CS 188 Fall 2021 — Introduction to Artificial Intelligence — Final

- You have approximately 170 minutes.
- The exam is open book, open calculator, and open notes.
- For multiple choice questions,
  - ☐ means mark **all options** that apply
  - ○ means mark a **single choice**

First name	
Last name	
SID	

For staff use only:

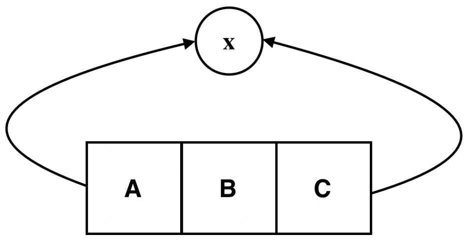
Q1.	Learning to Act	/15
Q2.	Fun with Marbles	/6
Q3.	Course Evaluations	/12
Q4.	Unsearchtainty	/5
Q5.	Phase <3	/11
Q6.	Particle Filtering on an MDP	/16
Q7.	Machine Learning	/10
Q8.	CSPs and Bayes Nets	/13
Q9.	Potpourri	/12
	Total	/100

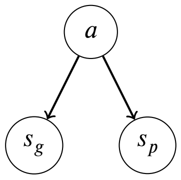
SID: ____________________________

Q1. [15 pts] Learning to Act

In lecture and discussion, we have mainly used the Naive Bayes algorithm to do binary classification, such as classifying whether an email is spam. However, we can also use Naive Bayes to learn how to act in an environment. This problem will explore learning good policies with Naive Bayes and comparing them to policies learned with RL.

We consider the following one-dimensional grid world environment with three squares, named A, B, and C from left to right. Pacman has two possible actions at each square: left and right. Taking the left action at square A or the right action at square C will transition to a terminal state x where no further action can be taken. At each timestep, Pacman observes his own position (s_p) as well as the ghost's position (s_g), and he uses these observations to decide on an action.





(a) In this part, Pacman has no idea about the transition probabilities of the ghost or the rewards it gets. However, Pacman has access to an expert demonstration dataset, which gives reasonably good actions to take in a number of scenarios. The dataset is divided into **training**, **validation**, and **test** datasets. The following is the **training set** of the dataset.

s_p	s_g	a
B	C	left
C	A	left
A	B	left
C	C	right
B	A	right
C	B	right

(i) [1 pt] **Q1.1** Using the standard Naive Bayes algorithm, what are the maximum likelihood estimates for the following conditional probabilities (encoded in the Bayes Net)?

$P(s_p = C \mid a = \text{left}) =$ ______________

$P(s_p = A \mid a = \text{right}) =$ ______________

$P(a = \text{left}) =$ ______________

(ii) [2 pts] **Q1.2** Using the standard Naive Bayes algorithm, which action should we choose in the following new scenarios?

$s_p = A, s_g = C$ ○ Left ○ Right

$s_p = C, s_g = B$ ○ Left ○ Right

(iii) [2 pts] **Q1.3** Suppose we want to add Laplace smoothing with strength k in the Naive Bayes algorithm. (There is no smoothing when $k = 0$.) Which of the following are true?

- ☐ To find the optimal value of k, we pick the value of k which gives the highest accuracy on the training set.
- ☐ To find the optimal value of k, we pick the value of k which gives the highest accuracy on the validation set.
- ☐ To find the optimal value of k, we pick the value of k which gives the highest accuracy on the test set.
- ☐ If $k = 0$, we may observe low accuracy on the test set due to overfitting.
- ☐ If k is a very large integer, the posterior probability for each action will be close to 0.5.
- ○ None of the above.

(iv) [2 pts] **Q1.4** We can extend this approach to supervised learning algorithms other than Naive Bayes, such as logistic regression and neural networks.

Which of the following are true if we use another machine learning algorithm instead of Naive Bayes on a much larger, arbitrary training and test set, assuming that the training set contains no conflicting labels (i.e. two identical data points must have the same class label)?

☐ If we use a binary perceptron, it might not terminate even after a sufficiently long time.

☐ If we use logistic regression, with some suitable hyperparameters it will classify all data points in the training set correctly.

☐ If we use a large neural network, with some suitable hyperparameters it will classify all data points in the training set correctly.

☐ If we use a large neural network, with some suitable hyperparameters we should expect it to classify all the data points in the test set correctly.

○ None of the above

(b) We now switch to using reinforcement learning to solve the problem. Specifically, we will use **model-based reinforcement learning** in this sub-part, though it is definitely possible to use model-free RL algorithms as well. We define a state as a tuple (s_p, s_g).

Assume that we have chosen a proper exploration policy, and collected 8 transition samples as follows: (Note: the data in the previous part should no longer be considered)

s	a	s'	r
(B, C)	right	(C, B)	-1
(C, B)	right	x	1
(B, C)	right	(C, C)	-5
(C, C)	right	x	0
(B, C)	right	(C, B)	-1
(C, B)	left	(B, A)	0
(B, A)	left	(A, A)	-5
(A, A)	left	x	0

(i) [1 pt] **Q1.5** What is the estimation of the following transition probabilities and rewards, based on the samples we collected?

$\hat{T}((B, C), right, (C, B)) =$ ______________

$\hat{R}((B, C), right, (C, B)) =$ ______________

(ii) [2 pts] **Q1.6** Which of the following statements are correct regarding model-based reinforcement learning vs. model-free reinforcement learning?

