Liquidity vs. Information Efficiency

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Abstract

I analyze a market with large and small traders with different values. I show that illiquidity and information efficiency are complements. Policy measures promoting liquidity might be harmful for information efficiency and vice versa. An increase in risk-bearing capacity may harm liquidity. An increase in the precision of information may harm information efficiency. Increasing market power or breaking up a centralized market into two separate exchanges might improve welfare. Multiple equilibria, in which higher liquidity is associated with lower information efficiency, are possible. Applied to crude oil market the model highlights (1) informational frictions and (2) market power of producers amplified by (1) as possible drivers of recent sharp price changes.

1 Introduction

In many modern markets, traders are heterogeneous along the following two dimensions. The first dimension is the price impact: there are large traders who are able to move prices and small traders whose effect on prices is negligible. For example, in financial markets there is evidence that large institutional investors (such as hedge, mutual and pension funds) have considerable price impact[1]. No such evidence exists for retail investors and smaller funds, and anecdotally, price impact is not an issue for these types of investors. The second dimension is heterogeneity in values. For example, in financial markets institutional and retail investors may have different values of an asset due to different trading needs or tax or risk-management considerations[2]. In this paper, I present a model that captures this heterogeneity and show that such heterogeneity has unexpected consequences for liquidity, information efficiency and welfare.

I consider a centralized market (modeled as a uniform-price double auction) populated by *large* and *small* traders. To capture the heterogeneity in price impacts, I assume that there is a countable number of large traders, whereas small traders form a continuum. The traders within each group are identical. I employ a linear-normal setting: traders have linear-quadratic objectives, and their values are distributed normally. To capture the second dimension of heterogeneity, I assume that the values

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[1]See, e.g., Chan and Lakonishok (1995), Keim and Madhavan (1995), Korajczyk and Sadka (2004), among others.

[2]Fund flows and fund managers' compensation relative to a benchmark can be conceptualized as endowment shocks. These endowment shocks create hedging needs that are specific to institutional investors. See Vayanos and Woolley (2013) for a treatment of the effect of fund flows. See Basak and Pavlova (2012) and Cuoco and Kaniel (2011) for a treatment of benchmarking.

of large and small traders are imperfectly correlated. For simplicity, I assume that the large traders know their value. However, for the information efficiency to play a role, the information concerning small traders value is dispersed among them. I show that the model provides a natural framework for considering asset, commodities, foreign exchange and product markets.

In my first set of results, I consider the interaction among liquidity, information efficiency and welfare. First, I show that a tension between liquidity and information efficiency might arise: policy measures intended to promote liquidity might be harmful for information efficiency and vice versa and changes in the market environment (such as risk-bearing capacity, number of large traders, information precision) can shift liquidity and information efficiency in opposite directions. Second, I show that a shock to the economic environment that has a positive direct effect on liquidity (an increase in risk-bearing capacity) may have a negative overall effect on liquidity (*liquidity paradox*). This is possible because the shock has a positive effect on information efficiency and there is a tension between the two. Similarly, a positive shock to information efficiency (an increase in the precision of the signals) might have a negative overall effect on information efficiency (*information aggregation paradox*). Third, when there is more competition between large traders, welfare might be lower. Moreover, all traders, even small ones, can be worse off as a result of more competition. This is possible because competition has negative effects on information efficiency. For a similar reason, breaking up a centralized market into two separate exchanges might improve welfare.

The above results are a consequence of an equilibrium mechanism that features a complementarity between illiquidity (price impact) and information efficiency. The mechanism is represented in Figure 1. A belief that the market is less liquid induces large investors to trade less aggressively (their demand is less sensitive to their information). It makes the price relatively less (more) sensitive to the values of large (small) traders. From the perspective of small traders, the price is more informative. Therefore, they provide less liquidity: if someone is buying and driving up the price, small traders are less willing to sell (decrease their demands) because they partly attribute higher prices to stronger fundamentals. In other words, when prices are more informative, small traders are less price-elastic. The latter confirms lower liquidity.

A direct consequence of such complementarity is the possibility of multiple equilibria driven by self-fulfilling beliefs concerning liquidity or information efficiency. I provide the sufficient conditions for the multiplicity to emerge. I show that the equilibria can be ranked in terms of liquidity and information efficiency and that the rankings are the opposite of one another: the equilibria with higher liquidity feature lower information efficiency and vice versa. I also provide a sufficient condition under which the equilibria can be ranked in terms of welfare: if the price does not provide much incremental information to the traders, the equilibria with higher liquidity are those with greater welfare.

I also explore the implications of the mechanism for market crashes. I understand the latter either as a switch between the two equilibria with different price levels or as a large change in price caused by a small change in the economic environment. The latter is possible because the complementarities provide a natural amplification mechanism. I show that, depending on whether the large traders are on the buy or sell side of the market, there are two scenarios consistent with a market crash, which differ in the behavior of information efficiency, liquidity, volatility, and trading volume. Under the sufficient

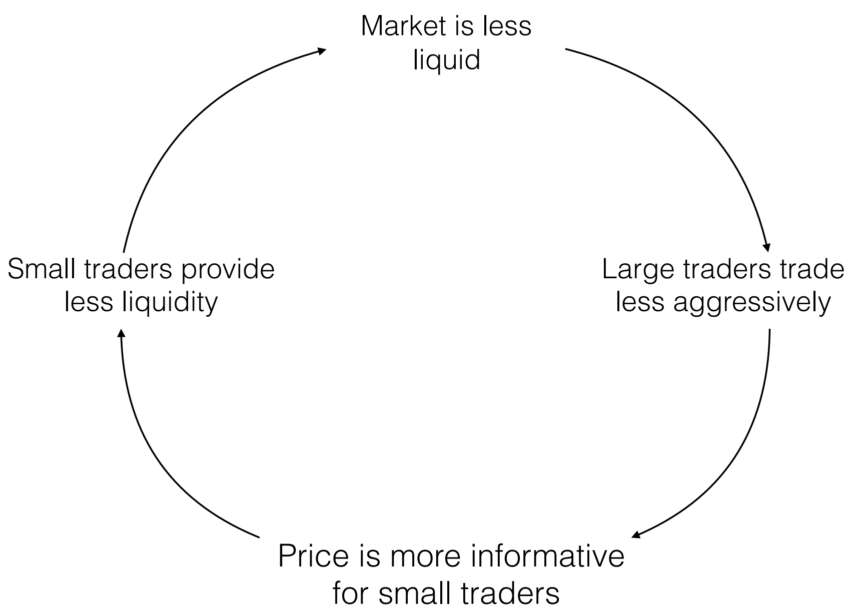

Figure 1: Equilibrium mechanism.

condition that price does not provide substantial incremental information, welfare decreases in only one scenario. Correspondingly, only one scenario suggests policy intervention.

I consider the implications of the model and empirical evidence in Section 7. Briefly, I consider two episodes that affected commodities markets, the 2008 boom/bust and the 2014 crash in oil prices, through the lens of the model. I emphasize the role of two forces: (1) informational frictions and (2) the market power of oil producers that is endogenously amplified because of (1). In asset markets, I seek evidence supporting the model's prediction that in a more liquid market, the price is more (less) reflective of the values of large (small) traders. I find suggestive evidence in the on-the-run treasury bonds and equity markets. I also discuss the policy implications concerning the effects of high-frequency traders and commodity index traders in the in asset and commodities markets and discuss the effects of competition on welfare.

On a technical side, I demonstrate how to perform a stability analysis in a strategic trading model with heterogeneous traders. The key idea is to represent the equilibrium as a fixed point that determines market liquidity. Given their beliefs concerning market liquidity, traders choose their demand schedules. In equilibrium, liquidity (which is determined by the slopes of the traders' demands) should be equal to assumed liquidity. The stability of equilibrium is associated with the stability of this fixed point[3]. This representation also allows me to characterize quantitatively the amplification through an *illiquidity multiplier*[4].

[3]The representation simplifies the stability analysis significantly, as mapping liquidity onto itself entails mapping $\mathbb{R}$ onto $\mathbb{R}$, whereas the best response mapping is $\mathbb{R}^4$ onto $\mathbb{R}^4$ in my model.

[4]The notion of an illiquidity multiplier is from Cespa and Foucault (2014). The idea of representing the equilibrium as a fixed point determining the price impact is from Weretka (2011) and Rostek and Weretka (2015).

Related literature This paper is related to two strands of literature: strategic trading/supply function equilibria and rational expectations models featuring multiple equilibria.

The first strand of literature can be further divided into two subgroups: the models with common values (Kyle (1989), Pagano (1989), Vayanos (1999), Rostek and Weretka (2015), and Malamud and Rostek (2015)) and the models with private values (Vives (2011), Rostek and Weretka (2012, 2014), Du and Zhu (2015), Kyle, Obizhaeva and Wang (2015), and Babus and Kondor (2015)). Technically, the common value models obviously lack the heterogeneity in trader' values, which I capture in my model. More important, given common values, the interaction between liquidity and information efficiency is in the opposite direction. In common value models, the price reflects traders' information and noise. If traders believe that the market is more liquid, they trade more aggressively on their information, and information efficiency improves. Consequently, the complementarity between illiquidity and information efficiency does not arise.

The private values model of Vives (2011), Rostek and Weretka (2012, 2014), Du and Zhu (2015) and Kyle, Obizhaeva and Wang (2015) capture the heterogeneity in traders' values; however, they focus on symmetric settings and there is no heterogeneity in price impact[5]. As a result, traders' behavior is affected by liquidity in a symmetric way, and the price reflects the same combination of their signals. Consequently, the complementarity uncovered in this paper does not arise. Babus and Kondor (2015) include the two dimensions of heterogeneity in their model. However, they focus on the over-the-counter markets, and the complementarity does not arise because of the bilateral interactions among the large traders.

The multiplicity of equilibria in REE models can arise for two reasons. First, due to demand nonlinearities, there can be multiple market-clearing prices (e.g., Gennotte and Leland (1990); Barlevy and Veronesi (2003); Yuan (2005)). In contrast, the equilibrium in this model is linear, and consequently, the market-clearing price is always unique.

The equilibrium multiplicity in this paper arises due to strategic complementarities, similar to Ganguli and Yang (2009), Goldstein, Li and Yang (2013), Cespa and Focault (2014), Cespa and Vives (2015), Rohi and Zigrand (2015), Huang (2015) and Bing et al. (2016). In these papers, the traders take prices as given, whereas the traders in my model account for their influence on prices. This difference is not merely technical: strategic behavior on the part of large traders is an integral component of the mechanism generating the complementarity in this paper. Moreover, price-taking behavior implies that traders regard the market as perfectly liquid; therefore, as my focus in the paper is liquidity, assuming the strategic behavior is desirable.

Through their focus on liquidity, the two most closely related papers among the above REE models with complementarities are Cespa and Foucault (2014) and Cespa and Vives (2015). These models also feature multiple equilibria that differ in liquidity and information efficiency. However, in these papers, the equilibria with higher liquidity are also those with higher information efficiency, which highlights the complementarity between *liquidity* and information efficiency (versus complementarity between *illiquidity* and information efficiency in this paper).

[5]Technically, there are small traders in Vives (2011). However, their behavior is not affected by either liquidity or information. The model predictions are the same if instead of small traders the model postulates an exogenously postulated demand curve.

The tension between liquidity and information efficiency can manifest as a comparative statics result in some other settings. For example, in Subrahamnyam (1991), increasing the variance of noise trading can increase liquidity but decrease information efficiency. However, in my paper this tension manifests through the potential coexistence of high liquidity/low efficiency and low liquidity/high efficiency equilibria. Bing et al. (2016) demonstrate that there might be a tension between the liquidity and information efficiency if noise traders chase liquidity: improvement in liquidity attracts more noise traders and may therefore harm the information efficiency. In my paper it is more aggresive trading, not the entry of traders, that have adverse effects on information efficiency.

The information aggregation paradox is reminiscent of the results of Banerjee et al. (2015), who show that reducing the cost of information acquisition (and, therefore, increasing signal precision in equilibrium) may not increase information efficiency. In their model, the traders may acquire information on asset fundamentals or on noise. They show that lowering the cost of information concerning the fundamentals may, under certain conditions, induce traders to learn more about noise. As a result, information efficiency may decrease. This mechanism differs from that in my paper, whereby more precise information improves liquidity and induces large traders to trade more, which is harmful for the price inference of small traders and, consequently, for information efficiency.

Rostek and Weretka (2014) show that increasing market size (the number of traders) does not necessarily increase welfare. They consider an equicommonal auction: a market with large traders who are heterogenous in their values, such that the average correlation of the value of each trader with the values of others is the same for all traders. They attribute the reduction in welfare to a decrease in gains from trade: in larger equicommonal markets, the values of traders are more aligned and, correspondingly, the gains from trade are lower. This mechanism is therefore different from that presented here, which emphasizes the negative externality that increased competition has on information efficiency[6].

2 The model

Consider a market for a divisible good in which two groups of agents, I and J, are trading. There are $N>1$ of I-traders, indexed by $i \in I \equiv \{1, 2, ..N\}$, and there is a unit continuum of J-traders, indexed by $j \in J \equiv [0,1]$. The traders within each group $k \in \{I, J\}$ are identical, and their preferences are given by a quasilinear-quadratic function

$$u_k = (v_k - p)\, x - \frac{w_k x^2}{2}, \tag{1}$$

where $w_k > 0$ is a constant, and the values $v_k \sim N\left(\bar{v}_k, \frac{1}{\tau_k}\right)$ are jointly normally distributed with

$$\operatorname{corr}(v_I, v_J) = \rho \in (-1, 1). \tag{2}$$

[6]In an equicommonal auction, the price always reflects the average of traders' signals. In contrast, in my model, the price is less (more) reflective of the value of small (large) traders when the competition among large traders increases.

The information structure is as follows. The I-traders know their value, but it is not known to J investors. The $J-$investors have dispersed information about their value. Each $j \in J$ receives a signal

$$s_j = v_J + \epsilon_j, \tag{3}$$

where $\epsilon_j \sim N\left(0, \frac{1}{\tau_s}\right)$, and for any $j, k \in J$, such that $k \neq j$ the noise ϵ_j is independent of v_I, v_J and ϵ_k. The parameter τ_s measures the precision of the signal. The information structure can be summarized by the information sets

$$\mathcal{F}_i = \{v_I\},\ \mathcal{F}_j = \{s_j\},\ \forall i \in I,\ j \in J.$$

In equilibrium, traders will also learn from prices.

The market is modeled as a uniform-price double auction. Each trader k submits his net demand schedule $x_k(p)$: $x_k(p) > 0$ ($x_k(p) < 0$) corresponds to a buy (sell) order. The market-clearing price p^* is such that the net aggregate demand is zero

$$\sum_{i=1}^{N} x_i\left(p^*\right) + \int_0^1 x_j\left(p^*\right) dj = 0. \tag{4}$$

In equilibrium, each trader is allocated

$$x_k^* = x_k\left(p^*\right).$$

The equilibrium concept is a symmetric linear Bayesian Nash Equilibrium (henceforth, equilibrium). A symmetric linear equilibrium is an equilibrium in which traders $i \in I$ and $j \in J$ have the following demand schedules

$$x_i = \alpha + \beta \cdot v_I - \gamma \cdot p \text{ and } x_j = \alpha_J + \beta_J \cdot s_j - \gamma_J \cdot p. \tag{5}$$

2.1 Examples

Below, I show that the model presented above provides a natural framework for considering at least four types of markets.

1. Securities markets.

In this example, the good being traded is a financial asset, such as a bond or stock. The I-traders are institutional investors. In the model, their distinguishing features are that they are large (can affect prices), and sophisticated/informed (know their value). It is therefore natural to interpret them in this manner[7]. The J-traders can be interpreted as retail investors.

The preference specification (1) is common in the securities markets context[8]. The quadratic component $\frac{w_k x^2}{2}$ represents an inventory cost that may come from the regulatory capital requirements, collateral requirements or risk-management considerations[9]. The difference in the values of I- and J-

[7]There is a vast empirical literature demonstrating that institutional investors have price impact and that the costs associated with it are considerable. Examples include Chan and Lakonishok (1995), Keim and Madhavan (1995), and Korajczyk and Sadka (2004), among others. Anecdotally, institutional investors are more informed because they have more resources to support a larger research division, pay for relevant data streams, etc. Hendershott et al. (2015) present empirical evidence supporting this point.

[8]E.g., Vives (2011), Rostek and Weretka (2012) and Du and Zhu (2015).

[9]See Du and Zhu(2015), Section 2.1 for a discussion.

traders may be due to the following reasons. The first is that along with the common value component v, representing the fundamental value of the security, investors may also care about a private value u_k, such that

$$v_k = v + u_k,\ k \in \{I, J\}.$$

The private values u_k, which differ between the two groups, may be due to different tax or risk-management considerations[10]. Assuming that v, u_I and u_J are normally distributed and not perfectly correlated, we obtain the setup with imperfectly correlated values described in the section above.

An alternative explanation is that the difference in v_I and v_J may represent uncertainty concerning the endowment shocks. Suppose that both types of investors care about the fundamental value of the security v. Suppose that I-investors receive a (normally distributed) endowment shock e that is known to them but unknown to J-investors. The J-investors receive no endowment shocks. In that case, the preference relation of I-investors can be written as $(v-p)\,x - \frac{w_I(e+x)^2}{2}$, or, dropping the constant $\frac{w_I e^2}{2}$,

$$(v - w_I e - p)\,x - \frac{w_I x^2}{2}.$$

Denoting $v_I \equiv v - w_I e$, the above becomes

$$(v_I - p)\,x - \frac{w_I x^2}{2},$$

which is consistent with the specification (1). Moreover, as long as e is not perfectly correlated with v, $v_J = v$ and $v_I = v - w_I e$ are imperfectly correlated, which is consistent with the setup described above.

2. Commodities or intermediate good markets.

In this example, the good being traded is a commodity, such as crude oil or aluminum. More generally, imagine any intermediate good, i.e., one that is an output for some firms while being an input for the others. The I-traders are commodity *producers*. The J-traders are *firms*, buying the commodity to produce the final good.

Commodity producers have a production technology characterized by a convex cost function

$$c \cdot y + \frac{w_I}{2} y^2, \tag{6}$$

where $c \sim N\left(\bar{c}, \frac{1}{\tau_I}\right)$ is a cost shock, which is known to producers but not to firms. The latter assumption captures that the producers are better informed about their own production technology. Producers are risk neutral and maximize their profit

$$p \cdot y - \left(cy + \frac{w_I}{2} y^2\right). \tag{7}$$

Note that in the above, y is the amount of commodity sold, i.e., the net supply. The net demand of producers is $x = -y$. With this change of variable, the above becomes

$$(c - p)\,x - \frac{w_I}{2} x^2, \tag{8}$$

[10]See, e.g., Duffie, Garleanu and Pedersen (2005) or Du and Zhu (2015) for a discussion of private values in the context of financial markets.

which is consistent with (1) with a value v_I equal to the cost shock c.

Firms $j \in [0,1]$ have a production technology characterized by a concave production function

$$Y(x) \equiv a \cdot x - \frac{w_J}{2} x^2. \tag{9}$$

In the above, $a \sim N\left(\bar{a}, \frac{1}{\tau_a}\right)$ is a productivity shock. The latter shock is common to all firms. The firms have dispersed information concerning a. In particular, each firm j is endowed with a signal

$$s_j = a + \epsilon_j,$$

where $\epsilon_j \sim N\left(0, \frac{1}{\tau_s}\right)$ and $\forall j, k$, such that $k \neq j$ the noise ϵ_j is independent of c, a and ϵ_k. Following Sockin and Xiong (2015), the productivity shock can be interpreted as the strength of the economy. Firms are risk neutral and maximize their profit

$$p_g \left(a \cdot x - \frac{w_J}{2} x^2 \right) - p \cdot x, \tag{10}$$

where $p_g = 1$ is the price of the final good (endogenized below) and p is the price of the commodity. The above is consistent with (1) with the value v_J equal to the productivity shock a.

I close the model and assume that the final good is sold to consumers $l \in [0,1]$, who have a linear Marshallian utility function over consumption of the final good z and residual money $m = m_0 - p_g z$

$$u_l(z, m) = z + m_0 - p_g z,$$

where m_0 is the endowment of money that each consumer has. The fact that there is a continuum of them implies that they are price takers. Therefore the price of the final good is equal to the marginal utility and, indeed, $p_g = 1$.

