Gebze Institute of Technology
Department of Computer Engineering

Computer Vision

CSE 665 463

What is Computer Vision?

Computer vision is extracting descriptions of the real world from images or images sequences.

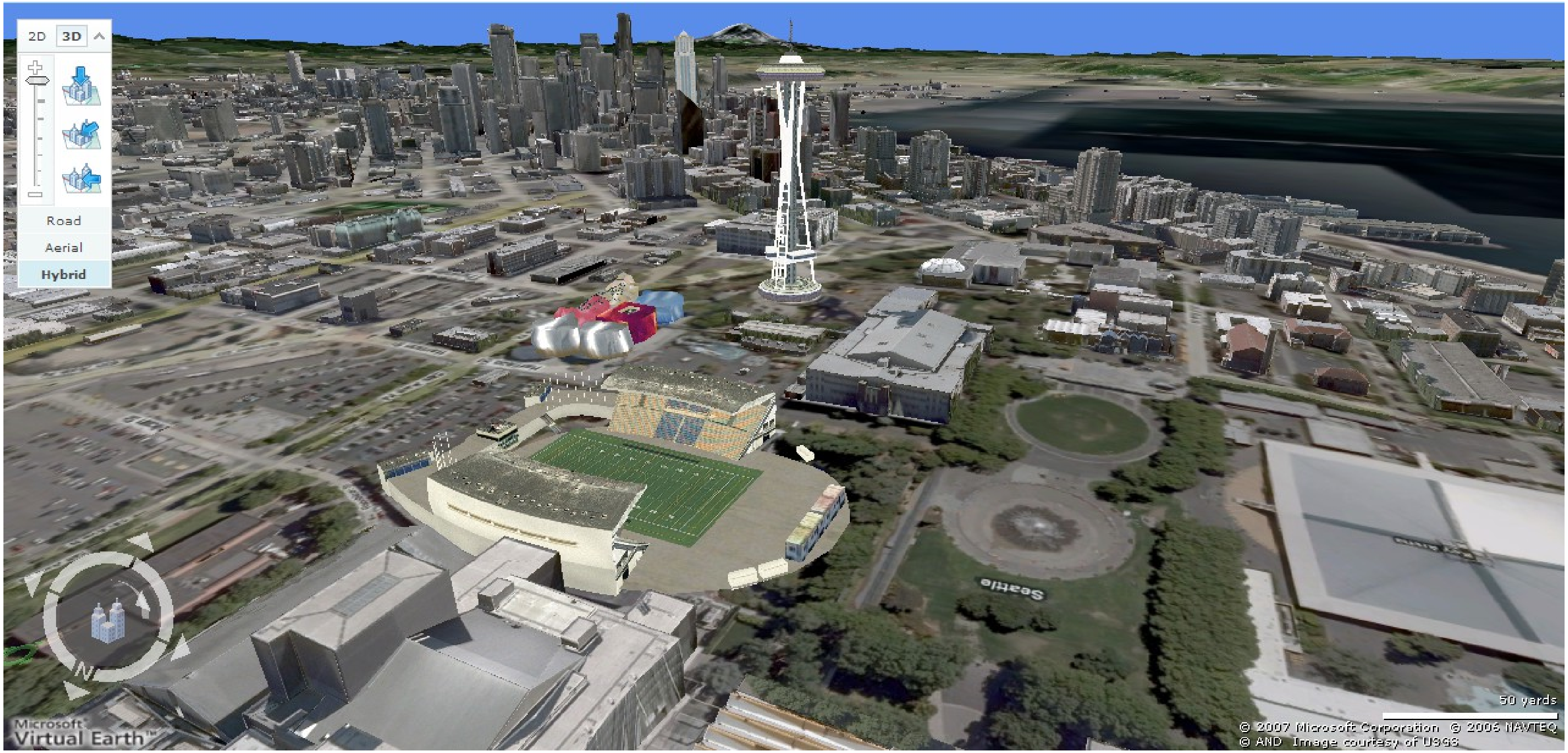
The extracted features are usually

- geometric (sizes or positions of objects),
- dynamic (velocities of objects)

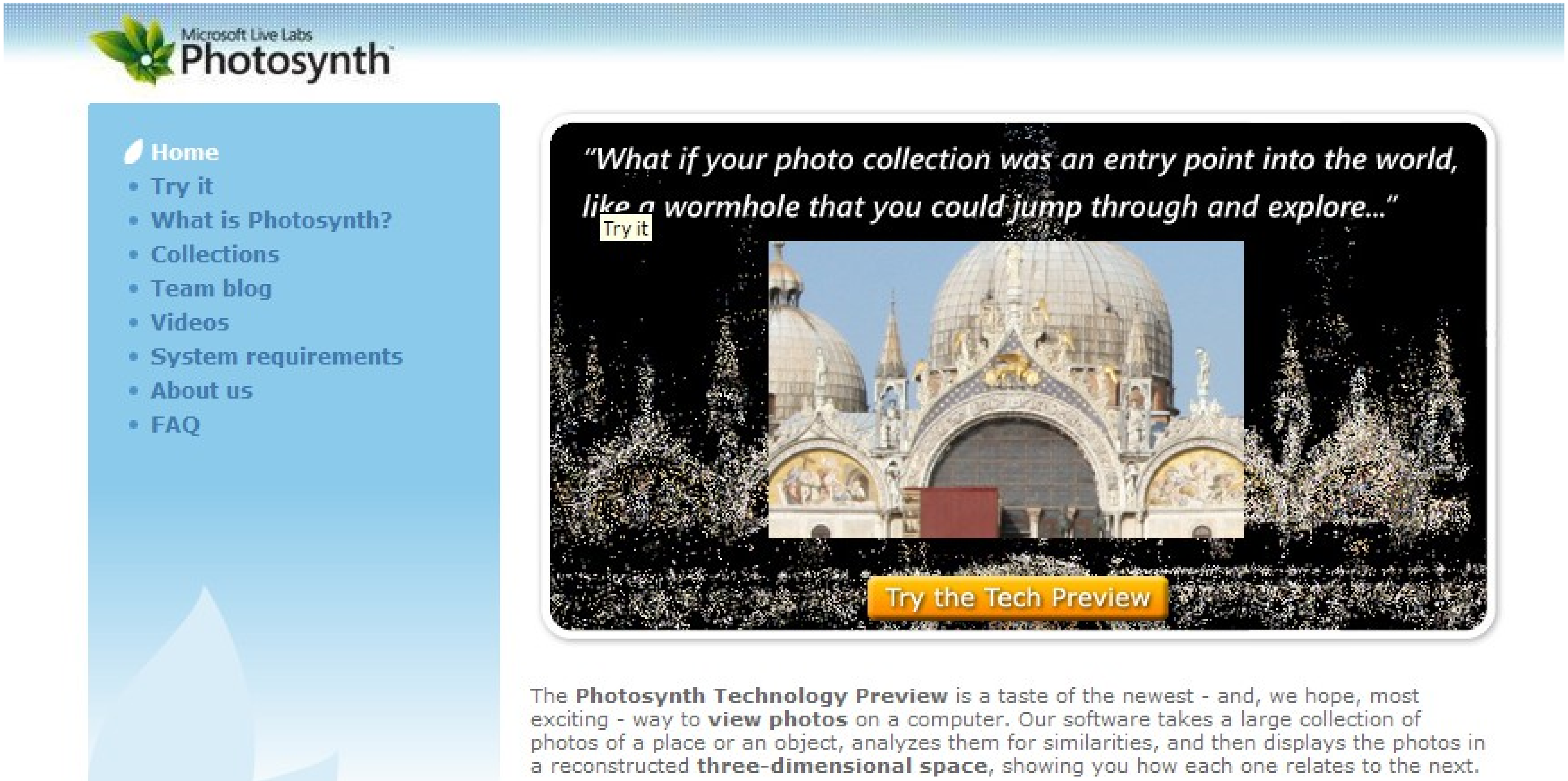
Computer Vision at Work

- Medical applications
- Augmented reality
- Object tracking
- Shape pose estimation
- Industrial inspection
- Object recognition

Earth viewers (3D modeling)



: Google Earth



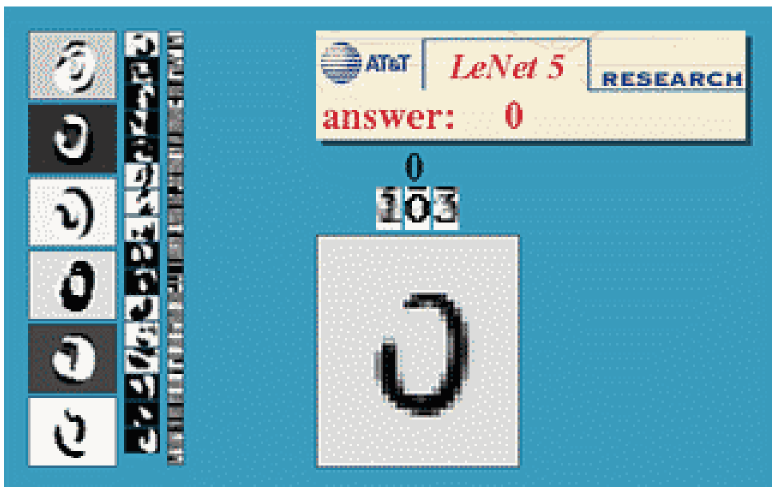
http://labs.live.com/photosynth/

Based on Photo Tourism technology developed here in CSE!
by Noah Snavely, Steve Seitz, and Rick Szeliski

Optical character recognition (OCR)

Technology to convert scanned docs to text

- If you have a scanner, it probably came with OCR software



Digit recognition, AT&T labs
http://www.research.att.com/~yann/

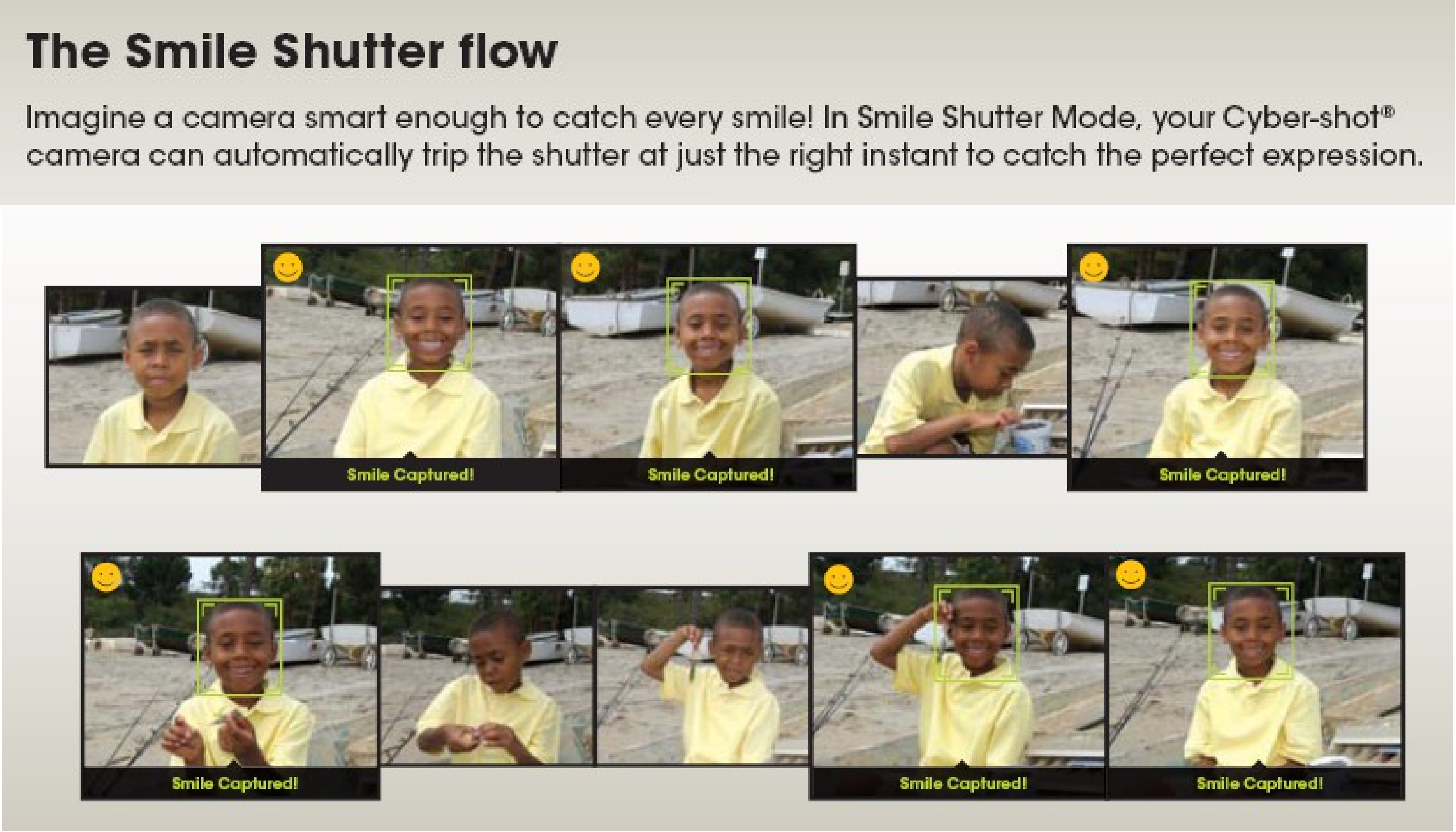
License plate readers
http://en.wikipedia.org/wiki/Automatic_number_plate_recognition

Face detection

Many new digital cameras now detect faces

– Canon, Sony, Fuji, …

Smile detection?



