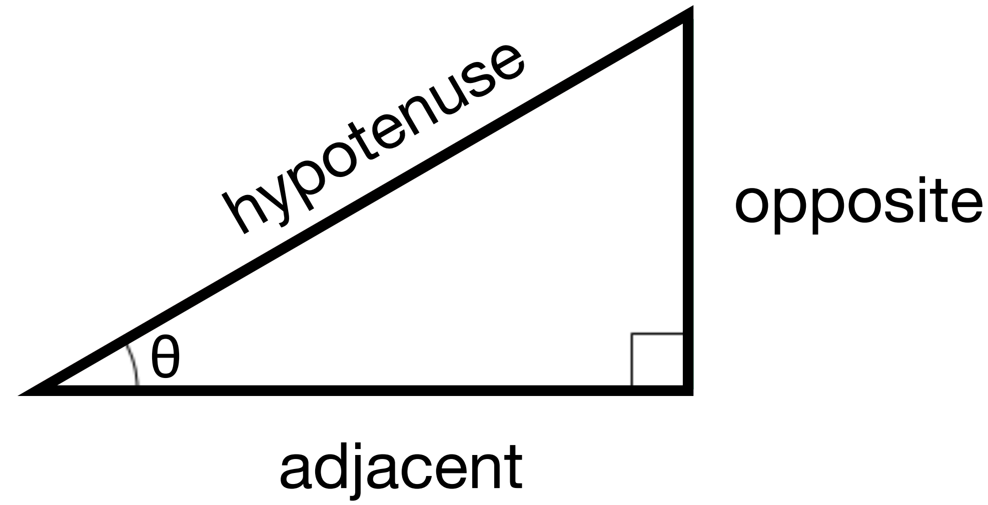
Trigonometry Review Material

Given a right triangle with one of the angle θ given like the one shown on the right, the ratios between different segments can be obtained by using trigonometry functions



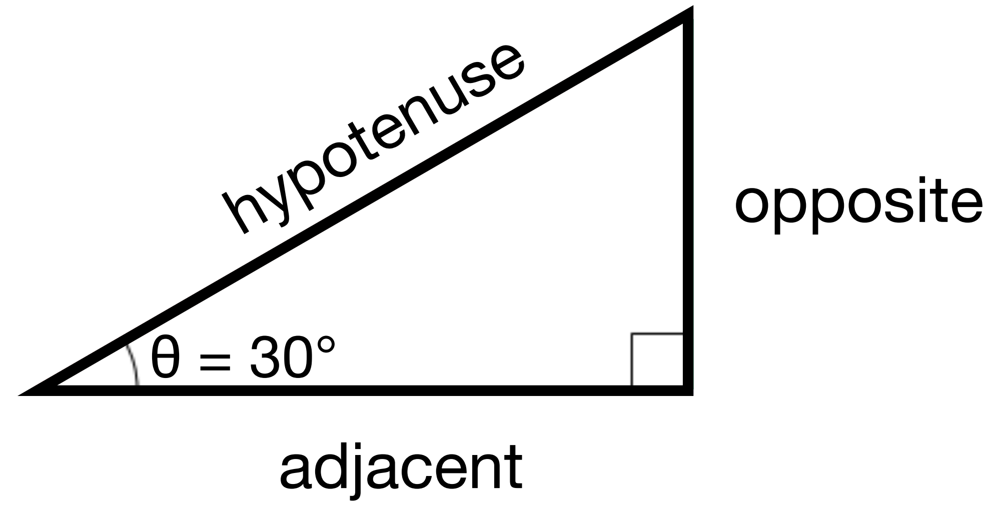
The three main trig functions are:

$$\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}} = \frac{\sin(\theta)}{\cos(\theta)}$$

Example:

If the right triangle has θ = 30°, then following are the ratios between the segment lengths

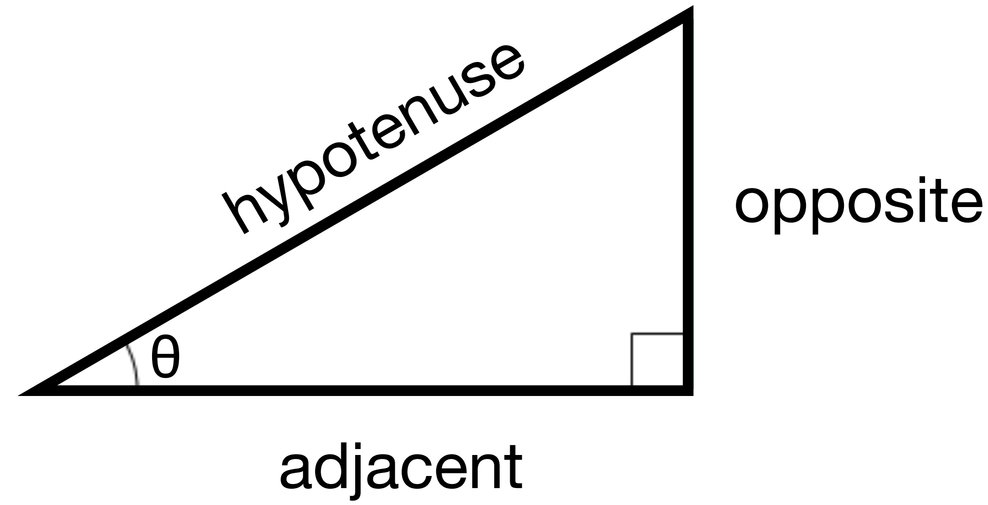


The three main trig functions are:

$$\cos(30°) = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{\sqrt{3}}{2} \qquad \sin(30°) = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{1}{2}$$

$$\tan(30°) = \frac{\text{opposite}}{\text{adjacent}} = \frac{\sin(30°)}{\cos(30°)} = \frac{1}{\sqrt{3}}$$

Relevance to PHY2048? Often we have to compute "*length of a segment given a length of another*". This happesn a lot in vectors, especially. Since we know the ratio between the segments via trig functions, we can compute them.



