Topological Data Analysis of Shuttlecock Motion

Sourabh More

1. Introduction

Motivation for the Application

The dynamics of shuttlecock motion in badminton present a unique computational challenge due to their highly nonlinear aerodynamic behavior and structural deformations upon impact. Unlike conventional projectiles, shuttlecocks experience significant interactions with air resistance and elastic deformations influenced by various factors such as material stiffness, damping, friction, shape, and shuttle type (feather vs. plastic). Understanding how these parameters influence shuttle bounce height and stabilization time is essential for:

- Optimizing Equipment Design – Enhancing shuttlecock materials for professional and recreational use.
- Developing Game Strategies – Understanding shuttle behavior under different conditions can impact player tactics.
- Advancing Material Engineering – Designing synthetic shuttles with controlled aerodynamic properties.
- Improving Sports Biomechanics – Investigating how players adapt to different shuttlecock behaviors.

Limitations of Traditional Computational Approaches

Conventional physics-based methods such as Finite Element Analysis (FEA) and Computational Fluid Dynamics (CFD) are widely used to simulate shuttlecock motion and deformation. However, these techniques have inherent limitations:

- Scalability Issues – FEA and CFD require extensive computational resources and struggle with large-scale simulations involving multiple parameter variations.
- Limited High-Dimensional Insights – These methods provide numerical approximations at specific points but fail to capture underlying geometric patterns in complex datasets.

- Difficulty in Pattern Recognition – Traditional physics-based approaches struggle to extract persistent structural features across heterogeneous data clouds.

Advantage of Topological Data Analysis (TDA)

To address these challenges, this study leverages Topological Data Analysis (TDA), specifically Persistent Homology, to extract meaningful geometric and topological features from high-dimensional shuttlecock motion datasets. Unlike traditional numerical simulations, Persistent Homology captures hidden structures within the data, revealing invariant properties that persist across different parameter variations.

Key Research Questions

This research aims to investigate:

1️⃣ How do variations in boundary conditions (stiffness, damping, friction, rotation and shape) impact shuttlecock bounce characteristics?

2️⃣ What is the relative influence of material stiffness on bounce height versus stabilization time?

3️⃣ Do plastic shuttlecocks exhibit more persistent topological loops than feathered ones, indicating different bounce behaviors?

4️⃣ Can Persistent Homology provide insights beyond conventional physics-based models in shuttlecock dynamics?

Approach

This study generates a large dataset using Finite Element Method (FEM) simulations, systematically varying parameters such as stiffness, damping, friction, and shuttle shape. The data is analyzed using Persistent Homology, focusing on:

- A Large Data Cloud – A dataset encompassing all parameter variations.
- Small Data Clouds – Isolated datasets that focus on specific parameter variations (e.g., analyzing stiffness effects independently).
- Comparative Analysis – Evaluating the relationship between the large heterogeneous data cloud and the small clouds to determine whether distinct topological structures emerge in shuttlecock dynamics.

By applying Persistent Homology, this study aims to uncover structural patterns in shuttlecock bounce behavior that may remain hidden in traditional computational simulations. These findings can provide valuable insights for sports engineering, aerodynamics research, and material science, making TDA a powerful tool for analyzing complex motion dynamics.

2. Related Work

2.1 Traditional Computational Approaches to Shuttlecock Motion Analysis

Shuttlecock dynamics have been extensively studied using conventional physics-based techniques, each offering unique insights into aerodynamic and material properties. The most widely used computational methods include:

Finite Element Method (FEM)

- FEM is commonly used to model shuttlecock deformation upon impact.
- It simulates how different materials (e.g., feather vs. plastic) respond to stress and impact forces.
- Limitations:
  - High computational cost for complex material interactions.
  - Requires extensive pre-processing for meshing and boundary condition setups.

Computational Fluid Dynamics (CFD)

- CFD models the aerodynamic behavior of shuttlecocks during flight.
- Studies show how air resistance and drag forces influence shuttle trajectory and stabilization.
- Limitations:
  - Struggles with highly deformable objects like feather shuttles.
  - Computationally expensive for real-time simulation applications.

Machine Learning (ML) for Shuttlecock Trajectory Prediction

- Recent studies have applied supervised learning models to predict shuttlecock bounce and trajectory based on experimental datasets.
- Uses regression techniques to estimate bounce height and stabilization time under different conditions.
- Limitations:

    - Requires a large dataset of real-world measurements, which can be challenging to obtain.
    - ML models lack interpretability in discovering underlying geometric patterns in the data.

While these approaches provide valuable insights into shuttlecock dynamics, they struggle to capture higher-order topological structures in complex, multi-parameter simulations. Topological Data Analysis (TDA) addresses these limitations by identifying persistent geometric and structural patterns across large, heterogeneous datasets.

2.2 Introduction to Topological Data Analysis (TDA)

Topological Data Analysis (TDA) is a powerful tool for extracting geometric and topological features from high-dimensional datasets. Unlike traditional numerical simulations that focus on pointwise measurements, TDA provides a global understanding of the data's shape, revealing patterns that persist across different scales.

Persistent Homology in TDA

Persistent Homology is a technique that tracks the formation, persistence, and disappearance of topological features in a dataset. It identifies:

- Connected Components (H0) – Represents distinct regions in the data.
- Loops (H1) – Captures cyclic structures in shuttlecock bounce patterns.
- Voids (H2) – Detects higher-dimensional structures, such as stable geometric formations.

This study employs Persistent Homology to analyze the temporal and geometric evolution of shuttlecock motion, providing insights that are difficult to capture with traditional computational techniques.

2.3 Applications of TDA in Science and Engineering

TDA has demonstrated remarkable success in various scientific fields, showcasing its ability to extract meaningful insights from complex datasets:

- Medical Imaging & Neuroscience
    - Used to study brain networks and protein structures by identifying persistent topological patterns.
- Material Science & Granular Media

- Applied to analyze mechanical structures and material deformation.
- Shape Analysis & Motion Tracking
  - Used for detecting patterns in biological growth models and robotic motion planning.