☐ In model-based RL we explicitly learn a model for the transition probabilities and rewards, while in model-free RL we do not.

☐ In model-based RL we do exploration, while in model-free RL we do not.

☐ Model-based RL is off-policy, while model-free RL is on-policy.

☐ Model-based RL involves pure offline computation, while model-free RL requires online interaction with the environment.

(iii) [2 pts] **Q1.7** We observe that a lot of the states are not covered in our small dataset of transitions. This is a general drawback of the naive model-based reinforcement learning algorithm. (e.g. When we have a larger grid, the size of the state space will be huge, and we need an even larger dataset of transitions).

Which of the following algorithmic improvements that we learned in lecture can be incorporated to alleviate this issue?

☐ Arc-consistency as an improvement of naive back-tracking

☐ Policy iteration as an improvement of value iteration

☐ Approximate Q-learning as an improvement of naive Q-learning

☐ Variable elimination as an improvement of inference by enumeration

(c) Now we have seen supervised learning (part (a)) and reinforcement learning (part (b)), two potential methods to solve the decision-making problem.

For each of the following quantities, determine whether they hold for supervised learning only, RL only, both, or neither.

(i) [1 pt] **Q1.8** This method usually requires data from a near-optimal policy (e.g. a human expert) to work well.
◯ Supervised Learning ◯ Reinforcement Learning ◯ Both ◯ Neither

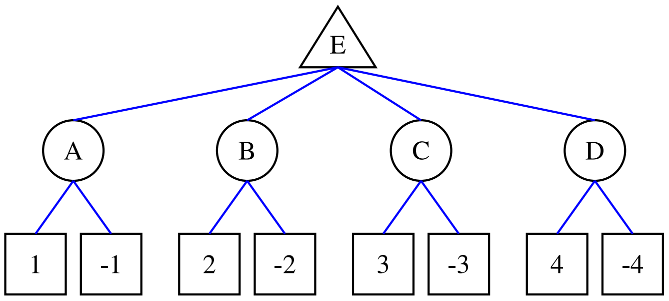
(ii) [1 pt] **Q1.9** This method requires us to know the transition probabilities of the MDP.
◯ Supervised Learning ◯ Reinforcement Learning ◯ Both ◯ Neither

(iii) [1 pt] **Q1.10** This method requires us to design a reward function.
◯ Supervised Learning ◯ Reinforcement Learning ◯ Both ◯ Neither

Q2. [6 pts] Fun with Marbles

Alice is playing a friendly, simplified game of marbles against Bob where each player starts with 10 marbles. Every turn, Alice chooses up to 4 of her marbles to bet, and Bob guesses whether the amount Alice is betting is even (2 or 4) or odd (1 or 3). If Bob guesses correctly, he gets to keep all the marbles that Alice bet and add them to his total count; else, Bob must give Alice the same amount of marbles that was bet. The game ends when one player has all 20 marbles.
Note that Alice and Bob never switch roles: Alice is always choosing marbles and Bob is always guessing at every turn.

(a) [2 pts] **Q2.1** Beginning the game, Alice wants to decide how much she should bet. She assumes that Bob has a 60% chance of picking even and a 40% chance of picking odd, and constructs the following depth 1 expectimax tree based on this first turn, modeling this as a zero-sum game where Alice is the maximizer. For the leaf nodes, she uses an evaluation function that outputs the **difference in marbles** she gets after Bob guesses:



Determine the value of all the nodes in the expectimax tree.

A: ______________

B: ______________

C: ______________

D: ______________

E: ______________

(b) [4 pts] Alice's friend Eve wants to make things more interesting, and offers a large cash prize to the winner of the game. However, for every turn that passes, Eve will reduce the prize by multiplying the amount remaining by some $\gamma \in (0, 1)$.

To model this, Alice incorporates γ into the tree, multiplying the values propagated up at each depth by γ.

For each letter **(A), (B), (C), (D), (E), (F), (G), (H)** select a single entry for the term corresponding to the correct equation to calculate the value of Alice's node represented as $V_A(s)$, where s represents the current Alice node. Denote the set of Alice's possible actions as a and the set of Bob's possible actions as b. Further, let $T_A(s, a, s')$ represent Alice's transition probability for taking an action a from state s to s' and let $T_B(s, b, s')$ represent Bob's transition probability of action b from some state s to s'.

$$V_A(s) = \textbf{(A)} \sum_{s'} \textbf{(B)}\ \textbf{(C)}\ \textbf{(D)}\ \textbf{(E)}\ \big[\textbf{(F)} + \textbf{(G)}\ V_A(\textbf{(H)})\big]$$

Q2.2 (A) ○ $\max_a$ ○ $\max_b$ ○ $\max_{s'}$ ○ $\max_{s''}$ ○ $\sum_a$ ○ $\sum_b$ ○ $\sum_{s'}$ ○ $\sum_{b,s''}$ ○ 1

Q2.3 (B) ○ $T_A(s, a, s')$ ○ $T_A(s', a, s'')$ ○ $T_B(s, b, s')$ ○ $T_B(s', b, s'')$ ○ 1

Q2.4 (C) ○ $\max_a$ ○ $\max_b$ ○ $\max_{s'}$ ○ $\max_{b,s''}$ ○ $\sum_a$ ○ $\sum_b$ ○ $\sum_{s'}$ ○ $\sum_{b,s''}$ ○ 1

