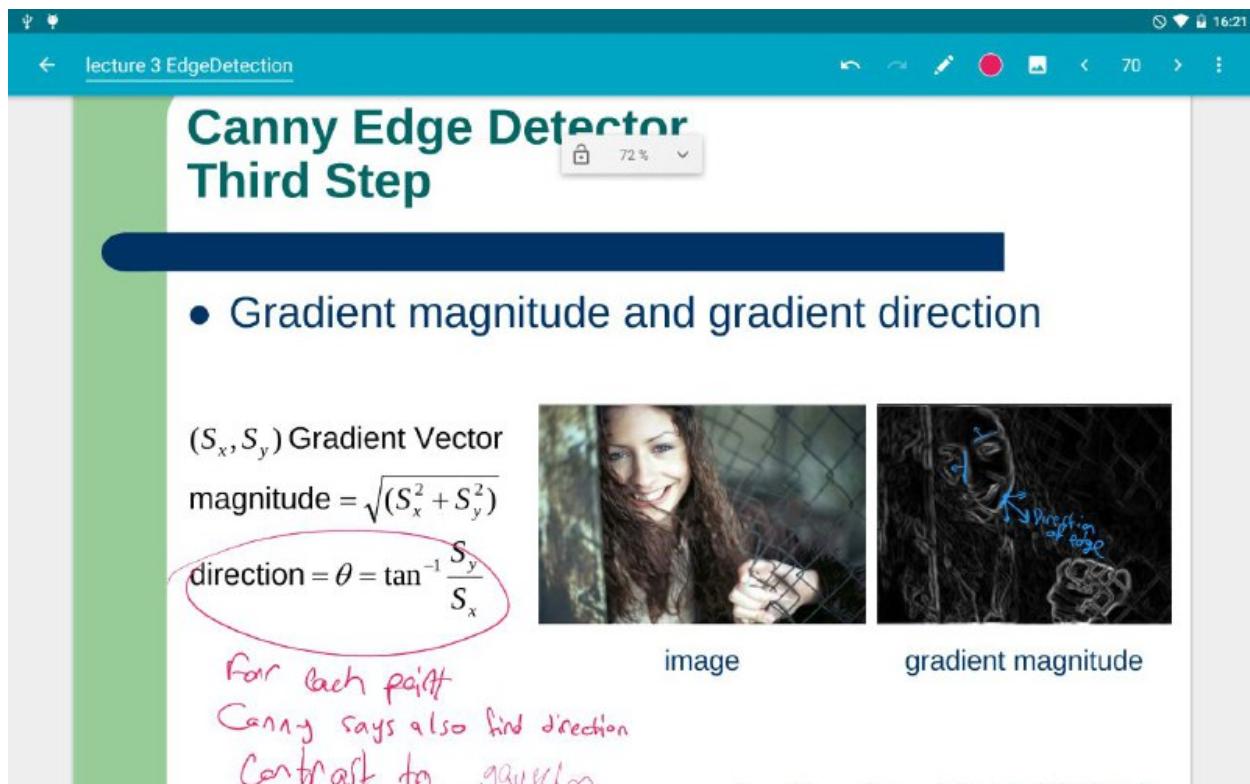
lecture 3 EdgeDetection

Canny Edge Detector Third Step

- Gradient magnitude and gradient direction

(S_x, S_y) Gradient Vector
magnitude = $\sqrt{S_x^2 + S_y^2}$
direction = $\theta = \tan^{-1} \frac{S_y}{S_x}$

For each point
Canny says also find direction
Contrast to gaussian



color shows orientation in each pixel

lecture 3 EdgeDetection

Canny Edge Detector Steps

1. Smooth image with Gaussian filter *→ Sigma parameter needed*
2. Compute derivative of filtered image
3. Find magnitude and orientation of gradient
4. Apply "Non-maximum Suppression" *→ No frontier*
5. Apply "Hysteresis Threshold" *→ H_{low}, H_{high}*
we Select 2 thresholds
50, 100 Alarm too strong
Below 50 is not good neither

Alper Yilmaz, Mubarak Shah Fall 2012, UCF

distinctive corner detectors are robust against rotation, lighting changes, scale changes



that's an edge because when you move, changes

Moving:

$$E(u, v) = \sum_{x, y} W(x, y) [I(x+u, y+v) - I(x, y)]^2$$

Annotations below the equation:

- Window function (points to the $W(x, y)$ term)
- Shifted intensity (points to the $I(x+u, y+v)$ term)
- Intensity (points to the $I(x, y)$ term)

$E(0,0)$ is 0

$E(1,1)$ Moving down and right

Taylor Series Expansion

Taylor's Theorem : Suppose f is continuous on the closed interval $[a, b]$ and has $n+1$ continuous derivatives on the open interval (a, b) . If x and c are points in (a, b) , then

The Taylor series expansion of $f(x)$ about c :

$$f(c) + f'(c)(x-c) + \frac{f^{(2)}(c)}{2!}(x-c)^2 + \frac{f^{(3)}(c)}{3!}(x-c)^3 + \dots$$

or

$$\text{Taylor Series} = \sum_{k=0}^{\infty} \frac{1}{k!} f^{(k)}(c) (x-c)^k$$

If the series converge, we can write:

$$f(x) = \sum_{k=0}^{\infty} \frac{1}{k!} f^{(k)}(c) (x-c)^k$$

lecture 4 harris corner spring 2020

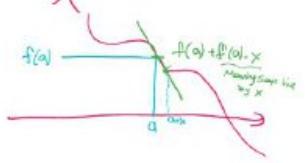
Change Approximation

If u and v are small, by Taylor theorem:

$$I(i+u, j+v) \approx I(i, j) + I_x u + I_y v$$

where $I_x = \frac{\partial I}{\partial x}$ and $I_y = \frac{\partial I}{\partial y}$

therefore

$$\begin{aligned} (I(i+u, j+v) - I(i, j))^2 &= (I(i, j) + I_x u + I_y v - I(i, j))^2 \\ &= (I_x u + I_y v)^2 \\ &= I_x^2 u^2 + 2 I_x I_y u v + I_y^2 v^2 \\ &= [u \quad v] \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} [u \quad v] \end{aligned}$$


$f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \dots$

$Ax = \lambda x$ | x is such a dimension/vector that when its multiplied by matrix A , X direction stays same.

$$A - \lambda I = 0$$

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Eigenvalue Analysis – simple case

First, consider case where:

$$C = \begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

If you have 2 eigenvalues, we have corner in this region.

This means dominant gradient directions align with x or y axis

If either λ is close to 0, then this is **not** a corner, so look for locations where both are large.

Slide credit: David Jacobs

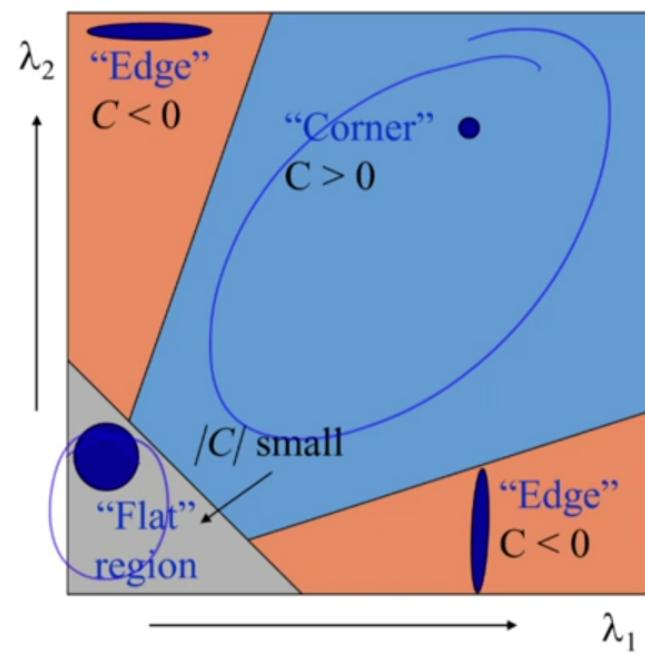
if we have 2 special direction we have corner; one \rightarrow edge

If no lambda be found then there is no corner . its a flat region

Cornerness score:

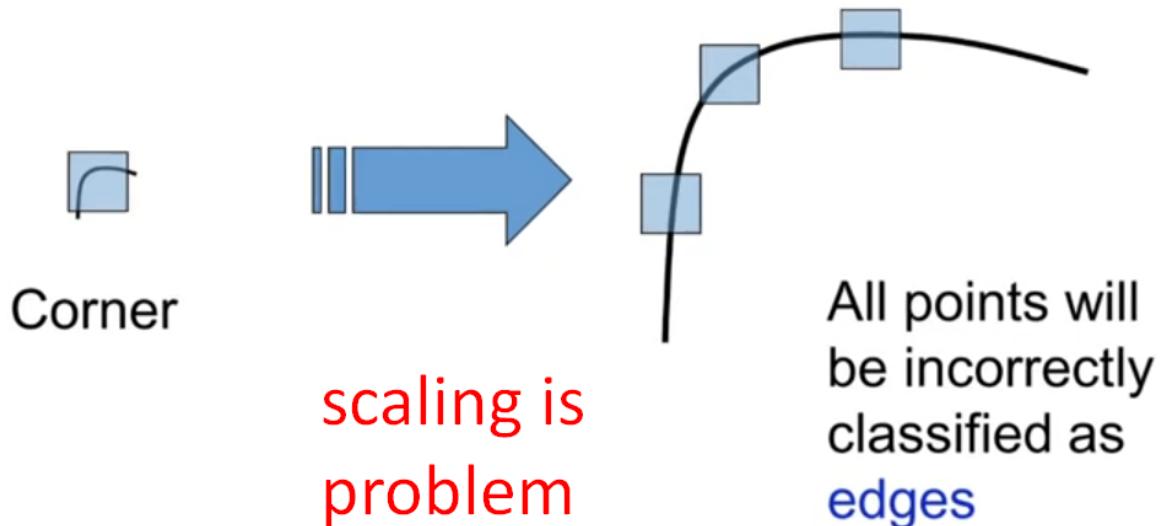
$$C = \lambda_1 \lambda_2 - \alpha(\lambda_1 + \lambda_2)^2$$

α : some constant (~ 0.04 to 0.06)



cornerlik hesaplama = Lambda1 * lambda2 - alfa(lambda1 + lambda2)²

Scaling



for ex.: a 128d **descriptor** can be used because robust to variations corresponding to typical viewing conditions.

Idea of SIFT → Scale Invariant [harris is not]

with descriptor you can Match points in different scale images

1 scale independence

Overall Procedure at a High Level

1. Scale-space extrema detection

Search over multiple scales and image locations.

2. Keypoint localization

Fit a model to determine location and scale.

Select keypoints based on a measure of stability.