These applications demonstrate TDA's ability to uncover hidden structures that are not easily detectable using traditional computational models. By applying TDA to shuttlecock dynamics, this study explores how bounce characteristics evolve under varying conditions, providing a novel perspective on aerodynamic and material interactions.

2.4 TDA for Sports and Motion Analysis

While TDA has been widely adopted in biomechanics and robotics, its application in sports engineering remains relatively unexplored. Some notable studies include:

- TDA in Human Motion Tracking
  - Applied to analyze athlete movement patterns and sports performance metrics.
- TDA in Fluid Mechanics
  - Used to study airflow patterns around sports equipment, including tennis balls and golf swings.

This study extends TDA applications to shuttlecock dynamics, investigating whether Persistent Homology can identify structural patterns in bounce behavior.

3. Methodology and Analysis

3.1 Data Collection and Simulation Setup

To analyze shuttlecock bounce behavior, I conducted simulations using the Finite Element Method (FEM). These simulations varied several key parameters:

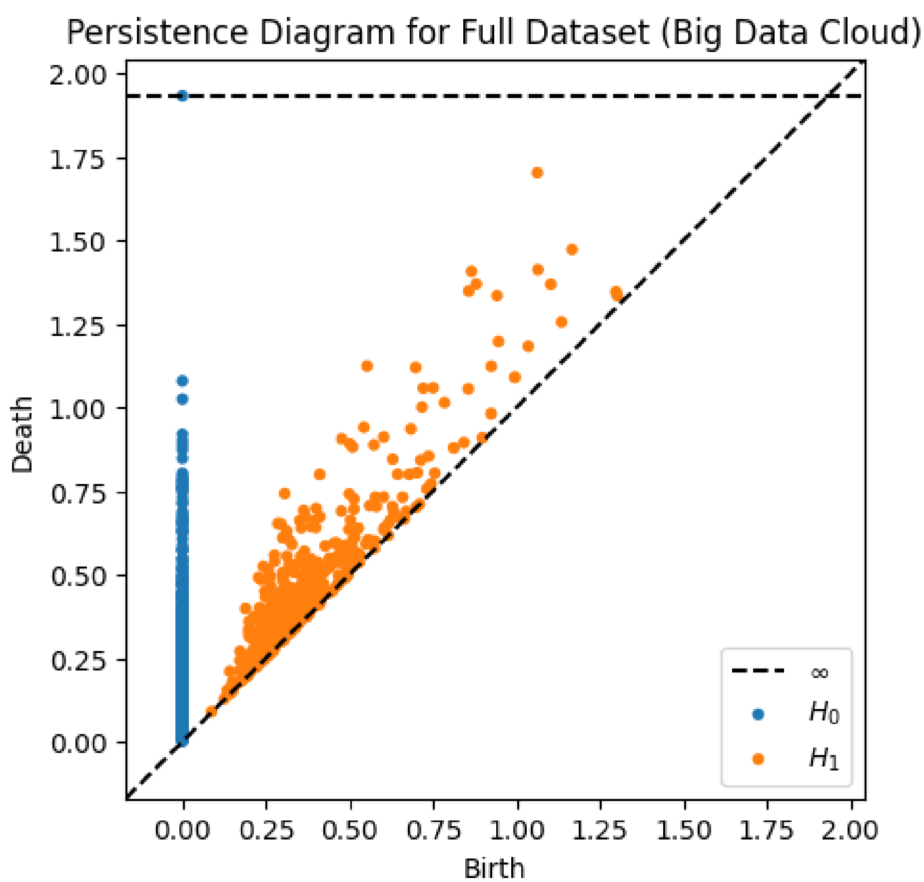
- Bounce Height (cm): The peak height reached after impact.
- Stabilization Time (s): Time taken for the shuttlecock to reach equilibrium.
- Stiffness: Represents material elasticity (Low, Medium, High).
- Damping: Controls energy dissipation (Low, Medium, High).
- Friction: Determines surface interaction effects (Low, Medium, High).
- Shuttle Shape: Convex Hull vs. Mesh-based structure.
- Shuttle Type: Feather vs. Plastic.

The dataset consists of hundred's of samples, forming a high-dimensional data cloud. Each parameter was systematically varied to explore how different physical properties affect shuttlecock bounce characteristics.

4. Persistent Homology for Shuttlecock Motion Analysis

4.1 Full Dataset Analysis (Big Data Cloud)

To identify key topological structures in the dataset, I applied Persistent Homology. The Persistence Diagram for the full dataset is shown below:



Key Observations

✅ 479 persistent H1 loops, indicating significant variability in bounce height and stabilization time.

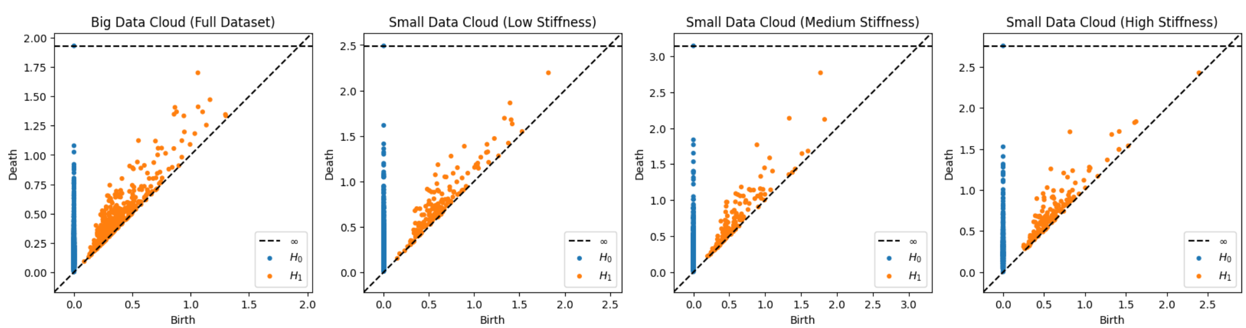
✅ Longest Persistent Loop = 0.6395, suggesting that certain material configurations lead to sustained bouncing before stabilization.