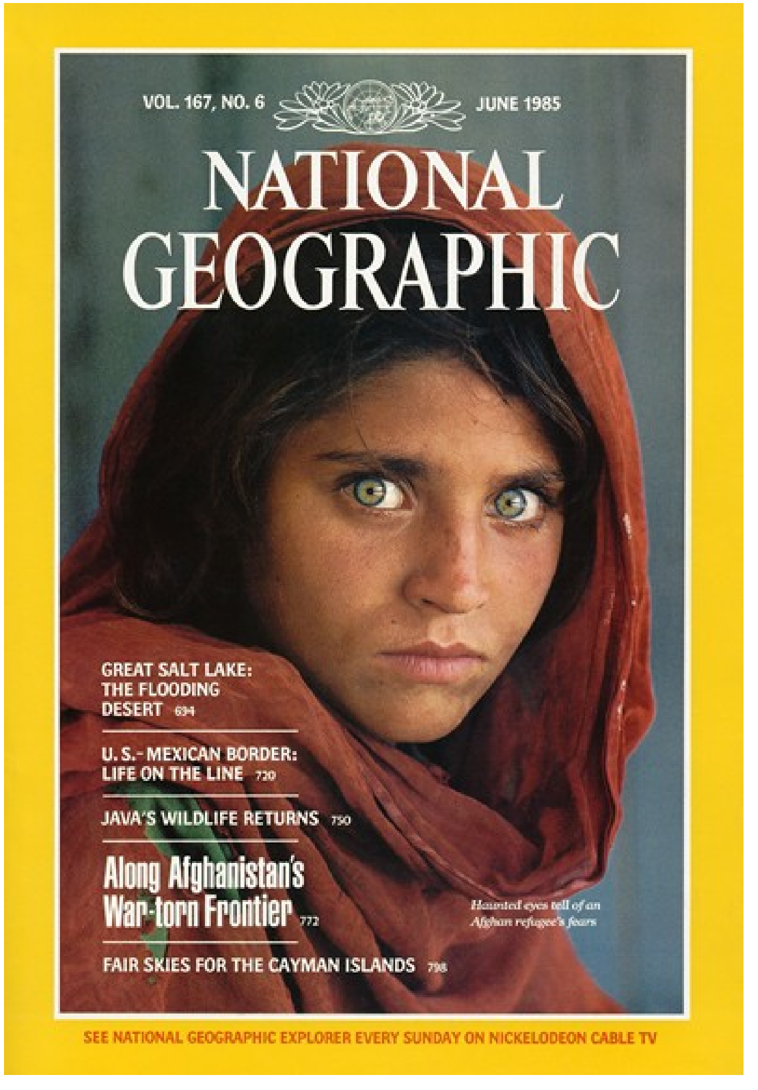
Sony Cyber-shot® T70 Digital Still Camera

Object recognition (in supermarkets)

LaneHawk by EvolutionRobotics

"A smart camera is flush-mounted in the checkout lane, continuously watching for items. When an item is detected and recognized, the cashier verifies the quantity of items that were found under the basket, and continues to close the transaction. The item can remain under the basket, and with LaneHawk,you are assured to get paid for it… "

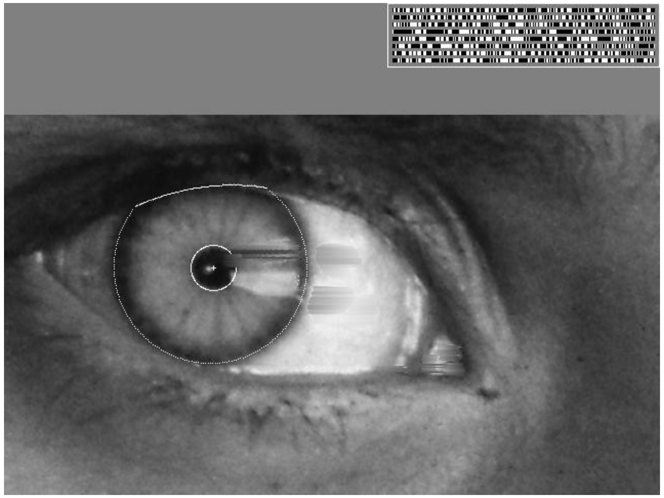
Face recognition

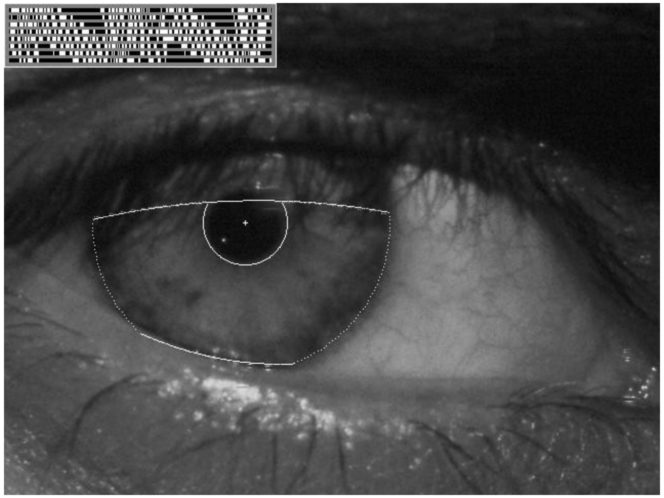


Who is she?

Vision-based biometrics

"How the Afghan Girl was Identified by Her Iris Patterns" Read the story





Login without a password…

Fingerprint scanners on
many new laptops,
other devices

Face recognition systems now
beginning to appear more widely
http://www.sensiblevision.com/

Object recognition (in mobile phones)

This is becoming real:

– Lincoln Microsoft Research

• Point & Find, Nokia

Special effects: shape capture

The Matrix movies, ESC Entertainment, XYZRGB, NRC

Special effects: motion capture

Pirates of the Carribean, Industrial Light and Magic

Click here for interactive demo

Sports

Sportvision first down line
Nice explanation on www.howstuffworks.com

Smart cars

Slide content courtesy of Amnon Shashua

Mobileye

- Vision systems currently in high-end BMW, GM, Volvo models
- By 2010: 70% of car manufacturers.

Vision-based interaction (and games)

Digimask: put your face on a 3D avatar.

Nintendo Wii has camera-based IR tracking built in. See Lee's work atCMU on clever tricks on using it to create a multi-touch display!

KINECT!!

"Game turns moviegoers into Human Joysticks", CNET
Camera tracking a crowd, based on this work.

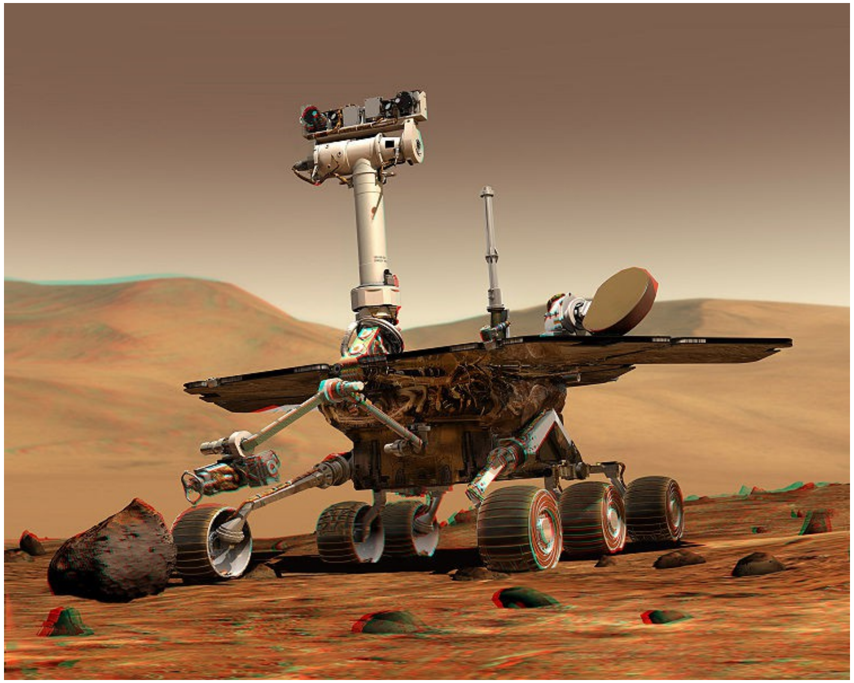
Vision in space

NASA'S Mars Exploration Rover Spirit captured this westward view from atop a low plateau where Spirit spent the closing months of 2007.

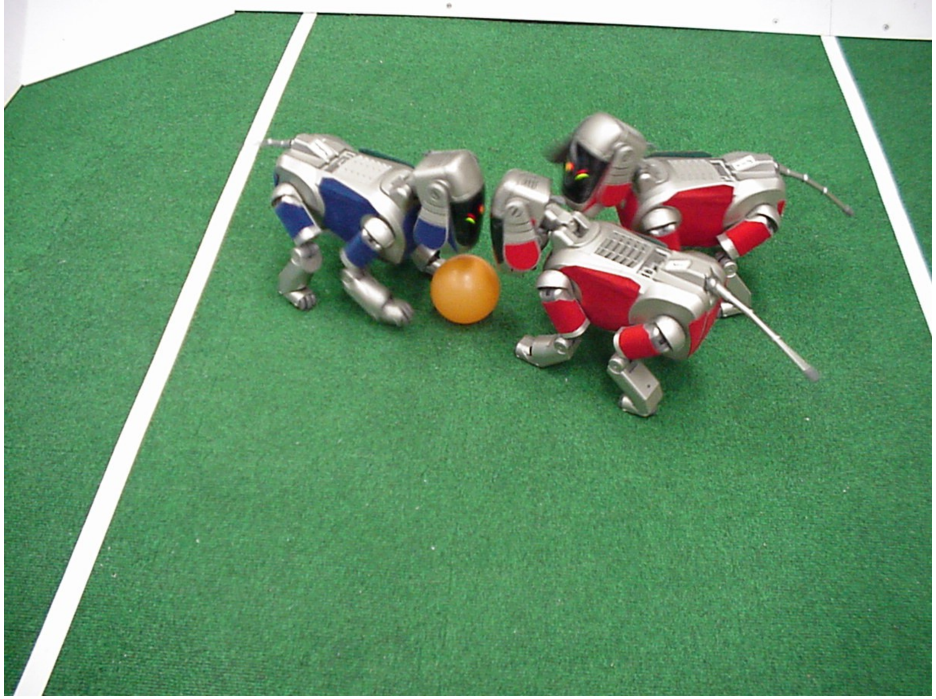
Vision systems (JPL) used for several tasks

- Panorama stitching
- 3D terrain modeling
- Obstacle detection, position tracking
- For more, read “Computer Vision on Mars” by Matthies et al.

Robotics

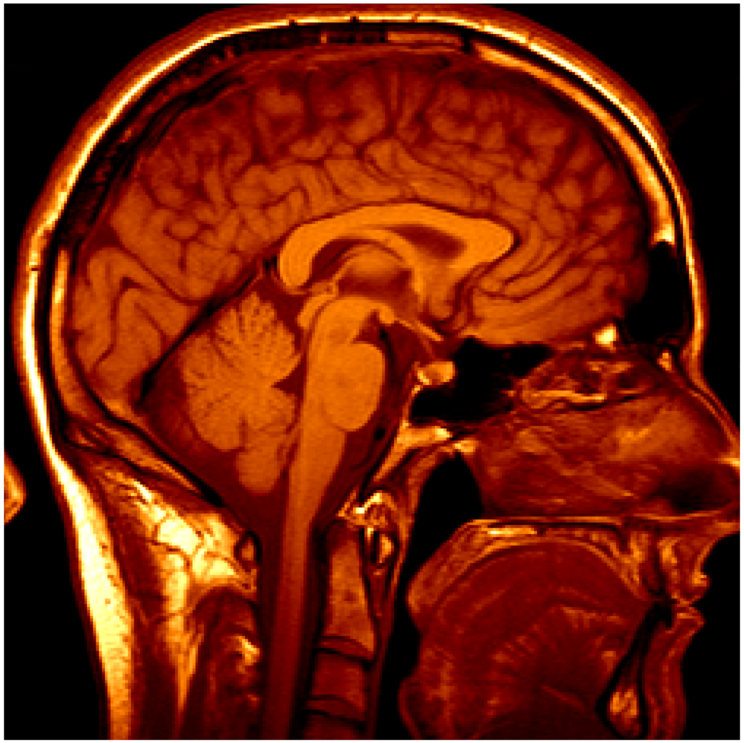


NASA's Mars Spirit Rover
http://en.wikipedia.org/wiki/Spirit_rover



http://www.robocup.org/

Medical imaging



3D imaging
MRI, CT

Image guided surgery
Grimson et al., MIT

Current state of the art

You just saw examples of current systems.

- Many of these are less than 5 years old

This is a very active research area, and rapidly changing

- Many new apps in the next 5 years

To learn more about vision applications and companies

- David Lowe maintains an excellent overview of vision companies
  - http://www.cs.ubc.ca/spider/lowe/vision.html

Relations to Other Fields

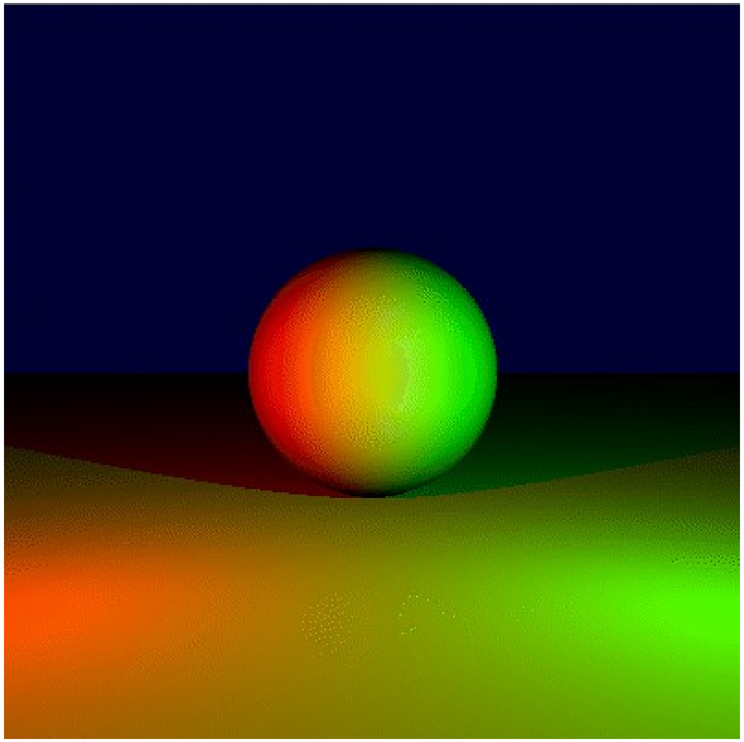
Image processing: Image properties or image to image transforms.

- Image enhancement, image compression, image restoration, …

Relations to Other Fields

Computer graphics: Producing (rendering) images using models.

Computer graphics is sometimes called “inverse vision”



Relations to Other Fields

Pattern recognition classifies patterns into finite set of prespecified classes. Usually it deals with 2D data.

Photogrammetry deals with obtaining reliable and accurate measurements from non-contact imaging. (Drawing maps using pictures)

Images and Image Formation

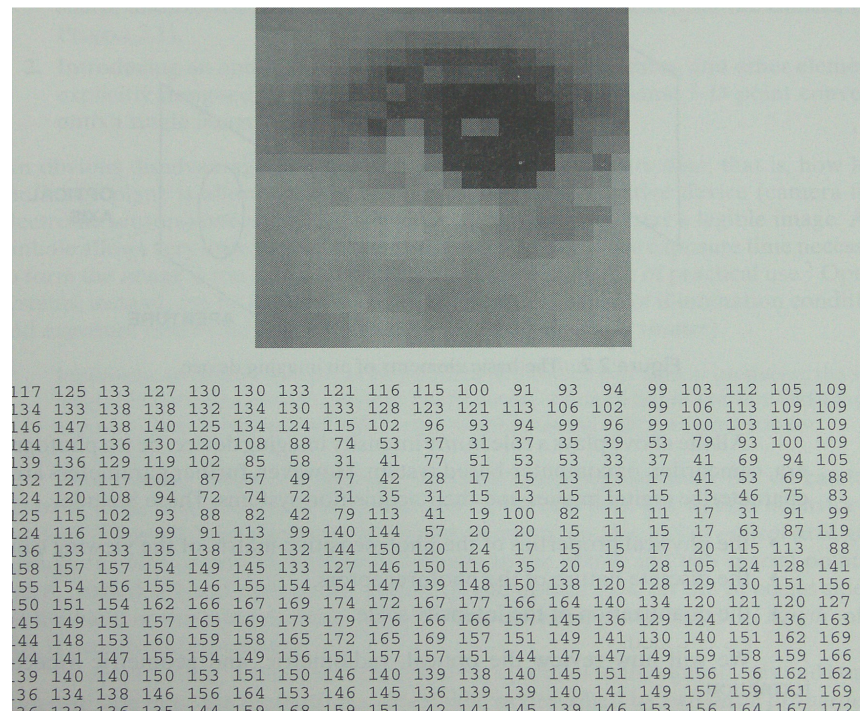
An image is a 2D matrix of numbers.