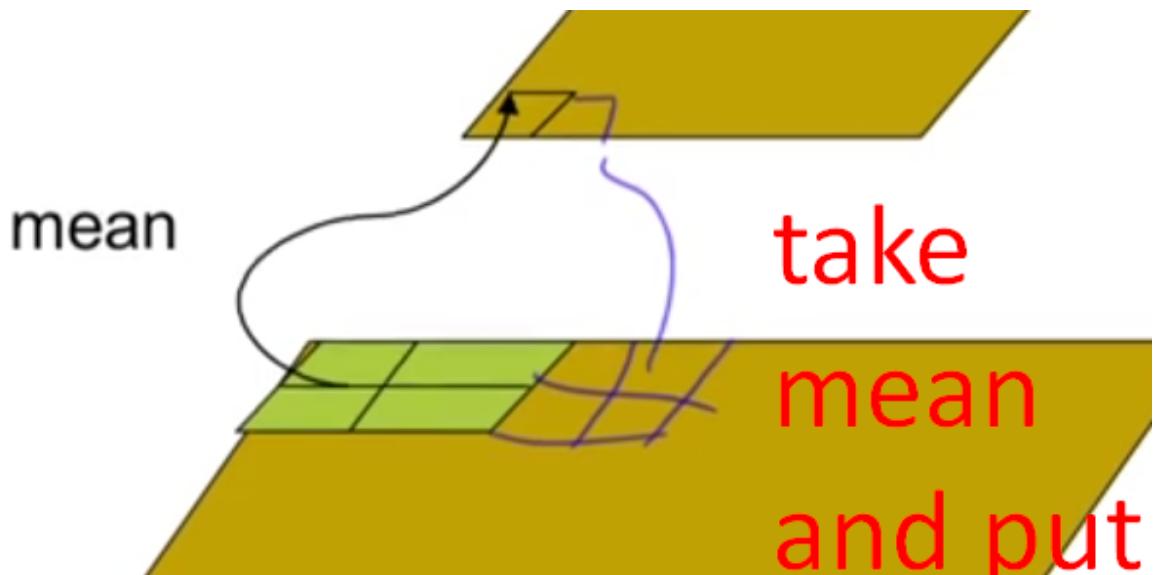
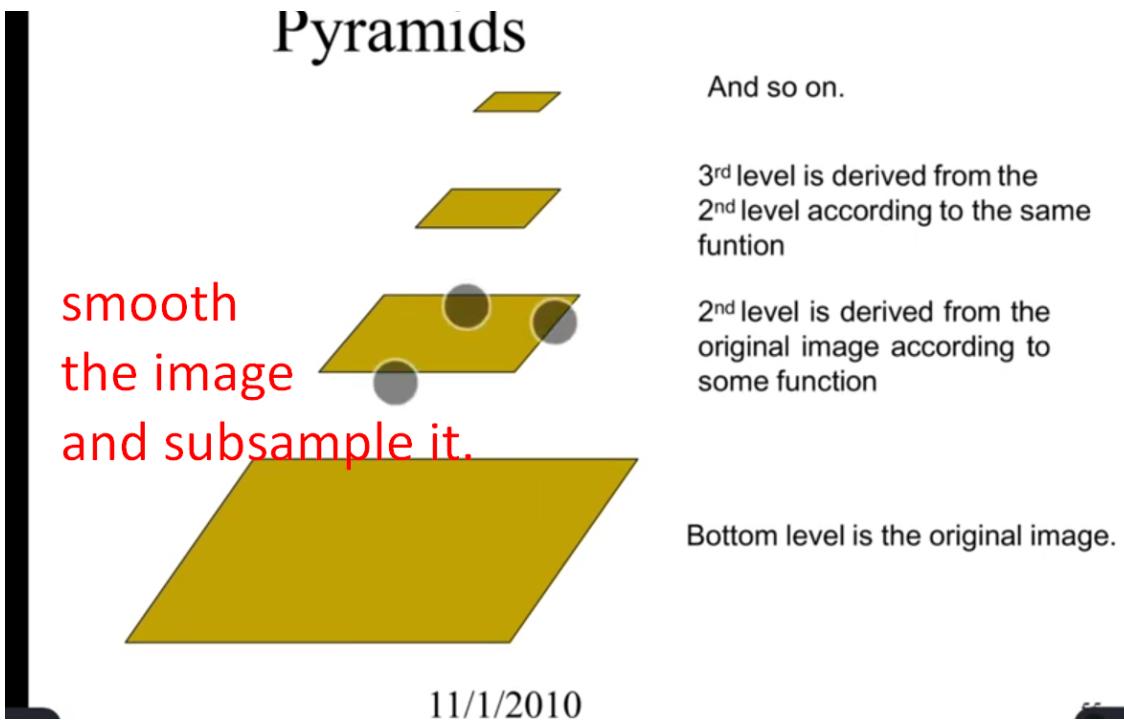
3. Orientation assignment

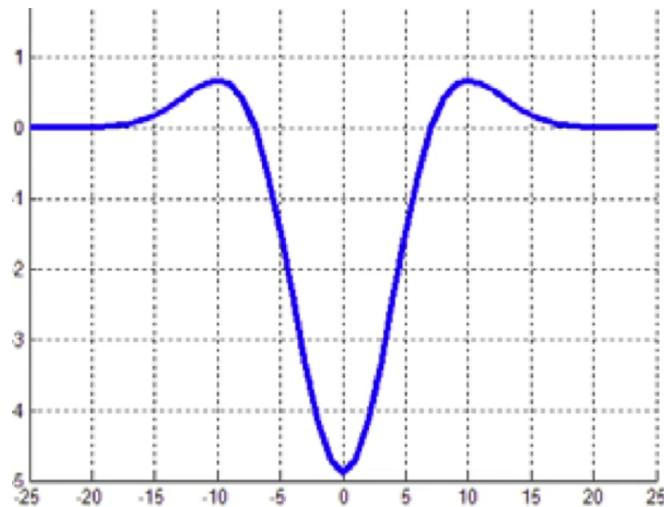
Compute best orientation(s) for each keypoint region.

4. Keypoint description

Use local image gradients at selected scale and rotation to describe each keypoint region.

Pyramids





Mexican hat: second derivative of gaussian

lecture 4 harris corner spring 2020 20:33 ← → 45 James Hays

Harris Corner Detector [Harris88]

0. Input image
We want to compute M at each pixel.

1. Compute image derivatives (optionally, blur first).

2. Compute M components as squares of derivatives.

3. Gaussian filter $g()$ with width σ
 $= g(I_x^2), g(I_y^2), g(I_x \circ I_y)$

4. Compute cornerness

$$C = \det(M) - \alpha \text{trace}(M)^2$$

$$= g(I_x^2) \circ g(I_y^2) - g(I_x \circ I_y)^2$$

Reminder: $a \circ b$ is Hadamard product (element-wise multiplication)

5. Threshold on C to pick high cornerness

6. Non-maximal suppression to pick peaks.

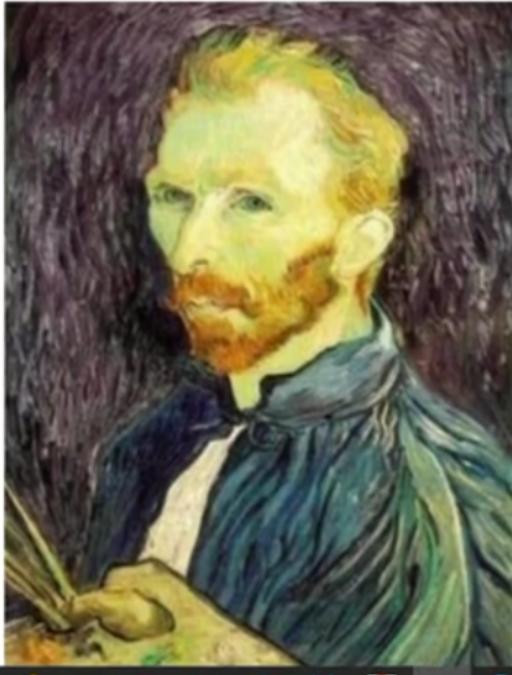
Non-maximal suppression

Harris can detect edge, but Conny is better for this

R

occlusion → engelleme / tikama

Example: Subsampling with Gaussian pre-filtering



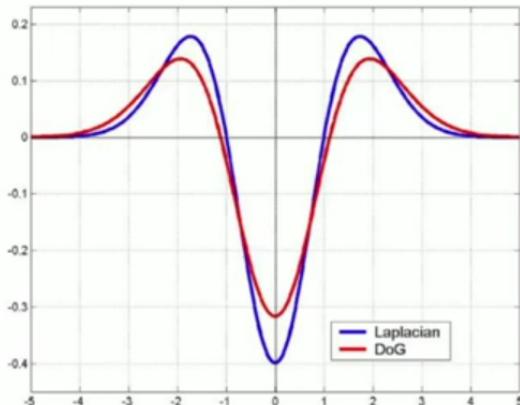
G 1/4

G 1/8

first → apply gaussian and then take average of and put that position

Lowe's Scale-space Interest Points: Difference of Gaussians

- Gaussian is an ad hoc solution of heat diffusion equation



$$\frac{\partial G}{\partial \sigma} = \sigma \nabla^2 G.$$

- Hence

$$G(x, y, k\sigma) - G(x, y, \sigma) \approx (k - 1)\sigma^2 \nabla^2 G.$$

- k is not necessarily very small in practice

Instead of taking laplacian(second derivative), for $k > 1$ subtract Gaussians above and it's similar with laplacian.

```

///- Step 1: Detect the keypoints using SURF Detector, compute the descriptors
int minHessian = 400;
Ptr<SURF> detector = SURF::create( minHessian );
std::vector<KeyPoint> keypoints1, keypoints2;
Mat descriptors1, descriptors2;
detector->detectAndCompute( img1, noArray(), keypoints1, descriptors1 );
detector->detectAndCompute( img2, noArray(), keypoints2, descriptors2 );

///- Step 2: Matching descriptor vectors with a FLANN based matcher
// Since SURF is a floating-point descriptor NORM_L2 is used
Ptr<DescriptorMatcher> matcher = DescriptorMatcher::create(DescriptorMatcher::FLANNBASED);
std::vector< std::vector<DMatch> > knn_matches;
matcher->knnMatch( descriptors1, descriptors2, knn_matches, 2 );

///- Filter matches using the Lowe's ratio test
const float ratio_thresh = 0.7f;
std::vector<DMatch> good_matches;
for (size_t i = 0; i < knn_matches.size(); i++)
{
    if (knn_matches[i][0].distance < ratio_thresh * knn_matches[i][1].distance)
    {
        good_matches.push_back(knn_matches[i][0]);
    }
}

///- Draw matches
Mat img_matches;
drawMatches( img1, keypoints1, img2, keypoints2, good_matches, img_matches, Scalar::all(-1),
Scalar::all(-1), std::vector<char>(), DrawMatchesFlags::NOT_DRAW_SINGLE_POINTS );

///- Show detected matches
imshow("Good Matches", img_matches );

