CHILE*

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Abstract

We study how wealth inequality affects market quality—specifically, liquidity and information efficiency. To this end, we introduce CHILE, an asset-pricing framework with asymmetric information, general utility functions, and arbitrary payoffs. It features a large economy (LE) with continuous-and-heterogeneous information (CHI). Making the rich richer and the poor poorer reduces information efficiency but improves liquidity. Making the rich more informed and the poor less informed has the same effect. With endogenous information, richer agents acquire more information, reinforcing the above outcomes. Overall, widening wealth inequality is a double-edged sword for market quality, increasing liquidity but harming information efficiency.

Keywords: inefficient markets, information aggregation, rational expectations with non-CARA preferences, wealth effects, competition

JEL Codes: D01, D53, D82, E19, G12, G14.

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1 Introduction

Market participants come in different sizes. Financial institutions differ in terms of the total value of assets they manage, while households and individuals differ in terms of wealth they invest in financial markets. As a growing literature establishes, these differences—hereinafter, "wealth inequality"—affect asset prices crucially, thereby also returns, risk premia, and risk-free rates (see Panageas, 2020, for a recent survey). Given that prices are but one outcome of financial trade, this literature suggests that wealth inequality may also matter for other outcomes, such as liquidity and informational efficiency. Collectively known as "market quality," these outcomes are vital for many economic decisions, including not only those faced by investors but also by policymakers gauging how well markets function. This raises the following question: How does wealth inequality affect market quality?

This question becomes especially relevant in light of research suggesting that many market participants are *granular*, meaning that idiosyncratic shocks to them, rather than averaging out, affect aggregate outcomes (Gabaix, 2011). In this view, granularity is linked to investor size—shocks to larger investors have a bigger effect on aggregate quantities. As a result, the distribution of investor sizes—i.e., wealth inequality—may play an important role in shaping how individual shocks aggregate and, in turn, how they influence market quality.[1]

To study how wealth inequality affects the aggregation of individual shocks and market quality, we need an asset-pricing framework where (i) wealth influences how investors trade, and (ii) individual shocks do not wash out in aggregation. Such models are hard to come by—especially under asymmetric information, which is essential for analyzing market quality. Most theories in this domain assume Constant Absolute Risk Aversion (CARA) preferences and thus exclude wealth effects, failing to meet the first requirement. Models that do incorporate wealth effects—notably Peress (2004)—rely on a continuum of investors for tractability, where individual shocks cancel out by the law of large numbers, as in Hellwig (1980). Consequently,

[1]Granularity is not merely a theoretical notion—it plays an active role in empirical asset pricing. Shocks to large (i.e., granular) investors are used to identify key asset pricing parameters. For instance, Gabaix and Koijen (2021) use sector-level demand shocks to estimate the elasticity of market demand. These types of idiosyncratic shocks form the basis of *granular instrumental variables*; see Gabaix and Koijen (2024) for the methodology and references therein for recent empirical applications.

these models fail to satisfy the second requirement and are not well suited to studying how wealth inequality shapes the aggregation of individual shocks.

To address these issues, we develop a tractable asset-pricing framework with asymmetric information and general, heterogeneous preferences that accommodate wealth effects, thereby fulfilling the first requirement. In our model, idiosyncratic shocks to investors' demands arise from noise in their private signals. Using an information structure described below, we construct a large economy in which this signal noise aggregates to a finite, non-zero random variable rather than washing out, thus satisfying the second requirement.

Beyond meeting these primary requirements, our framework has two additional advantages. First, the aggregated signal noise is the sole source of noise in the model, ensuring that prices remain partially revealing without the need for exogenous noise. The absence of exogenous noise is beneficial for our research question, as we avoid taking a stand on how such noise responds to changes in the wealth distribution. Second, our framework allows for general asset payoff distributions, sidestepping the well-known limited empirical appeal of models with Normal returns, which permit negative wealth—a valuable benefit given our focus on wealth inequality.

Our analysis begins by extending the notion of competitive equilibrium from Hellwig (1980) to our setting. The equilibrium objects in our economy are limits of quantities in economies with a finite number of traders (referred to as "discrete" economies). In any discrete economy, traders take prices as given, disregarding both their influence on the price level and on the informational content of prices—thereby sidestepping the "schizophrenia" issue highlighted in Hellwig (1980). We refer to the limit of this process as the *price-taking equilibrium*. We prove that a unique price-taking equilibrium exists, even in the absence of external noise in prices. Despite the generality of our primitives, our end-product is tractable, with all equilibrium objects in closed form.[2]

[2]In Avdis, Glebkin, and Peress (2025), we contrast the price-taking equilibrium with the competitive Rational Expectations Equilibrium (REE), where traders ignore their impact on price levels but not informational content, and the Bayesian Nash Equilibrium (BNE), where they account for both. These equilibria differ under our information structure. Here, we focus on wealth inequality and market quality, using the price-taking equilibrium as a natural competitive benchmark that abstracts from market power. Our ongoing work explores market power by analyzing the limiting BNE. For completeness, key results from Avdis et al. (2025) are summarized in Appendix F. In this appendix, we also demonstrate that our key results are not artifacts of the equilibrium concept, as they hold under both competitive REE and BNE.

Turning to our question of how wealth inequality affects market quality, we use our model to study the effect of reducing inequality, changing the population of traders à la "Robin Hood". More specifically, a Robin-Hood variation changes the distribution of wealth across traders, making the rich less rich and the poor less poor, without necessarily affecting aggregate wealth (for institutional investors, the changes are over fund size). Focusing on decreasing-absolute-risk-aversion (DARA) preferences, a plausibly realistic assumption, we show that reducing inequality makes prices more informative. Widening inequality has the opposite effect.

To see why narrowing inequality improves information efficiency, it helps to first develop a baseline for our intuition. We begin by pointing out that prices reveal the weighted average of all private signals, with weights proportional to agents' trading intensities.[3] We then ask the following: is there an "informationally ideal" signal-weighting scheme, however hypothetical, such that the corresponding price reveals maximum information? If so, we intuitively expect that signals of better quality get larger weights. As we show, a weighting scheme confirming this intuition does exist, weighing signals in proportion to their precision alone.

In contrast, the equilibrium weighting scheme is more involved. Accounting for wealth effects, the equilibrium scheme scales signals by risk tolerance on top of precision, placing more weight on the signals of more risk-tolerant traders. As risk tolerance increases in wealth under DARA, the signals of richer traders receive excessive weight, implying that the information content of prices is distorted due to wealth inequality. By transferring wealth from the rich to the poor, a Robin-Hood variation corrects this distortion, moving the equilibrium weighting scheme towards the ideal, making prices more informative.

The mechanism outlined above is empirically relevant at different economic timescales. For long timescales, our result can help interpret the long-term trend towards higher institutionalization, widening wealth inequality, and growing concentration of the asset management industry.[4] At shorter timescales, our model provides novel insights into recent evidence connecting changes in fund size distribution to variations in informational efficiency, as shown by Xiong,

[3]As we discuss below, the trading intensity of an agent is defined as the sensitivity of his demand to changes in private information.

[4]The empirical literature offers recent evidence on these trends. See Ben-David, Franzoni, Moussawi, and Sedunov (2021) on the increasing concentration of the asset-management industry and institutionalization, and Saez and Zucman (2016) on widening wealth inequality.

Yang, and Zheng (2024).[5]

We have, so far, assumed that our agents are endowed with signals and precisions. Given that, as Grossman and Stiglitz (1980) point out, our understanding of price efficiency rests critically on what we assume about the cost of information, one may wonder if our argument continues to hold if information is costly and endogenous. In fact, since people with different wealth may acquire different amounts of information, wealth variations turn on a new channel by changing signal precisions, one that requires more finessed comparative statics.

We isolate this new channel by studying what happens if we change precisions without changing wealth. A Robin Hood variation on precision—decreasing the precision of larger (as in "richer") agents and increasing that of smaller (as in "poorer") ones—again improves information efficiency. When we reduce the precision of larger agents, they end up trading less aggressively, their signals receive smaller weights, thereby pushing the price towards the informational ideal. The opposite happens on the other end of the population, but with the same end effect.

This result has a surprising corollary: prices can become less informative even if everyone receives weakly more information. More concretely, increasing the precision of sufficiently large traders without changing that of others exacerbates the distortion discussed above, because it pushes the signal-weighting scheme further away from the informational ideal. Coming off as an information-aggregation paradox, this property is, in fact, a hallmark of imperfect aggregation with heterogeneous wealth effects. In short, more can be less, because more badly-aggregated information is less information.

Returning to whether wealth effects change if information is endogenous, we revisit our comparative statics in an extension where traders acquire information in the spirit of Verrecchia (1982), with information costs convex in precision (see Appendix C). We show that large traders acquire more information than smaller ones. This is an intuitive result: due to trading more aggressively than others, large traders have stronger incentives to acquire more information because they make more use of it. Consequently, the overall response to a Robin Hood variation

[5]These shifts in fund size distribution may result, for example, from fund flows moving from smaller to larger funds or from mergers between funds.

in wealth combines two effects. One coming from wealth changes alone, improving efficiency directly; and another, coming from how precision changes in response to how wealth changes, improving efficiency indirectly by amplifying the direct effect.

A central mechanism in our analysis concerns how idiosyncratic shocks—arising from noise in traders' private signals—are aggregated into prices, and how this aggregation depends on wealth inequality. Earlier studies on the link between wealth and information efficiency adopt the standard large-economy framework introduced by Hellwig (1980), in which noise from individual signals washes out in the aggregate.[6] As a result, these models do not capture the effects we highlight. Their predictions also contrast with ours. Under Hellwig's information structure, redistributing wealth from rich to poor reduces the trading intensity of wealthy investors more than it increases that of poorer ones, ultimately lowering information efficiency.[7] Recent empirical evidence from Xiong et al. (2024) supports our predictions rather than those of prior models.

Turning to other aspects of market quality, we ask how wealth inequality affects liquidity. We show that narrowing wealth inequality induces two conflicting effects on the willingness of agents to provide liquidity, both of which are knock-on effects of prices becoming more efficient as discussed above. On the one hand, as efficiency improves, each trader puts more weight on commonly-observed price information, aligning his expectations closer to those of others. This effect decreases the agents' willingness to trade, reducing liquidity. On the other hand, as efficiency improves, prices deviate less from fundamentals and are thus de facto less volatile, reducing the risk that traders must absorb when they trade. This effect increases liquidity. As both effects are active in our model, wealth inequality affects liquidity non-monotonically. Nevertheless, we can separate out the risk component if we scale liquidity by return volatility, obtaining a globally monotone comparative static: as wealth inequality decreases, so does risk-

[6] In addition to Peress (2004), see Makarov and Schornick (2010), Kurlat and Veldkamp (2015) and Mihet (2022).

[7] Because signal noise cancels out in these models, they require the introduction of exogenous noise to prevent prices from becoming fully revealing. Consequently, information efficiency depends on the relative trading intensity of informed traders—which decreases after wealth redistribution—versus that of noise traders, which remains unchanged. Incorporating noise traders into our model would capture this additional effect, but it would not alter our main results, as the mechanism we highlight in the baseline model would remain the dominant force (see Appendix E for a detailed analysis incorporating noise). We exclude noise in the main model for parsimony, focusing on the novel mechanism in its simplest form while ensuring consistency with empirical findings.

adjusted liquidity.[8]

Our framework has tractability rarely seen beyond the case with homogeneous agents and CARA preferences.[9] What enables it in our case is the way we model information. In contrast to the traditional methodology for large markets (Hellwig, 1980; Admati, 1985, and consequent literature), we do not assume that traders have signals of finite precision, because that would imply that as the number of traders becomes large, so does the total amount of information.[10] What is more, with signals of finite precision, traders would make finite speculative trades, with the unfortunate consequence that aggregate demand would explode for large numbers of traders.

We instead use an assumption similar to that in Section 9 of Kyle (1989), whereby a finite amount of information is distributed among all traders. By definition, then, the total amount of information remains finite regardless of the number of traders. Moreover, as individual demands are based on signals with precision inversely related to the size of the economy, aggregate demand is finite, even for infinitely many traders. As we show, we can formally treat information structures with the aforementioned properties as diffusion processes running through a heterogeneous continuum, giving us structures we call continuous-and-heterogeneous information (CHI). Recognizing that our model also requires a large economy (LE), we adopt the name CHILE for the class of models described in this paper.

Working with CHILE yields several advantages. First, it unlocks the full arsenal of stochastic calculus, improving tractability similarly to when one switches from discrete-time to continuous-time formulations. Second, the noise in traders' signals aggregates into an Itô integral—a finite, non-zero random variable—rather than vanishing via the Law of Large Numbers, as in standard large-economy models. This type of aggregation yields prices that are noisy, ensuring

[8]This result is in line with empirical evidence in Glebkin, Malamud, and Teguia (2025).

[9]Within asymmetric-information asset pricing, papers that go beyond the CARA-Normal framework (imposing, however, other assumptions for tractability) include Peress (2004), Peress (2014), Breon-Drish (2015), Malamud (2015) and Chabakauri, Yuan, and Zachariadis (2022). Allowing for non-Normal distributions, Breon-Drish (2015) and Chabakauri et al. (2022) require CARA, and thus do not incorporate wealth effects. Malamud (2015) requires complete markets. Peress (2004) and Peress (2014) achieve tractability by requiring the risk of assets to be small. See Section 12 for details.

[10]As the total amount of information is the precision of the sufficient statistic of private signals, it equals the sum of signal precisions held by all traders. Thus, in economies where the precision of each signal is finite (as in "neither infinite nor infinitesimal," i.e., neither infinitely large nor infinitely small), the total amount of information becomes infinite with an infinite number of traders.

that our price-taking equilibrium is well-defined even without external noise. Third, unlike with a discrete economy, working with a continuous economy implies that our main primitives are functions. Consequently, we can carry out comparative statics through the calculus of variations, opening up questions that are otherwise hard to address. Finally, we note that the traditional approach to modeling large economies, which relies on aggregation via the Law of Large Numbers, falls short in fully capturing the effects of heterogeneity. As demonstrated in Appendix D, in such economies, it is possible to derive an aggregation result whereby an economy with investors differing in wealth, precision, and preferences appears observationally equivalent to one with homogeneous investors.[11]

2 Setup

We define a two-period economy on a fixed probability space $(\Omega, \mathcal{F}, \mathbb{P})$, with trading at $t = 1$ and consumption at $t = 2$.[12] There are two assets, one risky and one risk-free. As we use the risk-free asset as a numeraire, we normalize its gross return to 1. The risky asset pays off $V(v)$ in the second period, where $v \sim N(0, \tau_v^{-1})$ is the *fundamental*, and $V(v)$, a weakly increasing function of v, is the *payoff function*.[13]

The risky asset trades at price P, determined in the first period. Allowing for both price-inelastic and price-elastic supply components, we write the total supply of the risky asset as

$$\Theta(P) = \bar{\theta} + \theta(P),$$

where $\bar{\theta}$ is a constant, and $\theta(P)$ is a continuous function.

We consider a *large economy* (LE), meaning that the population of our agents is a continuum. A particular agent $a \in [0, 1)$ starts with wealth $W_0(a)$, and trades the risky asset in the first period, learning about the fundamental by observing prices and a private signal. He consumes

[11]Consequently, in economies with Law of Large Numbers aggregation, Robin Hood variations in wealth that leave aggregate trading intensity unchanged have *no* effect on market quality, unlike in CHILE.

[12]In Appendix C, we study information acquisition, for which we add a period at $t = 0$.

[13]The normalization $\mathbb{E}[v] = 0$ is made without loss of generality, as any mean can be incorporated into the general form of $V(\cdot)$.

all his post-trade wealth W in the second period, obtaining utility $u(W,a)$.[14] We assume that $u(W,a)$ is increasing, strictly concave, and thrice continuously differentiable over W in an open neighborhood around $W_0(a)$.

Our economy features *continuous-and-heterogeneous information* (CHI), an information structure in which the *cumulative signal up to agent* a is the Itô integral

$$s(a) = \int_0^a \left(v\, db + \frac{1}{\sqrt{t(b)}} dB(b) \right) = v\, a + \int_0^a \frac{1}{\sqrt{t(b)}} dB(b). \tag{1a}$$

Here, $B(b)$ is a standard Brownian Motion on $[0,1)$, independent of v. The *signal of agent* a, who lives in segment $[a, a+da)$, is the differential form of (1a),

$$ds(a) = v\, da + \frac{1}{\sqrt{t(a)}} dB(a). \tag{1b}$$

We stress that individual a does not observe $s(a)$ but only $ds(a)$, which can be thought of as an "infinitesimal slice" of $s(a)$.

To represent our CHI structure formally, we define the information set of agents in segment $[b,c)$ as the σ-algebra generated by the increments of the cumulative signal over $[b,c)$, $\mathcal{F}_{b,c} = \sigma\left(\{s(z) - s(b)\}_{b \le z < c}\right)$, and we assume that for any b and c, $0 \le b < c < 1$, the total information available to agents within $[b,c)$ is $\mathcal{F}_{b,c}$. That is, $\mathcal{F}_{b,c}$ is the information set we obtain if we combine the information owned by everyone in $[b,c)$. Under this definition, the information available in the entire economy is $\mathcal{F}_{0,1}$, denoted more compactly hereafter as $\mathcal{F}_1$.

For an arbitrary information set $\mathcal{F}$, we call the posterior gain in estimating v by observing $\mathcal{F}$, $\mathrm{Var}[v|\mathcal{F}]^{-1} - \mathrm{Var}[v]^{-1}$, the *cumulative precision of* $\mathcal{F}$. For the CHI process in (1), standard results in filtering theory imply that $\mathcal{F}_{0,a}$, the information available to agents in $[0,a)$, has cumulative precision $\int_0^a t(b)db$.[15] For signals of individual agents, we note that $ds(a)$ contributes $t(a)da$ to cumulative precision—we thus refer to the marginal change of cumulative precision of $\mathcal{F}_{0,a}$ at a, which is $\frac{d}{da}\left(\int_0^a t(b)db\right) = t(a)$, as the *precision of* $ds(a)$, or, if the context is clear,

[14] The agent location a is an index that tracks agents' individual characteristics, entering the utility function as a parameter. We thus refer to the agent and to his location interchangeably.

[15] See, e.g., Liptser and Shiryaev (2001), Theorem 10.1.

as *a's precision*.

To summarize, our agents are heterogeneous along three dimensions: initial wealth $W_0(a)$, preferences $u(W,a)$, and precisions $t(a)$. The profiles $W_0(a)$, $u^{(l)}(W_0(a),a)$, $l = 0$, 1, 2, 3, and $t(a)$ are arbitrary functions of a, continuous almost everywhere over $a \in [0,1)$.

Remark 1. Beyond assumptions on preferences and payoffs, there are two further distinctions between our economy and the economies in Hellwig (1980) and subsequent literature (hereinafter, "traditional large economies"). First, in our economy the total amount of information is $\int_0^1 t(a)da$, a finite quantity by construction. In traditional large economies, each of an infinite number of agents has a signal of finite precision, implying that the total amount of information is infinite. Consequently, with signals of finite precision, the infinitely many agents make finite speculative trades, implying that aggregate demand blows up. Second, as we explain in detail below, we do not use noise traders. In traditional economies, noise traders not only prevent prices from being fully-revealing, but they are also scaled in proportion to the number of agents, effectively assuming that in a large economy noise variance is infinite, too.[16]

Remark 2. Our agents can be interpreted either as individual traders or as fund managers. In the latter case, $W(a)$ represents the value of the assets managed by fund a, with a certain proportion of $W(a)$ counting as wages for the manager of the fund. To lighten notation, we subsume all coefficients of proportionality in parameter a of the utility profile.

3 Cumulative demand

There are two equilibrium objects of primary interest with all other objects derived from them: a cumulative demand function, $X_*(a,P)$, and a market-clearing price function, $\mathbf{P}_*$. We denote their non-equilibrium counterparts by dropping the asterisk. We discuss the former object here, leaving the latter for the next section. The equilibrium concept is introduced right after, in Section 5.

The *cumulative demand* is a function $X(a,P)$ such that for any b and c, $0 \le b < c < 1$, and for every $P \in \mathbb{R}$, the total number of shares of the risky asset that agents in $[b,c)$ are willing to

[16]See Hellwig (1980), Section 5, eq. (B.2).

buy at price P is $X(c,P)-X(b,P)$.[17] Normalizing $X(0,P)=0$, we have that for any $a\in[0,1)$, the total demand of agents in $[0,a)$ is $X(a,P)$, and $X(1,P)$ is the *aggregate demand*.[18] We focus on equilibria that obey the following restrictions:

Assumption 1. *For every $P\in\mathbb{R}$, the equilibrium cumulative demand $X_*(\cdot,P)$ satisfies*

(i) For any b and c, $0\le b<c<1$, $X_*(c,P)-X_*(b,P)$ is $\mathcal{F}_{b,c}$-measurable.

(ii) For any b and c, $0\le b<c<1$, $\mathbb{E}\left[(X_*(c,P)-X_*(b,P))^2\right]<\infty$.

(iii) $\mathbb{E}[X_*(a,P)]$ is differentiable over $a\in[0,1)$.

The first restriction above captures the intuition that the demand of traders in any given interval cannot depend on information they do not have. The other two restrictions are technical, ensuring that demand is "well-behaved." As the restrictions of Assumption 1 apply only to equilibrium demand, they do not constrain our agent's strategy space. In fact, they allow us to represent demand in the following manner.

Lemma 1 (Representation Lemma)**.** *Suppose that Assumption 1 holds. Then there exist deterministic functions $\beta(a,P):[0,1)\times\mathbb{R}\to\mathbb{R}$ and $\delta(a,P):[0,1)\times\mathbb{R}\to\mathbb{R}$ such that for any $b,c\in[0,1):b<c$,*

$$X_*(c,P)-X_*(b,P)=\int_b^c\delta(a,P)da+\int_b^c\beta(a,P)ds(a).$$

Lemma 1 implies that we can write the cumulative demand of agents in $[0,a)$ in differential form as

$$dX_*(a,P)=X_*(a+da,P)-X_*(a,P)=\delta(a,P)da+\beta(a,P)ds(a), \tag{2}$$

which is nothing other the individual demand of agent a. In what follows, we refer to $\beta(a,P)$, the sensitivity of a's demand to his private signal, as his *trading intensity.*

[17] We assume that demand is well-defined for every $P\in\mathbb{R}$, although for some special cases we technically have $\operatorname{supp}(\mathbf{P}_*)\subset\mathbb{R}$—for example, when $V(x)=\exp(x)$ the equilibrium is log-linear, and $\operatorname{supp}(\mathbf{P}_*)=\mathbb{R}_+$. This poses no loss of generality, as we can always extend demand to $P\in\mathbb{R}$ by letting $X(a,P)=0$ for $P\notin\operatorname{supp}(\mathbf{P}_*)$ without contradicting neither market clearing nor optimization.

[18] We use the term *aggregate* (resp., *cumulative*) for quantities aggregated over the whole (resp., a subset of) population.

There are two main takeaways from (2). First, like the cumulative signal, cumulative demand is also an Itô integral. Second, individual demands are linear in individual signals, a property that, as we discuss in the next section, makes learning from prices and market clearing tractable. Whereas previous literature achieves this property through particular combinations of preferences with payoff distributions—typically CARA-Normal—in our setting it holds generally.[19] See Section 6.2 for an intuitive discussion of why this property emerges.

4 Market clearing and price inference

Turning to market clearing, we require that the equilibrium price $\mathbf{P}_*$ is $\mathcal{F}_1$-measurable, as the price cannot reflect more information than what is available in the entire economy. Due to (2), when agents see the price realisation $\mathbf{P}_* = P$, they infer a sufficient statistic:

$$s_p \equiv \frac{\int_0^1 \beta(a,P)ds(a)}{\int_0^1 \beta(a,P)da}.$$

This statistic is derived from the market clearing condition as follows:

$$\int_0^1 dX_*(a,P)da = \int_0^1 \beta(a,P)ds(a) + \int_0^1 \delta(a,P)ds(a) = \Theta(P) \Rightarrow$$
$$s_p = \frac{\Theta(P) - \int_0^1 \delta(a,P)da}{\int_0^1 \beta(a,P)da} \equiv h(P). \tag{3a}$$

Moreover, s_p can be written as

$$s_p = \int_0^1 \omega(a,P)ds(a) = v + \int_0^1 \omega(a,P)\frac{dB(a)}{\sqrt{t(a)}} \tag{3b}$$

where

$$\omega(a,P) = \frac{\beta(a,P)}{\int_0^1 \beta(a,P)da}.$$

[19] The class of REE with linear demand functions has been recently enlarged by extending Normal payoffs to the exponential family. See Breon-Drish (2015) for the single-asset case and Chabakauri et al. (2022) for the case of many assets.

That is, the price reveals a weighted-average signal, made up by weighing the signal of each agent a by $\omega(a, P)$, a weight that is proportional to a's trading intensity, $\beta(a, P)$.[20]

Next, we assume the following applies to the function $h(P)$ in equilibrium.

Assumption 2. *The function $h(P)$ defined in (3a) is strictly monotone.*

Under Assumption 2, the equilibrium price $\mathbf{P}_*$ and the sufficient statistic s_p have the same informational content. The sufficient statistic s_p in (3b) has the familiar "truth plus noise" form. Moreover, both the truth, v, and the noise, represented by the Itô integral in (3b), are normally distributed. This allows one to use the standard Bayes rule with normal random variables, resulting in the characterization below.[21]

Lemma 2 (Price Inference)**.** *Suppose that Assumptions 1 and 2 hold. Then the conditional distribution of v given $\mathbf{P}_* = P$ is*

$$\mathcal{N}\left(\frac{\tau_p}{\tau} s_p, \frac{1}{\tau}\right).$$

The sufficient statistic s_p can be computed as $s_p = h(P)$, with $h(P)$ as in (3a). Here $\tau = \mathrm{Var}[v|P]^{-1} = \tau_p + \tau_v$, and τ_p is the precision of s_p, given as

$$\tau_p = Var[v|s_p]^{-1} - \mathrm{Var}[v]^{-1} = \left(\int_0^1 \frac{\omega(a, P)^2}{t(a)} da\right)^{-1}. \tag{4}$$

Remark 3. Our model does not require noise traders. Traditional large economies use noise traders to ensure that prices are not fully revealing—the noise entering prices through the aggregate signal is not enough, as it washes out in aggregation due to the exact Law of Large Numbers. In our economy, however, the aggregate signal does not reveal the fundamental, despite aggregating a large number of private signals. Instead, similarly to Avdis (2018), the aggregate noise entering prices is an Itô integral, namely, a random variable with positive and finite variance.[22]

[20] Thus, our economy features generalized linear equilibria, as in Breon-Drish (2015), Glebkin, Gondhi, and Kuong (2021), and others.

[21] See Section 6.3 for a heuristic derivation and an intuitive discussion of Lemma 2.

[22] Our price noise also ensures there is trade in our large economy in a way that, unlike Avdis (2018), is consistent with the notion of perfect competition in Hellwig (1980). See Remark 4 and our literature review for details.

5 Notion of price-taking equilibrium

We maintain Assumptions 1 and 2 from here on. Our equilibrium concept is as follows.

Definition 1. *A* ***price-taking equilibrium*** *is a cumulative demand function* $X_*(a, P)$ *and an* $\mathcal{F}_1$*-measurable price function* $\mathbf{P}_*$ *such that*

(i) $\mathbf{P}_*$ *clears the market, i.e.* $X_*(1, \mathbf{P}_*) = \Theta(\mathbf{P}_*)$*, and*

(ii) $X_*(a, P)$ *is optimal given* $\mathbf{P}_* = P$*.*

Definition 1 follows standard conventions in the literature, describing equilibrium as a juncture of market clearing and demand optimality. Nevertheless, we have not yet explained what optimality means for cumulative demand. To do so, we first note that in an economy with finitely many agents—we call such economies "discrete"—optimality of cumulative demand means that individual demands are optimal.

In our large economy, defining equilibrium using optimality for individual demands is tricky, as each individual demand is infinitesimal. As, however, cumulative demand is not, and optimality is well-defined in discrete economies, we can extend the notion of optimality for cumulative demand to large economies by continuity from the discrete setting.

We proceed in exactly that manner in Section 10 below. As a preview, we first define a sequence of discrete economies that are "neighbors" of the continuous economy introduced above, in the sense that the discrete economies converge to our continuous one. We then define optimal demands in the discrete economies. After that, we revisit demand optimality in our continuous economy, defining it as the limit of optimal cumulative discrete demands. As we explain below, our notion of optimality contains a crucial assumption: agents behave competitively, disregarding their influence on prices completely, in a way that sidesteps what Hellwig (1980) calls the "schizophrenia problem"—see section 10.3 for details.

While the rigorous construction of the equilibrium is presented in Section 10, the next section offers a heuristic derivation to build intuition.

6 Heuristic derivation of equilibrium

In this section, we sketch out the derivation of key equilibrium objects, applying familiar heuristics that have become available in CHILE due to its setup.[23]

6.1 Key stochastic calculus heuristics

We begin by recalling some properties of Brownian Motion and its increments. For $a \neq b \in [0, 1)$, we have

$$dB(a) \perp\!\!\!\perp dB(b) \qquad dB(a), dB(b) \sim \mathcal{N}(0, da),$$

allowing us to interpret the signals in (1) as a continuum of pointwise mutually-independent Normal random variables.

In what follows, we employ second-order Taylor expansions ignoring terms of orders higher than da, following a method commonly referred to as the "box calculus" (Steele, 2001, Chapter 8.4):

$$da \cdot da = 0, \quad dB(a) \cdot da = 0, \quad dB(a) \cdot dB(a) = da; \quad dB(a) \cdot dB(b) = 0. \tag{5}$$

6.2 Information continuity linearizes demand

As our Representation Lemma demonstrates, the demands *must* be linear in signals (see (2)). Suppose, by contradiction, that demands are non-linear in signals—for example, assume that (2) contained a squared-signal term. Denoting the coefficient of the quadratic signal as $\zeta(\cdot)$ and denoting the new version of the remaining quantities by adding tildes on top, we have

$$\begin{aligned} dX(a) &= \tilde{\beta}(P, a)ds(a) + \tilde{\delta}(P, a)da + \zeta(P, a)ds^2(a) \qquad (6) \\ &= \tilde{\beta}(P, a)ds(a) + \left[\tilde{\delta}(a, P) + \frac{\zeta^2(a, P)}{t(a)}\right]da, \end{aligned}$$

[23] See, e.g., Cochrane (2009), Appendix A.3, for an application of these heuristics over time, instead of over agents.

where the second equality follows by the box-calculus heuristic. What is more, by comparing (6) to (2) we can see that the demand function in (2) can subsume the extra term through its da term without affecting the signal term, giving us a contradiction. To summarize, the functional form in (2) already accounts for squared signals.[24]

6.3 Price inference

Equation (3) implies that information in the price is summarized by

$$s_p = v + \int_0^1 \omega(a, P) \frac{dB(a)}{\sqrt{t(a)}}.$$

This signal has a familiar "truth plus normally distributed noise" structure. Once the precision τ_p is known, the inference from prices, summarized in Lemma 2, follows from Bayes's rule for normal random variables. Here, we derive expression (4) for τ_p heuristically; see Appendix B.2 for a formal proof.