The numbers usually represent

Light intensities

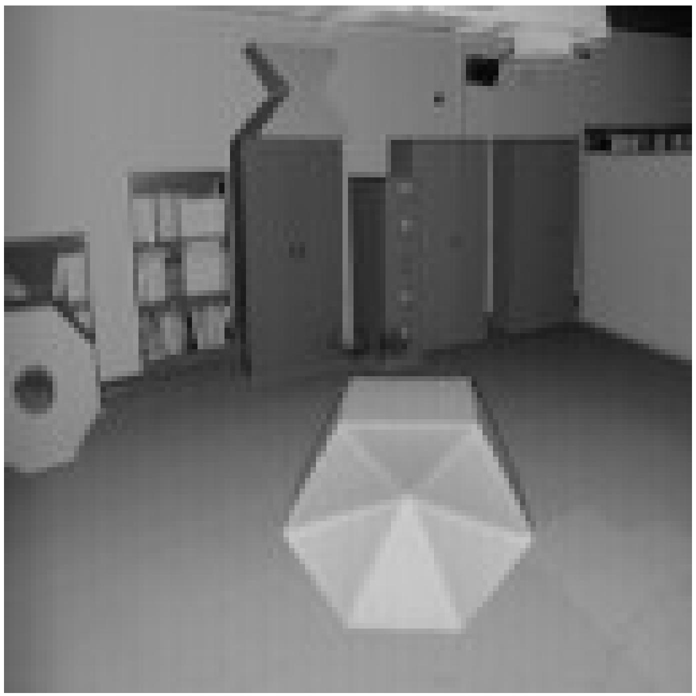
Position ranges

Physical properties

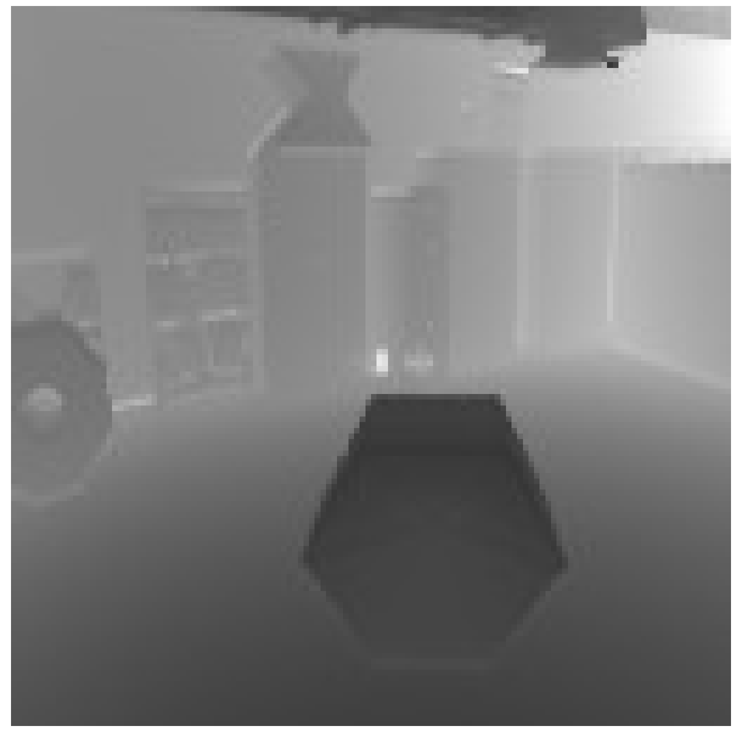


Images and Image Formation

Intensity image



Range image



Images and Image Formation

Physical property (MRI image)

Image Formation

For any computer vision task, need to know (model) how an image is formed. Why?

Three aspects of digital (intensity) image formation

- Image geometry
- Photometric parameters
- Quantization and sampling

Projection

Readings

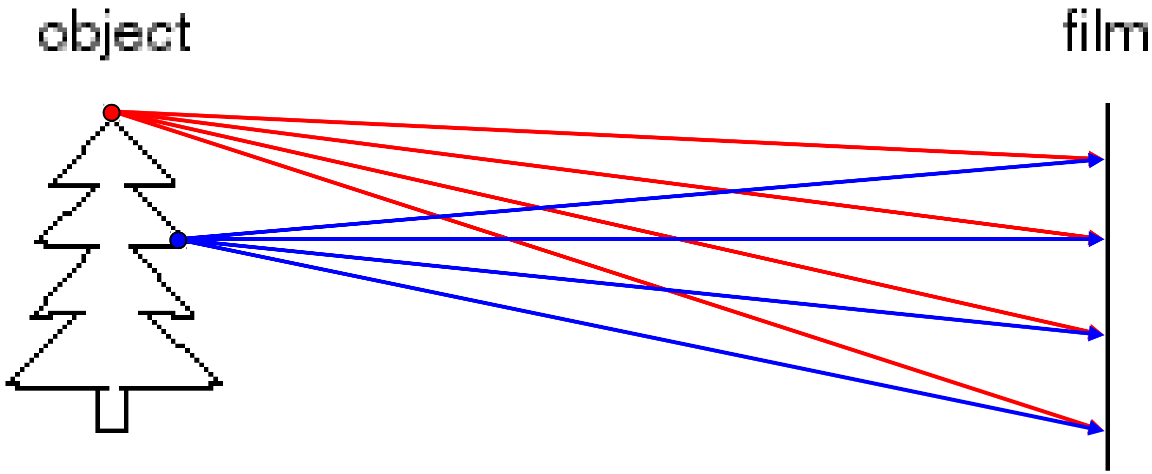
– Szeliski 2.1

Projection

Readings

– Szeliski 2.1

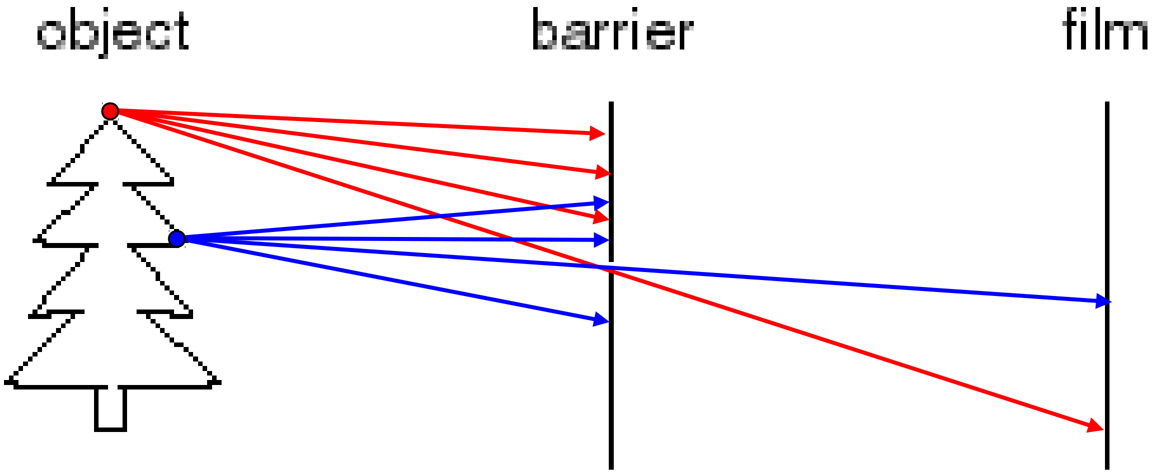
Image formation



Let's design a camera

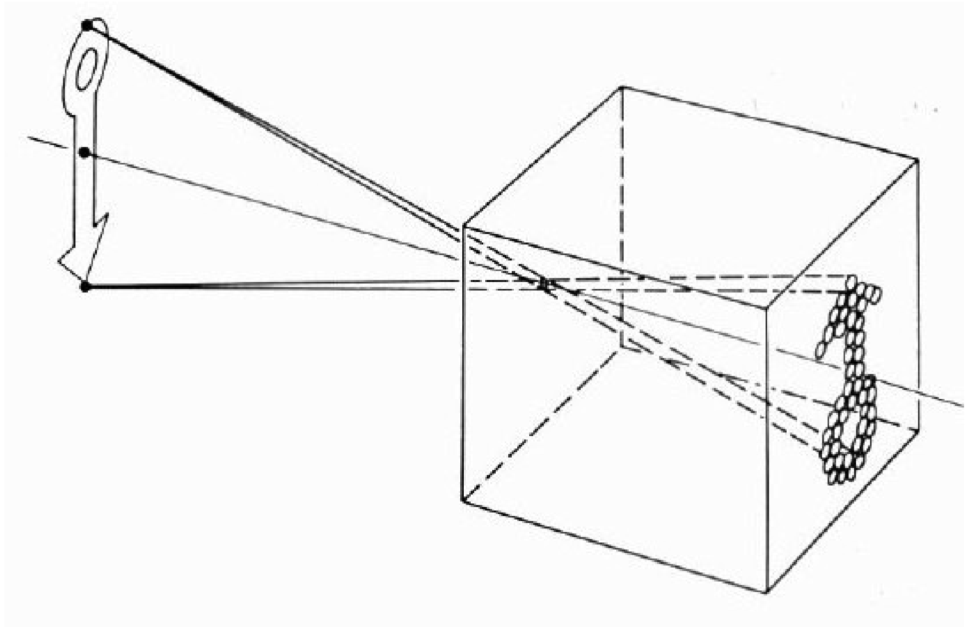
- Idea 1: put a piece of film in front of an object
- Do we get a reasonable image?

Pinhole camera



Add a barrier to block off most of the rays
- This reduces blurring
- The opening known as the **aperture**
- How does this transform the image?

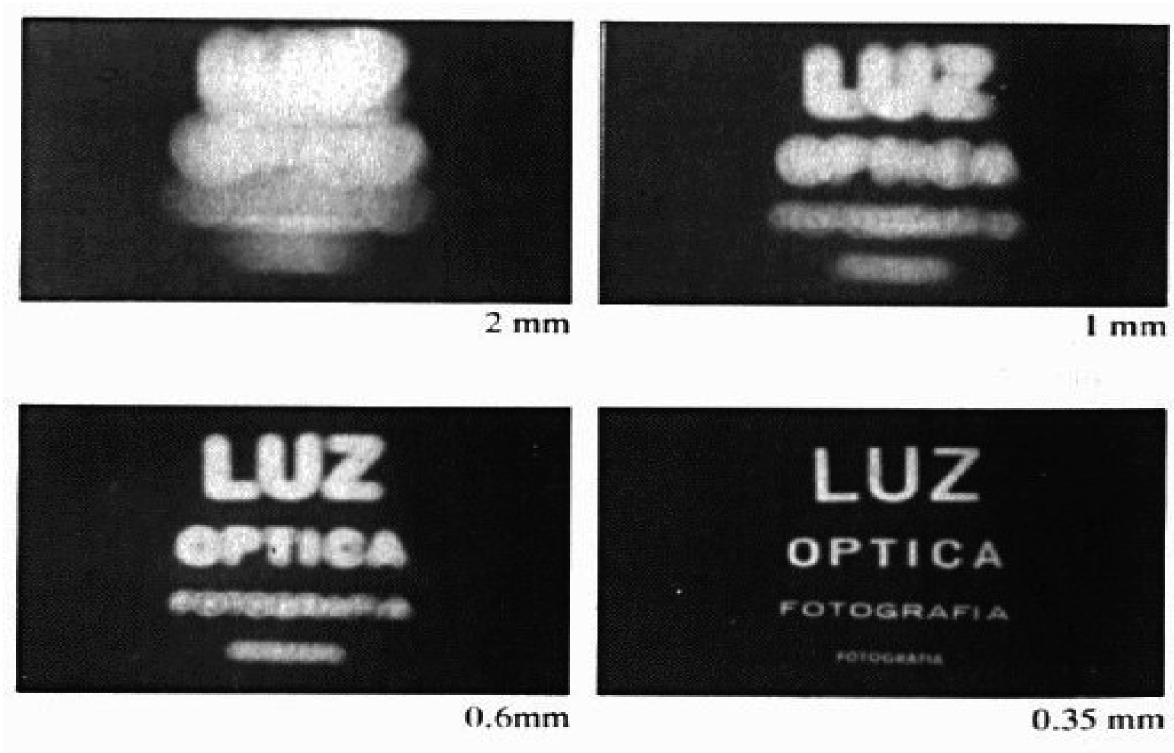
Camera Obscura



The first camera

- Known to Aristotle
- How does the aperture size affect the image?