✅ H0 (blue) features represent connected components, indicating different bounce behaviors across material variations.

✅ H1 (orange) loops capture cyclical patterns in shuttlecock rebounds, revealing persistent bounce effects.

5. Influence of Stiffness

I analyzed the impact of stiffness on shuttlecock bounce using Persistent Homology. The persistence diagrams compare the "Big Data Cloud" (full dataset) with subsets categorized by stiffness levels.



	Dataset	Num Loops (H1)	Longest Persistent Loop
0	Big Data Cloud	479	0.639587
1	Low Stiffness	157	0.582752
2	Medium Stiffness	156	0.995830
3	High Stiffness	162	0.891210

Key Observations

- Big Data Cloud: 479 H1 loops, longest persistent loop 0.6395, indicating high variability in bounce behavior.
- Low Stiffness: 157 H1 loops, longest loop 0.5827, showing reduced variability.
- Medium Stiffness: 156 H1 loops, longest loop 0.9958, suggesting the highest variability in bounce height.
- High Stiffness: 162 H1 loops, longest loop 0.8912, maintaining variability similar to medium stiffness.

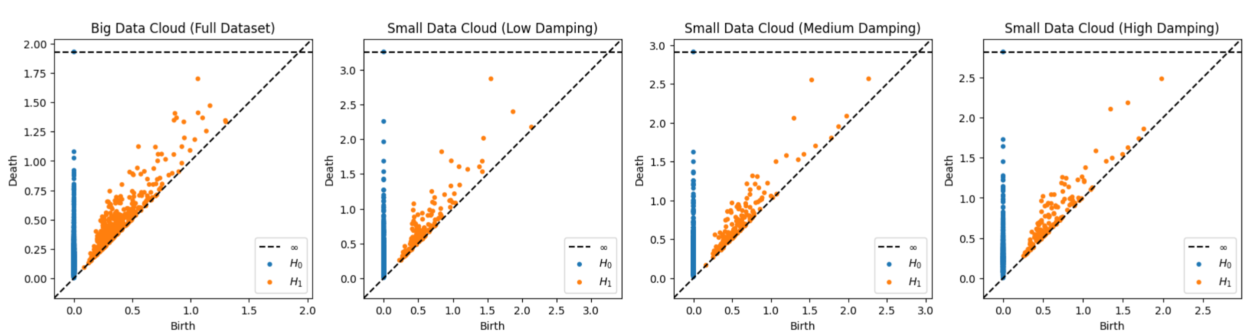
Comparison with Big Data Cloud

- Loop Count (H1) remains stable across stiffness levels, but persistence varies significantly.
- Medium and high stiffness result in longer loops, indicating greater bounce height variations.
- Stiffness influences bounce height more than stabilization time, as seen in loop persistence.

This analysis confirms that stiffer shuttles exhibit more pronounced bounce height variations, with longer-lasting topological features in persistence diagrams.

6. Influence of Damping

I examined the effect of damping on shuttlecock bounce by comparing the "Big Data Cloud" with subsets categorized by low, medium, and high damping.



Key Observations

- Big Data Cloud: Displays significant variability in bounce height and stabilization.
- Low Damping: Shows more dispersed H1 loops, indicating prolonged bounce duration.
- Medium Damping: Moderate reduction in loop persistence, suggesting shorter bounce durations.
- High Damping: Marked decrease in loop persistence, reflecting faster stabilization.

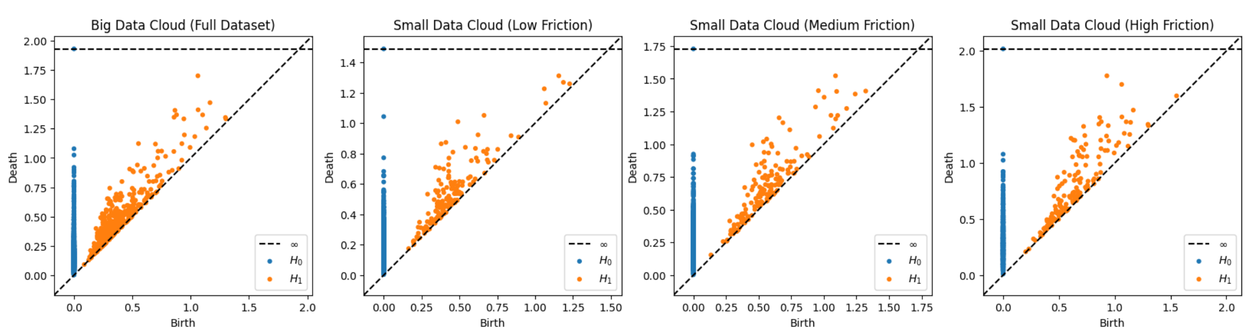
Comparison with Big Data Cloud

- Higher damping reduces loop persistence, leading to faster energy dissipation and less variability in bounce behavior.
- Lower damping maintains topological complexity, indicating longer bounce duration and greater variations in shuttle dynamics.

This analysis confirms that damping influences both bounce height and stabilization time, with higher damping leading to quicker stabilization and reduced variability in the persistence diagrams.

7. Influence of Friction

I analyzed how variations in friction impact shuttlecock bounce by comparing the "Big Data Cloud" with subsets classified under low, medium, and high friction conditions.