Following are examples relating one segment length to another

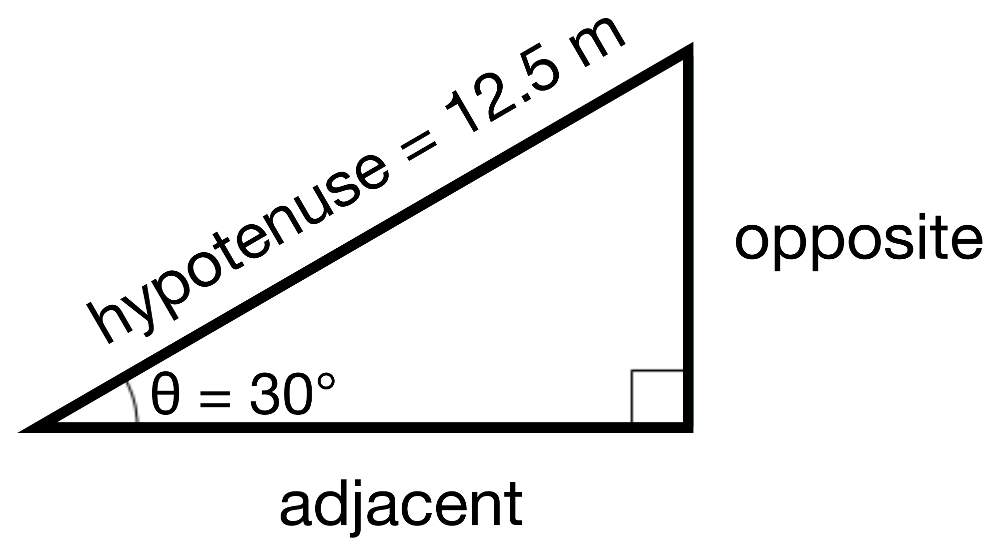
$$\text{hypotenuse} \times \cos(\theta) = \text{adjacent}$$

$$\text{hypotenuse} \times \sin(\theta) = \text{opposite}$$

$$\text{adjacent} \times \tan(\theta) = \text{opposite}$$

Example:

Given a right triangle with θ = 30°, and a known length of hypotenuse of 12.5 meter, find length of adjacent and opposite.



Following are examples relating one segment length to another

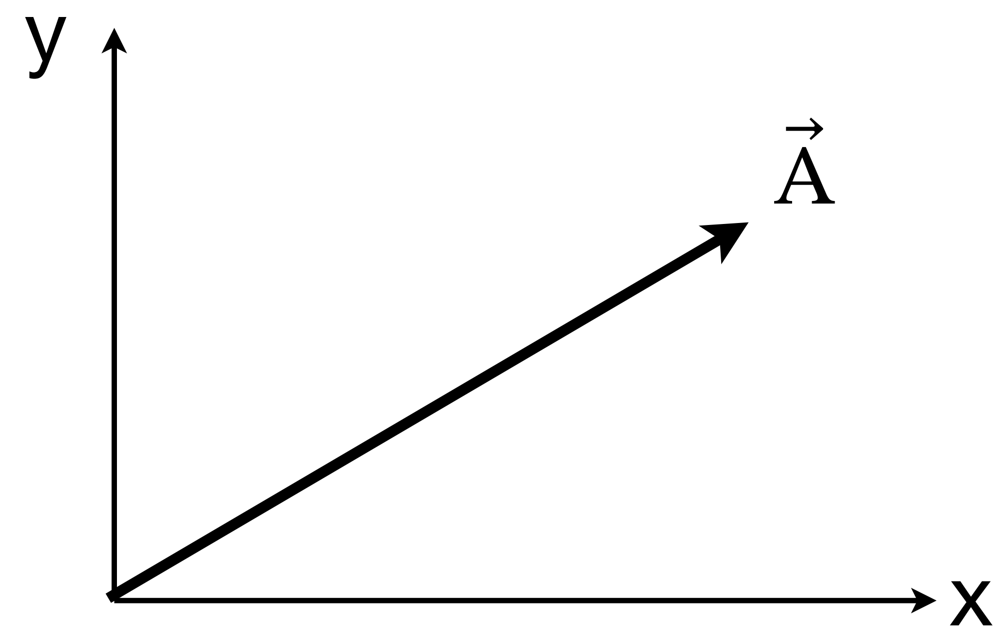
$$12.5\ \text{m} \times \cos(30°) = 12.5\ \text{m} \times \frac{\sqrt{3}}{2} = 21.7\ \text{m} = \text{adjacent}$$

$$12.5\ \text{m} \times \sin(30°) = 12.5\ \text{m} \times \frac{1}{2} = 6.25\ \text{m} = \text{opposite}$$

Knowing hypotenuse length, and one of the angle of right triangle we can compute the length of adjacent and opposite

Application: Vector component calculation

Given a vector, find the x and y component of the vector



To find the component of the vector in x and y axis, we need to ***project*** the vector onto each axis and compute the length of the projected segment.

$\Rightarrow$ This involves trigonometry!