The setting considered in this example is a natural framework to study commodities markets. The linear-quadratic specification of the cost and production functions is common in the commodities literature[11]. The information structure with a cost shock known to producers but not to firms and firms having dispersed information regarding the strength of the economy is the same as in Sockin and Xiong (2015), with an additional generality of allowing for correlation between c and a. The setting of this example can be considered a generalization of Sockin and Xiong (2015), in which I allow producers to have market power[12].

3. Product markets.

[11] E.g., Grossman (1977), Kyle (1984), Stein (1987), Goldstein and Yang (2015).

[12] The market power of producers is clearly relevant in commodities markets. E.g., in the crude oil market, OPEC accounts for more than 40% of world production (OPEC statistical bulletin (2015)); in the aluminum market, the 6 largest producers account for over 40% of world production (Nappi (2013)). Such concentration should not be surprising, and the possible reasons for it are twofold. First, for the energy and metals commodity classes, commodity-producing firms are typically monopolies in their home countries. Because there are few large commodity-producing countries, there are few large producers in the world. Second, even if there are many producers in a country (which is the case for agricultural commodities, for example) their actions in the global market are nevertheless orchestrated by their home governments through export quotas and tariffs.

This example is similar to the previous one, but the good being traded is a final good. The $I-$traders are the producers of that good. They have a cost function (6), with the same assumptions regarding the cost shocks distribution. The only difference is that J-traders are now consumers with concave utility

$$v_J x - \frac{w_J x^2}{2} - px.$$

The parameter v_J is interpreted as a quality of the product, and the consumers have dispersed information on quality in the form of signals (3). With cost c and quality v_J being imperfectly correlated, the example conforms to the setting presented in the section above.

4. Foreign exchange markets.

In this example, the good being traded is foreign currency. Suppose that the home currency is the pound and the foreign currency is the dollar. The price p is how many pounds one dollar is worth. The I-traders are exporters. The J-traders are importers. Exporters receive dollars from selling their goods. Importers need to buy dollars to purchase raw materials abroad. The supply and demand from those two groups determine the exchange rate.

The price of the good that the I-traders produce and export is denominated in dollars, and the exporters have no ability to influence it. Normalize it to one. Assume that the cost of production of y units of the export good is given by (6). The revenue from selling y units of the good is y dollars and $p \cdot y$ pounds. Therefore, the profit from a sale of y units (corresponding to the net demand of $x = -y$) is given by (8), just as in Example 2.

The J-traders need to import raw materials, the price of which is denominated in dollars and normalized to one, similar to the above. The cost of buying x units of raw materials is therefore x dollars and $p \cdot x$ pounds. With x units of raw materials, the importers can produce $Y(x)$ units of the good, where the production function $Y(x)$ is given by (9). The price of the good that the importers produce is denominated in pounds and normalized to one. The profit from selling x units of the good is therefore given by (10), just as in Example 2. The mapping to the general framework can therefore be established in the same way as in Example 2.

3 Equilibrium

In this section, I characterize the equilibrium in the model. I restrict myself to the case

$$\rho \geq 0. \tag{11}$$

This is a reasonable assumption in the securities[13], commodities[14] and product markets.[15] However, my main motivation to introduce it is to simplify exposition. The model with negative correlation is

[13] Indeed, under the traditional pure common value setup, the correlation is equal to one. If the departure from the pure common values is not too substantial, the correlation should still be positive.

[14] It is more general than the assumption of zero correlation of demand and supply shocks in Sockin and Xiong (2015) and Goldstein and Yang (2015).

[15] Indeed, the fact that a particular product is more expensive to produce is usually associated with that product having better quality.

still tractable but exhibits additional complementarities. To focus on the main mechanism, I consider the case (11). Theorem 1 characterizes the equilibria.

Theorem 1. *There exists at least one equilibrium. The closed-form expressions, up to a solution of a sextic equation, for the equilibrium coefficients* $(\alpha, \beta, \gamma, \alpha_J, \beta_J, \gamma_J)$ *are given by the equations (54-59) in the Appendix. The equilibrium is unique if*

$$\tau_I < \overline{\tau}_1. \tag{12}$$

Suppose that

$$w_I < \overline{w}, \ N > 4. \tag{13}$$

Then there exist thresholds $\underline{\tau_2}$ *and* $\overline{\tau_2}$ *such that* $\overline{\tau}_1 < \underline{\tau_2} < \overline{\tau_2}$*, and there are at least three equilibria if*

$$\underline{\tau_2} < \tau_I < \overline{\tau_2}. \tag{14}$$

The closed-form expressions for the thresholds are given by equations (75, 83-85) in the Appendix.

I present a detailed proof of the above theorem in the Appendix. Below I provide the most important steps.

Consider I-traders. They choose their demand schedules to maximize $(v_I - p)\, x - \frac{w_I}{2} x^2$. The first-order condition is given by

$$v_I - p - x\frac{\partial p}{\partial x} - w_I x = 0,$$

where the third term $x\frac{\partial p}{\partial x}$ reflects the fact that the I-traders realize that they can move prices.

In equilibrium, the price sensitivity $\frac{\partial p}{\partial x}$ is given by the slope of the inverse residual supply (Kyle's lambda) $\frac{\partial p}{\partial x} = \lambda$, where

$$\frac{1}{\lambda} = (N-1)\gamma + \gamma_J. \tag{15}$$

The above expression is intuitive: $1/\lambda$ is the slope of the (direct) residual supply function, and there are $(N-1)$ I-traders with supply elasticity γ and a unit mass of J-traders with demand elasticity γ_J contributing to it.

In what follows, I will refer to λ as a *price impact* and $1/\lambda$ as *liquidity.* Equation (15) provides the first takeaway: liquidity is directly related to price elasticities γ and γ_J. This enables me to use the following language: *if a trader increases (decreases) his price elasticity*, I say that *he provides more (less) liquidity.*

The above implies that the demand of the I-traders is given by

$$x_i = \frac{1}{w_I + \lambda}(v_I - p), \tag{16}$$

from which it follows, in particular, that

$$\beta = \gamma = \frac{1}{w_I + \lambda} > 0, \tag{17}$$

where λ is given by (15). The above equation provides the second takeaway: a higher price impact implies that I-traders trade less aggressively (β is lower) and provide less liquidity (γ is lower).

As there is a continuum of J-traders, they cannot move prices. Their optimization problem is given by

$$\max_x \left(E[v_J|s_j,p]-p\right)x-\frac{w_J x^2}{2},$$

implying an optimal demand of

$$x_j=\frac{1}{w_J}\left(E\left[v_J|s_j,p\right]-p\right). \tag{18}$$

It remains to understand the inference problem of the J-traders. In a linear equilibrium given by (5), the equilibrium price function is

$$p=\frac{1}{\Gamma}\left(N\beta v_I+\beta_J v_J\right)+c_p, \tag{19}$$

where $\Gamma=N\gamma+\gamma_J$ is a price elasticity of aggregate demand and c_p is a constant[16].

The values v_I and v_J are positively correlated, and hence without loss of generality, we may assume that

$$v_I=A+Bv_J+C\epsilon,$$

where $\epsilon\sim N(0,1)$ is independent of v_J and A, $B\geq 0$ and $C>0$ are some constants[17].

Substituting the above into (19), one can see that the price is informationally equivalent to the following sufficient statistic

$$\pi\equiv\frac{\Gamma p}{N\beta B+\beta_J}+\text{const}=v_J+\frac{N\beta C}{N\beta B+\beta_J}\epsilon,$$

where in the above and in what follows, I denote by const non-stochastic terms. Because ϵ and v_J are independent, the sufficient statistic π is an unbiased signal of v_J. This signal has a precision

$$\tau_\pi\equiv\mathrm{Var}[\pi|v_J]^{-1}=\frac{1}{C^2}\left(B+\frac{\beta_J}{N\beta}\right)^2>\left(\frac{B}{C}\right)^2. \tag{20}$$

From the Projection Theorem, the ex-post precision of v_J, measuring how much the J-traders can learn about their values, is

$$\tau=\mathrm{Var}[v_J|s_j,p]^{-1}=\tau_J+\tau_s+\tau_\pi.$$

Define *information efficiency* as[18]:

$$\mathcal{I}\equiv\frac{\mathrm{Var}(v_J)}{\mathrm{Var}(v_J|s_j,p)}=\frac{\tau_J+\tau_s+\tau_\pi}{\tau_J}.$$

[16] The exact value is $c_p=\frac{N\alpha+\alpha_J}{\Gamma}$.

[17] It is easy to express A, B and C through the parameters of the model. One can find $B=\rho\sqrt{\frac{\tau_J}{\tau_I}}$, $C=\sqrt{\frac{1-\rho^2}{\tau_I}}$ and $A=\bar{v}_I-B\bar{v}_J$.

[18] Intuitively, $\mathcal{I}$ measures the reduction in variance due to learning. As the I-traders know their value perfectly well, they do not contribute to $\mathcal{I}$.

This reveals the third takeaway: less aggressive trading by I-traders (lower β) makes the price more informative for J-traders (greater τ_π). Because the J-traders are the only ones who learn, the information efficiency of the market improves ($\mathcal{I}$ increases).

From the Projection Theorem, one can compute

$$\begin{aligned} E\left[v_J|s_j,p\right] &= \frac{\tau_s}{\tau}s_j + \frac{\tau_\pi}{\tau}\pi + \text{const} \\ &= \frac{\tau_s}{\tau}s_j + \frac{\tau_\pi}{\tau}\frac{\Gamma p}{(N\beta B+\beta_J)} + \text{const.} \end{aligned}$$

Substituting the above into (18) and comparing to (5) yields

$$\beta_J = \frac{1}{w_J}\frac{\tau_s}{\tau} > 0, \tag{21}$$

and, after some rearrangement,

$$\gamma_J = \frac{1}{w_J} - \frac{\Gamma}{\tau_s}\sqrt{\tau_\pi}\left(\sqrt{\tau_\pi} - \frac{B}{C}\right). \tag{22}$$

Intuitively, there are two effects determining the elasticity γ_J. The first is the expenditure effect: for a higher price, a trader would demand less because a higher price implies higher expenditure $p \cdot x$ from buying x units of the good. This effect corresponds to the first term in (22). The second is the information effect: a higher price may also signal a higher value of v_J, and a trader might wish to buy more for a higher price. The information effect therefore has the opposite sign and corresponds to the second term in (22). Intuitively, this effect is stronger the more informative the price is. This is why, as can be seen from (22), the price elasticity γ_J is decreasing in τ_π. This observation provides the last takeaway: greater price informativeness (higher τ_π) induces J-traders to provide less liquidity (decrease γ_J).

4 Strategic complementarities and multiplicity of equilibria

The four takeaways from the above section are the basis for the strategic complementarities in the model and the driver of the multiplicity of equilibria. The complementarities are represented in Figure 2, which depicts two feedback loops. The smaller one corresponds to complementarities *within* I-traders. If market is less liquid (λ is higher), I-traders provide less liquidity (γ is lower, cf. (17)). This, in turn, confirms a higher price impact (cf. (15)).

The larger loop corresponds to the complementarities *between* I- and J-traders. A higher price impact implies that I-traders are less aggressive (β is lower, cf. (17)). This implies that the price is more informative for J-traders (τ is higher, cf. (20)) and the market is more information efficient (as only J-traders learn from prices). Because the price is more informative, J-traders provide less liquidity (γ_J is lower): they are less willing to decrease their demand if the price increases because an increase in price may signal stronger fundamentals. This is confirmed by equation (21). The last step in the loop indicates that because J-traders provide less liquidity, the price impact is indeed higher (15).

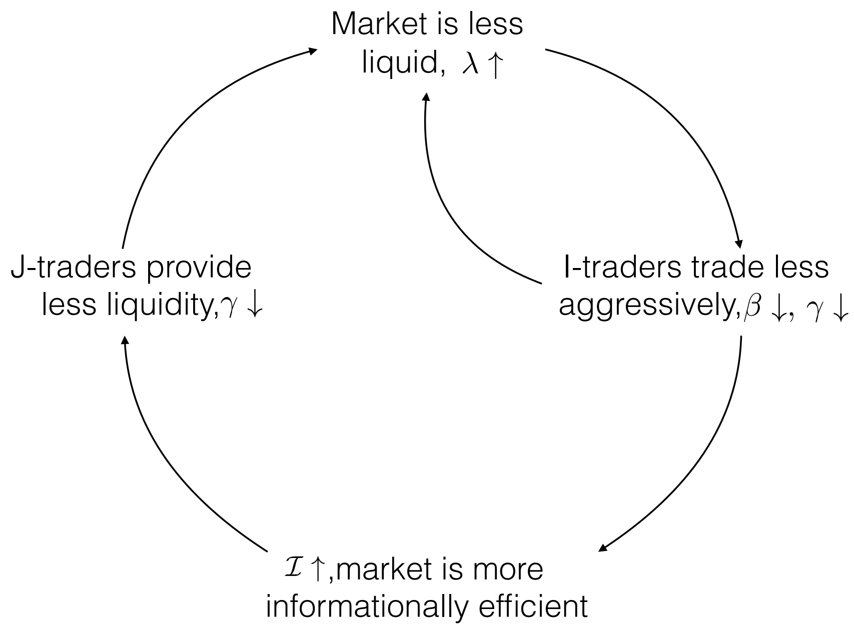

Figure 2: Equilibrium mechanism: two feedback loops. The smaller one corresponds to within-complementarities. The larger one corresponds to between-complementarities.

Complementarities may generate multiple equilibria driven by self-fulfilling beliefs regarding liquidity. Indeed, suppose that there is an equilibrium. Suppose that traders believe that the liquidity is actually lower. The I-traders will then trade less aggressively. This will make the price more informative for J-traders, who will provide less liquidity. The latter confirms lower liquidity and potentially allows traders to coordinate on another equilibrium. One can also interpret the multiplicity as being driven by self-fulfilling beliefs concerning information efficiency. The latter interpretation works as follows. Suppose that there is an equilibrium. Suppose that the J-traders believe that the price informativeness is actually higher than that in equilibrium. They will then provide less liquidity. This would lead to a higher price impact of I-traders, who will trade less aggressively, confirming the higher price informativeness and potentially justifying another equilibrium.

Theorem 1 provides sufficient conditions for uniqueness and multiplicity, which I discuss below. The complementarities between I- and J-traders are facilitated by the price inference of J-traders. The more informative the price is relative to the signal, the more J-traders rely on prices and the more the two groups of traders interact. Sufficient condition (12) ensures that the price is not too informative: if τ_I is low enough, there is enough noise in the price. This condition ensures that the between-complementarities (i.e., the larger loop in Figure 2) are not too strong to generate multiple equilibria. As is well known from the literature, the within-complementarities alone do not generate multiplicity[19], and hence no additional conditions to weaken the feedback in the small loop in Figure 2 are needed.

[19]Indeed, within-complementarities are present even in the pure common values setting of, e.g., Kyle (1989), Vayanos (1999) and Rostek and Weretka (2015). However, the equilibrium in those models is unique.

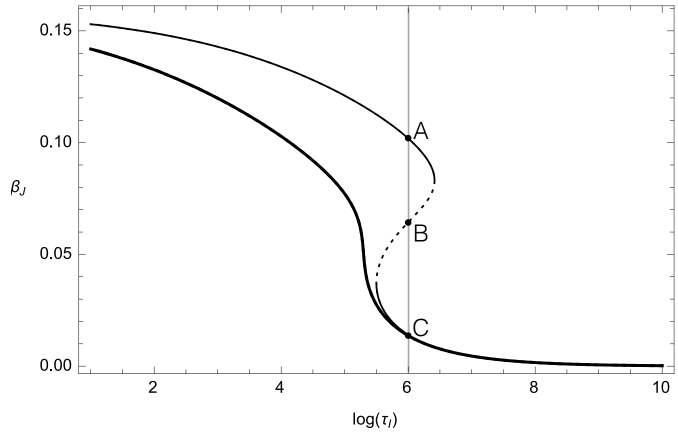

Figure 3: Multiple equilibria. The figure demonstrates dependence of β_J on $\log(\tau_I)$ for $w_I = 3$ (thick line) and $w_I = 1$ (thin line). Unstable equilibria are represented by dotted parts of the lines. The values of other parameters are $\rho = 0.9$, $\tau_J = 1$, $\tau_s = 1$, $N = 10$, $w_J = 1$.

The sufficient condition for multiplicity $\tau_I > \underline{\tau_2}$ ensures that price informativeness is high enough, such that the price inference channel, through which I- and J-traders interact, is important. The condition $\tau_I < \overline{\tau_2}$ ensures that price informativeness is not too high, and hence more/less aggressive trading by the I-traders can change the informativeness significantly. Thus, condition (14) ensures that the between-complementarities are strong enough. The condition that $w_I < \overline{w}$ ensures that the price elasticity γ is not too small (cf. (17)). Together with the condition that N is large enough, the former condition ensures that $(N-1)\gamma$ is not too small relative to γ_J, and hence the between-complementarities are an important determinant of the price impact (cf. (15)). Thus, condition (13) ensures that the within-complementarities are strong enough.

Figure 3 illustrates the multiplicity of equilibria in the model. It plots the equilibrium sensitivities β_J against τ_I. It should be read as follows: draw a vertical line corresponding to a particular value of the parameter τ_I. Each intersection of the vertical line with the plot in Figure 3 corresponds to an equilibrium. If the line intersects with a dashed part of the plot, the equilibrium is unstable[20]. For example the equilibria A and C in Figure 3 are stable, whereas equilibrium B is unstable. Observe, consistent with Theorem 1, that there is a unique equilibrium if τ_I is small enough and that when w_I is small enough, there are three equilibria for the intermediate values of τ_I.

4.1 Liquidity and information efficiency

In this section, I consider the case of equilibrium multiplicity and compare the equilibria in terms of liquidity and information efficiency. Recall the definitions of liquidity and information efficiency

$$\mathcal{L} = \frac{1}{\lambda} \text{ and } \mathcal{I} = \frac{\mathrm{Var}(v_J)}{\mathrm{Var}(v_J|s_j, p)}.$$

[20]The stability analysis is performed in Section A.

Recall that the multiplicity is driven by the complementarity between illiquidity and information efficiency: lower liquidity induces higher information efficiency (through I-traders being less aggressive); higher information efficiency confirms lower liquidity (through J-traders providing less liquidity). Therefore, given a particular equilibrium, traders can coordinate on another one with lower liquidity and higher information efficiency.

The above suggests that the equilibria can be ranked in terms of $\mathcal{L}$ and $\mathcal{I}$, with the equilibria that are more liquid being less information efficient and vice versa. This is confirmed in Proposition 1. If the traders were to pick an equilibrium, they would have to choose between two evils: the equilibrium with the highest liquidity is the one with the lowest information efficiency and vice versa. To resolve this tension, I compute the welfare $\mathcal{W}$ (defined as the sum of expected utilities of all traders) and provide a sufficient condition that allows me to rank equilibria in terms of welfare. See Proposition 1 below.

Proposition 1. *Suppose that there are multiple equilibria. For any two equilibria A and B: $\mathcal{L}_A > \mathcal{L}_B$ if and only if $\mathcal{I}_A < \mathcal{I}_B$. Moreover, there exists $\underline{\tau}_J$ such that if*

$$\tau_J > \underline{\tau}_J \text{ and } \tau_I < 1 - \rho^2 \tag{23}$$

holds, then $\mathcal{W}_A > \mathcal{W}_B$ if and only if $\mathcal{L}_A > \mathcal{L}_B$.

Condition (23) should be understood as follows: prices do not provide much incremental information. Indeed, τ_J being large enough ensures that J-traders face little uncertainty regarding their value. The condition that τ_I is small enough implies that the price is not too informative from the perspective of $J-$traders. If condition (23) holds, liquidity is more important and the equilibria with higher liquidity are those with higher welfare.

4.2 Crashes

In this section, I explore the implications of the mechanism presented above for price crashes and the associated changes in information efficiency, liquidity, volume, volatility and welfare.

In what follows, I refer to the expected price $E[p]$ simply as price. I refer to the standard deviation of the price simply as volatility and denote it as σ_p

$$\sigma_p \equiv \sqrt{\mathrm{Var}(p)}.$$

The expected trading volume (volume hereafter) is defined as

$$\mathcal{V} \equiv \frac{1}{2} \cdot E\left[\int_0^1 |x_j(p^*)|\, dj + N \cdot |x_I(p^*)|\right].$$

I define a *crash (jump)* in an endogenous variable such as price, volatility or volume as follows.