Q2.5 (D) ○ $P(a|s')$ ○ $P(b|s')$ ○ $P(a, b|s')$ ○ $P(a, b|s'')$ ○ 1

Q2.6 (E) ○ $T_A(s, a, s')$ ○ $T_A(s', a, s'')$ ○ $T_B(s, b, s')$ ○ $T_B(s', b, s'')$ ○ 1

Q2.7 (F) ○ γ ○ $T_A(s, a, s')$ ○ $T_A(s', a, s'')$ ○ $T_B(s, b, s')$ ○ $T_B(s', b, s'')$ ○ 0 ○ 1

Q2.8 (G) ○ γ ○ $T_A(s, a, s')$ ○ $T_A(s', a, s'')$ ○ $T_B(s, b, s')$ ○ $T_B(s', b, s'')$ ○ 0 ○ 1

Q2.9 (H) ○ s ○ s' ○ s'' ○ 1

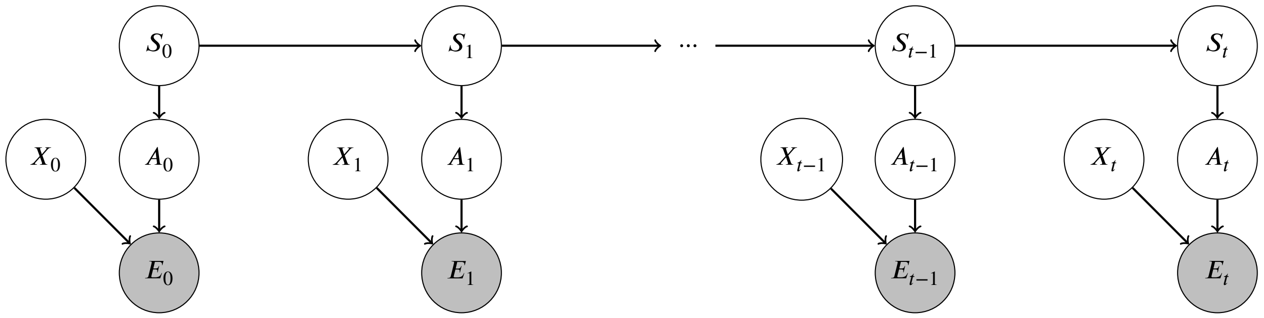
SID: ____________________________

Q3. [12 pts] Course Evaluations

Every semester we try to make CS 188 a little better. Let S_t represent the quality of the CS 188 offering at semester t, where $S_t \in \{1, 2, 3, 4, 5\}$. Changes to the course are incremental so between semester t and $t+1$, the value of S can only change by at most 1. Each possible transition occurs with equal probability. As examples: if $S_t = 1$, then $S_{t+1} \in \{1, 2\}$ each with probability 1/2. If $S_t = 2$, then $S_{t+1} \in \{1, 2, 3\}$ each with probability 1/3.

Let $E_t \in \{+e, -e\}$ represent the feedback we receive from student evaluations for the semester t, where $+e$ is generally positive feedback and $-e$ is negative feedback. Student feedback is dependent on whether released assignments were helpful for the given semester ($A_t \in \{+a, -a\}$) which is dependent on the quality of the semester's course offering (S_t). Additionally, student evaluations are also dependent on events external to the class ($X_t = \{+x, -x\}$).

The following HMM depicts the described scenario:



(a) Consider the above dynamic bayes net which ends at some finite timestep t. In this problem, we are trying to approximate the most likely value of S_t given all the evidence variables up to and including t. For each of the following subparts, first decide whether the given method can be used to solve this problem. Then, if yes, select all CPTs which must be known to run the algorithm.

(i) [1 pt] **Q3.1** Variable elimination
○ No ○ Yes: ☐ $P(S_0)$, $P(S_t|S_{t-1}), t > 0$ ☐ $P(E_t|X_t, A_t) \forall t$ ☐ $P(A_t|S_t) \forall t$ ☐ $P(X_t) \forall t$

(ii) [1 pt] **Q3.2** Value iteration
○ No ○ Yes: ☐ $P(S_0)$, $P(S_t|S_{t-1}), t > 0$ ☐ $P(E_t|X_t, A_t) \forall t$ ☐ $P(A_t|S_t) \forall t$ ☐ $P(X_t) \forall t$

(iii) [1 pt] **Q3.3** Gibbs sampling
○ No ○ Yes: ☐ $P(S_0)$, $P(S_t|S_{t-1}), t > 0$ ☐ $P(E_t|X_t, A_t) \forall t$ ☐ $P(A_t|S_t) \forall t$ ☐ $P(X_t) \forall t$

(iv) [1 pt] **Q3.4** Prior sampling
○ No ○ Yes: ☐ $P(S_0)$, $P(S_t|S_{t-1}), t > 0$ ☐ $P(E_t|X_t, A_t) \forall t$ ☐ $P(A_t|S_t) \forall t$ ☐ $P(X_t) \forall t$

(v) [1 pt] **Q3.5** Particle Filtering
○ No ○ Yes: ☐ $P(S_0)$, $P(S_t|S_{t-1}), t > 0$ ☐ $P(E_t|X_t, A_t) \forall t$ ☐ $P(A_t|S_t) \forall t$ ☐ $P(X_t) \forall t$

(b) For the HMM shown above, determine the correct recursive formula for the belief distribution update from $B(S_{t-1})$ to $B(S_t)$. Recall that the belief distribution $B(S_t)$ represents the probability $P(S_t|E_{0:t})$ and involves two steps: **(i)** Time elapse and **(ii)** Observation update.