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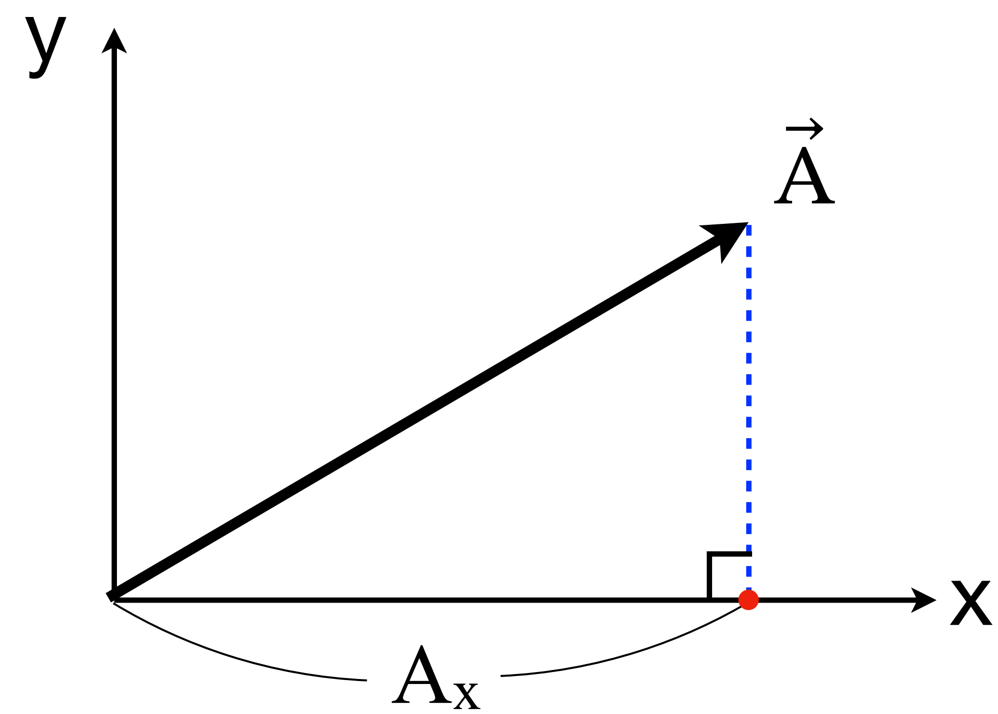

The projection of vector $\vec{A}$ onto x-axis is done via drawing a line perpendicular to the x-axis and finding the point that crosses on the x-axis. Then, we need to find the length indicated by the A_x

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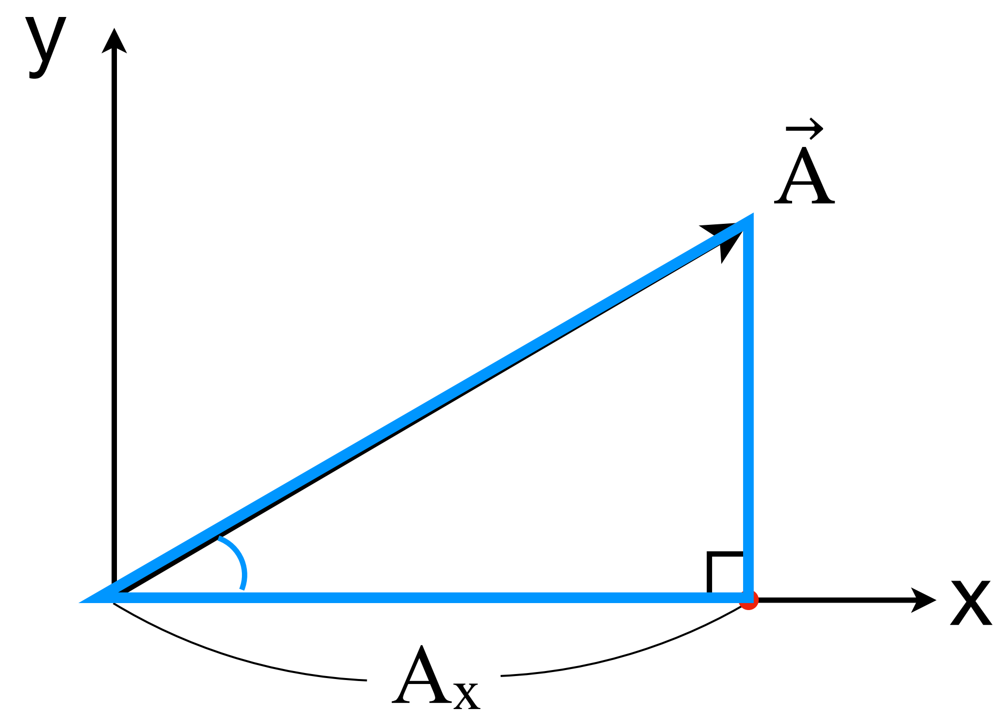

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<u>⇒ The act of projecting produces a right triangle!</u>