Definition 1. Suppose that there is an endogenous variable X and a parameter of the model $\zeta \in \{\tau_I,$ $\tau_J,$ $\tau_s,$ $\rho,$ $w_I,$ $w_J,$ $N\}$. A crash (jump) of X is either of the two situations. (1) There are multiple equilibria. A crash (jump) is a sunspot switch from the equilibrium in which X is high (low) to the equilibrium in which X is low (high). (2) There is unique equilibrium, in which $\frac{dX}{d\zeta} = -\infty$ ($\frac{dX}{d\zeta} = +\infty$).

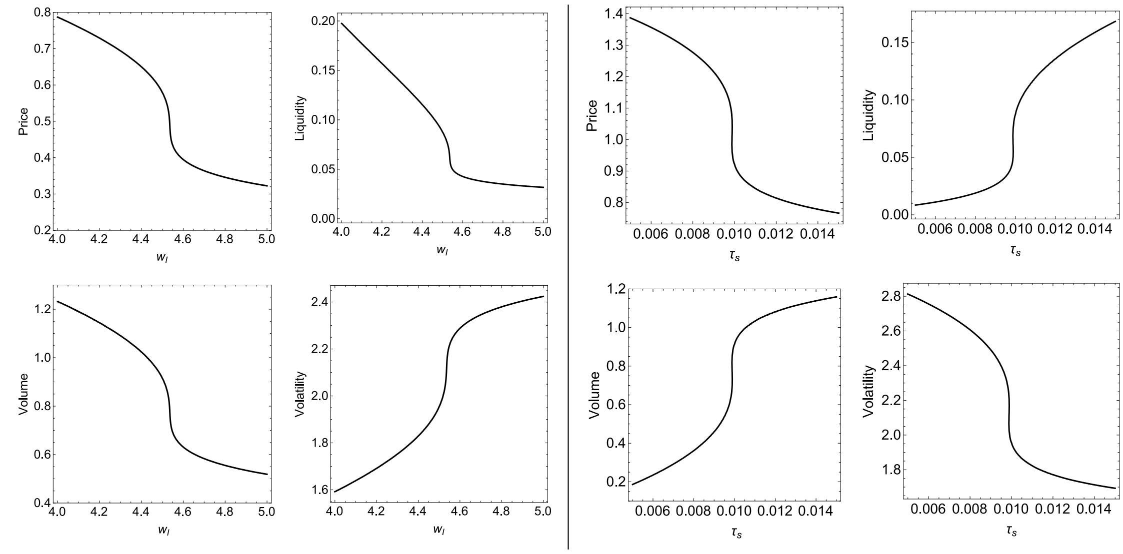

Figure 4: Two scenarios of a price crash. In the left panel, the I-traders are net buyers. The parameter values are $\bar{v}_I = 1.5$, $\bar{v}_J = 0$, and $\tau_s = 0.01$. In the right panel, the I-traders are net sellers. The parameter values are $\bar{v}_I = 0$, $\bar{v}_J = 1.5$, and $w_I = 4.5$. The values of the remaining parameters are the same for the two panels: $N = 10$, $w_J = 1$, $\rho = 0.9$, $\tau_J = 0.1$, and $\tau_I = 6.13$.

For example, the thick line in Figure 3 exhibits a crash of β_J when $\log(\tau_I)$ is close to 5.2, and a thin line exhibits a crash when $\log(\tau_I)$ is between 5.5 and 6.5. The proposition below characterizes the behavior of endogenous objects in the event of a price crash. I focus on the case of a price crash because in most markets, prices rarely jump up[21]. The corresponding statements for the case of jumps can be easily obtained in a way analogous to the proposition below.

Proposition 2. *Two scenarios are consistent with a price crash. (1) The price crash is associated with a liquidity crash, a jump in volatility and a jump in information efficiency, and if (23) holds, there is also a crash in the trading volume and welfare. This is the case when the I-traders are net buyers, i.e., $\bar{v}_I > \bar{v}_J$. (2) The price crash is associated with a jump in liquidity, a crash in volatility and a crash in information efficiency, and if (23) holds, there is also a jump in the trading volume. This is the case when the I-traders are net sellers, i.e., $\bar{v}_I < \bar{v}_J$.*

The above proposition identifies two scenarios consistent with a price crash. In the first scenario, the I-traders are net buyers. This scenario is represented in the left panel of Figure 4. Let us interpret it in the context of securities markets (Example 1). A small change in the risk-bearing capacity of I-traders (an increase in w_I) reduces liquidity. This initial liquidity shock is amplified due to two feedback loops. Due to the liquidity shock, I-traders provide less liquidity, which feeds back into a higher price impact. As I-traders also trade less aggressively, the price becomes more informative and the J-traders provide less liquidity. This also feeds back into a higher price impact. A small liquidity shock is amplified and results is a large overall drop in liquidity. Due to the increased price impact, I-traders buy less and the prices drop. Because liquidity is low, relatively small orders can cause large price changes, and hence volatility increases. The volume drops for two reasons. First, due to the higher price impact, the I-traders trade less. Second, after the crash, information efficiency increases (because I-traders trade

[21]The notable exception is currency markets: exchange rates do jump up.

less aggressively); therefore, the ex post values of J-traders $E[v_J|s_j,p]$ are closer to the true value v_J and are therefore more aligned. This implies less volume generated by J-traders[22]. The first scenario is associated with a drop in liquidity but an increase in information efficiency. If condition (23) holds (the price provides little incremental information), then such a crash is welfare-reducing and suggests a policy intervention.

In the second scenario, the I-traders are net sellers. This scenario is represented in the left panel of Figure 4. Let us interpret it in the context of commodities markets (Example 2). A small increase in the precision of information regarding the strength of the economy (an increase in τ_s) decreases the market power of producers (λ) . Due to the the mechanism discussed above, this reduction in market power is amplified and results in a substantial overall decrease in λ. Liquidity improves. Because the commodity producers have less market power, prices drop. The increase in liquidity means that volatility decreases. Volume increases because for two reasons. First, the commodity producers trade more, due to the lower price impact. Second, because there is less information, the ex post values of the firms are less aligned and there is an increase in the volume generated by them. The second scenario is associated with a drop in information efficiency but an increase in liquidity. If condition (23) holds (the informational role of price is not too important), then such a crash is welfare-improving, and no policy intervention is needed.

5 Comparative statics

In this section, I consider how information efficiency $\mathcal{I}$ and liquidity $\mathcal{L}$ are affected by changes in the model parameters. I focus on the following parameters: τ_s, which is related to informational frictions, N, which is related to the degree of competition, and w_I and w_J, which are related to liquidity. I consider two ways of obtaining comparative statics with respect to N.

1. No other parameters of the model change with N.
2. The convexity w_I is proportional to N, i.e., $w_I = w_1 N$, where w_1 is some constant. Other parameters are not affected by N.

The idea behind the second approach to obtain the comparative statics is as follows. Consider Example 2, in which the I-traders are producers. Decreasing (increasing) N in the second way corresponds to a merger (split) of existing producers[23]. Indeed, suppose that there are $N = n \cdot M$ producers with costs $C(x;N) = c \cdot x + \frac{w_I(N)}{2}x^2$. Suppose that every n producers have merged into 1. After the merger, there are M producers, each having n production units. To minimize the cost, producers will divide the production evenly across production units. Thus to obtain the output x, they will produce x/n

[22] There is, however, an effect that works in the opposite direction. The variation in the ex post value $\mathrm{Var}(E[v_J|s_j,p])$ may increase as a result of more information. To understand why, consider an extreme case in which the precision τ_s is zero. Without any information from price, the ex post value is $E[v_J|s_j] = \bar{v}_J$ and there is no variation in it. The more information the price provides, the closer the ex post value is to the true value v_J. Because the latter is stochastic, there will be more variation in the ex post value. The expected trading volume is an increasing function of the variance of the demand, which, in turn, depends on the ex post value $E[v_J|s_j,p]$. Therefore the above mechanism can lead to an increase in trading volume.

Condition (23) ensures that even without information from price, there is sufficient variation in the ex post value $E[v_J|s_j] = \beta_J w_J s_j$ that the above effect is not too strong.

[23] The first way of obtaining the comparative statics corresponds to entry/exit.

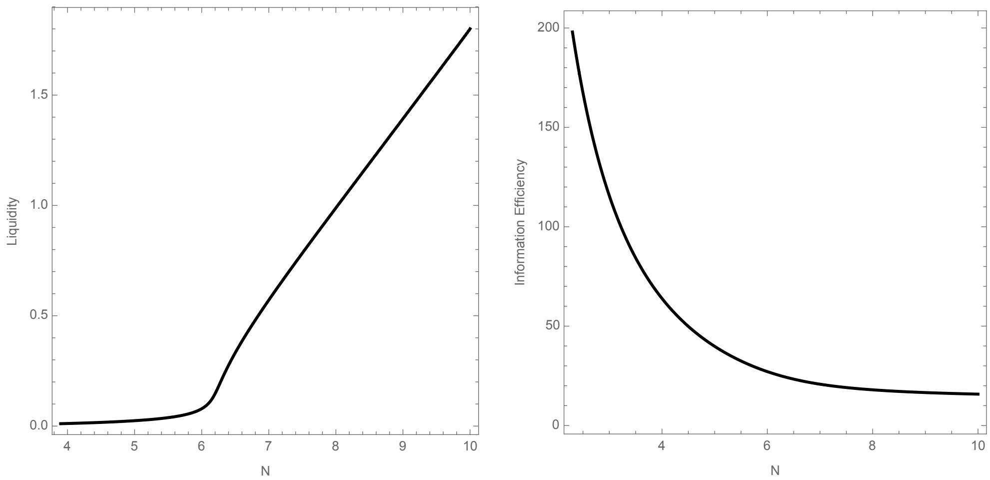

Figure 5: Tension between liquidity and information efficiency. Increasing N reduces the market power of I-traders and therefore improves liquidity, but because it induces I-traders to trade more aggressively, it reduces information efficiency.

units at each of the production units. Therefore, the cost function becomes $C(x; M) = nC(x/n; N) = c \cdot x + \frac{w_I(N)}{2n}x^2$. Therefore, $w_I(N/n) = w_I(N)/n$, and the coefficient w_I is indeed proportional to to the number of producers. In financial markets, the second approach to obtaining the comparative statics can regarded as a reduced-form approach to modeling the wealth effect (see Makarov and Schornick (2010)).

In the proposition below, I examine the comparative statics with respect to N[24].

Proposition 3. *In the unique equilibrium, irrespective of whether w_I does not depend on N, or $w_I = w_1 N$,*

$$\frac{d\mathcal{I}}{dN} < 0 \text{ and } \frac{d\mathcal{L}}{dN} > 0.$$

The proposition above implies that there is tension between liquidity and information efficiency. Increasing the number of I-traders improves liquidity: with more I-traders, each of them has less market power, and thus the price impact is lower. However, because more liquidity induces I-traders to trade more aggressively, it reduces information efficiency. This is illustrated in Figure 5.

Next, I examine the comparative statics with respect to τ_s.

Proposition 4. *In the unique equilibrium*

$$\frac{d\mathcal{L}}{d\tau_s} > 0,$$

for $\tau_s > \frac{1-2\rho^2}{1-\rho^2}\tau_J$. In particular, if $\rho > \frac{1}{\sqrt{2}}$, then $\frac{d\mathcal{L}}{d\tau_s} > 0$ for all τ_s.

The intuition is as follows. With more precise signals, the J-traders learn more from their signals and less from prices. Their price elasticities increase, and liquidity improves.

[24]Although, by definition, N takes discrete values, the quantities $\mathcal{I}$ and $\mathcal{L}$ are continuous functions of N, and hence I provide the results for the derivatives of those functions, rather than finite differences, to simplify exposition.

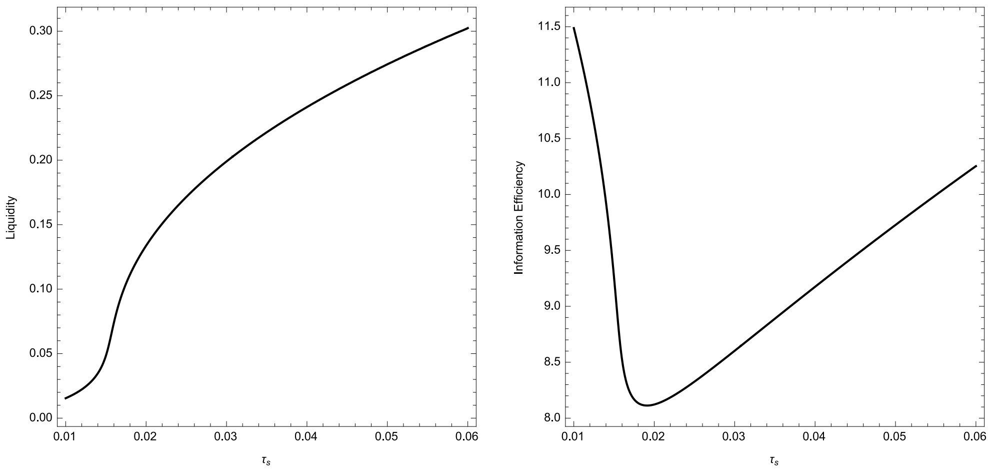

Figure 6: Left panel: liquidity is increasing in the precision of the signal τ_s. The higher the precision is, the less the J-traders learn from prices, the higher the price elasticity of their demand is and the greater the liquidity. Right panel: for small values of τ_s, there is an *information aggregation paradox*; the aggregation of more information yields less information ex post. The parameter values are $N = 10$, $w_I = 4.5$, $w_J = 1$, $\rho = 0.9$, $\tau_J = 0.1$, and $\tau_I = 7$.

The comparative statics for information efficiency are driven by two forces. On the one hand, increasing τ_s has a positive, direct effect on $\mathcal{I} \equiv \frac{\text{Var}(v_J)}{\text{Var}(v_J|s_j,p)} = \frac{\tau_J+\tau_s+\tau_\pi}{\tau_J}$. On the other hand, as the proposition above indicates, increasing τ_s improves liquidity and makes the I-traders trade more aggressively. This may have a negative effect on the precision τ_π of the price signal. If the second effect prevails, the *information aggregation paradox* obtains*:* aggregating an ex ante more precise information (higher τ_s) market conveys less information ex post (lower $\mathcal{I}$). Intuitively, the second force is stronger when the J-traders learn more from prices, which is the case when τ_s is low: this is illustrated in Figure 6.

Figure 6 illustrates that there is tension between liquidity and information efficiency when τ_s is small, such that there is an information aggregation paradox. When τ_s is large, there is no tension: improving the precision of information (i.e., by reducing the information acquisition costs) improves both liquidity and information efficiency.

I next examine the comparative statics with respect to w_I and w_J. An increase in w_I or w_J is interpreted as a decrease in risk-bearing capacity, which can be due to tightened of regulations or an external liquidity shock. Consider first the effect of a change in w_I and w_J on information efficiency. Intuitively, if w_J decreases, J-traders trade more aggressively on their signals and the price becomes more informative. An increase in w_I induces I-traders to trade less aggressively and therefore has a similar effect. This is intuition is confirmed in the proposition below.

Proposition 5. *In the unique equilibrium,* $\frac{d\mathcal{I}}{dw_J} < 0$ *and* $\frac{d\mathcal{I}}{dw_I} > 0$.

A decrease in the risk-bearing capacity of I-traders (an increase in w_I) has a direct negative effect on liquidity. It also has an indirect effect: an increase in w_I increases information efficiency, which has a negative effect on $\mathcal{L}$ because J-traders provide less liquidity. Therefore, the overall effect of the liquidity shock to I-traders on liquidity should be negative. This is confirmed in the proposition below[25].

[25]The proposition examines the effect of w_I on Γ. The numerical result is that $\mathcal{L}$ is also decreasing in w_I.

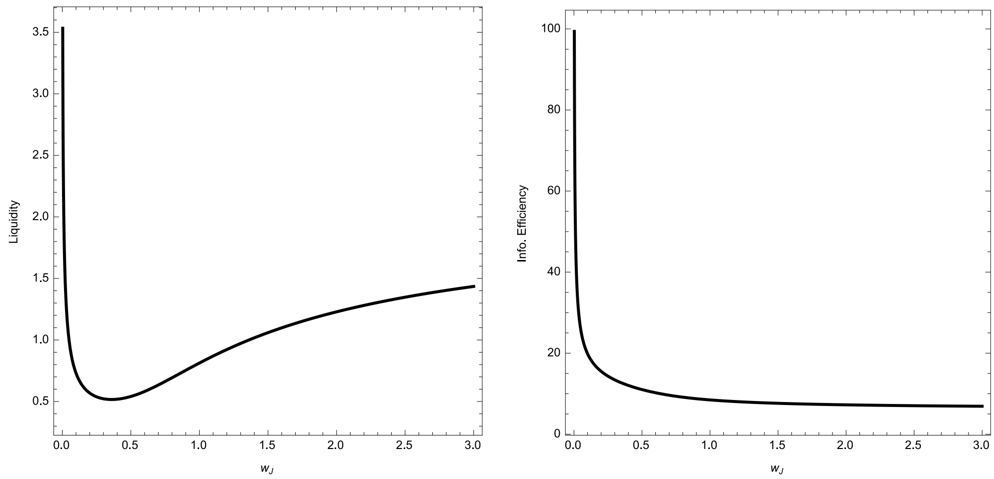

Figure 7: Left panel: liquidity paradox. An adverse shock to the risk-bearing capacity of J-traders may lead to an increase in liquidity. Right panel: information efficiency is decreasing in w_J. The parameter values are $N = 4$, $w_I = 1$, $\rho = 0.9$, $\tau_s = 0.1$, $\tau_J = 0.1$, and $\tau_I = 1$.

Proposition 6. *In the unique equilibrium,* $\frac{d\Gamma}{dw_I} < 0$*, where* $\Gamma = N\gamma + \gamma_J$ *is a slope of the aggregate demand.*

Combining the results of Propositions 5 and 6, it is clear that if the risk-bearing capacity of I-traders increases, liquidity improves but the information efficiency deteriorates. Thus, there is tension between liquidity and information efficiency.

A shock to the risk-bearing capacity of J-traders has conflicting effects on liquidity. The direct effect is negative: if w_J increases, J-traders provide less liquidity. However, an increase in w_J has a negative effect on information efficiency. This can induce J-traders to provide more liquidity. This effect can be amplified through I-traders being more aggressive. If the second effect dominates, a *liquidity paradox* obtains: an adverse liquidity shock leads to an improvement in liquidity. This is represented in Figure 7.

I finally consider the question of how combining two markets into one (or, alternatively, breaking up an existing exchange into two) affects the market quality of the combined market.

The exercise is as follows. Consider a market with N I-traders and a unit mass of J-traders. Divide this market into two: let $M < N$ I-traders trade with a proportional measure M/N of J-traders in "market 1"; let the remaining traders trade in "market 2". Markets 1 and 2 are completely segmented. I am interested in how the information efficiency and liquidity of markets 1 and 2 are related to those of a combined market ("1+2"). Intuitively, markets 1 and 2 are less competitive than a combined market $1+2$, and hence the liquidity should be lower. However, because I-traders are less aggressive in smaller markets, information efficiency can increase. The proposition below confirms this intuition: breaking up a market into two is bad for liquidity but is good for information efficiency.

Proposition 7. *Liquidity and information efficiency in markets* 1*,* 2 *and* $1 + 2$ *described above are related as follows:* $\mathcal{L}_{1+2} > \mathcal{L}_1$ *and* $\mathcal{L}_{1+2} > \mathcal{L}_2$*; however,* $\mathcal{I}_{1+2} < \mathcal{I}_1$ *and* $\mathcal{I}_{1+2} < \mathcal{I}_2$*.*

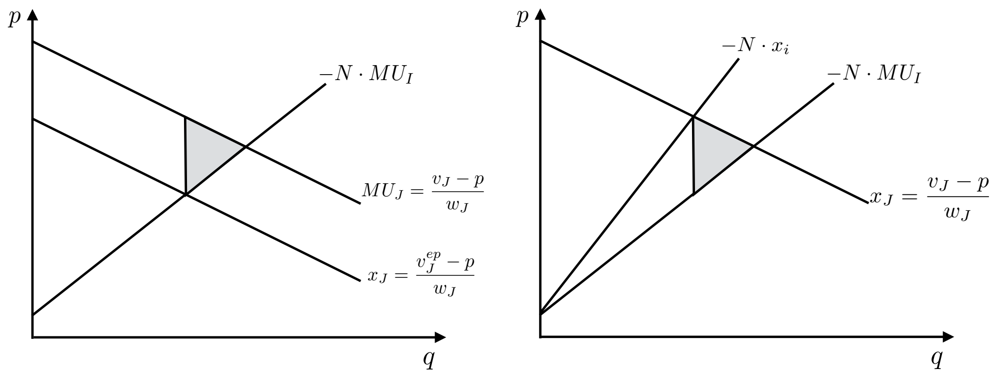

Figure 8: The role of liquidity and information efficiency in determining welfare: a gray triangle represents a welfare loss. Left panel: the role of information. In the economy with I-traders being price takers (such that liquidity plays no role), with more information the ex post value v_J^{ep} is less biased relative to v_J (state by state) which helps to reduce welfare loss. Right panel: the role of liquidity in determining welfare. In the economy in which J-traders know their value perfectly well (such that information plays no role), I-traders reduce their demands, which results in a welfare loss.