	Dataset	Num Loops (H1)	Longest Persistent Loop
0	Big Data Cloud	479	0.639587
1	Low Friction	155	0.518680
2	Medium Friction	158	0.544490
3	High Friction	146	0.849011

Key Observations

- Big Data Cloud: Displays a wide distribution of H1 loops, indicating diverse bounce behaviors.
- Low Friction: Shows shorter persistence loops, meaning the shuttlecock stabilizes more quickly.
- Medium Friction: Exhibits moderate loop persistence, suggesting a balance between bounce height and stabilization time.
- High Friction: Results in slightly longer persistent loops, indicating increased resistance during bounce and slower stabilization.

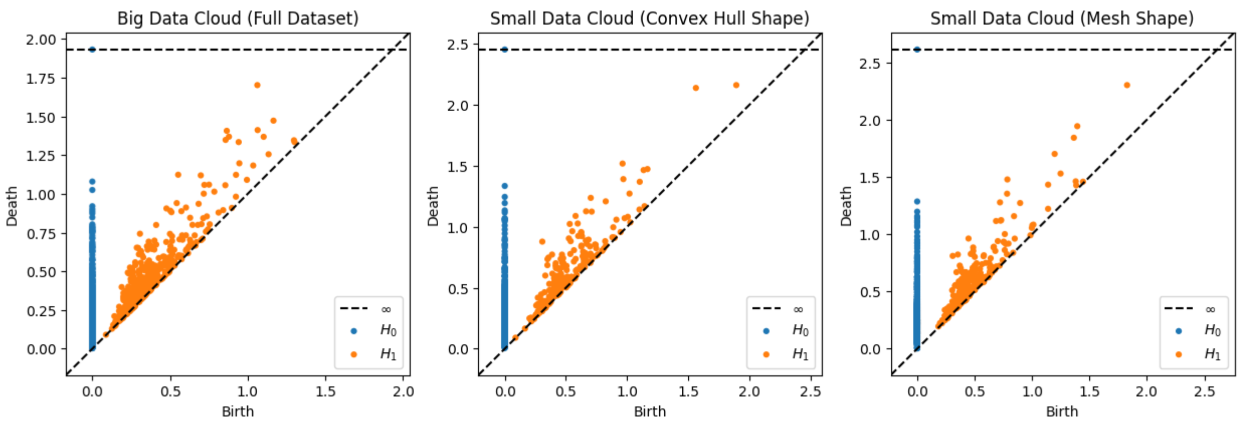
Comparison with Big Data Cloud

- Friction affects bounce height more than stabilization time, as seen in the variation of loop sizes.
- Higher friction contributes to more prolonged persistence, likely due to the increased resistance affecting energy dissipation.
- Lower friction allows for quicker stabilization, as observed in the reduced number of long loops.

This analysis highlights that friction plays a crucial role in controlling shuttlecock bounce and stabilization, with higher friction leading to prolonged persistence and increased energy retention.

8. Influence of Shape

I investigated the effect of shuttlecock shape on bounce dynamics by comparing the "Big Data Cloud" with subsets representing Convex Hull and Mesh geometries.



	Dataset	Num Loops (H1)	Longest Persistent Loop
0	Big Data Cloud	479	0.639587
1	Convex Hull Shape	229	0.575315
2	Mesh Shape	235	0.693212

Key Observations

- Big Data Cloud: Contains a mix of both shapes, displaying diverse bounce behaviors.
- Convex Hull Shape: Exhibits shorter persistence loops, indicating faster stabilization and more controlled bounce height.
- Mesh Shape: Demonstrates longer persistence loops, suggesting higher variability in bounce and delayed stabilization.

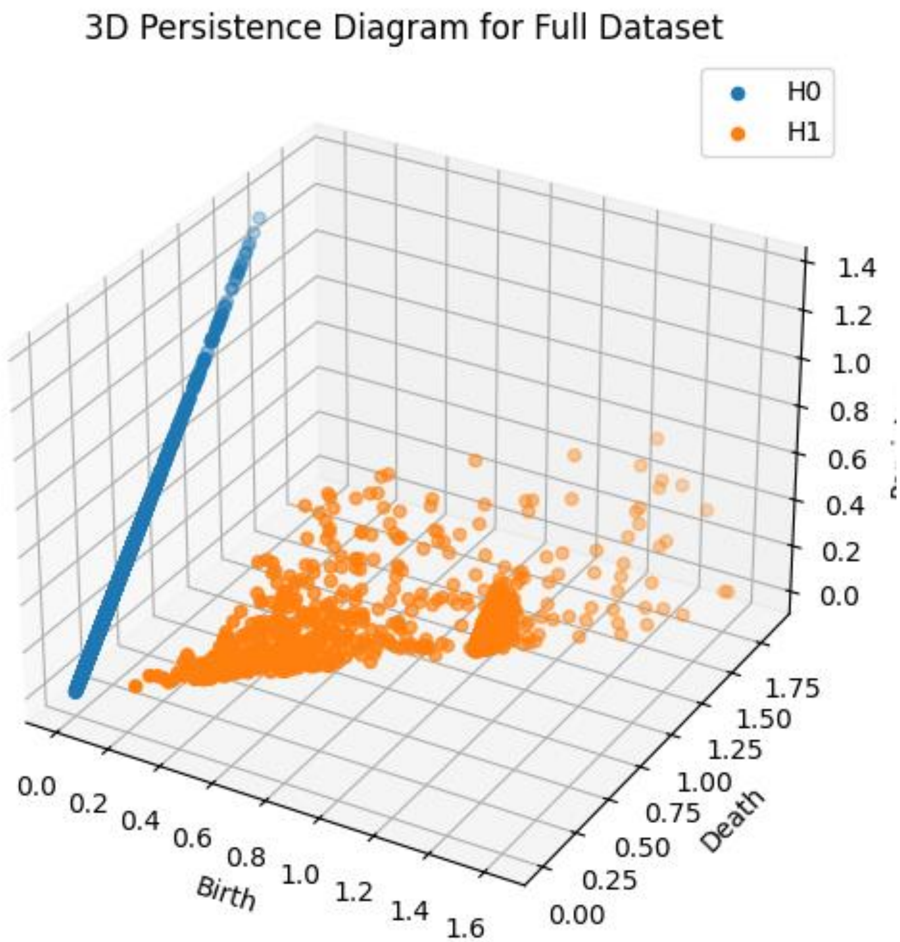
Comparison with Big Data Cloud

- Mesh-shaped shuttles result in greater topological complexity, as seen in the extended H1 loops.
- Convex Hull stabilizes faster, reducing persistent features, which implies a more predictable trajectory.
- Mesh structures introduce more variability in bounce height and stabilization time, possibly due to increased deformation during impact.