$$B(S_t) \propto \underline{\quad\textbf{(ii)}\quad} \cdot \underline{\quad\textbf{(i)}\quad}$$

(i) [1 pt] **Q3.6** Time elapse

○ $\sum_{S_{t-1}} P(S_t|S_{t-1})B(S_{t-1})$

○ $\sum_{S_{t-1}} \sum_{A_{t-1}} P(S_t|S_{t-1})P(A_{t-1}|S_{t-1})B(S_{t-1})$

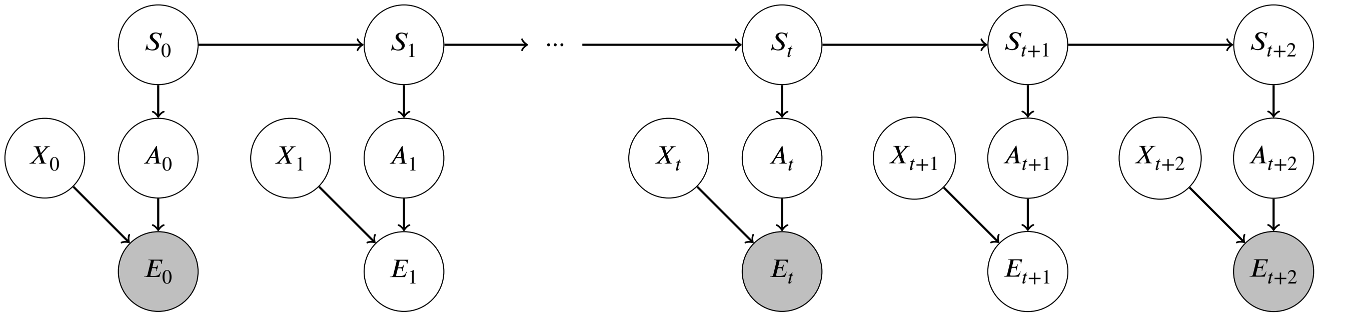
○ $\sum_{S_{t-1}} P(S_t|S_{t-1})P(A_t|S_t)B(S_{t-1})$

○ $\sum_{S_{t-1}} \sum_{A_{t-1}} P(S_t|S_{t-1})P(A_{t-1}|S_{t-1})P(A_t|S_t)B(S_{t-1})$

(ii) [1 pt] **Q3.7** Observation update

- ○ $P(E_t|X_t, A_t)$
- ○ $\sum_{x \in X_t} \sum_{a \in A_t} P(E_t|x, a)$
- ○ $\prod_{x \in X_t} \prod_{a \in A_t} P(E_t|x, a)$
- ○ $P(E_t|X_t, A_t)P(X_t)P(A_t|S_t)$
- ○ $\sum_{x \in X_t} \sum_{a \in A_t} P(E_t|x, a)P(x)P(a|S_t)$
- ○ $\prod_{x \in X_t} \prod_{a \in A_t} P(E_t|x, a)P(x)P(a|S_t)$

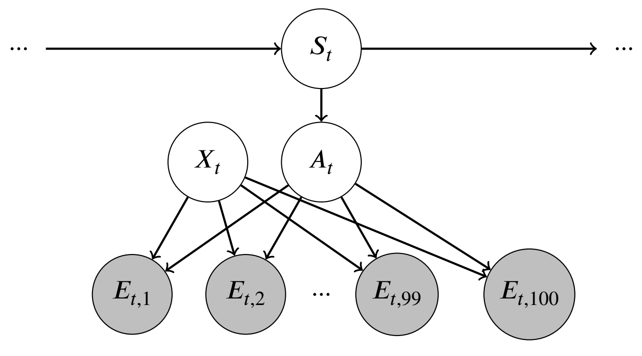
Due to the differences between CS188 offerings in the fall and spring semesters, we realize that only student evaluations from past fall semesters are accurate enough to be incorporated into our model. Assume that a fall semester occurs during an even timestep and that t is even in the diagram below. The new HMM can be repesented as follows:



(c) [2 pts] **Q3.8** In this question, we are trying to derive a recursive formula for the two-step belief distribution update from $B(S_t)$ to $B(S_{t+2})$ for the new problem described above. Which of the following steps represent the **correct and most efficient** method of performing HMM updates to get the belief distribution at S_{t+2} from the current belief at S_t?
For the following notation, let $B(S_t) = P(S_t|E_{0:t:2})$ and $B'(S_t) = P(S_t|E_{0:t-1:2})$ where $E_{0:i:2}$ represents the set of all evidence variables at even timesteps up to i. Further, let $O(E_t)$ represent the value of the observation update expression from the previous part **(ii)**. (Note that $O(E_{t+1})$ and $O(E_{t+2})$ would represent the appropriate observation update expressions for timestep $t + 1$ and $t + 2$ respectively.)

