CHILE

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Application: How does wealth distribution affect market quality (info efficiency, liquidity, trading volume, welfare)?
CHILE is uniquely suited, as one needs a model with

- Wealth effects
- Heterogeneity
- Asymmetric information

The model. Baseline setup. "Discrete economy"

- $t \in \{1, 2\}$.
- Risk-free asset, $R_f = 1$. Risky asset pays off $\exp(v)$, $v \sim N(0, \tau_v^{-1})$
- n traders of size (mass) $m = 1/n$.
  - Trader a lives in $[a, a + m)$
  - Observes signal $\Delta s(a) = v \cdot m + \frac{1}{\sqrt{t(a)}} \int_a^{a+m} dB$, precision $= t(a) \cdot m$

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The model. Baseline setup. “Continuum economy”

- $t \in \{1, 2\}$.
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- Continuum of traders $a \in [0, 1)$
  - Trader a lives in $[a, a + da)$
  - Observes signal $ds(a) = v \cdot da + \frac{1}{\sqrt{t(a)}} dB$, precision $= t(a) \cdot da$

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- n traders of size (mass) $m = 1/n$.
  - Trader a lives in $[a, a+m)$
  - Observes signal $\Delta s(a) = v \cdot m + \frac{1}{\sqrt{t(a)}} \int_a^{a+m} dB$, precision $= t(a) \cdot m$
  - Ignores his market impact by excluding himself from mkt clr. Solves

$$\max_{x(p, \Delta s(a))} E[u(W_0(a) + x(\cdot)(R-1);\ a)] \tag{1a}$$

$$\text{s.t.: } \sum_{j \neq i} x_j(p, \Delta s_j) = 0. \tag{1b}$$

- **Rich heterogeneity:** $\{W_0(a),\ t(a),\ u(\cdot, a)\}$, arbitrary functions of $a \in [0, 1)$. General utilities.

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- **Rich heterogeneity:** $\{W_0(a),\ t(a),\ u(\cdot, a)\}$, arbitrary functions of $a \in [0, 1)$. General utilities.
- Log-linear equilibrium. Let $p = \log P$. The dollar demand of trader a is

$$dx(a) = \alpha(a)da + \beta(a)ds(a) - \gamma(a)pda.$$

Preview of the results

Framework

- Tractable, log-linear equilibrium. Closed-form solutions.
- Closed-form solutions for info efficiency, liquidity, volume, and welfare.
- Invariant relationship linking info efficiency (harder to measure) to liquidity and volume (easier to measure).
- (Money-metric) welfare can be expressed via liquidity and volume

Application: wealth distribution and market quality

- Inequality is bad for info efficiency
- Inequality is good for liquidity, volume
- Ambiguous effect on welfare

Extensions and ongoing work

Some extensions:

- Information acquisition at $t = 0$. Appendix D.
- General payoffs $V(v)$, $v \sim N(0, \tau_v^{-1})$. Appendix C.

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Ongoing work:

- Kyle'89 setup. Traders do not ignore price impact in the discrete economy.
- Discriminatory price auction with heterogenous info
- Multi-asset model
- Dynamic, continuous-time CHILE
- $\cdots$

Note on ignoring market impact

- Each trader assumes he has no impact on the price
- No impact on price level (Kyle's $\lambda = 0$); no impact on info content of price ($\text{cov}(ds, p) = 0$).

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- No impact on price level (Kyle's $\lambda = 0$); no impact on info content of price ($\text{cov}(ds, p) = 0$).
- This is a small mistake. Given a trade $dx(a)$ price changes by λdx, where λ is finite. Similarly, $\text{cov}(ds, p) \sim da$
- These small mistakes aggregate and do not wash away
- Equilibrium is well defined even without noise traders

Key technical result: aggregation lemma

- Trader a observes $\Delta s(a) = vm + 1/\sqrt{t(a)}\Delta B(a)$. His demand is $x(\Delta s, m)$

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$$
\begin{aligned}
x(ds, da) &= x_s ds + 1/2 x_{ss} ds^2 \qquad + x_m da + 1/2 x_{mm} da^2 + x_{ms} dads \\
&= x_s ds + 1/2 x_{ss} da/t(a) + x_m da
\end{aligned}
$$

- Key: non-linear term ds^2 becomes non-stochastic.