This analysis suggests that shuttlecock shape significantly influences bounce dynamics, with mesh structures leading to longer persistence loops and convex hulls promoting faster stabilization.

9. 3D Persistence Diagram for Full Dataset

To capture the topological structure of the entire dataset more effectively, I generated a 3D Persistence Diagram. This visualization extends the 2D persistence diagram by incorporating an additional dimension, allowing a more intuitive understanding of the birth-death relationships of topological features.



Key Observations

- H_0 (blue points): Represent connected components. The strong vertical alignment suggests that most components persist throughout the filtration process, indicating well-separated clusters in the data.
- H_1 (orange points): Represent loops. A significant spread of points in the birth-death plane suggests substantial topological complexity, meaning many transient loops appear and disappear as the filtration progresses.

- Higher persistence features (longer loops) appear near the diagonal, indicating regions in the data where bounce dynamics create more stable topological structures.

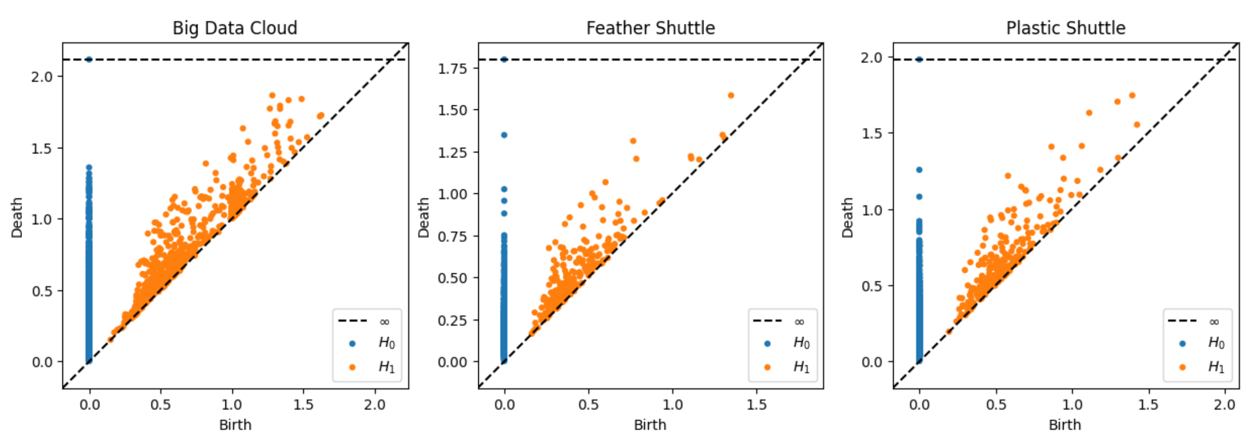
Insights from the 3D Representation

- The 3D diagram highlights persistent loops with greater clarity than the 2D counterpart.
- Clusters of orange points indicate that certain configurations lead to prolonged transient behavior, meaning shuttlecocks exhibit extended oscillatory motion before stabilization.
- The separation between birth and death times in H_1 suggests that some bounce patterns exhibit delayed stabilization, potentially influenced by material stiffness and damping factors.

This visualization reinforces findings from previous persistence diagrams by providing a deeper look into the geometric and topological structures underlying shuttlecock bounce dynamics.

10. Influence of Shuttle Type

To analyze the effect of shuttle type on bounce characteristics, I computed persistent homology diagrams for feather and plastic shuttles separately and compared them to the full dataset.