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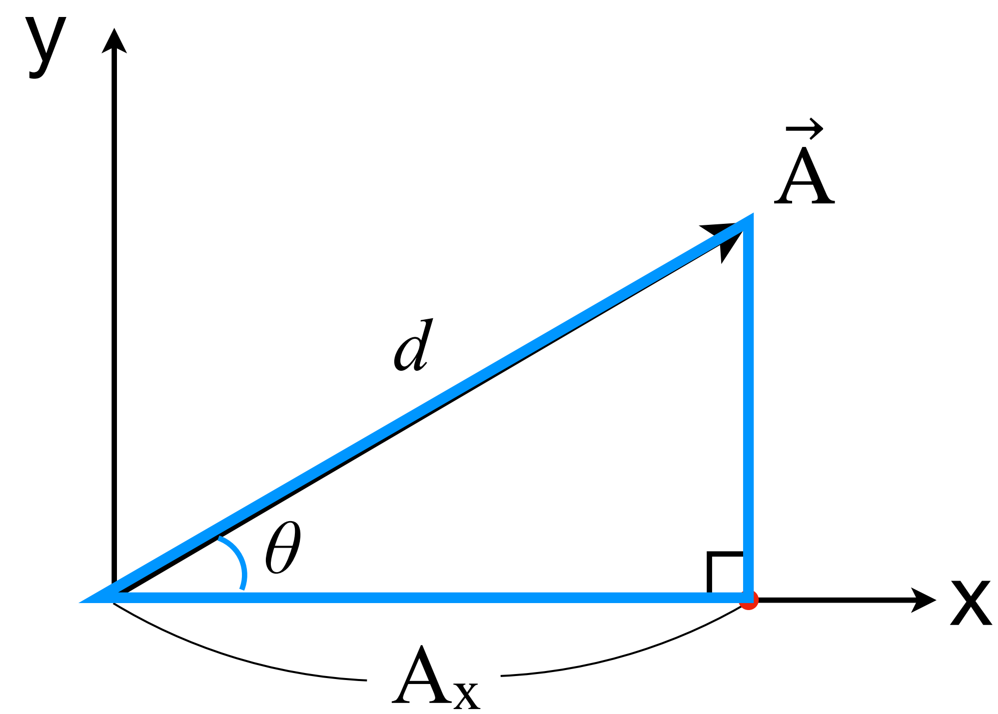

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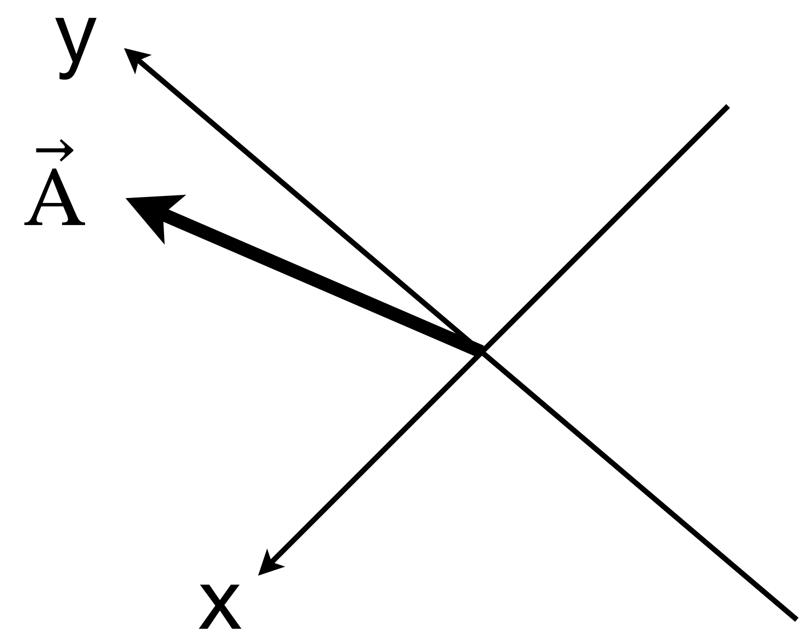
If the length of the vector is d and the angle between x-axis and the vector is θ, the "adjacent" length which is equal to A_x will be:

$$A_x = d\cos(\theta)$$

How it can get complicated:

Later it can get more complicated because the direction of the vector and the x and y-axes are sometimes rotated to be suitable for the physics problems, and therefore the act of projecting can get compliated



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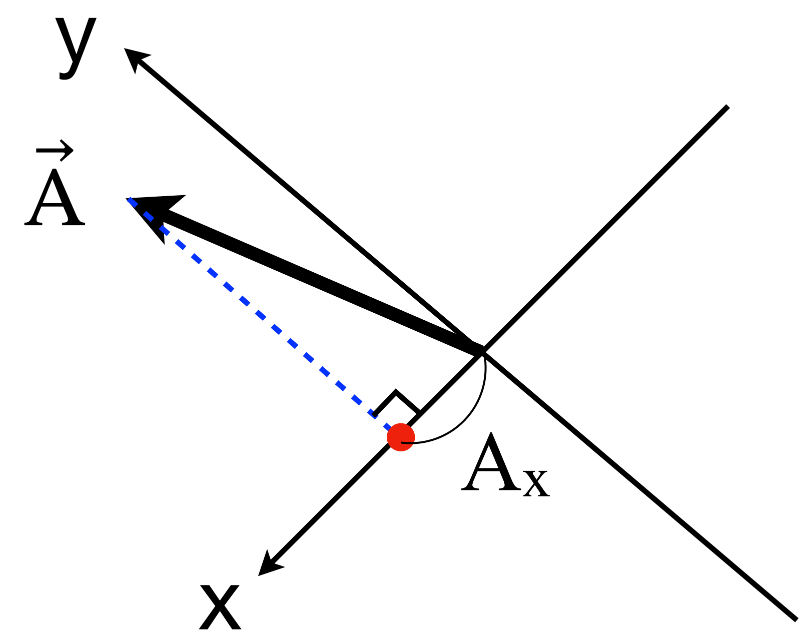