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- Key: non-linear term ds^2 becomes non-stochastic.

$$x(ds, da) = \qquad x_s ds + \qquad \left(\frac{x_{ss}}{2t(a)} + x_m \right) da \implies$$

$$\sum_{a<y} x(ds, da) \to \int_0^y x_s ds + \int_0^y \left(\frac{x_{ss}}{2t(a)} + x_m \right) da$$

Lemma. Suppose that demands $x(p, \Delta s, m, a)$ are well-behaved. Then aggregate demand $\sum_i x(\cdot)$ converges to the Ito process.

Key technical result: aggregation lemma

Lemma. Suppose that demands $x(p, \Delta s, m, a)$ are well-behaved. Then aggregate demand $\sum_i x(\cdot)$ converges to an Ito process

$$dX = \beta(p,a)ds(a) + \text{drift}(p,a)da, \text{ where}$$

$\beta(p,a) = x_s$ and $\text{drift}(p,a) = \frac{1}{2t(a)}x_{ss} + x_m$.

The aggregation lemma is key to the tractability of our analysis.

- Aggregate demand is linear in $ds(a) \Rightarrow$ the equilibrium is always generalized linear.
  Price $\propto \int_0^1 \beta(p,a)ds(a) = v \int_0^1 \beta(p,a)da + \int_0^1 \frac{\beta(p,a)dB(a)}{\sqrt{t(a)}}$
- Log-normal distribution yields even more tractability: $\beta(p,a) = \beta(a)$ and $\text{diffusion}(p,a) = \alpha(a) - \gamma(a)p$

Equilibrium

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Let $\rho(a)$ be absolute risk aversion, $\rho(a) = -u''(W_0(a), a)/u'(W_0(a), a)$.

Theorem. There exists a unique equilibrium.
$dx(a) = \alpha(a)da + \beta(a)ds(a) - \gamma(a)pda$, where

$$\beta(a) = \frac{t(a)/\tau}{\rho(a)\mathrm{Var}[R|p]}$$

with $\mathrm{Var}[R|p] = \exp(\tau^{-1}) - 1$ and $\tau = \tau_v + \tau_p$, where τ_p is the equilibrium price informativeness,

$$\tau_p = \frac{\left(\int_0^1 \frac{t(a)}{\rho(a)} da\right)^2}{\int_0^1 \frac{t(a)}{\rho(a)^2} da}.$$

Other coefficients are given in the closed form in the paper.

- Note: closed-from solutions, with non-CARA and rich heterogeneity!

Wealth distribution and information efficiency: first pass

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Proposition. Info efficiency is given by

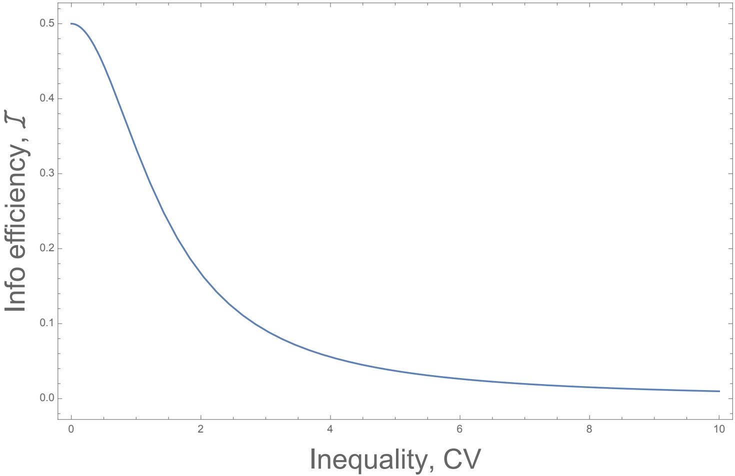
$$\mathcal{I} = \frac{\bar{t}}{\bar{t} + \tau_v(1 + CV^2)}, \text{ where}$$

$$CV = \text{standard deviation of wealth/average wealth}$$

is a *coefficient of variation*.

There is a negative relationship between inequality (CV) and information efficiency ($\mathcal{I}$).

Inequality and info efficiency.



Inequality and info efficiency. Intuition

- Price reflects the weighted average of signals. $p \propto \int \beta(a) ds(a)$
- Weights $\propto \beta(W_0) \propto W_0$
- More weight on wealthier traders
- What is more informative: $0.5(v + \epsilon_1) + 0.5(v + \epsilon_2)$ or $0.99(v + \epsilon_1) + 0.01(v + \epsilon_2)$?