The "noise" term $\int_0^1 \omega(a, P)/\sqrt{t(a)}\, dB(a)$ is normally distributed with mean 0 and variance $\int_0^1 \omega(a, P)^2/t(a)\, da$. This follows from the basic properties of stochastic integrals but can also be derived heuristically using the heuristics outlined at the beginning of this section. Indeed, the noise $\int_0^1 \omega(a, P)/\sqrt{t(a)}\, dB(a)$ accumulates terms $\omega(a, P)/\sqrt{t(a)}\, dB(a)$, each normally distributed with mean zero and variance $\omega(a, P)^2/t(a)\, da$, and independent from each other. As an accumulation of normal mean-zero independent random variables, the integral is normally distributed with a mean of zero and a variance that sums the variances of the individual terms, i.e.,

$$\mathrm{Var}\left[\int_0^1 \omega(a, P) \frac{dB(a)}{\sqrt{t(a)}}\right] = \int_0^1 \omega(a, P)^2/t(a)\, da.$$

This variance is the reciprocal of τ_p.

[24] Restricting attention to quadratic adjustments is without loss of generality, as any sufficiently differentiable non-linear function can be expanded into a polynomial of $ds(a)$, with terms of order three and above becoming zero due to (5).

6.3.1 Price inference for a price-taking agent

Note that equation (3b) can be expressed alternatively as:

$$s_p = v + \frac{\omega(a,P)}{\sqrt{t(a)}} dB(a) + \int_{b \in [0,1) \setminus [a,a+da)} \frac{\omega(b,P)}{\sqrt{t(b)}} dB(b).$$

However, a trader viewing the price in this manner would not be acting as a price taker. Consistent with Hellwig (1980)'s critique, such a trader would "account for the covariance between the 'noise' in their own information and the 'noise' embedded in the price." Indeed, using the heuristics presented above, one can obtain:

$$\mathrm{cov}(dB(a), s_p) = \frac{\omega(a,P)}{\sqrt{t(a)}} da.$$

A price-taking trader, who believes their signal does not influence the price, must perceive this covariance as zero.

We take a page right out of Hellwig (1980)'s critique: our agents assume that the noise in their private information is independent of the noise in the price. Specifically, trader a perceives the price as equivalent to a sufficient statistic $\hat{s}_p$, defined by

$$\hat{s}_p = v + \frac{\omega(a,P)}{\sqrt{t(a)}} d\hat{B}(a) + \int_{b \in [0,1) \setminus [a,a+da)} \frac{\omega(b,P)}{\sqrt{t(b)}} dB(b),$$

where the Brownian motions $\hat{B}$ and B are independent. Under this assumption, it is straightforward to see that

$$\mathrm{cov}(dB(a), \hat{s}_p) = 0.$$

This represents the minimal deviation in traders' beliefs consistent with price-taking behavior. Indeed, our traders correctly perceive their private signals $ds(a)$, the marginal distribution of s_p, and the joint distribution of s_p and v. Their only mistake lies in misunderstanding the correlation between the noise in their own signals and the noise embedded in the price.[25]

[25] As we demonstrate in Section 10, our results are robust to alternative approaches for capturing price-taking behavior. In a previous version of the paper, we assumed each trader a assumed away their impact on the price by excluding themselves from market clearing. (This amounts to assuming $\hat{s}_p = v +$

Consequently, the conditional distribution of v given $\hat{s}_p$ coincides with that of v given s_p. In particular, this implies that $\hat{s}_p$ and s_p have identical precision. We also emphasize that the mistakes agents make are infinitesimal. In the limit as $da \to 0$, their perceived price statistic $\hat{s}_p$ coincides exactly with the actual statistic s_p.

Remark 4 (Well-defined equilibrium without noise)*.* Our equilibrium exists and features trade despite lacking external noise—a property that may seem surprising given well-known results (Milgrom and Stokey, 1982; Tirole, 1982). By conjecturing that the noise in their signals is independent of that in the price, our agents treat their signals as incrementally informative relative to the price, in contrast to Grossman (1976)'s agents who treat their signals as dominated by the price. As a result, our agents employ their signals to formulate their demands, and trade ensues.

Nevertheless, we stress that our agents make the right price conjectures in the large limit. Although it may thus appear that trade would disappear in the limit, within the logic of our model the no-trade intuition works differently. As the economy grows, the size of individual demands shrinks to zero, but the number of agents grows, with cumulative demand counterbalancing their shrinking size by summing up more demand functions. This balance is maintained all the way to the large limit, guaranteeing that aggregate demand converges to a well-defined and non-trivial quantity. Restated colloquially, and perhaps paradoxically, our model features a no-trade theorem at the individual level—since each individual's demand converges to zero—but not at the aggregate level, as cumulative demands remain nonzero.

Remark 5 (Price-taking equilibrium is not essential to the economic mechanism)*.* While price-taking behavior is necessary for a well-defined equilibrium in the absence of external noise, the core economic mechanism outlined in Section 1 and analyzed in detail below is not merely an artifact of the equilibrium concept. In Appendix F, we show that this mechanism operates and our results hold under alternative equilibrium concepts, namely competitive REE and BNE.

$+ \int_{b\in[0,1)\setminus[a,a+da)} \omega(b,P)/\sqrt{t(b)}dB(b)$.) That approach yielded identical results. More generally, as long as traders' beliefs satisfy (1) $\hat{s}_p$ and s_p coincide in the $da \to 0$ limit, and (2) $\hat{s}_p$ and $ds(a)$ are conditionally independent given v, our equilibrium remains unchanged (see Section 10).

6.4 A market-efficiency result

Consider trader a, who has size da and receives signal ds. Trader a's optimal demand x satisfies the first-order condition:

$$\mathbb{E}[u'\left(W_0(a) + (V - P)x\right)(V - P)|ds(a), \hat{s}_p] = 0, \tag{7}$$

where we have dropped the dependence of the payoff function on v to lighten notation.

We note that demands must go to zero as the number of agents goes to infinity—if that was not the case, aggregate demand would sum up infinitely many terms of finite size, and it would explode. Consequently, $x = 0$ must solve (7) in the large economy limit. Further noting that the information set of $(ds(a), \hat{s}_p)$ collapses to that of just $\hat{s}_p$ in the limit, setting $x = 0$ in (7) yields

$$\mathbb{E}[u'\left(W_0(a)\right)(V - P)|\hat{s}_p] = 0 \iff P = \mathbb{E}[V|\hat{s}_p].$$

Since the conditional distribution of v given $\hat{s}_p$ coincides with that of v given s_p, we can replace $\hat{s}_p$ with s_p, yielding:

$$P = \mathbb{E}[V|s_p]. \tag{8}$$

This result implies that equilibrium prices in our model are weak-form efficient.

6.5 Deriving trading intensity

As we will show below, market quality is completely determined by the trading-intensity $\beta(a, P)$, on which we now focus. We Taylor-expand (7) over x up to the second order, and then we replace x by $dX(a)$, obtaining

$$\begin{aligned} 0 = u'\left(W_0(a)\right)\mathbb{E}[(V - P)|ds(a), \hat{s}_p] \\ + u''\left(W_0(a)\right)\mathbb{E}\left[(V - P)^2\middle|ds(a), \hat{s}_p\right] dX(a) \\ + \frac{1}{2}u'''\left(W_0(a)\right)\mathbb{E}\left[(V - P)^3\middle|ds(a), \hat{s}_p\right] dX(a)^2. \end{aligned} \tag{9}$$

To solve for trading intensity, we plug (2) into (9) and compare the result with (2). We then solve for $\beta(a, P)$ by matching $ds(a)$ coefficients (for $\delta(a, P)$, we can match da terms).

After some simplifications explained in detail in Appendix B.10, we obtain

$$0 \;=\; u'\left(W_0(a)\right)\mathbb{E}[(V-P)|ds(a),\hat{s}_p] + u''\left(W_0(a)\right)\mathbb{E}\big[(V-P)^2\big|s_p\big]\,\beta(a,P)ds(a) + \cdots, \quad (10)$$

where we have omitted terms that do not contain the signal, as they do not influence our subsequent calculations. Such "non-ds" terms are denoted by "$\cdots$" in the remainder of the subsection.

Next, note that by the market efficiency condition (8) we have $P = E[V|s_p]$ and so $\mathbb{E}[(V-P)^2|s_p] = \mathrm{Var}[V|s_p]$. Note also that we can replace $\rho(a) = -u''(W_0(a)/u'(W_0(a))$, where $\rho(a)$ is the risk-aversion coefficient of agent a. This simplifies the above even further, eventually yielding

$$\beta(a,P)ds(a) = \frac{\mathbb{E}[(V-P)|ds(a),\hat{s}_p]}{\rho(a)\,\mathbb{V}\mathrm{ar}(V|P)} + \cdots \quad (11)$$

Juxtaposing (11) with (2) brings forth a striking property: even though we use general preferences, the demand function depends on the signal as if preferences were mean-variance. We can think of this property as the converse of linearizing demand—by applying box calculus on the expanded first-order condition, all higher-order signal terms drop out, and thus the only way for signals to influence demand is through the first two terms in (9).[26]

To pin down $\beta(a, P)$ we need to separate out the $ds(a)$ term on the right-hand side of (11). To this end, we compute the $ds(a)$ term in the conditional expectation $\mathbb{E}[(V-P)|ds(a), \hat{s}_p]$. Note that for small $ds(a)$, this conditional expectation is linear in $ds(a)$ and so can be computed using a familiar linear regression formula[27]

$$\mathbb{E}[(V-P)|ds(a),\hat{s}_p] = \frac{\mathbb{C}\mathrm{ov}(V(v),ds(a)|\hat{s}_p)}{\mathbb{V}\mathrm{ar}(ds(a)|\hat{s}_p)}ds(a)+\cdots = \frac{\mathbb{C}\mathrm{ov}(v,ds(a)|\hat{s}_p)}{\mathbb{V}\mathrm{ar}(ds(a)|\hat{s}_p)}\,\mathbb{E}[V'(v)|\hat{s}_p]ds(a)+\cdots \quad (12)$$

[26]In equilibrium, preferences cannot be substituted by a mean-variance tradeoff locally approximating the utility. While higher-order moments of returns and higher-order marginal preferences (i.e. prudence) may not affect $\beta(a, P)$, they still affect $\delta(a, P)$.

[27]More precisely, the linearity of expectation allows to substitute it with the Best Linear Predictor formula (see, e.g., Goldberger (1991), Chapter 5.4.) that is written following the first equality in (12).

where the second equality follows from Stein's Lemma.[28]

Moreover, up to terms of order higher than da, we have

$$\mathbb{C}\text{ov}(v, ds(a)|\hat{s}_p) = \mathbb{C}\text{ov}(v, v\, da|\hat{s}_p) = da \cdot \mathbb{V}\text{ar}(v|\hat{s}_p) = \frac{da}{\tau}.$$

The first equality above uses the independence of $dB(a)$ and $\hat{s}_p$, and the third equality applies the identity $\tau \equiv 1/\mathbb{V}\text{ar}(v|\hat{s}_p)$.

Similarly, we compute

$$\mathbb{V}\text{ar}(ds(a)|\hat{s}_p) = \mathbb{V}\text{ar}(dB(a)/\sqrt{t(a)}|\hat{s}_p) = da/t(a).$$

Combining everything, (11) becomes

$$\beta(a, P)ds(a) = \frac{t(a)}{\tau} \frac{\mathbb{E}[V'(v)|\hat{s}_p]}{\rho(a)\,\mathbb{V}\text{ar}(V|\hat{s}_p)} ds(a) + \cdots \iff \beta(a, P) = \frac{t(a)}{\rho(a)} \frac{\mathbb{E}[V'(v)|s_p]}{\tau\,\mathbb{V}\text{ar}(V|s_p)}.$$

In the last step above, we have replaced $\hat{s}_p$ with s_p inside the terms $\mathbb{E}[V'(v)|s_p]$ and $\mathbb{V}\text{ar}(V|s_p)$, relying on the fact that the conditional distributions of v given $\hat{s}_p$ and v given s_p coincide.

Remark 6 (Price-taking equilibrium vs competitive REE). As highlighted in Hellwig (1980), another concern with traditional models of competition under asymmetric information lies within their version of noiseless equilibrium: the information conveyed by prices is unaffected by preferences (see also Grossman, 1976). As Hellwig points out, this is somewhat counter-intuitive: "One would expect that the weight with which agent i 's information I_i affects the equilibrium price should depend on the strength of agent i 's reaction to this information, which in turn should depend on his preferences. Presumably, it should make a difference whether the news of an increase in a firm's profits is passed to somebody who is almost risk-neutral and responds by buying a large number of shares or whether this piece of news is passed to a risk averter who hardly responds at all." As we discuss below, the informational content of prices *does* depend on preferences in our equilibrium, and in a way that aligns with the intuition in

[28]Stein's Lemma states that if X and Y are jointly normally distributed, then for any differentiable function g such that $\mathbb{E}|g'(X)|< \infty$, $\mathbb{C}\text{ov}(g(X), Y) = \mathbb{C}\text{ov}(X, Y) \cdot \mathbb{E}\left[g'(X)\right]$.

the quote.

7 Price-taking equilibrium in CHILE

We present the main theorem of the paper. The previous section provided a heuristic derivation of its key statements, while a rigorous proof is presented in the Appendix.

Theorem 1. *There exists a unique equilibrium. The equilibrium price function has the representation* $\mathbf{P}_* = \mathcal{P}(s_p)$, *where*

$$s_p = \int_0^1 \omega(a) ds(a) = v + \int_0^1 \frac{\omega(a)}{\sqrt{t(a)}} dB(a) \tag{13}$$

is the equilibrium sufficient statistic and $\omega(a)$ *is a weighting function given by*

$$\omega(a) = \frac{t(a)}{\rho(a)} \left(\int_0^1 \frac{t(b)}{\rho(b)} db \right)^{-1}. \tag{14}$$

The function $\mathcal{P}(x)$ *is given by*

$$\mathcal{P}(x) = \int V\left(\frac{\tau_p}{\tau} x + \frac{z}{\sqrt{\tau}} \right) d\Phi(z). \tag{15}$$

Here $\Phi(z)$ *denotes the standard normal cumulative distribution function (cdf). Consequently, the price function is completely determined by* $V(\cdot)$ *and two other quantities, the precision of* s_p, *given by*

$$\tau_p = \frac{\left(\int_0^1 \frac{t(a)}{\rho(a)} da \right)^2}{\int_0^1 \frac{t(a)}{\rho(a)^2} da}, \tag{16}$$

and the posterior precision of v, *given by* $\tau = \mathbb{V}ar(v|P)^{-1} = \tau_v + \tau_p$.

The equilibrium cumulative demand function has the representation $dX(a) = \beta(a,P)ds(a) + \delta(a,P)da$, *where*

$$\beta(a,P) = \frac{t(a)}{\rho(a)} \frac{\tau^{-1}\,\mathbb{E}[V'(v)|s_p]}{\mathrm{Var}\,[V(v)|s_p]}, \text{ and}$$

$$\delta(a,P) = \frac{\beta(a,p)^2}{2t(a)} \pi(a) \frac{\mathrm{Sk}[V(v)|s_p]}{\mathrm{Var}[V(v)|s_p]} - \beta(a,p) \frac{\mathbb{E}[v(V(v)-P)^2|s_p]}{\mathrm{Var}[V(v)|s_p]} + \frac{\psi(P)}{\rho(a)\mathrm{Var}[V(v)|s_p]}. \tag{17}$$

Here, $\rho(a)$ denotes the absolute risk aversion and $\pi(a)$ denotes the absolute prudence coefficient, defined as

$$\rho(a) = -\frac{u''(W_0(a))}{u'(W_0(a))}, \quad \pi(a) = -\frac{u'''(W_0(a))}{u''(W_0(a))}. \tag{18}$$

The sufficient statistic s_p is related to the price P as follows:

$$s_p = \mathcal{P}^{-1}(P),$$

where $\mathcal{P}^{-1}(\cdot)$ is the inverse of the function $\mathcal{P}(\cdot)$ defined in (15). The conditional moments of $V(v)$ and the function $\psi(P)$ are given in the closed form in Appendix B.4.

The theorem above highlights the notable tractability of CHILE. All equilibrium objects are available in closed form despite rich heterogeneity and the generality of preferences. We will make use of this tractability in Section 9, where we examine the effects of changes in wealth distribution on market quality. We now discuss the main features of our equilibrium.

Trading intensity $\beta(a, P)$. The trading intensity has an intuitive structure. First, $\beta(a, P)$ is proportional to $t(a)/\rho(a)$: More informed traders (those with higher $t(a)$) and more risk-tolerant ones (those with higher $1/\rho(a)$) trade more aggressively. Second, $\beta(a, P)$ is inversely proportional to $\mathrm{Var}(V(v)|P)$: Higher uncertainty makes all traders scale down their trading intensities. Third, $\beta(a, P)$ is proportional to $\mathbb{E}[V'(v)|P]$: Trading intensities are higher for assets with payoffs that are more sensitive to changes in fundamentals.[29]

The key results of our paper operate through *wealth effects*. With non-CARA utility, the risk tolerance $1/\rho(a)$ depends on the initial wealth $W_0(a)$. In a plausible case of decreasing absolute risk aversion, risk tolerance increases with wealth. In this scenario, Theorem 1 suggests that wealthier investors trade more aggressively, and their signals receive greater weight in the price.

Price function. The price reflects the weighted average of traders' signals $ds(a)$. Moreover, the weights are proportional to $t(a)/\rho(a)$: the signals of more informed and more risk-tolerant traders have greater weights. This is intuitive as such traders trade more aggressively.

[29] The last property is intuitive because higher $\mathbb{E}[V'(v)|P]$ implies that news about fundamentals is more payoff-relevant: signals about fundamental v tell more about payoffs $V(\cdot)$.

We also note that the closed-form expression (15) for the price function is a restatement of the efficiency condition (8). Indeed, Lemma 2 implies that conditional distribution of v given P is Normal with a mean of $\tau_p/\tau \cdot s_p$ and a variance of $1/\tau$. Thus, a random variable $z = \sqrt{\tau}(v - \tau_p/\tau \cdot s_p)$ has standard normal distribution. Therefore, one can write $v = \tau_p/\tau \cdot s_p + z/\sqrt{\tau}$. Substituting this into the efficiency condition yields $\mathbb{E}[V(v) \mid P] = \mathbb{E}[V(\tau_p/\tau \cdot s_p + z/\sqrt{\tau})] = \int V(\tau_p/\tau \cdot s_p + z/\sqrt{\tau})\, d\Phi(z)$. The same change of variable is used below to obtain closed-form expressions for other conditional moments of $V(v)$.

The coefficients $\delta(a, P)$. The first two terms in (17) indicate that our equilibrium is influenced by the higher moments of the payoff (skewness, $\mathrm{Sk}(V(v)|P)$), as well as by the higher derivatives of the utility function (prudence, $\pi(a)$). This contrasts with the approach of Peress (2004), where these effects do not play a role. Peress (2004) employs a "small risk" approximation (assuming the variance of the fundamental is small), rendering higher-order risk negligible in his model. In contrast, we use a "small information" approximation, where the risk faced by each trader remains substantial even in the limit.[30]

The last term in (17) represents the demand of a trader with no private signal, referred to as "uninformed demand" hereafter. To see this, note that if we set $t(a) = 0$, the term $\psi(P)/(\rho(a)\mathrm{Var}[V(v)|P])$ becomes the only non-zero component in the coefficients $\beta(a, P)$ and $\delta(a, P)$. To better understand this term, let us first discuss the risk premium in our economy.

Risk premium. At first glance, the efficiency condition (8) appears to imply the absence of a risk premium in our economy. However, a more accurate characterization is that the risk premium in CHILE is infinitesimal rather than exactly zero:

$$\mathbb{E}[V(v) - P \mid P] = \psi(P)\, da.$$

[30] Other papers that use a local second-order approximation to the utility function, such as Samuelson (1970), Campbell and Viceira (2002), Farboodi, Singal, Veldkamp, and Venkateswaran (2022b), and Mihet (2022), also do not account for higher-order effects. However, our results demonstrate that a second-order approximation is not always sufficient. Under small information asymptotics employed here, a third-order approximation to utility is necessary. Even under small-risk asymptotics, the second-order approximation may not be valid. For example, in the setting of Peress (2004), it is valid when traders learn about the mean of the fundamental, but it fails when they learn about the payoff itself (see Peress (2011)).

This mirrors how individual demands in our model are infinitesimal rather than strictly zero.

The expression above indicates that the risk premium in a discrete economy with trader mass $\mu = da$ is given by $\psi(P)\, da$.[31] The quantity $\psi(P)$ can be interpreted as the *aggregate risk premium* in the economy—that is, the total dollar excess return earned when all traders invest one dollar each in the risky asset. This contrasts with the traditional notion of a risk premium, which refers to the excess return earned by a *single* trader investing one dollar.

In CHILE, the risk premium is infinitesimal because a finite amount of aggregate risk is distributed across an infinite number of traders. While each trader earns an infinitesimal premium, these accumulate to a finite aggregate risk premium.

With this understanding, the demand of an uninformed trader (i.e., one with $t(a) = 0$) takes the form:

$$dX^u = \frac{\psi(P)}{\rho(a)\mathrm{Var}[V(v) \mid P]}\, da = \frac{\mathbb{E}[V(v) - P \mid P]}{\rho(a)\mathrm{Var}[V(v) \mid P]}.$$

This expression coincides with the standard mean-variance demand function.

8 Market quality

Our measure of information efficiency is based on how much prices reduce uncertainty about the fundamental and is defined as

$$\mathcal{I} = 1 - \frac{\mathbb{V}\mathrm{ar}(v|P)}{\mathbb{V}\mathrm{ar}(v)}.$$

This measure is common in both theoretical (e.g., Rostek and Weretka, 2012) and empirical work (e.g., Dessaint, Foucault, and Fresard, 2024; Dávila and Parlatore, 2023). In practice, $\mathcal{I}$ corresponds to the R^2 of predicting fundamentals by prices, where the fundamental v is typically proxied by earnings in empirical work.

[31] Formally, this corresponds to $\lim_{\mu \to 0} \int (V(v) - P)\, g(v, P, \mu)/\mu = \psi(P)$, as shown in the Appendix.

Our measure of liquidity is based on how much supply shocks can move prices

$$\mathcal{L} = -\left(\frac{\partial \mathcal{P}(s_p, \bar{\theta})}{\partial \bar{\theta}} \bigg|_{\mathcal{P}=P} \frac{1}{\mathbb{V}\mathrm{ar}(V|P)} \right)^{-1}. \tag{19}$$

The first term in (19) captures price sensitivity to unexpected price-inelastic supply shocks, a standard measure of liquidity in the theoretical literature (e.g. Vayanos and Wang, 2013). As our equilibrium is generally non-linear, this term depends on the realized price signal s_p, a stochastic quantity with few (if any) empirical counterparts. As we show next, scaling by $\mathbb{V}\mathrm{ar}(V|P)$ in (19) allows us to obtain the risk-adjusted liquidity measure that does not depend on the aggregate signal, and we thus adopt it as our primary notion of liquidity.[32]

Proposition 1. *The equilibrium expressions for information efficiency $\mathcal{I}$ and liquidity $\mathcal{L}$ are*

$$\mathcal{I} = \left(1 + \tau_v \frac{\int_0^1 \frac{t(a)}{\rho(a)^2} da}{\left(\int_0^1 \frac{t(a)}{\rho(a)} da \right)^2} \right)^{-1}, \quad \mathcal{L} = \frac{\int_0^1 \frac{t(a)}{\rho(a)^2} da}{\int_0^1 \frac{t(a)}{\rho(a)} da}.$$

As we see above, information efficiency and liquidity are in closed form. What is more, they are parsimoniously characterized by just three primitives of the economy, two profiles and one parameter: risk tolerances $1/\rho(a)$, precisions $t(a)$, and prior uncertainty $1/\tau_v$. This enables tractable comparative statics that we turn to next.

9 Comparative statics

We now explore how market quality is affected by inequality, starting with inequality in wealth, followed by inequality in precision. In more concrete terms, we consider an equilibrium object $\mathcal{O}$—our placeholder notation for either information efficiency $\mathcal{I}$ or liquidity $\mathcal{L}$—and we change the primitives $W_0(a)$ and $t(a)$ around those associated with the original equilibrium. That is,

[32] Our liquidity measure is the inverse of price impact per unit of risk. Price impact is proportional to variance, as shown in Kyle (1985) and Kyle (1989) for the single-asset case, and in Rostek and Weretka (2015) and Malamud and Rostek (2017) for the multi-asset case. Without normality, the price impact is proportional to risk-neutral variance, as demonstrated in Glebkin, Malamud, and Teguia (2023a). Therefore, scaling by variance, as in (19), allows for better cross-sectional comparisons consistent with empirical practice.

the task at hand for this section is to carry out comparative statics with respect to functions. The right tool for this job is the Gateaux derivative.

Definition 2. *The* ***Gateaux derivative*** *an equilibrium object* $\mathcal{O}$ *with respect to a parameter* $h(a)$ *in the direction* $h^{\Delta}(a)$ *is*

$$\mathcal{O}'\left(h(a)\right)\left[h^{\Delta}(a)\right] = \lim_{\varepsilon \to 0} \frac{\mathcal{O}\left(h(a) + \varepsilon h^{\Delta}(a)\right) - \mathcal{O}\left(h(a)\right)}{\varepsilon}.$$

To facilitate exposition, we use the following conventions and types of notation in our definition above. The *parameter* (namely, a function) with respect to which we differentiate appears inside round brackets, while the *direction* (another function) along which we perturb the parameter appears inside square brackets. Our notation also distinguishes the parameter from the direction by indicating the direction with a Δ superscript.[33]

Of particular interest are directions that correspond to reduced inequality. To this end, we refer to a variation of model parameters that makes poor agents better off (either richer or more informed) and rich agents worse off (either poorer or less informed) as a *Robin Hood variation*. We then index our agents in the same order as their wealth, implying that $W_0(a)$ *increases* in a, an assumption maintained hereafter without loss of generality. The poor and rich are defined with respect to thresholds $\underline{a}$ and $\bar{a}$, with the poor lying below $\underline{a}$ and the rich above $\bar{a}$.

Definition 3. *A* ***Robin-Hood variation*** *of parameter* $h(a)$ *is a direction* $h^{\Delta}(a)$*, bounded over* $a \in [0, 1)$*, and two associated thresholds* $\underline{a} < \bar{a}$ *such that*

(i) $h^{\Delta}(a) \geq 0$ *for* $a < \underline{a}$*,*

(ii) $h^{\Delta}(a) \leq 0$ *for* $a > \bar{a}$*,*

(iii) $h^{\Delta}(a) = 0$ *for* $a \in [\underline{a}, \bar{a}]$*, and*

(iv) $h^{\Delta}(a)$ *is not zero for some set of indices* a *with positive Lebesgue measure.*

[33] Once we fix a direction, computing a Gateaux derivative can be done with standard calculus. One differentiates $\mathcal{O}\left(h(a) + \varepsilon h^{\Delta}(a)\right)$ with respect to the scalar ε, evaluating the result at $\varepsilon = 0$.

Returning to our questions on wealth inequality, we can arrive at answers by considering how Robin-Hood variations in wealth affect market quality. As, however, wealth effects may also enter through information acquisition, we must do so with care, isolating different possible effects. We thus first take Robin-Hood variations with respect to wealth keeping precisions fixed, then with respect to precisions keeping wealth fixed, then allowing for both effects by letting precisions depend on wealth.

As we see below, this sequence of comparative statics not only uncovers useful intuition but also reveals some surprising results. Before we proceed, however, we establish an important benchmark for the discussion of later subsections.

9.1 Ideal information aggregation in CHILE

We begin by pointing out that prices reveal the weighted average of all private signals—see equation (13). Revisiting how information is aggregated in our economy, we ask: is there an "informationally ideal" version of the weighting function $\omega(a)$, in the sense that the corresponding price would reveal maximum information to the agents?

We answer this question for a hypothetical weighting function $\omega^H(a)$, using it to define the notion of an *aggregate signal* as

$$s\left[\omega^H(a)\right] = \int_0^1 \omega^H(b) ds(b), \qquad \omega^H(a) \geq 0, \qquad \int_0^1 \omega^H(a) da = 1.$$

Writing $s_p = s[\omega(a)]$ then shows that the equilibrium-price weights $\omega(a)$ are but one possibility for $\omega^H(a)$, with the more general $\omega^H(a)$ yielding an aggregate signal with precision

$$\mathbb{V}\mathrm{ar}\big(v \big| s\left[\omega^H(a)\right]\big)^{-1} - \mathbb{V}\mathrm{ar}(v)^{-1}.$$

The informationally-ideal weighting function maximizes this precision, and is characterized in the following result.

Lemma 3 (Ideal information aggregation)**.** *No aggregate signal can exceed the cumulative pre-*

cision of the entire economy, that is,

$$\mathbb{V}ar\big(v \big| s\left[\omega^H(a)\right]\big)^{-1} - \mathbb{V}ar(v)^{-1} \leq \int_0^1 t(a)da \tag{20}$$

for any weighting function $\omega^H(a)$. The weighting function that maximizes signal precision satisfies (20) as an equality, and is given by

$$\omega^I(a) = \frac{t(a)}{\int_0^1 t(a)da}. \tag{21}$$

As we can see in (21), if we were to design an aggregate signal with maximizing informativeness as our only goal, we should be weighing each signal in proportion to its precision. Comparing (21) with (14) now highlights the key to understanding information inefficiency: the informationally-ideal weights are proportional to precisions, $\omega^I \propto t(a)$, whereas the price weights ω are distorted by risk tolerances, $\omega \propto t(a)/\rho(a)$. With DARA, it is the wealthier agents that can tolerate more risk. Consequently, DARA preferences yield equilibria where prices overweigh the signals of the rich, underweighing those of the poor.