```



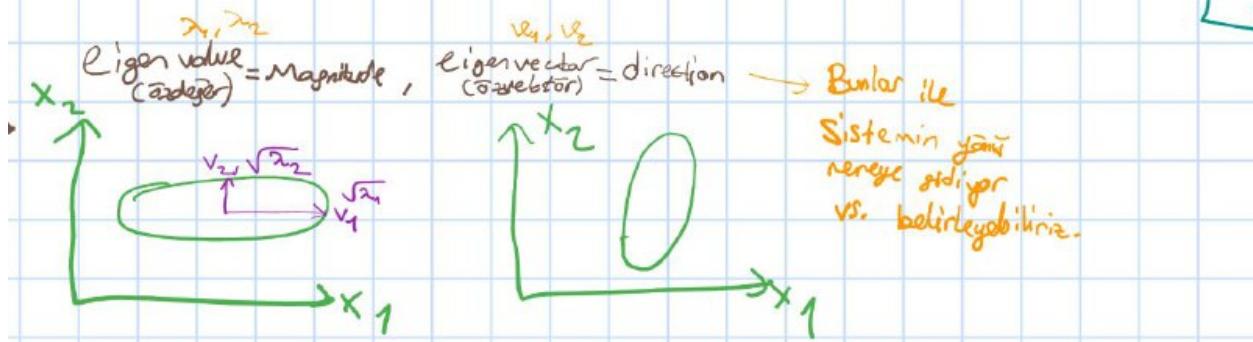
Assign random match, remove bad, do enough times and that's **RANSAC**

Flann is feature of Ransac[Feature Matching]

Non maximum supression → Edge'e dik yürü, intensity(yoğunluğun) peak noktasını al.
Gauss grafiği / ters paraboldaki max nokta.

- Aritmetik ortalama

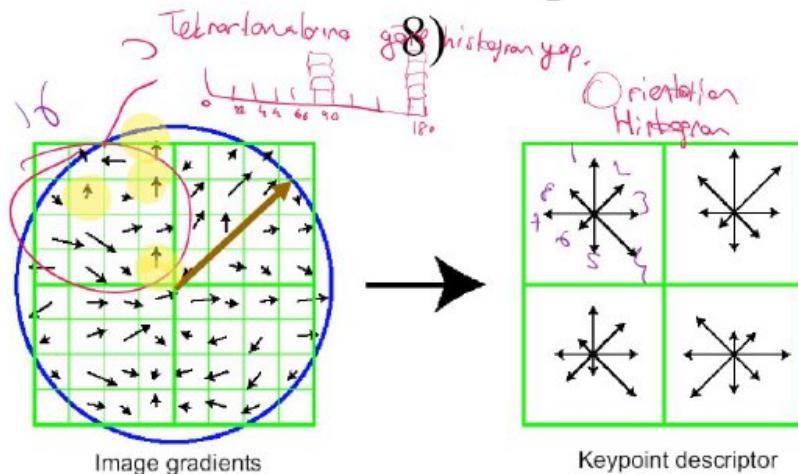
Standart Sapma (σ) \Rightarrow Verilerin aritmetik ortalamadan sapmalarının karelerinin aritmetik ortalamasının karekökü.



Ovaryans =) 2 deşeben arasındaki doğrusal ilişkinin deştekligini ölçen kavader.

Örnek mat desine nüf 30 adet not bir seri 1. not 1. serisine nüf 30 adet not birinci not

Lowe's Keypoint Descriptor (shown with 2 X 2 descriptors over 8 X

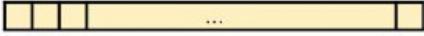


✓ In experiments, 4x4 arrays of 8 bin histogram is used, a total of 128 features for one keypoint

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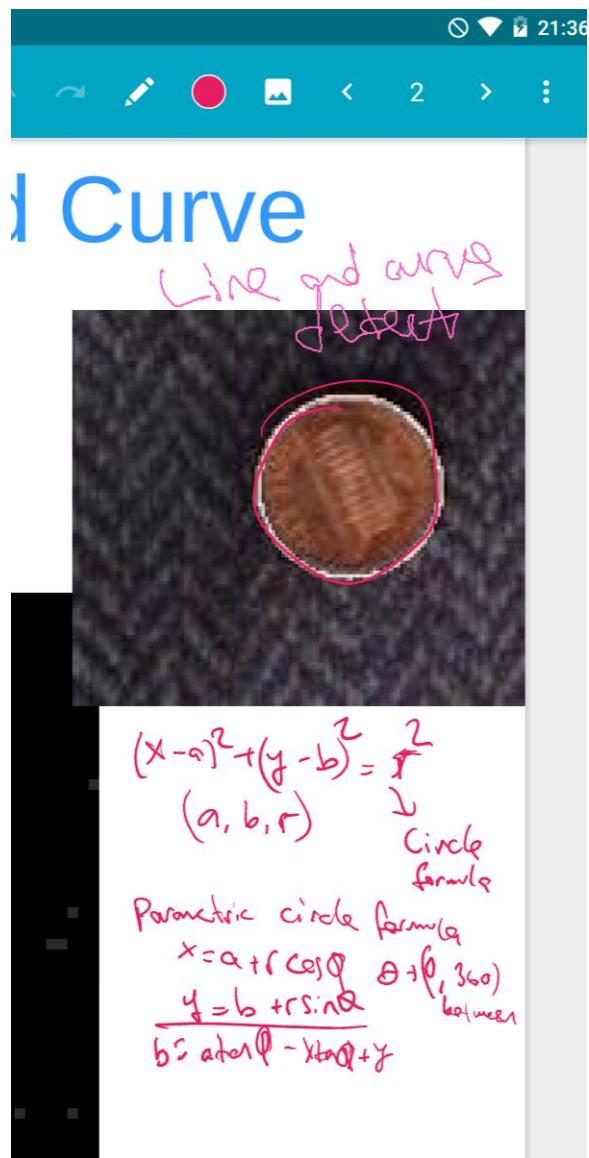
Lowe's Keypoint Descriptor

*Note
algorithm in
old. Many inspirers*

- use the **normalized** region about the keypoint
- compute gradient magnitude and orientation at each point in the region
- weight them by a **Gaussian** window overlaid on the circle
- create an **orientation histogram** over the 4×4 subregions of the window
- 4×4 descriptors over 16×16 sample array were used in practice. 4×4 times 8 directions gives a **vector of 128 values.** 

11/1/2010

78

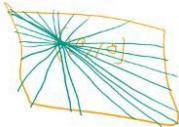


3D voting (a, b and r)

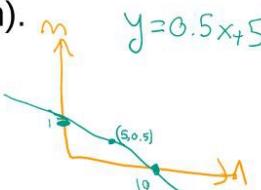
lecture 5 grouping and segmentation Spring 2020

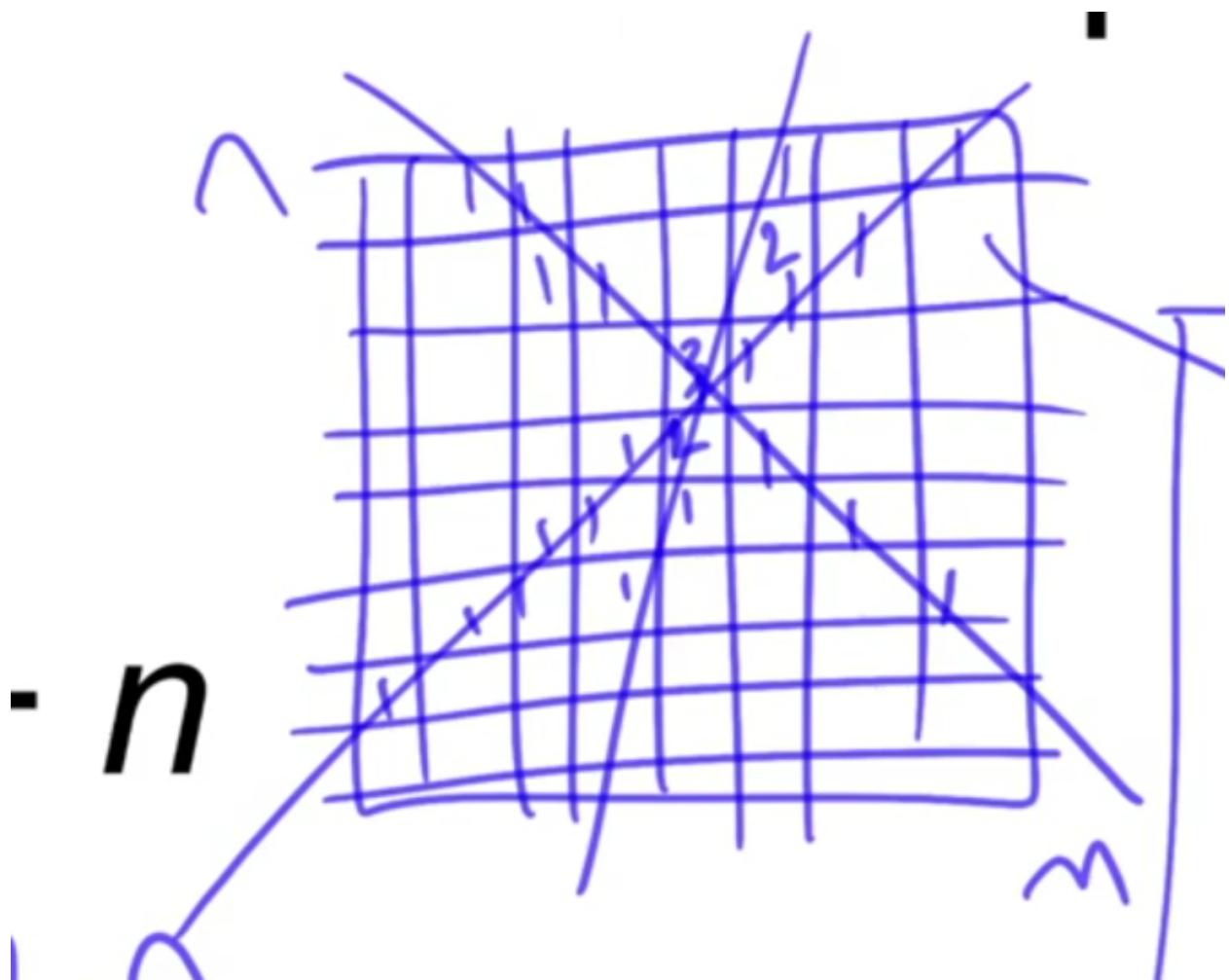
Line Detection by Hough Transforms (HT)

- Instead of going directly for the solution (as in template matching), we can use a voting scheme.
- Each possible point (edge) votes in the parameter space of a line. A line can be specified uniquely by two parameters m and n .