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- What is more informative: $0.5(v + \epsilon_1) + 0.5(v + \epsilon_2)$ or $0.99(v + \epsilon_1) + 0.01(v + \epsilon_2)$?
- Key effect: whose signal noise is reflected more in prices? $p \propto \int \beta(a) ds(a) = v \int \beta(a) da + \int \frac{\beta(a)}{\sqrt{t(a)}} dB(a)$
  Absent in LE a-la Hellwig (1980), signal noise is washed out by LLN

Wealth distribution and information efficiency

Suppose that $t(a) = \bar{t}$, and all traders are CRRA with the same RRA.

Corollary.

$$\mathcal{I} = \frac{\bar{t}}{\bar{t} + \tau_v(1 + CV^2)} \leq \frac{\bar{t}}{\bar{t} + \tau_v},$$

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Intuition.

- Price cannot reflect more than one could infer by seeing each signal: $\tau_p \leq \int_0^1 t(a)da$
- Suppose we have $s_1 = v + \frac{1}{\sqrt{t_1}}\epsilon_1$ and $s_2 = v + \frac{1}{\sqrt{t_2}}\epsilon_2$ (ϵ_i are standard normal). Known result: $\{s_1, s_2\}$ is info equivalent to $s = t_1 s_1 + t_2 s_2$
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- The best way to aggregate signals is with weights proportional to precisions.If $\beta(a) \propto t(a)$, $\tau_p = \int_0^1 t(a)da$
- But we have weights $\propto \beta(a) \propto t(a)/\rho(a)$.
- $\beta(a) \propto t(a)$ iff $\rho(a) = \bar{\rho}$ which is only possible when $W_0(a) = \overline{W}_0$

Wealth distribution and information efficiency: general case

Do our results still hold when utilities are heterogenous and non-CRRA?

When precisions are heterogenous?

When iprecisions are endogenous?

Wealth distribution and information efficiency: general case

How $\mathcal{I}$ changes when $W_0(a)$ changes?

Wealth distribution and information efficiency: general case

How $\mathcal{I}$ changes when $W_0(a)$ changes?

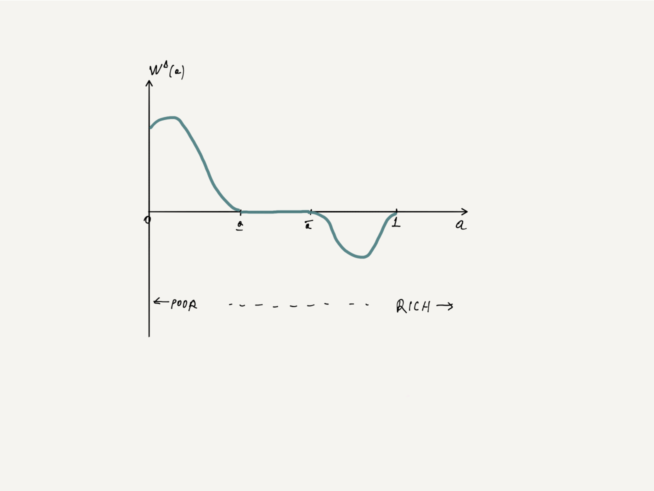
Definition. Gateaux derivative $\mathcal{I}'(W_0(a))[W_0^\Delta(a)]$ in the direction $W_0^\Delta(a)$ is

$$\mathcal{I}'(W_0(a))[W_0^\Delta(a)] = \lim_{\epsilon \to 0} \frac{\mathcal{I}(W_0(a) + \epsilon W_0^\Delta(a)) - \mathcal{I}(W_0(a))}{\epsilon}$$

Wealth distribution and information efficiency: general case

Assume $W_0(a) \uparrow$ in a (WLOG)

Definition. Robin Hood variation is a direction $W_0^{\Delta}(a) \neq 0$ such that $W_0^{\Delta}(a) \geq 0$ for $a < \underline{a}$ and $W_0^{\Delta}(a) \leq 0$ for $a > \bar{a}$.