9.2 Wealth inequality

Using the result above as an illuminating benchmark, we examine the effects of transferring wealth from the rich to the poor on market quality holding precisions fixed. To proceed, we need the following technical conditions.

Assumption 3. *The following hold for the profiles of wealth, $W_0(a)$, absolute risk tolerances, $1/\rho(a)$, and relative risk aversions, $\rho(a)/W_0(a)$.*

1. *The cross-sectional cdf of wealth is continuous, strictly increasing, and has support $[0,\infty)$;*
2. *The cross-sectional cdf of absolute risk tolerances $1/\rho(a)$ is continuous and strictly increasing;*
3. *There exists constants $\underline{\eta}$ and $\overline{\eta}$ such that $0 < \underline{\eta} \leq \rho(a)/W_0(a) \leq \bar{\eta} < \infty$.*[34]

[34] The cross-sectional cdf of wealth (resp., absolute risk tolerances) is defined as $F_{W_0}(x) = \Lambda(a : W_0(a) \leq x)$

We also assume $t(a) > 0$, which means that in our comparative statics exercises, we only consider wealth transfers among informed traders. Changing wealth of uninformed (i.e., those with $t(a) = 0$) has no effect on market quality as is clear from Proposition 1. As in earlier parts of the paper, we maintain the above assumption, from its statement onwards. We next state the main result of this section.

Proposition 2. *Suppose that agent preferences are DARA. Then, there exist thresholds* $0 < a_1^W \leq a_2^W < 1$, *such that for any Robin Hood variation* $W_0^\Delta(a)$ *with* $\underline{a} \leq a_1^W \leq a_2^W \leq \bar{a}$ *we have*

$$\mathcal{I}'(W_0(a))[W_0^\Delta(a)] > 0 \text{ and } \mathcal{I}'(W_0(a))[-W_0^\Delta(a)] < 0; \tag{22}$$

$$\mathcal{L}'(W_0(a))[W_0^\Delta(a)] < 0 \text{ and } \mathcal{L}'(W_0(a))[-W_0^\Delta(a)] > 0. \tag{23}$$

Reducing inequality by transferring wealth from the sufficiently rich to the sufficiently poor leads to higher informational efficiency. The intuition behind this is rooted in the distortion highlighted in the previous section: the price overweights the signals of the rich and underweights the signals of the poor. By redistributing wealth from the wealthy to the poor, this distortion is corrected, thereby improving informational efficiency.

We now turn to the liquidity result (23). There are two channels at play. First, a reduction in inequality is associated with increased information efficiency. When prices are more informative, traders become less willing to provide liquidity—for instance, by selling when prices increase—since higher prices are more likely to reflect stronger fundamentals. Second, there is the uncertainty reduction channel: higher information efficiency leads to less uncertainty about fundamentals, potentially decreasing $\mathrm{Var}[V(v)|P]$ and making traders more willing to provide liquidity. By examining the risk-adjusted measure $\mathcal{L}$, we isolate the first effect. As a result, a reduction in inequality negatively impacts $\mathcal{L}$.

Beyond providing comparative statics, our results connect with several sets of empirical observations. First, regarding secular trends, it is known that while stocks have become more liquid since the beginning of the 20th century (e.g., Chordia, Roll, and Subrahmanyam, 2001), the informativeness of the average US stock has deteriorated (Farboodi, Matray, Veldkamp,

(resp., $F_{1/\rho}(x) = \Lambda(a : 1/\rho(a) \leq x)$. Here $\Lambda(\cdot)$ denotes the Lebesgue measure.

and Venkateswaran, 2022a). This is puzzling given the improved data availability in modern markets.[35] Our model can explain both trends by appealing to the growing wealth inequality among individual investors (Saez and Zucman, 2016), or to the unequal distribution of assets under management for institutional investors (Ben-David et al., 2021).

Second, applied at a higher frequency, our results suggest that market quality responds to temporal changes in the size distribution of market participants. Relative to a market composed of traders with similar sizes, a market composed of traders with disparate sizes has noisier prices and higher liquidity. Given that noisier, more liquid markets constitute better trading environments for all investors irrespective of size, if large institutions ("whales") and small institutions or individuals ("small fry") were able to choose who to trade with, they would prefer those least like them.[36] Correspondingly, the trading volume clustering in McInish and Wood (1992) can be interpreted as an outcome of "coordination" among traders' choices of when—and thus against whom—to trade: small fry attract whales, and whales attract small fry.

Third, our results are consistent with recent empirical findings by Xiong et al. (2024), who document a negative relationship between wealth inequality—measured by the Herfindahl-Hirschman Index (HHI) of fund assets under management (AUM) or the ownership share of the top five investors—and information efficiency, as defined in Bai et al. (2016).[37] This relationship holds both at the market level and for individual stocks.

Importantly, our mechanism does not rely on market power, which plays a central role in the theoretical framework proposed by Xiong et al. (2024).[38]

[35] There is also evidence that the trend in price informativeness is not uniform: Bai, Philippon, and Savov (2016) find that the informativeness of S&P 500 firms has increased over time, while Farboodi et al. (2022a) show that the negative average trend is driven by small firms.

[36] Our model can be extended to incorporate two trading periods (or two trading venues), allowing investors to choose when (or where) to trade. We leave such extensions for future work.

[37] They also examine the effects of an exogenous increase in inequality driven by fund mergers.

[38] Their framework builds on Kacperczyk, Nosal, and Sundaresan (2024).

9.3 Information inequality

Here, we examine the effects of changing the distribution of information across agents on market quality holding the wealth profile fixed.

Proposition 3. *Suppose that traders have DARA utilities. Then, there exist thresholds* $0 < a_1^t \le a_2^t < 1$*, such that for any Robin Hood variation* $t^\Delta(a)$ *with* $\underline{a} \le a_1^t \le a_2^t \le \bar{a}$

$$\begin{aligned} &\mathcal{I}'(t(a))[t^\Delta(a)] > 0 \text{ and } \mathcal{I}'(t(a))[-t^\Delta(a)] < 0; \\ &\mathcal{L}'(t(a))[t^\Delta(a)] < 0 \text{ and } \mathcal{L}'(t(a))[-t^\Delta(a)] > 0. \end{aligned} \tag{24}$$

Making the rich less informed and the poor more informed improves information efficiency but reduces liquidity. We discuss the intuition for the information result, from which the liquidity result follows. Recall the key distortion highlighted in Section 9.1: the trading intensities of the rich are too large, while those of the poor are too small. To correct this inefficiency, one needs to increase the trading intensities of the poor relative to the rich. This can be achieved by making the poor more informed and the rich less informed.

Note that the definition of a Robin Hood variation allows for weak inequalities. Therefore, the proposition above applies to a variation that decreases the precisions of the rich while leaving other precisions unchanged.

Corollary 1 (An information-aggregation paradox)**.** *Suppose that traders have DARA utilities. Then, there exists a threshold* a_2^t *such that for any* $t^\Delta(a) \neq 0$ *such that* $t^\Delta(a) \ge 0$ *for* $a > a_2^t$*, and* $t^\Delta(a) = 0$ *otherwise,* $\mathcal{I}'(t(a))[t^\Delta(a)] > 0$ *and* $\mathcal{I}'(t(a))[-t^\Delta(a)] < 0$*.*

Weakly increasing the information of all traders can hurt information efficiency, while weakly decreasing it can have the opposite effect. Banerjee, Davis, and Gondhi (2018), Dugast and Foucault (2017), and Glebkin and Kuong (2023) also showed that improving the quality of private information can reduce information efficiency. In their studies, the mechanism is that giving more information to some traders invites more noise from another source. In our paper, we focus on a pure information aggregation channel: the only noise comes from the traders' signals themselves. Yet, reducing the noise in some traders' signals can harm information efficiency. This occurs because better information is aggregated less effectively: increasing the precisions of the rich makes them trade more aggressively, causing their signals to be even more overweighted in price, exacerbating the existing distortion.

The aforementioned papers also do not feature wealth effects, so the effect there is not specific to large traders (i.e., traders with large wealth). An implication of our result is that making large investors differentially more informed could harm information efficiency. One of the changes introduced in the MiFID II regulation was to unbundle investment research from trading costs. Simplifying, before MiFID, everyone who traded obtained information. After that, only whoever is willing to pay gets it. Since large traders have a higher value of information (see Section C), such regulation makes large traders differentially more informed, potentially negatively affecting price informativeness.

Remark 7 (Why quality of aggregation dominates quantity of information). The result of Corollary 1 arises from the balance of two effects: on the one hand, making the rich less informed decreases the overall quantity of information available in the economy; on the other hand, the information is aggregated more effectively. Why does the second effect dominate? To understand this, consider the contribution of trader a to price informativeness τ_p. We can express τ_p as $\tau_p = I_1^2/I_2$, where $I_1 = \int_0^1 t(a)/\rho(a)da$ and $I_2 = \int_0^1 t(a)/\rho(a)^2 da$. Price informativeness is related to the signal-to-noise ratio. The "signal" in this ratio is related to I_1, and trader a's contribution to it, $t(a)/\rho(a)da$, is proportional to their risk tolerance, $1/\rho(a)$. Their contribution to the "noise", captured in I_2, is $t(a)/\rho(a)^2$, which is proportional to the square of their risk tolerance, $1/\rho(a)^2$. For sufficiently wealthy traders, their contribution to the noise always dominates their contribution to the signal, resulting in an overall negative contribution to price informativeness.

9.4 Wealth inequality redux: the role of information acquisition

Do the results in Proposition 2 hold when the precisions are endogenous (i.e., can depend on $W_0(a)$)? Consider the information efficiency result. The mechanism there is that the signals of the rich are overweighted, while the signals of the poor are underweighted. With endogenous information acquisition, one can expect the rich to acquire higher quality signals: they trade more aggressively and thus have more use for their information, making them value it more.[39] Is overweighting the signals of better quality a bad idea?

The transfer of wealth from rich to poor has two effects, one direct and one indirect. Proposition 2 captures the direct one. Proposition 3 captures the indirect one, by decreasing the precisions of the rich and increasing that of the poor, via the dependence of precisions on wealth. Since both effects work in the same direction, the results (22)–(23) are reinforced in the presence of information acquisition. We summarize this in the proposition below.

Proposition 4. *Suppose that Assumption 3 holds. Suppose that traders have DARA utilities. Suppose that precisions are a function of wealth $t(W_0(a), a)$ and are increasing in $W_0(a)$. Then, for any Robin Hood variation $W_0^\Delta(a)$ with $\underline{a} \leq \min\{a_1^t, a_1^W\} \leq \max\{a_2^t, a_2^W\} \leq \bar{a}$, results (22)–(23) hold.*

The result of Proposition 4 is perhaps surprising. Since the rich acquire more information than the poor, a Robin Hood variation could lead to a smaller overall amount of information being produced in the economy. Why does the information efficiency improve? Similar to our information aggregation paradox (Corollary 1), the effect of better aggregation of information dominates: Less information, but aggregated better results in more information efficiency (see Remark 7).

10 The large economy as a limit of discrete economies

In this section, the terms "continuous" or "large" refer to the economy introduced in Section 2, while the terms "discrete" and "finite" refer to economies along a sequence, which, as mentioned above, converge to the one in Section 2, allowing us to extend optimality of demand to the continuum.

[39]In Section C, we consider the information acquisition problem and show that agent a's precision $t(a)$ is indeed an increasing function of his wealth $W_0(a)$.

10.1 Technical conditions on the primitives of the large economy

Our next assumption, which we maintain everywhere below, ensures that certain equilibrium objects are well-defined.

Assumption 4. *The following hold for the candidate price function* $\mathbf{P}$*, the payoff function* $V(v)$*, and the agent primitives.*

(i) The joint density of $\mathbf{P}$ *with* v*,* $f_{v,\mathbf{P}}(v, P)$*, is such that for any fixed* P*, there exist* $A > 0$ *and* $k > 0$ *(that may depend on* P*), such that* $f_{v,\mathbf{P}}(v, P) < A\exp(-kv^2)$*.*

(ii) $V(v)$ *satisfies* $\lim_{v\pm\infty} \frac{\ln|V(v)|}{v^2} \leq 0$*.*

(iii) The agent primitives jointly satisfy the regularity conditions

$$\int_0^1 \frac{1}{\rho(a)} da < \infty, \quad \int_0^1 \frac{t(a)}{\rho(a)} da < \infty, \quad \int_0^1 \frac{t(a)}{\rho^2(a)} da < \infty, \quad \int_0^1 \frac{t(a)\pi(a)}{\rho^2(a)} da < \infty.$$

The first two conditions refer to the risky asset. The first one requires that prices and fundamentals have a joint density dominated by a Gaussian function, to which the second one adds something similar, but for the shape of the payoff function. The last condition is a certain type of joint integrability for the agent primitives.[40]

10.2 Primitives of the discrete economies

The agents of the nth discrete economy live in n subintervals of size $\mu = 1/n$ that form a uniform partition of $[0, 1)$. A particular agent i, $i = 1, \ldots, n$, lives in segment $[a_i, a_{i+1})$, with $a_i = (i-1)\mu$. His initial wealth is $W_0(a_i)$, his precision is $t(a_i)$, and his utility function over terminal wealth W is $u(W; a_i)$. Thus, discrete primitives are obtained by "sampling" those in the continuous economy at the leftmost points of agent subintervals. His private signal is

$$\Delta s_i = v\,\Delta a_i + \frac{1}{\sqrt{t\,(a_i)}} \Delta B(a_i), \tag{25}$$

[40] Condition (i) holds in equilibrium by Lemma 2, (ii) holds if $V(v)$ has bounded support, while (iii) is easily met if risk aversion, prudence, and precision are continuous functions over $a \in [0, 1]$.

with $\Delta a_i = a_{i+1} - a_i = \mu$ and $\Delta B(a_i) = B(a_i + \mu) - B(a_i) \sim \mathcal{N}(0, \Delta a_i)$. Note that if we replace Δ with d, (25) becomes (1b).

Remark 8. Partitioning the agent interval uniformly and using the same profile of preferences in the discrete economy as in the continuous economy are intended to simplify exposition. Our results hold more generally, as long as the norm of the partition vanishes as $n \to \infty$, and the discrete preference profile converges to the continuous one. See Section B.4.1 for details.

10.3 Optimal competitive demands

A competitive economy is widely understood as one consisting of agents that have no market impact whatsoever. Existing work on noisy Rational Expectations Equilibria (REE) recognizes this idea explicitly, using the term "price taking" for how competitive behavior is modeled. And yet, as Hellwig (1980) points out, competition in REE is not entirely consistent with price taking: even though traders ignore their influence on the price level, they do account for their influence on the informational content of the price (a behavior that Hellwig (1980) dubs "schizophrenic").

Our approach differs from traditional REE in that our agents fulfill both aspects of price taking. The first aspect, that agents should not be able to influence price levels, is a standard property of any competitive equilibrium. The second aspect, that agents should not be able to influence price information, is the one that Hellwig finds problematic, identifying the crux as agents taking "account of the covariance between 'noise' in their own information and 'noise' in the price" (Hellwig, 1980, p. 478). Neither problem arises in our large market because, as (3) shows, changing $\beta(a, P)$ or $\delta(a, P)$ for a single agent a has no effect on the price.

The above notwithstanding, to extend demand optimality from discrete markets to continuous ones, we must first clarify the notion of competition among agents in our discrete economies. We conduct the same exercise as Hellwig (1980), passing from finite economies to the large limit. We fulfill the first aspect of competitive behavior by treating prices as fixed in the demand-choice problem of our agents. For the second aspect, we take a page right out of Hellwig's critique: our agents assume that the noise in their private information is independent of the noise in the price.

Formally, for each discrete economy, each agent assumes that his signal is independent of the price conditional on the fundamental.[41] More specifically, agent a, who in the nth discrete economy lives in subinterval $a \in [a_i, a_i + \mu)$, believes that the market-clearing price is the realization of a continuous random variable $\mathbf{P}_i^n$, which we call a *price-function conjecture*. These conjectures indexed over a, $\mathbf{P}^n(a) = \mathbf{P}_i^n$, for all $a \in [a_i, a_i + \mu)$ and each i, constitute a *profile of price-function conjectures*. We note that the profile $\mathbf{P}^n(a)$ can differ from the actual price function $\mathbf{P}$ for finite n, but it must converge to $\mathbf{P}$ for every a as n becomes large, guaranteeing that the price conjectures are correct in the limit. We summarize our assumptions on price conjectures below.

Assumption 5. *Agent i in the nth discrete economy*

(i) believes that the market-clearing prices are realizations of a continuous random variable $\mathbf{P}_i^n$ that has joint density $g(v, P, \mu)$ with v.

(ii) assumes that conditional on v, $\mathbf{P}_i^n$ and Δs_i are independent, and

(iii) makes the right price conjecture in the large economy, that is, $\mathbf{P}^n(a) \xrightarrow[n\to\infty]{\mathbb{P}} \mathbf{P}$, for all $a \in [0, 1)$.

We refer to the joint density of fundamentals and prices, $g(v, P, \mu)$, as the *price belief*. We make this belief symmetric across agents, a natural assumption given that it applies to quantities common to all agents.[42] Highlighting that we have defined the price belief using $\mu \equiv 1/n$ as an index for the sequence of economies instead of n, we also impose the following restrictions on it.[43]

Assumption 6. *The density $g(v, p, \mu)$ is continuously differentiable in μ for every $v \in \mathbb{R}$ and $P \in \mathbb{R}$. Moreover, for any fixed P, there exist constants A_1, $A_2 > 0$ and k_1, $k_2 > 0$ which may*

[41] The price and the signal cannot be independent unconditionally as they both reflect the fundamental v. Conditioning on v isolates the noise in both.

[42] Symmetry over price beliefs guarantees a unique solution for $\delta(a, P)$. Our main conclusions do not change without this symmetry, as our measures of market quality depend only on $\beta(a, P)$, which is uniquely determined even without symmetric price beliefs.

[43] Working with $\mu \in (0, 1)$, which is one-to-one with n, is more convenient because it has a bounded range.

depend on P, such that $g(v,P,\mu) < A_1\exp(-k_1v^2)$ and $g_\mu(v,P,\mu) < A_2\exp(-k_2v^2)$, for all $\mu \in (0,1)$.

With the price belief as above, our next Lemma characterizes the conditional density of fundamentals given the signal and price.

Lemma 4. *Suppose Assumptions 5 and 6 hold. The conditional density of v given $\mathbf{P}_i^n = P$ and $\Delta s_i = s$ can then be written as*

$$f_{v|\Delta s_i,\mathbf{P}_i^n}(v,s,P) = \frac{\exp\left(t(a_i)(sv - \frac{\mu}{2}v^2)\right) g(v,P,\mu)}{\int_{\mathbb{R}} \exp\left(t(a_i)(sy - \frac{\mu}{2}y^2)\right) g(y,P,\mu)dy}. \tag{26}$$

Lemma 4 demonstrates the conditional density of fundamentals given signals and price is completely determined by the price belief. As the conditional distribution of fundamentals in (26) is the only distribution we need to describe demand choice, we are now able to define demand optimality in the discrete economies.

Definition 4. *Given the realizations $\mathbf{P}_i^n = P$ and $\Delta s_i = s$, the demand of agent i in the nth discrete economy is **optimal** if it solves*[44]

$$x_i^*(s,P) = \arg\max_x \int_{\mathbb{R}} u\Big(W_0(a_i) + x\left(V(v) - P\right), a_i\Big) f_{v|\Delta s_i,\mathbf{P}_i^n}(v,s,P)dv. \tag{27}$$

Equation (27) defines a standard problem of maximizing expected utility, with fundamentals distributed as in Lemma 4. We maintain Assumptions 5 and 6 from here on.

Our last task in this section is to define optimal cumulative demand in the large economy as a limit of the same object in finite economies. To do so, we first establish if and when such limit exists. Hereafter, we denote $\mathbb{E}[\cdot|\mathbf{P} = P]$ as $\mathbb{E}[\cdot|P]$.

Lemma 5. *For a fixed realization $\mathbf{P} = P$, let*

$$X(a,P) = \operatorname*{plim}_{n\to\infty} \sum_{i:a_i<a} x_i^*(\Delta s_i, P) \tag{28}$$

[44] Given that preferences are strictly concave, the maximum in (27) is unique.

with $x_i^*(\cdot)$ as in (27). This limit exists if and only if $\mathbf{P}$ *satisfies*

$$\mathbb{E}[V(v)|P] = P. \tag{29}$$

If (29) holds, $X(a, P)$ satisfies Assumption 1 and there exist deterministic functions $\beta(a, P)$ and $\delta(a, P)$ such that

$$X(a, P) = \int_0^a \beta(b, P) ds(b) + \int_0^a \delta(b, P) db.$$

As the above result establishes, optimal cumulative demand is well-defined if and only if the market is weak-form efficient as in (29). In intuitive terms, the large market cannot accommodate non-infinitesimal trades from infinitely many agents: if, for example, $\mathbb{E}[V(v)|P] > P$ holds, all agents would buy finite amounts, making the cumulative demand explode. Consequently, price functions that violate (29) are not viable candidates for equilibrium, and thus we can focus on those that do satisfy (29).

Definition 5. *The limiting cumulative demand $X(a, P)$ in (28) is* ***optimal*** *given $\mathbf{P} = P$ for those price functions $\mathbf{P}$ which satisfy (29).*

To summarize, Definition 5 not only describes the notion of demand optimality in the large economy, but it also establishes that this notion is well-defined. Namely, an optimal cumulative demand exists, and it satisfies all necessary equilibrium restrictions, such as Assumption 1 and Lemma 1.

We note that the equilibrium concept is now well-defined. By specifying optimality in terms of cumulative demand, we have made Definition 1 operational. Theorem 1, rigorously proved in the Appendix, establishes that the equilibrium exists and is unique.

11 A summary of extensions and robustness exercises

This section summarizes additional robustness checks and model extensions, with full analyses presented in the Appendix.

- Section D shows that our main results are unique to the CHILE framework and do not extend to traditional large-economy models based on law-of-large-numbers (LLN) aggregation, such as Hellwig (1980), Admati (1985), and Peress (2004). The key insight is an aggregation result: even when investors differ in wealth, precision, and preferences, the economy remains observationally equivalent to one with homogeneous agents. As a result, Robin Hood variations that preserve aggregate trading intensity have no effect on market quality—unlike in CHILE.

- Section E incorporates noise traders into our main model. The resulting equilibrium captures both the classical mechanism—where inequality improves information efficiency via greater aggregate trading intensity—and the novel mechanism introduced in this paper, where inequality disrupts signal aggregation. The key result is that the latter effect dominates, making our core conclusion—that transferring wealth from rich to poor improves information efficiency but reduces liquidity—robust to the presence of noise.

- Section F compares three equilibrium concepts: (i) Bayes-Nash Equilibrium, where traders internalize both their price impact and their impact on the informational content of prices; (ii) Rational-Expectations Equilibrium, where they internalize only the latter; and (iii) the price-taking equilibrium, where both effects are ignored. We show that our main results—wealth redistribution from rich to poor improves information efficiency but lowers liquidity—hold under both competitive Rational Expectations and Bayes-Nash, and are not artifacts of the price-taking equilibrium concept.

12 Literature review

There are several branches of literature related to our paper. First, there is literature on REE models that go beyond the CARA-Normal framework.[45] Breon-Drish (2015) extends the CARA-Normal framework beyond normality in a single asset setup. Chabakauri et al.

[45] CARA-Normal framework is also used to model markets where traders have market power (see Rostek and Yoon (2020) for a review). Glebkin et al. (2023a) and Glebkin, Malamud, and Teguia (2023b) allow for, respectively, non-normal payoffs in a setup with CARA traders and non-normal payoffs in a setup that also allows for non-CARA utilities. These papers abstract from information frictions that are central to this paper.

(2022) further extends Breon-Drish (2015) by allowing for multiple assets. Albagli, Hellwig, and Tsyvinski (2021) consider a setup with general distribution and risk-neutral traders subject to position limits. All of these papers assume CARA utility and so abstract away from wealth effects that are central to our paper.[46]

Malamud (2015) considers an REE model with a continuum of assets. Central to the tractability of his framework is the assumption of market completeness.[47] In contrast, we have one asset and a continuum of states of the world, hence our market is incomplete. One of the central results in Malamud (2015) is that with non-CARA utility, the equilibrium is fully revealing.[48] In contrast, in our incomplete market setup, there is no full revelation for any utility function, thanks to the aggregate price noise.

Peress (2004) was the first (to our knowledge) to study wealth effects in noisy REE.[49] His model features log-normally distributed payoffs and non-CARA utilities. The key difference from our paper is that Peress (2004) relies on a "small risk" approximation, where the riskiness of the asset is small. In his limit, the variance of risky asset return is zero, making such an approach not suitable for quantitative work (as it would be hard to match variance). Our approximation is essentially "small information." In contrast to Peress (2004), in our model the asset stays risky even in the limit. Beyond different limit behaviors, our paper differs from Peress (2004) in several ways. First, in our model, the equilibrium quantities are affected by absolute risk aversion and absolute prudence, whereas in Peress (2004) only risk aversion matters. What is more; conditional skewness plays a role in our model, but not in Peress (2004). Finally, our model allows for generally-distributed asset payoffs.

Our paper is also related to asset pricing literature studying the implications of heterogeneity in preferences and wealth for asset prices. Examples include Dumas (1989), Gârleanu and Panageas (2015) and Gomez et al. (2016)–see Panageas (2020) for a review. This literature focuses on the implications of wealth heterogeneity on risk premia and risk-free rates but ab-

[46]Here, we consider risk-neutral preferences as a special case of CARA with risk aversion equal to zero.

[47]Relatedly, DeMarzo and Skiadas (1998) and DeMarzo and Skiadas (1999) analyze REE models, where the market is *quasi-complete.*

[48]See Theorem 2.1 in Malamud (2015). See also Chabakauri (2024) who shows that the results about full revelation with non-CARA utility are more nuanced.

[49]The wealth effect can arise even in CARA model, because the wealth may affect the tightness of financial constraints, as in Glebkin et al. (2021).

stracts away from informational frictions and does not derive implications for market quality that are central to our paper.

Next, our paper is also related to the literature on mean-field games. See Lasry and Lions (2007) and Achdou, Han, Lasry, Lions, and Moll (2022) and, for a review, Guéant, Lasry, and Lions (2011). As Achdou et al. (2022) note: "The name (Mean Field Games) comes from an analogy to the continuum limit taken in 'Mean Field theory' which approximates large systems of interacting particles by assuming that these interact only with the statistical mean of other particles." This analogy holds in our model. The effect of other traders on a trader of interest in our economy is summarized by several statistics of the cross-sectional distribution of traders' characteristics. These statistics can be viewed as a "mean field" that influences each trader's equilibrium behavior. As in Mean Field theory, other traders do not affect a trader of interest directly, but only through their (infinitesimal) contribution to the mean field.

On the technical side, our paper is also related to literature that uses stochastic calculus tools outside the domain of continuous time finance and economics. Examples include Malamud (2015) who models the noise in a continuum of assets as a cross-sectional stochastic process; Gârleanu, Panageas, and Yu (2015) who use a Brownian bridge to represent the dividends for firms located on a circle; and Glebkin et al. (2021) who use stochastic calculus techniques to derive a marginal value of information in a static model.[50] Finally, the most closely related paper is Avdis (2018), which introduces a model with continuous heterogeneous information, albeit with CARA preferences and, as a result, without wealth effects.

13 Conclusion

We introduce a new asymmetric-information asset-pricing framework called "Continuous-and-Heterogeneous Information in a Large Economy" (CHILE). In this economy, we study perfect competition with rich agent heterogeneity, arbitrary preferences, and general payoff distributions. A unique equilibrium features all quantities in closed form. Leveraging the tractability of our model and its ability to work with wealth effects, we show how changes in the distribution

[50] There is also a related econometric literature on the unit roots. A good example is Phillips (1987).

of wealth affect different aspects of market quality: information efficiency and liquidity.

There are many potentially fruitful extensions. While we focus on a competitive CHILE equilibrium in this paper, in ongoing and preliminary work we show that the strategic equilibrium has a different limit. Moreover, even though we model markets as uniform-price auctions, our techniques can also find applications in discriminatory-price auctions. Finally, our framework can help study several interesting environments, such as those with dynamic trading, feedback effects, and endogenous growth.