- ○ $B'(S_{t+1}) = \sum_{S_t} P(S_{t+1}|S_t)B(S_t)$
  $B'(S_{t+2}) = \sum_{S_{t+1}} P(S_{t+2}|S_{t+1})B'(S_{t+1})$
  $B(S_{t+2}) \propto O(E_{t+2})B'(S_{t+2})$
- ○ $B'(S_{t+1}) = \sum_{S_t} P(S_{t+1}|S_t)B(S_t)$
  $B(S_{t+1}) = \sum_{E_{t+1}} O(E_{t+1})B'(S_{t+1})$
  $B'(S_{t+2}) = \sum_{S_{t+1}} P(S_{t+2}|S_{t+1})B(S_{t+1})$
  $B(S_{t+2}) \propto O(E_{t+2})B'(S_{t+2})$
- ○ $B'(S_{t+2}) = \sum_{S_{t+1}} \sum_{S_t} P(S_{t+2}|S_{t+1})P(S_{t+1}|S_t)B(S_t)$
  $B(S_{t+2}) \propto O(E_{t+2})B'(S_{t+2})$
- ○ $B'(S_{t+1}) = \sum_{S_t} P(S_{t+1}|S_t)B(S_t)$
  $B(S_{t+1}) \propto O(E_{t+1})B'(S_{t+1})$
  $B'(S_{t+2}) = \sum_{S_{t+1}} P(S_{t+2}|S_{t+1})B(S_{t+1})$
  $B(S_{t+2}) \propto O(E_{t+2})B'(S_{t+2})$
- ○ None of the above.

(d) Now consider a scenario where instead of getting one general student feedback as evidence (E_t), we instead get individual student feedback from 100 students. Let the variable $E_{t,n}$ represent the evidence from student n at timestep t. Assume that the new evidence variables ($E_{t,n}$) can each take on the value of $+e$ or $-e$ with the same probability distribution as the single variable case (E_t).



(i) [2 pts] **Q3.9** Which of the following statements are true regarding this new setup?

☐ The evidence variables **within the same timestep** are independent of each other ($E_{t,j} \perp\!\!\!\perp E_{t,k}\ \forall j \neq k$).

☐ The evidence variables **between any two different timesteps** are independent of each other ($E_{t_1,j} \perp\!\!\!\perp E_{t_2,k}\ \forall t_1 \neq t_2$).

☐ The expression to calculate the time elapse step from S_t to S_{t+1} for this new setup will be the same as the time elapse expression in the case of one evidence from Q3.6.

○ None of the above.

(ii) [1 pt] **Q3.10** In the observation update at timestep t, we receive as evidence 60 positive evaluations ($+e$) and 40 negative evaluations ($-e$). Let x be the observation update probability at timestep t of observing one positive evaluation ($x = O(E_t = +e)$). Now, let $f(x)$ represent the new observation update expression for the case with the observed 100 evidence variables. Which of the following functions f gives the correct observation update for the new scenario? For this part only, regardless of your previous answer, please assume that each students' feedback is **independent** of each other.

○ $f(x) = x^{100}$

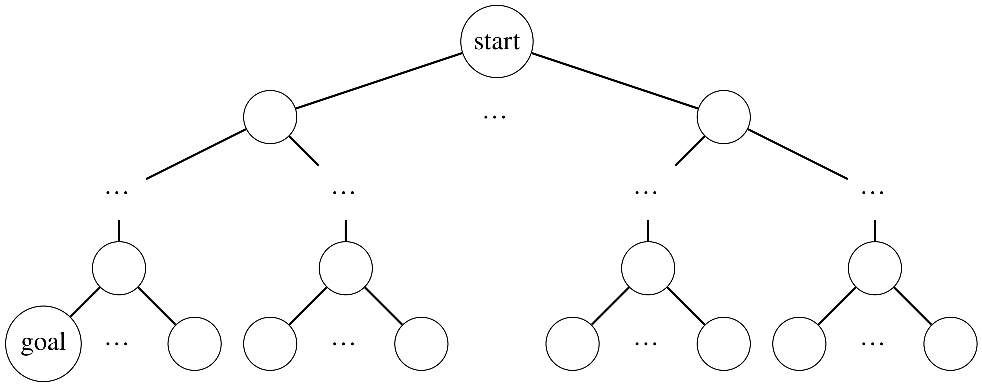
○ $f(x) = 60x \cdot 40(1 - x)$

○ $f(x) = x^{60} \cdot (1 - x)^{40}$

○ None of the above.

Q4. [5 pts] Unsearchtainty

(a) You have a tree-structured undirected graph with branching factor b and depth d as shown below (where we consider the root to be at depth 0). You want to run A* **graph** search on this graph where your start state is the root node and there is exactly one goal node, which is at a leaf. All edges have cost 1 to traverse. You can traverse each edge either way: from the parent to its child, or from the child to its parent.



For the following two subparts, determine how many nodes would be expanded if you used the given heuristics. Recall that $h^*(n)$ represents the true cost of reaching the goal from a node n.

(i) [1 pt] **Q4.1** $h(n) = 0$

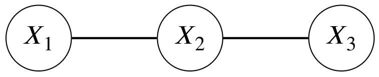
◯ $O(d)$	◯ $O(b^{d/2})$	◯ $O(b \log d)$
◯ $O(bd)$	◯ $O(b^{d/3})$	◯ $O((\log b)^d)$
◯ $O(b^d)$	◯ $O(b^{d/4})$	◯ $O(b^{\log d})$

(ii) [1 pt] **Q4.2** $h(n) = h^*(n)$

◯ $O(d)$	◯ $O(b^{d/2})$	◯ $O(b \log d)$
◯ $O(bd)$	◯ $O(b^{d/3})$	◯ $O((\log b)^d)$
◯ $O(b^d)$	◯ $O(b^{d/4})$	◯ $O(b^{\log d})$

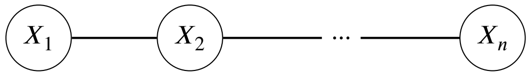
(b) This subpart is completely unrelated to the previous one. In this subpart, you are given a chain of nodes where the edge directions are unknown. For each of the following cases, assume that the direction of each edge is uniformly random and that the direction of any two edges is independent of each other.

