CHILE

Efstathios (Stathi) Avdis
(University of Alberta)

Sergei Glebkin
(INSEAD)

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(info efficiency, liquidity, trading volume, welfare)?

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Application: How does wealth distribution (distribution of sizes of market participants) affect market quality
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CHILE is uniquely suited, as one needs a model with

- Wealth effects
- Heterogeneity
- Asymmetric information

The model. Baseline setup. "Continuum economy"

- time $\in \{1, 2\}$.
- Risk-free asset, $R_f = 1$. Risky asset pays off $\exp(v)$, $v \sim N(0, \tau_v^{-1})$
- Continuum of traders $a \in [0, 1)$
  - Trader a lives in $[a, a + da)$
  - Observes signal $ds(a) = v \cdot da + \frac{1}{\sqrt{t(a)}} dB$, precision $= t(a) \cdot da$

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  - price level and
  - info content of prices
- Maximizes $E[u(W_0(a) + x(\cdot)(R - 1); a)|p]$

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- **Rich heterogeneity:** $\{W_0(a),\ t(a),\ u(\cdot, a)\}$, arbitrary functions of $a \in [0, 1)$. General utilities.
- Log-linear equilibrium. Let $p = \log P$. The dollar demand of trader a is

$$dx(a) = \alpha(a)da + \beta(a)ds(a) - \gamma(a)pda.$$

Preview of the results

Framework

- Tractable, log-linear equilibrium. Closed-form solutions.
- Closed-form solutions for info efficiency, liquidity, volume, and welfare.
- Invariant relationship linking info efficiency (harder to measure) to liquidity and volume (easier to measure).
- (Money-metric) welfare can be expressed via liquidity and volume

Application: wealth distribution and market quality

- Inequality is bad for info efficiency
- Inequality is good for liquidity, volume
- Ambiguous effect on welfare

Equilibrium

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Theorem. There exists a unique equilibrium.
$dx(a) = \alpha(a)da + \beta(a)ds(a) - \gamma(a)pda$, where

$$\beta(a) = \frac{t(a)/\tau}{\rho(a)\mathrm{Var}[R|p]}$$

with $\mathrm{Var}[R|p] = \exp(\tau^{-1}) - 1$ and $\tau = \tau_v + \tau_p$, where τ_p is the equilibrium price informativeness,

$$\tau_p = \frac{\left(\int_0^1 \frac{t(a)}{\rho(a)} da\right)^2}{\int_0^1 \frac{t(a)}{\rho(a)^2} da}.$$

Other coefficients are given in the closed form in the paper.

- Note: closed-from solutions, with non-CARA and rich heterogeneity!

Wealth distribution and information efficiency: first pass

Definition. Information efficiency $\mathcal{I} = 1 - \frac{\mathsf{Var}[v|p]}{\mathsf{Var}[v]} = \frac{\tau_p}{\tau_p + \tau_v}$.

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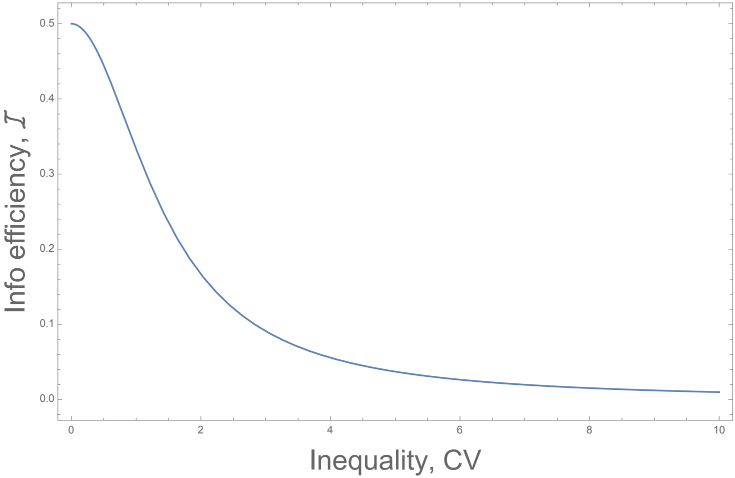
Proposition. Info efficiency is given by

$$\mathcal{I} = \frac{\bar{t}}{\bar{t} + \tau_v(1 + CV^2)}, \text{ where}$$

$$CV = \text{standard deviation of wealth/average wealth}$$

is a *coefficient of variation*.
There is a negative relationship between inequality (CV) and information efficiency ($\mathcal{I}$).

Inequality and info efficiency.