Note that in the competitive economy, proportionally scaling the measures of agents in the market has no effect on equilibrium, as it cancels out through market clearing. Comparing economy $1+2$ with either economy 1 or economy 2 provides another way of isolating the effect of competition on liquidity and information efficiency. Competition is good for liquidity but is bad for information efficiency.

6 Welfare

It is often argued that greater competition is associated with greater welfare. I examine the validity of this claim in the context of my model.

Define

$$\mathcal{U}_I \equiv E\left[(v_I - p)\,x_i(p) - \frac{w_I x_i(p)^2}{2}\right],$$

the expected utility of an I-trader, and

$$\mathcal{U}_J \equiv E\left[(v_J - p)\,x_j(p) - \frac{w_J x_j(p)^2}{2}\right],$$

the expected utility of a J-trader. The total welfare is then

$$\mathcal{W} \equiv N \cdot \mathcal{U}_I + \mathcal{U}_J.$$

With no informational frictions, the effect of competition on welfare is unambiguous (e.g., Tirole (1988)): with more competition, welfare improves. The intuition is as follows: when N increases, large traders have less market power and there is less reduction in their demands. This helps to reduce deadweight loss, and welfare increases. Because the large traders have less market power, prices are more favorable to small traders, and hence they also become better off.

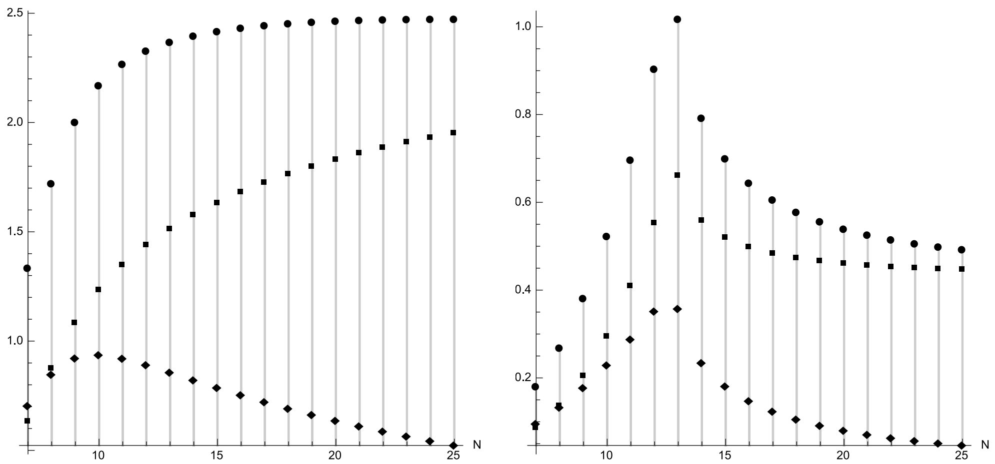

Figure 9: Welfare $\mathcal{W}$ (circles), expected utility of a $J-$trader $\mathcal{U}_J$ (squares) and the expected utility of I-traders $N \cdot \mathcal{U}_I$ (diamonds) . Left panel: $\tau_s = 0.1$. When the informational frictions are not too strong, the standard intuition applies: competition is good for welfare and for small traders. Right panel: $\tau_s = 0.01$. When informational frictions are strong, welfare becomes non-monotonic in N. Moreover, it is possible that everyone, including small traders, is worse off as a result of more competition. The remaining parameter values are $w_I = 8$, $w_J = 1$, $\rho = 0.3$, $\tau_J = 0.1$, and $\tau_I = 25$.

The proposition below provides a sufficient condition for the "usual" comparative statics for welfare: if prices provide little incremental information (condition (23) holds), the standard intuition applies.

Proposition 8. *Suppose that there is a unique equilibrium and (23) holds. Then,* $\frac{d\mathcal{W}}{dN} > 0$ *and* $\frac{d\mathcal{U}_I}{dN} > 0$*, irrespective of whether* w_I *does not depend on* N*, or* $w_I = w_1 N$ *.*

I show below that if condition (23) does not hold, increasing competition might actually reduce welfare. Moreover, this can be bad for everyone, including J-traders. I attribute this result to the tension between liquidity and information efficiency outlined in the section above. To proceed, it is necessary to understand the effects of liquidity and information efficiency on welfare. In the model, the two are tightly linked through the mechanism represented in Figure 2. Therefore, it is difficult to disentangle the roles of the two. To overcome this difficulty, I consider the following two thought experiments.

First, consider an economy in which the role of liquidity is "switched off". Suppose that traders behave as if the market were perfectly liquid, i.e., even I-traders take prices as given[26]. The informational frictions in this economy are the same as in the original economy. In this economy, welfare increases in information. First, with more information, the equilibrium quantities allocated to J-traders are less dispersed, which is good for welfare[27]. Second, with more information, the ex post values of

[26]This setting corresponds to REE in the model.

[27]It is easy to show that allocating the average quantity $x_J = \int x_j(p)dj$ to all traders (instead of allocating $x_j(p)$ to each of them) increases the ex post aggregate utility $\int_0^1 \left((v_J - p)\, x_j(p) - \frac{w_J x_j(p)^2}{2} \right) dj$ of J-traders. This is due to the concavity of their objective. The more dispersed $x_j(p)$ are relative to x_J, the smaller the ex post aggregate utility.

J-traders ($E[v_J|s_j, p]$) are less biased[28], which also increases welfare. Indeed, the maximum welfare in this economy is achieved when traders bid according to their marginal utilities

$$x_i = MU_I \equiv \frac{v_I - p}{w_I}, \text{ and } x_j = MU_J \equiv \frac{v_J - p}{w_J}.$$

However, the aggregate trade of J-traders is actually

$$\begin{aligned} x_J &\equiv \int x_j dj \\ &= \frac{v_J^{ep} - p}{w_J}, \text{ where } v_J^{ep} = \int E[v_J|s_j, p]dj. \end{aligned}$$

A bias between v_J^{ep} and the true value v_J results in a welfare loss. See the left panel of Figure 8.

Second, consider an economy in which the role of information is "switched off". Suppose that all traders know their values but that I-traders exercise their market power. This economy is, essentially, the textbook oligopoly model discussed above. The I-traders will reduce their demands, which will result in welfare loss. See the right panel of Figure 8.

Summarizing the above discussion, I conclude that taken in isolation, both liquidity and information efficiency are beneficial for welfare. This suggests that increasing competition can have an adverse effect on welfare through its adverse effects on information efficiency, as established in Section 5. Because the negative effect operates through the information channel, it should be more pronounced when informational frictions are high. This intuition is confirmed in Figure 9: when τ_s is small, welfare may be non-monotonic in N. Moreover, everyone, even small traders, can be worse off as a result of more competition.

I finally consider the question of the effects of breaking up an exchange on welfare. Common wisdom suggests that welfare should be higher in a centralized market. A centralized market should better aggregate information, and the traders should better share their risks in such a context. Figure 10 shows that this common wisdom may not be correct: it plots the welfare in an economy in which there are two identical segmented markets and the welfare when those two markets are combined into one relative to the number N of I-traders in either of the segmented markets. The right panel of the figure indicates that segmentation might be beneficial. The intuition is as follows: in two segmented markets, I-traders are less competitive, which results in lower liquidity; however, this is beneficial for information efficiency,[29] as I-traders trade less aggressively (Proposition 7). When N is large, such that the liquidity loss resulting from breaking up the market is less considerable, segmentation might be beneficial. This result is present when informational frictions are important (τ_s is low). A similar graph can be obtained for the surplus of J-traders instead of aggregate welfare.

[28] Indeed, define the bias as $E\left[E[v_J|s_j, p] - v_J|v_J\right] = \frac{\tau_J}{\tau}(\bar{v}_J - v_J)$. It goes to zero as $\tau \to \infty$.

[29] Malamud and Rostek (2015) show that breaking up an exchange can be beneficial through its effects on liquidity. Their setting does not feature asymmetric information and does not capture information efficiency. The result in this paper therefore complements that of Malamud and Rostek (2015). It would be desirable to incorporate the mechanism highlighted in this paper into the much more general market structure environment considered in Malamud and Rostek (2015).

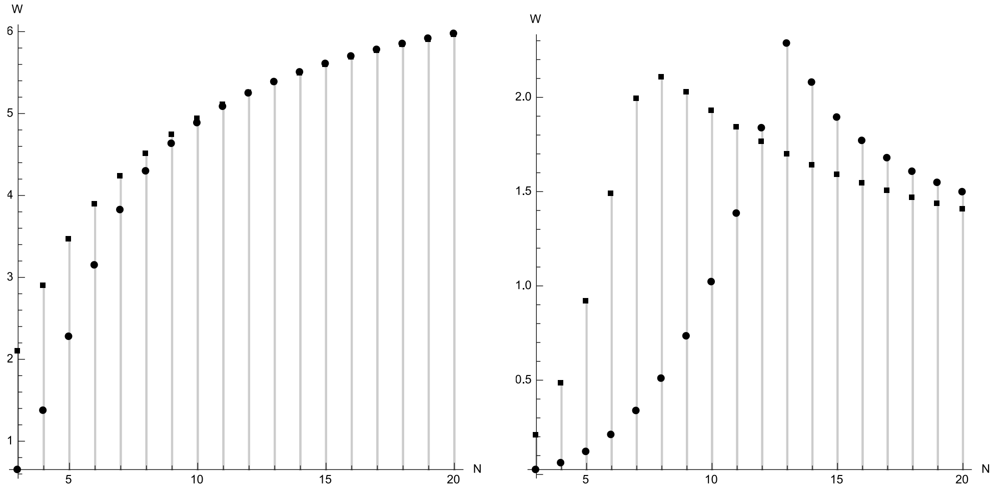

Figure 10: Welfare in the economy with 2 segmented markets (circles) and a combined market (squares). Left panel: $\tau_s = 0.3$. When the informational frictions are not too strong, the segmentation is bad for welfare. Right panel: $\tau_s = 0.01$. When informational frictions are strong, segmentation might be beneficial for welfare. The remaining parameter values are $w_I = 8$, $w_J = 1$, $\rho = 0.3$, $\tau_J = 0.1$, and $\tau_I = 25$.

7 Implications

In this section, I consider the implications of the model.

7.1 Commodities markets

I consider two episodes, the 2008 boom/bust in oil prices and the recent crash of oil prices, through the lens of the model.

2008 boom/bust in oil prices Oil prices reached an all-time high of $145 per barrel in July 2008, a 40% increase from the level in January of 2008, when the US and most other developed economies were entering a recession. It is difficult to explain such a sharp increase by a shift in either demand or supply: no major disruptions in supply occurred at that time; demand from developed economies was weaker, and it is unlikely that the demand from the developing economies (which were performing well at the time) could have offset this weakness.

My explanation for this episode emphasizes the role of two forces: (1) informational frictions and (2) the market power of oil producers being endogenously amplified because of (1). In the model, market power corresponds to λ, which measures the extent to which producers can drive up the price by reducing their supply. Proposition 4 implies that the producers' market power is higher when information concerning economic fundamentals is less precise ($1/\tau_s$ is higher). This is intuitive: when informational frictions are high, firms on the demand side rely on commodity prices as a signal of the strength of the economy. When commodity producers reduce their supply, they are driving up the price. Firms partly attribute the increase in price to stronger fundamentals and demand more, which amplifies the price impact of producers. Therefore, the boom in prices can be attributed to an increase in the market power of commodity producers caused by the uncertainty regarding the strength of the economy at the time.

Singleton (2014) presents empirical evidence that supports this explanation. He finds a strong, positive correlation between the dispersion of oil price forecasts (related to $1/\tau_s$ in the model) and the oil price level. Such a relationship can be explained by an increase in the market power of producers caused by higher uncertainty. This evidence therefore supports the mechanism discussed above.

In emphasizing the role of informational frictions, my explanation is closely related to that of Sockin and Xiong (2015)[30]. My explanation complements theirs by highlighting the role of commodity producers' market power and the role of informational frictions in amplifying the latter.

The bust can be explained by the price effect of the demand shock coming from CITs and hedge funds that unwounded their positions in the commodities markets during the financial crisis, as documented by Cheng et al. (2015). My model can help explain the magnitude of the price effect of this shock. Informational frictions amplified the illiquidity of the market, and a demand shock had a larger price effect.

2014 crash in oil prices Between January 2012 and October 2014, the oil price ranged from \$80 to \$110 per barrel. By the by the end of 2014, it had halved and remains in the range \$40 - \$60 to the present.

I attempt to explain this episode with assistance from my model. As illustrated in the right panel of Figure 4, a small change in the precision of information regarding the fundamentals of the economy (an increase in τ_s) can cause a sharp decrease in the market power of commodity producers (increase in $\mathcal{L}$) and a price crash in the model. This is because of the complementarity between the market power of producers and informational frictions discussed above. Therefore, the resolution of uncertainty regarding the strength of the world economy (e.g., news about "China's new normal"[31]) could have sharply decreased the market power of commodity producers and caused a price crash.

The above explanation attributes the sharp decrease in price to a decrease in the market power of oil producers. Consistent with this explanation, OPEC did not cut its production following the crash[32]. If the above story is true, after the crash, information efficiency should have decreased (Proposition 2): oil prices should have become worse barometers for the global economy. This can be tested, for example, by conducting the exercise in Hu and Xiong (2013) for the periods before and after the 2014 crash.

7.2 Asset markets

I showed in the Section 5 that changes in different aspects of the market environment (such as risk-bearing capacity, information precision, entry/exit or competition between large traders) can induce changes in liquidity and information efficiency in opposite directions. The latter implies, in particular, that when liquidity improves, the price better (worse) reflects the values of large (small) traders. In

[30]See Section V.C of Sockin and Xiong (2015) for more evidence and discussion on the importance of informational frictions.

[31]E.g., "Xi Says China Must Adapt to 'New Normal' of Slower Growth", Bloomberg, May 12, 2014

[32]See, e.g., "OPEC Pumps at Three-Year High Despite Oil Glut", WSJ Aug. 11, 2015

reality, one does not observe the values of the large traders. However, there are two settings in which the two can be proxied.

On-the-run treasury market and short-sellers In the treasury bonds market, the difference in the prices of the on-the-run bonds and off-the-run bonds is attributed to "specialness", the quality of the on-the-run bond of being better collateral. The difference between the price of the on-the-run and off-the-run bond can therefore be attributed to the values of the short sellers, who are buying the on-the-run bonds to use as collateral. Regard the short sellers as the I-traders; the model then implies that when the liquidity of the on-the-run market improves, the price of the on-the-run bond should increase. This is because in a more liquid market, the short sellers will trade more aggressively, driving up the price[33]. Because the off-the-run bonds are unaffected by the short sellers, the spread should increase. This is consistent with the evidence in Krishnamurthy (2002) and Banerjee and Graveline (2013)[34].

Equity markets and institutional investors Consider equity markets. Interpret I-traders as large institutional investors and J-traders as retail investors. Tension between liquidity and information efficiency implies that in a more liquid market, the price better reflects the value of institutional investors. While the latter is unobservable, arguably[35], it should be correlated with a benchmark relative to which the institutions are evaluated. Consequently, a stock traded by institutions should become more correlated with a benchmark when the liquidity of the stock improves. This prediction is testable.

Indirect evidence supporting the above prediction is provided by Chan et al. (2013), who show that stocks' co-movements with one another are positively related to market liquidity. The latter can be explained as follows. When there is more liquidity, the price of each stock better reflects the values of institutions and is thus more correlated with the benchmark against which the institutions are evaluated. The stocks' correlations with one another therefore also increase.

7.3 Policy

A recent policy debate centers on the effects of high-frequency traders (HFTs) on asset markets and commodity index traders (CITs) on commodities futures markets. It is often argued that the presence of those traders is beneficial because it improves market liquidity and, by incorporating the information those traders have into prices, information efficiency. The results in Section 5 confirm the beneficial effects of those groups of traders on liquidity (Proposition 3, interpret CITs and HFTs as I-traders[36]).

[33] More formally, this is a consequence of Lemma 10 in the Appendix.

[34] Vayanos and Weill (2008) describe similar implications in a search-based model. They show that if there are more short sellers in the market, both the spread and liquidity increase. The intuition in their model is as follows: if there are more short sellers in the on-the-run bond market, the price is more reflective of their values and is thus higher. Moreover, entry of short sellers also increases liquidity by relaxing search frictions. The main difference is that in my paper, the higher price is not necessarily a consequence of the entry of short sellers but rather more aggressive trading by existing ones due to higher liquidity.

[35] See, e.g., Basak and Pavlova (2012), who derive that the marginal utility of the fund manager should be increasing in the level of the benchmark in a moral hazard framework.

[36] Indeed, in asset markets, HFTs dominate trading at high frequencies. They also use quantitative strategies to account for their price impact. In this context, regard J-traders as retail investors.

In commodities futures markets, CITs and large hedge funds have large positions in commodities futures (Cheng et al. (2014)). They also have price impact, as shown in Cheng et al. (2014).

However, because those groups of traders may have different values from those of other traders[37] the beneficial effect on liquidity may feed back into an adverse effect on information efficiency: with more liquidity, the price will better reflect the values of HFTs and CITs as they trade more aggressively.

It is often argued that greater competition is beneficial for welfare. Moreover, this is a basis of antitrust policy around the world[38]. As shown in Propositions 3 and 7, promoting competition is beneficial for market liquidity but may be harmful for information efficiency. Consequently, the standard intuition holds when the price provides little incremental information to economic agents (Proposition 8). However, when the price is a valuable source of information, competition may harm price discovery (as the price will better reflect the values of large traders) and may have adverse effects on welfare. Moreover, all traders, large and small, could be worse off. Below, I discuss the economic environments in which the latter effects might be important.

In the context of product markets, consumers may have little information on the quality of new products. The price may be an important signal of quality. Consequently, consumers can benefit from patent protection, which restricts competition among producers (allowing them to recover the costs of designing a new product): with less competition, the price of a new product better reflects its quality.

In the context of financial markets, an important dimension of competition comes from the market structure: there are increasingly more trading venues that allow investors to execute their trades. I show that breaking up a centralized market into two separate exchanges can improve welfare because each of the two exchanges will be more informationally efficient. This result complements the results of Malamud and Rostek (2015), who show that breaking up the exchange can be beneficial because of the effects on liquidity.

8 Conclusion

This paper shows that traders' heterogeneity in their price impacts and values - a notable feature of many contemporaneous markets - might have unexpected consequences for liquidity, information efficiency and welfare.

I show that the heterogeneity results in a complementarity between illiquidity and information efficiency. A belief that the market is less liquid induces large traders to trade less aggressively. It makes the price relatively more sensitive to the values of small traders. From the latter's perspective, the price is more informative, and they provide less liquidity. Consequently, the market is less liquid. The complementarity has the following consequences.

First, tension might arise between liquidity and information: policy measures intended to promote liquidity may harm information efficiency and vice versa. Related to the latter, changes in the market environment (such as risk-bearing capacity, the number of large traders, information precision) can shift liquidity and information efficiency in opposite directions.

[37] HFTs may have a short-term horizon and care about the price in the near future. Retail investors may have a longer horizon and consequently care about the price in the more distant future.

CITs may have specific hedging needs arising from their positions in other markets. Evidence for this is provided in Cheng et al.

[38] See, e.g., US Federal Trade Commission introduction to antitrust laws https://www.ftc.gov/tips-advice/competition-guidance/guide-antitrust-laws.

Second, I show that an increase in risk-bearing capacity may have a negative effect on liquidity (*liquidity paradox*). This is possible because despite its direct negative effect on liquidity, it has a positive effect on information efficiency, and there is a tension between the two. Similarly, an increase in the precision of information might have a negative effect on information efficiency (*information aggregation paradox*).

Third, competition is not necessarily beneficial for welfare. This is possible because competition has negative effects on information efficiency. For a similar reason, breaking up a centralized market into two separate exchanges might improve welfare.

The evidence obtained from commodities and asset markets is broadly consistent with the predictions of the model.