References

Yves Achdou, Jiequn Han, Jean-Michel Lasry, Pierre-Louis Lions, and Benjamin Moll. Income and wealth distribution in macroeconomics: A continuous-time approach. The review of economic studies, 89(1):45–86, 2022. 41

Anat R Admati. A noisy rational expectations equilibrium for multi-asset securities markets. Econometrica: Journal of the Econometric Society, pages 629–657, 1985. 6, 39, 79

Elias Albagli, Christian Hellwig, and Aleh Tsyvinski. Dispersed information and asset prices. 2021. 40

Efstathios Avdis. Risk seekers: trade, noise, and the rationalizing effect of market impact on convex preferences. FTG Working Paper Series, (No. 00051-00), 2018. URL https://financetheory.org/public/storage/working_paper/00051-00.pdf. 12, 41

Efstathios Avdis, Sergei Glebkin, and Joel Peress. On competitive equilibria in large economies. Working paper, 2025. 2, 91

Jennie Bai, Thomas Philippon, and Alexi Savov. Have financial markets become more informative? Journal of Financial Economics, 122(3):625 – 654, 2016. ISSN 0304-405X. 30

Snehal Banerjee, Jesse Davis, and Naveen Gondhi. When transparency improves, must prices reflect fundamentals better? The Review of Financial Studies, 31(6):2377–2414, 2018. 31

Itzhak Ben-David, Francesco Franzoni, Rabih Moussawi, and John Sedunov. The granular nature of large institutional investors. Management Science, 67(11):6629–6659, 2021. 3, 30

Fischer Black. Studies of stock price volatility changes. Proceedings of the 1976 Meetings of the American Statistical Association, Business and Economic Statistics Section, pages 177–181, 1976. 84

Bradyn Breon-Drish. On existence and uniqueness of equilibrium in a class of noisy rational expectations models. The Review of Economic Studies, 82(3):868–921, 2015. doi: 10.1093/restud/rdv012. URL http://restud.oxfordjournals.org/content/82/3/868.abstract. 6, 11, 12, 39, 40

John Y Campbell and Luis M Viceira. Strategic asset allocation: portfolio choice for long-term investors. Clarendon Lectures in Economic, 2002. 23

Georgy Chabakauri. Informational efficiency and asset prices in large markets. Available at SSRN, 2024. 40

Georgy Chabakauri, Kathy Yuan, and Konstantinos E Zachariadis. Multi-asset noisy rational expectations equilibrium with contingent claims. The Review of Economic Studies, 89(5): 2445–2490, 2022. 6, 11, 39

Tarun Chordia, Richard Roll, and Avanidhar Subrahmanyam. Market liquidity and trading activity. The journal of finance, 56(2):501–530, 2001. 29

John Cochrane. Asset pricing: Revised edition. Princeton university press, 2009. 14

Samuel N Cohen and Robert James Elliott. Stochastic calculus and applications, volume 2. Springer, 2015. 49

Eduardo Dávila and Cecilia Parlatore. Identifying price informativeness. working paper, 2023. 24

P. DeMarzo and C. Skiadas. Aggregation, determinacy, and informational efficiency for a class of economies with asymmetric information. Journal of Economic Theory, 80(1):123–152, 1998. 40

P. DeMarzo and C. Skiadas. On the uniqueness of fully informative rational expectations equilibria. Economic Theory, 13(1):1–24, 1999. 40

Oolivier Dessaint, Thierry Foucault, and Laurent Fresard. Does alternative data improve financial forecasting? the horizon effect. The Journal of Finance, 79(3):2237–2287, 2024. 24

Jérôme Dugast and Thierry Foucault. Data abundance and asset price informativeness. Working paper, 2017. 31

Bernard Dumas. Two-person dynamic equilibrium in the capital market. The Review of Financial Studies, 2(2):157–188, 1989. 40

Maryam Farboodi, Adrien Matray, Laura Veldkamp, and Venky Venkateswaran. Where has all the data gone? The Review of Financial Studies, 35(7):3101–3138, 2022a. 29, 30

Maryam Farboodi, Dhruv Singal, Laura Veldkamp, and Venky Venkateswaran. Valuing financial data. Technical report, National Bureau of Economic Research, 2022b. 23, 79

Xavier Gabaix. The granular origins of aggregate fluctuations. Econometrica, 79(3):733–772, 2011. 1

Xavier Gabaix and Ralph SJ Koijen. In search of the origins of financial fluctuations: The inelastic markets hypothesis. Technical report, National Bureau of Economic Research, 2021. 1

Xavier Gabaix and Ralph SJ Koijen. Granular instrumental variables. Journal of Political Economy, 132(7):2274–2303, 2024. 1

Nicolae Gârleanu and Stavros Panageas. Young, old, conservative, and bold: The implications of heterogeneity and finite lives for asset pricing. Journal of Political Economy, 123(3): 670–685, 2015. 40

Nicolae Gârleanu, Stavros Panageas, and Jianfeng Yu. Financial entanglement: A theory of incomplete integration, leverage, crashes, and contagion. American Economic Review, 105(7):1979–2010, July 2015. doi: 10.1257/aer.20131076. URL http://www.aeaweb.org/articles?id=10.1257/aer.20131076. 41

Sergei Glebkin and John Chi-Fong Kuong. When large traders create noise. Journal of Financial Economics, 150(2):103709, 2023. 31

Sergei Glebkin, Naveen Gondhi, and John Chi-Fong Kuong. Funding constraints and informational efficiency. The Review of Financial Studies, 34(9):4269–4322, 2021. 12, 40, 41

Sergei Glebkin, Semyon Malamud, and Alberto Teguia. Illiquidity and higher cumulants. The Review of Financial Studies, 36(5):2131–2173, 2023a. 25, 39

Sergei Glebkin, Semyon Malamud, and Alberto Teguia. Strategic trading with wealth effects. 2023b. 39

Sergei Glebkin, Semyon Malamud, and Alberto Teguia. Benign granularity in asset markets. Working paper, 2025. 6

Arthur Stanley Goldberger. A course in econometrics. Harvard University Press, 1991. 19

Matthieu Gomez et al. Asset prices and wealth inequality. Unpublished paper: Princeton, 2016. 40

Sanford Grossman. On the efficiency of competitive stock markets where trades have diverse information. The Journal of finance, 31(2):573–585, 1976. 17, 20

Sanford J. Grossman and Joseph E. Stiglitz. On the impossibility of informationally efficient markets. The American Economic Review, 70(3):393–408, June 1980. 4

Olivier Guéant, Jean-Michel Lasry, and Pierre-Louis Lions. Mean field games and applications. In Paris-Princeton lectures on mathematical finance 2010, pages 205–266. Springer, 2011. 41

Martin F. Hellwig. On the aggregation of information in competitive markets. Journal of Economic Theory, 22(3):477–498, 1980. URL http://linkinghub.elsevier.com/retrieve/pii/0022053180900563. 1, 2, 5, 6, 9, 12, 13, 16, 20, 35, 39, 79, 81

Marcin Kacperczyk, Jaromir Nosal, and Savitar Sundaresan. Market power and price informativeness. Review of Economic Studies, page rdae077, 2024. 30

Pablo Kurlat and Laura Veldkamp. Should we regulate financial information? Journal of Economic Theory, 158:697–720, 2015. 5, 91

Albert S. Kyle. Continuous auctions and insider trading. Econometrica, 53(6):1315–1335, 1985. URL http://www.jstor.org/stable/1913210. 25

Albert S. Kyle. Informed speculation with imperfect competition. Review of Economic Studies, 56:317–356, 1989. 6, 25, 99, 100

Jean-Michel Lasry and Pierre-Louis Lions. Mean field games. Japanese journal of mathematics, 2(1):229–260, 2007. 41

Robert S. Liptser and Albert N. Shiryaev. Statistics of Random Processes: I. General Theory. Springer, 2001. 8, 49, 50

Dmitry Makarov and Astrid V Schornick. A note on wealth effect under cara utility. Finance Research Letters, 7(3):170–177, 2010. 5, 91

Semyon Malamud. Noisy arrow-debreu equilibria. Available at SSRN 2572881, 2015. 6, 40, 41

Semyon Malamud and Marzena Rostek. Decentralized exchange. American Economic Review, 107(11):3320–3362, 2017. 25

Thomas H McInish and Robert A Wood. An analysis of intraday patterns in bid/ask spreads for nyse stocks. the Journal of Finance, 47(2):753–764, 1992. 30

Roxana Mihet. Financial technology and the inequality gap. Swiss Finance Institute Research Paper, (21-04), 2018. 91

Roxana Mihet. Financial information technology and the inequality gap. 2022. 5, 23, 79

P. Milgrom and N. Stokey. Information, trade and common knowledge. Journal of Economic Theory, 26(1):17–27, 1982. 17

Stavros Panageas. The implications of heterogeneity and inequality for asset pricing. Foundations and Trends in Finance, 12(3):199–275, 2020. ISSN 1567-2395. doi: 10.1561/0500000057. URL http://dx.doi.org/10.1561/0500000057. 1, 40

Joel Peress. Wealth, information acquisition, and portfolio choice. The Review of Financial Studies, 17(3):879–914, 2004. 1, 5, 6, 23, 39, 40, 79

Joel Peress. Erratum: Wealth, information acquisition and portfolio choice, a correction. The Review of Financial Studies, 24(9):3187–3195, 2011. 23, 79, 80

Joel Peress. Learning from stock prices and economic growth. The Review of Financial Studies, 27(10):2998–3059, 2014. 6

Peter CB Phillips. Time series regression with a unit root. Econometrica: Journal of the Econometric Society, pages 277–301, 1987. 41

Daniel Revuz and Marc Yor. Continuous martingales and Brownian motion, volume 293. Springer Science & Business Media, 2013. 55

Marzena Rostek and Marek Weretka. Price inference in small markets. Econometrica, 80(2): 687–711, 2012. 24

Marzena Rostek and Marek Weretka. Dynamic thin markets. The Review of Financial Studies, 28(10):2946–2992, 2015. 25

Marzena J Rostek and Ji Hee Yoon. Equilibrium theory of financial markets: Recent developments. Available at SSRN 3710206, 2020. 39

Emmanuel Saez and Gabriel Zucman. Wealth inequality in the united states since 1913: Evidence from capitalized income tax data. The Quarterly Journal of Economics, 131(2):519–578, 2016. 3, 30

Paul A Samuelson. The fundamental approximation theorem of portfolio analysis in terms of means, variances and higher moments. The Review of Economic Studies, 37(4):537–542, 1970. 23

J Michael Steele. Stochastic calculus and financial applications, volume 1. Springer, 2001. 14

Jean Tirole. On the possibility of speculation under rational expectations. Econometrica, 50(5): 1163–1181, 1982. ISSN 00129682, 14680262. URL http://www.jstor.org/stable/1911868. 17

Dimitri Vayanos and Jiang Wang. Market liquidity—theory and empirical evidence. In Handbook of the Economics of Finance, volume 2, pages 1289–1361. Elsevier, 2013. 25

Robert E Verrecchia. Information acquisition in a noisy rational expectations economy. Econometrica: Journal of the Econometric Society, pages 1415–1430, 1982. 4

Yan Xiong, Liyan Yang, and Zexin Zheng. Institutional ownership concentration and informational efficiency. Available at SSRN, 2024. 3, 5, 30

Appendices

A Summary of notation

Notation	Explanation
	General mathematical notation
$X = \operatorname{plim}_{n\to\infty} X_n$ or $X_n \xrightarrow[n\to\infty]{\mathbb{P}} X$	X_n converges to X in probability
$dB(a)$	An increment of the Brownian motion
$u^{(l)}(\cdot)$	l-th derivative of $u(\cdot)$
a, b	Index of agents in the continuous economy
i, j	Index of agents in the discrete economy
n	Number of agents in the discrete economy
m_i	Mass of an agent in the discrete economy
$\mu = 1/n$	Average mass of an agent in the discrete economy
$\mathrm{Sk}[Y] = \frac{\mathbb{E}[(Y-E[Y])^3]}{\mathrm{Var}[Y]^{3/2}}$	Skewness of r.v. Y
$\Phi(\cdot)$	Standard normal cdf
$\mathcal{O}'\left(h(a)\right)\left[h^{\Delta}(a)\right]$	Gateaux derivative of an equilibrium object $\mathcal{O}$ with respect to a parameter $h(a)$ in the direction $h^{\Delta}(a)$
	Model variables
$v \sim N(0, \tau_v^{-1})$	Fundamental, with ex-ante precision τ_v
$V(v)$	Payoff function
$\Theta(P) = \bar{\theta} + \theta(P)$	Supply of risky asset
$ds(a) = v\,da + \frac{1}{\sqrt{t(a)}}dB(a)$	Trader a's signal, with precision $t(a)$
$\mathcal{F}_{b,c} = \sigma\left(\{s(z) - s(b)\}_{b\le z<c}\right)$	Information available to agents living in $[b, c)$
$\mathcal{F}_1 = \mathcal{F}_{0,1}$	Information in the entire economy
$dX(a) = \beta(a, P)ds(a) + \delta(a, P)da$	Demand of trader a; $\beta(a, P)$ is a's trading intensity

Notation	Explanation
$\rho(a) = -u''(a)/u'(a)$	Absolute risk aversion of trader a
$\pi(a) = -u'''(a)/u''(a)$	Absolute prudence of trader a
$s\left[\omega^H(a)\right] = \int_0^1 \omega^H(b)ds(b)$	Aggregate signal; $\omega^H(a)$ is a weighting function
$s_p = s[\omega(a)] = \int_0^1 \omega(a)ds(a)$	Equilibrium sufficient statistic; $\omega(a)$ is equilibrium weighting function
$\mathcal{P}(s_p)$	Equilibrium price function
$h(\cdot)$	The inverse of $\mathcal{P}(\cdot)$

B Derivations and proofs

B.1 The Representation Lemma

Proof of Lemma 1.

For ease of notation, we denote $X(a,P)$ simply as X_a. We fix some b and c, $0 \le b < c < 1$. Consider a filtration $\mathbb{F}_{b,c} = \{\mathcal{F}_{b,z}\}_{z\in[b,c)}$. Denote

$$\mu_b(z) = E[v|\mathcal{F}_{b,z}].$$

Note that

$$B^s(z) = \int_b^z \sqrt{t(a)}\,(ds(a) - \mu_b(a)da)$$

is a Brownian motion with respect to $\mathbb{F}_{b,c}$ (Liptser and Shiryaev (2001), Theorem. 8.1).

Consider a martingale

$$Y_z = E[X_c - X_b|\mathcal{F}_{b,z}].$$

By Martingale Representation Theorem (Cohen and Elliott (2015), Theorem 14.5.1), there exists an $\mathbb{F}_{b,c}$-adapted process $\zeta(a,P)$, denoted by $\zeta(a)$ hereafter, such that Y_z can be written as

$$Y_z = Y_0 + \int_b^c \zeta(a)\sqrt{t(a)}\,(ds(a) - \mu_b(a)da)\,.$$

Letting $z \to c$ and substituting the definition of Y we obtain

$$Y_c = X_c - X_b = E[X_c - X_b] + \int_b^c \zeta(a)\sqrt{t(a)}\,(ds(a) - \mu_b(a)da)\,.$$

Lemma 6 (to follow) shows that $\int_b^c \zeta(a)\sqrt{t(a)}\mu_b(a)da = \int_b^c \hat{\zeta}(a)ds(a)$ for some $\mathbb{F}_{b,c}$ - adapted process $\hat{\zeta}(a)$, explicitly given in the Lemma.[51]

[51]The process $\hat{\zeta}(a)$ may depend on P, but we drop the P argument for compactness of notation, as we do

Thus, we have

$$Y_c = X_c - X_b = E[X_c - X_b] + \int_b^c \beta(a, P)ds(a),$$

where $\beta(a, P) = \zeta(a)\sqrt{t(a)} - \hat{\zeta}(a)$.

It remains to prove that $\beta(a, P)$ must be deterministic. First, we argue that the process $\beta(a, P)$ is the same for any chosen b and c. Denote $\beta_{b,c}(a)$ the process $\beta(a, P)$ constructed for some fixed b and c (again, we drop the argument P for compactness of notation). Then, for any $0 \le b < c < 1$, we can write $X_1 - X_0 = X_1 - X_c + X_c - X_b + X_b - X_0 = E[X_1 - X_0] + \int_0^1 \beta_{0,1}(z)ds(z) = E[X_1 - X_0] + \int_0^b \beta_{0,b}(z)ds(z) + \int_b^c \beta_{b,c}(z)ds(z) + \int_c^1 \beta_{c,1}(z)ds(z)$. Then, we must have $\int_0^b(\beta_{0,1}(z) - \beta_{0,b}(z))ds(z) + \int_b^c(\beta_{0,1}(z) - \beta_{b,c}(z))ds(z) + \int_c^1(\beta_{0,1}(z) - \beta_{c,1}(z))ds(z) = 0$, for any b and c. This is only possible when $\beta_{b,c}(z) = \beta_{0,1}(z)$ for any $z \in [b, c)$.

Second, note that $\mathcal{F}_{b,c}$-measurable $\beta(z, P)$ is deterministic at $z = b$. Since b is arbitrary, $\beta(z, P)$ is deterministic for any $z \in [0, 1)$.

Finally, note that since $E[X_c - X_b]$ is differentiable, Leibniz's rule implies that $X(c) - X(b) = \int_b^c \delta(a, P)da$, with $\delta(a, P)$ being the derivative of $E[X(a, P)]$ with respect to a $\blacksquare$

Lemma 6. *We have* $\int_b^c \zeta(a)\sqrt{t(a)}\mu_b(a)da = \int_b^c \hat{\zeta}(a)ds(a)$, *where* $\hat{\zeta}(a)$ *is given by* (32).

Proof. Denote $\tau_b(z) = Var[v|\mathcal{F}_{b,z}]^{-1}$. By Kalman-Bucy filtering equations (Liptser and Shiryaev (2001), Theorem 10.1), we have

$$d\mu_b(z) = \frac{t(z)}{\tau_b(z)}\left(ds(z) - \mu_b dz\right) \tag{30}$$

$$\frac{d}{dz}\left(\tau_b(z)\right) = t(z).$$

Moreover, the SDE (30) can be solved explicitly, as follows:

$$\mu_b(z) = \frac{\int_b^z t(a)ds(a)}{\tau_v + \int_b^z t(a)da}. \tag{31}$$

with the $\zeta(a)$.

Let, for this proof only, $h(a) = \zeta(a)/(1 + t(a)/\tau_b(a))$ and write

$$\begin{aligned}\int_b^c h(a)\sqrt{t(a)}\mu_b(a)da &= \mu_b(c)\int_b^c h(a)\sqrt{t(a)}da - \int_b^c h(a)\sqrt{t(a)}d\mu_b \\ &= \mu_b(c)\int_b^c h(a)\sqrt{t(a)}da - \int_b^c h(a)\sqrt{t(a)}\frac{t(a)}{\tau_b(a)}\left(ds(a) - \mu_b(a)da\right)\end{aligned}$$

We integrate by parts to obtain the first transition and then substitute $d\mu_b$ with (30).

Rearranging, we obtain

$$\begin{aligned}\int_b^c h(a)\sqrt{t(a)}\left(1 + \frac{t(a)}{\tau_b(a)}\right)\mu_b(a)da &= \mu_b(c)\int_b^c h(a)\sqrt{t(a)}da - \int_b^c h(a)\sqrt{t(a)}\frac{t(a)}{\tau_b(a)}ds(a) \\ &= \int_b^c \left(\frac{t(a)\int_b^c h(a)\sqrt{t(a)}da}{\tau_v + \int_b^c t(a)da} - h(a)\sqrt{t(a)}\frac{t(a)}{\tau_b(a)}\right)ds(a).\end{aligned}$$

Here, in the second line, we substituted (31). Now note that, by construction, $h(a)(1 + t(a)/\tau_b(a)) = \zeta(a)$ and so the statement of the lemma holds with

$$\hat{\zeta}(a) = \frac{t(a)\int_b^c h(a)\sqrt{t(a)}da}{\tau_v + \int_b^c t(a)da} - h(a)\sqrt{t(a)}\frac{t(a)}{\tau_b(a)}. \tag{32}$$

■

B.2 Proof of Lemma 2

Proof of Lemma 2. By the properties of the Ito integral, $\int_0^1 \omega(a)/\sqrt{(}t(a))dB(a)$ is distributed normally with a mean of zero. By Ito isometry, the variance of the integral is given by $\int_0^1 \frac{w(a)^2}{t(a)}da$. The statements of the lemma then follow from the standard results on Bayes rule with normal random variables. ■

B.3 Proof of Lemma 4

Proof of Lemma 4.

The conditional density of v given $\mathbf{P}_i^n$ and Δs_i can be written as

$$f_{v|\Delta s_i,\mathbf{P}_i^n}(v,s,P) = \frac{f_{\Delta s_i|v}(v,s)g(v,P,\mu)}{\int_{\mathbb{R}} f_{\Delta s_i|v}(u,s)g(u,P,\mu)du} \tag{33}$$
$$= c(s,P,\mu)\exp((sv - mv^2/2)t(a_i))g(v,P,\mu).$$

In the first line, we write in the numerator the joint density $f_{v,\Delta s_i,\mathbf{P}_i^n}(\cdot) = f_{\Delta s_i|v,\mathbf{P}_i^n}(\cdot)f_{v|\Delta s_i,\mathbf{P}_i^n}(\cdot) = f_{\Delta s_i|v,\mathbf{P}_i^n}(s,v,P)g(v,P,\mu)$ and then use the conditional independence of Δs_i and $\mathbf{P}_i^n$ given v to substitute $f_{\Delta s_i|v,\mathbf{P}_i^n}(\cdot) = f_{\Delta s_i|v}(\cdot)$. In the second line, we substitute the density $f_{\Delta s_i|v}(\cdot)$. It follows from (25), that this density is Gaussian, with mean mv and variance $m/t(a_i)$. We then collect all terms that do not depend on v into a function $c(s,P)$ (this subsumes the denominator of the fraction in (33)). The function $1/\int_{\mathbb{R}} \exp((sv - mv^2/2)t(a_i))g(v,P,\mu)dv$ is finite since Assumption 6 implies that the integrand $\exp((sv - mv^2/2)t(a_i))g(v,P,\mu)$ is dominated by a function $A\exp(-kv^2)\exp((sv - mv^2/2)t(a_i))$, which is integrable. ■

B.4 Proof of Lemma 5 and Theorem 1

The structure of this section is as follows. First, we introduce the most general technical restrictions on the sequences of discrete economies. Second, we prove the aggregation lemma that is central to proofs here. Finally, we prove Lemma 5 and Theorem 1 with the proof split into parts, such as deriving the efficiency condition, deriving $\beta(a,P)$, etc. Subsections up to B.4.4 prove Lemma 5, whereas the rest of the sections prove Theorem 1.

In what follows, we use circumflex to denote objects in the discrete economy.

B.4.1 Technical conditions on the sequence of discrete economies

Here, we describe the most general sequence of discrete economies. The agents of the n-th discrete economy are located in n disjoint neighboring segments, forming a partition of the interval $[0,1)$. The size of subinterval i of the partition is m_i. A particular agent i, $i = 1,\ldots,n$, lives in segment $[a_i, a_{i+1})$, with $a_i = \sum_{j<i} m_j$. His initial wealth is $\hat{W}_0(a_i)$, his precision is $\hat{t}(a_i)$, and his utility function over terminal wealth W is $\hat{u}(W; a_i)$. His private signal is given by (25). We denote $m = \max_i m_i$ and we consider any partition such that $m \to 0$ as $n \to \infty$. We impose the following restrictions on the primitives of the discrete economies.

Assumption 7. *We have*

- *For every $a \in [0, 1)$, $\hat{u}(W, a)$ is thrice continuously differentiable in W in the neighborhood of $W = W_0(a)$.*
- *For $l = 1, 2, 3$, $\hat{u}^{(l)}(W_0(a), a)$ converges uniformly to $u^{(l)}(W_0(a), a)$, a.e., over $a \in [0, 1)$.*
- *For any fixed P, for small enough ϵ, and for all n there exist $M(v)$ such that $|\hat{u}^{(l)}(\hat{W}_0(a) + \epsilon(V(v) - P))| < M(v)$ for $l = 1, 2, 3$ and $M(v)$ is such that $\lim_{v\to\infty} \frac{\ln M(v)}{v^2} \leq 0$.*
- *$\hat{t}(a)$ converges uniformly to $t(a)$ and $\hat{W}(a)$ converges uniformly to $W(a)$, a.e., over $a \in [0, 1)$*

B.4.2 The Aggregation Lemma

Lemma 7. ***(Aggregation lemma)*** *Consider a sequence of processes $d\hat{s}^n(a) = v\, da + \frac{1}{\sqrt{\hat{t}^n(a)}} dB(a)$. Consider a sequence of functions $\hat{x}^n(P, s, m, a) : \mathbb{R} \times \mathbb{R} \times [0, 1] \times [0, 1] \to \mathbb{R}$ with partial derivatives $\hat{x}^n_s(P, s, m, a)$, $\hat{x}^n_{ss}(P, s, m, a)$ and $\hat{x}^n_m(P, s; m, a)$ that are continuous functions of s and m in the neighbourhood of $s = m = 0$ for every $P \in \mathbb{R}$ and $a \in [0, 1)$. Suppose that $1/\hat{t}^n(a)$, $\hat{x}^n_s(P, 0, 0, a)$, $\hat{x}^n_s(P, 0, 0, a)$, $\hat{x}^n_{ss}(P, 0, 0, a)$ and $\hat{x}^n_m(P, 0, 0, a)$ are continuous functions of a, a.e., and converge uniformly, a.e., over $a \in [0, y]$ for every $P \in \mathbb{R}$ and $0 < y < 1$. Denote the respective limits by $1/t(a)$, $x_s(P, a)$, $x_s(P, a)$, $x_{ss}(P, a)$ and $x_m(P, a)$. Let*

$$\beta(P, a) = x_s(P, a) \text{ and } \delta(P, a) = \frac{1}{2t(a)} x_{ss}(P, a) + x_m(P, a).$$

For a fixed $y \in (0,1)$ take a partition $[0,y] = \cup_{i=1}^{n}[a_i, a_i + m_i]$, and let $\bar{m} = \max_i m_i$, $\Delta s_i = vm_i + \frac{1}{\sqrt{\hat{t}(a_i)}}(B(a_i+m_i) - B(a_i))$. Assume that $\int_0^y \beta(a,P)^2/t(a)da < \infty$, $\int_0^y \beta(a,P)da < \infty$, and $\int_0^y \delta(a,P)da < \infty$ for every $P \in \mathbb{R}$. For any partition sequence such that $\bar{m} \to 0$ as $n \to \infty$, we have that

$$\sum_{i:\, a_i<y} (\hat{x}^n(P, \Delta s_i, m_i, a_i) - \hat{x}^n(P, 0, 0, a_i)) \xrightarrow[n\to\infty]{p} \int_0^y \beta(P,a)ds(a) + \int_0^y \delta(P,a)da.$$

Proof of Lemma 7. Fix P and let $\hat{x}(\Delta\hat{s}_i, m_i, a_i)$ denote $\hat{x}_n(P, \Delta\hat{s}_i, m_i, a_i)$. Thus, we don't indicate the dependence of $\hat{x}_n(\cdot)$ on P and n explicitly. The sequence $\hat{x}(\cdot)$ can be distinguished from its' limit $x(\cdot)$ by the presence of circumflex in the former. By Itô's Lemma,

$$\begin{aligned}\hat{x}(\Delta\hat{s}_i, m_i, a_i) - \hat{x}(0,0,a_i) &= \int_{a_i}^{a_{i+1}} \hat{x}_s\Big(\hat{s}(a) - \hat{s}(a_i), a - a_i, a_i\Big)\left(v\,da + dB(a)/\sqrt{\hat{t}^n(a_i)}\right) \\ &+ \int_{a_i}^{a_{i+1}} \left[\hat{x}_m\Big(\hat{s}(a) - \hat{s}(a_i), a - a_i, a_i\Big) + \frac{1}{2\hat{t}(a_i)}\hat{x}_{ss}\Big(\hat{s}(a) - \hat{s}(a_i,n), a - a_i, a_i\Big)\right] da.\end{aligned}$$

For the n-th element of a partition sequence, denote $\hat{a}_n(a) = \sum_{i=1}^{n-1} a_i \mathbb{1}_{[a_i, a_{i+1})}(a)$, so that for any $a \in [0,1)$, $\hat{a}_n(a)$ equals the left point of the segment $[a_i, a_{i+1})$ that a belongs to. Similarly, denote $\check{a}_n(a) = \sum_{i=1}^{n-1} a_{i+1} \mathbb{1}_{[a_{i-1}, a_{i+1})}(a)$, so that for any $a \in [0,1)$, $\check{a}_n(a)$ equals the right point of the segment $[a_i, a_{i+1})$ that a belongs to. With this notation, we can write

$$\begin{aligned}&\sum_{i:\, a_i<y} \hat{x}(\Delta\hat{s}_i, m_i, a_i) - \hat{x}(0,0,a_i) \\ &\quad= \int_0^{\check{a}_n(y)} \hat{x}_s\Big(s(a) - s(\hat{a}_n(a)), a - \hat{a}_n(a), \hat{a}_n(a)\Big)\left(v\,da + dB(a)/\sqrt{\hat{t}^n(\hat{a}_n(a))}\right) \\ &\qquad+ \int_0^{\check{a}_n(y)} \hat{x}_m\Big(s(a) - s(\hat{a}_n(a)), a - \hat{a}_n(a), \hat{a}_n(a)\Big)da \\ &\qquad+ \int_0^{\check{a}_n(y)} \frac{1}{2\hat{t}^n(\hat{a}_n(a))}\hat{x}_{ss}\Big(s(a) - s(\hat{a}_n(a)), a - \hat{a}_n(a), \hat{a}_n(a)\Big)da. \qquad (34)\end{aligned}$$

The proof is concluded by passing to the limit in the integrals above. Note that as $n \to \infty$,

$\hat{a}_n(a) \rightrightarrows a$ and $\check{a}_n(a) \rightrightarrows a$ over $a \in [0,1)$.[52] Then, in the limit, the partials of $\hat{x}(\cdot)$ are substituted by the respective partials of $x(\cdot)$, and the terms $\check{a}_n(y)$, $s(a) - s(\hat{a}_n(a))$, and $a - \hat{a}_n(a)$ are respectively substituted by y, 0, and 0. We obtain

$$\sum_{i:\, a_i<y} \left(\hat{x}(\Delta\hat{s}_i, m_i, a_i) - \hat{x}(0,0,a_i)\right) \xrightarrow[n\to\infty]{p} \int_0^y x_s(P,a)ds(a) + \int_0^y \left[x_m(P,a) + \frac{1}{2t(a)}x_{ss}(P,a)\right] da.$$

The remainder of the proof justifies passing to the limit in (34). Note that since the points of discontinuity of $1/\hat{t}^n(a)$, $\hat{x}^n_s(P,0,0,a)$, $\hat{x}^n_s(P,0,0,a)$, $\hat{x}^n_{ss}(P,0,0,a)$ and $\hat{x}^n_m(P,0,0,a)$ have a Lebesgue measure zero, we can assume, without loss of generality that these functions are continuous over $a \in [0,y]$. Similarly, we can assume that these functions converge uniformly over $a \in [0,y]$. For the Lebesgue integrals in (34), passing to the limit is justified by the Uniform Convergence Theorem. The hypotheses of the UCT hold since Lemma 8 (to follow) implies that the sequence of integrands converges uniformly to $x_s(P,a)v + x_m(P,a) + \frac{1}{2t(a)}x_{ss}(P,a)$. For the stochastic integral, the passage to the limit follows from Theorem IV.2.12 in Revuz and Yor (2013).[53] The hypotheses of the Theorem hold, because the integrands $\hat{y}^n(\cdot) = \hat{x}_s(s(a) - s(\hat{a}_n(a)), a - \hat{a}_n(a), \hat{a}_n(a))/\sqrt{\hat{t}^n(\hat{a}^n(a))}$ are bounded (by the Extreme Value Theorem) and Lemma 8 implies that the sequence $\hat{y}^n(\cdot)$ converges uniformly. Then, the sequence $\hat{y}^n(\cdot)$ is uniformly bounded. ■

Lemma 8. *Consider a continuous function $Y(a) : [0,1) \to \mathbb{R}^d$. Consider a sequence of functions $f_n(y,a) : \mathbb{R}^d \times [0,1) \to \mathbb{R}$ such that: (1) $f_n(0,a)$ converges uniformly to $f(0,a)$ over $a \in [0,1)$, (2) $f_n(y,a)$ is continuous in y in the neighborhood of 0 for every $a \in [0,1)$ and for every n, and (3) $f(0,a)$ is continuous in a. Take a partition $[0,y] = \cup_{i=1}^n [a_i, a_i + m_i]$, and let $\bar{m} = \max_i m_i$ and $\hat{a}_n(a) = \sum_{i=1}^{n-1} a_i \mathbb{1}_{[a_i,a_{i+1})}(a)$. For any partition sequence such that $\bar{m} \to 0$ as $n \to \infty$, we have that $f_n\Big(Y(a) - Y(\hat{a}_n(a)), \hat{a}_n(a)\Big)$ converges uniformly to $f\Big(0,a\Big)$*

[52] The notation $\rightrightarrows$ stands for uniform convergence.