Inequality and info efficiency. Intuition

- Price reflects the weighted average of signals. $p \propto \int \beta(a) ds(a)$
- Weights $\propto \beta(W_0) \propto W_0$
- More weight on wealthier traders
- What is more informative: $0.5(v+\epsilon_1)+0.5(v+\epsilon_2)$ or $0.99(v+\epsilon_1)+0.01(v+\epsilon_2)$?

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- What is more informative: $0.5(v + \epsilon_1) + 0.5(v + \epsilon_2)$ or $0.99(v + \epsilon_1) + 0.01(v + \epsilon_2)$?
- Key effect: whose signal noise is reflected more in prices?
  Absent in LE a-la Hellwig (1980), signal noise is washed out by LLN

Wealth distribution and information efficiency

Suppose that $t(a) = \bar{t}$, and all traders are CRRA with the same RRA.

Corollary.

$$\mathcal{I} = \frac{\bar{t}}{\bar{t} + \tau_v(1 + CV^2)} \leq \frac{\bar{t}}{\bar{t} + \tau_v},$$

maximum $\mathcal{I}$ is attained when $CV = 0$.

Maximum info efficiency is attained when there is no inequality.

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Intuition.

- Price cannot reflect more than one could infer by seeing each signal: $\tau_p \leq \int_0^1 t(a)da$
- Suppose we have $s_1 = v + \frac{1}{\sqrt{t_1}}\epsilon_1$ and $s_2 = v + \frac{1}{\sqrt{t_2}}\epsilon_2$ (ϵ_i are standard normal). Known result: $\{s_1, s_2\}$ is info equivalent to $s = t_1 s_1 + t_2 s_2$
- The best way to aggregate signals is with weights proportional to precisions.

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- The best way to aggregate signals is with weights proportional to precisions.If $\beta(a) \propto t(a)$, $\tau_p = \int_0^1 t(a)da$
- But we have weights $\propto \beta(a) \propto t(a)/\rho(a)$.
- $\beta(a) \propto t(a)$ iff $\rho(a) = \bar{\rho}$ which is only possible when $W_0(a) = \overline{W}_0$

Wealth distribution and information efficiency: general case

Do our results still hold when utilities are heterogeneous and non-CRRA?

When precisions are heterogenous?

When precisions are endogenous?

Wealth distribution and information efficiency: general case

How $\mathcal{I}$ changes when $W_0(a)$ changes?

Wealth distribution and information efficiency: general case

How $\mathcal{I}$ changes when $W_0(a)$ changes?

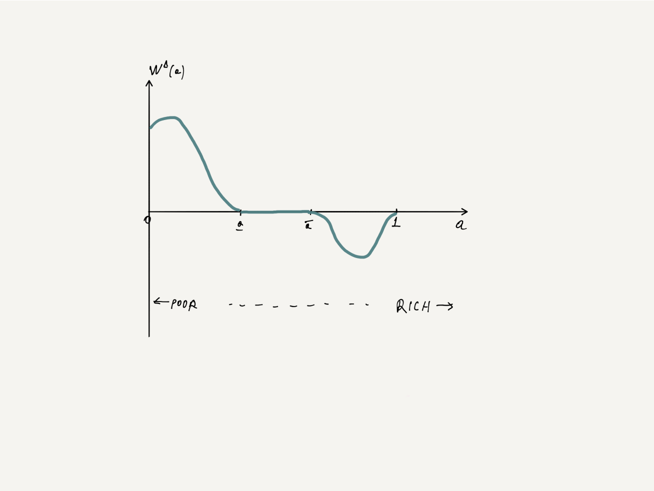
Definition. Gateaux derivative $\mathcal{I}'(W_0(a))[W_0^\Delta(a)]$ in the direction $W_0^\Delta(a)$ is

$$\mathcal{I}'(W_0(a))[W_0^\Delta(a)] = \lim_{\epsilon \to 0} \frac{\mathcal{I}(W_0(a) + \epsilon W_0^\Delta(a)) - \mathcal{I}(W_0(a))}{\epsilon}$$

Wealth distribution and information efficiency: general case

Assume $W_0(a) \uparrow$ in a (WLOG)

Definition. Robin Hood variation is a direction $W_0^{\Delta}(a) \neq 0$ such that $W_0^{\Delta}(a) \geq 0$ for $a < \underline{a}$ and $W_0^{\Delta}(a) \leq 0$ for $a > \bar{a}$.