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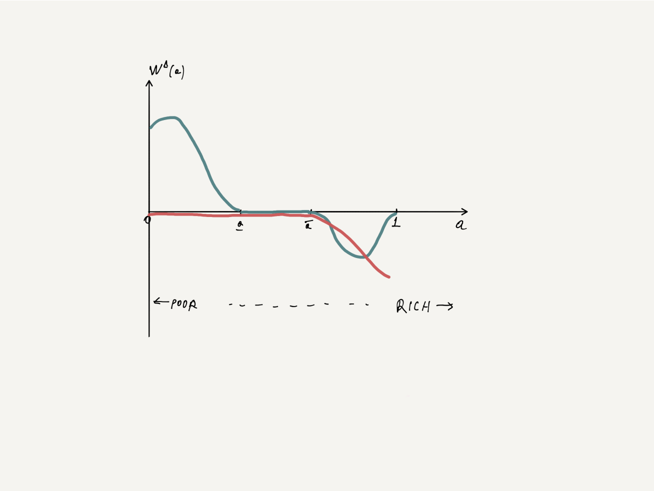

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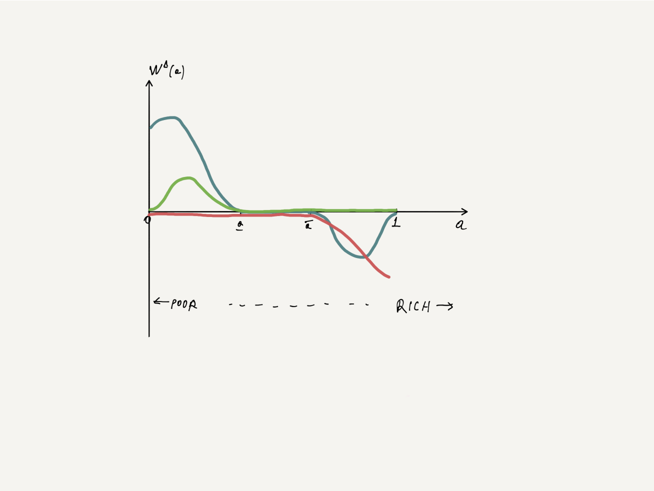

Wealth distribution and information efficiency: general case

How $\mathcal{I}$ changes when $W_0(a)$ changes? When $t(a)$ changes?

The sequence of exercises:

1. Vary $W_0(a)$ keeping $t(a)$ fixed
2. Vary $t(a)$ keeping $W_0(a)$ fixed
3. Vary both

Wealth distribution and information efficiency: general case

Proposition. Assume DARA utility, exogenous precisions+technical conditions. There exists $0 < a^* < 1$ such that for all Robin Hood $W^\Delta(a)$ with $\underline{a} \leq a^* \leq \bar{a}$:

$$\mathcal{I}'[W_0^\Delta(a)] > 0, \quad \mathcal{I}'[-W_0^\Delta(a)] < 0.$$

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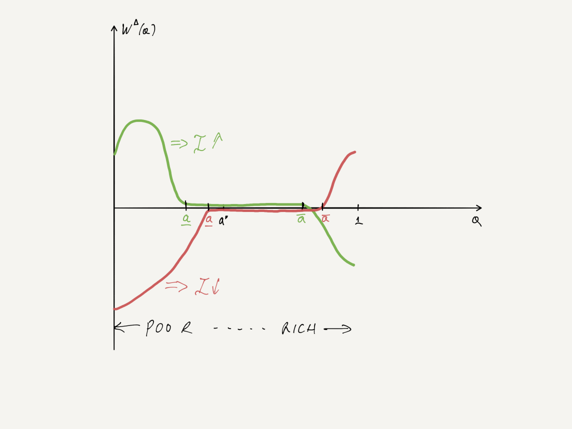

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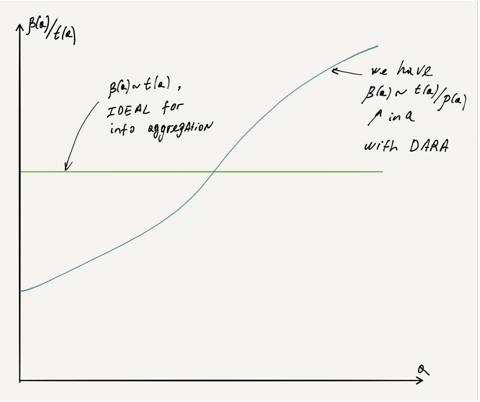

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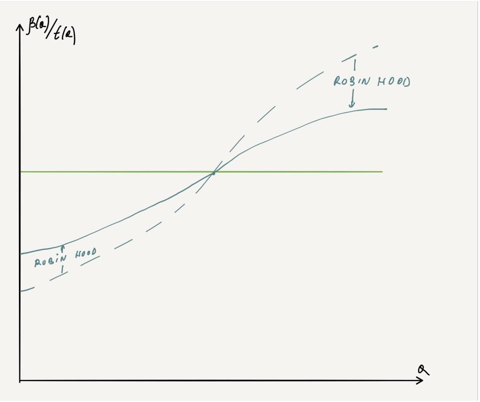