The model can be extended in multiple directions. Allowing for multiple assets would allow for a treatment of asset-class effects in the presence of institutional investors and interactions with market liquidity and information efficiency. Considering a dynamic extension is also interesting. Consider a large trader, who can trade in several periods. If he trades more aggressively in the first period, the price will be less informative for small traders, and they will provide more liquidity. Trading more aggressively will then be less costly for a large trader. Consequently, in the presence of the complementarity highlighted in this paper, large investors may trade faster. These extensions are left for future work.

A Stability and Amplification

One of the notions of stability in game theory is associated with a stability of a fixed point of the best-response mapping determining the equilibrium[39]. This notion is commonly adopted in REE models[40].

The idea behind this notion is the following. Suppose that agents make a small deviation from the profile of the equilibrium strategies S_0. Denote the perturbed strategy profile by S_1. Let agents play S_2, which is a best response to S_1. Let them play a best response (denote it S_3) to the strategy profile S_2, and so on. If, as a result of such tâtonnement process, the strategies will converge back to the initial equilibrium (i.e. $S_n \to S_0$ as $n \to \infty$) the equilibrium is called stable.

Such analysis is tractable in symmetric models, because the best response typically maps a scalar (the equilibrium strategy of other agents) onto a scalar (the best reply of a particular agent).

In this model, however, there are two groups of traders, and the best response depends on a vector of strategies. For example the best response sensitivity β_J depends on β and γ chosen by I-traders and β_J and γ_J chosen by the other J-traders. In other words the best response maps $\mathbb{R}^4$ onto $\mathbb{R}^4$, which complicates the stability analysis of its' fixed point.

To overcome this difficulty, I represent the equilibrium as being a fixed point of a mapping characterizing the market as a whole. The characteristic I focus on is illiquidity, i.e. price impact[41].

[39]See e.g. Fudenberg and Tirole (1994) ch. 1.2.5 in a context of Cournot duopoly.

[40]E.g. Cespa and Vives (2015).

[41]The idea of equilibrium in a model with market power being a fixed point of a mapping of price impact to itself is due to Weretka (2011). The representation of the supply function equilibrium as a fixed point of a mapping of price impact to itself is due to Rostek and Weretka (2015).

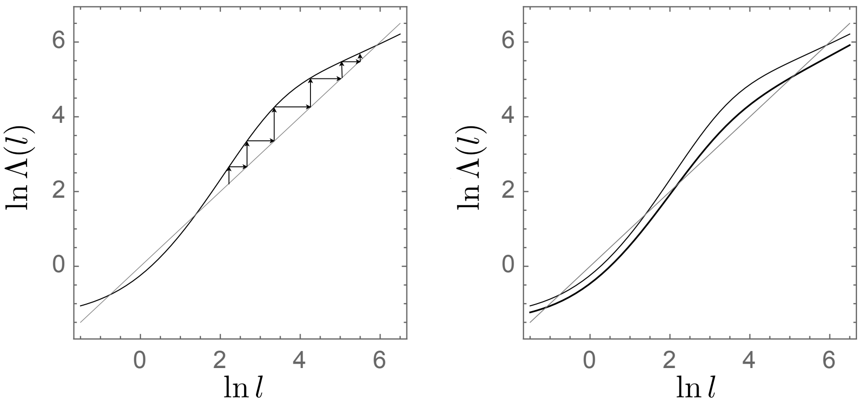

Figure 11: Stability analysis. The left panel illustrates the concept of stability. The middle equilibrium is unstable: a small deviation from it leads to further deviation, until the system converges to the top equilibrium, which is stable. The right panel shows that stable and unstable equilibria provide opposite comparative statics results: an increase in the variance of I-traders' value decreases the price impact in the stable equilibria and increases it in the unstable one. The parameter values are $\rho = 0.99$, $\tau_J = 1$, $\tau_s = 1$, $N = 10$, $w_J = 1$, $w_I = 1$, $1/\tau_I = 0.006\,(0.01)$ (thin (thick) line).

To understand how the price impact mapping onto itself is determined, consider the following logic. Suppose that traders believe that the price impact is l. Given this belief they optimally choose their strategies. Those strategies determine the "true" price impact, i.e. the slope of the inverse residual supply $\Lambda(l)$. In equilibrium, the assumed price impact should be equal to the "true" one. In other words, the fixed point condition $l = \Lambda(l)$ should hold.

I will call the equilibrium stable if the fixed point $l = \Lambda(l)$ is stable. The $\Lambda(l)$ maps $\mathbb{R}$ onto $\mathbb{R}$, so the stability of fixed point of this function is easier to analyze. The intuition behind such definition of stability is as follows. Suppose that the equilibrium price impact is l_0. Suppose that traders hold slightly incorrect belief l_1 about the price impact. With this belief they choose their strategies, which determines the slope of residual supply $l_2 = \Lambda(l_1)$. Now the traders realize that the price impact is l_2, not l_1. They choose their strategies, which determine the new price impact $l_3 = \Lambda(l_2)$. If the iteration of that process brings them back to the l_0, then the equilibrium is stable. The latter is equivalent to the stability of the fixed point $l = \Lambda(l)$.

I derive the mapping $\Lambda(l)$ and formally define stability below.

Suppose that the traders believe that the price impact is equal to l such that $2l + w_I > 0$[42]. The I-traders' strategy is then given by (16), implying

$$\beta = \gamma = g(l) \equiv \frac{1}{w_I + l}. \tag{24}$$

The strategy of each of J-traders given l is determined as best-response to the strategies (24) of the I-traders and to the strategies of other J-traders. Denote those by $\beta_J = b_J(l)$ and $\gamma_J = g_J(l)$.

[42]The minimal price impact that can be sustained in equilibrium is $\lambda = -w_I/2$. This is because the second-order conditions would be violated otherwise. See Lemma 1 in the Appendix.

Combining (20) and (21) one can find the implicit expression for $\beta_J = b_J(l)$

$$\frac{1}{\beta_J} = w_J \left(\frac{\tau_J + \tau_s}{\tau_s} + \frac{1}{\tau_s C^2} \left(B + \frac{\beta_J}{N \cdot g(l)} \right)^2 \right). \tag{25}$$

The left-hand side of the above is strictly decreasing while the right-hand side is strictly increasing in β_J for $\beta_J > 0$, therefore there is a unique solution $\beta_J = b_J(l)$ to (25). Moreover, differentiating the above implicitly, one can see that this function is strictly decreasing in l.

Given $\beta_J = b_J(l)$ and $\beta = g(l)$, the precision of the signal π is determined by (20), implying

$$\sqrt{\tau_\pi} = t(l) \equiv \frac{1}{C} \left(B + \frac{b_J(l)}{N \cdot g(l)} \right). \tag{26}$$

Given the above one can find the elasticity γ_J from (22)

$$\gamma_J = \frac{1}{w_J} - \frac{1}{\tau_s} t(l) \left(t(l) - \frac{B}{C} \right) (\gamma_J + N g(l)). \tag{27}$$

In the above I used the fact that the aggregate elasticity $\Gamma = N\gamma + \gamma_J$ is equal to $\gamma_J + \frac{N}{w_I + l}$. Expressing γ_J from the equation (27) yields

$$\gamma_J = g_J(l) \equiv \frac{\tau_s - w_J N t(l) \left(t(l) - \frac{B}{C} \right) g(l)}{\tau_s w_J + w_J \cdot t(l) \left(t(l) - \frac{B}{C} \right)}. \tag{28}$$

We now know how the price elasticities $g(l)$ and $g_J(l)$ are determined. Those, in turn, determine the "true" price impact $\Lambda(l)$:

$$\Lambda(l) \equiv \frac{1}{(N-1)\, g(l) + g_J(l)}. \tag{29}$$

In equilibrium the price impact assumed by traders should be equal to the slope of the residual supply, i.e. $l = \Lambda(l)$.

The above argument is justified by the following Theorem.

Theorem 2. *The equilibrium price impact is equal to l if and only if it solves the fixed point problem $l = \Lambda(l)$ and $2l + w_I > 0$.*

We are now ready to formally define stability.

Definition 2. The equilibrium is called stable if and only if the price impact λ in that equilibrium is a stable fixed point of the function $\Lambda(\cdot)$, that is, iff the λ satisfies $|\Lambda'(\lambda)| < 1$.

The left panel of Figure 11 illustrates the above definition[43]. According to the definition the middle equilibrium on the Figure should be unstable: the function $\Lambda(l)$ crosses the 45 degree line from below, therefore its' slope is greater than one. The figure illustrates the tâtonnement process by a sequence of arrows: a small deviation from the middle equilibrium leads to further deviation, until the system

[43] The Figure applies log-log scale, because otherwise the top equilibrium is too far apart from the two bottom ones and the plot does not read well. Since in equilibrium $l = \Lambda(l)$, one may write $\frac{d \ln \Lambda(\lambda)}{d \ln l} = \frac{l}{\Lambda(l)} \Lambda'(l) = \Lambda'(l)$. Therefore the stability can be seen graphically on the Figure 11: the equilibrium is stable iff $|\ln \Lambda'(l)| < 1$.

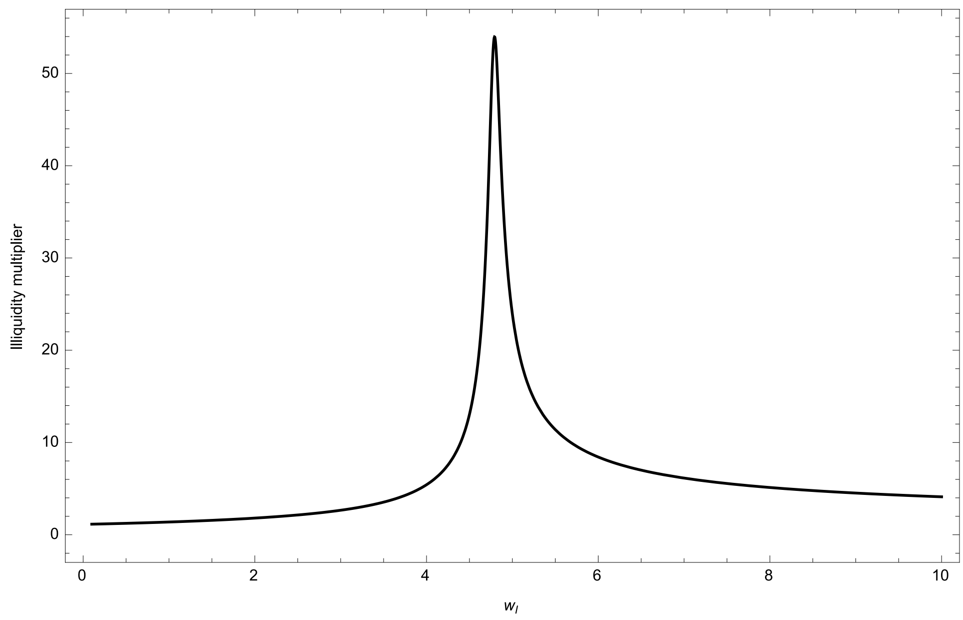

Figure 12: Illiquidity multiplier: when w_I is close to a resonant value of 4.8, the economy exhibits sharply larger amplification. The parameter values are $N = 10$, $w_J = 1$, $\rho = 0.9$, $\tau_s = 0.01$, $\tau_J = 0.1$, $\tau_I = 6$.

converges to the top equilibrium, which is stable. The right panel illustrates that the stable and unstable equilibria yield the opposite comparative statics results. An increase in the variance of I-traders' value shifts the curve $\Lambda(l)$ down (with greater variance the prices are more noisy for J-traders, so they learn less from prices and their price elasticity is higher). Two equilibrium points corresponding to the crossing of $\Lambda(l)$ and the 45 degree line from above shift down, while the other equilibrium point shifts up.

A.1 Illiquidity multiplier

Consider an equilibrium with a price impact equal to λ. Suppose that there is an unexpected shock to a parameter of the model so that the curve $\Lambda(l)$ shifts up by $d\Lambda$ at $l = \lambda$. Since the shock is unexpected, the traders will still behave as if the price impact was λ and the change of the "true" price impact is then $d\Lambda$, the direct effect of the shock.

If the shock is expected, the traders should adjust their behavior. Consider a stable equilibrium and the tâtonnement process described above. In the first step traders realize that the "true" price impact is higher by $d\Lambda$, so they will adjust their strategies as if the price impact was $\lambda + d\Lambda$. The "true" price impact corresponding to such a belief is $\Lambda(\lambda + d\Lambda) \approx \lambda + \Lambda'(\lambda)d\Lambda$. In the second step they realize that the "true" price impact is higher by $\Lambda'(\lambda)d\Lambda$. The "true" price impact corresponding to such a belief is $\Lambda(\lambda + \Lambda'(\lambda)d\Lambda) \approx \lambda + (\Lambda'(\lambda))^2 d\Lambda$ and so on. The total change in of the equilibrium price impact is the sum of changes of price impact at each step, so we get

$$\text{change of equilibrium price impact} = d\Lambda \cdot \mathcal{M},$$

where

$$\mathcal{M} \equiv 1 + \Lambda'(\lambda) + (\Lambda'(\lambda))^2 + (\Lambda'(\lambda))^3 + ... \tag{30}$$

is *illiquidity multiplier*[44].

Intuitively, the direct effect $d\Lambda$ gets amplified and the total effect of the shock to a parameter is $\mathcal{M} \cdot d\Lambda$. Therefore the illiquidity multiplier $\mathcal{M}$ characterizes quantitatively the amplification mechanism of the model.

It is well know that the geometric series (30) converge if and only if $|\Lambda'(\lambda)| < 1$, i.e. iff the equilibrium is stable. Applying the formula for the sum of the geometric series, one can get that in the stable equilibrium with price impact equal to λ the multiplier is given by

$$\mathcal{M} = \frac{1}{1 - \Lambda'(\lambda)}.$$

Repeating formally the above logic for unstable equilibria, one would get that the series (30) diverge. Therefore one can think of those equilibria as the ones with extreme amplification, so that the multiplier $\mathcal{M}$ explodes.

I summarize the above discussion by formally defining the multiplier.

Definition 3. Consider a stable equilibrium with a price impact equal to λ. The illiquidity multiplier is $\mathcal{M} \equiv \frac{1}{1-\Lambda'(\lambda)}$.

Figure 12 plots the illiquidity multiplier against w_I. The figure reminds of a resonance in physics: when w_I is close to a "resonant" value of 4.8, the economy exhibits sharply larger amplification.

A.2 Stability analysis

The stability is easily analyzed numerically. One have to evaluate $|\Lambda'(\lambda)|$ and see whether it is smaller than 1. In the Appendix B.10 I provide a closed form expression, up to a solution of a cubic equation, for the derivative $\Lambda'(\lambda)$.

The stability analysis is represented in the Figure 3. We see that all equilibria in which $\beta_J(\tau_I)$ is increasing ("upward-sloping" equilibria) are unstable. To understand why, consider an increase in τ_I and examine its' effects on the strategies of J-traders. Suppose first that I-traders' strategies are unchanged. An increase in τ_I makes the prices more informative so the J-traders learn less from their signals and more from prices. The sensitivity β_J should therefore decrease. Therefore, in order for the equilibrium β_J to increase following an increase in τ_I, the I-traders demand should become more sensitive to their value, i.e. β should increase. It implies that the equilibrium price impact λ should decrease.

From the above discussion we know that an increase in τ_I in the "upward-sloping" equilibria should lead to a decrease in λ. I will demonstrate that this is only possible if the λ corresponds to the intersection of $\Lambda(l)$ and the 45 degree line from below, which implies that such an equilibrium is unstable.

[44]This definition is analogous to the concept of illiquidity multiplier in Cespa and Foucault (2014).

Indeed, an increase in τ_I corresponds to an upward shift of the curve $\Lambda(l)$: given the same belief l about the price impact (and therefore the same strategies of I-traders) the prices become more informative, so J-traders reduce their elasticity which increases the "true" price impact $\Lambda(l)$. But if the curve $\Lambda(l)$ shifts up, its' intersection with a 45 degree line can shift down only if $\Lambda(l)$ this intersection is from below.

B Proofs

In this section I will use the following notation:

$$\theta \equiv \frac{\tau_J + \tau_s}{\tau_s} > 1,\ \xi \equiv \rho\sqrt{\frac{\tau_J}{\tau_I}},\ \varkappa \equiv \sqrt{\frac{\tau_I/\tau_s}{1-\rho^2}} > 0,$$

$$\psi \equiv \frac{w_I}{Nw_J} > 0,\ \phi \equiv \varkappa\xi = \frac{\rho}{\sqrt{1-\rho^2}}\sqrt{\frac{\tau_J}{\tau_s}}.$$

Unless stated otherwise, the proofs are not restricted to the case $\rho \geq 0$.

B.1 Proof of Theorem 1

B.1.1 Derivation of equilibrium

The proof is split into 4 steps.

Step 1. Guess.

I conjecture that I- and J-traders have the following linear schedules in equilibrium

$$x_i = \alpha + \beta v_I - \gamma p \text{ and } x_j = \alpha_J + \beta_J s_j - \gamma_J p. \tag{31}$$

Step 2. Residual supply and the equilibrium price function.

The above guess and the market-clearing rule (4) implies that I-traders face the following inverse residual supply

$$p = \iota + \lambda \cdot x, \tag{32}$$

where the stochastic intercept ι has full support[45] and the slope is given by

$$\frac{1}{\lambda} = (N-1)\gamma + \gamma_J. \tag{33}$$

The J-traders are atomistic, so their inverse residual supply does not depend on the quantity they trade. Substituting (31) to (4) we find that it is given by

$$p = \frac{1}{\Gamma}\left(N\beta v_I + \beta_J v_J\right) + c_p, \tag{34}$$

[45] The exact expression is $\iota = \lambda\left((N-1)\alpha + \alpha_J + (N-1)\beta v_I + \beta_J v_J\right)$.

where

$$\Gamma = N\gamma + \gamma_J, \text{ and } c_p = \frac{N\alpha + \alpha_J}{\Gamma}. \tag{35}$$

Equation (34) is also the equilibrium price function.

Step 3. Verify.

I-traders. The approach to solving I-traders problem relies on the idea of maximizing against the residual supply (Kyle (1989), Klemperer and Meyer (1989)). For a given realization of ι we find an optimal price-quantity pair $(x^*(\iota), p^*(\iota))$ on the residual supply curve (32) that maximizes the utility of a trader. Trader's optimal schedule is given parametrically by $(x^*(\iota), p^*(\iota))$ as a function of ι. The problem of finding an optimal price-quantity pair on the residual supply curve can be written as

$$\max_{(x,p)} (v_I - p)\, x - \frac{w_I}{2} x^2 \tag{36}$$

$$\text{s.t.: } p = \iota + \lambda \cdot x. \tag{37}$$

Taking the first order condition and eliminating ι using (37) yields the following expression for the optimal schedule of a trader i

$$x_i = \frac{1}{w_I + \lambda}(v_I - p). \tag{38}$$

The second order conditions require

$$w_I + 2\lambda > 0. \tag{39}$$

In what follows I assume that the second-order condition holds, which is justified by the Lemma 1. Comparing (38) and (31) yields

$$\beta = \gamma = \frac{1}{w_I + \lambda} > 0 \text{ and } \alpha = 0. \tag{40}$$

The guess for I-traders is therefore verified.

J-traders, being atomistic, have no price impact. For a given realisation of price they find the optimal quantity x, solving

$$\max_x \left(E[v_J | s_J, p] - p\right) x - \frac{w_J x^2}{2}.$$

The first order necessary and sufficient condition implies

$$x_j = \frac{1}{w_J}\left(E\left[v_J | s_j, p\right] - p\right). \tag{41}$$

To solve the inference problem of a J-trader I apply Lemma 2 according to which the price is informationally equivalent to a signal

$$\pi \equiv \frac{\Gamma p}{\beta_J + N\beta\xi} + c_\pi, \tag{42}$$

Where the expression for the constant c_π is given by (60). This signal can also be written as

$$\pi = v_J + \frac{1}{\sqrt{\tau_\pi}}\epsilon_\pi, \tag{43}$$

where $\epsilon_\pi \sim N(0,1)$ is independent of v_J and the noize ϵ_j in the signal s_j. Therefore, the signal π is an unbiased signal of v_J. According to Lemma 2 the precision of the signal is given by

$$\tau_\pi = \varkappa^2 \tau_s \left(\frac{\beta_J}{N\beta} + \xi\right)^2. \tag{44}$$

Applying the Projection Theorem one gets

$$E\left[v_J | s_j, p\right] = \frac{\tau_J}{\tau}\bar{v}_J + \frac{\tau_s}{\tau}s_j + \frac{\tau_\pi}{\tau}\pi, \tag{45}$$

where

$$\tau \equiv \tau_J + \tau_s + \tau_\pi.$$

Substituting (45) and (42) to (41) and comparing to (31) one gets

$$\beta_J = \frac{1}{w_J}\frac{\tau_s}{\tau}, \tag{46}$$

$$\gamma_J = \frac{1}{w_J}\left(1 - \frac{\tau_\pi}{\tau}\frac{\Gamma}{\beta_J + N\beta\xi}\right), \tag{47}$$

$$\alpha_J = \frac{1}{w_J}\left(\frac{\tau_J}{\tau}\bar{v}_J + \frac{\tau_\pi}{\tau}c_\pi\right),$$

which verifies the guess for J-traders.