[53] We are grateful to Christoph Frei for suggesting this reference.

over $a \in [0, y]$.

Proof. We need to establish that

$$\lim_{n\to\infty} \sup_{a\in[0,y]} \left| f_n\Big(Y(a) - Y(\hat{a}_n(a)), \hat{a}_n(a)\Big) - f\,(0, a) \right| = 0$$

Note that, for a given n,

$$\begin{aligned}
&\sup_{a\in[0,y]} \left| f_n\Big(Y(a) - Y(\hat{a}_n(a)), \hat{a}_n(a)\Big) - f_n\,(0, a) \right| = \\
&\max_i \sup_{a\in[a_i,a_{i+1}]} \left| f_n\Big(Y(a) - Y(\hat{a}_n(a)), \hat{a}_n(a)\Big) - f\,(0, a) \right| = \\
&\max_i \left| f_n\Big(Y(a_i^*) - Y(a_i), a_i\Big) - f\,(0, a_i^*) \right|
\end{aligned}$$

Here $a_i^* \in [a_i, a_{i+1}]$ is maximand in the optimization over $a \in [a_i, a_{i+1}]$ above. The a_i^* exists by Weiestrass's extreme value theorem. Note that since $a_i^* \in [a_i, a_{i+1}]$ and $a_{i+1} - a_i = m_i \to 0$ as $n \to \infty$, we have $a_i^* - a_i \to 0$ as $n \to \infty$. Note further that

$$\begin{aligned}
&\max_i \left| f_n\Big(Y(a_i^*) - Y(a_i), a_i\Big) - f\,(0, a_i^*) \right| \le \\
&\max_i \left| f_n\Big(Y(a_i^*) - Y(a_i), a_i\Big) - f_n\,(0, a_i) \right| + \max_i \left| f_n\Big(0, a_i\Big) - f\,(0, a_i) \right| + \max_i \left| f\Big(0, a_i\Big) - f\,(0, a_i^*) \right|.
\end{aligned} \tag{35}$$

The proof is concluded by noting that for any $\epsilon > 0$ there exists a large enough $N(\epsilon)$ such that for all $n > N(\epsilon)$, each of the three terms in (35) are less then $\epsilon/3$. Indeed, if, on the contrary, there exists an $\epsilon > 0$ such that $\max_i \left| f_n\Big(Y(a_i^*) - Y(a_i), a_i\Big) - f_n\,(0, a_i) \right| > \epsilon/3$ for all large enough n, a contradiction with the hypothesis (2) would occur. Similarly, if $\max_i \left| f_n\Big(0, a_i\Big) - f\,(0, a_i) \right| > \epsilon/3$ a contradiction with the hypothesis (1) would occur. Finaly, if $\max_i \left| f\Big(0, a_i\Big) - f\,(0, a_i^*) \right| > \epsilon/3$, a contradiction with the hypotheis (3) would occur. ■

B.4.3 Optimal demand in a discrete economy

Consider a trader a living in the interval $[a, a+m)$ in the n-th discrete economy. The price belief in that economy is $g(v, P, \mu)$. Denote trader a'a signal realization by s. We denote the optimal demand of the trader a by $x^*(P, s, m, a, \mu)$. (Note that we can index economies by $\mu = 1/n$ instead of n as there is a one-to-one correspondence between the two.) Given the strict concavity of the utility function, the optimality condition (27) holds if and only if $x^*(P, s, m, a, \mu)$ satisfies the first-order condition

$$\int \hat{u}' \left(\hat{W}_0(a) + x^*(P, s, m, a, \mu)(V(v) - P),\, a \right) (V(v) - P) \\ \cdot \exp((sv - mv^2/2)\hat{t}(a))g(v, P, \mu)dv = 0. \quad (36)$$

Note that we dropped the constant $c(s, P, \mu)$ in the conditional density as it does not affect maximization.

For the n-th discrete economy, let $\hat{X}(y)$ be the cumulative demand, defined as

$$\hat{X}(y) = \sum_{i:\, a_i < y} x^*(P, \Delta s_i, m_i, a_i, \mu).$$

We can decompose the cumulative demand as follows

$$\begin{aligned} \hat{X}(y) = & \sum_{i:\, a_i < y} x^*(P, \Delta s_i, m_i, a_i, \mu) = \\ & \sum_{i:\, a_i < y} (x^*(P, \Delta s_i, m_i, a_i, \mu) - x^*(P, 0, 0, a_i, \mu)) + \\ & \sum_{i:\, a_i < y} (x^*(P, 0, 0, a_i, \mu) - x^*(P, 0, 0, a_i, 0)) + \\ & \sum_{i:\, a_i < y} x^*(P, 0, 0, a_i, 0). \end{aligned}$$

By Aggregation Lemma, we have

$$\sum_{i:\,a_i<y}\left(x^*(P,\Delta s_i,m_i,a_i,\mu)-x^*(P,0,0,a_i,\mu)\right)\xrightarrow[n\to\infty]{\mathbb{P}}$$
$$\int_0^y x_s^*(P,0,0,a,0)ds(a)+\int_0^y\left(x_m^*(P,0,0,a,0)+\frac{1}{2}x_{ss}^*(P,0,0,a,0)\right)da \quad (37)$$

The hypotheses of the Lemma hold. The fact that $x_s^*(P,s,m,a,\mu)$, $x_{ss}^*(P,s,m,a,\mu)$ and $x_m^*(P,s,m,a,\mu)$ are continuous functions of s and m in the neighborhood of $s=m=0$ follows via the Implicit Function Theorem. We demostrate below that $x_s^*(P,0,0,a,\mu)$, $x_{ss}^*(P,0,0,a,\mu)$ and $x_m^*(P,0,0,a,\mu)$ converge uniformly to, respectively, $x_s^*(P,0,0,a,0)$, $x_{ss}^*(P,0,0,a,0)$ and $x_m^*(P,0,0,a,0)$ over $a\in[0,y]$, using Lemma 12 (to follow).

Similarly, by Aggregation Lemma, we have

$$\sum_{i:\,a_i<y}\left(x^*(P,0,0,a_i,\mu)-x^*(P,0,0,a_i,0)\right)\xrightarrow[n\to\infty]{p}\int_0^y x_\mu^*(P,0,0,a,0)da. \quad (38)$$

Similarly to the previous step, the hypotheses of the lemma follow via the Implicit Function Theorem and Lemma 12.

We show below that the limiting integrals in (37) and (38) are finite. Thus, we must have $\lim_{n\to\infty}\sum_{i:a_i<y}x^*(P,0,0,a_i,0)<\infty$ for the limit of $\hat{X}(y)$ to be well-defined. This gives rise to the efficiency condition, as we derive below.

B.4.4 Efficiency condition

We start by characterizing $x^*(P,a)=\lim_{n\to\infty}x^*(P,0,0,a,\mu)$. Substituting $m=s=0$ to (36) and taking the $n\to\infty$ limit we obtain

$$\int u'\left(W_0(a)+x^*(P,a)(V(v)-P),\,a\right)(V(v)-P)\cdot g(v,P,0)dv=0. \quad (39)$$

To get from (36) to (39), we first passed the limit inside the integral, which is justified by the Dominated Convergence Theorem. The hypotheses of the DCT hold since the Assumption 7 implies that the integrand in (36) admits an integrable majorant of the form $A\exp(-kv^2)$ with some constants A, $k \geq 0$, provided that $|x^*(\cdot)|$ is small enough. We then pass to the limiting functions (i.e., we go from $\hat{u}(\cdot)$ to $u(\cdot)$ and from $g(\cdot,\mu)$ to $g(\cdot,0)$), which the Uniform Limit Theorem justifies. The $|x^*(\cdot)|$ is indeed small enough and is, in fact, zero in the limit, as implied by the Lemma below.

Lemma 9. *For a fixed $y \in (0,1)$ take a partition $[0,y] = \cup_{i=1}^{n}[a_i, a_i+m_i]$, and let $m = \max_i m_i$. For any partition sequence such that $m \to 0$ as $n \to \infty$, we have that $\lim_{n\to\infty}\sum_{i:a_i<y} x^*(P,a_i) < \infty$ if, and only if, $x^*(P,a) = 0$ for any $a \in [0,y]$ and*

$$\int (V(v) - P) \cdot g(v,P,0)dv = 0. \tag{40}$$

Proof of Lemma 9. Suppose, on the contrary, that $\int (V(v) - P) \cdot g(v,P,0)dv > 0$. (The case of the opposite inequality is considered analogously and is omitted for brevity.) Then, for (39) to hold, $x^*(P,a_k)$ must be positive for any fixed $a_k < y$. Thus, all terms in the sum $\lim_{n\to\infty}\sum_{i:a_i<y} x^*(P,a_i)$ are positive. By continuity of $x^*(P,a)$ there exist a δ such that for all $a : |a - a_k| < \delta$ we have $x^*(P,a) > \epsilon$, for some $\epsilon > 0$ and some fixed k. We then have $\sum_{i:a_i<y} x^*(P,a_i) \geq \sum_{i:|a_i-a_k|<\delta} x^*(P,a_i)$. Since $\#\{i : |a_i - a_k| < \delta\} \to \infty$ and each $x^*(P,a_i) > \epsilon > 0$ we have that $\sum_{i:|a_i-a_k|<\delta} x^*(P,a_i) \to \infty$. A contradiction. ■

Note that (40) and the efficiency condition (29) are equivalent.

B.4.5 $\beta(a, P)$

By the Aggregation Lemma, $\beta(a, P)$ is given by the limit of $x_s^*(P, 0, 0, a, \mu)$ as $n \to \infty$. This partial can be computed by differentiating (36) implicitly, as follows

$$\lim_{n\to\infty} \frac{\partial}{\partial s} \int \hat{u}' \left(\hat{W}_0(a_i) + x^*(\cdot)(V(v) - P) \right) (V(v) - P) \cdot \exp((sv - mv^2/2)\hat{t}(a))g(v, P, \mu)dv = 0.$$

We then interchange the limit and differentiation with the integral, which, as in the previous steps, is justified by the Dominated Convergence Theorem, and evaluate the resulting expression at $s = m = \mu = 0$.

We get:

$$\int \lim_{n\to\infty} \frac{\partial}{\partial s} \left(\hat{u}' \left(\hat{W}_0(a) + x^*(\cdot)(V(v) - P) \right) (V(v) - P) \exp((sv - mv^2/2)\hat{t}(a))g(v, P, \mu) \right) dv =$$
$$x_s^*(P, 0, 0, a, 0)\, u''(W_0(a))(V(v) - P)^2 g(v, P, 0)+$$
$$u'(W_0(a))(V(v) - P)t(a)\, v\, g(v, P, 0). \quad (41)$$

The terms in the second line of (41) arise from differentiating $x^*(\cdot)$, whereas the terms in (41) arise from differentiating the density. We accounted for the fact that $x^*(P, 0, 0, a, 0)$ is zero (Lemma 9). The transitions $\lim_{n\to\infty} \hat{u}' (W_0(a) + x^*(\cdot)(V(v) - P)) = u'(W_0(a))$—and similarly for $\hat{u}''(\cdot)$—are due to the Uniform Limit Theorem. The convergence $\lim_{n\to\infty} x_s(P, 0, 0, a, \mu) = x_s(P, 0, 0, a, 0)$ is uniform over $a \in [0, y]$ by Lemma 12.

Integrating over v and accounting for the fact that $E[V(v)-P|P] = \int_{\mathbb{R}}(V(v)-P)g(v, P, 0)dv = 0$ (Lemma 9) yields

$$\beta(a, P) = \frac{t(a)}{\rho(a)} \frac{E[v(V(v) - P)|P]}{\mathrm{Var}[V(v)|P]}.$$

We summarize in the following lemma.

Lemma 10. *$\beta(a, P)$ is multiplicatively separable, that is, $\beta(a, P) = \beta_a(a)\beta_P(P)$ where*

$$\beta_a(a) = \frac{t(a)}{\rho(a)} \text{ and } \beta_P(P) = \frac{E[v(V(v) - P)|P]}{\text{Var}[V(v)|P]}. \tag{42}$$

If $v|\mathbf{P}_*$ is distributed normally with variance τ^{-1}, we can write

$$\beta_P(P) = \frac{\tau^{-1}E[V'(v)|P]}{\text{Var}[V(v)|P]}.$$

The last equation of the Lemma follows via integration by parts.[54] In the following part of the proof, we proceed by deriving the equilibrium price function. This derivation is greatly simplified by the multiplicative separability of $\beta(a, P)$.

B.4.6 The price function

Given that $\beta(a, P)$ is of the multiplicatively separable (Lemma 10), we have

$$\beta(a, P) = \beta_a(a)\beta_p(P).$$

The informational content of the price is summarized by

$$s_p = \frac{\int_0^1 \beta_a(a)ds(a)}{\int_0^1 \beta_a(a)da} = v + \int_0^1 \frac{\omega(a)}{\sqrt{t(a)}}dB(a).$$

[54]Indeed:

$$\begin{aligned}
E[v(V(v) - P)|P] &= \int_{\mathbb{R}} v(V(v) - P)\frac{1}{\sqrt{2\pi/\tau}} \exp\left(-\frac{(v - E[v|P])^2\tau}{2}\right)dv \\
&= \int_{\mathbb{R}} (v - E[v|P])(V(v) - P)\frac{1}{\sqrt{2\pi/\tau}} \exp\left(-\frac{(v - E[v|P])^2\tau}{2}\right)dv \\
&= -\tau^{-1}\int_{\mathbb{R}} (V(v) - P)d\left[\frac{1}{\sqrt{2\pi/\tau}} \exp\left(-\frac{(v - E[v|P])^2\tau}{2}\right)\right] \\
&= \tau^{-1}\int_{\mathbb{R}} \frac{1}{\sqrt{2\pi/\tau}} \exp\left(-\frac{(v - E[v|P])^2\tau}{2}\right)V'(v)dv.
\end{aligned}$$

Here

$$\omega(a) = \frac{\beta_a(a)}{\int_0^1 \beta_a(a)da}.$$

The inference from price is then the same as summarized in the Lemma 2. In particular, the conditional distribution of v given $\mathbf{P}_* = P$ is Normal with mean $\tau_p/\tau\, s_p$ and variance $1/\tau$. This means that v can be written as

$$v = \frac{\tau_p}{\tau} s_p + \frac{1}{\sqrt{\tau}} z.$$

Here z is a standard Normal random variable. The condition $E[V(v) - P|P] = 0$ (Lemma 9) can be then be rewritten

$$\int V\left(\frac{\tau_p}{\tau} s_p + \frac{z}{\sqrt{\tau}}\right) d\Phi(z) = P. \tag{44}$$

This gives the explicit expression for the function $\mathcal{P}(s_p)$. We summarize in the lemma below.

Lemma 11. *The equilibrium price function is given by* $\mathbf{P}_* = \mathcal{P}\left(s_p\right)$*, where* $s_p = v + \int_0^1 \frac{\beta_a(a)}{\sqrt{t(a)}} dB(a)$ *and* $\mathcal{P}(x)$ *is a strictly increasing function defined by*

$$\int V\left(\frac{\tau_p}{\tau} x + \frac{z}{\sqrt{\tau}}\right) d\Phi(z) = \mathcal{P}(x).$$

Moreover, $\tau = \tau_v + \tau_p$ *and* τ_p *is given by* $\tau_p = \frac{\left(\int_0^1 \beta(a)da\right)^2}{\int_0^1 \frac{\beta(a)^2}{t(a)} da}$*. Let* $h(P)$ *be the inverse of* $\mathcal{P}(\cdot)$*. For a function* $f(v, P)$ *we have*

$$E[f(v, P)|P] = \int f\left(\frac{\tau_p}{\tau} h(P) + \frac{z}{\sqrt{\tau}}, P\right) d\Phi(z).$$

The last expression of the lemma follows by the same change of variables that yields (44). It allows us to compute the conditional moments of $V(v)$. In particular, for k-th moment, we have:

$$\mathbb{E}[(V(v) - \mathbb{E}[V(v)|P])^k|P] = \mathbb{E}[(V(v) - P)^k|P] = \int \left(V\left(\frac{\tau_p}{\tau} h(P) + \frac{z}{\sqrt{\tau}}\right) - P\right)^k d\Phi(z). \tag{45}$$

B.4.7 $\delta(a,p)$

By Aggregation Lemma,

$$\delta(a,p)=\lim_{n\to\infty}\frac{1}{2t(a)}x_{ss}(P,0,0,a,\mu)+x_m(P,0,0,a,\mu)+x_\mu(P,0,0,a,\mu).$$

To compute these partials, we follow the same steps as in Section B.4.5: We differentiate the first-order condition in (36) implicitly and then take the $n\to\infty$ limit. We split the calculation into several steps.

Step 1. Calculating $\lim_{n\to\infty}\frac{1}{2t(a)}x_{ss}(P,0,0,a,\mu)+x_m(P,0,0,a,\mu)$.

Differentiating (36) twice with respect to s, dividing by $2t(a)$, adding the derivative of (36) with repect to m (i.e., we compute $\frac{1}{2t(a)}\frac{\partial^2}{\partial s^2}(36)+\frac{\partial}{\partial m}(36)$), passing to the limit we get:

$$\begin{aligned}\frac{1}{2t(a)}u'''(W_0(a))x_s^*(P,0,0,a,0)^2\int(V(v)-P)^3g(v,P,0)dv&\\ +u''(W_0(a))x_s^*(P,0,0,a,0)\int v(V(v)-P)^2g(v,P,0)dv&\\ +u''(W_0(a))\left(\frac{1}{2t(a)}x_{ss}^*(P,0,0,a,0)+x_m^*(P,0,0,a,0)\right)\int(V(v)-P)^2g(v,P,0)dv&=0.\end{aligned}$$

The convergence $\lim_{n\to\infty}x_{ss}(P,0,0,a,\mu)=x_s(P,0,0,a,0)$ and $\lim_{n\to\infty}x_m(P,0,0,a,\mu)=x_m(P,0,0,a,0)$ is uniform over $a\in[0,y]$ by Lemma 12.

We then note that $-u'''(W_0(a))/u''(W_0(a))=\pi(a)$, $x_s^*(P,0,0,a,0)=\beta(a,P)$, $\int(V(v)-P)^3g(v,P,0)dv=\mathbb{E}[(V(v)-\mathbb{E}[V(v)|P])^3|P]=\mathrm{Sk}[V(v)|P]\mathrm{Var}[V(v)|P]^{3/2}$ and $\int(V(v)-P)^2g(v,P,0)dv=\mathrm{Var}[V(v)|P]$. With this, we obtain

$$\begin{aligned}\frac{1}{2t(a)}x_{ss}^*(P,0,0,a,0)+x_m^*(P,0,0,a,0)=&\\ \frac{\beta(a,p)^2}{2t(a)}\pi(a)\frac{\mathrm{Sk}[V(v)|P]}{\mathrm{Var}[V(v)|P]}-\beta(a,p)\frac{E[v(V(v)-P)^2|P]}{\mathrm{Var}[V(v)|P]}.&\end{aligned}$$

Step 2. Calculating $\lim_{n\to\infty} x_\mu(P,0,0,a,\mu)$.

Differentiating (36) with respect to μ and passing to the limit we obtain

$$u'(W_0(a))\int (V(v)-P)g_\mu(v,P,0) + u''(W_0(a))x_\mu(P,0,0,a,0)\int (V(v)-P)^2 g(v,P,0) = 0$$

Now note that $-u''(W_0(a))/u'(W_0(a)) = \rho(a)$, $\int (V(v)-P)^2 g(v,P,0)dv = \text{Var}[V(v)|P]$ and

$$\int (V(v)-P)g_\mu(v,P,0) = \frac{\partial}{\partial\mu}\int (V(v)-P)g(v,P,\mu)dv\bigg|_{\mu=0} = \frac{\partial}{\partial\mu}E[V(v)-P|P] = \psi(P).$$

With this, we obtain

$$x_\mu(P,0,0,a,0) = \frac{\psi(P)}{\rho(a)\text{Var}[V(v)|P]}.$$

Combining everything together, we obtain the following expression for $\delta(a,p)$:

$$\delta(a,P) = \frac{\beta(a,p)^2}{2t(a)}\pi(a)\frac{\text{Sk}[V(v)|P]}{\text{Var}[V(v)|P]} - \beta(a,p)\frac{E[v(V(v)-P)^2|P]}{\text{Var}[V(v)|P]} + \frac{\psi(P)}{\rho(a)\text{Var}[V(v)|P]}.$$

The function $\psi(P)$ can be uniquely pinned down from the market clearing condition

$$\int \delta(a,P)da + h(P)\int \beta(a,P)da = \Theta(p).$$

B.4.8 Existence

Our derivation so far shows that an equilibrium is unique if it exists. We still need to show that a sequence of discrete economies satisfying the technical conditions in Section B.4.1 exists.

To this end, we consider a sequence of economies with the primitives given by $\hat{t}(a) = t(a)$, $\hat{W}(a) = W(a)$ and $\hat{u}''(w;a) = \text{Tr}(u''(w;a), u''(W_0(a)-\epsilon;a), u''(W_0(a)+\epsilon,a))$ for some fixed ϵ, where $\text{Tr}(x,l_1,l_2)$ denotes the function that truncates x to lower limit $\min\{l_1,l_2\}$ and upper limit $\max\{l_1,l_2\}$.

We next consider a uniform partition of the unit interval with $a_i = (i-1)/n$ and $m_i = \mu = 1/n$ for all i. To define the price beliefs, consider

$$s_{p,i}^n \equiv v + k_i \int_{-i} \beta(a)/\sqrt{t(a)}dB(a).$$

where

$$k_i = \sqrt{\frac{\int_0^1 \beta(a)^2/t(a)da}{\int_{-i} \beta(a)^2/t(a)da}}.$$

Here $-i \equiv [0,1) \setminus [a_i, a_i + m_i)$. Note that k_i is chosen such that the distribution on $s_{p,i}^n|v$ is the same for all i. Moreover, we have $k_i \to 1$ as $n \to \infty$ and so $s_p = \text{plim}_{n\to\infty} s_{p,i}^n$. We define $\mathbf{P}_i^n = \mathcal{P}^n(s_{p,i}^n)$, where the function $\mathcal{P}^n(\cdot)$ is such that its inverse is $h^n(P) = h(P) + \mu q(P)$, for some function $q(P)$ that will be linked to $\psi(P)$ in equilibrium. One can verify that such a sequence satisfies all technical conditions. In particular, the symmetry of the joint density of v and $\mathbf{P}_i^n$ follows from the symmetry of the conditional distribution of $s_{p,i}^n|v$ and we have $\mathbf{P}_* = \text{plim}_{n\to\infty} \mathbf{P}_i^n$ by Continuous Mapping Theorem.

B.4.9 Auxiliary lemmata

Lemma 12. *Consider a sequence of functions $\hat{x}(P,a)$ and $\hat{q}(P,a)$ that converge, as $n \to \infty$ to 0 and $q(P,a)$, respectively, for all $a \in [0,1)$ and all $P \in \mathbb{R}$. Suppose that for any fixed P, there exist A and k that may depend on P, such that $\hat{q}(v,p) < A\exp(-kv^2)$, for all n. Take a partition $[0,y] = \cup_{i=1}^n [a_i, a_i + m_i]$, and let $m = \max_i m_i$ and $\hat{a}_n(a) = \sum_{i=1}^{n-1} a_i \mathbb{1}_{[a_i,a_{i+1})}(a)$. For any partition sequence such that $m \to 0$ as $n \to \infty$, we have that*

$$\int_{\mathbb{R}} \hat{u}^{(l)} \left(\hat{W}_0(a) + \hat{x}(P,a)(V(v) - P),\, a \right) \hat{q}(v,P)dv$$

converges uniformly to

$$u^{(l)} \left(W_0(a),\, a \right) \int_{\mathbb{R}} q(v,P)dv$$

over $a \in [0, y]$.

Proof of Lemma 12. Note that, for a given n,

$$\sup_{a\in[0,y]} \left| \int_{\mathbb{R}} \hat{u}^{(l)} \left(\hat{W}_0(a) + \hat{x}(P,a)(V(v) - P),\, a \right) \hat{q}(v,P) dv - \int_{\mathbb{R}} u^{(l)} \left(W_0(a),\, a \right) q(v,P) dv \right| \leq$$
$$\sup_{a\in[0,y]} \int_{\mathbb{R}} \left| \hat{u}^{(l)} \left(\hat{W}_0(a) + \hat{x}(P,a)(V(v) - P),\, a \right) \hat{q}(v,P) - u^{(l)} \left(W_0(a),\, a \right) q(v,P) \right| dv \leq$$
$$\int_{\mathbb{R}} \sup_{a\in[0,y]} \left\{ \left| \hat{u}^{(l)} \left(\hat{W}_0(a) + \hat{x}(P,a)(V(v) - P),\, a \right) - u^{(l)} \left(W_0(a),\, a \right) \right| \right\} |\hat{q}(v,P)| dv +$$
$$\int_{\mathbb{R}} \sup_{a\in[0,y]} \left\{ \left| u^{(l)} \left(W_0(a),\, a \right) \right| \right\} \left| \hat{q}(v,P) - q(v,P) \right| dv.$$

Now consider the $n \to \infty$ limit of the above. We have

$$\lim_{n\to\infty} \int_{\mathbb{R}} \sup_{a\in[0,y]} \left\{ \left| \hat{u}^{(l)} \left(\hat{W}_0(a) + \hat{x}(P,a)(V(v) - P),\, a \right) - u^{(l)} \left(W_0(a),\, a \right) \right| \right\} |\hat{q}(v,P)| dv =$$
$$\int_{\mathbb{R}} \lim_{n\to\infty} \sup_{a\in[0,y]} \left\{ \left| \hat{u}^{(l)} \left(\hat{W}_0(a) + \hat{x}(P,a)(V(v) - P),\, a \right) - u^{(l)} \left(W_0(a),\, a \right) \right| \right\} |q(v,P)| dv = 0 \quad (46)$$

In the first transition, we passed the limit under the integral sign, which is permitted by the Dominated Convergence Theorem. We then noted that in the last equation above, $\lim_{n\to\infty} \sup_{a\in[0,y]} \{\cdots\} = 0$. Here $\cdots$ denote the term in the curly brackets in (46). The fact that the limit is zero can be proved using the the"$\epsilon/3$" argument as in the proof of Lemma 8.

We also have that

$$\lim_{n\to\infty} \int_{\mathbb{R}} \sup_{a\in[0,y]} \left\{ \left| u^{(l)} \left(W_0(a),\, a \right) \right| \right\} \left| \hat{q}(v,P) - q(v,P) \right| dv = 0.$$

Similarly to previous calculation, the limit can be passed under the integral by DCT, the sup is finite by the Extreme Value Theorem and $\hat{q}(\cdot) \to q(\cdot)$ by the hypothesis of the Lemma. ■

B.5 Proof of Proposition 1

Proof of Proposition 1. From definition of $\mathcal{I}$ we have $\mathcal{I} = \tau_p/\tau$. Substituting (16) into the last equation and rearranging, we obtain the stated expression for $\mathcal{I}$.

We turn to deriving expression for $\mathcal{L}$. The market clearing price $\mathcal{P}(s_p, \bar{\theta})$ is P that solves

$$s_p = \frac{\bar{\theta} + \theta(P) - \int \delta(a, P)da}{\int \beta(a, P)da}.$$

Then define

$$\begin{aligned}\text{Liquidity} &\equiv -\left(\frac{\partial}{\partial \bar{\theta}} \mathcal{P}(s_p, \bar{\theta})\right)^{-1} \\ &= -s_p \frac{\partial}{\partial P}\left(\int \beta(a, P)da\right) - \frac{\partial}{\partial P}\left(\int \delta(a, P)da - \theta(P)\right)\end{aligned}$$

On the other hand,

$$\begin{aligned}\frac{\partial}{\partial s_p} \mathcal{P}(s_p, \bar{\theta}) &= \frac{-\int \beta(a, P)da}{s_p \frac{\partial}{\partial P}\left(\int \beta(a, P)da\right) + \frac{\partial}{\partial P}\left(\int \delta(a, P)da - \theta(P)\right)} \\ &= \text{Liquidity}^{-1} \cdot \int \beta(a, P)da.\end{aligned}$$

From (15) we get that

$$\frac{\partial}{\partial s_p} \mathcal{P}(s_p, \bar{\theta}) = \frac{\tau_p}{\tau} \int V'\left(\frac{\tau_p}{\tau} s_p + \frac{z}{\sqrt{\tau}}\right) d\Phi(z) = \frac{\tau_p}{\tau} E[V'(v)|P].$$

Combining the two preceding equations and substituting the expression for $\beta(a, P)$ we get

$$\frac{\tau_p}{\tau} E[V'(v)|P] = \text{Liquidity}^{-1} \cdot \int \frac{t(a)}{\rho(a)} \frac{\tau^{-1}\,\mathbb{E}[V'(v)|P]}{\text{Var}\,[V(v)|P]} da \implies$$

$$\text{Liquidity} = \frac{1}{\tau_p \text{Var}[V(v)|P]} \int \frac{t(a)}{\rho(a)} da.$$

Multiplying by $\text{Var}[V(v)|P]$ and substituting (16) we get the stated equation for $\mathcal{L}$. ■

B.6 Proof of Lemma 3

Proof of Lemma 3. First, from Lemma 2 we have $\tau_{agg} \equiv \mathbb{V}\text{ar}\big(v \big| s\left[\omega^H(a)\right]\big)^{-1} - \mathbb{V}\text{ar}(v)^{-1} = 1/\Big(\int_0^1 \omega^H(a)^2/t(a)da\Big)$. Second, we apply the Cauchy-Bunyakovsky-Schwartz inequality

$$\left(\int_0^1 f(a)g(a)da\right)^2 \leq \int_0^1 f(a)^2 da \int_0^1 g(a)^2 da \tag{47}$$

with $f(a) = \omega^H(a)/\sqrt{t(a)}$ and $g(a) = \sqrt{t(a)}$ to obtain $\tau_{agg}(b(a)) \leq \int_0^1 t(a)da$. The equality in (47) is attained if, and only if, $f(a)$ and $g(a)$ are linearly dependent, i.e., when $\omega^H(a)/\sqrt{t(a)} = c\sqrt{t(a)}$. The constant c is pinned down by the condition $\int_0^1 w(a)da = 1$. ■

B.7 Proof of Proposition 2

Proof of Proposition 2. Denote absolute risk tolerance $y(a) \equiv 1/\rho(a)$. Without loss of generality, index traders such that $y(b)$ increases in b. (This is in contrast to index a, which is such that $W_0(a)$ is increasing in a.) We first compute the Gateaux derivatives of $\mathcal{I}$ and $\mathcal{L}$ with respect to $y(b)$. We then show that under DARA utilities and the technical conditions imposed, the signs of the derivatives with respect to $y(b)$ and $W_0(a)$ are the same. We start by proving the following statement.