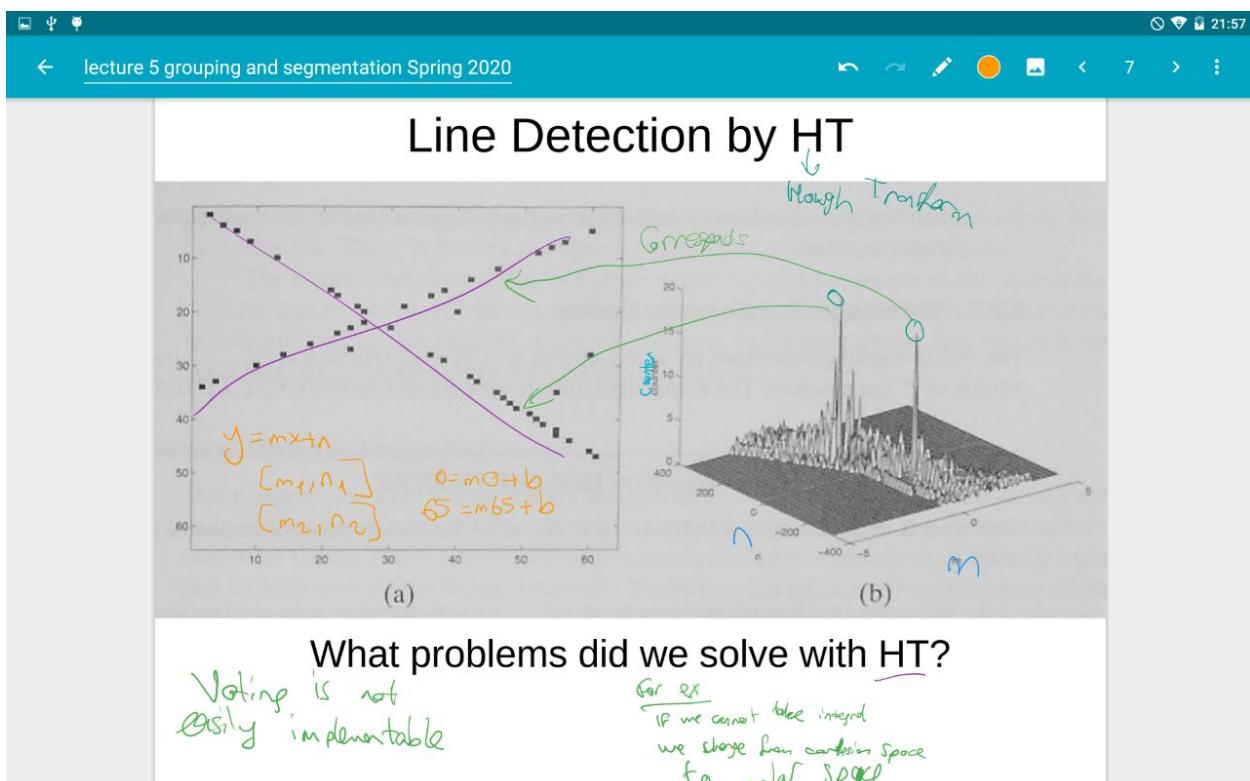
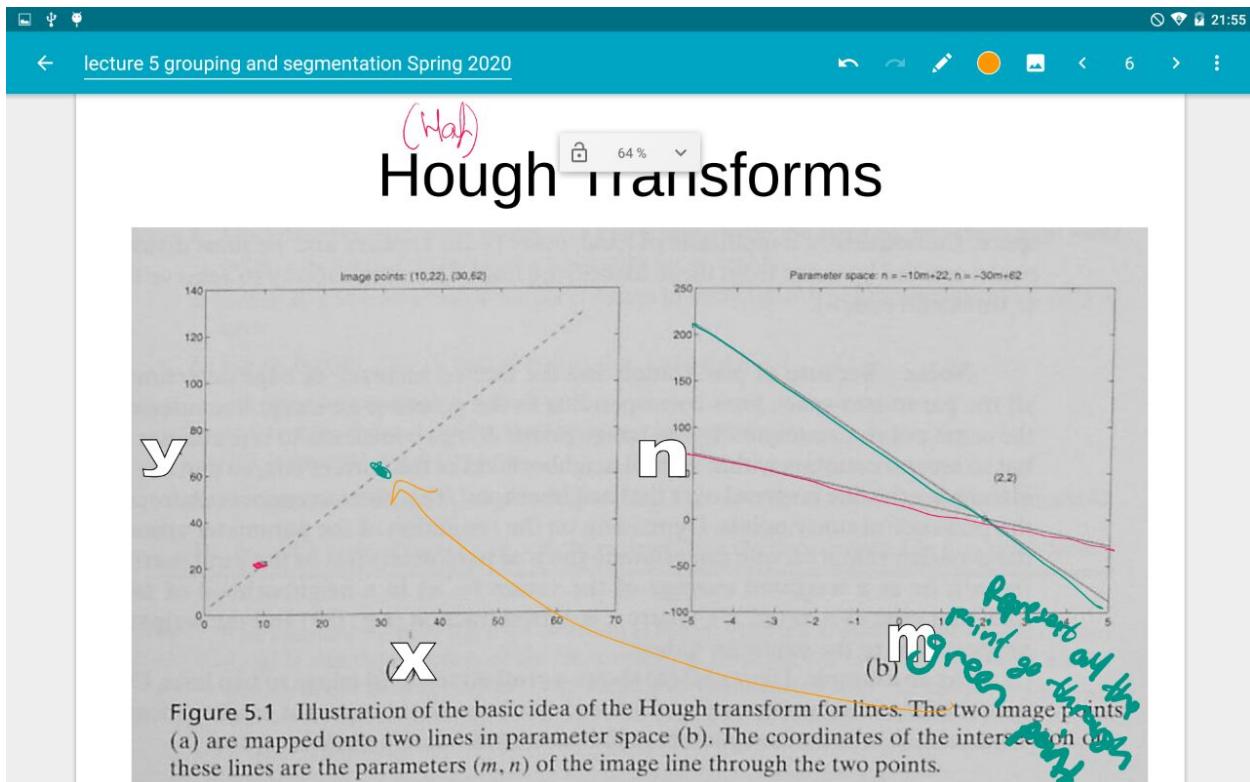
$$y = m \cdot x + n$$
$$\{ \alpha = m \cdot \theta + n$$


Parameter space of 2 dimensions (m and n).





Voting the highest number. When many lines cross the same point, then increment the counter for it. (LINE DETECTION BY HOUGH TRANSFORMS)



Hough sayesinde eğer integral alamıysak, kartezyen uzaydan polar uzaya geçebilirz

Circle Detection With Hough Transforms

$$(x-a)^2 + (y-b)^2 = r^2$$

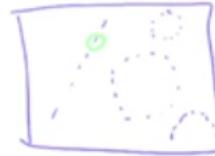
is the formula for a circle.

We can use the parametric version

$$\begin{aligned} x &= a + r \cos \theta & \rightarrow \theta = 0 \rightarrow 360 \\ y &= b + r \sin \theta \end{aligned}$$

When we eliminate r from the above formula, we would have

$$b = a \tan \theta - x \tan \theta + y$$



Circle Detection With Hough Transforms

nicelendir $b = a \tan \theta - x \tan \theta + y$

1. Quantize the parameter space for parameters a and b
2. Zero the accumulator array $M(a,b)$
3. Compute the edges in the image and their angles.
4. For each edge point, increment all points in the accumulator array $M(a,b)$ along the line

$$b = a \tan \theta - x \tan \theta + y \quad \text{voting}$$

1. Local maxima in the accumulator array will give you the centers of the circles. **biriktirici**

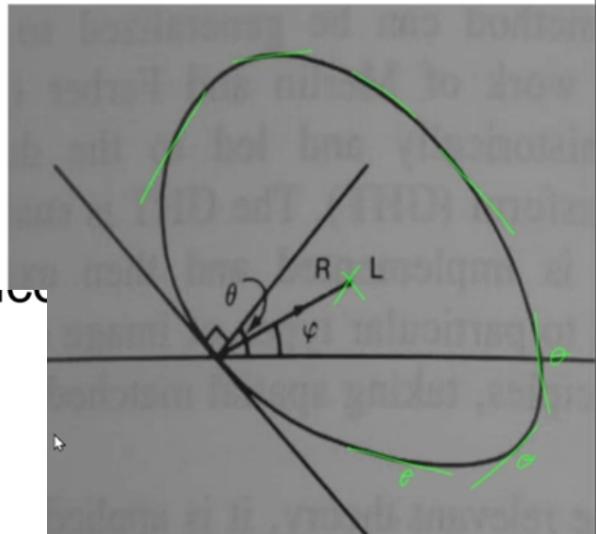
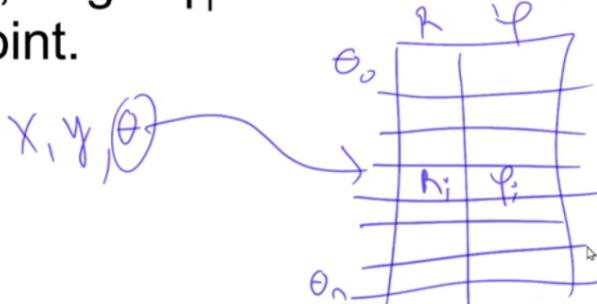
bir fonksiyonun kısmi türevlerinin vektörüne gradyan denir. türev, fonksiyonun bir noktadaki eğimini bulmak için kullanılıyordu. birden fazla türev de yönlü, yani vektörel bir eğim belirtir. dolayısıyla bu tip vektörler, "bir tepeden en hızlı şekilde aşağı inmek için hangi yönlerde ilerlemeliyim?" gibi sorularda işimize yarar.

item durduruldu We can use HT to localize arbitrary shaped objects.

18 / 33

On the object to be localized:

- Pick a reference point on the pattern object
- Compute the angles θ_i along the object.
- For each θ_i , store the distance R_i , angle φ_i from the reference point.