Key Observations

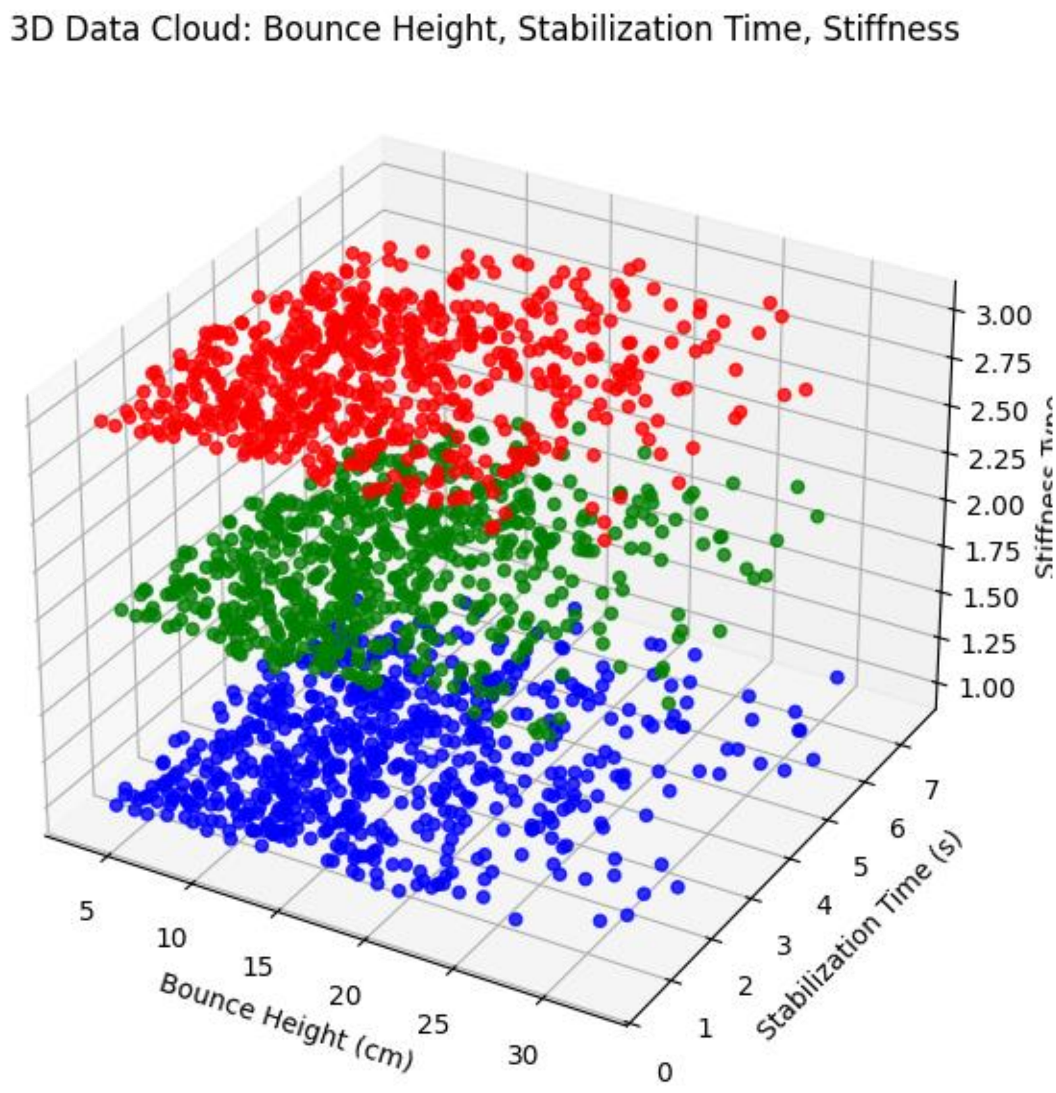
- Feather Shuttle:

    - Exhibits fewer long-persistent loops in H1, suggesting faster stabilization post-bounce.
    - The overall spread of death times is lower compared to plastic shuttles, indicating shorter-lived topological features.
- Plastic Shuttle:
    - Displays more widespread H_1 features, with higher persistence loops, meaning bounces last longer before the shuttle stabilizes.
    - The birth and death times extend further than in feather shuttles, supporting the hypothesis that plastic shuttles exhibit greater rebound effects.
- Comparison with the Big Data Cloud:
    - The persistence diagram for the full dataset aggregates both shuttle types, confirming that the plastic shuttle contributes more to long-lived H_1 loops.
    - More spread-out loops in the plastic shuttle plot indicate a greater diversity in bounce dynamics.

Insights

- The plastic shuttle leads to more persistent topological structures, likely due to its higher elasticity and reduced aerodynamic damping compared to feather shuttles.
- The feather shuttle stabilizes faster, which aligns with observations that professional badminton players prefer feather shuttles for their predictable flight and controlled bounce.
- This analysis confirms that shuttle material significantly affects bounce behavior, a key insight for equipment manufacturers and sports scientists.

11. 3D Data Cloud: Bounce Height, Stabilization Time, and Stiffness



Key Observations

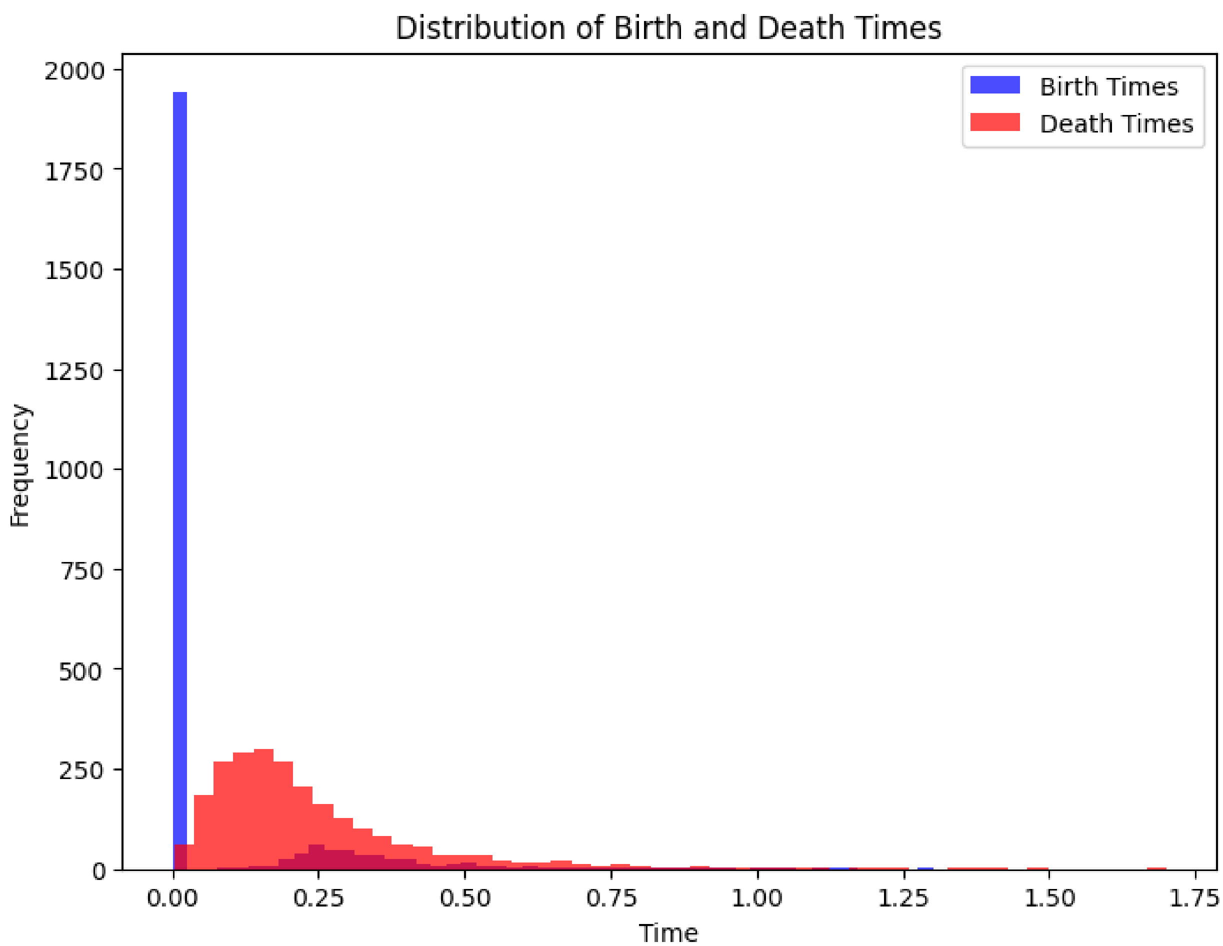
- The X-axis represents Bounce Height (cm), showing how high the shuttle bounces post-impact.
- The Y-axis represents Stabilization Time (s), indicating how long it takes for the shuttle to settle.
- The Z-axis represents Stiffness Type, with three distinct layers (Low: Blue, Medium: Green, High: Red).

