Benign Granularity in Asset Markets

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FTG Meeting at UT Austin (Fall 2025)

Motivation and what we do

1. **Markets are granular (concentrated)**
   - Institutional ownership rose from 29% (1980) to 76% (2015); the top 1% now hold 30% of market cap (Lewellen & Lewellen, 2021).
   - Top-10 managers hold ∼25% of equity AUM (Ben-David et al., 2021).
2. **Markets are illiquid**
   - Koijen & Yogo (2019); Gabaix & Koijen (2025): demand shocks move prices **a lot**.
3. **How do granularity and illiquidity — and their interaction — shape equilibrium?**
   - **Need:** a model with (i) strategic trading (illiquidity), (ii) wealth effects (AUM/wealth distribution matters), (iii) heterogeneous wealth (non-degenerate distributions).
   - We develop such a model and link granularity to **returns, volatility, liquidity, and welfare.**

Preview of main results

1. **Tractable framework:** tractability(our model) $\sim$ tractability(CARA-normal).
   - Yet we allow for: (i) wealth effects and (ii) general distributions.
2. **Liquidity & strategic trading**
   - Strategic traders provide **more** liquidity than price-takers.
   - $\Rightarrow$ Aggregate demand is more elastic; **liquidity improves** with concentration.
3. **Prices, returns, volatility**
   - Non-competitive equilibrium outcomes are scaled by a common wedge $\varphi > 1$.
   - $\mu/\mu^c = \sigma/\sigma^c = \Lambda^c/\Lambda = \varphi$.
     $\Rightarrow$ **Concentration $\uparrow$ $\Rightarrow$ returns $\uparrow$, volatility $\uparrow$, liquidity $\uparrow$**.
4. **Welfare & benign granularity**
   - Higher concentration (merger, flows from small to large) can raise **all agents' utility**.
   - $\Rightarrow$ **Granularity can be benign**, not always harmful.

The Model

1. **Timing: two-period economy**
   - $t = 0$: traders submit demand schedules, market clears at P^*.
   - $t = 1$: dividends $\boldsymbol{\delta}$ realized.
2. **Agents & endowments**
   - Liquidity Providers (LPs) with wealth shares α_i, $\sum_i \alpha_i = 1$.
   - Liquidity Demanders (LDs) with aggregate market order Q.
3. **Preferences (LPs)**
   - Epstein–Zin with EIS $= 1$, RRA $\gamma > 0$:

$$U_i(c_0^i, c_1^i) = \log(\underbrace{\alpha_i w_0 - q^\top P}_{c_0^i}) + \log\left(E\left[(\underbrace{q^\top \delta}_{c_1^i})^{1-\gamma}\right]^{\frac{1}{1-\gamma}}\right).$$

4. **Assets**
   - $N{+}1$ assets, payoffs $\boldsymbol{\delta}$.
   - Asset 0 is risk-free with $\delta_0 = 1$.

Trading Mechanism & Equilibrium

- **Trading mechanism (uniform-price double auction):**
  - LPs $k \in \{1, \ldots, L\}$ submit *demand schedules* $D^k : \mathbb{R}^{N+1} \to \mathbb{R}^{N+1}$.
  - LDs submit an *market orders*. Aggregate market order $Q \in \mathbb{R}^{N+1}$.
  - **Market clearing at a uniform price** P^*: $\sum_{k=1}^{L} D^k(P^*) = Q$
  - All traders are fully rational and **take their price impact into account**

Key technical problem

1. **(Generic) strategic trading FOC:**

$$I_i(q_i) + \Lambda_i(q_i)q_i = \text{Marginal Utlity}(q_i).$$

2. **Key problematic term:** $\Lambda_i(q_i)q_i$
   - $\Lambda_i(q_i)$ is a *slope* of inverse residual demand, contains derivatives of demands other traders
   - With many heterogenous traders and many assets the FOC is a system of PDEs

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3. **Ways out:**
   - Canonical way out: CARA-Normal, $\Lambda_i(q_i) = \Lambda_i = const$
   - Some progress can be made without heterogeneity: FOC is a single PDE
     - Glebkin, Malamud, and Teguia (2025): PDE can be reduced to a first-order ODE
     - Glebkin, Malamud, and Teguia (2023): with CARA the ODE is linear even without normality $\implies$ closed form solutions
   - Neither symmetry nor CARA works: need wealth effects and wealth heterogeneity

Key technical trick

Ansatz:

- *Scale symmetry:* $I_i(q) = \beta_i \, I(q)$ (common shape, scaled by a scalar).
- *Homogeneity:* $I(tq) = t^{-1} I(q)$ (scale up quantities $\Rightarrow$ prices scale down).