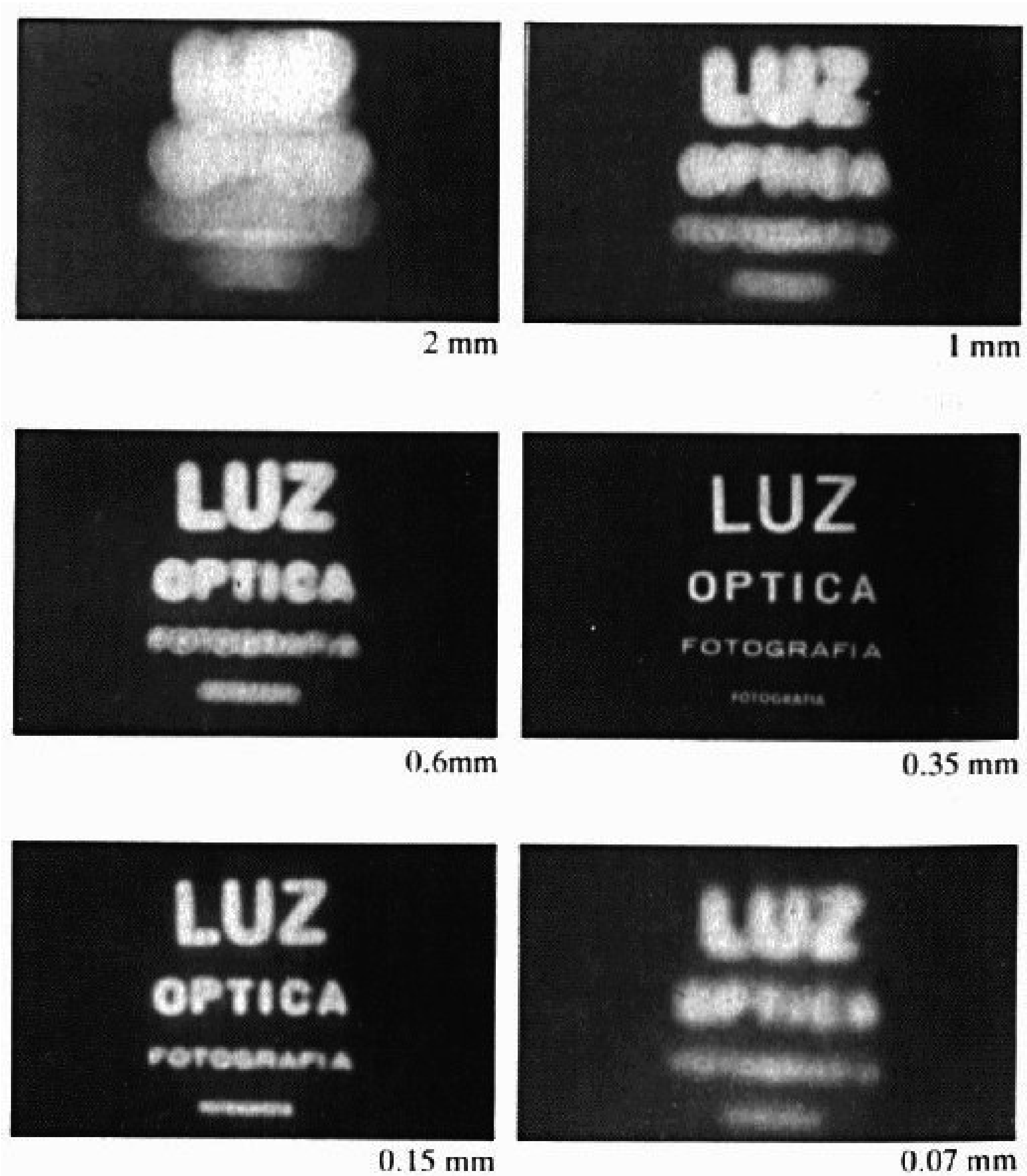
Shrinking the aperture



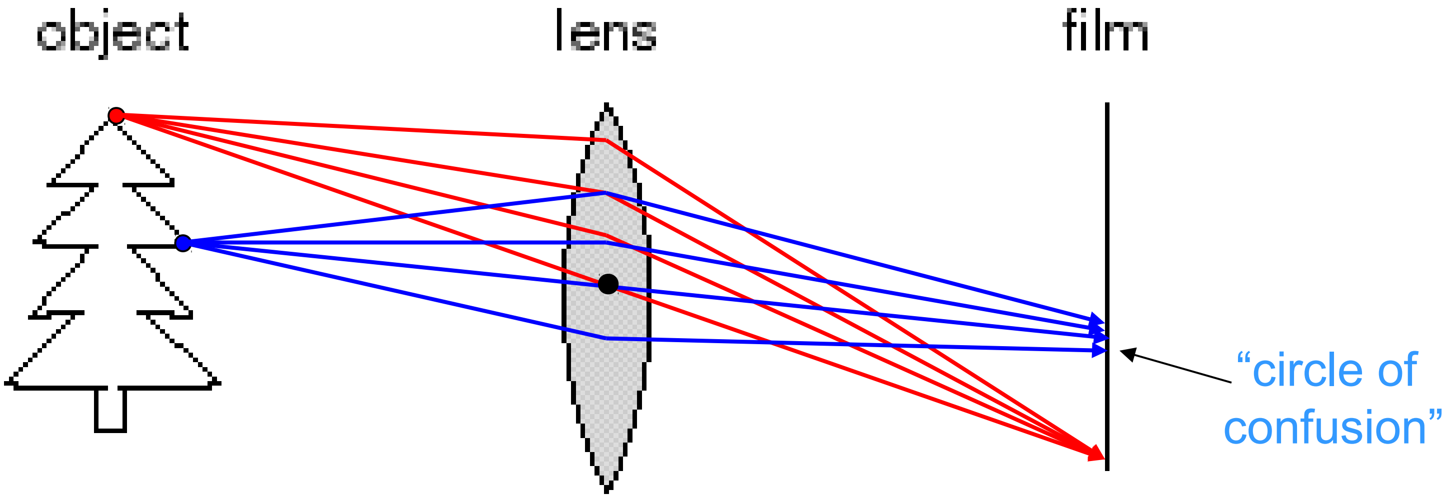
Why not make the aperture as small as possible?

- Less light gets through
- *Diffraction* effects...

Shrinking the aperture

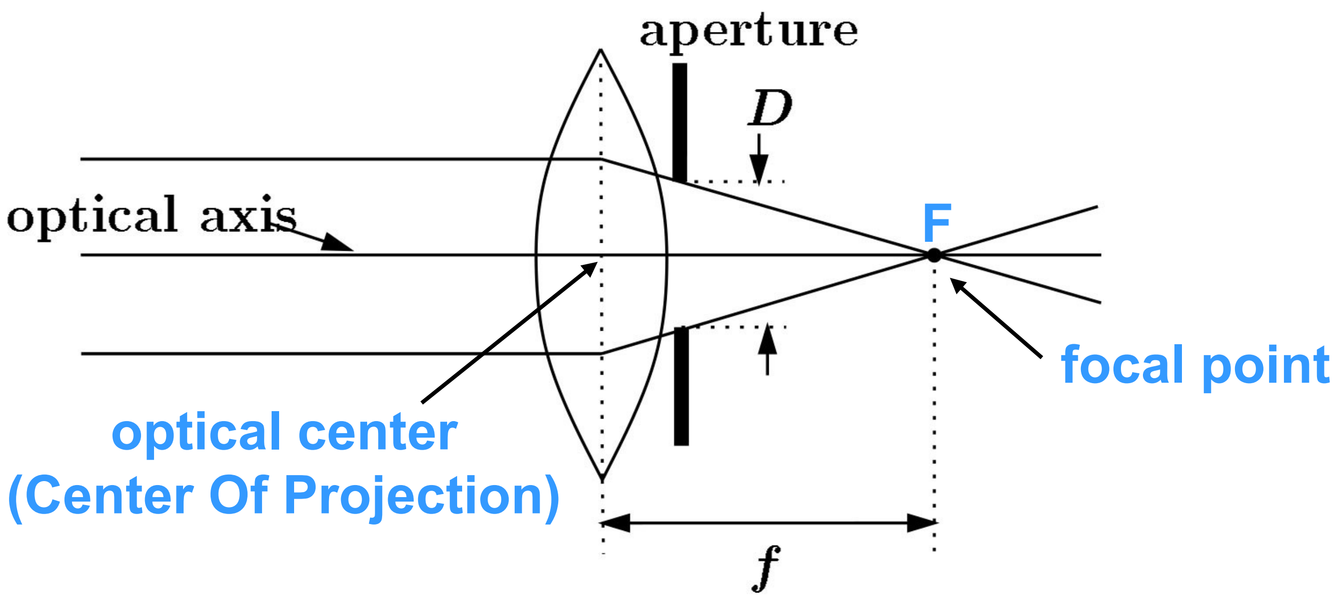


Adding a lens



- A lens focuses light onto the film
  - There is a specific distance at which objects are “in focus”
    - other points project to a “circle of confusion” in the image
  - Changing the shape of the lens changes this distance

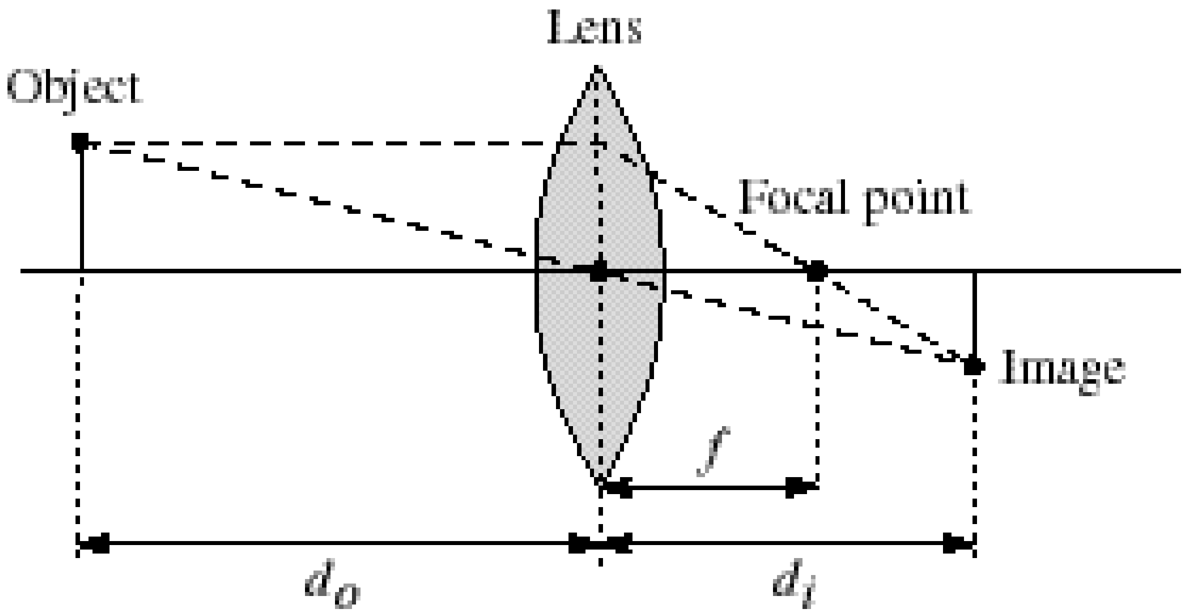
Lenses



A lens focuses parallel rays onto a single focal point

- focal point at a distance *f* beyond the plane of the lens
  - *f* is a function of the shape and index of refraction of the lens
- Aperture of diameter D restricts the range of rays
  - aperture may be on either side of the lens
- Lenses are typically spherical (easier to produce)

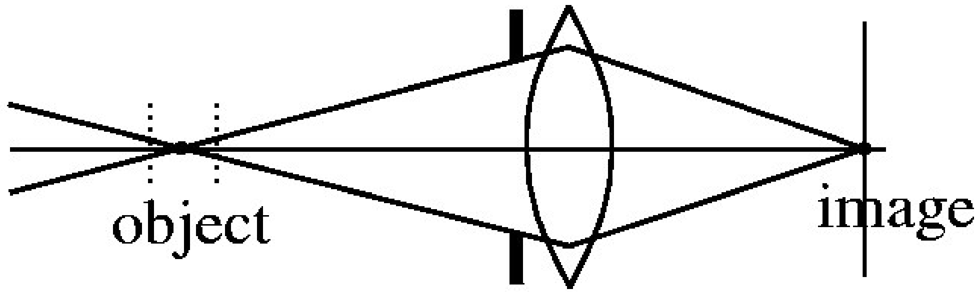
Thin lenses



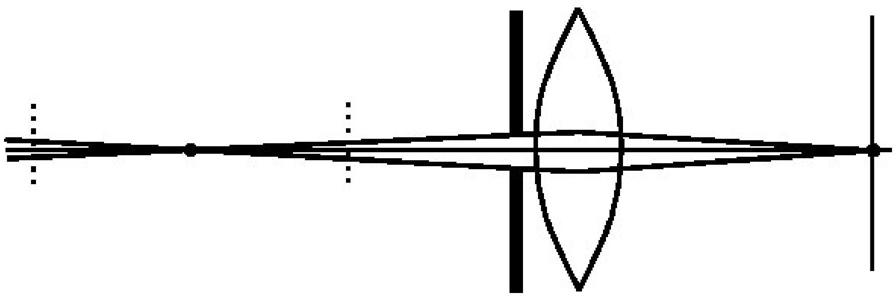
Thin lens equation: $$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

- Any object point satisfying this equation is in focus
- What is the shape of the focus region?
- How can we change the focus region?
- Thin lens applet: http://www.phy.ntnu.edu.tw/java/Lens/lens_e.html (by Fu-Kwun Hwang)

Depth of field



f / 5.6



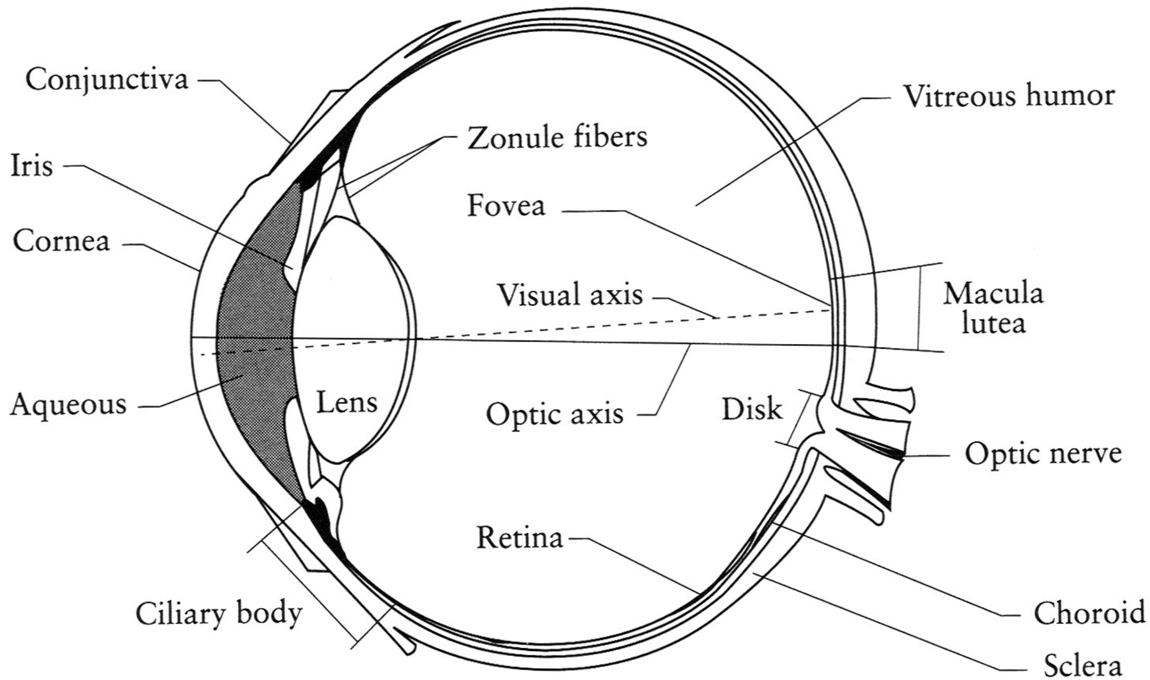
f / 32

Changing the aperture size affects depth of field

– A smaller aperture increases the range in which the object is approximately in focus