Step 1. There exist thresholds $0 < b_1^y \leq b_2^y < 1$, such that for for any Robin Hood variation $y^\Delta(b)$ with $\underline{b} \leq b_1^y \leq b_2^y \leq \bar{b}$

$$\begin{aligned} \mathcal{I}'(y(b))[y^\Delta(b)] > 0 \text{ and } \mathcal{I}'(y(b))[-y^\Delta(b)] < 0; \\ (\mathcal{L})'(y(b))[y^\Delta(b)] < 0 \text{ and } (\mathcal{L})'(y(b))[-y^\Delta(b)] > 0. \end{aligned} \tag{48}$$

From (16) we obtain

$$\tau_p = \left(\int_0^1 y(b)t(b)db\right)^2 \Big/ \left(\int_0^1 y(b)^2 t(b)db\right). \tag{49}$$

Substituting this expression into $\mathcal{I} = \tau_p/(\tau_p + \tau_v)$ and computing the Gateaux derivative (this entails substituting $y(b)+\epsilon y^{\Delta}(b)$ instead of $y(b)$, differentiating with respect to ϵ, and evaluating the resulting expression at $\epsilon = 0$) yields:

$$\mathcal{I}'(y(b))[y^{\Delta}(b)] = C_{\mathcal{I}} \int_0^1 t(b) y^{\Delta}(b)(I_2 - I_1 y(b))\, db.$$

Here, $C_{\mathcal{I}} > 0$ is positive (we have the closed-form expressions for $C_{\mathcal{I}}$ via parameters of the model, but it is not important here), $I_1 = \int_0^1 t(b)y(b)\, db$ and $I_2 = \int_0^1 t(b)y(b)^2\, db$. Lemma 13 (to follow) implies that there exists a unique b_y^* such that $I_2 - I_1 y(b) \gtreqless 0$ iff $b \lesseqgtr b_*^y$. Then, for a $y^{\Delta}(b)$ that is Robin Hood with $\underline{b} < b_*^y < \bar{b}$, $y^{\Delta}(b)(I_2 - I_1 y(b)) > 0$ and the first statement of this step follows by letting $b_1^y = b_2^y = b_*^y$.

One can obtain $\mathcal{L} = \int_0^1 t(b)y(b)db/\tau_p$. Substituting (49) in this equation and computing the Gateaux derivative yields

$$(\mathcal{L})'(y(b))[y^{\Delta}(b)] = C_{\mathcal{L}} \int_0^1 t(b) y^{\Delta}(b)(2I_1 y(b) - I_2)\, db.$$

Here, $C_{\mathcal{L}} > 0$ is positive, and Lemma 13 (to follow) implies that there exists a unique $b_{**}^y > b_*^y$ such that $2I_1 y(b) - I_2 \gtreqless 0$ iff $b \gtreqless b_{**}^y$. Then, for a $y^{\Delta}(b)$ that is Robin Hood with $\underline{b} < b_{**}^y < \bar{b}$, $y^{\Delta}(b)(2I_1 y(b) - I_2) > 0$ and the first three statements of this step follow by letting $b_1^y = b_*^y$ and $b_2^y = b_{**}^y$.

Step 2. There exist thresholds $0 < a_1^W \leq a_2^W < 1$*, such that for for any Robin Hood variation* $W_0^{\Delta}(a)$ *with* $\underline{a} \leq a_1^W \leq a_2^W \leq \bar{a}$ (48) *follows.*

Let a_2^W be a unique solution to $W_0(a) = \underline{\eta} b_2^y$. Note that for any $a > a_2^W$, we have $y(a) > W_0(a)/\underline{\eta} > b_2^y$. Thus, by the previous step of the proposition, decreasing risk tolerances for traders $a > a_2^W$ leads to improvement in information efficiency and reduction in liquidity. Note also that decreasing wealth for traders $a > a_2^W$ induces their risk tolerances to decrease as well (DARA utilities). Similarly, letting a_1^W be a unique solution to $W_0(a) = \overline{\eta} b_2^y$, we get that for any $a < a_1^W$, $y(a) < b_1^y$ and that increasing wealth for traders $a < a_a^W$ induces their risk tolerances to increase as well (DARA utilities). Then, by the previous step, the statement of the proposition holds. ■

Lemma 13. *Assume* $0 < \underline{\eta} \leq \eta(a) \leq \bar{\eta} < \infty$. *Assume that absolute risk tolerance* $y(a)$ *is a continuous and strictly increasing function of* a. *For any* $c > 0$, *there exists a unique solution* $\hat{a}(c)$ *to* $y(a) = c$, *moreover,* $\hat{a}(c)$ *increases in* c *and* $0 < \hat{a}(c) < 1$ *for any* $0 < c < \infty$.

Proof of Lemma 13. Since $y(a)$ increases in a, at most one solution to $y(a) = c$ exists. Monotonicity also implies $\hat{a}(c)$ increases in c. For risk tolerance $y(a)$ we can write $y(a) = W_0(a)/\eta(a)$. We have $y(a) < W_0(a)/\underline{\eta}$ and so $0 \leq \lim_{a\to 0} y(a) = \inf y(a) \leq \inf\{W_0(a)\}/\underline{\eta} = 0$. Thus, $\lim_{a\to 0} y(a) = 0$. One can show analogously that $\lim_{a\to\infty} y(a) = \infty$. Then, by Intermediate Value Theorem, $\hat{a}(c)$ exists and $0 < \hat{a}(c) < 1$. ■

B.8 Proof of Proposition 3

Proof of Proposition 3. This proof follows the same steps as the proof of Proposition 2. Without loss of generality, index traders such that $y(b)$ increases in b.

Step 1. There exist thresholds $0 < b_1^t \leq b_2^t < 1$, *such that for for any Robin Hood variation* $t^\Delta(b)$ *with* $\underline{b} \leq b_1^t \leq b_2^t \leq \bar{b}$

$$\mathcal{I}'(t(b))[t^\Delta(b)] > 0 \text{ and } \mathcal{I}'(t(b))[-t^\Delta(b)] < 0;$$
$$(\mathcal{L})'(t(b))[t^\Delta(b)] < 0 \text{ and } (\mathcal{L})'(t(b))[-t^\Delta(b)] > 0.$$

Here, the proof is identical to step 1 of Proposition 2, with the difference of expressions for the Gateaux derivatives, which we reproduce below:

$$\mathcal{I}'(t(b))[t^\Delta(b)] = C_\mathcal{I} \int_0^1 y(b)t^\Delta(b)(2I_2 - I_1 y(b))\, db;$$

$$(\mathcal{L})'(t(b))[t^{\Delta}(b)] = C_{\mathcal{L}} \int_0^1 y(b)t^{\Delta}(b)(I_1 y(b) - I_2)\, db;$$

Step 2. There exist thresholds $0 < a_1^W \le a_2^W < 1$, such that for for any Robin Hood variation $W_0^{\Delta}(a)$ with $\underline{a} \le a_1^W \le a_2^W \le \bar{a}$ (24) holds.

Let a_2^W be a unique solution to $W_0(a) = \underline{\theta} b_2^t$. Note that for any $a > a_2^W$, we have $y(a) > W_0(a)/\underline{\theta} > b_2^t$. Similarly, letting a_1^W be a unique solution to $W_0(a) = \overline{\theta} b_2^t$, we get that for any $a < a_1^W$, $y(a) < b_1^t$. Then, by the previous step, the statement of the proposition holds. ∎

B.9 Proof of Proposition 4

Proof of Proposition 4. With endogenous information acquisition and DARA utilities, the precision of information is increasing in wealth. The transfer of wealth from rich to poor has two effects. The indirect one of decreasing the precisions of the rich and increasing that of the poor (via endogenous information acquisition) is captured by the Proposition 3. Proposition 2 captures the direct one. By taking the thresholds $\underline{a} \le \min\{a_1^t, a_1^W\}$ and $\bar{a} \ge \max\{a_2^t, a_2^W\}$ we make sure that both propositions apply ∎

B.10 Completing heuristic derivation in Section 6

Here, we justify the transition from (9) to (10). Note that by the stochastic calculus heuristics (5), $dX(a)^2 = \beta(a,P)^2/t(a)da$ and so $\mathbb{E}[(V-P)^3|ds(a), \hat{s}_p]\, dX(a)^2$ does not contain any $ds(a)$ terms. (Indeed, the terms in $\mathbb{E}[(V-P)^3|ds(a), \hat{s}_p]$ that contain $ds(a)$ will be zeroed out after multiplying by $dX(a)^2 = \beta(a,P)^2/t(a)da$ and applying (5).)

Similarly, the term $\mathbb{E}[(V-P)^2|ds(a), \hat{s}_p]$ in (9) can be replaced by $\mathbb{E}[(V-P)^2|s_p]$ in (10). The difference $\mathbb{E}[(V-P)^2|ds(a), \hat{s}_p] - \mathbb{E}[(V-P)^2|\hat{s}_p]$ contains a leading $ds(a)$ term, which will become a da term after being multiplied by $\beta(a,P)ds(a)$ in (10). Finally, $\mathbb{E}[(V-P)^2|\hat{s}_p]$ can be replaced by $\mathbb{E}[(V-P)^2|s_p]$ since conditional distributions $v|\hat{s}_p$ and $v|s_p$ are the same.

B.11 Proof of Proposition 5

Proof of Proposition 5.

Fix a trader i. Given the price P, his realized utility at time $t = 2$ is

$$\mathcal{U}_{i,t=2}\left(\Delta s_i, m_i\right) = u\left(W_0(a_i) + x(\Delta s_i, m_i; a_i)(V(v) - P)\right).$$

The Δs_i is a finite increment of a diffusion process

$$ds(b) = vdb + \frac{1}{\sqrt{t(a_i)}}dB(b)$$

between $b = a_i$ and $b = a_i + m_i$. Similarly, $\mathcal{U}_{i,t=2}\left(\Delta s_i, m_i\right) - \mathcal{U}_{i,t=2}\left(0,0\right)$ can be viewed as a finite increment of a diffusion process driven by $ds(b)$, between $b = a_i$ and $b = a_i + m_i$. By Ito's lemma, we can write

$$\mathcal{U}_{i,t=2}\left(\Delta s_i, m_i\right) - \hat{\mathcal{U}}_{i,t=2}\left(0,0\right) = \int_0^m \mu_u(b)db + \int_0^m \sigma_u(b)dB,$$

where μ_u and σ_u denote the drift (the "db" coefficient) and the diffusion coefficients (the "$dB(b)$" coefficient) of $\mathcal{U}_{i,t=2}$ process.

By Lemma 14 (to follow), the optimal precision solves

$$t(a_i) \in \arg\max_t \left\{ \frac{\partial\, \mathbb{E}[\mathcal{U}_{i,t=2}]}{\partial m_i}\bigg|_{m_i=0} \right\}. \tag{50}$$

Thus, $t(a_i)$ maximized the expected drift of drift $\hat{\mathcal{U}}_{i,t=2}$ at $m_i = 0$, $\mathbb{E}[\mu_u(0)]$. The expected drift is then computed as in Section C.1, with the first order (necessary and sufficient) condition in (50) reducing to (57). ∎

Lemma 14. *Consider a continuously differentiable function $f(t,m)$ such that $f(t,0)$ does not depend on t. Consider $t(m) \in \arg\max_t f(t,m)$. Suppose that $t(m)$ is bounded for small enough*

m. Then, $t(0) \in \arg\max_t f_m(t, 0)$.

Proof of Lemma 14. Suppose, on the contrary, that there exists some $\check{t}$ such that $f_m(\check{t}, 0) > f_m(t(0), 0)$. Then, by continuity, there exists $\bar{m}$ such that

$$f_m(\check{t}, m) > f_m(t(m), m) \text{ for } m < \bar{m}. \tag{51}$$

Integrate (51) with respect to m:[55]

$$f(\check{t}, m) - f(\check{t}, 0) > f(t(m), m) - f(t(0), 0).$$

Since $f(t, 0)$ does not depend on t we have $f(t(0), 0) = f(\check{t}, 0)$ and so

$$f(\check{t}, m) > f(t(m), m).$$

We obtained a contradiction with $t(m) \in \arg\max_t f(t, m)$. ■

[55]To integrate the right-hand side we use the Envelope Theorem $\frac{df(t(m),m)}{dm} = f_m(t(m), m)$

C Information acquisition

Information acquisition happens at $t = 0$. The precisions $t(a)$ are endogenized by requiring them to be optimal, given an information acquisition cost. We define the notion of optimality of the profile of precisions $t(a)$ similarly to how we defined the optimality of demands: precision profile $t(a)$ is optimal in CHILE if it is a limit of optimal precisions in the discrete economies. We make this definition precise after we describe the information acquisition in the discrete economy. We assume that agents are uncertain about the distribution of wealth $W_0(a)$ (but they know their wealth).[56] This distribution becomes known before the start of the trade at $t = 1$.

The new primitive in CHILE is the information acquisition cost $c(t, a)$. We assume that the cost of acquiring an infinitesimal signal $ds(a) = vda + 1/\sqrt{t}dB(a)$ for a trader a is $c(t; a)da$, where $c(\cdot)$ is continuous, strictly increasing and convex function of t. Thus, the cost of acquiring a finite signal

$$\Delta s = \int_{a_i}^{a_i+m_i} vda + \int_{a_i}^{a_i+m_i} \frac{1}{\sqrt{t(a)}} dB(a) \text{ is } \int_{a_i}^{a_i+m_i} c(t(a), a_i)da.$$

A finite signal can be split into a collection of infinitesimal ones, with associated costs. Using Jensen's inequality, one can show that it is not optimal to split a finite signal into infinitesimal ones of varying precisions. If a trader wants to get information of precision tm_i, he should acquire a signal

$$\Delta s = \int_{a_i}^{a_i+m_i} vda + \frac{1}{\sqrt{t}} dB(a) = vm_i + \frac{B(a_i + m_i) - B(a_i)}{\sqrt{t}} \text{ at a cost } c(t; a_i)m_i. \tag{52}$$

Thus, without loss of generality, we restrict finite signals in the discrete economy to be of the form (52).

[56] Done this way, the information choice of agent a will only depend on his wealth, but not the wealth of others, which helps to simplify some of the analysis.

Denote $\mathcal{U}_i(W_0^i, t_i, m_i)$ the maximum utility in (27) for a given precision t_i and initial wealth W_0^i.[57] Define $c(t_i, a_i)m_i$ as a (monetary) cost of acquiring signal Δs_i. We require that precisions t_i are *optimal*, i.e.

$$t_i = \arg\max_t \left\{ \int \mathcal{U}_i(W_0^i - c(t, a_i)m_i, t, m_i) g(v, P, \mu) dv dP \right\}. \tag{53}$$

Here $g(\cdot)$ denotes the PDF of the joint distribution of $\mathbf{P}_i^n$ and Δs_i. We denote $t^n(a)$ the profile of optimal precisions in the discrete economy, $t^n(a) = t_i$, for all $a \in [a_i, a_i + m_i)$, where t_i solves (53).

We introduce some technical restrictions on $c(t; a)$.

Assumption 8. *The information acquisition cost $c(t; a)$ is such that there exists M_t such that for any i, $\hat{t}_i > M_t$ is not optimal.*

The additional technical restriction allows us to make a choice set in (53) compact. Thus, without loss of generality, we assume $t \in [0, M_t]$ everywhere in the sequel. We define the notion of optimal profile of precisions $t(a)$ by continuity, analogously to Section 10.

Definition 6. *A profile $t(a)$ is* ***optimal*** *if, for every a, the optimal precision in discrete economy $t^n(a)$ converges to $t(a)$ as $n \to \infty$.*

Finally, we define the information acquisition equilibrium at $t = 0$.

Definition 7. *A profile $t(a)$ is an information acquisition* ***equilibrium*** *if for every a, $t(a)$ is optimal.*

With these definitions at hand, we are ready to state this section's main result in the subsection below.

[57] Here we follow the setup of the main paper, where the profile of preferences in the discrete and continuous economy are the same. This approach can be generalized to allow them to differ for finite n but coincide in the $n \to \infty$ limit. Such an alternative approach could allow to lift the technical restrictions from the primitives of CHILE and "shift" those restrictions to the primitives of discrete economies instead.

C.1 Information acquisition: heuristic derivation

It is illuminating to start with the heuristic derivation. Consider a change in trader a's time-2 realized utility due to trade

$$\begin{aligned} d\mathcal{U}_{t=2}(dX(a);a) &= u(W_0(a) - c(t(a))da + dX(a)(V(v) - P);a) - u(W_0(a);a) \\ &= u'(W_0(a))\left(dX(a)(V(v) - P) - c(t(a))da + \frac{\rho(a)}{2}dX(a)^2(V(v) - P)^2\right) \end{aligned}$$

In the second line, we Taylor expanded the $u(\cdot)$, up to terms of order da. Due to the heuristics of Section 6, we have $dX(a)^2 = \beta(a,P)^2/t(a)da$. Now substitute $P = \mathbf{P}(a)$ (trader a's conjecture about the market-clearing price) and take the expectation:

$$\frac{\mathbb{E}[d\mathcal{U}_{t=2}(dX(a);a)]}{u'(W_0(a);a)} = \mathbb{E}[dX(a)(V(v) - \mathbf{P}(a)] - c(t(a))da + \frac{\rho(a)}{2t(a)}\mathbb{E}[\beta(a,\mathbf{P}(a))^2(V(v) - \mathbf{P}(a))^2]da. \tag{54}$$

Now simplify

$$\begin{aligned} \mathbb{E}[dX(a)(V(v) - \mathbf{P}(a))] &= da\,\mathbb{E}[\beta(a,\mathbf{P}(a))v(V(v) - \mathbf{P}(a))] \\ &+ \mathbb{E}[dB(a)\beta(a,\mathbf{P}(a))(V(v) - \mathbf{P}(a))] \\ &+ da\,\mathbb{E}[\delta(a,\mathbf{P}(a))(V(v) - \mathbf{P}(a))] \\ &= da\,\mathbb{E}[\beta(a,\mathbf{P}(a))v(V(v) - \mathbf{P}(a))]. \end{aligned} \tag{55}$$

In the first transition, we substituted $dX(a) = \beta(a,P)ds(a) + \delta(a,P)da$. To get (55), we substituted $\mathbb{E}[dB(a)\beta(a,\mathbf{P}(a))(V(v) - \mathbf{P}(a))] = 0$ ($dB(a)$ is independent of $V(v)$ and $\mathbf{P}(a)$ as the traders are price takers and assume the noise in their signals in independent from that in the price) and $\mathbb{E}[\delta(a,\mathbf{P}(a))(V(v) - \mathbf{P}(a))] = 0$ (market efficiency condition).

To proceed further, we divide (54) by da and pass to $da \to 0$ limit. In that limit, $\mathbf{P}(a)$ becomes the market clearing price $\mathbf{P}_*$, which for ease of notation we simply denote P going forward. We can further simplify (55) by noting that $\mathbb{E}[v(V(v) - P)|P] = \beta(a,P)Var[V(v)|P]\rho(a)/t(a)$

(see (42)). Then, after substituting $\mathbb{E}[(V(v)-P)^2|P] = \mathrm{Var}[V(v)|P]$, and $\beta(a,P) = t(a)/\rho(a)\beta_P(P)$ we can finally obtain

$$\frac{\mathbb{E}[d\mathcal{U}_{t=2}(dX(a);a)]}{u'(W_0(a);a)da} = \frac{t(a)}{2\rho(a)}\,\mathbb{E}[\beta_P(P)^2\mathrm{Var}[V(v)|P]] - c(t(a)) \tag{56}$$

The optimal precision maximizes (56) with respect to $t(a)$. We summarize in the Proposition below. The rigorous proof is in the Appendix B.11.

Proposition 5. *The optimal precision choice for a trader a solves*

$$c'(t,a) = \frac{1}{2\rho(a)}\,\mathbb{E}[\beta_P(P)^2\mathrm{Var}[V(v)|P]]. \tag{57}$$

Here $\beta_P(P) = \frac{E[v(V(v)-P)|P]}{\mathrm{Var}[V(v)|P]}$. *When* $V(v) = \exp(v)$, *the optimal precision solves*

$$c'(t,a) = \frac{1}{2\rho(a)}\,\mathbb{E}\left[\frac{1}{\tau^2\left(\exp\left(\tau^{-1}\right)-1\right)}\right]. \tag{58}$$

Provided that the equilibrium exists, the equilibrium precision for trader a is an increasing function of his wealth $W_0(a)$ under DARA preferences.

Note that the expectations in (57) and (58) are over prices P, distribution of aggregate signal paths $\{s(a)\}_a$, and potential wealth distributions.

Online appendix

D A benchmark model with LLN aggregation

This section demonstrates that our main results do not obtain if one adopts the traditional approach to modeling a large economy based on law of large numbers (LLN) aggregation, as in Hellwig (1980), Admati (1985), Peress (2004), among others. The key finding here is an aggregation result: an economy with investors differing in wealth, precision, and preferences appears observationally equivalent to one with homogeneous investors. Consequently, in such economies, Robin Hood variations in wealth that leave aggregate trading intensity unchanged have no effect on market quality, unlike in CHILE.

This section derives our main results using the model of Peress (2011), which is, to the best of our knowledge, the only model in the literature with micro-founded wealth effects and heterogeneous information.[58] We present the Peress (2011) model using notation similar to ours and provide brief derivations of the main results. For more details, we refer the reader to the original paper.

The economy unfolds over two time periods, $t = 1$ and $t = 2$. It consists of a continuum of agents of total mass one. There are two assets: one risk-free and one risky, with the returns of both assets, realized at $t = 2$. The risk-free asset has a perfectly elastic supply and gross return normalized to 1, while the risky asset has a liquidation value of V that is log-normally distributed.[59]

Agents trade at $t = 1$, determining the price P of the risky asset. There are rational traders indexed by $a \in [0, 1)$ and noise traders. Noise traders submit price-inelastic demand $\theta \sim N(0, \tau_\theta^{-1})$ (measured in units of stocks). Agent a has initial wealth $W_0(a)$, and he trades a

[58] It is straightforward to derive the results of this section within a mean-variance preference framework that includes ad-hoc wealth effects, where the "variance" coefficient depends on wealth, as in Farboodi et al. (2022b) and Mihet (2022).

[59] It is also straightforward to derive our results for the case of normal distribution of V. However, existing methods are limited in their ability to accommodate the broad range of payoff functions that CHILE supports.

total amount of x *dollars*—i.e. price per share times number of shares. His realized utility is

$$u(W, a),$$

where his wealth at time $t = 2$ is

$$W = W_0(a) + x(R - 1),$$

and where

$$R = V/P.$$

Eaach trader has a private signal $s(a) = v + Z(a)/\sqrt{t(a)}$, where $Z(a)$ are i.i.d. standard Normal random variables.

The analytical tractability is achieved by employing a "small risk" approximation. It is assumed that

$$\ln V = \zeta \cdot v + v_n, \tag{59}$$

where $v \sim N(0, \tau_v^{-1})$ is a learnable component of asset payoff, and $v_n \sim N(0, \zeta\tau_{v,n}^{-1})$ is a non-learnable component.[60] The approach consists of characterizing the equilibrium asymptotics as $\zeta \to 0$.

Remark 9. Relative to the model in the paper, the model here features two new ingredients. The first is noise θ. The second one is the unlearnable component v_u of asset payoff. Both are essential for the equilibrium here to be well-defined. Without θ, there will be no noise in the price, and the equilibrium will not exist. Without v_u, the asymptotics of equilibrium as $\zeta \to 0$ will not be well-behaved (see Peress (2011)). Our model in the paper is well-defined without these two elements, so we did not introduce them there. However, we verified that introducing them does not change our main conclusions. Thus, the comparison to the benchmark here is

[60] This is equivalent to Peress (2011) formulation, saying that agents learn about the mean of log-payoff as opposed to log-payoff itself. Indeed our assumption can equivalently be formulated as $\ln V \sim N(\zeta v, \zeta\tau_{v,u}^{-1})$, the mean $v \sim N(0, \tau_v^{-1})$ is not known, and agents learn about mean v as opposed to about log-payoff $\ln V$.

fair.

Remark 10. The aggregation here will be performed by computing $\int x(a)da$. It should be noted that $\int x(a)da$ represents the average, not aggregate demand. Indeed, the integral is approximately $\sum_a x(a)\frac{1}{n}$, which is average demand. (Here, n is the number of traders in the economy.) In the large economy here, each demand is not infinitesimal $1/x(a) < \infty$, so their sum is infinite. Starting from Hellwig (1980), the literature solves this problem by equalizing the average, not aggregate demand $\int x(a)da$ to the noisy supply θ. Consequently, the actual aggregate noisy supply, $\theta \cdot n$, has infinite variance.

D.1 Equilibrium

Solving for equilibrium involves taking the standard steps. We postulate that for small ζ, traders have the following conjecture about the price function

$$\ln P \equiv p = \hat{p}\zeta, \text{ where } \hat{p} = c_0 + c_1 v - c_2\theta. \tag{60}$$

Given such conjecture, agents form their demands, which then results in an equilibrium price function $p^{eq}(\zeta)$. Equilibrium requires traders' conjecture to be consistent in the $\zeta \to 0$ limit: $p^{eq}(\zeta) = \hat{p}\zeta + o(\zeta)$.

Substituting (59) and conjecture (60) into agent a's first order condition $E_a[u'(W_0(a) + x(\exp(\ln V - p) - 1)(\exp(\ln V - p) - 1)] = 0$, expanding it up to the terms of order ζ, we obtain $x(p) = \hat{x}(\hat{p}) + o(1)$, where

$$\hat{x}(\hat{p}) = \frac{E_a[v] - \hat{p} + 0.5\tau_{v,n}^{-1}}{\rho(a)\tau_{v,n}^{-1}},$$

where $\rho(a) = -u''(W_0(a))/U'(W_0(a))$ is trader a's absolute risk aversion and

$$E_a[v] = \frac{t(a)}{\tau_v + \tau_p + t(a)}s(a) + \frac{\tau_p}{\tau_v + \tau_p + t(a)}\frac{\hat{p} - c_0}{c_1}, \text{ and } \tau_p \equiv \mathrm{Var}[v|p]^{-1} - \tau_v = \frac{c_1^2}{c_2^2}\tau_\theta.$$

Applying market clearing, we get

$$v \cdot \tau_{v,n} \int_{\nu}^{1} \frac{t(a)/\rho(a)}{\tau_v + \tau_p + t(a)} da + (\text{affine function of } p) = \theta,$$

and hence

$$\frac{c_1}{c_2} = \sqrt{\frac{\tau_p}{\tau_\theta}} = \tau_{v,n} \int_0^1 \frac{t(a)/\rho(a)}{\tau_v + \tau_p + t(a)} da.$$

The main result of this section follows.

Theorem 2. *There exists a unique equilibrium. The equilibrium demands can be written as* $\hat{x}(\hat{p}) = \alpha(a) + \beta(a)s(a) - \gamma(a)\hat{p}$, *where*

$$\beta(a) = \frac{t(a)\tau_{v,n}}{\rho(a)(\tau_v + \tau_p + t(a))}.$$

The price informativeness τ_p *can be written as*

$$\tau_p = \left(\int_0^1 \beta(a) da \right)^2 \tau_\theta$$

and is the unique solution to

$$\sqrt{\frac{\tau_p}{\tau_\theta}} = \tau_{v,n} \int_0^1 \frac{t(a)/\rho(a)}{\tau_v + \tau_p + t(a)} da.$$

The price function and the aggregate demand is the same as in the economy with homogeneous investors with absolute risk aversion ρ *and precision* t *satisfying* $\int_0^1 \frac{t(a)/\rho(a)}{\tau_v + \tau_p + t(a)} da = \frac{t/\rho}{\tau_v + \tau_p + t}$.

The economy with LLN aggregation allows for a "representative agent" formulation. Aggregate quantities, such as the price function and informativeness, remain consistent between the original heterogeneous economy and the equivalent homogeneous one. In this case, heterogeneity is irrelevant, unlike in CHILE. Consequently, we also get the following result, further illustrating the distinctions of CHILE and traditional economies in terms of comparative statics of information efficiency.

Corollary 2. *Aggregate trading intensity* $\int_0^1 \beta(a)da$ *is a sufficient statistic of price informativeness. Changes in wealth distribution that leave the aggregate trading intensity unchanged leave price informativeness unchanged.*

In CHILE, under the assumptions of Proposition 2, *any* Robin Hood variation of wealth with thresholds $\underline{a} \leq a_1^W \leq a_2^W \leq \bar{a}$ results in increased information efficiency. This implies that the result holds even for redistributions that leave aggregate trading intensity unchanged. This distinction underscores the difference between CHILE and traditional large economies: changes in parameters that do not affect the parameters of an equivalent homogeneous economy have no impact on traditional economies, but they do in CHILE.

E A model with noise traders

In this section, we introduce noise traders. The resulting equilibrium captures both the mechanism emphasized in previous literature—where inequality enhances information efficiency by increasing the aggregate trading intensity of informed traders—and the new mechanism highlighted previously in this paper, which shows that inequality hinders efficiency by disrupting the optimal aggregation of informed traders' signals. The main result is that the new mechanism tends to dominate, ensuring that our findings from the main part of the paper remain robust to the introduction of noise.

E.1 Setup

The model follows the setup in Section 2, with the addition of noise traders. There is a measure ν of these traders, and we index all traders by $a \in [0, 1+\nu)$. Traders in the range $a \in [0,1)$ are rational, while those in $a \in [1, 1+\nu)$ are noise traders.