x-axis projection: (1) guideline, (2) crossing point, (3) component

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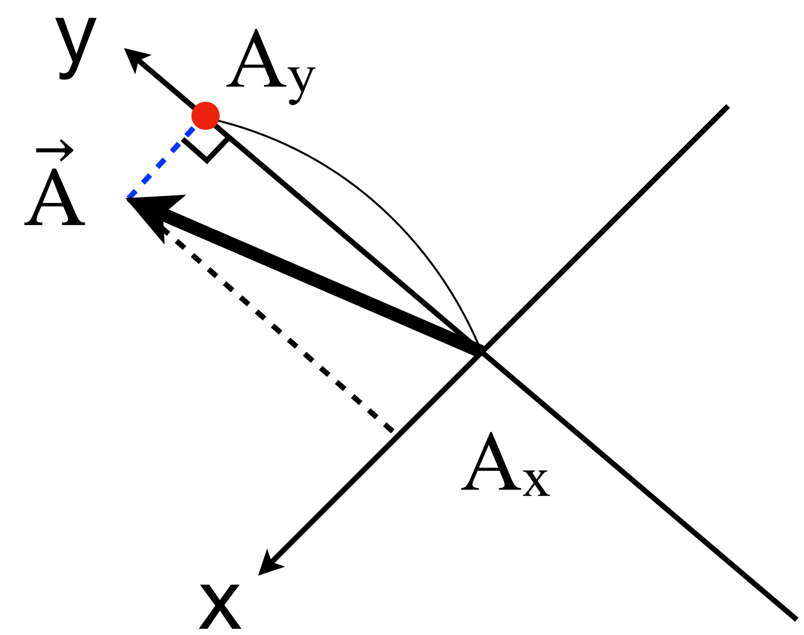

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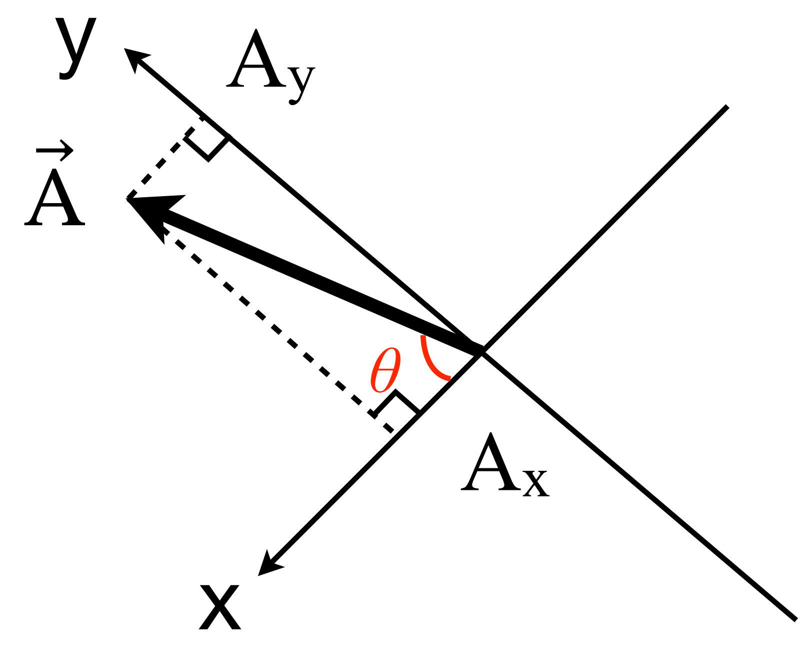

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If the given angle is θ, and the length of the vector is $|A|$, then…

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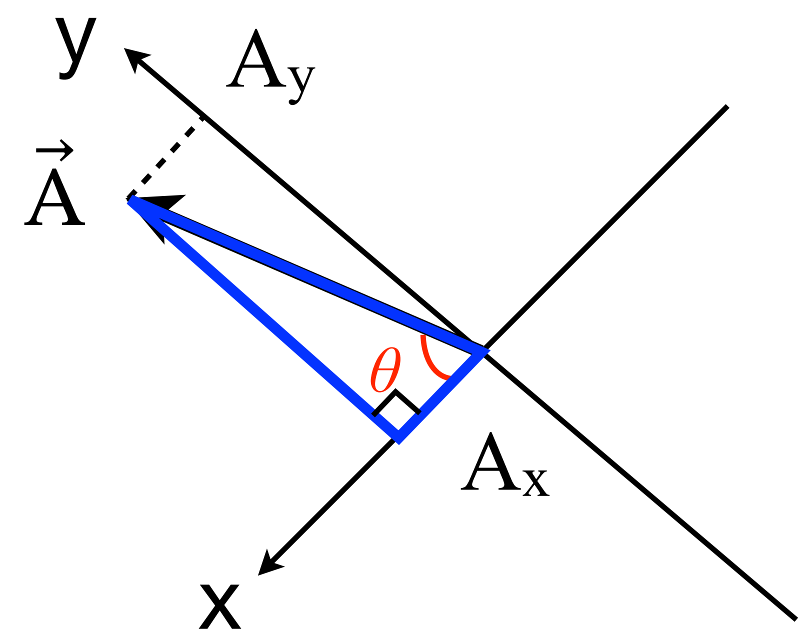

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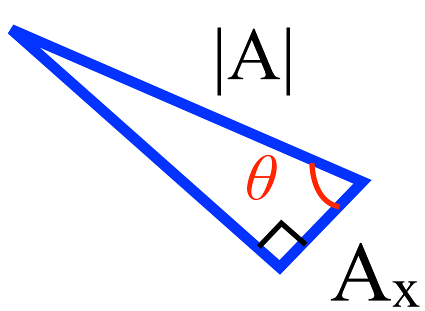