Flower images from Wikipedia http://en.wikipedia.org/wiki/Depth_of_field

The eye



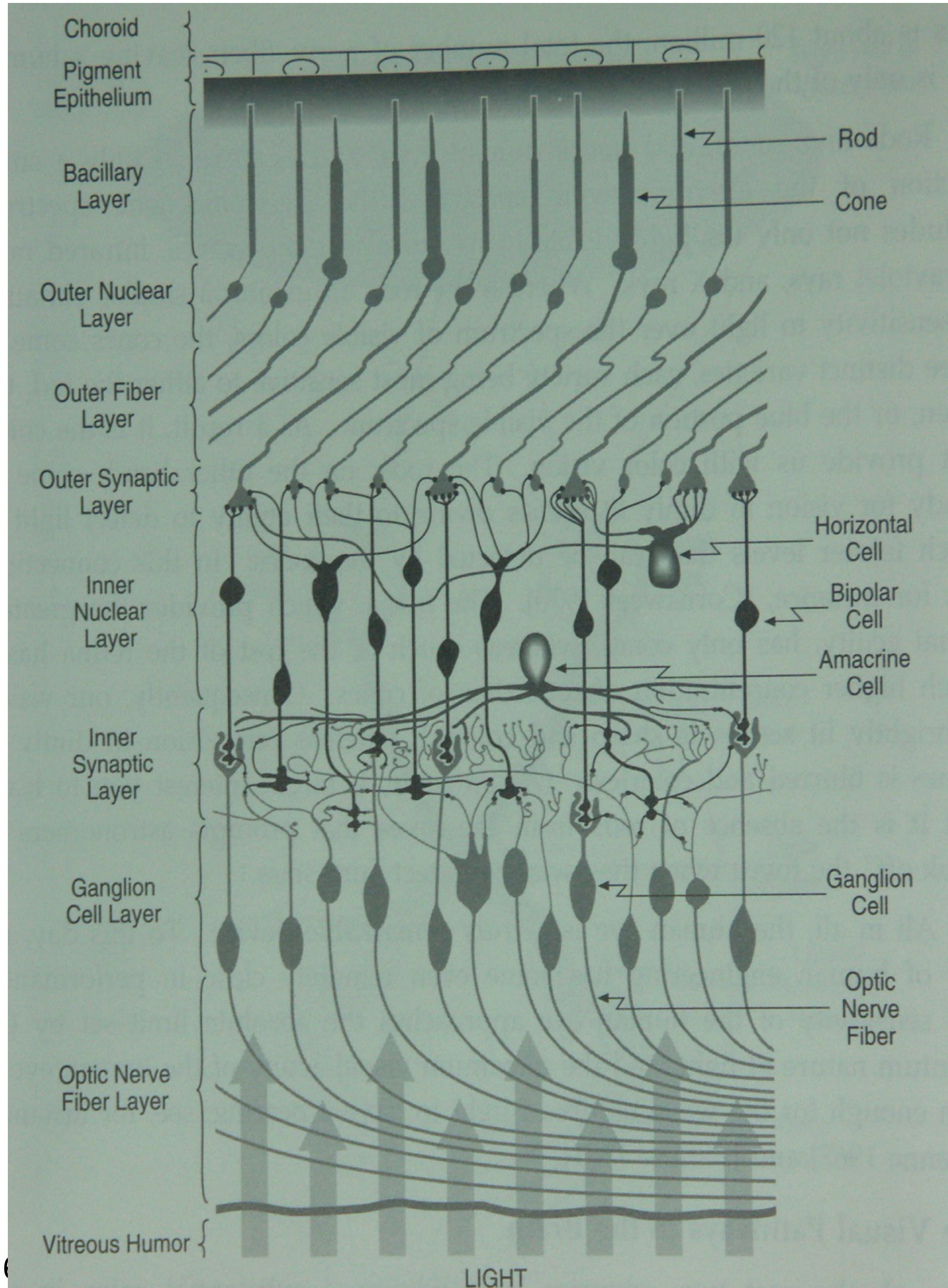
The human eye is a camera

- **Iris -** colored annulus with radial muscles
- **Pupil -** the hole (aperture) whose size is controlled by the iris
- What's the "film"?
  - photoreceptor cells (rods and cones) in the **retina**

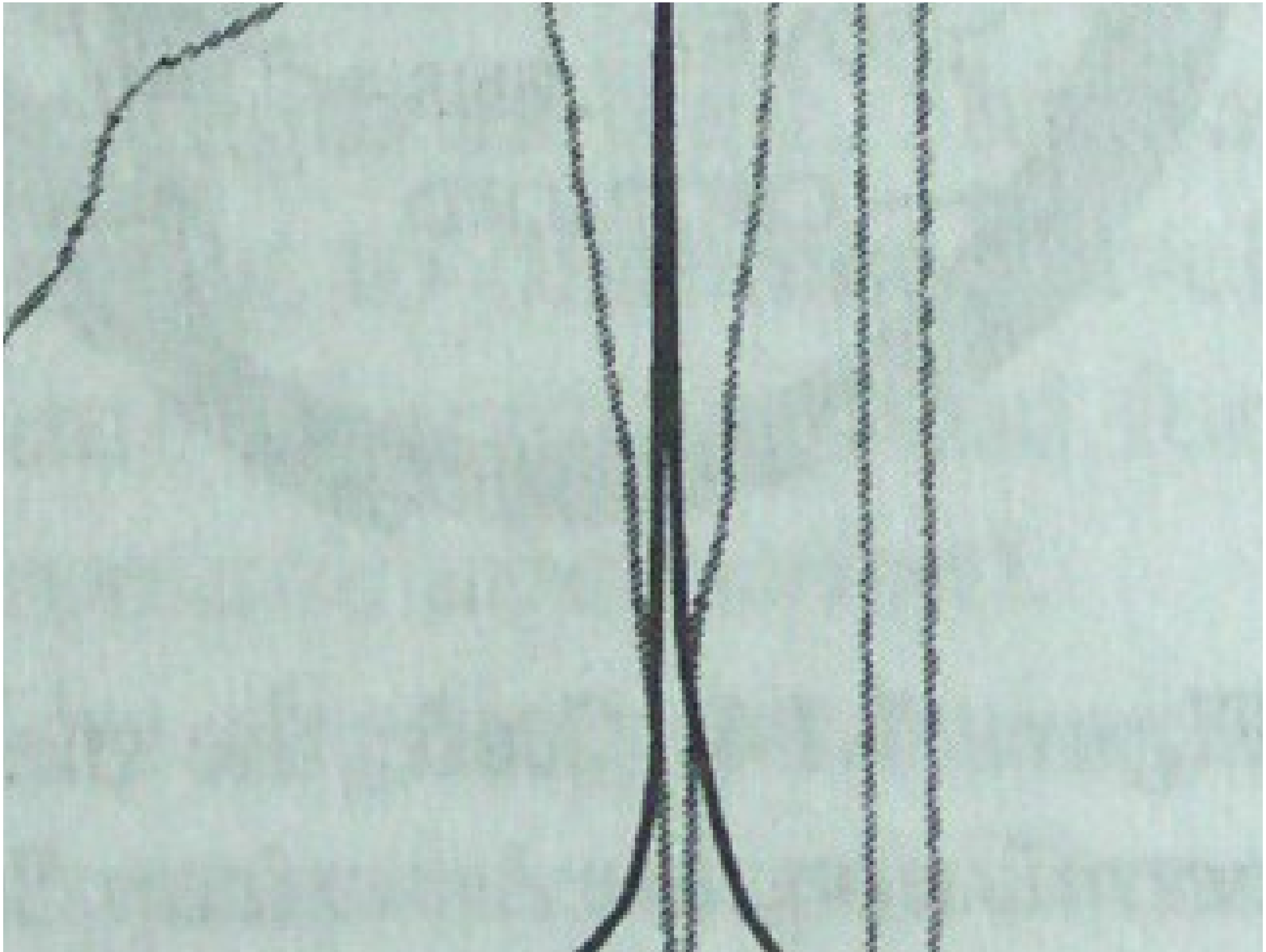
Retina

Rods and cones are photosensors.

- Cones are 3 types, red, green and blue sensitive.
- Rods are sensitive to all light spectrum. They can work in low light.
- Fovea has cones only.



Retina



Digital camera

A digital camera replaces film with a sensor array

- Each cell in the array is a **C**harge **C**oupled **D**evice
  - light-sensitive diode that converts photons to electrons
  - other variants exist: CMOS is becoming more popular
- http://electronics.howstuffworks.com/digital-camera.htm

Issues with digital cameras

Noise
- big difference between consumer vs. SLR-style cameras
- low light is where you most notice noise

Compression
- creates artifacts except in uncompressed formats (tiff, raw)

Color
- color fringing artifacts from Bayer patterns

Blooming
- charge overflowing into neighboring pixels

In-camera processing
- oversharpening can produce halos

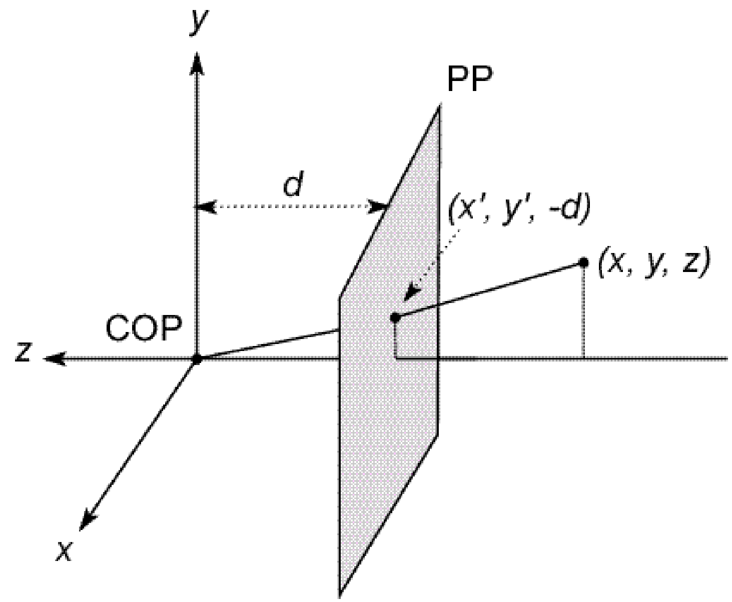
Interlaced vs. progressive scan video
- even/odd rows from different exposures

Are more megapixels better?
- requires higher quality lens
- noise issues

Stabilization
- compensate for camera shake (mechanical vs. electronic)

More info online, e.g.,
- http://electronics.howstuffworks.com/digital-camera.htm
- http://www.dpreview.com/

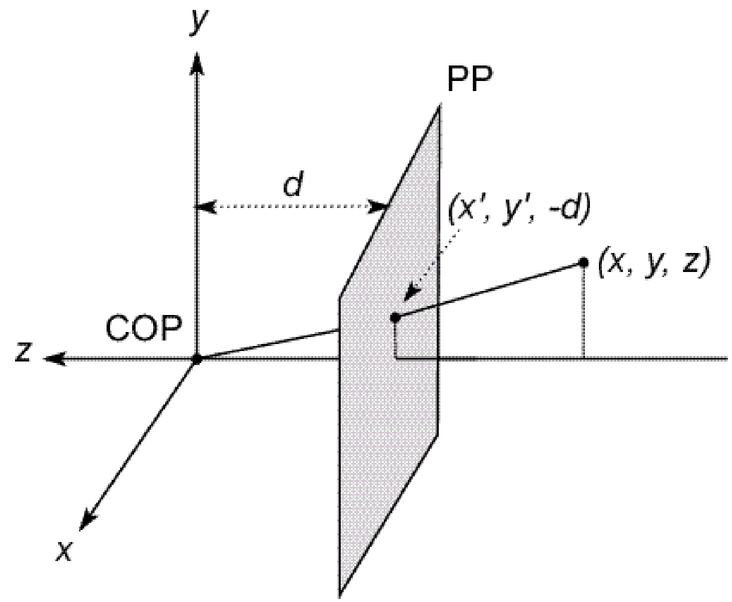
Modeling projection



The coordinate system

- We will use the pin-hole model as an approximation
- Put the optical center (**C**enter **O**f **P**rojection) at the origin
- Put the image plane (**P**rojection **P**lane) *in front* of the COP
  - Why?
- The camera looks down the *negative* z axis
  - we need this if we want right-handed-coordinates

Modeling projection



Projection equations

- Compute intersection with PP of ray from (x,y,z) to COP
- Derived using similar triangles

$$(x, y, z) \rightarrow (-d\frac{x}{z},\ -d\frac{y}{z},\ -d)$$

- We get the projection by throwing out the last coordinate:

$$(x, y, z) \rightarrow (-d\frac{x}{z},\ -d\frac{y}{z})$$

Homogeneous coordinates

Is this a linear transformation?

- no—division by z is nonlinear

Trick: add one more coordinate:

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

homogeneous image coordinates

$$(x, y, z) \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

homogeneous scene coordinates

Converting *from* homogeneous coordinates

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w)$$

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \Rightarrow (x/w, y/w, z/w)$$

Perspective Projection

Projection is a matrix multiply using homogeneous coordinates:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1/d & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ -z/d \end{bmatrix} \Rightarrow (-d\frac{x}{z}, \quad -d\frac{y}{z})$$

divide by third coordinate

This is known as **perspective projection**

- The matrix is the **projection matrix**
- Can also formulate as a 4x4 (today's reading does this)

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1/d & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ -z/d \end{bmatrix} \Rightarrow (-d\frac{x}{z}, \quad -d\frac{y}{z})$$

divide by fourth coordinate

Perspective Projection

How does scaling the projection matrix change the transformation?

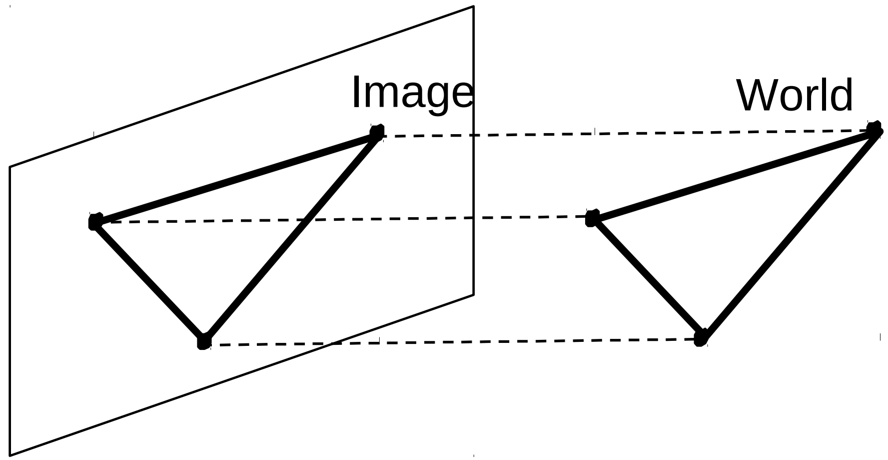
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1/d & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ -z/d \end{bmatrix} \Rightarrow (-d\frac{x}{z}, \quad -d\frac{y}{z})$$

$$\begin{bmatrix} -d & 0 & 0 & 0 \\ 0 & -d & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} -dx \\ -dy \\ z \end{bmatrix} \Rightarrow (-d\frac{x}{z}, \quad -d\frac{y}{z})$$

Orthographic projection

Special case of perspective projection

- Distance from the COP to the PP is infinite

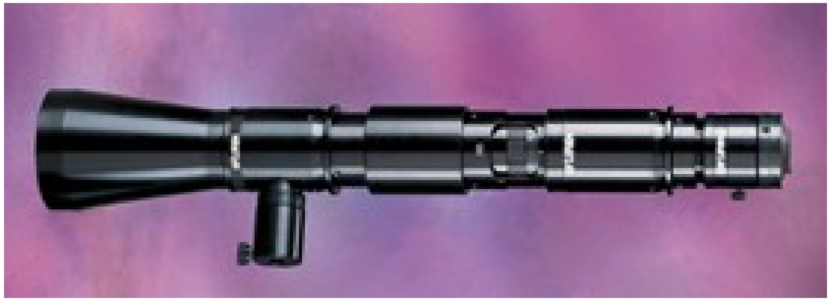


Good approximation for telephoto optics

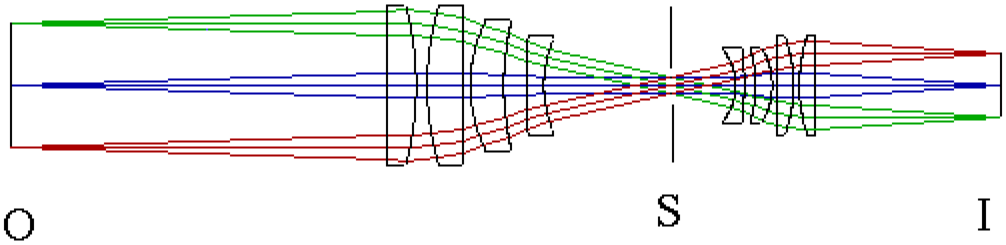
- Also called “parallel projection”: (x, y, z) → (x, y)
- Wh n ?