Distribution of precisions and information efficiency

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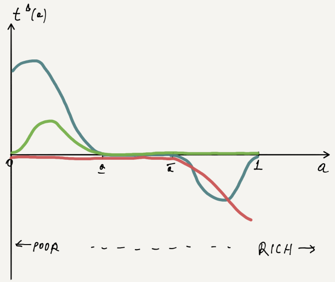
Definition. Robin Hood variation of $t(a)$ is a direction $t^\Delta(a) \neq 0$ such that $t^\Delta(a) \geq 0$ for $a < \underline{a}$ and $t^\Delta(a) \leq 0$ for $a > \bar{a}$.



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Intuition

- Rich trade too aggressively, poor too passively
- Giving less info to the rich makes them less aggressive; analogously, for poor

Distribution of precisions and information efficiency

Corollary. Assume DARA utility+technical conditions. For any $t^{\Delta}(a) \neq 0$ such that $t^{\Delta}(a) \geq 0$ for $a > a^{**}$ and $t^{\Delta}(a) = 0$ otherwise:

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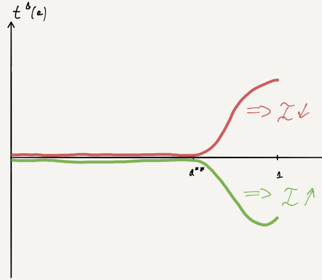

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Implication

- MIFID (unbundling of research fees and trading commissions) makes it harder for small funds to acquire info compared to large. Potentially detrimental effects for info efficiency

Distribution of wealth and information efficiency with endogenous information

Proposition. Assume DARA utility, info $\text{cost}(t) = t^c$, $c > 1$, technical conditions. There exists a unique overall equilibrium. There exists $0 < a^* < a^{**} < 1$ such that for all Robin Hood $W^\Delta(a)$ with $\underline{a} \leq a^* < a^{**} \leq \bar{a}$:

$$\mathcal{I}'[W_0^\Delta(a)] > 0, \quad \mathcal{I}'[-W_0^\Delta(a)] < 0.$$

- Combination of two previous exercises
- Robin Hood variation:
  - Flattens the distribution of risk tolerances
  - Decreases the info of the rich and increases the info of the poor via endogenous info acquisition (rich acquire more info)

Information efficiency, liquidity and volume

Definition

- Liquidity $\mathcal{L} = \int_0^1 \gamma(a) da$
- Volume $\mathcal{V}^2 = \int_0^1 dx(a)^2$

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$$\mathcal{I}(1-\mathcal{I})\frac{\mathcal{L}^2}{\mathcal{V}^2}\sigma_v^2 = 1.$$

Key underlying equation

- To have small equilibrium demands, must have demand elasticity $\sim da$
- Suppose price $\downarrow$ by 1%. Asset is cheaper, demand $\uparrow$ (cost component). Perhaps fundamental is lower, demand $\downarrow$ (info component).

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$$\text{Cost component} = \text{information component} + O(da)$$

$$1 = \frac{\tau_p}{\tau_p + \tau_v} \cdot \frac{\int_0^1 \gamma(a) da}{\int_0^1 \beta(a) da}$$

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Implicaton

$$\underbrace{\mathcal{I}(1-\mathcal{I})}_{\text{hard to measure}} = \underbrace{\frac{\mathcal{V}^2}{\sigma_v^2 \mathcal{L}^2}}_{\text{easier to measure}}.$$

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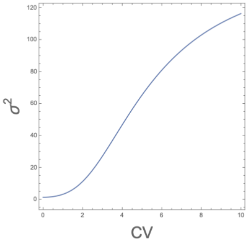

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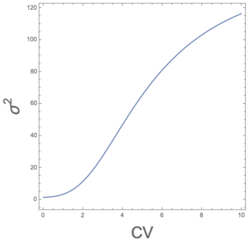

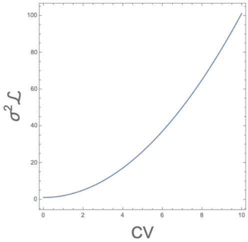

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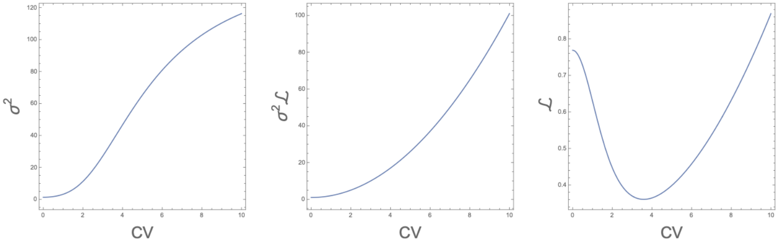
Proposition. The volatility $\sigma^2 = \mathrm{Var}[R|p]$ is increasing in CV. Risk-scaled liquidity $\mathcal{L}\sigma^2$ is increasing in CV. Liquidity is U-shaped in CV.



