Funding Constraints and Informational Efficiency

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INSEAD

FIRS 2019

Motivation

Important feature of markets: prices reflect information

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Prices reflect information. How?

- Investors acquire information about future asset values
- Through trading, information gets impounded into prices

However:

1. Trading requires funding
2. In reality, investors face funding constraints

Q1: How funding constraints affect info. efficiency?

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- Financiers providing funding to investors are concerned about the risk of financing a trade
- Price provides useful information to assess this risk

Q2: Does info. efficiency affect funding constraints?

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Q2: Does info. efficiency affect funding constraints?

To answer these questions, we need a model where
info. efficiency and funding constraints are jointly determined.

We present and analyze such a model
Study asset pricing implications of the interaction between
constraints and informational efficeincy.

Main results - 1

We present and analyze a tractable REE model that allows for **general price-dependent** portfolio constraints

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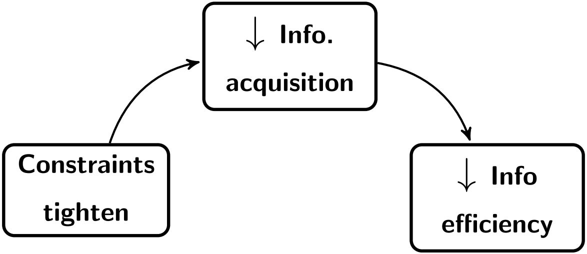
- Key to tractability: irrelevance result. For a given quality of investors' private info, info. efficiency is can be found by solving the model without constraints.
- As constraints tighten, investors have less incentive to acquire information $\Longrightarrow$ lower informational efficiency

Main results - 2

- How funding constraints affect informational efficiency?



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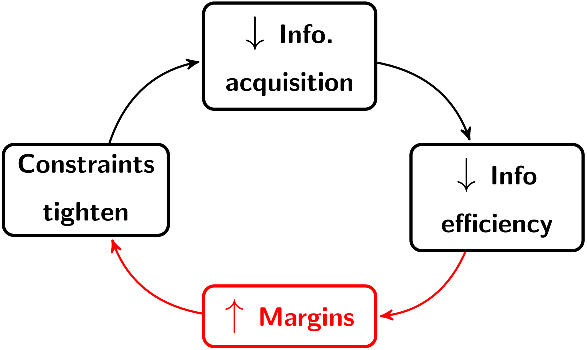

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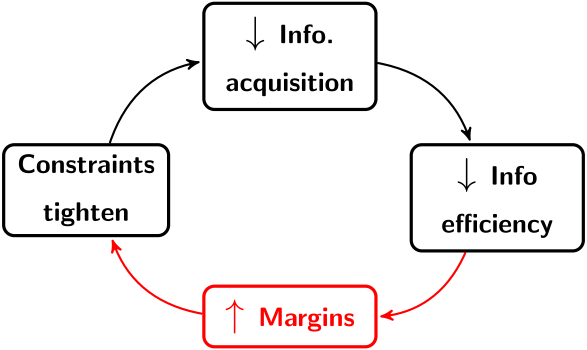
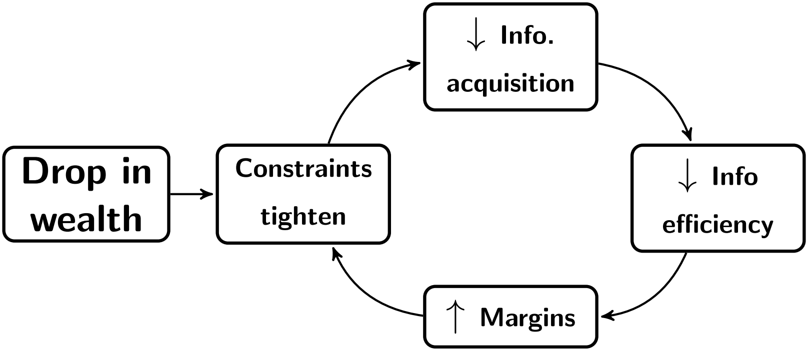


- We uncover a novel **information spiral.**

Main results - 3

Asset pricing implications: a small shock to investors' wealth can lead to large increase in

- risk premium
- volatility
- Sharpe ratio



An REE model with general portfolio constraints

- $t \in \{0, 1, 2\}$
- *risk-free bond*: $r = 0$
- *risky-asset:* pays off $f = v + \theta$ at $t = 2$. Supply $= 1$.
  Learnable $v \sim N(\bar{v}, \tau_v^{-1})$;   Unlearnable $\theta \sim N(0, \tau_\theta^{-1})$
- at $t = 2$, investors $i \in [0, 1]$ receive endowment

$$b_i = e_i\theta \qquad e_i = z + u_i$$

- at $t = 1$, i receives signal $s_i = v + \epsilon_i$ and solves:

$$\max_{x_i(p; s_i, e_i)} E[-e^{-\gamma W_i} | s_i, e_i, p]$$

s.t.: $W_i = W_0 + x_i(v - p) + e_i\theta + \eta_i$, and $a(p) \leq x_i \leq b(p)$