To maintain analytical tractability, we model noise traders following Black (1976), who describes them as traders "trading on noise as if it were information." Specifically, we assume that noise traders observe

$$ds(a) = u\,da + \frac{1}{\sqrt{t(a)}}dB(a), \quad a \in [1, 1+\nu). \tag{61}$$

Here, $u \sim N(u, \tau_v^{-1})$ is distributed identically to v but is independent of it. Although their signals follow (61), noise traders act as if they were receiving signals given by (1b). Noise traders are identical to rational traders in all aspects except for the information they receive and how they respond to it.

E.2 Derivation of equilibrium

Derivation identical to that of Theorem 1 yields that the demand of noise traders is given by

$$dX(a) = \beta(a, P)ds(a) + \delta(a, P)da,$$

Where

$$\beta(a, P) = \frac{t(a)}{\rho(a)} \frac{\tau^{-1}\,\mathbb{E}[V'(v)|s_p]}{\mathrm{Var}\,[V(v)|s_p]}.$$

From market clearing, agents can extract the sufficient statistic

$$s_p = v + \frac{\int_1^{1+\nu} \frac{t(a)}{\rho(a)} da}{\int_0^1 \frac{t(a)}{\rho(a)} da} u + \int_0^{1+\nu} \frac{t(a)/\rho(a)}{\int_0^1 t(a)/\rho(a) da} \frac{dB(a)}{\sqrt{t(a)}}, \tag{62}$$

which has precision

$$\tau_p = \left(\left(\frac{\int_1^{1+\nu} \frac{t(a)}{\rho(a)} da}{\int_0^1 \frac{t(a)}{\rho(a)} da} \right)^2 \tau_v^{-1} + \left(\frac{1}{\int_0^1 t(a)/\rho(a) da} \right)^2 \int_0^{1+\nu} \frac{t(a)}{\rho(a)^2} da \right)^{-1}. \tag{63}$$

As in the main model, prices must be weak-form efficient, which implies

$$P = \mathbb{E}[v \mid s_p] = \frac{\tau_p}{\tau} s_p.$$

The conditional distribution of v given P is normal with mean $(\tau_p/\tau) \cdot s_p$ and variance $1/\tau$. Defining the standardized variable $z = \sqrt{\tau}(v - (\tau_p/\tau) \cdot s_p)$, we note that z follows a standard normal distribution.

Thus, we can express v as

$$v = \frac{\tau_p}{\tau} s_p + \frac{z}{\sqrt{\tau}}.$$

Substituting this into the efficiency condition gives

$$\mathbb{E}[V(v) \mid P] = \mathbb{E}\left[V\left(\frac{\tau_p}{\tau}s_p + \frac{z}{\sqrt{\tau}}\right)\right] = \int V\left(\frac{\tau_p}{\tau}s_p + \frac{z}{\sqrt{\tau}}\right) d\Phi(z),$$

where $\Phi(z)$ is the standard normal cumulative distribution function.

The rest of the derivation follows the steps of Theorem 1, leading to the final result.

Theorem 3. *There exists a unique equilibrium. The equilibrium price function has the representation* $\mathbf{P}_* = \mathcal{P}(s_p)$*, where the equilibrium sufficient statistic* s_p *is given by* (62)*. The function* $\mathcal{P}(x)$ *is given by*

$$\mathcal{P}(x) = \int V\left(\frac{\tau_p}{\tau}x + \frac{z}{\sqrt{\tau}}\right) d\Phi(z). \tag{64}$$

Here $\Phi(z)$ *denotes the standard normal cumulative distribution function (cdf). Consequently, the price function is completely determined by* $V(\cdot)$ *and two other quantities, the precision of* s_p*, given by* (63) *and the posterior precision of* v*, given by* $\tau = \mathbb{V}ar(v|P)^{-1} = \tau_v + \tau_p$*.*

The equilibrium cumulative demand function has the representation $dX(a) = \beta(a,P)ds(a) + \delta(a,P)da$*, for all* $a \in [0, 1+\nu)$*, where*

$$\beta(a,P) = \frac{t(a)}{\rho(a)} \frac{\tau^{-1}\,\mathbb{E}[V'(v)|s_p]}{\mathrm{Var}\,[V(v)|s_p]}, \text{ and}$$

$$\delta(a,P) = \frac{\beta(a,p)^2}{2t(a)}\pi(a)\frac{\mathrm{Sk}[V(v)|s_p]}{\mathrm{Var}[V(v)|s_p]} - \beta(a,p)\frac{\mathbb{E}[v(V(v)-P)^2|s_p]}{\mathrm{Var}[V(v)|s_p]} + \frac{\psi(P)}{\rho(a)\mathrm{Var}[V(v)|s_p]}.$$

Here $\rho(a)$ *and* $\pi(a)$ *denote the absolute risk aversion and prudence coefficients, defined in* (18)*. The sufficient statistic* s_p *is related to the price* P *as follows:*

$$s_p = \mathcal{P}^{-1}(P).$$

Here $\mathcal{P}^{-1}(\cdot)$ *is the inverse of the function* $\mathcal{P}(\cdot)$ *defined in* (64)*. The conditional moments of* $V(v)$ *and the function* $\psi(P)$ *are given in the closed form in* (45)*.*

E.3 Market quality

Our definitions of liquidity $\mathcal{L}$ and information efficiency $\mathcal{I}$ follow those in the main paper. Using the derivation steps from Proposition 1, we obtain

$$\mathcal{I} = \frac{\tau_p}{\tau_v + \tau_p},$$

and

$$\mathcal{L} = \frac{1}{\tau_p} \int_0^1 \frac{t(a)}{\rho(a)} da.$$

Since τ_p is given by (63), this leads to the following result.

Proposition 6. *The equilibrium information efficiency and liquidity are given by*

$$\mathcal{I} = \frac{\tau_p}{\tau_v + \tau_p}, \quad \mathcal{L} = \frac{1}{\tau_p} \int_0^1 \frac{t(a)}{\rho(a)} da,$$

where τ_p is given by (63).

E.4 Comparative statics

To ensure consistency with previous literature, which assumes exogenous noise remains unaffected by the parameters of interest, we impose that comparative statics do not influence noise traders. Formally, our definition of the Robin Hood variation assumes $h^{\Delta}(a) = 0$ for all $a \in [1, 1 + \nu)$ and coincides with Definition 3 in all other respects.

E.4.1 Wealth inequality

First, we examine changes in the wealth distribution $W_0(a)$ while keeping the precisions $t(a)$ fixed. This leads to the following proposition, which exactly matches Proposition 2.

Proposition 7. *Suppose that agent preferences are DARA. Then, there exist thresholds* $0 <$

$a_1^W \le a_2^W < 1$, *such that for any Robin Hood variation* $W_0^\Delta(a)$ *with* $\underline{a} \le a_1^W \le a_2^W \le \bar{a}$ *we have*

$$\mathcal{I}'(W_0(a))[W_0^\Delta(a)] > 0 \text{ and } \mathcal{I}'(W_0(a))[-W_0^\Delta(a)] < 0; \tag{65}$$

$$\mathcal{L}'(W_0(a))[W_0^\Delta(a)] < 0 \text{ and } \mathcal{L}'(W_0(a))[-W_0^\Delta(a)] > 0. \tag{66}$$

The mechanism highlighted in previous literature—where inequality enhances information efficiency by increasing the aggregate trading intensity of informed traders—does not alter Proposition 2 in the presence of noise. However, this does not imply that exogenous noise is irrelevant. Its presence may influence the thresholds a_1^W and a_2^W.

E.4.2 Information inequality

Here, we examine the effects of changing the distribution of information across agents on market quality holding the wealth profile fixed.

Proposition 8. *Suppose that traders have DARA utilities. Then, there exist thresholds* $0 < a_1^t \le a_2^t < 1$, *such that for any Robin Hood variation* $t^\Delta(a)$ *with* $\underline{a} \le a_1^t \le a_2^t \le \bar{a}$

$$\mathcal{I}'(t(a))[t^\Delta(a)] > 0 \text{ and } \mathcal{I}'(t(a))[-t^\Delta(a)] < 0;$$

$$\mathcal{L}'(t(a))[t^\Delta(a)] < 0 \text{ and } \mathcal{L}'(t(a))[-t^\Delta(a)] > 0.$$

The formulation of Proposition 3 remains unchanged. Corollary 1 also holds with noise.

Corollary 3 (An information-aggregation paradox)**.** *Suppose that traders have DARA utilities. Then, there exists a threshold* a_2^t *such that for any* $h^\Delta(a) \neq 0$ *such that* $h^\Delta(a) \ge 0$ *for* $a > a_2^t$, *and* $h^\Delta(a) = 0$ *otherwise,* $\mathcal{I}'(t(a))[t^\Delta(a)] > 0$ *and* $\mathcal{I}'(t(a))[-t^\Delta(a)] < 0$.

E.4.3 The role of information acquisition

Combining the results of Propositions 7 and 8, we conclude that the statement of Proposition 4 remains unchanged.

Proposition 9. *Suppose that Assumption 3 holds. Suppose that traders have DARA utilities. Suppose that precisions are a function of wealth $t(W_0(a), a)$ and are increasing in $W_0(a)$. Then, for any Robin Hood variation $W_0^\Delta(a)$ with $\underline{a} \leq \min\{a_1^t, a_1^W\} \leq \max\{a_2^t, a_2^W\} \leq \bar{a}$, results (65)–(66) hold.*

E.5 Proofs for Section E

E.6 Proof of Proposition 7

Proof of Proposition 7. Following the proof of Proposition 2, we denote absolute risk tolerance $y(a) \equiv 1/\rho(a)$. Without loss of generality, index traders such that $y(b)$ increases in b. (This is in contrast to index a, which is such that $W_0(a)$ is increasing in a.) We first compute the Gateaux derivatives of $\mathcal{I}$ and $\mathcal{L}$ with respect to $y(b)$.

Substituting (63) into $\mathcal{I} = \tau_p/(\tau_p + \tau_v)$ and computing the Gateaux derivative (this entails substituting $y(b) + \epsilon y^\Delta(b)$ instead of $y(b)$, differentiating with respect to ϵ, and evaluating the resulting expression at $\epsilon = 0$) yields:

$$\mathcal{I}'(y(b))[y^\Delta(b)] = C_{\mathcal{I}} \int_0^1 t(b) y^\Delta(b)(K_2 - K_1 y(b))\, db.$$

Here, $C_{\mathcal{I}} > 0$ is positive (we have the closed-form expressions for $C_{\mathcal{I}}$ via parameters of the model, but it is not important here), $I_1 = \int_0^1 t(b)y(b)\, db$, $I_2 = \int_0^{1+\nu} t(b)y(b)^2\, db$, and $I_3 = \int_1^{1+\nu} t(b)y(b)\, db$, $K_1 = I_3^2 + I_2\tau_v$, and $K_2 = I_1\tau_v$.

Lemma 13 implies the existence of a unique b_y^* such that $K_2 - K_1 y(b) \gtreqless 0$ if and only if $b \lesseqgtr b_y^*$. The remainder of the proof follows the same steps as Proposition 2 and is omitted for brevity.

We now turn to the liquidity results. Substituting (63) in $\mathcal{L} = \int_0^1 t(b)y(b)db/\tau_p$ and computing the Gateaux derivative yields

$$(\mathcal{L})'(y(b))[y^{\Delta}(b)] = C_{\mathcal{L}} \int_0^1 t(b)y^{\Delta}(b)(k_1 y(b) - k_2)\, db.$$

Here, $C_{\mathcal{L}} > 0$ is positive, $k_1 = 2\tau_v I_1$ and $k_2 = I_2\tau_v + I_3^2$. Lemma 13 implies that there exists a unique $b_{**}^y > b_*^y$ such that $k_1 y(b) - k_2 \gtreqless 0$ iff $b \gtreqless b_{**}^y$. The remainder of the proof follows the same steps as Proposition 2 and is omitted for brevity. ■

E.7 Proof of Proposition 8

Proof of Proposition 8.

This proof follows the same steps as the proof of Proposition 2. Without loss of generality, we index traders such that $y(b)$ is increasing in b. The proof remains identical to that of Proposition 2, except for the expressions for the Gâteaux derivatives, which we reproduce below:

$$\mathcal{I}'(t(b))[t^{\Delta}(b)] = C_{\mathcal{I}} \int_0^1 y(b)t^{\Delta}(b)(K_2 - K_1 y(b))\, db;$$

$$\mathcal{L}'(t(b))[t^{\Delta}(b)] = C_{\mathcal{L}} \int_0^1 y(b)t^{\Delta}(b)(k_1 y(b) - k_2)\, db.$$

Here, $K_2 = 2I_3^2 + 2I_2\tau_v$, $K_1 = I_1\tau_v$, $k_1 = I_1\tau_v$, and $k_2 = K_2/3$. The notation I_1, I_2, and I_3 follows the definitions in the proof of Proposition 7. ■

F Price taking equilibrium, competitive REE and BNE

In this section, we contrast three equilibrium concepts: (i) the Bayesian Nash Equilibrium (BNE), in which traders account for both (a) their individual impact on the price level and (b) the informational content of the price; (ii) the competitive Rational Expectations Equilibrium (REE), where traders internalize (b) but ignore (a); and (iii) the price-taking equilibrium, where traders disregard both. A large portion of the material presented here is drawn from Avdis et al. (2025) and is included for completeness.

A full treatment of BNE and competitive REE under general preferences and payoff structures is the subject of our ongoing work. In this section, we focus on the CARA-normal framework, modeling wealth effects in an ad hoc manner by assuming that the absolute risk aversion coefficient $\rho(a)$ decreases with initial wealth $W_0(a)$.[61]

F.1 Setup

We consider a sequence of economies with a finite number n of traders and examine the behavior of equilibrium quantities in the large economy limit as $n \to \infty$. The economy features a risky asset with payoff $v \sim N(0, \tau_v^{-1})$ and a riskless asset that serves as the numéraire.

Each trader i lives in an interval $[a_i, a_i + m)$, where $a_i = \frac{i-1}{n}$ and $m = \frac{1}{n}$. Traders exhibit Constant Absolute Risk Aversion (CARA), with individual absolute risk aversion coefficients denoted by $\rho(a_i)$. Trader i receives a private signal of the form:

$$\Delta s_i = v\,m + \frac{1}{\sqrt{t(a_i)}}\Delta B(a_i),$$

where Δs_i has precision tm.

[61] See Makarov and Schornick (2010), Kurlat and Veldkamp (2015), and Mihet (2018), who adopt a similar ad hoc approach to modeling wealth effects.

In the large economy limit, we can write the above as

$$ds(a) = v\,da + \frac{1}{\sqrt{t(a)}} dB(a).$$

Our objective is to study the limiting behavior of equilibria as $n \to \infty$. To support the existence of BNE and competitive REE, we introduce noise traders in addition to the informed traders described above. Noise traders submit a random quantity:

$$u \sim N(0, \tau_u^{-1}).$$

We begin by analyzing standard equilibrium concepts: (i) the competitive Rational Expectations Equilibrium (REE), and (ii) the Bayesian Nash Equilibrium (BNE) in demand schedules. We then turn to the price-taking equilibrium.

F.2 Market quality

Our definitions of liquidity and information efficiency are unchanged from the main text. We define information efficiency as:

$$\mathcal{I} = 1 - \frac{\text{Var}[v \mid P]}{\text{Var}[v]},$$

which measures the fraction of uncertainty about the fundamental value v that is eliminated by observing the price P.

Our measure of liquidity is based on the sensitivity of prices to price-inelastic shocks. Formally, it is defined as:

$$\mathcal{L} = -\left(\frac{\partial P}{\partial u} \cdot \frac{1}{\text{Var}[v \mid P]}\right)^{-1}.$$

In our discrete economies, equilibrium is linear, with individual demands taking the form:

$$x_i^n = \beta^n(a_i)\,\Delta s_i - \gamma^n(a_i)\,p\,m.$$

Under this structure, the liquidity expression simplifies to:

$$\mathcal{L} = \mathrm{Var}[v \mid P] \cdot \sum_{i=1}^{n} \gamma(a_i)\,m. \tag{67}$$

F.3 Competitive REE

We begin by defining the notion of equilibrium in the discrete economy:

Definition 8. *In the discrete economy n, a competitive REE consists of a price function P^n and a collection of demand schedules $\{x_i^n(p, \Delta s_i)\}$, such that the following two conditions are satisfied:*

1. *Optimality: For each trader i,*

$$x_i^n(p, \Delta s_i) \in \arg\max_x \mathbb{E}\left[-\exp\left(-\gamma_i(v-p)x\right) \mid \Delta s_i, P^n = p\right];$$

2. *Market clearing:*

$$\sum_i x_i^n(P^n, \Delta s_i) + u = 0.$$

We employ a guess-and-verify approach to solve for the equilibrium. We conjecture that the equilibrium strategies in the discrete economy n take the linear form:

$$x_i^n = \beta^n(a_i)\,\Delta s_i - \gamma^n(a_i)\,p\,m, \tag{68}$$

where $\beta^n(a_i)$ and $\gamma^n(a_i)$ are constants specific to each trader.

In the large economy limit, the corresponding strategy becomes:

$$dX(a) = \beta(a)\, ds(a) - \gamma(a)\, p\, da.$$

From market clearing, traders can compute the sufficient statistic:

$$s_p^n = v + \sum_{i=1}^{n} \frac{w^n(a_i)}{\sqrt{t(a_i)}} \Delta B(a_i) + \frac{u}{\sum_{i=1}^{n} \beta(a_i) m} = P \cdot \frac{\sum_{i=1}^{n} \gamma(a_i) m}{\sum_{i=1}^{n} \beta(a_i) m},$$

where we introduce the weights

$$w^n(a_i) = \frac{\beta^n(a_i)}{\sum_{j=1}^{n} \beta(a_j) m}.$$

In the large economy limit, these expressions become:

$$s_p = v + \int_0^1 \frac{w(a)}{\sqrt{t(a)}}\, dB(a) + \frac{u}{\int_0^1 \beta(a)\, da} = P \cdot \frac{\int_0^1 \gamma(a)\, da}{\int_0^1 \beta(a)\, da}, \tag{69}$$

and the weight function is defined as

$$w(a) = \frac{\beta(a)}{\int_0^1 \beta(a)\, da}.$$

F.3.1 Heuristic derivation of equilibrium

Following the approach in Section 6, it is straightforward to derive that optimal demands take the form:

$$dX(a) = \frac{\mathbb{E}[v - P \mid ds(a), s_p]}{\rho(a)\ \mathbb{V}\mathrm{ar}(v|s_p)}. \tag{70}$$

To identify $\beta(a)$, we isolate the contribution of $ds(a)$ on the right-hand side of (70), which requires computing the conditional expectation $\mathbb{E}[v \mid ds(a), s_p]$.

To facilitate this calculation, we rewrite the conditional expectation as:

$$\mathbb{E}[v \mid ds(a), s_p] = \mathbb{E}[v \mid ds(a), s_p - w(a)\, ds(a)],$$

where $w(a) = \beta(a)/\int_0^1 \beta(a)\, da$. Since $ds(a)$ and $s_p - w(a)\, ds(a)$ are conditionally independent given v, we can apply the standard linear projection formula to obtain:

$$\begin{aligned}\mathbb{E}[v \mid ds(a), s_p - w(a)\, ds(a)] &= \frac{t(a)}{\tau}\, ds(a) + \frac{\tau_p}{\tau}\,(s_p - w(a)\, ds(a)) \\ &= \left(\frac{t(a)}{\tau} - \frac{\tau_p}{\tau} w(a)\right) ds(a) + \cdots,\end{aligned}$$

where $\tau \equiv \mathbb{V}\mathrm{ar}(v|s_p)^{-1}$, τ_p is the precision of s_p, and "$\cdots$" denotes terms that do not depend on $ds(a)$.

Matching the coefficients of $ds(a)$ in (70), and substituting for $w(a)$, we obtain:

$$\beta(a) = \frac{1}{\rho(a)} \left(t(a) - \tau_p \cdot \frac{\beta(a)}{\int_0^1 \beta(a)\, da} \right). \tag{71}$$

To solve for $\beta(a)$, we introduce the shorthand:

$$\kappa \equiv \frac{\tau_p}{\int_0^1 \beta(a)\, da}. \tag{72}$$

Then equation (80) becomes:

$$\beta(a) = \frac{1}{\rho(a)}\,(t(a) - \kappa\, \beta(a)) \quad \iff \quad \beta(a) = \frac{t(a)}{\rho(a) + \kappa}.$$

Thus, once κ is determined, $\beta(a)$ is fully characterized. To solve for κ, we compute precision

τ_p, by deriving the variance of the noise term in the sufficient statistic s_p:

$$\tau_p^{-1} = \mathrm{Var}\left(\int_0^1 \frac{w(a)}{\sqrt{t(a)}}\, dB(a) + \frac{u}{\int_0^1 \beta(a)\, da}\right)$$
$$= \int_0^1 \frac{w(a)^2}{t(a)}\, da + \frac{1}{\left(\int_0^1 \beta(a)\, da\right)^2},$$

where the second equality follows from Itô isometry and can be derived heuristically as in Section 6.

Substituting $w(a) = \beta(a)/\int_0^1 \beta(a)\, da$, we obtain:

$$\tau_p = \frac{\left(\int_0^1 \beta(a)\, da\right)^2}{\int_0^1 \frac{\beta(a)^2}{t(a)}\, da + \tau_u^{-1}}. \tag{73}$$

Hence,

$$\kappa = \frac{\int_0^1 \beta(a)\, da}{\int_0^1 \frac{\beta(a)^2}{t(a)}\, da + \tau_u^{-1}}. \tag{74}$$

Substituting $\beta(a) = t(a)/(\rho(a) + \kappa)$ into (74), we obtain an equation for κ alone:

$$\kappa = \frac{\int_0^1 \frac{t(a)}{\rho(a)+\kappa}\, da}{\int_0^1 \frac{t(a)}{(\rho(a)+\kappa)^2}\, da + \tau_u^{-1}}.$$

We summarize this heuristic derivation below and provide a rigorous proof in Appendix F.7.

Theorem 4. *Let κ be the unique positive solution to*

$$\kappa = \frac{\int_0^1 \frac{t(a)}{\rho(a)+\kappa}\, da}{\int_0^1 \frac{t(a)}{(\rho(a)+\kappa)^2}\, da + \tau_u^{-1}}. \tag{75}$$

There exists a unique competitive REE with

$$\beta(a) = \frac{t(a)}{\rho(a) + \kappa}.$$

F.3.2 Market quality

The following proposition characterizes liquidity and information efficiency in equilibrium.

Proposition 10. *The equilibrium information efficiency and liquidity are given by*

$$\mathcal{I} = \frac{\tau_p}{\tau_v + \tau_p}, \quad \mathcal{L} = \frac{1}{\kappa},$$

where

$$\tau_p = \kappa \int_0^1 \frac{t(a)}{\rho(a) + \kappa} \, da, \tag{76}$$

and κ solves (75).

With the above proposition in hand, we are now ready to examine how changes in the wealth distribution affect market quality.

F.3.3 Comparative statics

We assume that absolute risk aversion is a decreasing function of initial wealth. With a slight abuse of notation, we write it as $\rho(W_0(a))$, where $\rho(\cdot)$ is a decreasing function. In addition, we continue to maintain Assumption 3 from the main text and assume that traders are indexed so that $W_0(a)$ is increasing in a.

Since endogenous information acquisition is not modeled here, we conduct comparative statics with respect to $W_0(a)$ only, and take $t(a)$ as exogenous.

Proposition 11. *Suppose that agent preferences are DARA. Then there exist thresholds $0 < a_1^W \leq a_2^W < 1$ such that for any Robin Hood variation $W_0^\Delta(a)$ with $\underline{a} \leq a_1^W \leq a_2^W \leq \bar{a}$, the following holds:*

$$\mathcal{L}'(W_0(a))[W_0^\Delta(a)] < 0 \quad \text{and} \quad \mathcal{L}'(W_0(a))[-W_0^\Delta(a)] > 0.$$

Now suppose in addition that $\int_0^1 W_0^{\Delta}(a)\, da = 0$*, and that the function* $a \mapsto t(a)\, |\rho'(W_0(a))|/(c+\rho(W_0(a)))^2$ *is decreasing in* a *for every* $c > 0$*, at least for* a *sufficiently close to 0 and 1. Then, there exist thresholds* $0 < a_1^W \leq a_2^W < 1$ *such that for any Robin Hood variation* $W_0^{\Delta}(a)$ *with* $\underline{a} \leq a_1^W \leq a_2^W \leq \bar{a}$*, we have:*

$$\mathcal{I}'(W_0(a))[W_0^{\Delta}(a)] > 0 \quad \textit{and} \quad \mathcal{I}'(W_0(a))[-W_0^{\Delta}(a)] < 0.$$

Our results on liquidity extend directly from the main paper without any modification. In the case of information efficiency, we impose additional technical assumptions in order to complete the proof. However, the main economic message remains unchanged: transferring wealth from the rich to the poor improves information efficiency but reduces liquidity.

The proof is more intricate here due to the interaction between liquidity and information efficiency—an effect that was absent in the baseline model. As seen in equation (76), the precision τ_p depends on $\kappa = 1/\mathcal{L}$, introducing a feedback loop between the two measures.

To ensure the result goes through analytically, we impose two auxiliary conditions:

1. *Budget balance*: The requirement $\int_0^1 W_0^{\Delta}(a)\, da = 0$ guarantees that the Robin Hood variation is not wasteful—all wealth taken from the rich is transferred to the poor.

2. *Monotonicity*: We assume that the function

$$y(a) = \frac{t(a)\, |\rho'(W_0(a))|}{(c + \rho(W_0(a)))^2}$$

is decreasing in a. This condition is satisfied, for instance, when $\rho(a) = \eta/W_0(a)$ (emulating CRRA preferences with RRA equal to η) and $t(a) = 1$.[62] In that case,

$$y(a) = \frac{\eta}{(cW_0(a) + \eta)^2}$$

[62]This example can be generalized to allow for non-constant signal precisions $t(a)$.

is decreasing in a.

While these assumptions are used for technical tractability, we expect that the underlying comparative statics continue to hold more broadly.

F.4 BNE

In this section, we consider an equilibrium with strategic traders, as in Kyle (1989).

Definition 9. *A Bayesian Nash Equilibrium (BNE) is a collection of demand schedules $\{x_i^n(p, \Delta s_i)\}_{i=1}^n$ such that for each trader i, the schedule x_i^n solves:*

$$\mathbb{E}\left[-\exp\left(\rho(a_i)(v - p(x_i^n, x_{-i}^n)) \cdot x_i^n\right)\right] \geq \mathbb{E}\left[-\exp\left(\rho(a_i)(v - p(y, x_{-i}^n)) \cdot y\right)\right]$$

for any alternative demand schedule y.

Here, x_{-i}^n denotes the vector of equilibrium demand schedules for traders $j \neq i$, and $p(y, x_{-i}^n)$ is the market-clearing price when trader i's schedule is y, and other traders follow x_{-i}^n.

We employ a guess-and-verify approach to solve for the equilibrium. We conjecture that the equilibrium strategies in the discrete economy with n traders take the linear form:

$$x_i^n = \beta^n(a_i)\, \Delta s_i - \gamma^n(a_i)\, p\, m, \tag{77}$$

where $\beta^n(a_i)$ and $\gamma^n(a_i)$ are constants specific to each trader.

In the large economy limit, the corresponding strategy becomes:

$$dX(a) = \beta(a)\, ds(a) - \gamma(a)\, p\, da.$$

Following the derivation in Kyle (1989), the optimal demand for trader i is given by:

$$x_i^n = \frac{\mathbb{E}[v \mid \Delta s_i, s_p^n] - P}{\rho(a_i)\,\mathrm{Var}[v \mid \Delta s_i, s_p^n] + \lambda_i^n}.$$

Here, λ_i denotes the individual price impact, i.e., the slope of the inverse residual supply curve faced by trader i, given by:

$$\lambda_i^n = \left(\sum_{j \neq i} m\,\gamma^n(a_j)\right)^{-1}.$$

In the large economy limit, we write, heuristically:

$$dX(a) = \frac{\mathbb{E}[v - P \mid ds(a), s_p]}{\rho(a)\ \mathbb{V}\mathrm{ar}(v|s_p) + \lambda}, \tag{78}$$

where

$$\lambda = \left(\int_0^1 \gamma(a)\,da\right)^{-1} = \mathbb{V}\mathrm{ar}(v|s_p) \cdot \mathcal{L}^{-1}.$$

In the last equality, we used the limiting form of the liquidity expression from equation (67), replacing the discrete sum $\sum_{i=1}^n \gamma(a_i)\,m$ with its continuous counterpart $\int_0^1 \gamma(a)\,da$.

F.4.1 Heuristic derivation of equilibrium

We focus our derivation on the coefficient $\beta(a)$. Define the constant κ as in Section F.3, via equation (72). We will show below that the relationship $\kappa = 1/\mathcal{L}$ continues to hold in the Bayesian Nash Equilibrium (BNE). This allows us to rewrite the first-order condition (79) as:

$$dX(a) = \frac{\mathbb{E}[v - P \mid ds(a), s_p]}{\mathrm{Var}[v \mid s_p] \cdot (\rho(a) + \kappa)}. \tag{79}$$

Proceeding as in Section F.3.1 we obtain:

$$\mathbb{E}[v \mid ds(a), s_p - w(a)\, ds(a)] = \left(\frac{t(a)}{\tau} - \frac{\tau_p}{\tau} w(a)\right) ds(a) + \cdots,$$

where $\tau = \mathrm{Var}[v \mid s_p]^{-1}$, and $\cdots$ denotes terms independent of $ds(a)$.

Matching the coefficients of $ds(a)$ in equation (79), and substituting for $w(a) = \beta(a)/\int_0^1 \beta(a)\, da$, we obtain:

$$\beta(a) = \frac{1}{\rho(a) + \kappa}\left(t(a) - \tau_p \cdot \frac{\beta(a)}{\int_0^1 \beta(a)\, da}\right). \tag{80}$$

Substituting the definition of κ from (72) into the equation above gives:

$$\beta(a) = \frac{t(a) - \kappa \cdot \beta(a)}{\rho(a) + \kappa} \quad \Longleftrightarrow \quad \beta(a) = \frac{t(a)}{\rho(a) + 2\kappa}.$$

Moreover, equation (74) continues to hold in BNE. Substituting the above expression for $\beta(a)$ into it, we obtain the equation that determines κ:

$$\kappa = \frac{\int_0^1 \frac{t(a)}{\rho(a)+2\kappa}\, da}{\int_0^1 \frac{t(a)}{(\rho(a)+2\kappa)^2}\, da + \tau_u^{-1}}.$$

We summarize this heuristic derivation below and provide a rigorous proof in Appendix F.7.