Step 4. Solve for coefficients.

To solve the model I introduce the quantity

$$\delta \equiv \varkappa\left(\frac{\beta_J}{N\beta} + \xi\right) > \phi, \tag{48}$$

and express all coefficients through it. The inequality is true because both β and β_J are positive (cf. (46) and (40)). From (44) we get

$$\frac{\tau_\pi}{\tau_s} = \delta^2,$$

which allows to rewrite (46) as

$$\beta_J = \left(w_J\left(\theta + \delta^2\right)\right)^{-1}. \tag{49}$$

One can rewrite (48) as

$$\frac{1}{\beta} = w_I + \lambda = \frac{N}{\beta_J}\left(\frac{\delta}{\varkappa} - \xi\right)$$

Substituting (49) into which yields

$$\lambda = \frac{N w_J}{\varkappa}\left(\delta - \phi\right)\left(\theta + \delta^2\right) - w_I. \tag{50}$$

One can express all coefficients of I-traders through δ by substituting the above expression for λ to (40).

After some algebra γ_J can be expressed as

$$\gamma_J = \frac{1}{w_J} - \delta\left(\delta - \phi\right)\Gamma. \tag{51}$$

Combining the above and $\Gamma = \gamma_J + \frac{N}{w_I+\lambda}$ one gets

$$\Gamma = \frac{\frac{1}{w_J} + \frac{N}{w_I+\lambda}}{1+\delta\left(\delta-\phi\right)}. \tag{52}$$

After some algebra, for α_J one can get the following expression:

$$\alpha_J = \frac{\beta\beta_J N(\tau_J + \delta\varkappa\tau_s(\xi\overline{v}_J - \overline{v}_I))}{\tau_s(\beta_J\delta\varkappa + \beta N)}.$$

Finally, we get the expression to pin down δ. It can be done substituting (52) to $\Gamma = \frac{1}{\lambda} + \frac{1}{w_I+\lambda}$ which yields, after some algebra

$$\lambda\left(w_I + Nw_J + \lambda\right) - w_J\left(1 + \delta\left(\delta - \phi\right)\right)\left(w_I + 2\lambda\right) = 0. \tag{53}$$

Substituting (50) to the above yields the following sextic equation in δ:

$$\begin{aligned} N\left(\left(\delta^2+\theta\right)\left(\delta-\phi\right)+\varkappa\right)\left(Nw_J\left(\delta^2+\theta\right)\left(\delta-\phi\right)-\varkappa w_I\right) &- \\ -\kappa\left(\delta\left(\delta-\phi\right)+1\right)\left(2Nw_J\left(\delta^2+\theta\right)\left(\delta-\phi\right)-\varkappa w_I\right) &= 0. \end{aligned} \tag{54}$$

The analysis of the number of equilibria is performed in the section B.1.2.

I summarize the results for BNE. Given δ that solves (54) and satisfies $\frac{Nw_J}{\varkappa}\left(\delta-\phi\right)\left(\theta+\delta^2\right) > \frac{w_I}{2}$, the coefficients in BNE are given by

$$\beta = \gamma = \frac{1}{w_I+\lambda} > 0,\ \alpha = 0,\ \text{where} \tag{55}$$

$$\lambda = \frac{Nw_J}{\varkappa}\left(\delta-\phi\right)\left(\theta+\delta^2\right) - w_I. \tag{56}$$

$$\beta_J = \left(w_J\left(\theta+\delta^2\right)\right)^{-1} > 0,\ \gamma_J = \frac{1}{w_J} - \delta\left(\delta-\phi\right)\Gamma, \tag{57}$$

$$\alpha_J = \frac{\beta\beta_J N(\overline{v}_J(\delta\phi\tau_s + \tau_J) - \delta\varkappa\tau_s\overline{v}_I)}{\tau_s(\beta_J\delta\varkappa + \beta N)}, \tag{58}$$

where

$$\Gamma = \frac{\frac{1}{w_J} + \frac{N}{w_I+\lambda}}{1+\delta\left(\delta-\phi\right)} > 0. \tag{59}$$

Lemma 1. *There is no equilibrium in which* $2\lambda + w_I \leq 0$.

Proof. Suppose $w_I + 2\lambda < 0$. In that case for any realisation of ι the profit maximizing quantity in the problem (36) is infinite and the market will not clear. Therefore such an equilibrium does not exist.

In the case $w_I + 2\lambda = 0$, the problem is (38) is linear and the demand of I-itraders is only finite when $p = v_I$ which is the case only when at least one of the traders submits perfectly price elastic schedule. In the latter case the price impact of other I-traders is $\lambda = 0$, not $\lambda = -\frac{w_I}{2}$, a contradiction. $\square$

Lemma 2. *In a linear equilibrium characterised by the schedules (5) the price function is informationally equivalent to the sufficient statistic*

$$\pi \equiv \frac{\Gamma p}{\beta_J + N\beta\xi} + c_\pi,$$

where

$$c_\pi \equiv \frac{\xi \bar{v}_J - \bar{v}_I - \Gamma \frac{c_p}{N\beta}}{\frac{\beta_J}{N\beta} + \xi}. \tag{60}$$

The sufficient statistic π *can be written as*

$$\pi = v_J + \frac{1}{\varkappa\sqrt{\tau_s}\left(\frac{\beta_J}{N\beta} + \xi\right)}\epsilon_\pi, \tag{61}$$

where

$$\epsilon_\pi \equiv \varkappa\sqrt{\tau_s}\left(v_I - \bar{v}_I - \xi\left(v_J - \bar{v}_J\right)\right).$$

Moreover $\epsilon_\pi \sim N(0,1)$ *and, for any* j, ϵ_π *is independent of* v_J *and the noize* ϵ_j *in the signal* s_j. *The precision of* π *is given by*

$$\tau_\pi \equiv Var[\pi | v_J]^{-1} = \varkappa^2 \tau_s \left(\frac{\beta_J}{N\beta} + \xi\right)^2.$$

Proof. The π is a linear transformation of, and hence is informationally equivalent to, the price p.

Given the price function (34), it can be checked by a direct calculation that (61) holds.

It is clear that ϵ_π is distributed noirmally with mean zero. The variance can be computed as

$$\begin{aligned}
\mathrm{Var}[\epsilon_\pi] &= \varkappa^2 \tau_s \mathrm{Var}[v_I - \bar{v}_I - \xi\left(v_J - \bar{v}_J\right)] \\
&= \frac{\tau_I}{1-\rho^2}\left(\frac{1}{\tau_I} + \xi^2 \frac{1}{\tau_J} - 2\xi \frac{\rho}{\sqrt{\tau_I \tau_J}}\right) \\
&= \frac{\tau_I}{1-\rho^2}\left(\frac{1}{\tau_I} + \rho^2 \frac{\tau_J}{\tau_I}\frac{1}{\tau_J} - 2\rho\sqrt{\frac{\tau_J}{\tau_I}}\frac{\rho}{\sqrt{\tau_I \tau_J}}\right) \\
&= 1.
\end{aligned}$$

The ϵ_π is independent of ϵ_j, because ϵ_π is a linear combination of v_I and v_J and the two are independent of ϵ_j. To see that ϵ_π is independent of v_J compute

$$\begin{aligned}\frac{\operatorname{cov}(\epsilon_\pi, v_J)}{\varkappa\sqrt{\tau_s}} &= \operatorname{cov}(v_I - \bar{v}_I - \xi\left(v_J - \bar{v}_J\right), v_J) \\ &= \operatorname{cov}(v_I, v_J) - \rho\sqrt{\frac{\tau_J}{\tau_I}}\frac{1}{\tau_J} \\ &= 0,\end{aligned}$$

which, given joint normality of v_I and v_J (and hence ϵ_π and v_J) implies independence.

The formula for precision follows immediately from (61). □

B.1.2 Existence and sufficient conditions for uniqueness and multiplicity of equilibria

Existence

Lemma 3. *If $\phi > -\sqrt{3\theta}$ there exists at least one BNE. In particular, if $\rho \geq 0$ there always exists at least one BNE.*

Proof. According to the Theorem 1 the BNE exists if and only if there is a solution δ to the sextic equation (54) that satisfies

$$\frac{Nw_J}{\varkappa}\left(\delta - \phi\right)\left(\theta + \delta^2\right) > \frac{w_I}{2}. \tag{62}$$

Denote by $\check{\delta}$ the solution to

$$\frac{Nw_J}{\varkappa}\left(\check{\delta} - \phi\right)\left(\theta + \check{\delta}^2\right) = \frac{w_I}{2}. \tag{63}$$

Such a solution is unique and the left hand side of the above is increasing in δ for $\delta > \hat{\delta}$[46].

After substituting $\delta = \check{\delta}$ to (54) it becomes

$$-N\left(\left(\check{\delta}^2 + \theta\right)\left(\check{\delta} - \phi\right) + \varkappa\right)\frac{\varkappa w_I}{2} < 0, \tag{64}$$

i.e. the polynomial is negative at $\delta = \check{\delta}$. On the other hand, the leading coefficient of the polynomial (54) is $N^2 w_J > 0$, therefore it becomes positive for δ large enough. By the Intermediate Value Theorem there should be a solution $\delta^* > \check{\delta}$ to (54). Since for $\delta > \check{\delta}$ and the function $\frac{Nw_J}{\varkappa}\left(\delta - \phi\right)\left(\theta + \delta^2\right)$ is strictly increasing in δ, the second order condition (62) holds for $\delta = \delta^*$. □

Sufficient conditions for the uniqueness

Lemma 4. *The BNE is unique if $\xi > \underline{\xi}$, where $\underline{\xi} < 1$ is given by (74). In the case $\rho \geq 0$ the sufficient condition can be written as $\tau_I < \overline{\tau}_1$, where $\overline{\tau}_1$ is given by (75).*

[46] The solution exists, because LHS is less than RHS at $\delta = \phi$ and is greater than RHS for δ large enough. Since for $\delta < \phi$ the LHS is negative, there are no solutions in that region and we may consider only $\delta > \phi$. If $\phi > 0$, then the LHS is strictly increasing for $\delta > \phi$. If $-\sqrt{3\theta} < \phi < 0$ then the LHS is stricly increasing for all δ. In any case there is at most one solution and LHS is increasing in δ for $\delta > \hat{\delta}$.

Proof. The BNE corresponds to a solution of a system of equations (50) and (53) satisfying the second order condition $\lambda > -w_I/2$. After the change of variables

$$l \equiv \frac{2\lambda + w_I}{2Nw_J} > 0, \tag{65}$$

the system becomes

$$l = l(\delta) \equiv \frac{\left(\delta^2 + \theta\right)(\delta - \phi)}{\varkappa} - \frac{\psi}{2} \tag{66}$$

$$l(Nl + N - 2\left(1 + \delta(\delta - \phi)\right)) = N\left(\left(\frac{\psi}{2}\right)^2 + \frac{\psi}{2}\right). \tag{67}$$

We are now looking for the solutions to the above system satisfying $l > 0$.

Denote

$$y \equiv \delta(\delta - \phi).$$

From (67) one can express y through l as follows

$$y = \frac{4l^2n + 4l(N-2) - N\psi(\psi + 2)}{8l}. \tag{68}$$

The equation (66) can be written as

$$l = \frac{\delta y + \theta(\delta - \phi)}{\varkappa} - \frac{\psi}{2},$$

which allows to get an expression of δ through y and l:

$$\delta = \frac{2\theta\phi + \varkappa\psi + 2\varkappa l}{2(\theta + y)}.$$

Substituting y from (68) the above becomes after some algebra

$$\delta = \delta(l) \equiv \frac{2\varkappa}{N}\frac{l\left(l + \theta\xi + \frac{\psi}{2}\right)}{(l - l^+)(l - l^-)}, \tag{69}$$

where

$$l^{\pm} \equiv \frac{-G \pm \sqrt{G^2 + F}}{2},$$

$$G \equiv 1 + \frac{2(\theta - 2)}{N} > 1,$$

$$F \equiv 2\psi + \psi^2 > 0.$$

One can show that

$$0 < l^+ < \frac{\psi}{2},\ l^- < 0.$$

The idea for proceeding further is the following. Equation (66) gives an explicit expression for the function $l(\delta)$. Equation (69) expresses explicitly the function $\delta(l)$. The sufficient conditions for uniqueness can be obtained by analyzing how many times the two functions intersect. In what follows

I consider the behavior of the two curves on the coordinate plane in which l is a vertical and δ is a horiozontal axis.

I make the following additional assumption

$$\xi > -\min\left(\frac{\psi}{2\theta}, \frac{\sqrt{3\theta}}{\varkappa}\right). \tag{70}$$

Note that when $\rho \geq 0$ the above condition always holds.

Condition (70) implies $\xi > -\frac{\psi}{2\theta}$, which implies that $\delta(l)$ is positive for $l > l^+$. Condition $\xi > -\frac{\sqrt{3\theta}}{\varkappa}$ together with $\xi > -\frac{\psi}{2\theta}$ implies that in the region $l > 0$ the curve $l(\delta)$ lies to the right of the vertical line $\delta = 0$[47]. The two curves can therefore intersect only in the region $l > l^+$ and $\delta > 0$. Since $l(\delta) < 0 < l^+$ for $\delta < \phi$ we may actually restrict our attantion to the region $l > l^+$ and $\delta > \max(\phi, 0)$.

It is easy to show that for $\delta > \max(\phi, 0)$ the function $l(\delta)$ is strictly increasing.

Lemma 5 implies that for l such that $\delta(l) > \frac{2\varkappa}{N}$ the function $\delta(l)$ is strictly decreasing. Suppose that

$$\phi > \frac{2\varkappa}{N}. \tag{71}$$

The intersection of the two curves can only occur in the region $\delta > \phi > \frac{2\varkappa}{N} > 0$ and $l > l^+$. In this region $\delta(l)$ is strictly decreasing, whereas $l(\delta)$ is strictly increasing so they intersect in at most one point. Condition (71) is equivalent to

$$\xi > \frac{2}{N}. \tag{72}$$

We also know from the Lemma 5 that if $\xi > \frac{4(\theta-1)+N(2-\psi)}{2\theta N}$, then the function $\delta(l)$ is strictly decreasing in l for all $l > l^+$. Therefore

$$\xi > \frac{4(\theta-1)+N(2-\psi)}{2\theta N} \tag{73}$$

is also a sufficient condition for unqueness. Combining (70), (72) and (73) one gets

$$\xi > \underline{\xi} = \max\left(-\min\left(\frac{\psi}{2\theta}, \frac{\sqrt{3\theta}}{\varkappa}\right), \min\left(\frac{2}{N}, \frac{4(\theta-1)+N(2-\psi)}{2\theta N}\right)\right), \tag{74}$$

moreover, since $N > 1$ we have

$$\underline{\xi} \leq 1.$$

In the case $\rho \geq 0$ the condition (74) can be written as

$$\tau_I < \overline{\tau}_1 \equiv \left(\frac{\rho\sqrt{\tau_J}}{\min\left(\frac{2}{N}, \frac{4(\theta-1)+N(2-\psi)}{2\theta N}\right)}\right)^2. \tag{75}$$

[47] Note that for $l(\delta)$ can not be positive if $\delta \leq \phi$. If $\xi \geq 0$ then the condition $\delta > \phi = \kappa\xi > 0$ ensures that the statement is true. In the case $\xi < 0$ the condition $\xi > -\frac{\sqrt{3\theta}}{\varkappa}$ implies that the function $l(\delta)$ is strictly increasing. Consider $l(0)$. One can compute $l(0) = -\theta\xi - \frac{\psi}{2}$, which is negative since (70) implies that $\xi > -\frac{\psi}{2\theta}$. Therefore the function $l(\delta)$ can only be positive for strictly positive δ.

□

Lemma 5. *Suppose that (70) holds. Then: 1) in the region $l > l^+$ the function $\delta(l)$ is strictly decreasing in l for l such that $\delta(l) > \frac{2\varkappa}{N}$; 2) the function $\delta(l)$ is strictly decreasing in l for all $l > l^+$, provided that*

$$\xi > \frac{4(\theta-1)+N(2-\psi)}{2\theta N}. \tag{76}$$

Proof. One can find that for $l > l^+$ the condition $\delta(l) > \frac{2\varkappa}{N}$ is equivalent to

$$l\left(l^+ + l^- + \theta\xi + \psi/2\right) > l^+ l^-. \tag{77}$$

Computing the derivative of $\delta(l)$ one gets

$$\begin{aligned}\delta'(l) &= \frac{\varkappa\left(2l\left(l^- l^+ - l(\theta\xi + l^- + l^+ + \psi/2)\right) + l^- l^+(2\theta\xi + \psi + 2l)\right)}{N(l-l^-)^2(l-l^+)^2} \\ &\equiv \frac{n(l)}{N(l-l^-)^2(l-l^+)^2}.\end{aligned}$$

The above is negative provided that (77) holds, which proves the first claim.

The sign of $\delta'(l)$ is the same as the sign of its' nominator $n(l)$. Below I prove that $n(l^+) < 0$ and that $n(l)$ is decreasing provided that (76) holds, which proves the second claim. Indeed,

$$n(l^+) = \varkappa l^+(l^- - l^+)(2\left(\theta\xi + \psi/2\right) + 2l^+) < 0.$$

The derivative of $n(l)$ is given, by

$$n'(l) = \kappa(4l^- l^+ - 2l(2(\theta\xi + l^- + l^+) + \psi)),$$

which is negative if

$$2(\theta\xi + l^- + l^+) + \psi > 0.$$

The above is equivalent to $\xi > \frac{4(\theta-1)+N(2-\psi)}{2\theta N}$.

□

Sufficient conditions for multiplicity of BNE, $\rho \geq 0$

Lemma 6. *Suppose $N > 4$. There are at least three equilibria if $\underline{\tau_2} < \tau_I < \overline{\tau_2}$ and $w_I < \overline{w}$, where $\overline{\tau}_1 < \underline{\tau_2} < \overline{\tau_2}$ and the expressions for the tresholds $\left(\underline{\tau_2}, \overline{\tau_2}, \overline{w}\right)$ are given by (83-85).*

Proof. Denote

$$Q \equiv -4N\xi + 8\xi + 4\psi$$

$$T \equiv 16N^2\xi\psi\left(\xi - \frac{2}{N}\right)(\psi + 2).$$

Assume that

$$Q < 0,\ \xi < \frac{1}{N},\ Q^2 + T > 0,\ \psi < 1,\ N > 4. \tag{78}$$

Consider all solutions to

$$\delta(l) = \phi. \tag{79}$$

If the conditions (78) hold then there exist two solutions to (79) given by

$$L^{\pm} = \frac{-Q \pm \sqrt{Q^2 + T}}{8N\left(\frac{2}{N} - \xi\right)}$$

Moreover, both solutions $L^{\pm} > l^{+}$[48]. The fact that there are two solutions to (79) implies that the function $\delta(l)$ attains local minimum in the region $l > l^{+}$ and this minimum is less than ϕ.

Consider also all solutions to

$$\delta(l) = \frac{\varkappa}{N}.$$

Given that (78) holds there are two solutions to the above. Denote the maximal of them by L_m. One can calculate

$$L_m = \frac{1}{2}\left(Q_m + \sqrt{Q_m^2 + T_m}\right) > L^{+}, \text{ where}$$

$$Q_m \equiv \frac{2(\theta - 1)}{N} + 1 - \frac{2\theta\phi}{\varkappa} - \psi,$$

$$T_m \equiv -\left(\psi^2 + 2\psi\right)$$

If

$$L_m < l\left(\frac{\varkappa}{N}\right) = \frac{\left(\varkappa^2 + \theta N^2\right)(\varkappa - N\phi)}{\varkappa N^3} - \frac{\psi}{2} \equiv l_m, \tag{80}$$

then there are at least three equilibria.