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \Rightarrow (x, y)$$

Orthographic ("telecentric") lenses



Navitar telecentric zoom lens



http://www.lhup.edu/~dsimanek/3d/telecent.htm

Variants of orthographic projection

Scaled orthographic

– Also called "weak perspective"

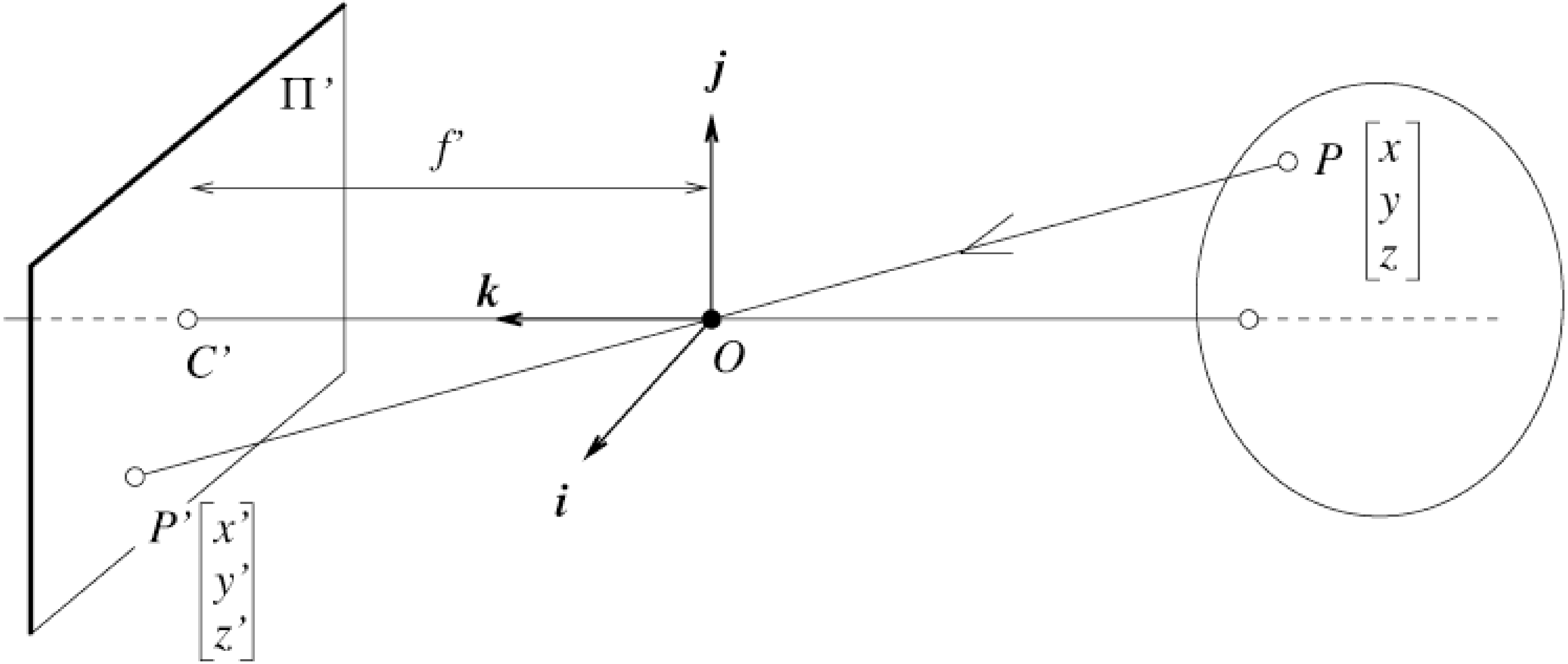
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1/d \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1/d \end{bmatrix} \Rightarrow (dx, dy)$$

Affine projection

– Also called "paraperspective"

$$\begin{bmatrix} a & b & c & d \\ e & f & g & h \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

The equation of projection



$$\begin{cases} x' = \lambda x \\ y' = \lambda y \\ f' = \lambda z \end{cases} \iff \lambda = \frac{x'}{x} = \frac{y'}{y} = \frac{f'}{z},$$

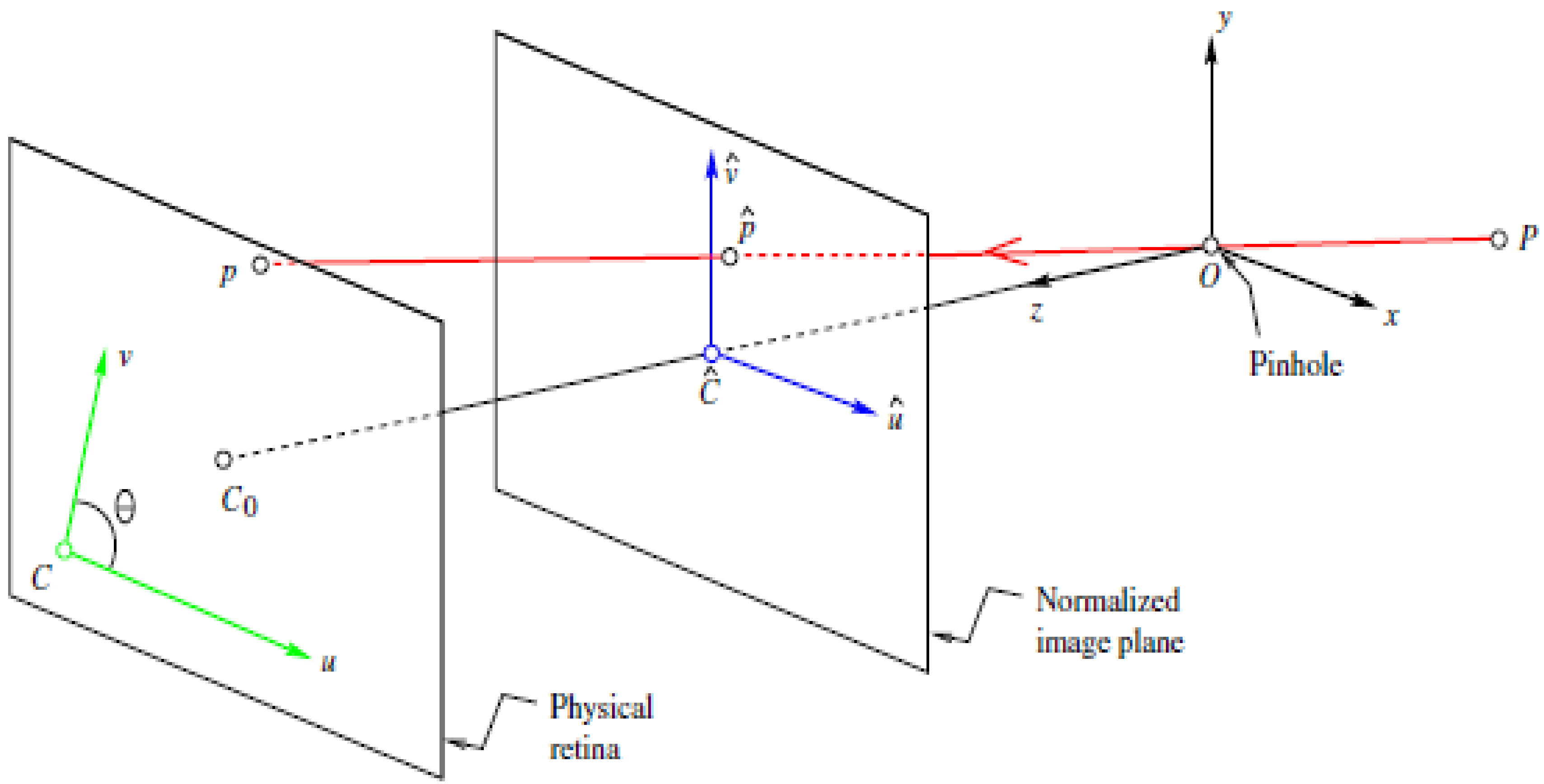
$$\begin{cases} x' = f' \dfrac{x}{z}, \\ y' = f' \dfrac{y}{z}. \end{cases}$$

Homogeneous coordinates

- Add an extra coordinate and use an equivalence relation
- for 2D
  - equivalence relation k*(X,Y,Z) is the same as (X,Y,Z)
- for 3D
  - equivalence relation k*(X,Y,Z,T) is the same as (X,Y,Z,T)

- Basic notion
  - Possible to represent points "at infinity"
    - Where parallel lines intersect
    - Where parallel planes intersect
  - Possible to write the action of a perspective camera as a matrix

The camera parameters



Intrinsic Camera Parameters

$$\begin{cases} \hat{u} = \dfrac{x}{z} \\ \hat{v} = \dfrac{y}{z} \end{cases} \iff \hat{\boldsymbol{p}} = \frac{1}{z} \begin{pmatrix} \mathrm{Id} & \mathbf{0} \end{pmatrix} \begin{pmatrix} \boldsymbol{P} \\ 1 \end{pmatrix},$$

$$\begin{cases} u = k f \dfrac{x}{z}, \\ v = l f \dfrac{y}{z}. \end{cases}$$

The parameters k, l and f are not independent, and they can be replaced by the magnifications $\alpha = kf$ and $\beta = lf$ expressed in pixel units.

$$\begin{cases} u = \alpha \dfrac{x}{z} + u_0, \\ v = \beta \dfrac{y}{z} + v_0. \end{cases}$$

$$\begin{cases} u = \alpha \dfrac{x}{z} - \alpha \cot\theta \dfrac{y}{z} + u_0, \\ v = \dfrac{\beta}{\sin\theta} \dfrac{y}{z} + v_0. \end{cases}$$

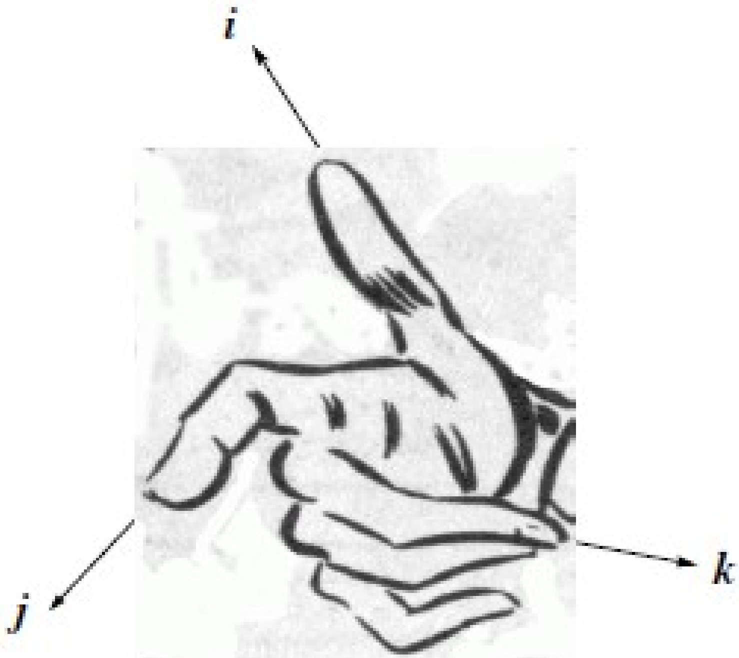
Intrinsic Camera Params

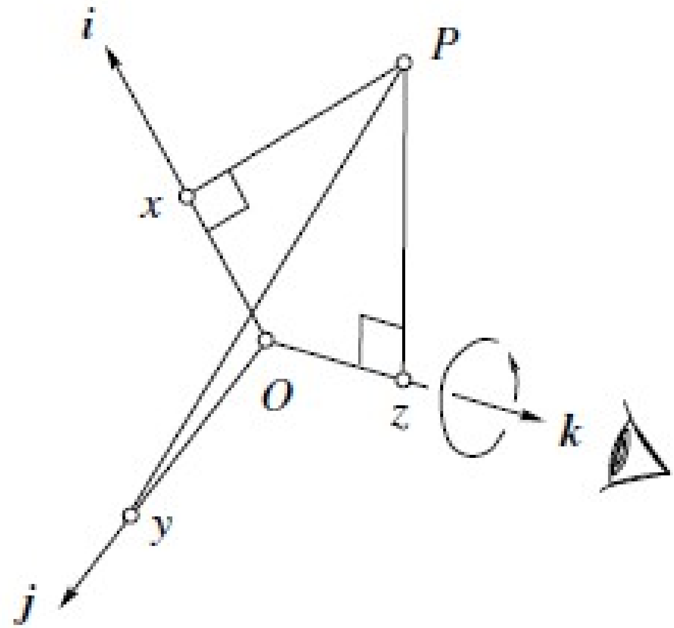
$$\boldsymbol{p} = \mathcal{K}\hat{\boldsymbol{p}}, \quad \text{where} \quad \boldsymbol{p} = \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} \quad \text{and} \quad \mathcal{K} \stackrel{\text{def}}{=} \begin{pmatrix} \alpha & -\alpha \cot\theta & u_0 \\ 0 & \dfrac{\beta}{\sin\theta} & v_0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Putting it all together, we obtain

$$\boldsymbol{p} = \frac{1}{z}\mathcal{M}\boldsymbol{P}, \quad \text{where} \quad \mathcal{M} \stackrel{\text{def}}{=} \begin{pmatrix} \mathcal{K} & 0 \end{pmatrix}$$

Coordinate System Changes and Rigid Transformations





$$^{F}P = {}^{F}\overrightarrow{OP} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \iff \overrightarrow{OP} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}.$$

The coordinate vector of the point P (resp. vector v) in the frame (F)