(i) [1 pt] **Q4.3** Consider the following chain of 3 nodes.



What is the probability that X_1 and X_3 are guaranteed to be independent? ______________

(ii) [2 pts] **Q4.4** Now consider the same graph but with a chain of n nodes as shown below.



How many directed graphs can be formed where X_1 and X_n are **not guaranteed** to be independent? (You may use the variable n in your answer.)

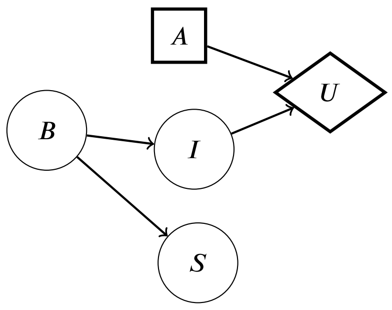
____

Q5. [11 pts] Phase <3

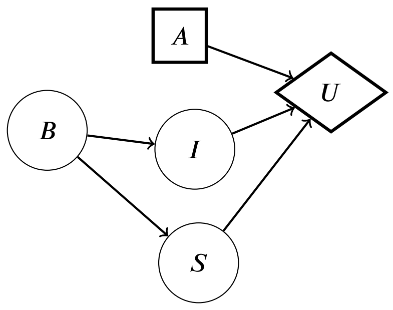
The EECS department decides to offer an exciting new course, CS1888, next semester. You and your friends are deciding if you want to take it or not, and decide to draw inspiration from CS188 and model this using decision networks. For all of the following parts, A represents the action of taking CS1888 or not, and U represents the utility function of a specific student.

(a) [1 pt] **Q5.1** Your friend Lexy will only take CS1888 if a specific instructor will be teaching it (I), and doesn't care about the course size or curriculum. However, due to uncertainty in instructor hiring practices in the university, the instructor chosen to teach is affected by the outcome of instructor collective bargaining efforts (B). The collective bargaining outcome will also affect the size of the course (S).

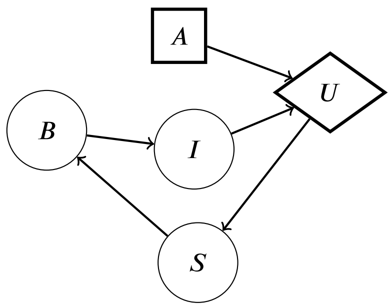
Select all of the decision networks which can represent Lexy's decision.



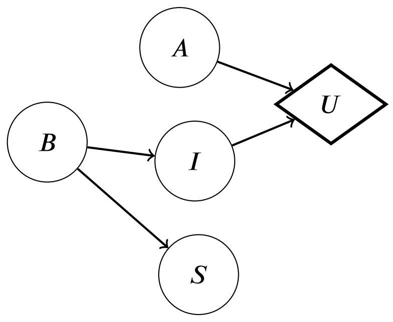
(1)



(2)



(3)

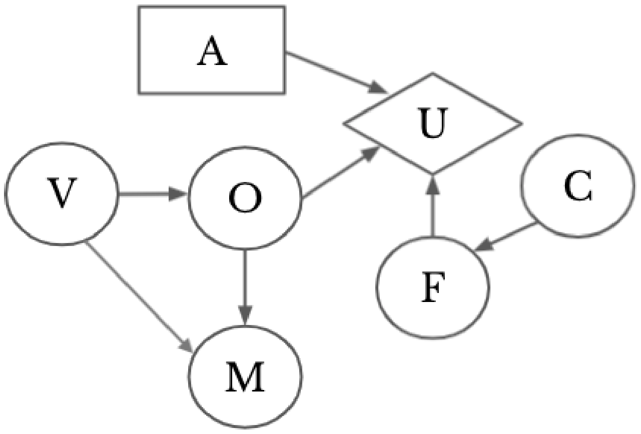


(4)

☐ **(1)**
☐ **(2)**
☐ **(3)**
☐ **(4)**
○ None of the above

Your friend Varun is also making the same decision. His decision is represented by the following decision network, where the chance nodes correspond to the following random variables:

- O: if classes will be online next semester
- V: the current virus situation
- M: the chancellor's message about how optimistic the university is about the virus situation
- F: whether Varun's friend takes CS1888
- C: whether Varun's friend's crush takes CS1888



Each variable has a binary domain. The conditional probability tables of variables are known, but not represented here.

(b) [2 pts] **Q5.2** We want $P(F, O|c)$, where c is a value that C can take on. Which of the following algorithms could be used for calculating this?

☐ Forward Algorithm
☐ Prior Sampling
☐ Laplace Smoothing
☐ Inference by Enumeration
☐ Particle Filtering
◯ None of the Above

(c) Varun wants to reason about the VPI of observing different evidence variables. For the following statements, select if they are always, sometimes, or never true.

(i) [1 pt] **Q5.3** $VPI(F) \geq VPI(C)$

◯ Always true
◯ Sometimes true
◯ Never true

(ii) [1 pt] **Q5.4** $VPI(O|V) + VPI(V|O) > VPI(V, O)$

◯ Always true
◯ Sometimes true
◯ Never true

(d) [2 pts] **Q5.5** Varun peeks at his friend's crush's CalCentral and sees whether she is taking CS1888 ($C = c'$). Which of the following formulas represents the highest utility he has now?