The condition $Q < 0$ is equivalent to

$$\xi > \frac{\psi}{N - 2}. \tag{81}$$

The condition $Q^2 + T > 0$ holds provided that[49]

$$\xi > \frac{2\psi(N(\psi + 3) - 2)}{N\left(N(\psi + 1)^2 - 4\right) + 4} \text{ and } N\left(N(\psi + 1)^2 - 4\right) + 4 > 0. \tag{82}$$

The second part of the above holds given (78). Note that

$$\frac{2\psi(N(\psi + 3) - 2)}{N\left(N(\psi + 1)^2 - 4\right) + 4} < \frac{8\psi}{N - 4} > \frac{\psi}{N - 2}$$

[48] It is easy to see that both solutions are positive. But $\delta(L) = \phi > 0$ is positive only if $L > l^{+}$.

[49] Indeed

$$Q^2 + T = 16\xi^2\left(N\left(N(\psi + 1)^2 - 4\right) + 4\right) - 32\xi\psi(N(\psi + 3) - 2) + 16\psi^2.$$

Condition (82) ensures that the first two terms are positive.

Therefore (81) and (82) hold if the weaker condition holds:

$$\xi > \underline{\xi_1} \equiv \frac{8\psi}{N-4}.$$

The above can be written as

$$\tau_I < \frac{\rho^2 \tau_J}{\underline{\xi}_1^2} \equiv \overline{\tau}_2. \tag{83}$$

Suppose that

$$l_m - Q_m > 0.$$

Then (80) holds[50]. The above can be written as

$$\left(\frac{\varkappa^2}{N^2} - \theta\right)\left(\frac{1}{N} - \xi\right) > 1 - \frac{2}{N} - \frac{\psi}{2}.$$

Assume

$$\xi < \frac{1}{2N}.$$

Then the LHS of the above is greater than $\left(\frac{\varkappa^2}{N^2} - \theta\right)\frac{1}{2N}$ and the constraint holds provided that

$$\frac{\varkappa^2}{N^2} - \theta > 2N - 4 - N\psi,$$

which is equivalent to

$$\tau_I > (1-\rho^2)\tau_s N^2 \left(2N - 4 - N\psi + \theta\right).$$

The above holds if the stricter inequality holds:

$$\tau_I > (1-\rho^2)\tau_s N^2 \left(2N - 4 + \theta\right).$$

The constraint that $\xi < \frac{1}{2N}$ implies that

$$\tau_I > 4N^2\rho^2\tau_J.$$

The above two conditions hold provided that

$$\tau_I > \underline{\tau_2} \equiv \max\left(4N^2\rho^2\tau_J, (1-\rho^2)\tau_s N^2 \left(2N - 4 + \theta\right)\right). \tag{84}$$

It is clear that

$$\underline{\tau_2} > 4N^2\rho^2\tau_J > \overline{\tau}_1.$$

[50] Indeed (80) is equivalent to

$$Q_m^2 + T_m - (2l_m - Q_m)^2 = 2l_m(2Q_m - 2l_m) + T_m < 0,$$

which is true.

We finaly derive the conditions when $\underline{\tau_2} < \overline{\tau}_2$. One gets

$$\sqrt{\underline{\tau_2}} < \frac{\rho\sqrt{\tau_J}}{\underline{\xi}_1} = \frac{\rho\sqrt{\tau_J}}{8\psi}(N-4),$$

which is equivalent to

$$w_I < \overline{w} \equiv w_J \rho \frac{N(N-4)}{8}\sqrt{\frac{\tau_J}{\underline{\tau_2}}}. \tag{85}$$

□

B.2 Proof of Proposition 1

According to Lemmas 7-11 the equilibrium objects considered in the Proposition can be written in terms of the model parameters and

$$\delta \equiv \varkappa\left(\frac{\beta_J}{N\beta} + \xi\right) = \sqrt{\frac{\tau_\pi}{\tau_s}}. \tag{86}$$

The liquidity is decreasing in δ, whereas the information efficiency is increasing in δ. The two have the opposite rankings. It is established in the proof of Proposition 8 that the welfare is decreasing in δ if (23) holds.

B.3 Proof of Proposition 2

Throughout the proof I maintain the assumption that $\rho \geq 0$.

I first start with the case of crash(jump) understood as a switch from one equilibrium to another. According to Lemmas 7-11 the equilibrium objects considered in the Proposition can be written in terms of the model parameters and

$$\delta \equiv \varkappa\left(\frac{\beta_J}{N\beta} + \xi\right) = \sqrt{\frac{\tau_\pi}{\tau_s}}. \tag{87}$$

When there is a switch from one equilibrium to another, parameters of the model obviously do not change, and the only thing that changes is δ. The change in delta leads to a change to all equilibrium quantities. Therefore, given the monotonicity of the equilibrium objects in δ (Lemmas 7-11), once we understand how the δ changes if there is a price crash we know how the equilibrium objects change.

From the Lemma 10 it is clear that if there is a price crash, there is a crash (jump) in δ if $\bar{v}_I < \bar{v}_J$ ($\bar{v}_I > \bar{v}_J$). The statements of the proposition then follow from the monotonicity of λ, σ_p, $\mathcal{I}$ and V in δ which follow from Lemmas 7-11.

If the crash(jump) is understood as a situation in which the sensitivity of endogenous object to model parameters is infinite, the proof works as follows. Suppose the model parameter of interest is τ_s. We can write

$$\lambda = \lambda(\delta, \tau_s).$$

The sensitivity can be computed as

$$\frac{\partial \lambda}{\partial \tau_s} = \lambda_\delta(\delta, \tau_s)\frac{\partial \delta}{\partial \tau_s} + \lambda_{\tau_s}(\delta, \tau_s).$$

It can be seen by direct computation that $\lambda_\delta(\delta, \tau_s)$ and $\lambda_{\tau_s}(\delta, \tau_s)$ are both finite as long as δ is finite. In general, all equilibrium quantities are smooth functions of δ and the model parameters. The δ latter is finite, because the only case when iti is not is when $\beta = 0$, which is not possible due to second-order condition $w_I + 2\lambda > 0$. Therefore $\frac{\partial \lambda}{\partial \tau_s} = -\infty$ iff $\frac{\partial \delta}{\partial \tau_s} = -\infty$. Analogous statements can be formulated for σ_p, $\mathcal{I}$, $E[p]$ and V.

B.3.1 Expressing the endogenous objects through δ and the parameters of the model

In this section I show that the equilibrium objects considered in the Proposition 2 can be expressed through δ given by (87) and the parameters of the model.

Price impact

Lemma 7. *One can write the price impact λ as a function of δ and the parameters of the model, moreover $\frac{\partial \lambda}{\partial \delta} > 0$.*

Proof. Equation (56) implies that

$$\lambda = \frac{N w_J}{\varkappa}(\delta - \phi)\left(\theta + \delta^2\right) - w_I,$$

which is an increasing function of for $\delta > \phi$, if $\rho \geq 0$. □

Volatility

Lemma 8. *One can write the volatiltiy σ_p as a function of δ and the parameters of the model, moreover $\frac{\partial \sigma_p}{\partial \delta} > 0$.*

Proof. The volatility can be computed as follows. Using Lemma 2 one can write

$$\begin{aligned}
\mathrm{Var}(p) &= \mathrm{Var}\left(\frac{\beta_J + N\beta\xi}{\Gamma}\pi\right) \\
&= \mathrm{Var}\left(\frac{N\beta}{\Gamma}\frac{\delta}{\varkappa}\left(v_J + \frac{1}{\sqrt{\tau_s}\delta}\epsilon_\pi\right)\right) \\
&= \left(\frac{N\beta}{\Gamma}\frac{\delta}{\varkappa}\right)^2\frac{1}{\tau_J} + \left(\frac{N\beta}{\Gamma\varkappa}\right)^2\frac{1}{\tau_s}.
\end{aligned}$$

One can compute

$$\begin{aligned}\frac{N\beta}{\Gamma} &= \frac{N}{(w_I+\lambda)\left(\frac{1}{\lambda}+\frac{1}{w_I+\lambda}\right)}\\ &= \frac{N}{\frac{w_I}{\lambda}+2}.\end{aligned}$$

The above is positive and is increasing in λ and, given the monotonicity of $\lambda(\delta)$, also in δ. Hence, $\sigma_p'(\delta) > 0$. $\square$

Information efficiency

Lemma 9. *One can write the information efficiency $\mathcal{I}$ as a function of δ and the parameters of the model, moreover $\frac{\partial \mathcal{I}}{\partial \delta} > 0$.*

Proof. The information efficiency can be written as

$$\begin{aligned}\mathcal{I} &\equiv \frac{\mathrm{Var}(v_J)}{\mathrm{Var}(v_J|s_j,p)}\\ &= \frac{\tau_J+\tau_s+\tau_\pi}{\tau_J}\\ &= \frac{\tau_J+\tau_s(1+\delta^2)}{\tau_J},\end{aligned}$$

which is increasing in δ. $\square$

Price

Lemma 10. *One can write the price $E[p]$ as a function of δ and the parameters of the model, moreover $sign\left(\frac{\partial E[p]}{\partial \delta}\right) = sign\left(\bar{v}_J - \bar{v}_I\right)$.*

Proof. Expected price sets net expected demand to zero. Expected total demand of I traders is given by

$$X = \frac{N}{w_I+\lambda}\left(\bar{v}_I - E[p]\right). \tag{88}$$

The expected demand of J-traders is given by

$$\begin{aligned}x_J &= \frac{1}{w_J}E\left[\frac{\tau_s}{\tau}v_J+\frac{\tau_\pi}{\tau}\pi+\frac{\tau_J}{\tau}\bar{v}_J - p\right]\\ &= \frac{1}{w_J}\left(\bar{v}_J - E[p]\right).\end{aligned}$$

Equalising the expected demand and supply we find

$$E[p] = \frac{Nw_J\bar{v}_I+(w_I+\lambda)\,\bar{v}_J}{w_I+\lambda+Nw_J}. \tag{89}$$

Taking the derivative of the above one can find

$$\operatorname{sign}\left(\frac{\partial E[p]}{\partial \lambda}\right) = \operatorname{sign}\left(\bar{v}_J - \bar{v}_I\right),$$

which, given the monotonicity of λ in δ proves the Lemma. $\square$

Trading volume

Lemma 11. *The trading volume is given by (97). There exists $\underline{\tau}_J$ such that if*

$$\tau_I < 1 - \rho^2, \text{ and } \tau_J > \underline{\tau}_J \tag{90}$$

the trading volume is a decreasing function of δ.

Proof. Denote

$$X = N x_I\left(p^*\right),$$

the aggregate trade of I-traders. Denote also

$$u_j \equiv E[v_J|s_j, p]dj = \frac{\tau_\pi}{\tau}\left(v_J + \epsilon_\pi\right) + \frac{\tau_s}{\tau}\left(v_J + \epsilon_j\right) + \frac{\tau_J}{\tau}\bar{v}_J$$

the ex-post value of a trader j. Denote

$$u_J \equiv \int_0^1 u_j dj = \frac{\tau_\pi}{\tau}\left(v_J + \epsilon_\pi\right) + \frac{\tau_s}{\tau} v_J + \frac{\tau_J}{\tau}\bar{v}_J$$

the aggregate ex-post value of J-traders. One can also write the above in a more convinient way

$$u_J = \frac{\tau_\pi + \tau_s}{\tau}\left(v_J - \bar{v}_J\right) + \frac{\tau_\pi}{\tau}\epsilon_\pi + \bar{v}_J.$$

The market clearing condition can be written as

$$N\frac{v_I - p^*}{w_I + \lambda} + \frac{u_J - p^*}{w_J} = 0.$$

From the above one can express the market-clearing price and the aggregate trade of I-traders

$$p^* = \frac{v_I N w_J + u_J(\lambda + w_I)}{\lambda + N w_J + w_I},$$

$$X = G\left(v_I - u_J\right) \text{ where } G \equiv \frac{N}{\lambda + N w_J + w_I}. \tag{91}$$

One can also compute

$$x_j\left(p^*\right) = -X + \beta_J \epsilon_j. \tag{92}$$

According to the Lemma 2 one can write

$$v_I = \overline{v}_I + \xi(v_J - \overline{v}_J) + \frac{1}{\varkappa\sqrt{\tau_s}}\epsilon_\pi,$$

which allows to rewrite

$$v_I - u_J = \overline{v}_I - \overline{v}_J + c_v(v_J - \overline{v}_J) + c_\epsilon \epsilon_\pi, \text{ where} \tag{93}$$

$$c_v \equiv \xi - \frac{\tau_\pi + \tau_s}{\tau}, \text{ and } c_\epsilon \equiv \frac{1}{\varkappa\sqrt{\tau_s}} - \frac{\tau_\pi}{\tau}. \tag{94}$$

Substituting $\frac{\tau_s}{\tau} = w_J\beta_J$ the above expressions become

$$c_v = \xi - 1 + \frac{\tau_J}{\tau_s} w_J\beta_J \text{ and } c_\epsilon \equiv \frac{1}{\varkappa\sqrt{\tau_s}} - 1 + \frac{\tau_J + \tau_s}{\tau_s} w_J\beta_J, \tag{95}$$

and it is clear that both c_v and c_ϵ are decreasing in δ since $\beta_J = \frac{1}{w_J(\theta+\delta^2)}$ is decreasing in δ.

I next use the well-known fact that for $Y \sim N(\mu, \sigma^2)$ the mean of $|Y|$ is given by

$$E[|Y|] = M\left(\mu, \sigma^2\right) \equiv \sigma\sqrt{\frac{2}{\pi}} \exp\left(\frac{-\mu^2}{2\sigma^2}\right) - \mu \cdot \operatorname{erf}\left(\frac{-\mu}{\sqrt{2}\sigma}\right), -1 unexpected'' in math \tag{96}$$

where $\operatorname{erf}(z) \equiv \frac{2}{\sqrt{\pi}} \int_0^z e^{-t^2}\, \mathrm{d}t$ is the error function. The trading volume then can be written as

$$V = \frac{1}{2}\left(M\left(\mu_X, \sigma_X\right) + M\left(\mu_{x_j}, \sigma^2_{x_j}\right)\right),$$

where

$$\mu_X \equiv E[X] = G\left(\overline{v}_I - \overline{v}_J\right),$$

$$\mu_{x_j} \equiv E\left[x_j\left(p^*\right)\right] = -\mu_X,$$

$$\sigma^2_X \equiv \operatorname{Var}[X] = G^2\left(c_v^2/\tau_J + c_\epsilon^2\right),$$

$$\sigma^2_{x_j} \equiv \operatorname{Var}[x_j\left(p^*\right)] = \sigma^2_X + \beta_J^2/\tau_s.$$

Applying the Lemma 12 one can write

$$V = \frac{1}{2}\left(M\left(|\mu_X|, \sigma^2_X\right) + M\left(|\mu_X|, \sigma^2_X + \beta_J^2/\tau_s\right)\right). \tag{97}$$

If c_v and c_ϵ are positive, then $|\mu_X|$, σ^2_X and $\sigma^2_X + \beta_J^2/\tau_s$ are all decreasing in δ. Lemma 12 then implies that the trading volume is a decreasing function of δ.

The sufficient condition for c_v and c_ϵ to be positive can be found from (94):

$$\xi - \frac{\tau_\pi + \tau_s}{\tau_J + \tau_\pi + \tau_s} > 0 \text{ and } \frac{1}{\varkappa\sqrt{\tau_s}} - \frac{\tau_\pi}{\tau} > 0 \tag{98}$$

The inequalities hold if

$$\xi > \frac{\tau_s}{\tau_J}\left(1 + \overline{\delta}^2\right) \text{ and} \tag{99}$$

$$\frac{1}{\varkappa\sqrt{\tau_s}} > 1, \tag{100}$$

where the value of $\bar{\delta}$ is given by Lemma 13.

Consider τ_J that solves

$$\inf\left\{\tau_J \geq 0 : \xi > \frac{\tau_s}{\tau_J}\left(1+\bar{\delta}^2\right)\right\}.$$

Denote the solution $\underline{\tau}_J$. The solution exists and is unique, since the left-hand side of $\xi > \frac{\tau_s}{\tau_J}\left(1+\bar{\delta}^2\right)$ is increasing in τ_J and is unbouneded, whereas the right-hand side is decreasing in τ_J. The inequality (99) holds if $\tau_J > \underline{\tau}_J$.

The inequality (100) holds if

$$\tau_I < 1-\rho^2.$$

□

Lemma 12. *The function $M\left(\mu,\sigma^2\right)$ defined by (??) is symmetric in μ, i.e. $M\left(-\mu,\sigma^2\right) = M\left(\mu,\sigma^2\right) = M\left(|\mu|,\sigma^2\right)$, is incresing in μ if $\mu \geq 0$ and is increasing in σ.*

Proof. The fact that $M\left(-\mu,\sigma^2\right) = M\left(\mu,\sigma^2\right)$ follows from the definition of $M(\cdot)$ and the symmetry of the error function $\operatorname{erf}(-z) = -\operatorname{erf}(z)$. Taking the derivatives of $M(\cdot)$ one gets

$$\frac{\partial}{\partial\mu}M\left(\mu,\sigma^2\right) = \operatorname{erf}\left(\frac{\mu}{\sqrt{2}\sigma}\right),$$

$$\frac{\partial}{\partial\sigma}M\left(\mu,\sigma^2\right) = \sqrt{\frac{2}{\pi}}e^{-\frac{\mu^2}{2\sigma^2}},$$

which proves the Lemma. □

Lemma 13. *In equilibrium $\delta < \bar{\delta}$, where $\bar{\delta}$ is given by (101); $\bar{\delta}/\sqrt{\tau_J}$ decreases in τ_J.*

Proof. From the definition of δ

$$\delta \equiv \varkappa\left(\frac{\beta_J}{N\beta}+\xi\right) = \varkappa\left(\frac{\beta_J}{N}(w_I+\lambda)+\xi\right) < \varkappa\left(\frac{1}{Nw_J\theta}(w_I+\lambda)+\xi\right) < \varkappa\left(\frac{1}{Nw_J}(w_I+\lambda)+\xi\right).$$

We next find an upper bound for λ. One can write

$$\frac{1}{\lambda} = \Gamma - \gamma < \Gamma < \frac{1}{w_J} + \frac{N}{w_I+\lambda},$$

where the last inequality follows from (59). Since $\lambda > -\frac{w_I}{2}$ we have $\frac{N}{w_I+\lambda} < \frac{2N}{w_I}$ and

$$\lambda < \left(\frac{1}{w_J}+\frac{2N}{w_I}\right)^{-1} = \frac{w_I w_J}{w_I+2Nw_J}.$$

Thus we get the following expression for $\bar{\delta}$

$$\bar{\delta} = \frac{\varkappa}{Nw_J}\left(w_I + \frac{w_I w_J}{w_I+2Nw_J}\right) + \phi. \tag{101}$$

The $\bar{\delta}/\sqrt{\tau_J}$ is given by $\frac{\varkappa}{Nw_J\sqrt{\tau_J}}\left(w_I+\frac{w_Iw_J}{w_I+2Nw_J}\right)+\frac{\rho}{\sqrt{1-\rho^2}}\frac{1}{\sqrt{\tau_s}}$. Clearly, it is decreasing in τ_J. $\square$

B.4 Proof of Proposition 3

The equilibrium is a solution to the system (50-53), which can be written as follows

$$\lambda = L(\delta;N) \equiv \frac{Nw_J}{\varkappa}(\delta-\phi)\left(\theta+\delta^2\right)-w_I, \text{ and} \tag{102}$$

$$\delta = D(\lambda;N) \equiv h\left(\frac{\lambda\left(w_I+Nw_J+\lambda\right)}{w_J\left(w_I+2\lambda\right)}\right), \tag{103}$$

where $h(x)$ is the inverse of $1+\delta(\delta-\phi)$, i.e. it solves

$$x = 1+h(x)\left(h(x)-\phi\right).$$

Lemma 14 implies that in equilibrium $\lambda>0$. This is not possible if $\delta<\phi$, therefore we may look for the nitersection of the two curves in the region $\delta>\phi$ and $\lambda>0$.

Since for $\delta>\phi$ the function $1+\delta(\delta-\phi)$ is strictly increasing, the function $h(x)$ is well-defined and is strictly increasing as well.