Theorem 5. *Let k be the unique positive solution to*

$$\kappa = \frac{\int_0^1 \frac{t(a)}{\rho(a)+2\kappa}\, da}{\int_0^1 \frac{t(a)}{(\rho(a)+2\kappa)^2}\, da + \tau_u^{-1}}. \tag{81}$$

There exists a unique equilibrium with

$$\beta(a) = \frac{t(a)}{\rho(a) + 2\kappa}.$$

F.4.2 Market quality

The characterization of liquidity and information efficiency in equilibrium remains unchanged from the competitive REE case.[63] For completeness, we restate the result below.

Proposition 12. *The equilibrium information efficiency and liquidity are given by*

$$\mathcal{I} = \frac{\tau_p}{\tau_v + \tau_p}, \quad \mathcal{L} = \frac{1}{\kappa},$$

where

$$\tau_p = \kappa \int_0^1 \frac{t(a)}{\rho(a) + 2\kappa}\, da,$$

and κ is the unique solution to equation (81).

We now turn to the comparative statics analysis.

F.4.3 Comparative statics

We assume that absolute risk aversion is a decreasing function of initial wealth. With a slight abuse of notation, we write it as $\rho(W_0(a))$, where $\rho(\cdot)$ is a decreasing function. In addition, we continue to maintain Assumption 3 from the main text and assume that traders are indexed so that $W_0(a)$ is increasing in a.

The comparative statics results in the Bayesian Nash Equilibrium closely mirror those in the competitive REE case. However, the technical conditions that were previously required only for proving the results on information efficiency $\mathcal{I}$ are now also necessary for establishing the results on liquidity $\mathcal{L}$. As discussed earlier, the core economic message remains unchanged: transferring wealth from the rich to the poor increases information efficiency but decreases liquidity.

[63] The only difference lies in the expression for $\beta(a)$, which is given by $\beta(a) = t(a)/(\rho(a) + 2\kappa)$ in BNE, as opposed to $\beta(a) = t(a)/(\rho(a) + \kappa)$ in REE.

Proposition 13. *Suppose that agent preferences exhibit decreasing absolute risk aversion (DARA). Assume, in addition, that the function*

$$a \mapsto \frac{t(a)\,|\rho'(W_0(a))|}{(c+\rho(W_0(a)))^2}$$

is decreasing in a for every $c > 0$, at least for a sufficiently close to 0 and 1. Then, there exist thresholds $0 < a_1^W \leq a_2^W < 1$ such that for any Robin Hood variation $W_0^\Delta(a)$, satisfying $\int_0^1 W_0^\Delta(a)\,da = 0$ and $\underline{a} \leq a_1^W \leq a_2^W \leq \bar{a}$, the following comparative statics hold:

$$\mathcal{I}'(W_0(a))[W_0^\Delta(a)] > 0 \text{ and } \mathcal{I}'(W_0(a))[-W_0^\Delta(a)] < 0;$$

$$\mathcal{L}'(W_0(a))[W_0^\Delta(a)] < 0 \text{ and } \mathcal{L}'(W_0(a))[-W_0^\Delta(a)] > 0.$$

F.5 Price-taking equilibrium

The definition of equilibrium and the derivation are as in the main part of the paper. For completeness, we state the characterization of the equilibrium in the theorem below.

Theorem 6. *There exists a unique price-taking equilibrium with*

$$\beta(a) = \frac{t(a)}{\rho(a)}.$$

F.5.1 Market quality

Substituting $\beta(a) = t(a)/\rho(a)$ into equation (73), we obtain the expression for the precision of the sufficient statistic of the price:

$$\tau_p = \frac{\left(\int_0^1 \frac{t(a)}{\rho(a)}\,da\right)^2}{\int_0^1 \frac{t(a)}{\rho(a)^2}\,da + \tau_u^{-1}}.$$

The expression for information efficiency $\mathcal{I}$ then follows from:

$$\mathcal{I} = \frac{\tau_p}{\tau_v + \tau_p}.$$

Similarly, substituting $\beta(a) = t(a)/\rho(a)$ into equation (74), and using the identity $\kappa = 1/\mathcal{L}$, we obtain:

$$\mathcal{L} = \frac{\int_0^1 \frac{t(a)}{\rho(a)^2}\, da + \tau_u^{-1}}{\int_0^1 \frac{t(a)}{\rho(a)}\, da}.$$

We summarize these expressions in the proposition below.

Proposition 14. *The equilibrium expressions for information efficiency $\mathcal{I}$ and liquidity $\mathcal{L}$ are given by:*

$$\mathcal{I} = \left(1 + \tau_v \cdot \frac{\int_0^1 \frac{t(a)}{\rho(a)^2}\, da + \tau_u^{-1}}{\left(\int_0^1 \frac{t(a)}{\rho(a)}\, da\right)^2}\right)^{-1}, \qquad \mathcal{L} = \frac{\int_0^1 \frac{t(a)}{\rho(a)^2}\, da + \tau_u^{-1}}{\int_0^1 \frac{t(a)}{\rho(a)}\, da}. \tag{82}$$

We are ready to proceed to comparative statics.

F.5.2 Comparative statics

We assume that absolute risk aversion is a decreasing function of initial wealth. With a slight abuse of notation, we write it as $\rho(W_0(a))$, where $\rho(\cdot)$ is a decreasing function. In addition, we continue to maintain Assumption 3 from the main text and assume that traders are indexed so that $W_0(a)$ is increasing in a.

The comparative statics results remain unchanged from the main part of the paper. The proof, however, requires a slight adjustment to account for the presence of noise traders; the modified argument is presented in Section F.7.

Proposition 15. *Suppose that agent preferences are DARA. Then, there exist thresholds* $0 <$

$a_1^W \leq a_2^W < 1$, *such that for any Robin Hood variation* $W_0^\Delta(a)$ *with* $\underline{a} \leq a_1^W \leq a_2^W \leq \bar{a}$ *we have*

$$\mathcal{I}'(W_0(a))[W_0^\Delta(a)] > 0 \text{ and } \mathcal{I}'(W_0(a))[-W_0^\Delta(a)] < 0;$$
$$\mathcal{L}'(W_0(a))[W_0^\Delta(a)] < 0 \text{ and } \mathcal{L}'(W_0(a))[-W_0^\Delta(a)] > 0.$$

F.6 Comparing Price-Taking Equilibrium, REE, and BNE

Juxtaposing Theorems 4, 5, and 6, we can summarize the equilibrium trading intensities as follows:

$$\beta^{PT}(a) = \frac{t(a)}{\rho(a)}, \quad \beta^{REE}(a) = \frac{t(a)}{\rho(a) + \kappa}, \quad \beta^{BNE}(a) = \frac{t(a)}{\rho(a) + 2\kappa},$$

corresponding to the price-taking equilibrium, competitive Rational Expectations Equilibrium (REE), and Bayesian Nash Equilibrium (BNE), respectively.

Here, $\kappa = 1/\mathcal{L}$ represents the inverse of market liquidity and serves as a measure of price impact. The only difference across the expressions lies in the denominator, which reflects the extent to which traders internalize their influence on the market.

- Price-Taking Equilibrium: Traders behave competitively and ignore both how they affect the informational content of the price and their direct price impact. Hence, no adjustment is made to the denominator.

- Rational Expectations Equilibrium (REE): Traders internalize that they affect the informational content of prices but not their impact on the price level. This introduces a correction term $+\kappa$.

- Bayesian Nash Equilibrium (BNE): Traders account for both how they affect the information in the price and their influence on the price level. This leads to a total adjustment of $+2\kappa$.

Thus, the denominator can be interpreted as $\rho(a) +$ (number of effects internalized) $\cdot\, \kappa$.

F.7 Proofs for Section F

F.7.1 Proof of Theorem 4

Proof of Theorem 4.

Given the strategies in (77), the market-clearing price function takes the form:

$$P^n = k_s^n \left(\sum_i w^n(a_i)\, \Delta s_i \right) + k_u^n\, u,$$

where the constants k_s^n and k_u^n are given by:

$$k_s^n = \frac{\sum_{i=1}^n \beta(a_i)m}{\sum_{i=1}^n \gamma(a_i)m}, \qquad k_u^n = \frac{1}{\sum_{i=1}^n \gamma(a_i)m}.$$

The optimal demand of trader i is:

$$x_i^n = \frac{\mathbb{E}[v \mid \Delta s_i, s_p^n] - p}{\rho(a_i)\, \mathrm{Var}[v \mid \Delta s_i, s_p^n]}.$$

We decompose the aggregate demand as follows:

$$\sum_i x_i^n = \sum_i \frac{1}{\rho(a_i)\, \mathrm{Var}[v \mid \Delta s_i, s_p^n]} \left(\mathbb{E}[v \mid \Delta s_i, s_p^n] - \mathbb{E}[v \mid s_p^n] + \underbrace{\mathbb{E}[v \mid s_p^n] - p}_{\text{risk premium}} \right).$$

To compute the first term, we apply the standard Bayesian updating formulas (see Lemma 15) and use the Aggregation Lemma. In the limit $n \to \infty$, this yields:

$$\lim_{n\to\infty} \sum_i \frac{1}{\rho(a_i)\, \mathrm{Var}[v \mid \Delta s_i, s_p^n]} \left(\mathbb{E}[v \mid \Delta s_i, s_p^n] - \mathbb{E}[v \mid s_p^n] \right) = \tag{83}$$
$$\int_0^1 \frac{t(a) - w(a)\, \tau_p}{\rho(a)}\, ds(a) + \int_0^1 \frac{s_p\, \tau_p\, (-t(a)^2 + t(a)w(a)(\tau_p + 3\tau_v) + \tau_p \tau_v w(a)^2)}{\rho(a)\, t(a)\, (\tau_p + \tau_v)}\, da.$$

Here, all quantities without superscript n denote the $n \to \infty$ limit of their discrete counterparts.

To compute the risk premium term, we apply $\mathbb{E}[\cdot \mid s_p]$ to both sides of the market-clearing condition:

$$\lim_{n\to\infty} \sum_i \frac{\mathbb{E}[v \mid s_p^n] - P^n}{\rho(a_i)\,\mathrm{Var}[v \mid \Delta s_i, s_p^n]} = -\mathbb{E}[u \mid s_p].$$

Thus, the risk premium is of order $1/n$ and can be written as:

$$\mathbb{E}[v \mid s_p^n] - p = \frac{1}{n}\psi(s_p) + \text{higher-order terms},$$

where the function $\psi(s_p)$ is determined by market clearing:

$$(\tau_v + \tau_p)\,\psi(s_p) = -\mathbb{E}[u \mid s_p] \int_0^1 1/\rho(a)\,da.$$

Substituting into (86) leads to (80), and the rest of the proof follows the heuristic derivation.

To establish uniqueness of the positive solution κ in (75), note that the equation can be rewritten as:

$$\tau_u^{-1} = \int_0^1 \frac{t(a)\,\gamma(a)}{(\gamma(a)+\kappa)^2\,\kappa}\,da. \tag{84}$$

The right-hand side is a continuous, strictly decreasing function of κ, with limits $+\infty$ as $\kappa \to 0$, and 0 as $\kappa \to \infty$. Therefore, by the Intermediate Value Theorem, a unique positive solution exists. ■

Lemma 15. *We have*

$$E[v|s_p^n] = \frac{\tau_p^n}{\tau_p^n + \tau_v} s_p^n;$$

$$E[v|s_p^n, \Delta s_i] = \frac{t(a_i) - w^n(a_i)\tau_{p,i}^n/(1 - mw^n(a_i))}{\tau_v + \tau_{p,i}^n + t(a_i)m} \Delta s_i + \frac{\tau_{p,i}^n/(1 - mw^n(a_i))}{\tau_v + \tau_{p,i}^n + t(a_i)m} s_p^n;$$

$$\mathrm{Var}[v|s_p, \Delta s_i]^{-1} = \tau_v + \tau_{p,i}^n + t(a_i)m.$$

Here we denoted

$$(\tau_p^n)^{-1} = \sum_j \frac{(w^n(a_j))^2 m}{t(a_j)} + \left(\frac{k_u^n}{k_s^n}\right)^2 \tau_u^{-1} \text{ and } (\tau_{p,i}^n)^{-1} = \sum_{j \neq i} \frac{(w^n(a_j))^2 m}{t(a_j)(1 - mw^n(a_i))^2} + \left(\frac{k_u^n}{k_s^n}\right)^2 \frac{\tau_u^{-1}}{(1 - mw^n(a_i))^2}.$$

Proof of Lemma 15. First, we construct the statistic $s_{p,i}^n$ that is conditionally uncorrelated with Δs_i:

$$s_{p,i}^n = (s_p - w^n(a_i)\Delta s_i)/(1 - mw^n(a_i)) = v + \frac{\sum_{j \neq i} w^n(a_j)\Delta B_j/\sqrt{t(a_j)}}{(1 - mw^n(a_i))} + \frac{k_u^n}{k_s^n} \frac{u}{1 - mw^n(a_i)}.$$

It has precision, $\tau_{p,i}^n$ where $(\tau_{p,i}^n)^{-1} = \sum_{j \neq i} \frac{(w^n(a_j))^2 m}{t(a_j)(1 - mw^n(a_i))^2} + \left(\frac{k_u^n}{k_s^n}\right)^2 \frac{\tau_u^{-1}}{(1 - mw^n(a_i))^2}$. Using the standard formulas, we can write $E[v|s_p^n, \Delta s_i] = E[v|s_{p,i}^n, \Delta s_i] = \frac{t(a_i)\Delta s_i}{\tau_v + \tau_{p,i}^n + t(a_i)m} + \frac{\tau_{p,i}^n s_{p,i}^n}{\tau_v + \tau_{p,i}^n + t(a_i)m}$. Substituting the relationship between s_p and $s_{p,i}^n$ we obtain the stated result. ■

F.7.2 Proof of Proposition 10

Proof of Proposition 10. First, recall from (69) that

$$s_p = P \cdot \frac{\int_0^1 \gamma(a)\, da}{\int_0^1 \beta(a)\, da}.$$

The price impact λ is defined as the reciprocal of the slope of aggregate demand:

$$\lambda^{-1} = \int_0^1 \gamma(a)\, da.$$

Substituting into the expression for s_p, we get:

$$s_p = \frac{1}{\lambda \int_0^1 \beta(a)\, da} \cdot P.$$

In the large economy limit, prices are weak-form efficient:

$$P = \mathbb{E}[v \mid s_p] = \frac{\tau_p}{\tau_v + \tau_p} \cdot s_p.$$

Substituting the expression for s_p in terms of P, we find:

$$\begin{aligned}\mathbb{E}[v \mid s_p] - P &= \frac{\tau_p}{\tau_v + \tau_p} \cdot s_p - P \\ &= \left(\frac{\tau_p}{\tau_v + \tau_p} \cdot \frac{1}{\lambda \int_0^1 \beta(a)\, da} - 1 \right) P.\end{aligned}$$

Since this expression must be zero for equilibrium prices, we conclude:

$$\frac{\tau_p}{\tau_v + \tau_p} \cdot \frac{1}{\lambda \int_0^1 \beta(a)\, da} = 1.$$

Rewriting this identity, we obtain:

$$\kappa = \frac{\tau_p}{\int_0^1 \beta(a)\, da} = \lambda(\tau_p + \tau_v) = \frac{\lambda}{\mathrm{Var}[v \mid s_p]} = \frac{1}{\mathcal{L}}.$$

The expression for $\mathcal{I}$ follows directly from the definition of information efficiency, while the expression for $\mathcal{L}$ follows from the above identity and the definition of κ given in (72). ■

F.7.3 Proof of Proposition 11

Proof of Proposition 11. Note that the equation (87) that pins down κ in equilibrium can be rewritten as:

$$\kappa\tau_u^{-1} = \int_0^1 \frac{t(a)\,\rho(W_0(a))}{(\rho(W_0(a))+\kappa)^2}\,da. \tag{85}$$

We now differentiate this equation implicitly with respect to the wealth distribution $W_0(a)$. Specifically, we consider a perturbation of the form $W_0(a)+\epsilon W_0^{\Delta}(a)$, differentiate both sides of (85) with respect to ϵ, assuming $\kappa=\kappa(\epsilon)$, and evaluate the derivative at $\epsilon=0$, thereby obtaining the Gateaux derivative $\kappa'(W_0(a))[W_0^{\Delta}(a)]$.

This yields:

$$\kappa'(W_0(a))[W_0^{\Delta}(a)] = C_\kappa \int_0^1 \frac{t(a)\,\rho'(W_0(a))}{(\rho(W_0(a))+\kappa)^2}\left(1-\frac{2\rho(W_0(a))}{\rho(W_0(a))+\kappa}\right)W_0^{\Delta}(a)\,da,$$

where

$$C_\kappa = \left(\tau_u^{-1} + 2\int_0^1 \frac{t(a)\,\rho(W_0(a))}{(\kappa+\rho(W_0(a)))^2}\,da\right)^{-1} > 0.$$

is a positive constant.

By Lemma 13, there exists a unique threshold a^* such that

$$1-\frac{2\rho(W_0(a))}{\rho(W_0(a))+\kappa} \gtrless 0 \quad \text{if and only if} \quad a \lessgtr a^*.$$

Therefore, for any Robin Hood variation $W_0^{\Delta}(a)$ with $\underline{a} < a_* < \bar{a}$, we have

$$\rho'(W_0(a))\,W_0^{\Delta}(a)\left(1-\frac{2\rho(W_0(a))}{\rho(W_0(a))+\kappa}\right) > 0,$$

and hence $\kappa'(W_0(a))[W_0^{\Delta}(a)] > 0$.

We now turn to the comparative statics of $\mathcal{I}$. It suffices to show that

$$\tau_p'(W_0(a))[W_0^\Delta(a)] > 0$$

for a Robin Hood variation with sufficiently low $\underline{a}$ and sufficiently high $\bar{a}$.

Applying the chain rule, we obtain:

$$\tau_p'(W_0(a))[W_0^\Delta(a)] = \tau_p'(W_0(a);\bar{\kappa})[W_0^\Delta(a)] + \frac{\partial \tau_p}{\partial \kappa} \cdot \kappa'(W_0(a))[W_0^\Delta(a)],$$

where the bar in $\tau_p'(W_0(a);\bar{\kappa})$ denotes the derivative of τ_p with respect to $W_0(a)$ while keeping κ fixed.

Differentiating $\tau_p = \kappa \int_0^1 \frac{t(a)}{\rho(a)+\kappa}\, da$, we compute:

$$\tau_p'(W_0(a);\bar{\kappa})[W_0^\Delta(a)] = -\bar{\kappa} \int_0^1 \frac{t(a)\, \rho'(W_0(a))\, W_0^\Delta(a)}{\left(\bar{\kappa} + \rho(W_0(a))\right)^2}\, da.$$

By Lemma 16, if the integrand

$$y(a) = -\frac{t(a)\, \rho'(W_0(a))}{\left(\bar{\kappa} + \rho(W_0(a))\right)^2}$$

is decreasing in a, then

$$\tau_p'(W_0(a);\bar{\kappa})[W_0^\Delta(a)] > 0.$$

Since $\frac{\partial \tau_p}{\partial \kappa} > 0$ and $\kappa'(W_0(a))[W_0^\Delta(a)] > 0$ by earlier results, the full derivative $\tau_p'(W_0(a))[W_0^\Delta(a)]$ is also positive. This completes the proof of the proposition. ■

Lemma 16. *Let $h^\Delta(a)$ be a Robin Hood variation of a parameter $h(a)$, with thresholds $0 < \underline{a} < \bar{a} < 1$, and suppose that $\int_0^1 h^\Delta(a)\, da = 0$. Let $y(a)$ be a decreasing function on $[0,1]$. Then:*

$$\int_0^1 y(a)\, h^\Delta(a)\, da > 0.$$

Proof of Lemma 16. Let $a^* \in (\underline{a}, \bar{a})$. Since $y(a)$ is decreasing, the function $y(a) - y(a^*)$ is positive for $a < a^*$ and negative for $a > a^*$. The same sign pattern holds for $h^\Delta(a)$, by the definition of a Robin Hood variation. Hence, the product $(y(a) - y(a^*))h^\Delta(a)$ is non-negative everywhere and strictly positive on a set of positive measure. It follows that:

$$\int_0^1 (y(a) - y(a^*))h^\Delta(a)\, da > 0.$$

Using linearity of the integral, we rewrite the left-hand side:

$$\int_0^1 (y(a) - y(a^*))h^\Delta(a)\, da = \int_0^1 y(a)\, h^\Delta(a)\, da - y(a^*) \int_0^1 h^\Delta(a)\, da.$$

Since $\int_0^1 h^\Delta(a)\, da = 0$ by assumption, the second term vanishes, and we conclude:

$$\int_0^1 y(a)\, h^\Delta(a)\, da > 0,$$

as claimed. ■

F.7.4 Proof of Theorem 5

Proof of Theorem 5.

The proof follows closely that of Theorem 4.

Given the strategies in (77), the market-clearing price function takes the form:

$$P^n = k_s^n \left(\sum_i w^n(a_i)\, \Delta s_i \right) + k_u^n\, u,$$

where the constants k_s^n and k_u^n are given by:

$$k_s^n = \frac{\sum_{i=1}^n \beta(a_i) m}{\sum_{i=1}^n \gamma(a_i) m}, \quad k_u^n = \frac{1}{\sum_{i=1}^n \gamma(a_i) m}.$$

The optimal demand of trader i is:

$$x_i^n = \frac{\mathbb{E}[v \mid \Delta s_i, s_p^n] - p}{\rho(a_i)\,\mathrm{Var}[v \mid \Delta s_i, s_p^n] + \lambda_i^n}.$$

We decompose the aggregate demand as follows:

$$\sum_i x_i^n = \sum_i \frac{1}{\rho(a_i)\,\mathrm{Var}[v \mid \Delta s_i, s_p^n] + \lambda_i^n}\left(\mathbb{E}[v \mid \Delta s_i, s_p^n] - \mathbb{E}[v \mid s_p^n] + \underbrace{\mathbb{E}[v \mid s_p^n] - p}_{\text{risk premium}}\right).$$

To compute the first term, we apply the standard Bayesian updating formulas (see Lemma 15) and use the Aggregation Lemma. In the limit $n \to \infty$, this yields:

$$\lim_{n\to\infty} \sum_i \frac{1}{\rho(a_i)\,\mathrm{Var}[v \mid \Delta s_i, s_p^n] + \lambda_i^n}\left(\mathbb{E}[v \mid \Delta s_i, s_p^n] - \mathbb{E}[v \mid s_p^n]\right) = \tag{86}$$

$$\int_0^1 \frac{t(a) - w(a)\,\tau_p}{\rho(a) + \kappa}\,ds(a) + \int_0^1 \frac{s_p\,\tau_p\,(-t(a)^2 + t(a)w(a)(\tau_p + 3\tau_v) + \tau_p\tau_v w(a)^2)}{(\rho(a) + \kappa)\,t(a)\,(\tau_p + \tau_v)}\,da.$$

Here, all quantities without superscript n denote the $n \to \infty$ limit of their discrete counterparts.

To compute the risk premium term, we apply $\mathbb{E}[\cdot \mid s_p]$ to both sides of the market-clearing condition:

$$\lim_{n\to\infty} \sum_i \frac{\mathbb{E}[v \mid s_p^n] - P^n}{\rho(a_i)\,\mathrm{Var}[v \mid \Delta s_i, s_p^n] + \lambda_i^n} = -\mathbb{E}[u \mid s_p].$$

Thus, the risk premium is of order $1/n$ and can be written as:

$$\mathbb{E}[v \mid s_p^n] - p = \frac{1}{n}\psi(s_p) + \text{higher-order terms},$$

where the function $\psi(s_p)$ is determined by market clearing:

$$(\tau_v + \tau_p)\,\psi(s_p) = -\mathbb{E}[u \mid s_p] \int_0^1 1/\rho(a)\,da.$$

Substituting into (86) leads to (80), and the rest of the proof follows the heuristic derivation.

To establish uniqueness of the positive solution κ in (75), note that the equation can be rewritten as:

$$\tau_u^{-1} = \int_0^1 \frac{t(a)}{(\rho(a)+2\kappa)^2} \cdot \left(\frac{\rho(a)}{\kappa} + 1\right) da. \tag{87}$$

The right-hand side is a continuous, strictly decreasing function of κ, with limits $+\infty$ as $\kappa \to 0$, and 0 as $\kappa \to \infty$. Therefore, by the Intermediate Value Theorem, a unique positive solution exists. ■

F.7.5 Proof of Proposition 13

Proof of Proposition 13. Note that the equation (87), which determines κ in equilibrium, can be rewritten as:

$$\tau_u^{-1} = \int_0^1 \frac{t(a)\,\rho(W_0(a))}{\left(\rho(W_0(a))+\kappa\right)^2} \left(\frac{\rho(W_0(a))}{\kappa} + 1\right) da. \tag{88}$$

We now differentiate this equation implicitly with respect to the wealth distribution $W_0(a)$. Specifically, we consider a perturbation of the form $W_0(a) + \epsilon W_0^{\Delta}(a)$, differentiate both sides of (88) with respect to ϵ, assume $\kappa = \kappa(\epsilon)$, and evaluate the derivative at $\epsilon = 0$. This yields the Gâteaux derivative $\kappa'(W_0(a))[W_0^{\Delta}(a)]$. We assume that $W_0^{\Delta}(a)$ is a Robin Hood variation satisfying $\int_0^1 W_0^{\Delta}(a)\,da = 0$.

Let the right-hand side of equation (88) be denoted by $f(\kappa; W_0(a))$. Then the result of the differentiation can be written as:

$$0 = \frac{\partial f}{\partial \kappa} \cdot \kappa'(W_0(a))[W_0^{\Delta}(a)] + f'(W_0(a), \bar{\kappa})[W_0^{\Delta}(a)], \tag{89}$$

where the bar notation indicates that κ is held fixed while differentiating with respect to $W_0(a)$.

Computing the second term yields:

$$f'(W_0(a),\bar{\kappa})[W_0^{\Delta}(a)] = -\int_0^1 \frac{t(a)\, W_0^{\Delta}(a)\, \rho(W_0(a))\, \rho'(W_0(a))}{\kappa\,(2\kappa + \rho(W_0(a)))^3}\, da.$$

Note that the monotonicity condition

$$a \mapsto -\frac{t(a)\, \rho'(W_0(a))}{(2c + \rho(W_0(a)))^2}$$

being decreasing in a is sufficient to ensure that the integrand

$$a \mapsto -\frac{t(a)\, \rho(W_0(a))\, \rho'(W_0(a))}{(2c + \rho(W_0(a)))^3}$$

is also decreasing in a. Then, by Lemma 16, we have

$$f'(W_0(a),\bar{\kappa})[W_0^{\Delta}(a)] > 0.$$

Since $\partial f/\partial\kappa < 0$, it follows from equation (89) that

$$\kappa'(W_0(a))[W_0^{\Delta}(a)] > 0.$$

We now turn to the comparative statics of $\mathcal{I}$. It suffices to show that

$$\tau_p'(W_0(a))[W_0^{\Delta}(a)] > 0$$

for a Robin Hood variation with sufficiently low $\underline{a}$ and sufficiently high $\bar{a}$.

Applying the chain rule, we obtain:

$$\tau_p'(W_0(a))[W_0^{\Delta}(a)] = \tau_p'(W_0(a);\bar{\kappa})[W_0^{\Delta}(a)] + \frac{\partial \tau_p}{\partial \kappa}\cdot \kappa'(W_0(a))[W_0^{\Delta}(a)],$$

where the bar in $\tau_p'(W_0(a);\bar{\kappa})$ denotes the derivative of τ_p with respect to $W_0(a)$ while keeping κ fixed.

Differentiating $\tau_p = \kappa \int_0^1 \frac{t(a)}{\rho(a)+2\kappa}\, da$, we compute:

$$\tau_p'(W_0(a);\bar{\kappa})[W_0^\Delta(a)] = -\bar{\kappa}\int_0^1 \frac{t(a)\,\rho'(W_0(a))\,W_0^\Delta(a)}{(2\bar{\kappa}+\rho(W_0(a)))^2}\, da.$$

By Lemma 16, if the integrand

$$y(a) = -\frac{t(a)\,\rho'(W_0(a))}{(2\bar{\kappa}+\rho(W_0(a)))^2}$$

is decreasing in a, then the integral is strictly positive for any Robin Hood variation $W_0^\Delta(a)$ with thresholds $\underline{a}$ sufficiently small and $\bar{a}$ sufficiently large. Therefore,

$$\tau_p'(W_0(a);\bar{\kappa})[W_0^\Delta(a)] > 0.$$

Since $\frac{\partial \tau_p}{\partial \kappa} > 0$ and $\kappa'(W_0(a))[W_0^\Delta(a)] > 0$ by earlier results, the full derivative $\tau_p'(W_0(a))[W_0^\Delta(a)]$ is also positive. This completes the proof of the proposition. ■

F.7.6 Proof of Proposition 15

Proof of Proposition 15. Differentiating the expressions for information efficiency and liquidity in equation (82), we obtain:

$$\mathcal{I}'(W_0(a))[W_0^\Delta(a)] = \frac{2\tau_v}{K_1^2\left(1+\frac{K_2}{K_1^2}\tau_v\right)^2}\int_0^1 \frac{t(a)}{\rho(W_0(a))^2}\,\rho'(W_0(a))\,W_0^\Delta(a)\left(\frac{1}{\rho(W_0(a))}-\frac{K_2}{K_1}\right) da,$$

where

$$K_1 \equiv \int_0^1 \frac{t(a)}{\rho(W_0(a))}\, da, \qquad K_2 \equiv \frac{1}{\tau_u} + \int_0^1 \frac{t(a)}{\rho(W_0(a))^2}\, da.$$

Similarly, the derivative of liquidity with respect to a variation in initial wealth is:

$$\mathcal{L}'(W_0(a))[W_0^{\Delta}(a)] = \frac{1}{K_1}\int_0^1 \frac{t(a)}{\rho(W_0(a))^2}\,\rho'(W_0(a))\,W_0^{\Delta}(a)\left(\frac{K_2}{K_1} - \frac{2}{\rho(W_0(a))}\right)da.$$

The remainder of the proof proceeds analogously to that of Proposition 2, invoking Lemma 13 to sign the expressions.

■