Why is it a reasonable guess? Start with a competitive case

$$\sup_q \left\{ \log(\alpha_i w_0 - q^\top P) + \log\left(E\left[\left(q^\top \delta \right)^{1-\gamma} \right]^{\frac{1}{1-\gamma}} \right) \right\}.$$

FOC:

$$\left(\alpha_i w_0 - q^\top P\right) \frac{E\left[\left(q^\top \delta \right)^{-\gamma} \delta \right]}{E\left[\left(q^\top \delta \right)^{1-\gamma} \right]} = P \quad \Longrightarrow \quad I_i(q) = \frac{\alpha_i w_0}{2} \frac{E\left[\left(q^\top \delta \right)^{-\gamma} \delta \right]}{E\left[\left(q^\top \delta \right)^{1-\gamma} \right]}.$$

Key technical trick

1. **(Generic) strategic FOC:**

$$I_i(q_i) + \Lambda_i(q_i) q_i = \text{Marginal Utlity}(q_i).$$

   - With many heterogenous traders and many assets the FOC is a system of PDEs

2. **Add scale-symmetry:** $I_i(q) = \beta_i\, I(q)$ (common shape, scaled by a scalar).

   FOC becomes: $I(q) \underbrace{-k\nabla I(q)}_{=\Lambda(q)} q = \text{Marginal Utlity}(q)$, where k is a scalar

   - single PDE for a common inverse demand $I(q)$ that determines *all* demands via a simple rescaling

3. **Key problematic term** is now $\nabla I(q) q$

4. **Add homogeneity:** $I(tq) = t^{-1} I(q)$
   - **Euler's theorem.** Differentiate wrt to t: $\nabla I(tq)q = -t^{-2} I(q)$. Evaluate at $t = 1$: $\nabla I(q)q = -I(q)$. PDE becomes a linear equation!

Non-competitive equilibrium

Theorem

There exists a unique scale-symmetric equilibrium with a homogeneous $I(Q)$. The inverse demands are given by $I_i(q) = I(q/\beta_i)$. The function $I(q)$ is given by

$$I(q) = \frac{w_0}{2\phi} \frac{E\left[\left(\delta^\top q\right)^{-\gamma} \delta\right]}{E\left[\left(\delta^\top q\right)^{1-\gamma}\right]}.$$

The scaling constants are given by

$$\beta_i = \alpha_i \phi + 1 - \sqrt{(\alpha_i \phi)^2 + 1}.$$

The constant ϕ is the unique positive solution to

$$\sum_{i=1}^{L} \left(\alpha_i \phi + 1 - \sqrt{(\alpha_i \phi)^2 + 1} \right) = 1.$$

Interpretation: β_i is both LP i's *turnover share* and *liquidity provision share.*

Aggregate comparison: strategic vs. competitive

1. **Single scaling wedge:**

$$\frac{\mu_k}{\mu_k^c} = \frac{\sigma_k}{\sigma_k^c} = \frac{\Lambda_{kl}^c}{\Lambda_{kl}} = \varphi \; > \; 1$$

   - Returns ↑ and volatility ↑ under market power (tilt in favor of LPs).

2. **Surprising part — and key mechanism — liquidity improves:**

$$\frac{\Lambda_{kl}^c}{\Lambda_{kl}} = \varphi \; > \; 1$$

   Similar result when the concentration improves. Empirical evidence:

   - Aggregate Amihud's lambda is positively associated with changes in concentration (HHI of mutual funds AUM) ← (See empricail section)
   - Hedge fund closures lead to improved liquidity ← (Pugachev, 2022)

Mechanism: Why liquidity improves

Strategic FOC:

$$I_i(q_i) = \text{Marginal Utility}_i(q_i) - \Lambda_i(q_i)q_i.$$

1. $\Lambda_i(q_i)q_i$ increasing in q $\Rightarrow$ strategic inverse demand steeper than competitive $\Rightarrow$ imperfect competition = less liquidity
2. $\Lambda_i(q_i)q_i$ decreasing in q $\Rightarrow$ strategic inverse demand flatter than competitive $\Rightarrow$ imperfect competition = more liquidity
3. **CARA–Normal:** $\Lambda_i = \text{const}$ $\Rightarrow$ imperfect competition = less liquidity (conventional wisdom).
4. **Our case:** $\Lambda_i(q_i)q_i$ decreases in q_i $\Rightarrow$ imperfect competition = more liquidity
   - Unlike CARA, EZ utility implies $c_0^i, c_1^i > 0$. $\Rightarrow$ Expenditure $q_i^\top I_i(q_i)$ must be bounded. $\Rightarrow$ Expenditure reduction $q_i^\top \Lambda_i q_i$ must also be bounded. $\Rightarrow$ Impossible if $\Lambda_i(q_i)q_i$ increases in q.