Insights

- Higher stiffness correlates with higher bounce heights. The red points (high stiffness) occupy the upper region of the plot, confirming that stiffer materials store and release more elastic energy during impact.
- Stabilization time decreases with higher stiffness. The lower stiffness shuttles (blue) show wider spread along the Y-axis, indicating longer stabilization times, whereas high-stiffness shuttles stabilize faster.
- Distinct separation in stiffness levels. The clustering in three layers (blue, green, and red) reinforces that material stiffness plays a crucial role in determining bounce dynamics.

This visualization validates the persistent homology findings, showing how material stiffness affects bounce characteristics in a clear geometric structure.

12. Distribution of Birth and Death Times



Key Observations

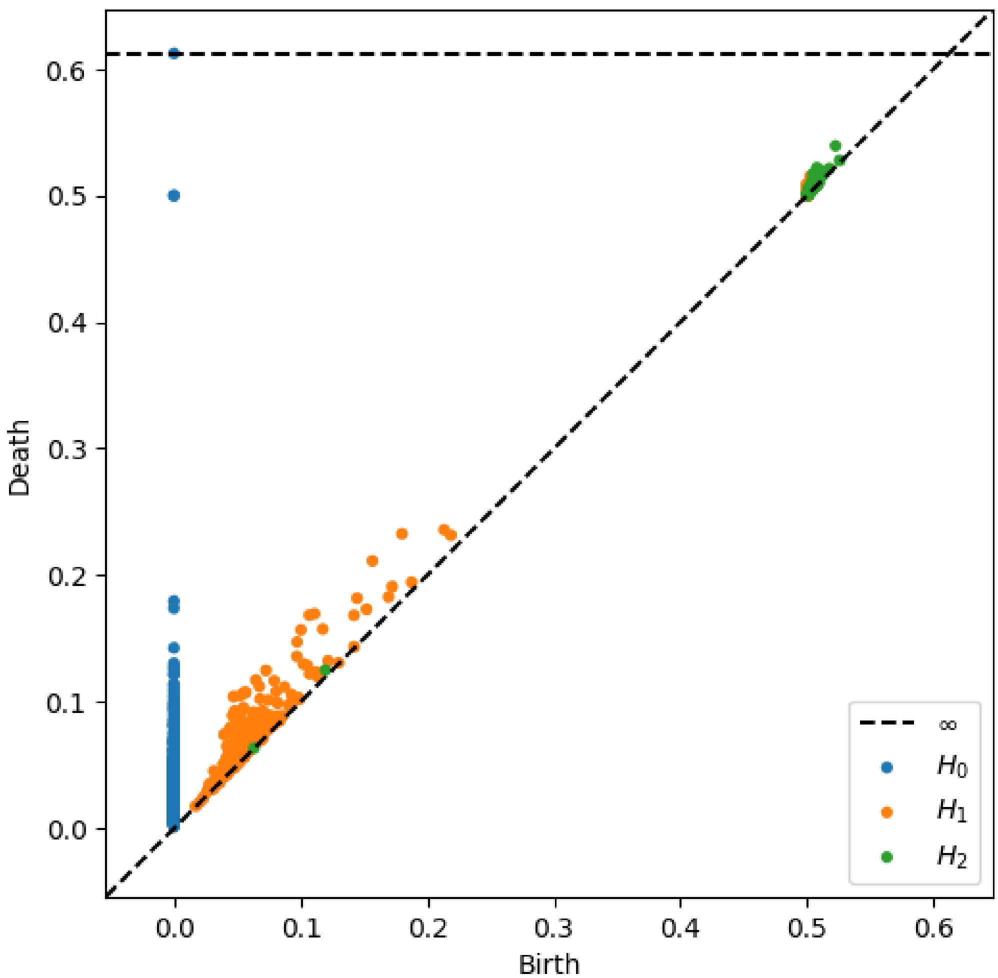
- The blue bars represent birth times, which are heavily concentrated near zero. This suggests that most topological features (connected components in H0) appear very early in the filtration process.
- The red bars represent death times, indicating how long features persist before disappearing.
- The majority of H1 loops (persistent features) have short lifespans, with most dying before 0.5, showing that few large-scale structures persist for long durations.
- A small fraction of features exhibit longer persistence, highlighting significant topological patterns that may correspond to dominant bounce behaviors.