◯ $\max_a \sum_f \sum_o P(f, o|c')U(f, o, a)$

◯ $\max_a \sum_f P(f|c')U(f, a)$

◯ $\sum_c P(c)[\max_a \sum_f \sum_o P(f, o|c')U(f, o, a)]$

◯ $\sum_c P(c)[\max_a \sum_f P(f|c')U(f a)]$

(e) In the following question, we will investigate the idea of sampling applied in both Particle Filtering and Likelihood weighting applications.

(i) [2 pts] **Q5.6** Which of the following statements are true? Recall that in the particle filtering algorithm, we resample each particle at every timestep according to its weight.

☐ HMMs require keeping track of probabilities over time, and particle filtering provides a more efficient method than the forward algorithm to approximate the true probability distribution.

☐ HMMs have a **constant** transition and sensor model across all timesteps, which is necessary to perform resampling at every iteration of particle filtering.

☐ Resampling is necessary in particle filtering to ensure particles start each iteration with weight 1 while still representing a correct observation-updated distribution.

☐ Solving an HMM using particle filtering without the resampling step (instead storing the product of a particle's weight across all timesteps as the probability of that particle's state) would be equivalent to running the forward algorithm on an HMM.

○ None of the above.

(ii) [2 pts] **Q5.7** Consider the following modification of likelihood weighting: after getting an initial n samples and corresponding weights, resample a new set of n samples from the normalized weight distribution of the original samples. For the resulting n resampled samples, consider the following options:

- Method 1: Reweight each resampled sample with weight 1.
- Method 2: Keep the original weights of each resampled sample (ie. each sample still has weight equal to the probability of evidence given parents)

Select all the statements below that are true regarding either resampling method applied to likelihood weighting.

☐ Method 1 would give a sampling method which is **consistent** with the underlying distribution.

☐ Method 2 would give a sampling method which is **consistent** with the underlying distribution.

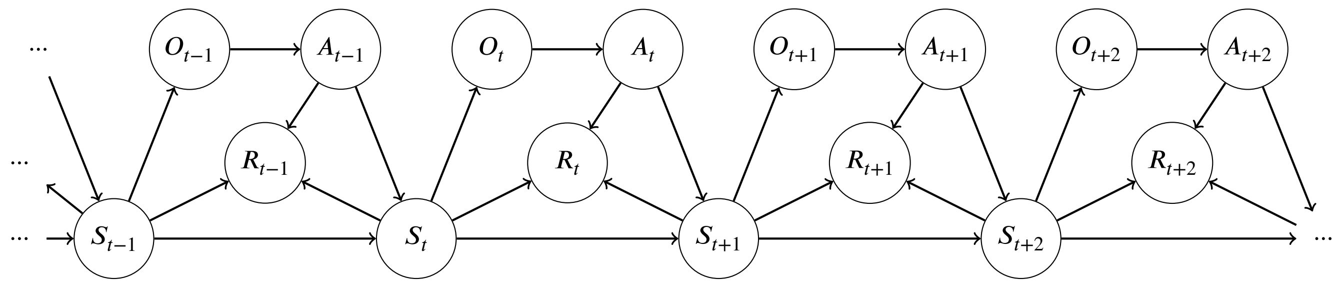
☐ Using resampling reduces the potentially negative effect of downstream evidence on likelihood weighting calculations.

○ None of the above.

Q6. [16 pts] Particle Filtering on an MDP

An agent is acting in an environment where they get noisy observations about their underlying state. The agent makes actions in a fixed, stochastic way based on their observation in the given timestep. The reward depends on the current state, the action taken, and the next state; however, unlike the MDPs we've seen in lecture, the reward function is **non-deterministic**.

We can represent this scenario by the Dynamic Bayes Net below:



(a) **(i)** [3 pts] **Q6.1** Which of the following independence assumptions are guaranteed to be true?

☐ $S_{t-1} \perp\!\!\!\perp S_{t+1} \mid S_t$

☐ $O_{t-1} \perp\!\!\!\perp O_{t+1} \mid O_t$

☐ $A_{t-1} \perp\!\!\!\perp A_{t+1} \mid A_t$

☐ $R_{t-1} \perp\!\!\!\perp R_{t+1} \mid R_t$

☐ $A_{t-1} \perp\!\!\!\perp A_{t+1} \mid A_t, O_{0:t}$

☐ $A_{t-1} \perp\!\!\!\perp A_{t+1} \mid A_t, O_{0:t+1}$

☐ $A_{t-1} \perp\!\!\!\perp A_{t+1} \mid A_t, O_{0:\infty}$

☐ $R_{t-1} \perp\!\!\!\perp R_{t+1} \mid S_t$

(b) The agent wants to create a recursive algorithm to estimate its underlying state S_t. The agent knows what the observations, previous actions, and previous rewards are, but doesn't know the underlying states.

Remember, at a given timestep, the agent uses the observation for that timestep and (probabilistically) chooses the action to perform. It then observes the reward.