Cross-section of investor behavior: Results

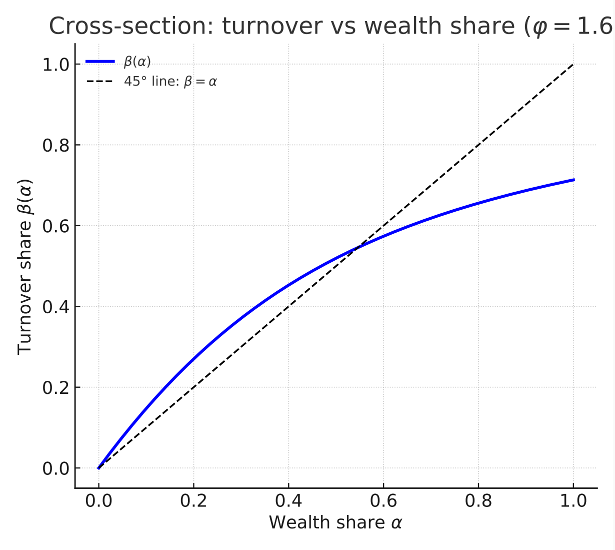
1. Larger LPs (α_i big) make larger trades and supply more liquidity in absolute terms, but have a higher price impact.

2. But their **turnover share** β_i grows more slowly than their wealth share α_i.

3. Turnover vs. wealth share: for the largest LPs, $\beta_i < \alpha_i$; for the smallest, $\beta_i > \alpha_i$.

4. **Empirical match:** Koijen & Yogo (2019) show that the biggest funds' turnover share is below their wealth share.

Cross-section of investor behavior: Illustration



Large-market limit and HHI

1. **Herfindahl–Hirschman Index (HHI):**

$$\mathrm{HHI}(L) = \sum_{i=1}^{L} \alpha_i^2, \qquad \mathrm{HHI}(\infty) = \lim_{L\to\infty} \mathrm{HHI}(L).$$

2. **Proposition:**
   - If $\mathrm{HHI}(\infty) = 0$: $\varphi(\infty) = 1 \Rightarrow$ competitive limit.
   - If $\mathrm{HHI}(\infty) > 0$: $\varphi(\infty) > 1 \Rightarrow$ wedge persists; market power survives in large markets.

3. **Regulatory tool:**
   - HHI provides a *structural link* between ownership concentration and the degree of competition in equilibrium.

Large-market limit — illustrative examples

1. **Example 1: Equal shares**
   - $\alpha_i = 1/L$ for all i.
   - $\mathrm{HHI}(L) = 1/L \ \rightarrow \ 0$.
   - Market converges to competitive benchmark.

2. **Example 2: One large + many small**
   - One LP has fixed share $s > 0$, others fragmented.
   - $\mathrm{HHI}(L) \rightarrow s^2$ as $L \rightarrow \infty$.
   - Wedge persists: $\varphi(\infty) > 1$.

3. **Example 3: Fat-tailed wealth distribution**
   - Wealth shares follow a heavy-tailed law (e.g., Pareto).
   - HHI does not vanish even as $L \rightarrow \infty$.
   - Persistent wedge: large investors retain market power.

Benign granularity: when concentration is welfare-improving

1. Conventional view: concentration $\uparrow$ raises volatility, systemic risk, fragility.

2. Our result: concentration $\uparrow$ also improves liquidity, via more elastic aggregate demand.

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3. Welfare effect:

   - In sufficiently non-competitive markets
   - a merger of two funds can raise the utility of **all** participants.
   - Mechanism: liquidity improves $\Rightarrow$ better risk-sharing and intertemporal smoothing; prices tilt in favor of remaining LPs.

   Conversely, in sufficiently competitive markets, increased concentration reduces welfare (conventional view).

4. This is what we call **benign granularity**.

Conclusion

Granularity in asset markets can be benign.

- Concentration raises returns and volatility, but can **improve liquidity**.
- Welfare effects are **non-monotone**: in sufficiently non-competitive markets, more inequality can benefit all participants.
- Empirics: changes in mutual-fund HHI predict volatility (VIX $\uparrow$) and liquidity (Amihud $\downarrow$).
- **Bridge:** connects strategic trading models with the empirical institutional liquidity literature.
- **Technical contribution:** extend strategic trading to EZ preferences $\rightarrow$ tractable strategic equilibrium with wealth effects.