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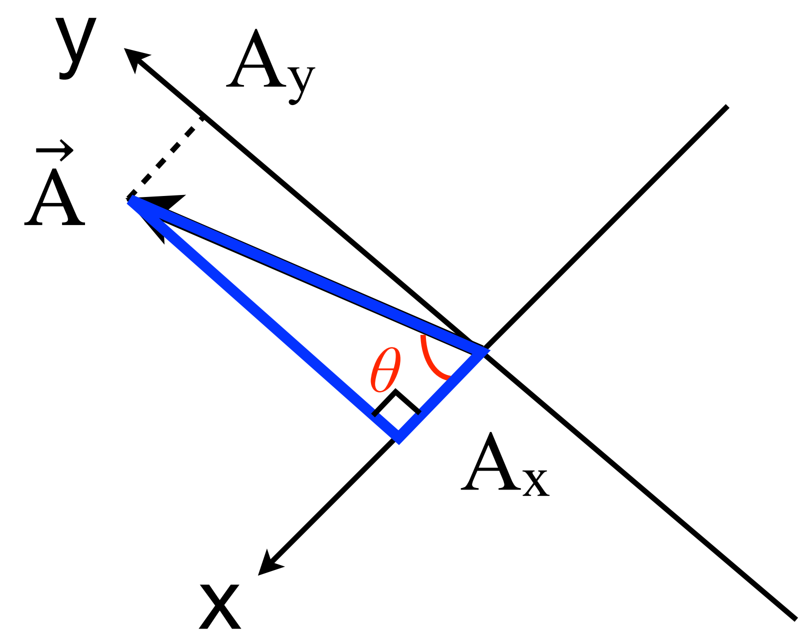

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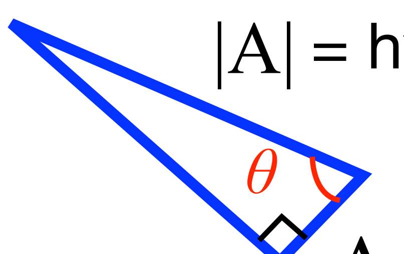

$|A|$ = hypotenuse of the blue triangle

A_x = adjacent of the blue triangle

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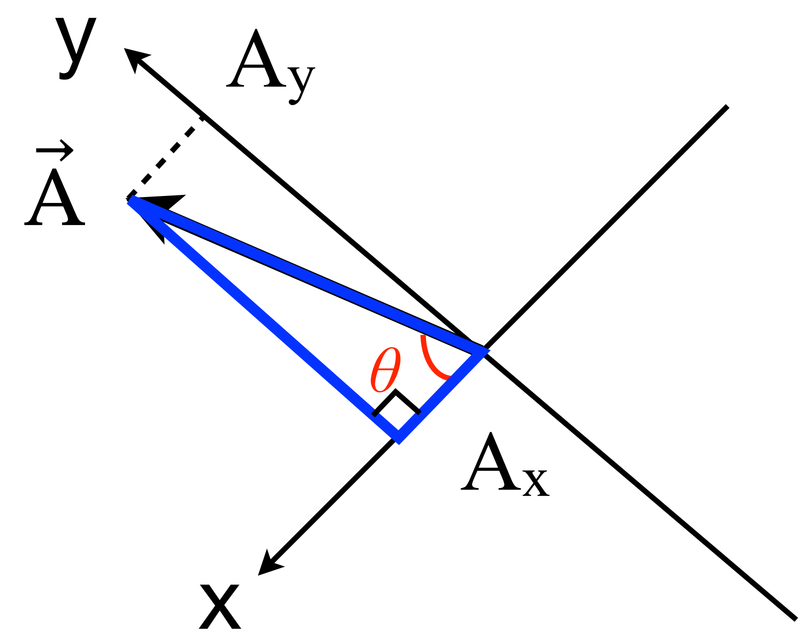

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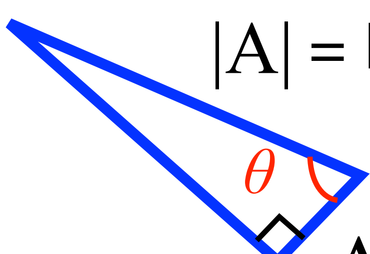

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$$\Rightarrow |A| \cos(\theta) = A_x$$

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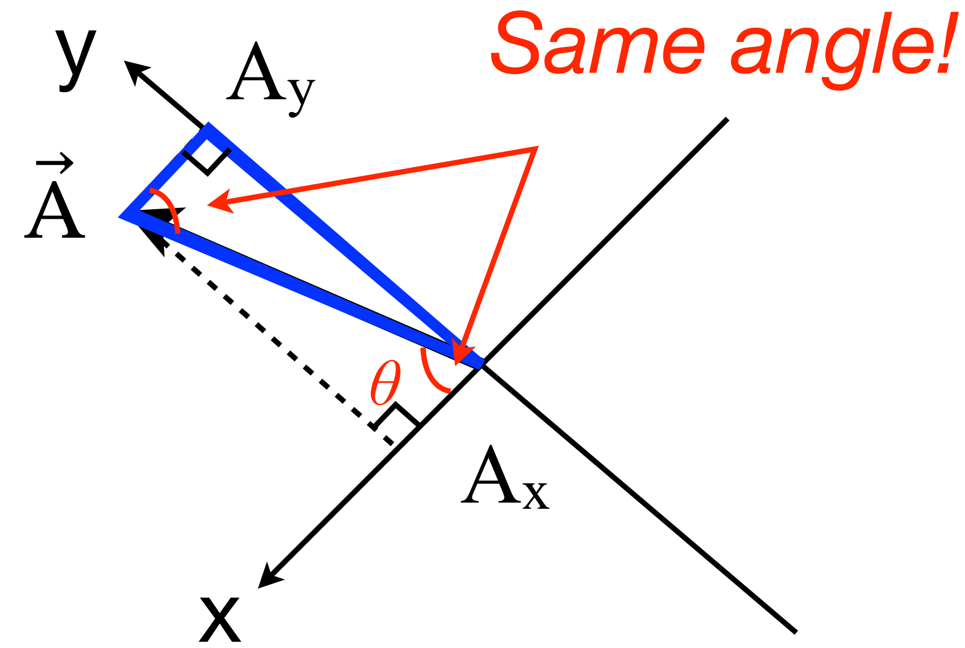

