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CST SPL 02
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TUTORIAL 2

Q1
```
void fun (int n) {
    int j=1, i=0;
    while (i<n) {
        i+=j;
        j++; }
}
```

Ans

$j=1 \quad i=1$

$j=2 \quad i=1+2$

$j=3 \quad i=1+2+3$

$j=4 \quad i=1+2+3+4$

$j=k \quad i=1+2+3+ \dots +k$

$\therefore$ as $i<n$,

sum of k cons. integer $= \frac{k(k+1)}{2}$

Thus, $\frac{k(k+1)}{2} < n$

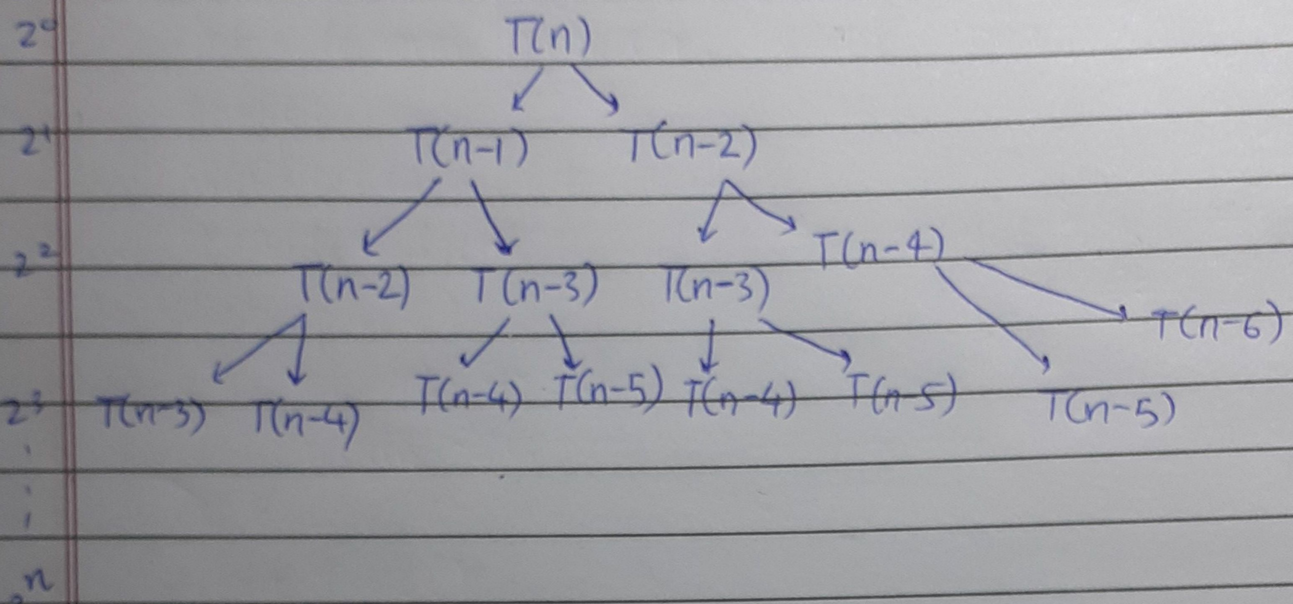
$\frac{k^2+k}{2} < n$ removing constant

$k < \sqrt{n}$

$\therefore \boxed{T(n) = O(\sqrt{n})}$

② Recurrence Relation of fibonacci series:-

$$T(n) = T(n-1) + T(n-2) + 1$$



$$T(n) = 2^0 + 2^1 + 2^2 + 2^3 + \cdots + 2^n$$

Sum of GP $= \dfrac{a(r^n - 1)}{(r-1)}$ Here $a = 2^0 = 1$

$r = 2$

Sum $= \dfrac{1(2^n - 1)}{(2-1)} \Rightarrow 2^n \cancel{-1}$ removing constant

$\therefore \boxed{T(n) = 2^n}$

Space Complexity: Since space complexity depends on the maximum depth of the tree so.

$\boxed{S.C = O(n)}$

③ (a) $T(n) = O(n^3)$

```
for(i=1 ; i<=n ; i++)
 { for(j=1 ; j<=n ; j++)
   { for (k=1 ; k<=n ; k++)
      {  printf("#"); }}}
```

(b) $T(n) = O(n\log n)$

```
for(i=1; i<=n ; i*=2)
 {for(j=1; j <=n; j++)
   { printf("#");
   }
 }
```

(c) $T(n) = O(\log\log(n))$

~~for(i=1; i<=n; i*=2)~~
~~{ for(j=n; j>=1; j=j/2)~~
~~{ printf("#") }}~~

```
for (i= 2, i<=n, i=i^2)
  { printf("#"); }
```

Q4 $T(n) = T(n/4) + T(n/2) + cn^2$

removing lower order terms.

$T(n) = T(n/2) + cn^2$

Using Master's Theorem,

$T(n) = aT(n/b) + f(n)$, $a \geq 1$, $b > 1$

Here $a = 1$, $b = 2$

Now $c = \log_b a = \log_2 1 = 0$

$\therefore n^c = n^0 = 1$

$\therefore n^2 > n^0$

Thus, $T(n) = \Theta(f(n)) = \boxed{\Theta(n^2)}$

6. for (i=2 ; i<=n ; i= i^k)

{ //O(1) }

TC = $2, 2^k, 2^{k^2}, \ldots \ldots 2^{\cancel{k\log_k(\log n)}\, k^{\log_k(\log n)}}$

~~Since~~ $\cancel{2^{k\log_k(\log n)}} = \cancel{2^{\log n} = n}$

Since $2^{k^{\log_k(\log n)}} = 2^{\log n} = n$

So there are in total $\log_k(\log n)$ iterations

$\therefore T(n) = O(\log_k \log(n))$.

(8) (a) $\log(\log n) < 100 < \log(n) < \log^2 n < \sqrt{n} < n < n\log n < n^2 < 2^n$
$< 2^{2^n} = 4^n < \log(n!) < n!$

(b) $1 < \log\log n < \sqrt{\log n} < \log n < \log 2n < 2\log n < n < 2n < 4n <$
$n\log n < n^2 < \log(n!) < 2^{n+1} < \cancel{n} ! \, n!$

(c) $96 < \log_8(n) < \log_2(n) < 5n < n\log_6 n < n\log_2 n \cancel{< n!} < \log(n!)$
$< n! < 8^{2n}$