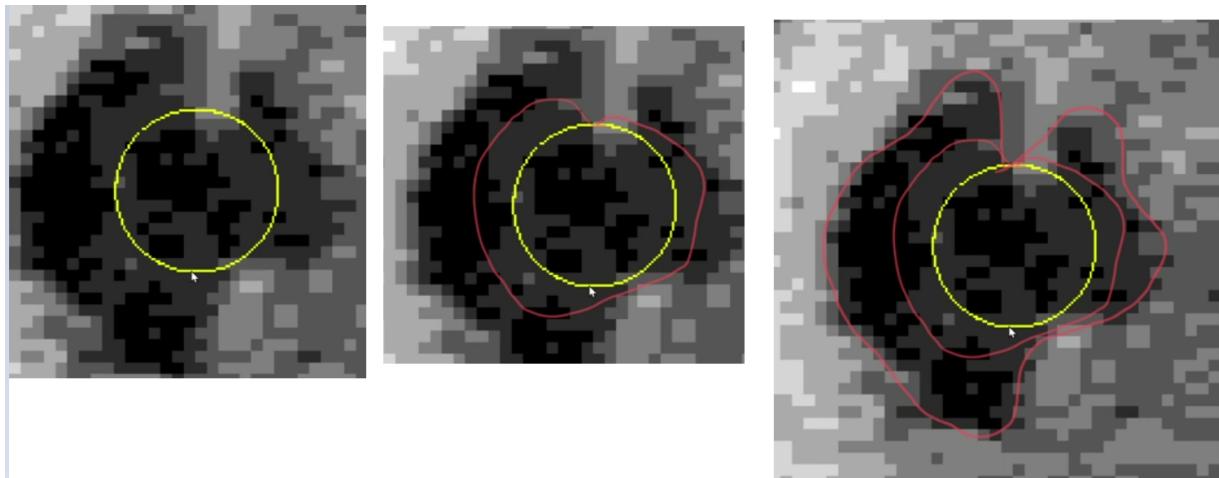


- For each edge point (x, y) with gradient angle θ , the possible locations of the reference point are given by

$$a = x - R(\theta) \cos(\varphi(\theta))$$

$$b = y - R(\theta) \sin(\varphi(\theta))$$

- The peaks on the voting space is the location of the reference point.



IT DOESN'T KNOW WHICH OBJECT LOOKING FOR

[DEFORMABLE CONTOUR SNAKE OPTIMIZATION]

You release a snake and it tries to expand . when sees high gradient image areas, expand

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Algorithm HOUGH_LINES

The input is E , a $M \times N$ binary image in which each pixel $E(i, j)$ is 1 if an edge pixel, 0 otherwise. Let ρ_d, θ_d be the arrays containing the discretized intervals of the ρ, θ parameter spaces ($\rho \in [0, \sqrt{M^2 + N^2}]$, $\theta \in [0, \pi]$), and R, T , respectively, their number of elements.

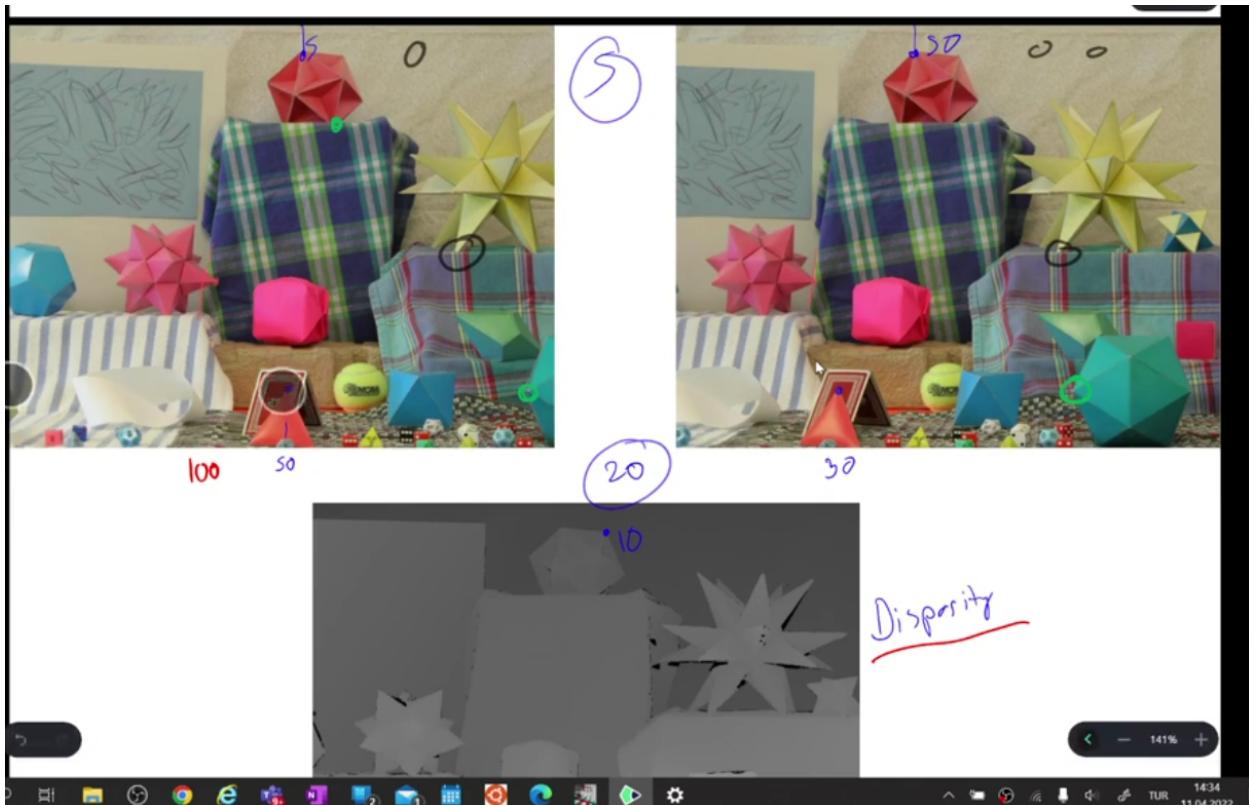
1. Discretize the parameter spaces of ρ and θ using sampling steps $\delta\rho, \delta\theta$, which must yield acceptable resolution and manageable size for ρ_d and θ_d .
2. Let $A(R, T)$ be an array of integer counters (accumulators); initialize all elements of A to zero.
3. For each pixel, $E(i, j)$, such that $E(i, j) = 1$ and for $h = 1 \dots T$:
 - let $\rho = i \cos \theta_d(h) + j \sin \theta_d(h)$; *Each h , will give me a row...*
 - find the index, k , of the element of ρ_d closest to ρ ; *From 0 to R*
 - increment $A(k, h)$ by one.
4. Find all local maxima (k_p, h_p) such that $A(k_p, h_p) > \tau$, where τ is a user-defined threshold. *From 0 to T*

The output is a set of pairs $(\rho_d(k_p), \theta_d(h_p))$, describing the lines detected in E in polar form.

disparity = uyumsuzluk

Computer stereo vision

Bilgisayar stereo görüşü, bir CCD kamera tarafından elde edilenler gibi dijital görüntülerden 3B bilgilerin çıkarılmasıdır. Bir sahne hakkında bilgileri iki bakış noktasından karşılaştırarak, iki paneldeki nesnelerin göreceli konumlarını inceleyerek 3B bilgiler çıkarılabilir.



All correspondences are on the same line. [**Epipolar constraint**]

DISPARITY: let's say that's my left and right eye. disparity shows how they differ.

If points are close to us, disparity will be larger. Calculating disparity between left and right images will give depth from stereo.

Thus both the Essential and Fundamental matrices completely describe the geometric relationship between corresponding points of a stereo pair of cameras. The only difference between the two is that **the former deals with calibrated cameras, while the latter deals with uncalibrated cameras**.

<http://robotics.stanford.edu/~birch/projective/node20>

Essential and fundamental matrices

2 Öne çıkan sınıflar hakkında • 1 Geri bildirim

Derivative in Two-Dimensions

- **Definition**

$$\frac{\partial f(x, y)}{\partial x} = \lim_{\varepsilon \rightarrow 0} \left(\frac{f(x + \varepsilon, y) - f(x, y)}{\varepsilon} \right) \quad \frac{\partial f(x, y)}{\partial y} = \lim_{\varepsilon \rightarrow 0} \left(\frac{f(x, y + \varepsilon) - f(x, y)}{\varepsilon} \right)$$

- **Approximation**

$$\frac{\partial f(x, y)}{\partial x} \approx \frac{f(x_{n+1}, y_m) - f(x_n, y_m)}{1} \quad \frac{\partial f(x, y)}{\partial y} \approx \frac{f(x_n, y_{m+1}) - f(x_n, y_m)}{1}$$

- **Convolution kernels**

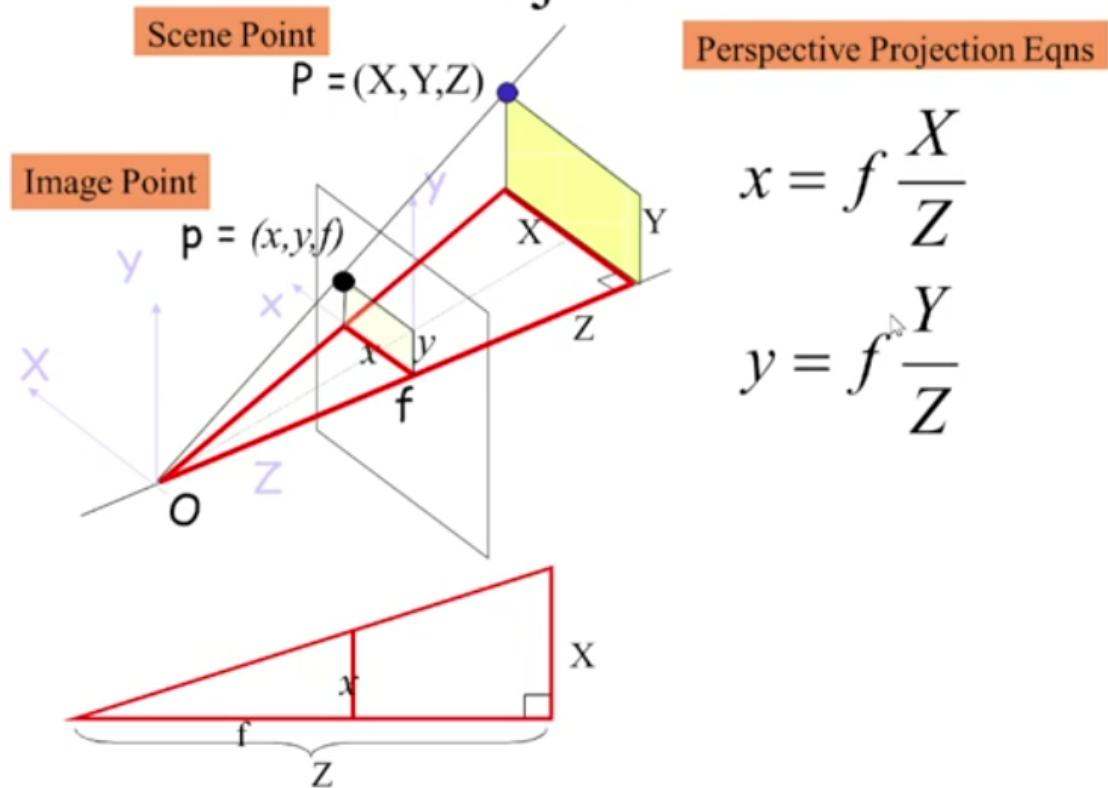
$$f_x = [1 \quad -1]$$

$$f_y = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Alper Yilmaz, Mubarak Shah Fall 2012UCF