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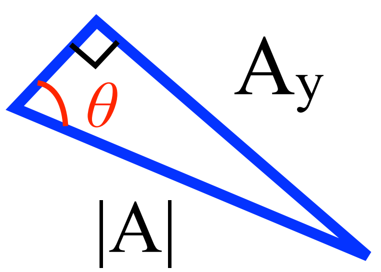
we repeat for A_y

If the given angle is θ, and the length of the vector is $|A|$, then…



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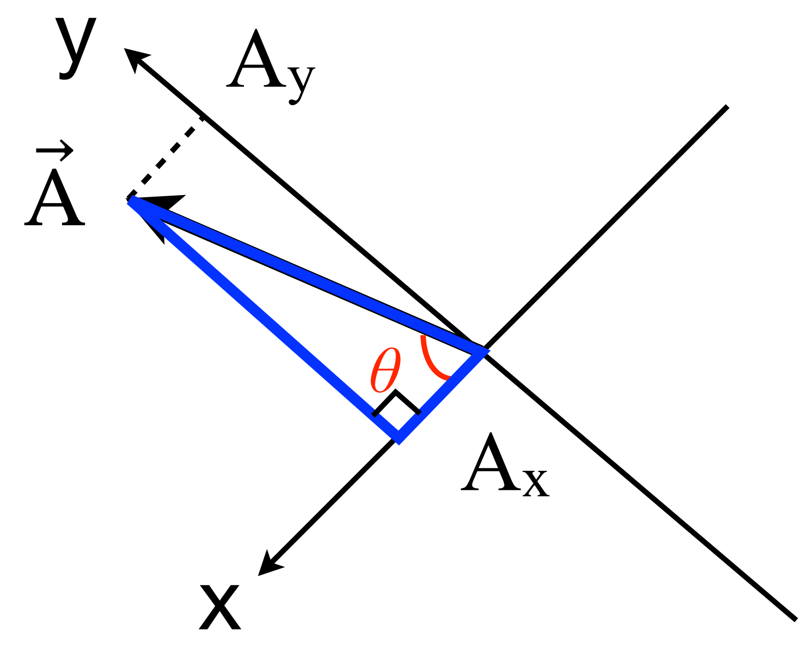

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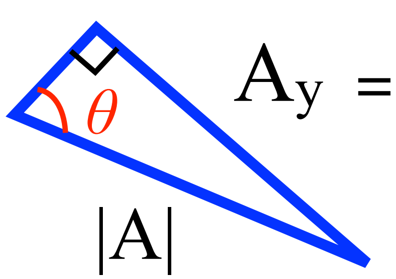

A_y = opposite of the blue triangle

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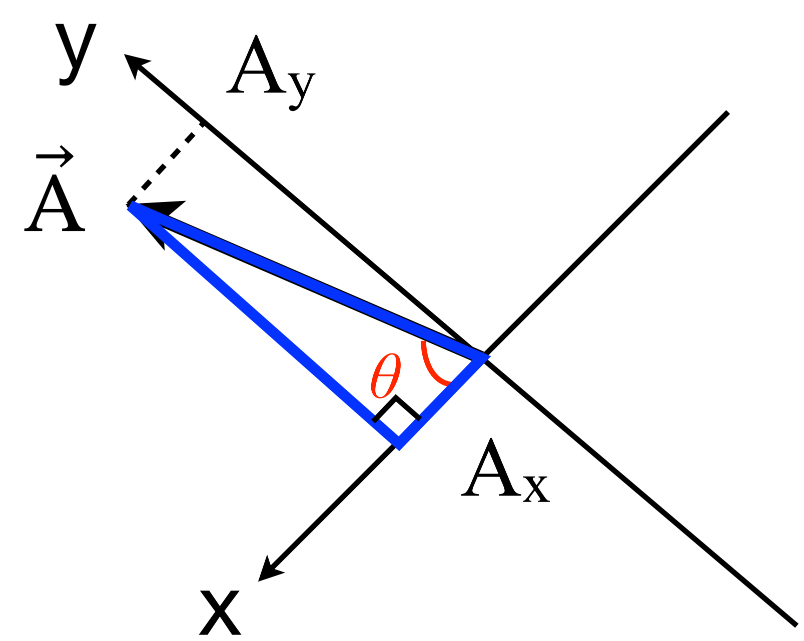

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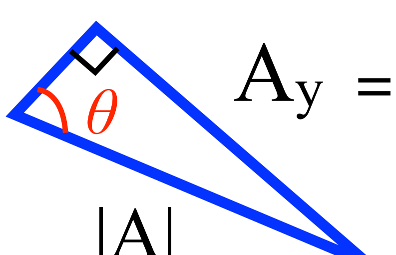


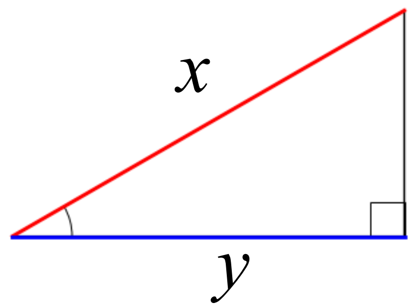
A_y = opposite of the blue triangle

$|A|$ = hypotenuse of the blue triangle

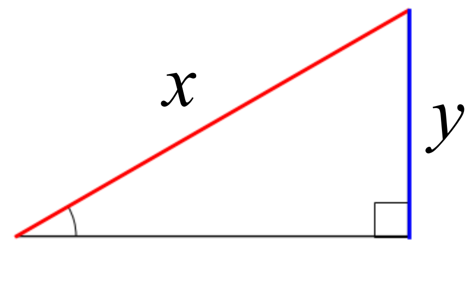
$$\Rightarrow |A| \sin(\theta) = A_y$$

As we have to do this kind of identifying right triangle and apply cos, sin or tan in the right way many many times, it is crucially important that one can apply the trig function properly to find the length of the vector components for this course!