Pure Translation

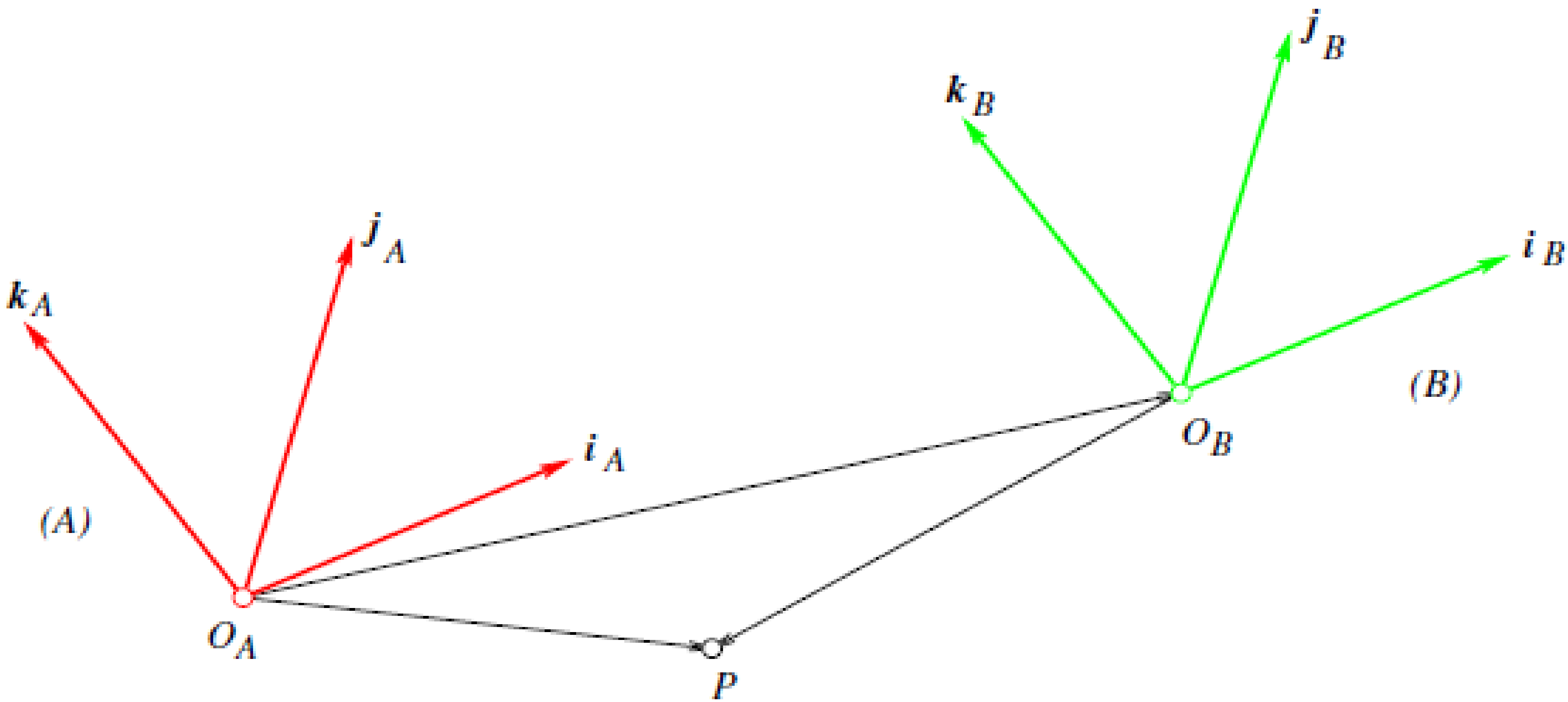
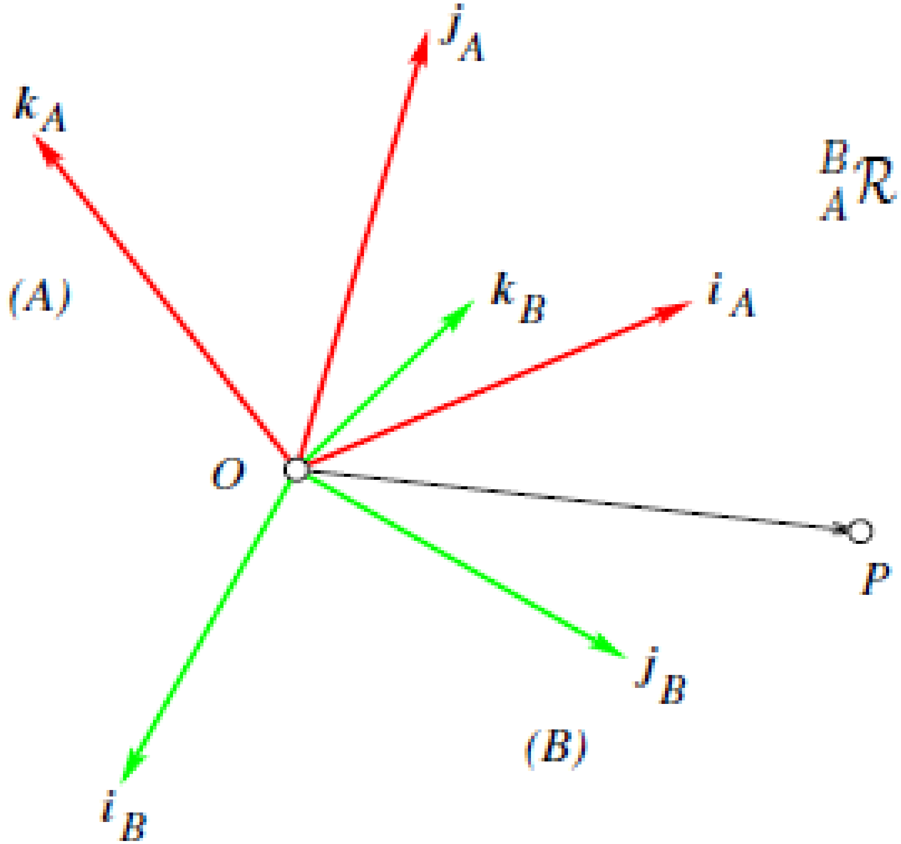


Figure 5.4. Change of coordinates between two frames: pure translation.

$$^{B}P = {}^{A}P + {}^{B}O_A.$$

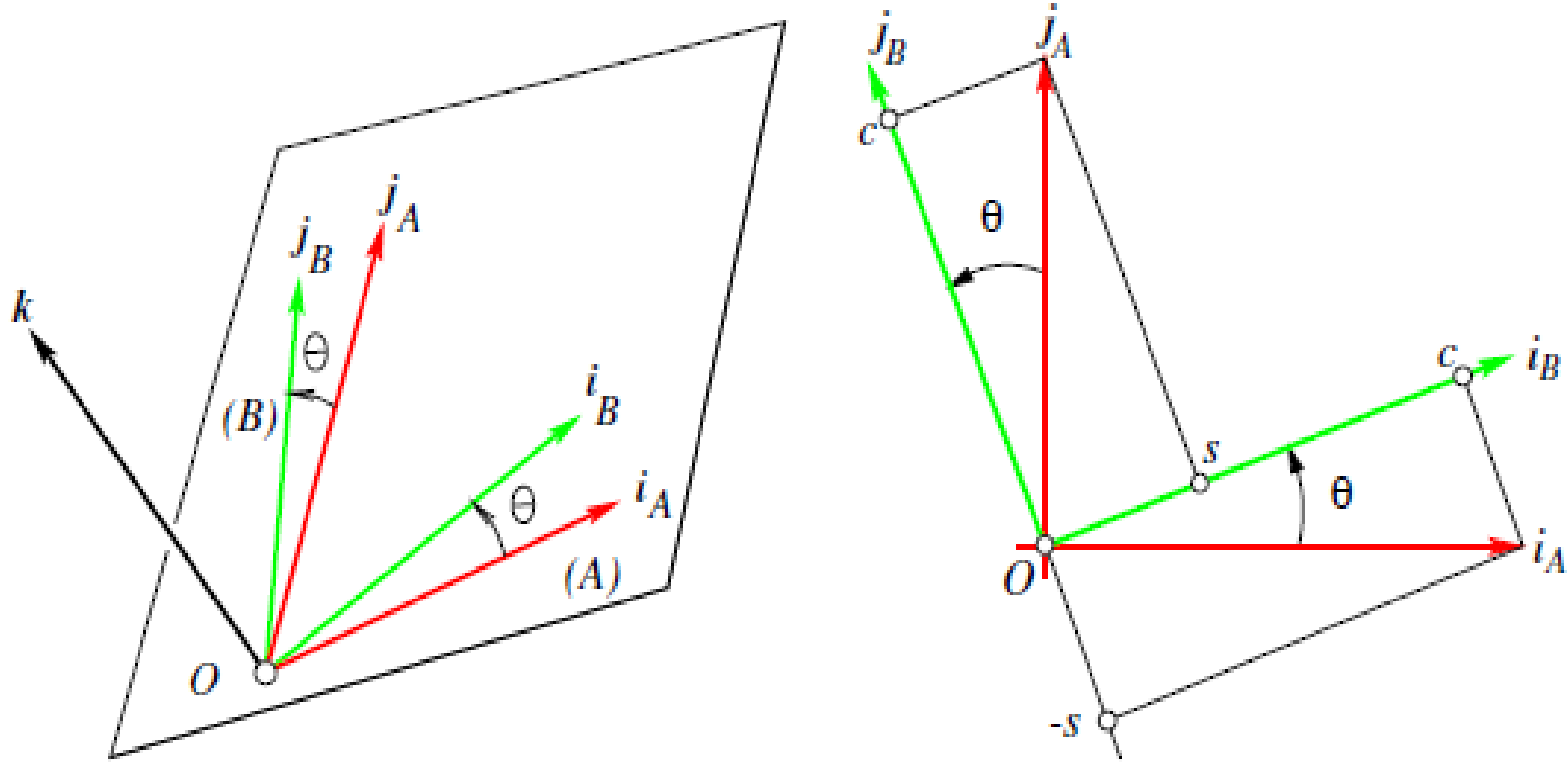
Pure Rotation



$$ {}^{B}_{A}\mathcal{R} \stackrel{\text{def}}{=} \begin{pmatrix} \boldsymbol{i}_A \cdot \boldsymbol{i}_B & \boldsymbol{j}_A \cdot \boldsymbol{i}_B & \boldsymbol{k}_A \cdot \boldsymbol{i}_B \\ \boldsymbol{i}_A \cdot \boldsymbol{j}_B & \boldsymbol{j}_A \cdot \boldsymbol{j}_B & \boldsymbol{k}_A \cdot \boldsymbol{j}_B \\ \boldsymbol{i}_A \cdot \boldsymbol{k}_B & \boldsymbol{j}_A \cdot \boldsymbol{k}_B & \boldsymbol{k}_A \cdot \boldsymbol{k}_B \end{pmatrix}. $$

$$ {}^{B}P = {}^{B}_{A}\mathcal{R}\,{}^{A}P $$

$$ {}^{A}_{B}\mathcal{R} = {}^{B}_{A}\mathcal{R}^{T}. $$



$$ {}^{B}_{A}\mathcal{R} = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}. $$

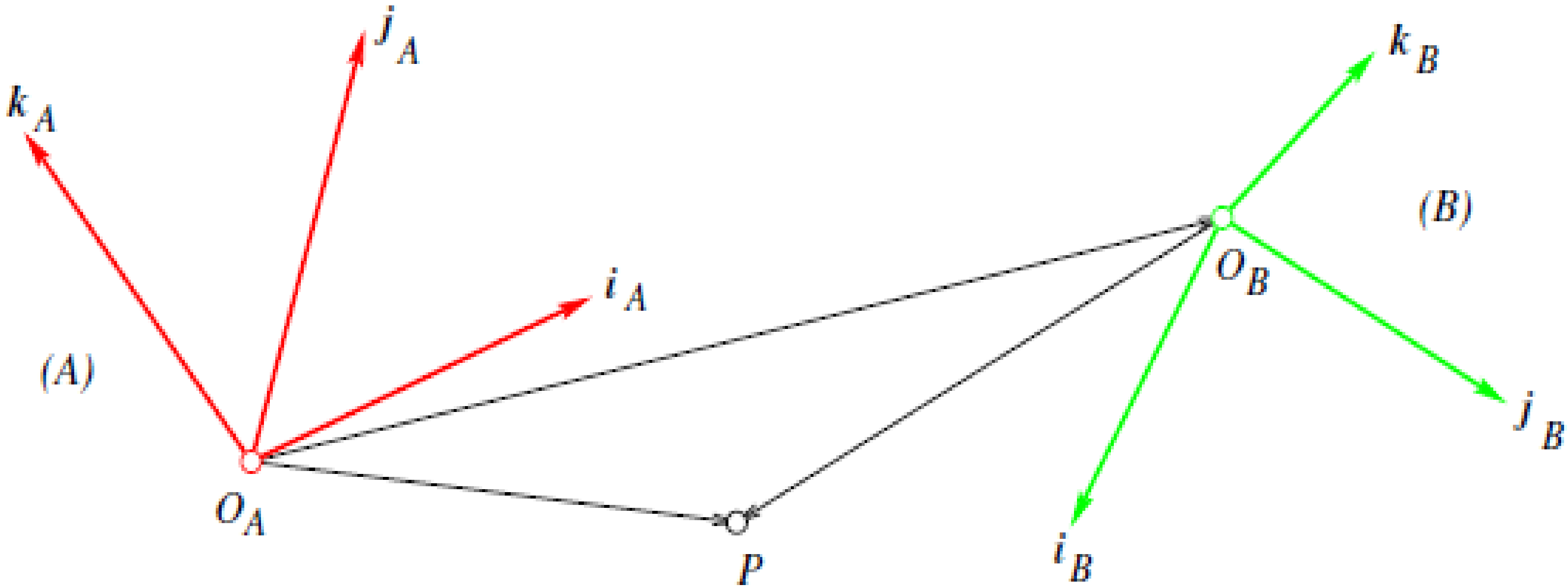


Figure 5.7. Change of coordinates between two frames: general rigid transformation.

$$^{B}P = {}^{B}_{A}\mathcal{R}\,{}^{A}P + {}^{B}O_{A},$$

Homogeneous Coords.

$$\mathcal{A} = \begin{pmatrix} \mathcal{A}_{11} & \mathcal{A}_{12} \\ \mathcal{A}_{21} & \mathcal{A}_{22} \end{pmatrix} \quad \text{and} \quad \mathcal{B} = \begin{pmatrix} \mathcal{B}_{11} & \mathcal{B}_{12} \\ \mathcal{B}_{21} & \mathcal{B}_{22} \end{pmatrix},$$

$$\mathcal{A}\mathcal{B} = \begin{pmatrix} \mathcal{A}_{11}\mathcal{B}_{11} + \mathcal{A}_{12}\mathcal{B}_{21} & \mathcal{A}_{11}\mathcal{B}_{12} + \mathcal{A}_{12}\mathcal{B}_{22} \\ \mathcal{A}_{21}\mathcal{B}_{11} + \mathcal{A}_{22}\mathcal{B}_{21} & \mathcal{A}_{21}\mathcal{B}_{12} + \mathcal{A}_{22}\mathcal{B}_{22} \end{pmatrix}.$$

$$\begin{pmatrix} {}^{B}P \\ 1 \end{pmatrix} = {}^{B}_{A}\mathcal{T} \begin{pmatrix} {}^{A}P \\ 1 \end{pmatrix}, \quad \text{where} \quad {}^{B}_{A}\mathcal{T} \stackrel{\text{def}}{=} \begin{pmatrix} {}^{B}_{A}\mathcal{R} & {}^{B}O_{A} \\ 0^{T} & 1 \end{pmatrix}$$

$${}^{F}P' = \mathcal{R}\,{}^{F}P + \boldsymbol{t} \Longleftrightarrow \begin{pmatrix} {}^{F}P' \\ 1 \end{pmatrix} = \begin{pmatrix} \mathcal{R} & \boldsymbol{t} \\ 0^{T} & 1 \end{pmatrix} \begin{pmatrix} {}^{F}P \\ 1 \end{pmatrix},$$

Extrinsic Camera Params

Let us now consider the case where the camera frame (C) is distinct from the world frame (W). Noting that

$$^{C}P = \left({}^{C}_{W}\mathcal{R} \quad {}^{C}O_{W} \right) \begin{pmatrix} {}^{W}P \\ 1 \end{pmatrix}$$

and substituting in (5.2.6) yields

$$\boldsymbol{p} = \frac{1}{z}\mathcal{M}\boldsymbol{P}, \quad \text{where} \quad \mathcal{M} = \mathcal{K} \left(\mathcal{R} \quad \boldsymbol{t} \right), \tag{5.2.7}$$

$\mathcal{R} = {}^{C}_{W}\mathcal{R}$ is a rotation matrix, $\boldsymbol{t} = {}^{C}O_{W}$ is a translation vector, and $\boldsymbol{P} = {}^{W}P$ denotes the homogeneous coordinate vector of P in the frame (W).