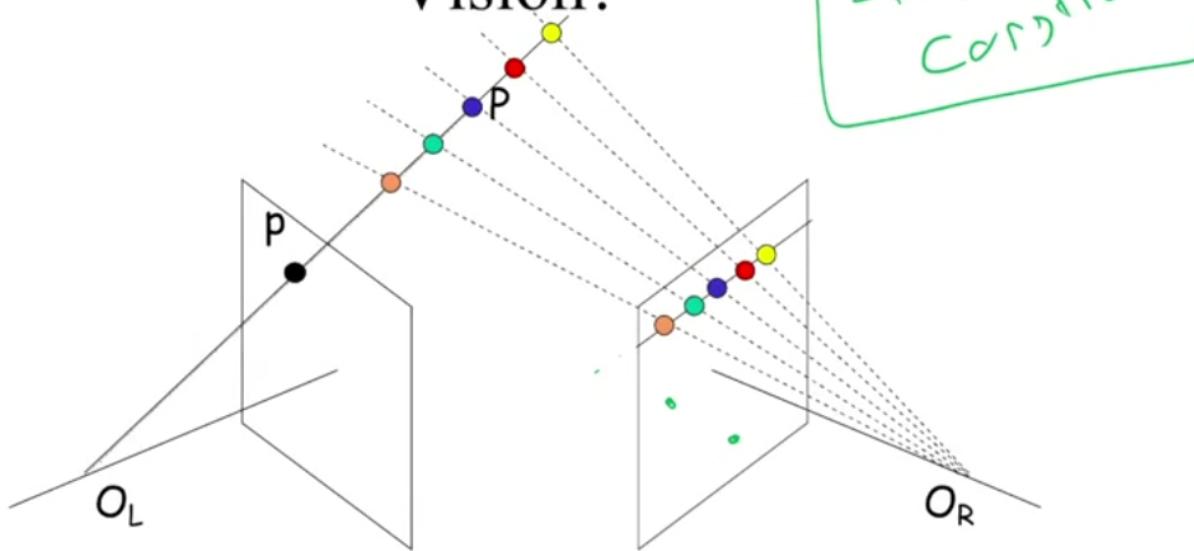
When object that we are looking is far away, disparity is less.

Basic Perspective Projection



f is like focal length

Why Stereo Vision?



A second camera can resolve the ambiguity, enabling measurement of depth via triangulation.

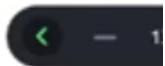
Epipolar constraint \rightarrow we don't have to search all image for P . one line is enough.

Parallax \rightarrow Points at different depths displace differently

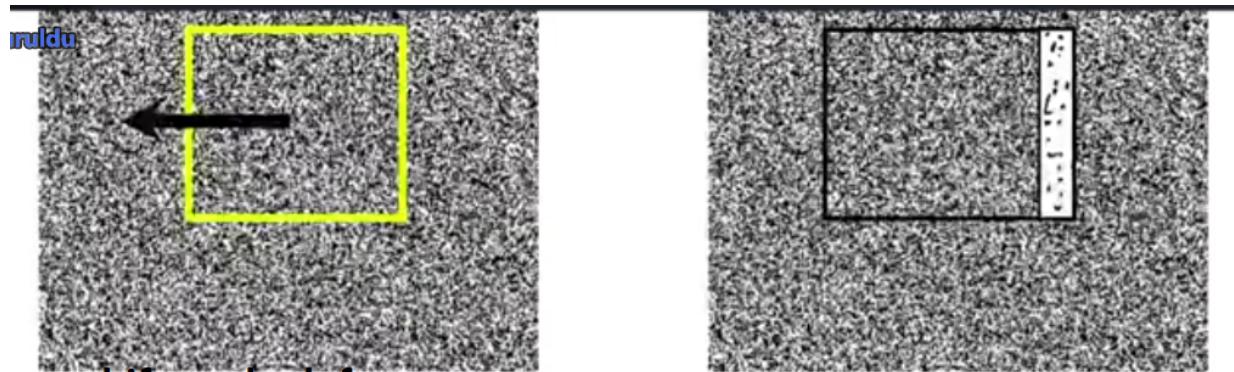
Anaglyphs (3d gözlük) \rightarrow are a way of encoding parallax in a single picture. actually there is no color[just channels] info but we see it as red and cyan(green+blue)



Occluded contours
(perceptual completion)



Occlusion gives info about which one is closer.



shift to the left



**when we wear glasses
feel 3d
[without using vanishing
points etc.]**

$$D = f \frac{x}{z} - f \frac{x - T_x}{z}$$

$$D = \frac{f}{z} (x - (x - T_x))$$

$$D = \frac{f}{z} \frac{T_x}{x}$$

ight

amera

located at

$T_x, 0, 0$)

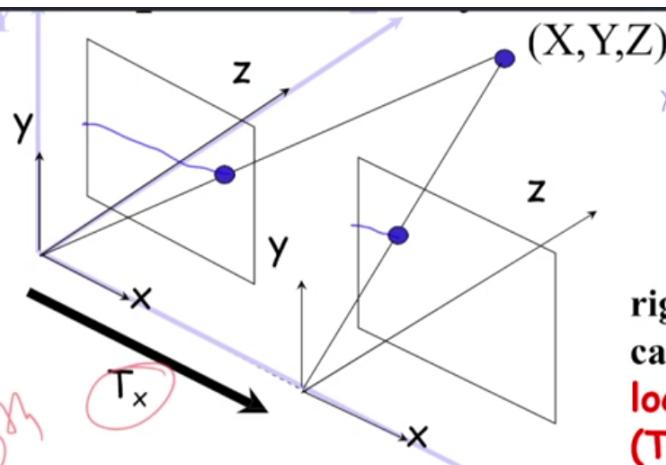
z = depth

disparity and depth
ters orant

natüm durduruldu

left
camera
located at
(0,0,0)

Baseline
length



$$x_l - x_r = D$$
$$D = f \frac{X}{Z} - f \frac{X - T_x}{Z}$$
$$D = \frac{f}{Z} (X - (X - T_x))$$
$$D = f \frac{T_x}{Z}$$

right
camera
located at
(T_x, 0, 0)

Image coords of point (X, Y, Z)

in Left Camera: $x_l = f \frac{X}{Z}$ $y_l = f \frac{Y}{Z}$

What are image coords of that same point
in the Right Camera?

$$x_r = f \frac{X - T_x}{Z}$$

T_x = Baseline length

Z = depth

D = disparity

$$Z = \frac{f \cdot T_x}{D}$$

When baseline is large, it will cause more depth and more disparity and baseline will be away from 0. [better Z]

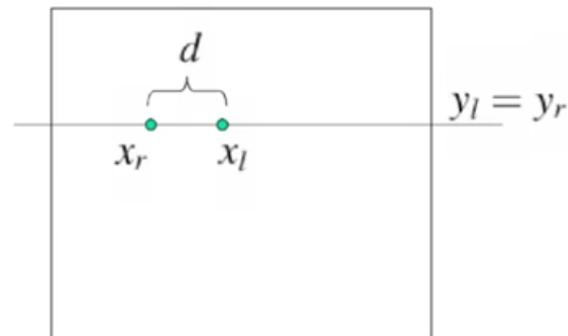
Disparity

Left camera

$$x_l = f \frac{X}{Z} \quad y_l = f \frac{Y}{Z}$$

Right camera

$$x_r = f \frac{X - T_x}{Z} \quad y_r = f \frac{Y}{Z}$$



Stereo Disparity

$$d = x_l - x_r = f \frac{X}{Z} - (f \frac{X}{Z} - f \frac{T_x}{Z})$$

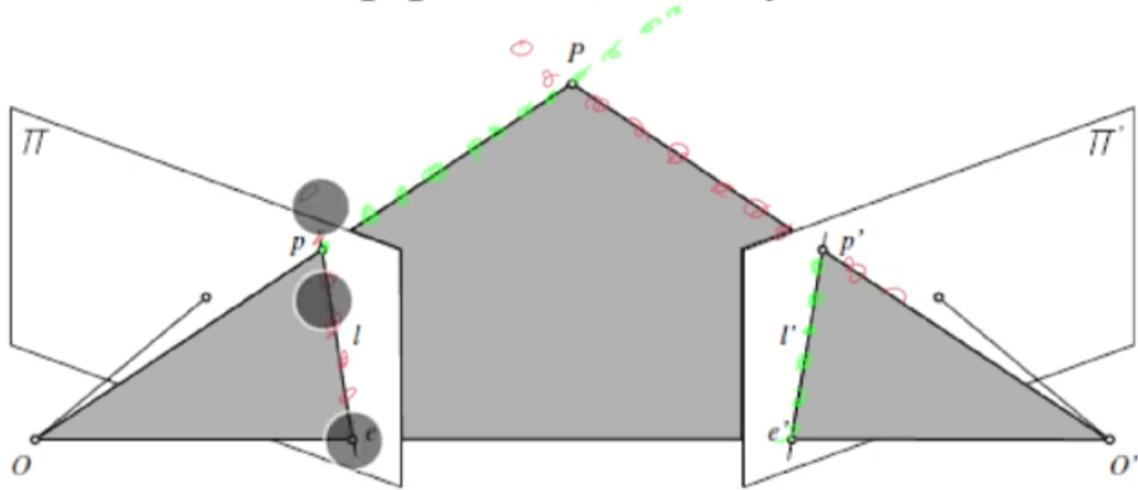
$$d = f \frac{T_x}{Z}$$

depth $Z = \frac{f T_x}{d}$ baseline
disparity

Important equation!