The equilibrium is therefore the intersection of the two curves, $\lambda=L(\delta;N)$ and $\delta=D(\lambda;N)$, moreover it is easy to see that $\frac{\partial L}{\partial\delta}>0$ and $\frac{\partial D}{\partial\lambda}>0$ for $\delta>\phi$, so both curves are strictly upward-sloping for a given N. We next compute

$$\frac{\partial L}{\partial N} = \frac{w_J\left(\delta^2+\theta\right)(\delta-\phi)}{\varkappa}-w_I'(N),$$

which is positive both if w_I does not depend on N, and if $w_I=w_1N$.

Analogously, we compute

$$\frac{\partial D}{\partial N} = h'(\cdot)\times\begin{cases}\frac{\lambda}{2\lambda+w_I} & \text{, if } w_I \text{ does not depend on } N\\ \frac{\lambda^2(w_1+2w_J)}{w_J(2\lambda+Nw_1)^2} & \text{, if } w_I=w_1N.\end{cases}$$

The above is positive.

Therefore an infinitesimal increase in N shifts the curve $L(\delta;N)$ up and the curve $D(\lambda;N)$ to the right. Their new intersection will be below and to the left from the old one[51]. Thus, we have

$$\frac{d\lambda}{dN}<0,\ \frac{d\delta}{dN}<0.$$

[51] It can be shown that the curve $\lambda=L(\delta;N)$ has to intersect the curve $\delta=D(\lambda;N)$ from below, since for $\lambda=0$ the curve $\lambda=L(\delta;N)$ is to the right of the curve $\delta=D(\lambda;N)$

Since $\mathcal{I} = \frac{\tau_J + \tau_s(1+\delta^2)}{\tau_J}$ is increasing in δ and does not directly depend on N, and $\mathcal{L}$ is inversely related to λ, we have

$$\frac{d\mathcal{I}}{dN} < 0 \text{ and } \frac{d\mathcal{L}}{dN} > 0.$$

Lemma 14. *[If $\rho \geq 0$, the equilibrium price impact is positive.*

Proof. Rewrite (53) as follows

$$\lambda = \frac{w_J \left(1 + \delta\left(\delta - \phi\right)\right)\left(w_I + 2\lambda\right)}{\left(w_I + N w_J + \lambda\right)}.$$

The $\delta > \phi$, because otherwise $\lambda < -w_I$ and the second order condition $2\lambda + w_I > 0$ does not hold. Therefore $1 + \delta\left(\delta - \phi\right) > 0$. Other terms in the above are positive due to the second order condition $w_I + 2\lambda > 0$. ☐

B.5 Proof of Proposition 4

I will use the fixed point condition $l = \Lambda(l)$ to analyze the comparative statics of price impact.

Recall that

$$1/\Lambda(l;\tau_s) \equiv (N-1)\,g(l) + g_J(l;\tau_s).$$

Denote

$$\begin{aligned} z(l;\tau_s) &\equiv \frac{1}{\tau_s} t(l;\tau_s)\left(t(l;\tau_s) - \frac{B}{C}\right) \\ &= \frac{1}{\tau_s} t(l;\tau_s)\frac{b_J(l;\tau_s)}{N g(l) C}. \end{aligned}$$

Recall that $C = \sqrt{\frac{1-\rho^2}{\tau_I}}$ does not depend on τ_s. With this notation one can write

$$1/\Lambda(l;\tau_s) = \frac{g(l) N w_J + 1}{w_J z(l;\tau_s) + w_J} - g(l). \tag{104}$$

I am interested in $\frac{\partial \Lambda}{\partial \tau_s}$. The only term which depend on τ_s directly is $z(l;\tau_s)$. Substituing (26) and differentiating implicitly (25) one can find

$$\frac{\partial}{\partial \tau_s}\left(\frac{t(l;\tau_s) b_J(l;\tau_s)}{\tau_s}\right) =$$

$$= \frac{-b_J(l;\tau_s)\left(g(l)^2 N^2 b_J(l;\tau_s)\left(B^2 + C^2(\tau_s - \tau_J)\right) + 2BNg(l) b_J(l;\tau_s)^2 + BC^2 g(l)^3 N^3 \tau_s + b_J(l;\tau_s)^3\right)}{CNg(l)\tau_s^2\left(g(l)^2 N^2\left(B^2 + C^2(\tau_J + \tau_s)\right) + 4BNg(l) b_J(\tau_s) + 3 b_J(\tau_s)^2\right)}.$$

The above is negative provided that $B^2 + C^2(\tau_s - \tau_J) > 0$, which is equivalent to

$$\tau_s > \frac{1 - 2\rho^2}{1 - \rho^2}\tau_J.$$

Therefore, provided that the above holds, the nominator (denominator) of (104) is increasing (decreasing) in τ_s, which implies that (the two are positive since $\Lambda(\lambda)$ is positive) $\frac{\partial \Lambda}{\partial \tau_s} < 0$. Therefore an increase in τ_s shifts the function $\Lambda(l)$ up, and its' new intersection with a 45 degree line will shift up as well ($\Lambda(l)$ intersects the 45 degree line from above, see Lemma 15).

Lemma 15. *In the unique equilibrium* $\Lambda'(\lambda) < 1$.

Proof. I derive the sign of $\Lambda'(\lambda)$ from the comparative statics with respect to τ_I.

We first prove that $\frac{\partial \lambda}{\partial \tau_I} > 0$ in the unique equilibrium. Indeed changes of τ_I has no effect on the curve (103) but shifts down the curve (102). Their intersection occurs at a point with greater λ.

Second, we prove that $\frac{\partial \Lambda(\lambda;\tau_I)}{\partial \tau_I} > 0$. Indeed, observe that in (104) only the term z depend on τ_I through $t(l;\tau_I)b_J(l;\tau_I)$. Moreover, it is easy to prove that $t(l;\tau_I)b_J(l;\tau_I)$ is decreasing in τ_I. Thereofre the denominator of (104) is decreasing in τ_I which implies that $\frac{\partial \Lambda}{\partial \tau_I} > 0$. Therefore an increase in τ_I shifts the function $\Lambda(l)$ up. Since $\frac{\partial \lambda}{\partial \tau_I} > 0$, the intersection of $\Lambda(l)$ and the 45 degree line should be from above, which proves the Lemma. □

B.6 Proof of Propositions 5 and 6

Make a change of variable

$$m \equiv \frac{\lambda + w_I}{N w_J} = \frac{\left(\delta^2 + \theta\right)(\delta - \phi)}{\varkappa}.$$

With this change of variable (54) becomes

$$(m+1)\frac{m-\psi}{2m-\psi} = \frac{\delta(\delta-\phi)+1}{N}.$$

The left hand side of the above is increasing in δ and is decreasing in ψ. The right hand side is increasing in δ and does not depend on ψ. An infinitisemal increase in w_I (or a decrease in w_J) shifts the LHS down and the new intersection of LHS and RHS occurs at a point with greater δ and thus greater $\mathcal{I}$.

Since m is increasing in δ we have that $\lambda + w_I$ is increasing in w_I. The $\Gamma = \frac{\frac{1}{w_J} + \frac{N}{w_I+\lambda}}{1+\delta(\delta-\phi)}$ is therefore decreasing in w_I.

B.7 Proof of Proposition 7

Given Lemma 16 scaling the measure of J traders by a factor $\mu = M/N$ is equivalent to scaling w_J by a factor $\frac{1}{\mu} = N/M$. Therefore λ and δ in the economy 1 solve (cf. (102-103))

$$\lambda = L(\delta) \equiv \frac{N w_J}{\varkappa}(\delta-\phi)\left(\theta+\delta^2\right) - w_I, \text{ and} \tag{105}$$

$$\delta = D(\lambda;\mu) \equiv h\left(\mu \frac{\lambda\left(w_I + N w_J + \lambda\right)}{w_J\left(w_I + 2\lambda\right)}\right). \tag{106}$$

The first curve is unaffected by changes in μ. Decreasing μ shifts the curve to the left and the new point of intersection has lower δ and greater λ. Therefore, the liquidity decreases, whereas the information efficiency increases.

Lemma 16. *Consider an economy with N I-traders and a measure μ of J-traders with utility $u_J = (v_J - p)\, x - \frac{\hat{w}_J x^2}{2}$. Call this economy $\hat{\mathcal{E}}$. Any equilibrium in this economy is an equilibrium in the economy with N I-traders and a unit measure of J-traders with utility $u_J = (v_J - p)\, x - \frac{w_J x^2}{2}$,where $w_J = \frac{\hat{w}_J}{\mu}$. Call this economy $\mathcal{E}$. Conversely, any equilibrium in $\mathcal{E}$ is also an equilibrium in $\hat{\mathcal{E}}$.*

Proof. Guess that the equilibrium demands in the economy $\hat{\mathcal{E}}$ are given by

$$x_i = \alpha + \beta \cdot v_I - \gamma \cdot p \text{ and } x_j = a_J + b_J \cdot s_j - g_J \cdot p.$$

Denote $\alpha_J = \mu a_J$,$\beta_J = \mu b_J$ and $\gamma_J = \mu g_J$. Following the steps of the proof of Theorem 1 one can see that α, β, γ,α_J, β_J and γ_J satisfy the same equations as the corresponding soefficients in the economy $\mathcal{E}$. □

B.8 Proof of Proposition 8

Define

$$\mathcal{U}_J^{ep} \equiv E\left[(v_J - p)\, x_j(p) - \frac{w_J x_j(p)^2}{2} \middle| s_j, p\right]$$

the ex-post utility of a J-trader. Using the notation introduced in the Proof of the Lemma 11 the above can be written as

$$\mathcal{U}_J^{ep} = (u_j - p)\, x_j(p) - \frac{w_J x_j(p)^2}{2}.$$

Compute

$$\begin{aligned} x_j(p) &= \frac{u_j - p}{w_J} \\ &= \frac{u_J - p}{w_J} + \beta_J \epsilon_j \\ &= -X + \beta_J \epsilon_j, \end{aligned}$$

where the second line substitutes $u_j = u_J + w_J \beta_J \epsilon_j$ and the third line uses the market cleraing conditon $X + \int_0^1 x_j(p) dj = 0$. The expected utility of a J-trader can now be written as

$$\begin{aligned} \mathcal{U}_J &= E\left[\mathcal{U}_J^{ep}\right] \\ &= E\left[(u_J - p + w_J \beta_J \epsilon_j)(-X + \beta_J \epsilon_j) - \frac{w_J(-X + \beta_J \epsilon_j)^2}{2}\right] \\ &= E\left[-(u_J - p)\, X - \frac{w_J X^2}{2}\right] + \frac{w_J \beta_J^2}{2\tau_s}, \end{aligned}$$

where the third line computes the expectation with respect to ϵ_j. Substituting $X = G(v_I - u_J)$ and

$$\begin{aligned} u_J - p &= \frac{Nw_J(u_J - v_I)}{Nw_J + w_I} \\ &\equiv K_J(u_J - v_I) \end{aligned}$$

the above becomes

$$\begin{aligned} \mathcal{U}_J &= E\left[GK_J\left(u_J - v_I\right)^2 - \frac{w_J G^2 \left(u_J - v_I\right)^2}{2}\right] + \frac{w_J \beta_J^2}{2\tau_s} \\ &\equiv H_J E\left[\left(u_J - v_I\right)^2\right] + \frac{w_J \beta_J^2}{2\tau_s}, \end{aligned}$$

where I have denoted

$$H_J \equiv G\left(K_J - \frac{w_J G}{2}\right). \tag{107}$$

As in the Proof of the Lemma 11 one can write

$$v_I - u_J = \overline{v}_I - \overline{v}_J + c_v(v_J - \overline{v}_J) + c_\epsilon \epsilon_\pi,$$

which allows to calculate

$$\mathcal{U}_J = H_J\left(\left(\overline{v}_I - \overline{v}_J\right)^2 + c_v^2/\tau_J + c_\epsilon^2\right) + \frac{w_J \beta_J^2}{2\tau_s}.$$

Lemma 17 implies that H_J is increasing in N. From Proposition 3 we know that δ is decreasing in N, hence β_J is increasing in N. From the proof of Lemma 11 we know that if (90) holds, c_v^2 and c_ϵ^2 are both decreasing in δ and hence increasing in N. Therefore, the if (90) holds, $\mathcal{U}_J$ is increasing in N.

The expected utility of an I-trader can be calculated as

$$\mathcal{U}_I = \frac{1}{N} E\left[\left(v_I - p\right) X - \frac{w_I X^2}{2N}\right].$$

Substituting $X = G(v_I - u_J)$ and

$$v_I - p = \left(1 - K_J\right)\left(v_I - u_J\right)$$

the expected utility of an I-trader becomes

$$\mathcal{U}_I = H_I E\left[\left(u_J - v_I\right)^2\right],$$

where

$$H_I \equiv \frac{1}{N} G\left(\left(1 - K_J\right) - \frac{w_I}{2N} G\right).$$

Computing the expectation we get

$$\mathcal{U}_I = H_I\left(\left(\overline{v}_I - \overline{v}_J\right)^2 + c_v^2/\tau_J + c_\epsilon^2\right).$$

We finally compute the welfare

$$\begin{aligned}\mathcal{W} &= N\mathcal{U}_I + \mathcal{U}_J \\ &= H\left((\overline{v}_I - \overline{v}_J)^2 + c_v^2/\tau_J + c_\epsilon^2\right) + \frac{w_J\beta_J^2}{2\tau_s}.\end{aligned}$$

Where I denoted

$$H \equiv NH_I + H_J = G - \frac{1}{2}\left(\frac{w_I}{N} + w_J\right)G^2. \tag{108}$$

From Proposition 3 we know that δ is decreasing in N, hence β_J is increasing in N. From Lemma 17 we know that H is increasing in N. Finally, from Lemma 11 we know that if (90) holds, c_v^2 and c_ϵ^2 are both decreasing in δ and hence increasing in N. Therefore, the if (90) holds, the welfare is increasing in N.

Lemma 17. *The H_J and H given by (107) and (108) is increasing in N.*

Proof. Write

$$H(G,N) = G - \frac{1}{2}\left(\frac{w_I}{N} + w_J\right)G^2.$$

Compute

$$\frac{dH}{dN} = \frac{\partial H}{\partial G}\frac{dG}{dN} + \frac{1}{2}\frac{w_I}{N^2}G^2.$$

Compute

$$\frac{\partial H}{\partial G} = 1 - \frac{\frac{w_I}{N} + w_J}{1/G} = 1 - \frac{\frac{w_I}{N} + w_J}{\frac{w_I}{N} + w_J + \frac{\lambda}{N}} > 0.$$

From

$$G = \frac{1}{\frac{w_I}{N} + w_J + \frac{\lambda}{N}}$$

it is clear that G is increasing in N (recall that according to Proposition 3 λ is decreasing in N).

Analogously,

$$\frac{dH_J}{dN} = \frac{\partial H_J}{\partial G}\underbrace{\frac{dG}{dN}}_{>0} + \frac{\partial H_J}{\partial K_J}\underbrace{\frac{dK_J}{dN}}_{>0}.$$

Compute

$$\frac{\partial H_J}{\partial G} = K_J - w_J G = \frac{\lambda N w_J}{(Nw_J + w_I)(\lambda + Nw_J + w_I)} > 0$$

and

$$\frac{\partial H_J}{\partial K_J} = G > 0.$$

Thus, $\frac{dH_J}{dN} > 0$. □

B.9 Proof of Theorem 2

The values of price impact l such that $2l + w_I > 0$ does not hold are ruled out by the Lemma 1.

For the if part, we should show that if $l = \Lambda(l)$ then there exists an equilibrium such that l is equal to the slope of the inverse residual supply. A natural candidate is an equilibrium with $\beta = g(l)$, $\gamma = g(l)$ and $\beta_J = b_J(l)$, $\gamma_J = g_J(l)$, where the function $g(l)$ is given by (24), $b_J(l)$ is given by (25) and $g_J(l)$ is given by (28).

We shall show that if $l = \Lambda(l)$ then the above strategies constitute mutual best responses. By definition $\Lambda(l) = \frac{1}{(N-1)g(l)+g_J(l)}$ therefore by the Lemma 18 $\beta = g(l)$ and $\gamma = g(l)$ are the best responses to other I-traders playing $\beta = g(l)$ and $\gamma = g(l)$ and J-traders playing $\beta_J = b_J(l)$ and $\gamma_J = g_J(l)$.

By consrtruction $\beta_J = b_J(l)$ and $\gamma_J = g_J(l)$ are the best responses to I-traders playing $\beta = g(l)$ and $\gamma = g(l)$ and J-traders playing $\beta_J = b_J(l)$ and $\gamma_J = g_J(l)$, which proves the if part.

For the only if part we shall show that if there is an equilibrium with a price impact l such that $2l + w_I > 0$, then $l = \Lambda(l)$ should hold.

From Lemma 18 we know that in this equilibrium I-traders play $\beta = g(l)$ and $\gamma = g(l)$. Since it is an equilibrium, the strategies of J-traders should constitute mutual best responses and be a best response to $\beta = g(l)$ and $\gamma = g(l)$ played by the I-traders. By definition of $b_J(l)$ and $g_J(l)$ the J-traders should play $\beta_J = b_J(l)$ and $\gamma_J = g_J(l)$

Since the price impact l is given by $\frac{1}{(N-1)\gamma+\gamma_J}$ and we have $\gamma = g(l)$ and $\gamma_J = g_J(l)$, we get that $l = \Lambda(l)$, which proves the only if part.

Lemma 18. *The strategy $x_i(p)$ of a trader $i \in I$ is a best response to the profile of symmetric linear strategies of traders $j \in J$ and $k \in I$, $k \neq i$ characterised by the coefficients $(\alpha_J, \beta_J, \gamma_J)$ and (α, β, γ) such that $\frac{2}{(N-1)\gamma+\gamma_J} + w_I > 0$ if and only if it is given by*

$$x_i(p) = \frac{v_I - p}{w_I + l}, \text{ where } l = \frac{1}{(N-1)\gamma + \gamma_J}.$$

Proof. As it is discussed in the proof of the Theorem 1 the best response of a trader i solves

$$\max_{(x,p)} (v_I - p)\, x - \frac{w_I}{2} x^2 \tag{109}$$

$$\text{s.t.: } p = \iota + l \cdot x. \tag{110}$$

Taking the first order condition and eliminating ι using (110) yields the following expression for the best response of the trader i

$$x_i = \frac{1}{w_I + l}(v_I - p). \tag{111}$$

By assumption the second order condition $w_I + 2\lambda > 0$ holds, therefore the above first order necessary condition is also sufficient. $\square$

B.10 Stability. The expression for $\Lambda'(l)$.

First recall that $B = \rho\sqrt{\frac{\tau_J}{\tau_I}}$, $C = \sqrt{\frac{1-\rho^2}{\tau_I}}$ implying that

$$B = \xi,\ C = \frac{1}{\sqrt{\tau_s \varkappa}},$$

in the notation adopted in the appendix.

We first differentiate implicitly (25) to get the expression for $b'_J(l)$:

$$b'_J(l) = -\frac{2\varkappa^2 b_J(l)^2(b_J(l)(l+w_I)+N\xi)}{\varkappa^2 b_J(l)(l+w_I)(3b_J(l)(l+w_I)+4N\xi)+N^2\left(\theta+\varkappa^2\xi^2\right)} < 0. \tag{112}$$

Differentiation of (29) yields

$$\Lambda'(l) = \frac{\text{nominator}}{\text{denominator}},$$

where

$$\text{denominator} = \frac{1}{w_J}\left(N^2(l+(N-1)w_J+w_I) - \varkappa^2 w_J b_J(l)(l+w_I)(b_J(l)(l+w_I)+N\xi)\right)^2,$$

$$\begin{aligned}
\text{nominator} \quad = \quad & \varkappa^2 N^2 b_J(l)^2(l+w_I)^2\left(2l+w_J\left(-\varkappa^2\xi^2+3N-2\right)+2w_I\right)+(N-1)N^4 w_J - \\
- \quad & 2\varkappa^4 N\xi w_J b_J(l)^3(l+w_I)^3 - \varkappa^4 w_J b_J(l)^4(l+w_I)^4 + \\
+ \quad & \varkappa^2 N^3\xi(l+w_I)^2 b'_J(l)(l+Nw_J+w_I) + \\
+ \quad & b_J(l)\left(2\varkappa^2 N^2(l+w_I)^3 b'_J(l)(l+Nw_J+w_I)+\varkappa^2\xi N^3(l+w_I)(l+2(N-1)w_J+w_I)\right)
\end{aligned}$$

Substituting (112) to the above two expressions yields a closed form solution for $\Lambda'(l)$, up to a solution of a cubic equation determining $b_J(l)$:

$$1 = w_J b_J\left(\theta+\left(\phi+\kappa\left(w_I+l\right)\frac{b_J}{N}\right)^2\right).$$

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