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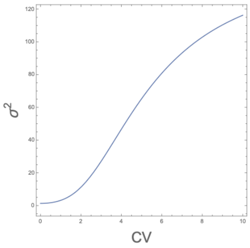

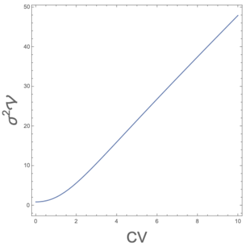

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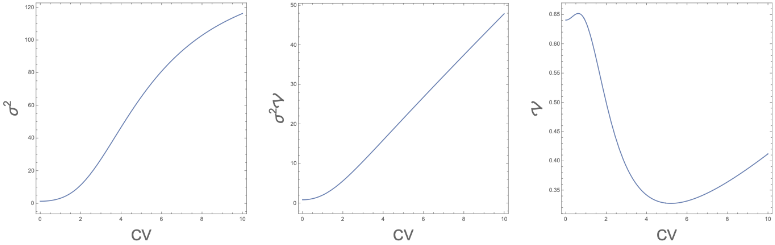
Proposition. The volatility $\sigma^2 = \mathrm{Var}[R|p]$ is increasing in CV. Risk-scaled volume $\mathcal{V}\sigma^2$ is increasing in CV. Volume is non-monotone in CV.



Welfare and wealth distribution

Welfare:

- Money-metric welfare loss
- How much money do we need to give to a to make him as happy as in the first best?
- Aggregate. Take the $m \to 0$ limit

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Suppose that $t(a) = \bar{t}$, and all traders are CRRA with the same RRA.

Proposition. $\text{sign}\left(\mathcal{W}_{loss}(CV)'\right) = \text{sign}\left(\frac{1+CV^2}{\bar{t}+\tau_v(1+CV^2)} - 1.6\right)$.

- Using invariant relationships, can write $\mathcal{W} = \frac{\sigma_v^2}{2}\mathcal{I}\left(1-\mathcal{I}\right)\mathcal{L}$
- Large info frictions (small $\bar{t}$, or small τ_v): inequality is bad for welfare. (Effect of inequality on $\mathcal{I}$ dominates)
- Small info frictions (large $\bar{t}$, or large τ_v): inequality is good for welfare. (Effect of inequality on $\mathcal{L}$ dominates)

Conclusion

- A new heterogeneous information asset pricing framework
- Tractable. General utilities. Rich heterogeneity. Closed-form solutions
- Allows to analyse how wealth distribution affects market quality
- Active follow-ups:
  - Kyle in CHILE
  - Discriminatory price auction/ static limit order book
  - Continuous-time CHILE
  - Multi-asset CHILE
  - $\cdots$

How we solved for equilibrium

1. Consider a discrete economy where each trader a believes other traders' demands are $d\hat{x}(b) = \hat{\alpha}(b, m) + \hat{\beta}(b, m)ds(b) - \hat{\gamma}(b, m)p$
   Solves

$$x^{BR}(a, \Delta s, p, m) = \arg \max_{x(p, \Delta s(a))} E[u(W_0(a) + x(\cdot)(R-1);\ a)] \tag{2a}$$

$$\text{s.t.:} \quad \int_{-a} d\hat{x}(b) = 0. \tag{2b}$$

2. Use aggregation lemma to compute $\lim_{m \to 0} \sum x^{BR}(a, p, m)$. (Involves implicitly differentiating FOC to get x_s, x_{ss} etc)
3. Require consistency
   - $\lim_{m \to 0} d\hat{x} = \alpha(b) + \beta(b)ds(b) - \gamma(b)p$
   - $\lim_{m \to 0} dx^{BR} = \alpha(b) + \beta(b)ds(b) - \gamma(b)p$
4. We've shown this procedure yields the limiting equilibrium in the discrete economy

back

Technical conditions

Technical conditions = x-sectional distribution of wealth (relative risk aversion) has unbounded (compact) support. back