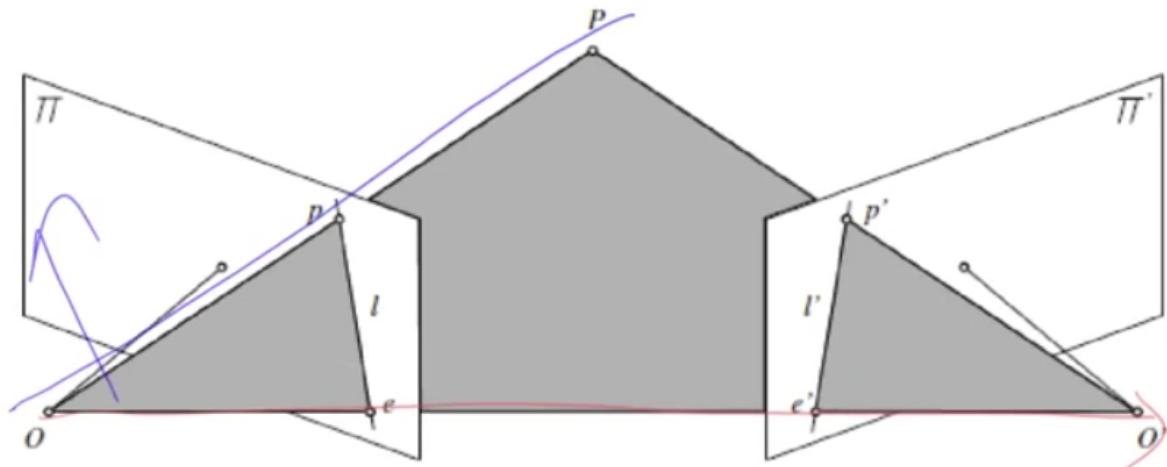
Epipolar geometri stereo görüşün geometrisidir. İki kamera bir 3D sahnesini iki farklı konumdan görüntülediğinde, 3B noktalar ile bunların 2B görüntülere yansımaları arasında görüntü noktaları arasında kısıtlamalara yol açan bir dizi geometrik ilişki vardır.

Epipolar Geometry



Epipolar plane, conjugated epipolar lines, epipoles, baseline
konjuge = eşlenik

Epipolar Geometry

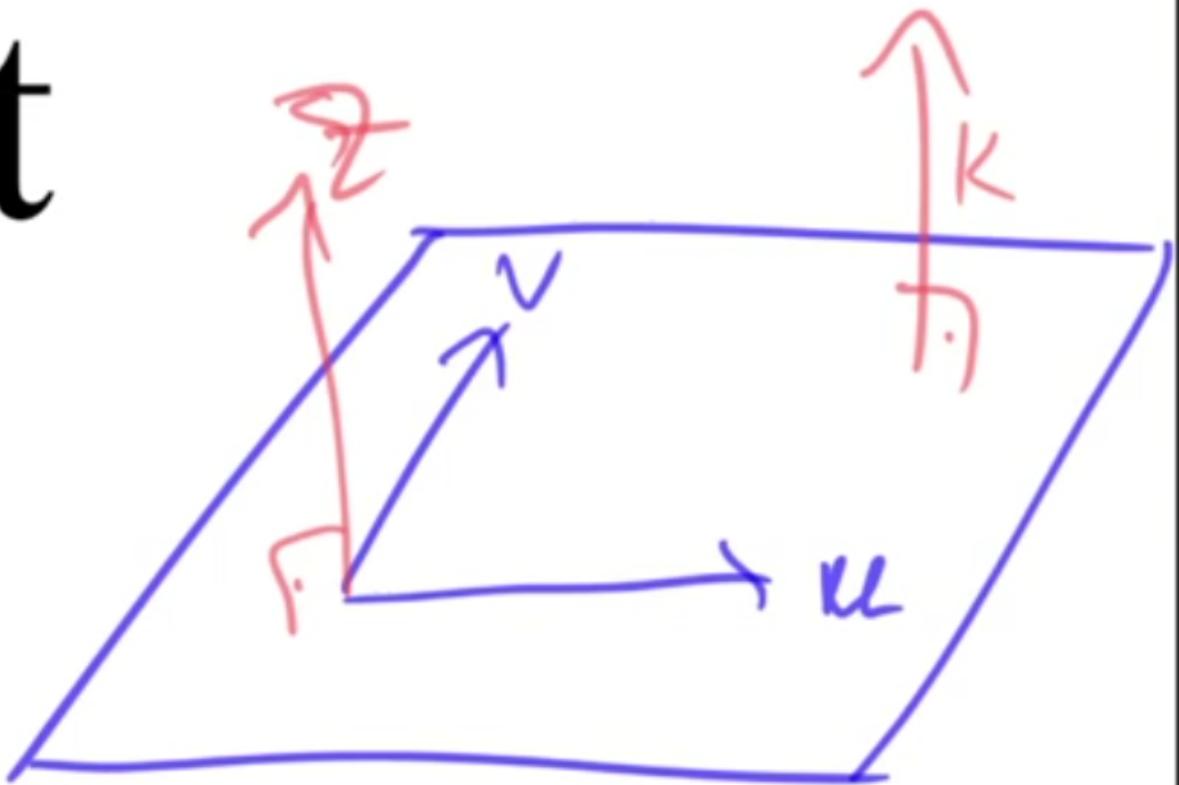


Epipolar plane, conjugated epipolar lines, epipoles, baseline

$$(O \times O') \cdot O' P = 0$$

we like where epipoles intersect at infinity (like this birthday image.)

It



$$v \times u = r$$

$$v \cdot k = 0$$

$$u \cdot k = 0$$

if perpendicular
cross product is 0

Cross product in matrix form

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}. \quad \text{a} \times \text{a}$$

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} \mathbf{i} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} \mathbf{j} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \mathbf{k}$$

The vector cross product also can be expressed as the product of a skew-symmetric matrix and a vector:

$$\mathbf{a} \times \mathbf{b} = [\mathbf{a}]_{\times} \mathbf{b} = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \quad [\mathbf{a}]_{\times} \stackrel{\text{def}}{=} \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}.$$

Calibrated Epipolar Equations



Clearly, the epipolar constraint implies that the three vectors $\vec{O}p$, $\vec{O}'p'$, and $\vec{OO'}$ are coplanar. Equivalently, one of them must lie in the plane spanned by the other two, or

$$\vec{O}p \cdot [\vec{OO'} \times \vec{O}'p'] = 0.$$

We can rewrite this coordinate-independent equation in the coordinate frame associated to the first camera as

$$p \cdot [t \times (\mathcal{R}p')],$$

where p and p' denote the homogenous image coordinate vectors, t is the coordinate vector of the translation, and R is the rotation matrix such that a free vector with coordinates w in the second coordinate system has coordinates Rw in the first one.

coplanar = aynı düzlemdedir

Mathematics [\[edit\]](#)

The gradient of an image is a vector of its [partials](#):^{[2]: 165}

$$\nabla f = \begin{bmatrix} g_x \\ g_y \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix},$$

rectification (doğrultma/düzelme) = özel bir işlemden geçirilerek correspondencelerin aynı satır hizasına gelmesini sağlama

Görüntünün büyüklüğünü ne kadar hızla değiştiğine bölersen **TIME TO COLLISION** bulunur.

$$\frac{D(t)}{v} = T$$

Görüntünün veya objenin ne kadar uzakta olduğunu bilmeye gerek yok, ne de objenin ne hızla yaklaştığını bilmeye.

Projection eşitliği

$$I(t) = fL / D(t)$$

imagelar arası stitch olabilmesi için aralarında baseline olmaması lazım

Motion field → gerçek dünyadaki cisimlerin hız vektörlerinin görüntü üzerine izdüşümü direkt izdüşüm hesaplamak çok zor bu yüzden optical flow kullan (OF)

Optical flow → görüntü üzerindeki parlaklıkların yer değişimi

Aperture problem

$$(\nabla E)^T \cdot v + E_t = 0$$

Eğer v 'nin yönüyle gradyanın yönü çok benzeşiyorsa o zaman elimde image gradient'i olduğu, yönleri aynı olduğu ve uzunlugunu da bulduğum için o zaman v 'yi elde edebilirim.

Optical flow - Optik akış, video kareleri arasındaki içerik farklılıklarını analiz ederek hareketli nesnelerin bilgisayarlı izlemesini açıklar. Bir video'da, hem nesne hem de gözlemci hareket halinde olabilir; bilgisayar, fotoğrafların sınırlarını, kenarlarını ve bölgelerini işaretleyen ipuçlarını bulabilir. İllerlemelerini tespit etmek, bilgisayarın zaman ve uzayda bir nesneyi takip etmesini sağlar.



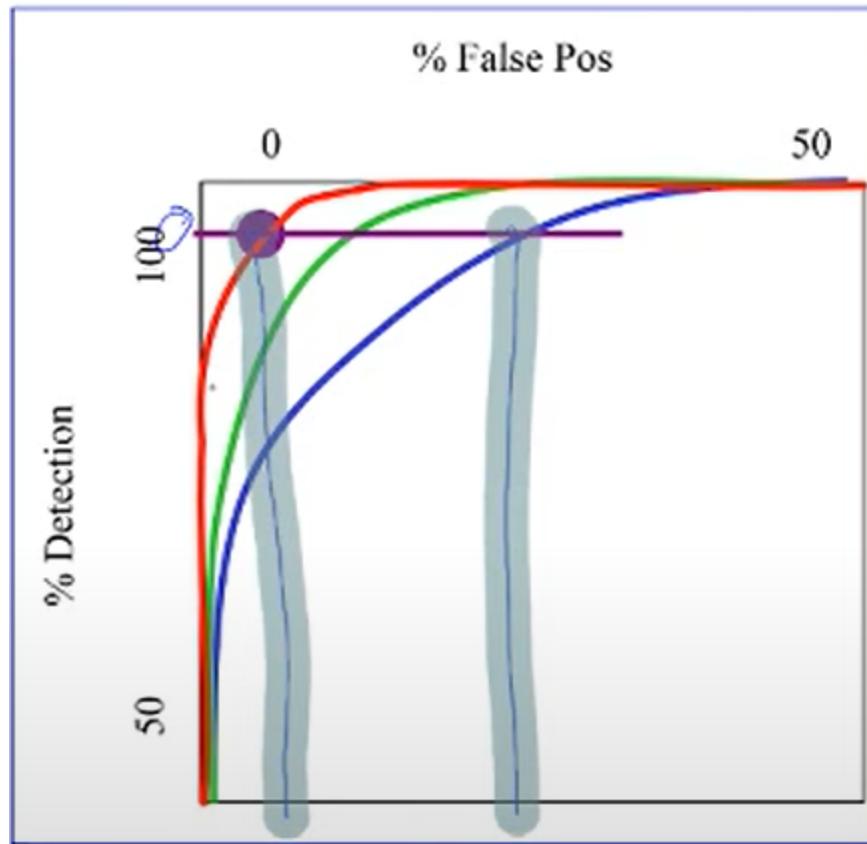
Okların büyüklükleri optic flowun vektörlerinin büyüklüklerini gösteriyor. renkler de yön gösteriyor.