The matrix $\mathcal{M}$ can be rewritten explicitly as a function of the intrinsic and extrinsic parameters of the camera, namely

$$\mathcal{M} = \begin{pmatrix} \alpha \boldsymbol{r}_1^T - \alpha \cot\theta \boldsymbol{r}_2^T + u_0 \boldsymbol{r}_3^T & \alpha t_x - \alpha \cot\theta t_y + u_0 t_z \\ \dfrac{\beta}{\sin\theta} \boldsymbol{r}_2^T + v_0 \boldsymbol{r}_3^T & \dfrac{\beta}{\sin\theta} t_y + v_0 t_z \\ \boldsymbol{r}_3^T & t_z \end{pmatrix}, \tag{5.2.9}$$

where $\boldsymbol{r}_1^T$, $\boldsymbol{r}_2^T$ and $\boldsymbol{r}_3^T$ denote the three rows of the matrix $\mathcal{R}$ and t_x, t_y and t_z are the coordinates of the vector $\boldsymbol{t}$. If $\mathcal{R}$ is written as the product of three elementary rotations, the vectors $\boldsymbol{r}_i$ $(i = 1, 2, 3)$ can of course be written explicitly in terms of the corresponding three angles.

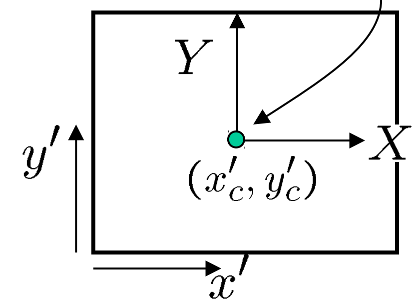
Camera parameters

A camera is described by several parameters

- Translation T of the optical center from the origin of world coords
- Rotation R of the image plane
- focal length f, principle point (x'_c, y'_c), pixel size (s_x, s_y)
- blue parameters are called "extrinsics," red are "intrinsics"

Projection equation

$$\mathbf{x} = \begin{bmatrix} sx \\ sy \\ s \end{bmatrix} = \begin{bmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \mathbf{\Pi X}$$



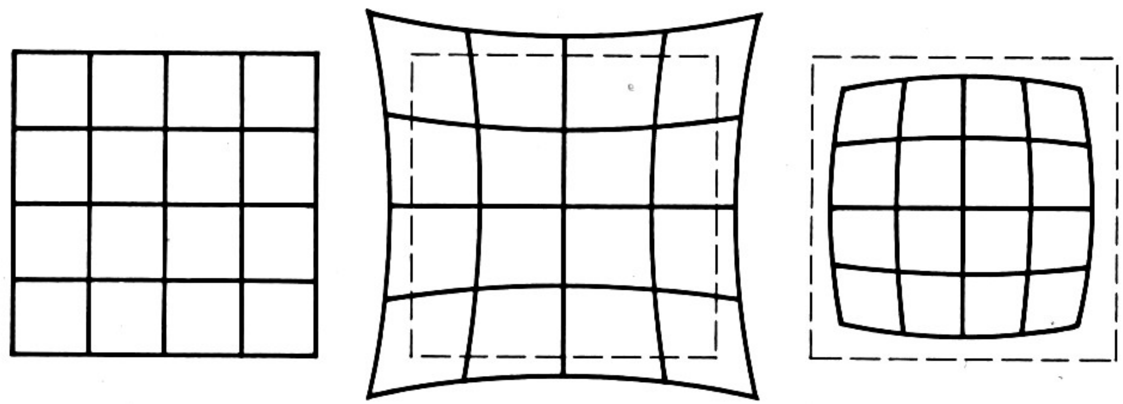
- The projection matrix models the cumulative effect of all parameters
- Useful to decompose into a series of operations

identity matrix

$$\mathbf{\Pi} = \underbrace{\begin{bmatrix} -fs_x & 0 & x'_c \\ 0 & -fs_y & y'_c \\ 0 & 0 & 1 \end{bmatrix}}_{\text{intrinsics}} \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{\text{projection}} \underbrace{\begin{bmatrix} \mathbf{R}_{3x3} & \mathbf{0}_{3x1} \\ \mathbf{0}_{1x3} & 1 \end{bmatrix}}_{\text{rotation}} \underbrace{\begin{bmatrix} \mathbf{I}_{3x3} & \mathbf{T}_{3x1} \\ \mathbf{0}_{1x3} & 1 \end{bmatrix}}_{\text{translation}}$$

- The definitions of these parameters are **not** completely standardized
  - especially intrinsics—varies from one book to another

Distortion



No distortion Pin cushion Barrel

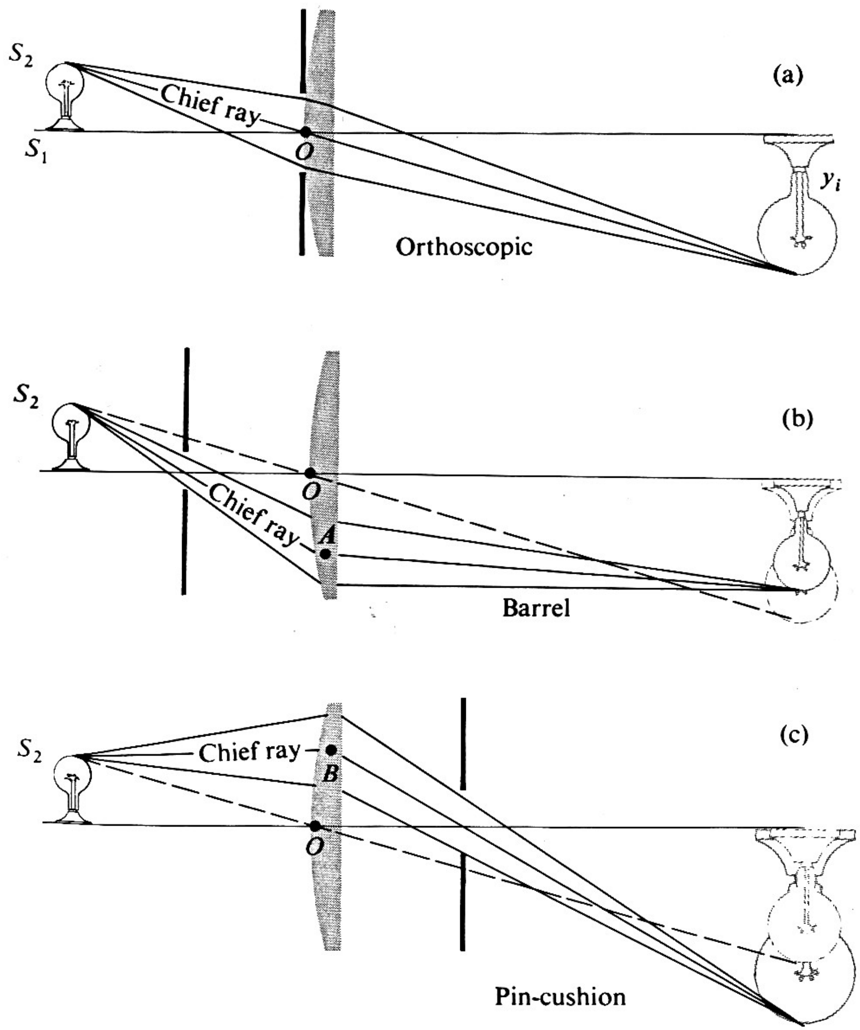
Radial distortion of the image

- Caused by imperfect lenses
- Deviations are most noticeable for rays that pass through the edge of the lens

Correcting radial distortion

from Helmut Dersch

Distortion



S_2
(a)
Chief ray
S_1
O
y_i
Orthoscopic
S_2
(b)
O
Chief ray
A
Barrel
(c)
S_2
Chief ray
B
O
Pin-cushion

Modeling distortion

Project $(\widehat{x}, \widehat{y}, \widehat{z})$ to "normalized" image coordinates

$$
\begin{aligned}
x'_n &= \widehat{x}/\widehat{z} \\
y'_n &= \widehat{y}/\widehat{z}
\end{aligned}
$$

Apply radial distortion

$$
\begin{aligned}
r^2 &= {x'_n}^2 + {y'_n}^2 \\
x'_d &= x'_n(1 + \kappa_1 r^2 + \kappa_2 r^4) \\
y'_d &= y'_n(1 + \kappa_1 r^2 + \kappa_2 r^4)
\end{aligned}
$$

Apply focal length translate image center

$$
\begin{aligned}
x' &= f x'_d + x_c \\
y' &= f y'_d + y_c
\end{aligned}
$$

To model lens distortion

- Use above projection operation instead of standard projection matrix multiplication

Other types of projection

Lots of intriguing variants…

(I’ll just mention a few fun ones)

360 degree field of view…

Basic approach

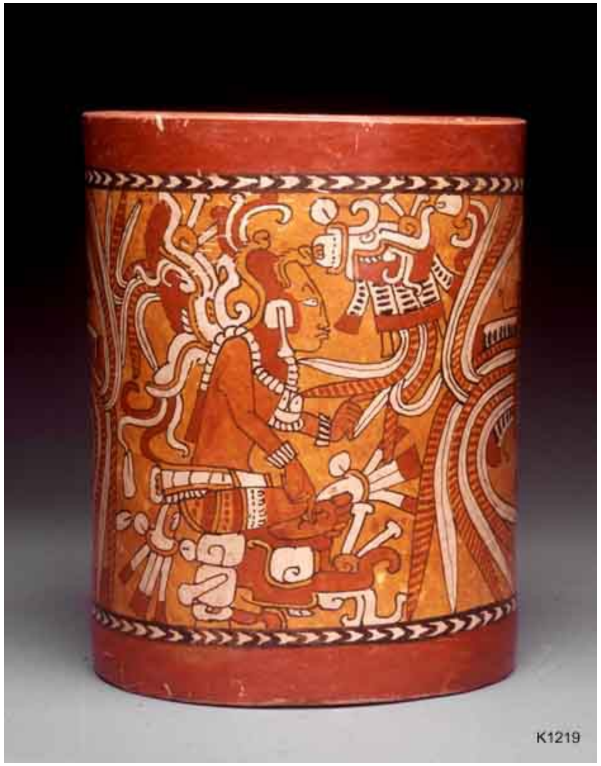
- Take a photo of a parabolic mirror with an orthographic lens (Nayar)
- Or buy one a lens from a variety of omnicam manufacturers…
  - See http://www.cis.upenn.edu/~kostas/omni.html

Tilt-shift

http://www.northlight-images.co.uk/article_pages/tilt_and_shift_ts-e.html

Titlt-shift images from Olivo Barbieri
and Photoshop imitations

Rotating sensor (or object)

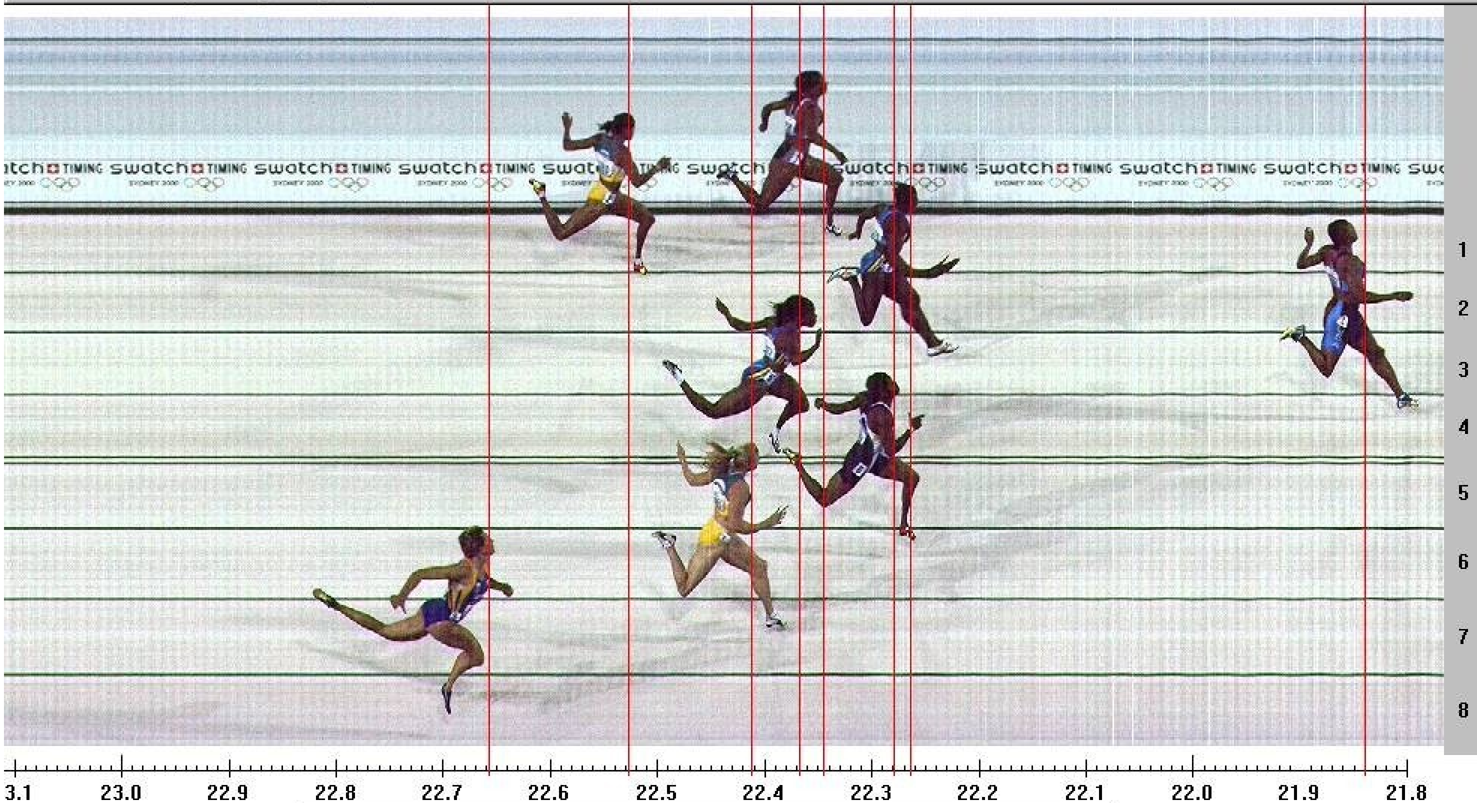


Rollout Photographs © Justin Kerr
http://research.famsi.org/kerrmaya.html

Also known as “cyclographs”, “peripheral images”

Photofinish

The 2000 Sydney Olympic Games - 200m Women Final



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3.	6	3058	Jayasinghe Susanthika	SRI	22.28	0.207
4.	1	2291	McDonald Beverly	JAM	22.35	0.151
5.	5	1178	Ferguson Debbie	BAH	22.37	0.196
6.	7	1111	Gainsford-Taylo Melinda	AUS	22.42	0.178
7.	2	1110	Freeman Cathy	AUS	22.53	0.235
8.	8	3239	Pintusevych Zhanna	UKR	22.66	0.190

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