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  The equilibrium price clears the market

$$\int x_i(p, s_i, e_i)di + x_m(p) = 1$$

Benchmark: No constraints

Proposition. Suppose investors have identical signal precision τ_ϵ and $\gamma > \bar{\gamma}$. There exists unique linear equilibrium in which:

- Sufficient statistic of price $\phi^u = f_0 + f_1 p$, $\phi^u = v - (\beta^u)^{-1} z$
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In this equilibrium,

$$\mathbb{V}(v|p) = (\tau_v + (\beta^u)^2 \tau_z)^{-1}$$

Informational efficiency is β^u

- increases in τ_ϵ, investor's signal precision

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$$f'(p) = \frac{c_p^m + \pi_2 c_p - \pi_1 a'(p) - \pi_3 b'(p)}{c_\phi^m + \pi_2 c_\phi}$$

where π_1, π_2 and π_3 - fraction of investors constrained by $a(p)$, unconstrained and constrained by $b(p)$.

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- **Irrelevance result:** $\beta = \beta^u$

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Constraints affect both trading intensity and hedging intensity

- Info efficiency is determined by $\int \frac{\partial x_i}{\partial s_i} di / \int \frac{\partial x_i}{\partial e_i} di$
- Both $\int \frac{\partial x_i}{\partial s_i} di$ and $\int \frac{\partial x_i}{\partial e_i} di$ are reduced with constraints
- We show $\int \frac{\partial x_i}{\partial s_i} di / \int \frac{\partial x_i}{\partial e_i} di$ remains unchanged

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Assumptions needed:
continuum of investors;
noise comes from endowment shocks

Information acquisition incentives

At date 0, we assume that investors preferences are given by

$$U_0 = E_0 \left[E_1 \left[-e^{-\gamma(W_2 - C(\tau_{\epsilon,i}))} \right] \right]$$

Proposition. The foc for investor i

$$C'(\tau_{\epsilon,i}) = \frac{\tau_i}{2\tau_{v,i}^2 \gamma} \underbrace{\frac{E[-e^{-\gamma CE_1} \mathbb{I}_{\text{uncons.}}]}{E[-e^{-\gamma CE_1}]}}_{\text{the term due to constraints}} \tag{1}$$

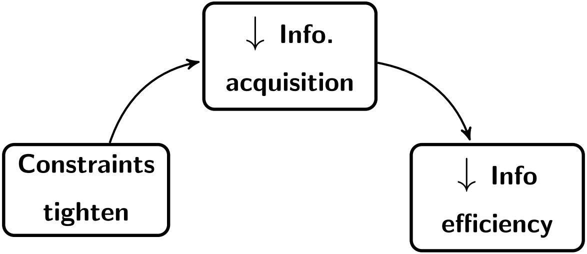
where $\tau_i = \mathbb{V}(f|s_i, e_i, p)^{-1}$ and $\tau_{v,i} = \mathbb{V}(v|s_i, e_i, p)^{-1}$.

Information acquisition incentives

Assume $\gamma_m = 0$.

Result. As constraints tighten (i.e., $a(p)$ increases, $b(p)$ decreases $\forall p$) investors acquire less information and equilibrium info. efficiency decreases.

Intuition: info. is less valuable if one can trade less on it



Portfolio constraint from margin requirements

Margin requirements:

- To buy (sell short) asset at price p one has to set aside $m^+(p) \geq 0$ ($m^-(p) \geq 0$) per unit
- Funding constraint:

$$m^-(p)[x]^- + m^+(p)[x]^+ \leq W_0$$

- Implies $a(p) = -\frac{W_0}{m^-(p)}$ and $b(p) = \frac{W_0}{m^+(p)}$.

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Assumption: Margins are set based on Value-at-Risk (VaR)

- m^+ is such that $Pr(p - f > m^+(p)|p) = 1 - \alpha$
- m^- is such that $Pr(f - p > m^-(p)|p) = 1 - \alpha$

Common in practice. See Brunnermeier and Pedersen (2009).

How informational efficiency affects constraints?

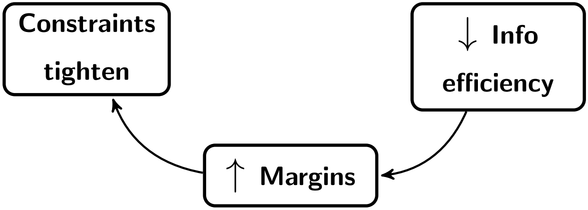
Assume $\gamma_m = 0$.