AdaBoosting

Diyelim ki elimde 8-10 sınıflandırıcı var

Her bir weak classifier ile sınıflandırır, kötü sınıflandırmaların weight'ini artır böylece sonraki sınıflandırıcı ile bir daha sınıflandırılabilisin. (diğer sınıflandırıcı ağırlığı fazla olanda hata yaparsa cezalandırılacak gibi)

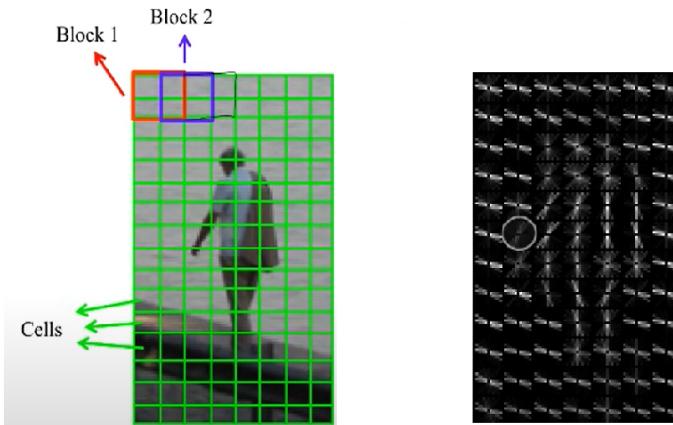
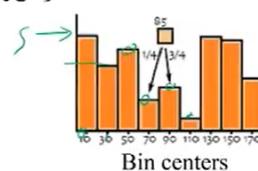
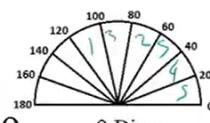
Cascading classifier



Yüz var mı diye maviye sor, mavi evet derse yeşile sor, yeşil evet derse kırmızıya sor.
Kırmızı güvenilir ama maviye göre daha yavaş

Tri-linear Interpolation

- Each block consists of 2x2 cells with size 8x8 ↴
- Quantize the gradient orientation into 9 bins (0-180)
 - The vote is the gradient magnitude



Evaluation Metrics

$$\text{precision} = \frac{\text{GT} \cap \text{RM}}{\text{RM}} = \frac{\text{TP}}{\text{RM}}$$

$$\text{recall} = \frac{\text{GT} \cap \text{RM}}{\text{GT}} = \frac{\text{TP}}{\text{GT}}$$

$$F_{\text{score}} = \frac{2 \text{P} \text{R}}{\text{P} + \text{R}}$$

G

F-score 'un iyi olması iyi.

Precision ve recall en fazla 1.0 olabilir ve istedigimiz de bu.

Aperture problem - Açıklık sorunu, bir çubuk veya kenar gibi tek boyutlu bir uzaysal yapının hareketinin, uyaran uçlarının görülemeyeceği şekilde küçük bir açıktıktan bakıldığından açık bir şekilde belirlenemeyeceği gerçeğine atıfta bulunur.

In general, corners can be used as informative features that are consistently identifiable over time. For motion estimation, the presence of the two intersecting edges that define a corner disambiguates the unobservable motion in the direction parallel to a single edges orientation, i.e. it solves the aperture problem.

Weak perspective, what we are saying is that **the difference of depth over an object or over the range of an object, is very small compared to the difference in depth from the object to the center of projection**. And if that's true, then you can basically say that each object has its own scale factor. In weak perspective we assume that all of our objects are z_0 away from us.

With using SVD, we can reduce redundancy in the motion matrices.

$$W = M * S \text{ (Camera motion * scene structure)}$$

Optical flow

- Definition: optical flow is the apparent motion of brightness patterns in the image
- Ideally, optical flow would be the same as the motion field

Geometric camera calibration, estimates the parameters of a lens and image sensor of an image or video camera. You can use these parameters to correct for lens distortion, measure the size of an object in world units, or determine the location of the camera in the scene. These tasks are used in applications such as machine vision to detect and measure objects. They are also used in robotics, for navigation systems, and 3-D scene reconstruction.

Formally, an image gradient is defined as **a directional change in image intensity**. Or put more simply, at each pixel of the input (grayscale) image, a gradient measures the change in pixel intensity in a given direction.

A sift keypoint is circular image region with an orientation. Keypoint center coordinates x and y, its scale and its orientation. It's local feature of image . Keypoints are scale & rotation invariant

Disadvantage of Harris corner

detector is that it is not scale invariant and though it is rotational invariant it is not that much reliable[9]. There are some scale and rotation invariant feature detector or descriptors for ex: SIFT, SURF, BRIEF etc. whose descriptor can be used for further feature matching.

Also, Lowe aimed to create a descriptor that was robust to the variations corresponding to typical viewing conditions. The descriptor is the most-used part of SIFT.

The **rectification** process can be implemented by projecting the original pictures onto the new image plane .

With an appropriate choice of coordinate system, the rectified epipolar lines are scanlines of the new images, and they are also parallel to the baseline

The algorithm consists of four steps:

- Rotate the left camera so that the epipole goes to infinity along the horizontal axis.
- Apply the same rotation to the right camera to recover the original geometry.
- Rotate the right camera by R .
- Adjust the scale in both camera reference frames.

Algorithm RECTIFICATION

The input is formed by the intrinsic and extrinsic parameters of a stereo system and a set of points in each camera to be rectified (which could be the whole images). In addition, Assumptions 1 and 2 above hold.

1. Build the matrix R_{rect} as in (7.22);
2. Set $R_l = R_{rect}$ and $R_r = RR_{rect}$;
3. For each left-camera point, $\mathbf{p}_l = [x, y, f]^\top$ compute

$$R_l \mathbf{p}_l = [x', y', z']$$

and the coordinates of the corresponding rectified point, \mathbf{p}'_l , as

$$\mathbf{p}'_l = \frac{f}{z'} [x', y', z'].$$

4. Repeat the previous step for the right camera using R_r and \mathbf{p}_r .

The output is the pair of transformations to be applied to the two cameras in order to rectify the two input point sets, as well as the rectified sets of points.

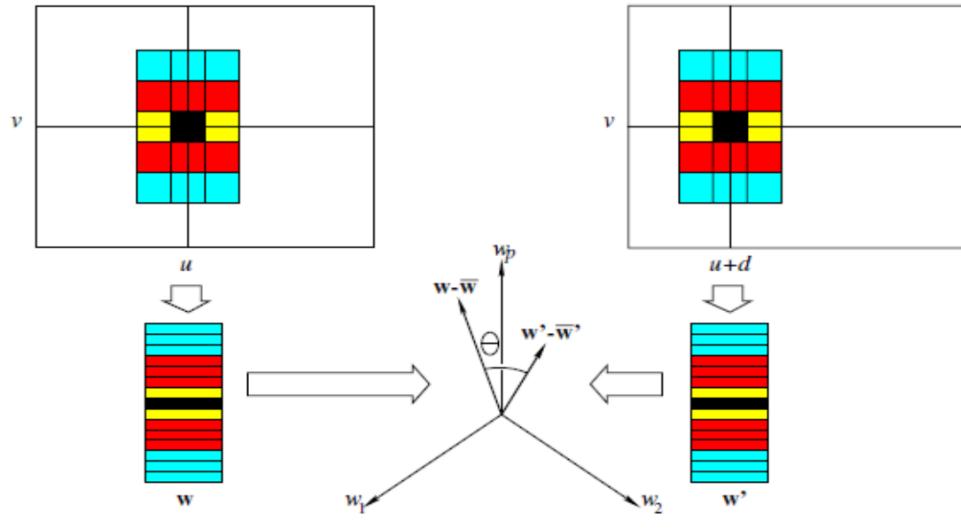
Fundamental matrix 3x3

7 degrees of freedom

rank = 2

$$x'^T F x = 0$$

Normalized Cross Correlation



Line detection

$$r_0 = x \cdot \cos \theta + y \cdot \sin \theta$$

Görüntünün büyüklüğünü ne kadar hızla değiştiğine bölersen **TIME TO COLLISION** bulunur.

$$\frac{D(t)}{v} = T$$

The image of the object has size $l(t)$:

$$l(t) = \frac{fL}{D(t)}$$

HARRIS CORNER DETECTOR

- 1 - compute image derivatives
- 2 - compute M components as squares of derivatives
- 3 - Gaussian filter $g()$ with width σ .
 $g(Ix^2) g(Iy^2) g(Ix \circ Iy)$
- 4 Croner score $\lambda_1^* \lambda_2 - \alpha (\lambda_1 + \lambda_2)^2$
- 5 threshold on score
- 6 Then apply non maximal suppression.

Aperture Problem describes a situation where the size of a window is too small to obtain useful information about changes in the image. The way I like to think about it is that there is an appropriate window size for every feature and image.

<https://elvers.us/perception/aperture/>

Corners are interest points which satisfy some of the above properties. They also help solve another problem called the Aperture problem. Given a region in the image with just Edge or flat region it is impossible to identify it uniquely. There could be multiple matching patches to this given patch.

HOUGH TRANSFORM

uniform değil

$$P^T F p' = 0$$

$$K^{-T} \epsilon K'^{-1}$$

$$\text{ESSENTIAL} = T_x^* R$$

Thus both the Essential and Fundamental matrices completely describe the geometric relationship between corresponding points of a stereo pair of cameras. The only difference between the two is that **the former deals with calibrated cameras, while the latter deals with uncalibrated cameras**

Both matrices relate corresponding points in two images. The difference is that in the case of the Fundamental matrix, the points are in pixel coordinates, while in the case of the Essential matrix, the points are in "normalized image coordinates". Normalized image coordinates have the origin at the optical center of the image, and the x and y coordinates are normalized by F_x and F_y respectively, so that they are dimensionless.

The two matrices are related as follows: $E = K' * F * K$, where K is the intrinsic matrix of the camera.

F has 7 degrees of freedom, while E has 5 degrees of freedom, because it takes the camera parameters into account. That's why there is an 8-point algorithm for computing the fundamental matrix and a 5-point algorithm for computing the essential matrix.

One way to get a 3D position from a pair of matching points from two images is to take the fundamental matrix, compute the essential matrix, and then to get the rotation and translation between the cameras from the essential matrix. This, of course, assumes that you know the intrinsics of your camera. Also, this would give you up-to-scale reconstruction, with the translation being a unit vector.

In a stereo image pair, the fundamental matrix encodes the rigidity constraint of the scene. It

combines the internal parameters of both cameras (which can be the same) and their relative

position and orientation. It associates to image points in one view the so-called epipolar line in

the other view, which is the locus of projection of the same 3D point, whose particular position

on the straight line is determined by its depth. Reducing the correspondence search to a 1D line

instead of the 2D image is a large benefit enabling the computation of the dense 3D scene. The

estimation of the matrix depends on at least seven pairs of corresponding points in the images.

The epipolar geometry depends on the relative pose and calibration of the cameras

This can be computed using the fundamental matrix

Once this is computed, we can use the epipolar lines to restrict the search space to a 1D search

The fundamental matrix expresses the same constraint, but when two cameras are characterized by general (and unknown) camera matrices K

and K' , respectively. Therefore the derivation of F includes the camera matrices for both cameras.