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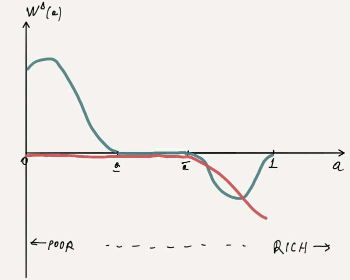

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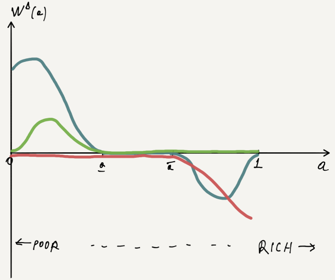

Wealth distribution and information efficiency: general case

How $\mathcal{I}$ changes when $W_0(a)$ changes? When $t(a)$ changes?

The sequence of exercises:

1. Vary $W_0(a)$ keeping $t(a)$ fixed
2. Vary $t(a)$ keeping $W_0(a)$ fixed
3. Vary both

Wealth distribution and information efficiency: general case

Proposition. Assume DARA utility, exogenous precisions+technical conditions. There exists $0 < a^* < 1$ such that for all Robin Hood $W^\Delta(a)$ with $\underline{a} \leq a^* \leq \bar{a}$:

$$\mathcal{I}'[W_0^\Delta(a)] > 0, \quad \mathcal{I}'[-W_0^\Delta(a)] < 0.$$

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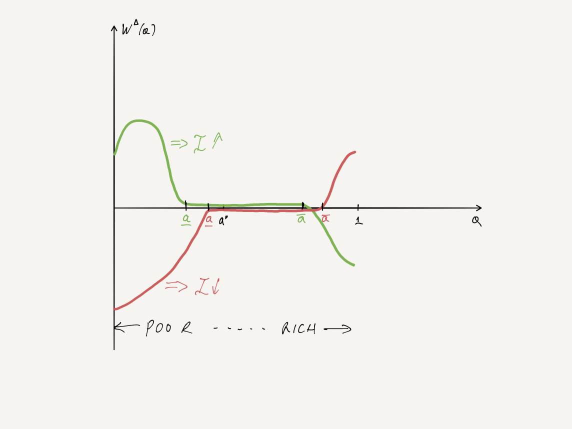

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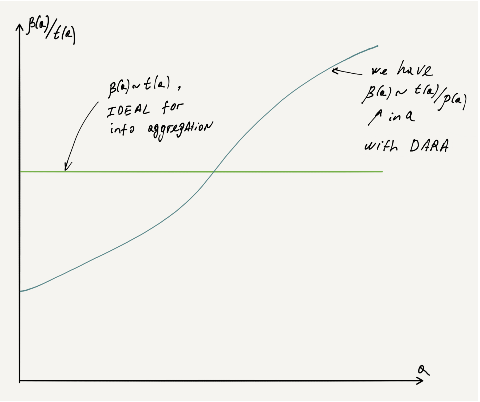

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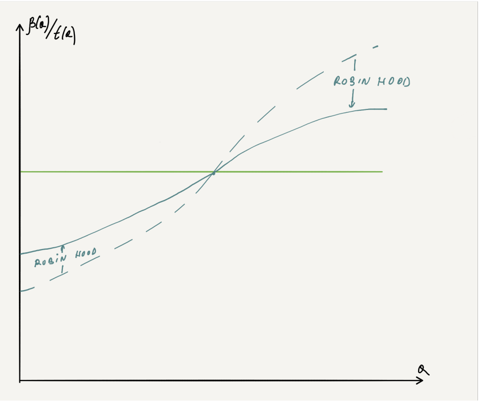
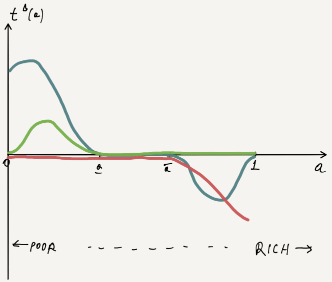


Distribution of precisions and information efficiency

Assume $W_0(a) \uparrow$ in a (WLOG). Here, we will vary $t(a)$ keeping $W_0(a)$ fixed.

Definition. Robin Hood variation of $t(a)$ is a direction $t^{\Delta}(a) \neq 0$ such that $t^{\Delta}(a) \geq 0$ for $a < \underline{a}$ and $t^{\Delta}(a) \leq 0$ for $a > \bar{a}$.