Proposition. Equilibrium margins satisfy

$$m^+ = m^- = \frac{\Phi^{-1}(\alpha)}{\sqrt{\tau_v + \beta^2 \tau_z}}.$$

As $\beta \downarrow$, margins increase $\implies$ constraints $\left[-\frac{W_0}{m^-}, \frac{W_0}{m^+}\right]$ tighten.

Intuition: with lower info. efficiency, financiers face higher residual risk of financing a trade. Hence, set higher margins.



Information spiral.

Proposition. As constraints tighten, investors acquire less information and equilibrium informational efficiency(β) $\downarrow$.

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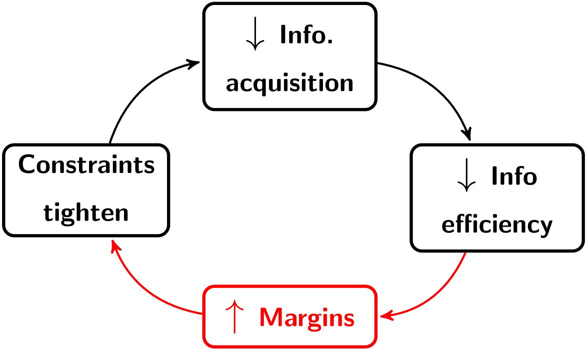


Information Spiral Implications: Complementarity

Substuitability: In traditional REE models (GS 1980), the value of acquiring information decreases as others acquire more information

Additional channel (complementarity): As others acquire more information, prices become more informative, margins become lower and increases the agents incentive to acquire information

Proposition. When W_0 is low enough, there is complementairt in information acquisition for τ_ϵ and τ_{ϵ_i} such that

$$\frac{d}{d\tau_\epsilon} \underbrace{log\left(\frac{W_0}{m(\tau_\epsilon)}\right)}_{\text{effect of a change in the constraints}} > -\frac{d}{d\tau_\epsilon} \underbrace{log\left(\frac{\tau_i}{2\gamma\tau_{v,i}^2}\right)}_{\text{GS effect}}.$$

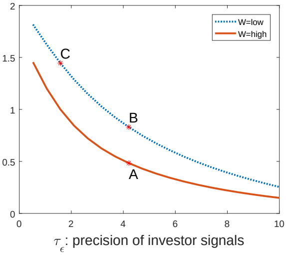
Asset pricing implications: risk premium.

Assume market maker is risk averse, $\gamma_m > 0$.

Risk premium $RP := \mathbb{E}[v - p]$.

Suppose W_0 drops (crisis)

- Exogenous private info. $W_0 \downarrow$: capacity to go long and short is diminished (tighter constraints)$\Rightarrow RP \uparrow$.
  plot: Move from A to B.



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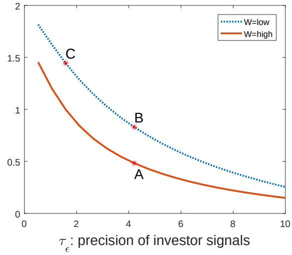
- With endogenous private info. Constraints tighten $\Rightarrow \tau_\epsilon \downarrow$.
  plot: Move from B to C. Amplification.



Volatility: Unintended consequences

Often argued: tighter margins should lower volatility.

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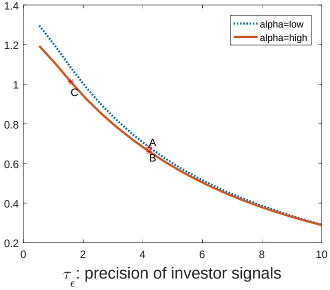

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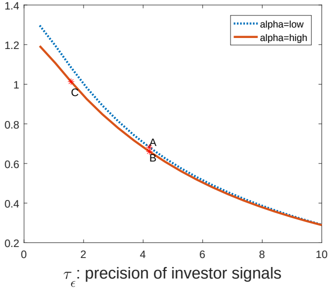
- Tighter margin requirements can *increase* volatility.



Robustness and additional results (new appendices!)

- We show that our informational spiral holds
  1. In a setting with GS information structure and noise traders, where irrelevance result does not hold
     - new chanel: information aggregation. As constraints tighten for informed investors but not for noise traders, less info is embedded into price
  2. In a setting where investors are initially endowed with risky asset, not cash
- Analytical conditions under which our information spiral holds with risk-averse market maker
- We attempt to microfound VaR-based margins

Conclusion

- We developed a tractable REE model that allows for general portfolio constraints
- Portfolio constraints affect info. efficiency only through info. acquisition channel

When portfolio constraints arise due to margin requirements

- Wealth of investors matters for asset prices and info. efficiency, unlike in traditional CARA models
- Due to novel **informational spiral**, wealth effects are amplified which has important asset pricing implications