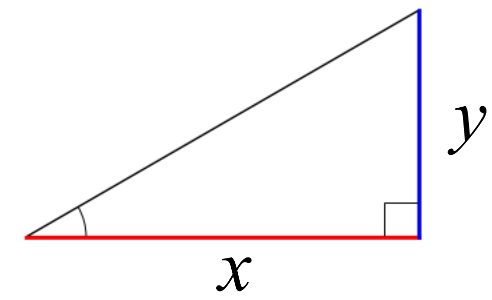
Here are all the six possibilities of how the length relation can be used.
Imagine the length of the red segment is given in each six situations.
I need to find the length of the blue segments.
How is blue length related to red? (N.B. angle marked with the arc is angle = θ)



$$x \cos(\theta) = y$$

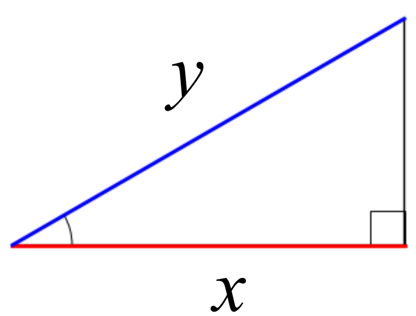


$$x \sin(\theta) = y$$

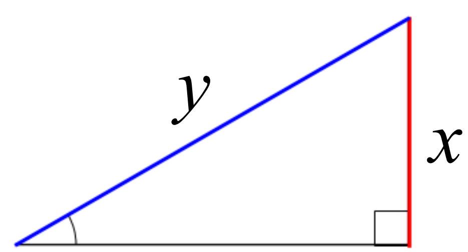


$$x \tan(\theta) = y$$

below are inverse examples ⇒ divide by trig function instead



$$\frac{x}{\cos(\theta)} = y$$



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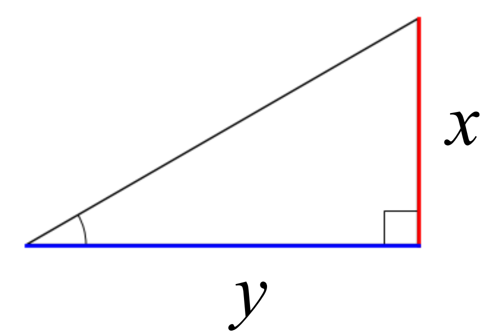
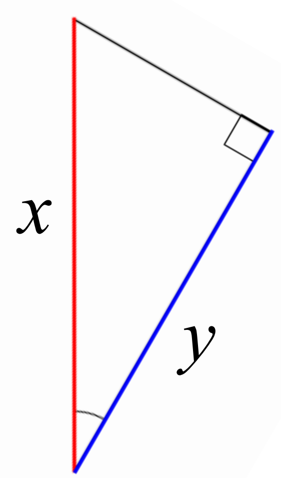


$$\frac{x}{\tan(\theta)} = y$$

Now practice to be able to apply the correctly even if the triangles are rotated!



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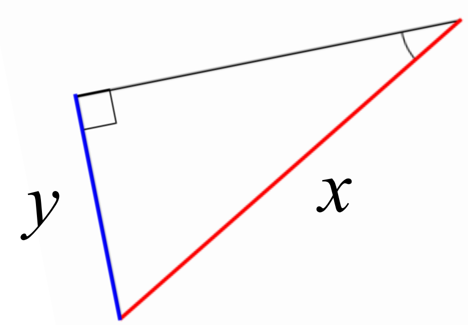

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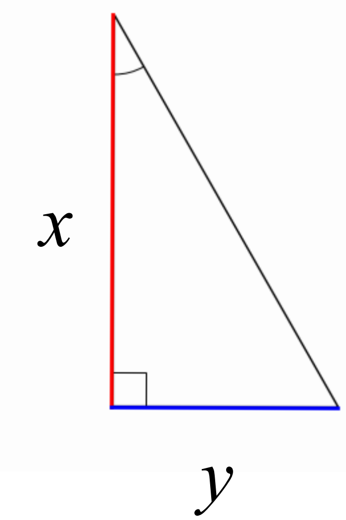

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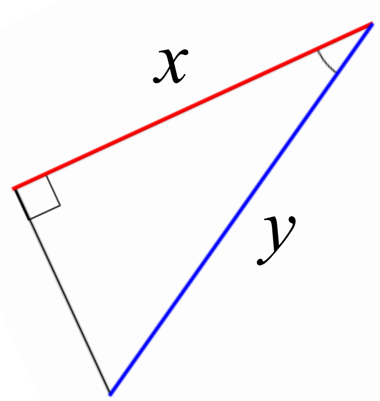

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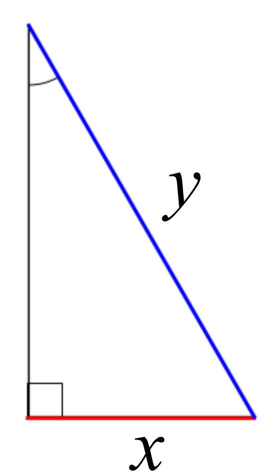

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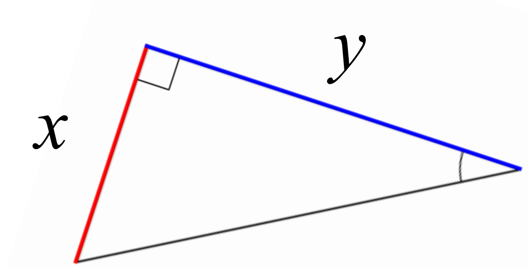

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