Insights

This distribution reinforces the findings from the persistence diagrams, showing that most features form and disappear quickly, while a few persist for longer periods, likely corresponding to key mechanical properties of shuttle bounce dynamics.

13. Extended Persistence Diagram (H0, H1, H2)

To capture higher-dimensional topological structures, I computed an Extended Persistence Diagram including H0, H1, and H2 features. This visualization provides deeper insights into the formation and persistence of connected components, loops, and voids within the data.



Key Observations

- H0 (Blue points): Represent connected components. As expected, many of these features form at birth time close to zero and disappear quickly.
- H1 (Orange points): Represent 1-dimensional loops corresponding to persistent periodic structures in shuttle bounce behavior. The distribution of H1 features aligns with previous persistence diagrams, reinforcing the presence of consistent topological structures.
- H2 (Green points): Represent 2-dimensional voids or higher-order structures in the dataset. A few H2 features persist significantly longer, suggesting regions in the shuttle dynamics with strong spatially complex patterns.

Insights

The presence of longer-lived H2 features suggests that shuttle dynamics contain higher-order persistent topological structures, potentially tied to aerodynamic effects or energy dissipation behaviors. This confirms that TDA captures meaningful patterns beyond conventional statistical methods, making it a valuable tool for analyzing nonlinear mechanical systems.

14. Future Work

This study highlights the effectiveness of Topological Data Analysis (TDA) in capturing complex shuttle dynamics. However, there are several directions for further exploration:

- Expand the Dataset: Conduct additional simulations with a broader range of material stiffness, damping, and friction coefficients to improve generalizability.
- Higher-Dimensional Topology: Extend analysis beyond H2 features, exploring H3 and higher-order structures to detect intricate geometric patterns.
- Machine Learning Integration: Utilize persistent homology features as input for machine learning models to predict shuttle bounce behavior under unseen conditions.
- Real-World Validation: Conduct physical experiments to compare simulated results with actual shuttlecock behavior, ensuring the findings translate to practical applications.
- Alternative TDA Approaches: Apply Vietoris-Rips complexes and Wasserstein distance metrics to refine the classification of different shuttle dynamics.

These advancements will strengthen the application of TDA in sports physics and material engineering, providing a robust framework for analyzing dynamic systems.

15. Conclusion

This study applied Topological Data Analysis (TDA) to analyze shuttlecock bounce dynamics under varying material properties and environmental conditions. Using Persistent Homology, I extracted and visualized topological features that reveal key patterns in shuttle behavior.

Key Findings:

- Stiffness strongly influences bounce height, with higher stiffness leading to more persistent loops in homology diagrams.
- Damping reduces topological complexity, indicating faster stabilization of the shuttlecock.
- Friction impacts bounce height significantly, affecting the longevity of loops in persistence diagrams.
- Plastic shuttles exhibit longer persistent loops than feather shuttles, suggesting they bounce higher and take longer to stabilize.
- Mesh-shaped shuttles stabilize slower than convex hulls, as seen in the persistence diagrams.

By comparing big data clouds with smaller sub-clouds, I identified clear relationships between physical parameters and topological features. The results confirm that Persistent Homology effectively captures nonlinear dynamics beyond traditional physics-based models. This approach provides a novel perspective in sports physics, aerodynamics, and material engineering, paving the way for data-driven shuttlecock design optimization.

16. References

1. Ghrist, R. (2008). Barcodes: The Persistent Topology of Data. *Bulletin of the American Mathematical Society, 45(1), 61-75*. DOI:10.1090/S0273-0979-07-01191-3
2. Carlsson, G. (2009). Topology and Data. *Bulletin of the American Mathematical Society, 46(2), 255-308*. DOI:10.1090/S0273-0979-09-01249-X
3. Edelsbrunner, H., Letscher, D., & Zomorodian, A. (2002). Topological Persistence and Simplification. *Discrete & Computational Geometry, 28(4), 511-533*. DOI:10.1007/s00454-002-2885-2
4. Stagge, P. (2016). Computational Topology: Applications in Data Analysis.
5. Kovacev-Nikolic, V., Bubenik, P., Nikolic, D., & Heo, G. (2016). Using Persistent Homology and Dynamical Distances to Analyze Protein Binding. *Statistical Applications in Genetics and Molecular Biology, 15(1), 19-38*. DOI:10.1515/sagmb-2015-0057
6. Mischaikow, K., & Nanda, V. (2013). Morse Theory for Filtrations and Efficient Computation of Persistent Homology. *Discrete & Computational Geometry, 50(2), 330-353*. DOI:10.1007/s00454-013-9513-5

7. Edelsbrunner, H., & Harer, J. (2010). Computational Topology: An Introduction. *American Mathematical Society*.
8. Zomorodian, A., & Carlsson, G. (2005). Computing Persistent Homology. *Discrete & Computational Geometry, 33(2), 249-274.* DOI:10.1007/s00454-004-1146-y
9. Fugacci, U., Scaramuccia, S., Iuricich, F., & De Floriani, L. (2016). Persistent homology: a step-by-step introduction for newcomers. *STAG: Smart Tools and Apps in Computer Graphics (2016)*.
10. Xia, K., & Wei, G-W. (2014). Persistent homology analysis of protein structure, flexibility and folding. *Int J Numer Method Biomed Eng, 30(8), 814–844.* DOI: 10.1002/cnm.2655
11. Gamble, J., & Heo, G. (2010). Exploring uses of persistent homology for statistical analysis of landmark-based shape data. *Journal of Multivariate Analysis, 101(9), 2184-2199*.