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Intuition

- Rich trade too aggressively, poor too passively
- Giving less info to the rich makes them less aggressive; analogously, for poor

Distribution of precisions and information efficiency

Corollary. Assume DARA utility+technical conditions. For any $t^{\Delta}(a) \neq 0$ such that $t^{\Delta}(a) \geq 0$ for $a > a^{**}$ and $t^{\Delta}(a) = 0$ otherwise:

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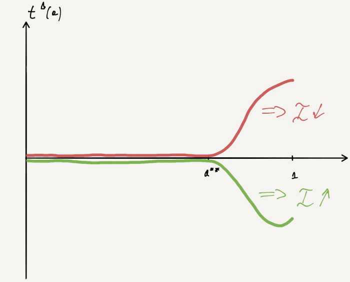

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  - there: giving more info (reducing noise in signals) invites more noise coming from another source
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Implication

- MIFID (unbundling of research fees and trading commissions) makes it harder for small funds to acquire info compared to large. Potentially detrimental effects for info efficiency

Distribution of wealth and information efficiency with endogenous information

Proposition. Assume DARA utility, info $\text{cost}(t) = t^c$, $c > 1$, technical conditions. There exists a unique overall equilibrium. There exists $0 < a^* < a^{**} < 1$ such that for all Robin Hood $W^\Delta(a)$ with $\underline{a} \leq a^* < a^{**} \leq \bar{a}$:

$$\mathcal{I}'[W_0^\Delta(a)] > 0, \quad \mathcal{I}'[-W_0^\Delta(a)] < 0.$$

- Combination of two previous exercises
- Robin Hood variation:
  - Flattens the distribution of risk tolerances
  - Decreases the info of the rich and increases the info of the poor via endogenous info acquisition (rich acquire more info)

Information efficiency, liquidity and volume

Definition

- Liquidity $\mathcal{L} = \int_0^1 \gamma(a) da$
- Volume $\mathcal{V}^2 = \int_0^1 dx(a)^2$

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Proposition. Let $\sigma_v^2 = \text{Var}[v]$. For any primitives of the economy, the following invariant relationship holds

$$\mathcal{I}(1-\mathcal{I})\frac{\mathcal{L}^2}{\mathcal{V}^2}\sigma_v^2 = 1.$$

Key underlying equation

- To have small equilibrium demands, must have demand elasticity $\sim da$
- Suppose price $\downarrow$ by 1%. Asset is cheaper, demand $\uparrow$ (cost component). Perhaps fundamental is lower, demand $\downarrow$ (info component).

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$$\text{Cost component} = \text{information component} + O(da)$$

$$1 = \frac{\tau_p}{\tau_p + \tau_v} \cdot \frac{\int_0^1 \gamma(a) da}{\int_0^1 \beta(a) da}$$

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Implicaton

$$\underbrace{\mathcal{I}(1-\mathcal{I})}_{\text{hard to measure}} = \underbrace{\frac{\mathcal{V}^2}{\sigma_v^2\mathcal{L}^2}}_{\text{easier to measure}}.$$

Conclusion

- A new heterogeneous information asset pricing framework
- Tractable. General utilities. Rich heterogeneity. Closed-form solutions
- Allows to analyse how wealth distribution affects market quality
- Active follow-ups:
    - Kyle in CHILE
    - Discriminatory price auction/ static limit order book
    - Continuous-time CHILE
    - Multi-asset CHILE
    - $\cdots$

How we solved for equilibrium

1. Consider a discrete economy where each trader a believes other traders' demands are $d\hat{x}(b) = \hat{\alpha}(b,m) + \hat{\beta}(b,m)ds(b) - \hat{\gamma}(b,m)p$

   Solves

$$x^{BR}(a, \Delta s, p, m) = \arg \max_{x(p,\Delta s(a))} E[u(W_0(a) + x(\cdot)(R-1);\ a)] \qquad (1a)$$

$$\text{s.t.:} \int_{-a} d\hat{x}(b) = 0. \qquad (1b)$$

2. Use aggregation lemma to compute $\lim_{m\to 0} \sum x^{BR}(a, p, m)$. (Involves implicitly differentiating FOC to get x_s, x_{ss} etc)
3. Require consistency
   - $\lim_{m\to 0} d\hat{x} = \alpha(b) + \beta(b)ds(b) - \gamma(b)p$
   - $\lim_{m\to 0} dx^{BR} = \alpha(b) + \beta(b)ds(b) - \gamma(b)p$
4. We've shown this procedure yields the limiting equilibrium in the discrete economy

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Technical conditions

Technical conditions = x-sectional distribution of wealth (relative risk aversion) has unbounded (compact) support. back