(i) [3 pts] **Q6.2** Determine an expression for the recursive relationship of $P(S_t \mid o_{1:t}, a_{1:t}, r_{1:t-1})$ from the previous timestep $P(s_{t-1} \mid o_{1:t-1}, a_{1:t-1}, r_{1:t-2})$. For the following equation, select the **minimum number** of answer choices that should be multiplied together to fill in the blank. Assume $t > 2$.

$$P(S_t \mid o_{1:t}, a_{1:t}, r_{1:t-1}) \propto \sum_{s_{t-1}} P(s_{t-1} \mid o_{1:t-1}, a_{1:t-1}, r_{1:t-2}) \cdot \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

☐ $P(S_t \mid s_{t-1})$

☐ $P(S_t \mid s_{t-1}, a_{t-1})$

☐ $P(S_t \mid s_{t-1}, a_{t-1}, r_{t-1}, r_{t+1})$

☐ $P(r_{t-2} \mid s_{t-2}, a_{t-2}, S_{t-1})$

☐ $P(r_{t-1} \mid s_{t-1}, a_{t-1}, S_t)$

☐ $P(r_t \mid s_t, a_t, S_{t+1})$

☐ $P(r_{t-1} \mid r_{t-2})$

☐ $P(o_t \mid S_t)$

☐ $P(o_{t-1} \mid S_{t-1})$

☐ $P(a_t \mid o_t)$

☐ $P(S_{t+1} \mid s_t)$

☐ $P(S_{t+1} \mid s_t, a_t)$

(ii) [2 pts] We adapt particle filtering to approximate inference on this Dynamic Bayes Net. Below is pseudocode for the body of our loop. Fill in the blanks!

- **Q6.3** Elapse time: For each particle s_t, sample successor s_{t+1} from ________________

  ○ $P(S_{t+1} \mid s_t)$ ○ $P(S_{t+1} \mid s_t, a_t)$ ○ $P(S_{t+1} \mid s_t, a_t, r_t)$ ○ $P(S_{t+1} \mid s_t, a_t, r_t, r_{t+1})$

- **Q6.4** Incorporate Evidence: For each particle s_{t+1}, weight the sample by ________________

  ○ $P(o_{t+1} \mid s_{t+1})$ ○ $P(r_t \mid s_t, a_t, s_{t+1})$ ○ $P(a_{t+1} \mid o_{t+1})$

  ○ $P(o_{t+1} \mid s_{t+1})P(r_t \mid s_t, a_t, s_{t+1})$ ○ $P(o_{t+1} \mid s_{t+1})P(a_{t+1} \mid o_{t+1})$ ○ None of these

- Resample particles from the normalized weighted particle distribution distribution.

(c) Now, we are the agent's Swiss banker. We can only see the rewards (which the agent deposits into the bank) and can't see the agent's actions or observations.

(i) [3 pts] For each letter **(A), (B), (C), (D), (E), (F)** select a single entry for the term corresponding to the **correct and most efficient** recursive expression to determine $P(S_t, O_t, A_t \mid r_{1:t-1})$ from $P(s_{t-1}, o_{t-1}, a_{t-1} \mid r_{1:t-2})$.

$$P(S_t, O_t, A_t \mid r_{1:t-1}) \propto \textbf{(A) (B) (C) (D) (E) (F)}\ P(s_{t-1}, o_{t-1}, a_{t-1} \mid r_{1:t-2})$$

Q6.5 (A):	○ 1	○ $P(O_t \mid S_t)$	○ $P(O_t \mid O_{t-1})$	○ $P(r_{t-1} \mid S_{t-1}, a_{t-1}, S_t)$	○ $P(r_{t-1} \mid r_{t-2})$
Q6.6 (B):	○ 1	○ $P(A_t \mid O_t)$	○ $P(A_t \mid A_{t-1})$	○ $P(S_t \mid S_{t-1}, A_{t-1})$	○ $P(S_t \mid S_{t-1})$
Q6.7 (C):	○ 1	○ $\sum_{s_{t-1}, a_{t-1}}$	○ $\sum_{s_{t-1}, a_{t-1}, o_{t-1}}$	○ $\sum_{a_{t-1}}$	○ $\sum_{s_{t-1}}$	○ $\sum_{o_{t-1}}$
Q6.8 (D):	○ 1	○ $P(a_t \mid O_t)$	○ $P(a_t \mid a_{t-1})$	○ $P(S_t \mid s_{t-1}, a_{t-1})$	○ $P(S_t \mid s_{t-1})$
Q6.9 (E):	○ 1	○ $P(O_t \mid S_t)$	○ $P(O_t \mid O_{t-1})$	○ $P(r_{t-1} \mid s_{t-1}, a_{t-1}, S_t)$	○ $P(r_{t-1} \mid r_{t-2})$
Q6.10 (F):	○ 1	○ $\sum_{s_{t-1}}$	○ $\sum_{o_{t-1}}$	○ $\sum_{a_{t-1}}$	○ $\sum_{r_{t-1}}$

(ii) [4 pts] We again want to adapt particle filtering to approximate inference. Below is the pseudocode for the body of our loop. Each particle contains a state s_t, observation o_t, and action a_t.

- **Q6.11** Elapse Time: For each particle (s_t, o_t, a_t), sample the successor state s_{t+1} from ______________________________

  ○ $P(S_{t+1} \mid s_t)$ ○ $P(S_{t+1} \mid s_t, a_t)$ ○ $P(S_{t+1} \mid s_t, a_t, r_t)$ ○ $P(S_{t+1} \mid s_t, a_t, r_t, r_{t+1})$

- **Q6.12** Incorporate evidence: For each new s_{t+1}, assign weight ____

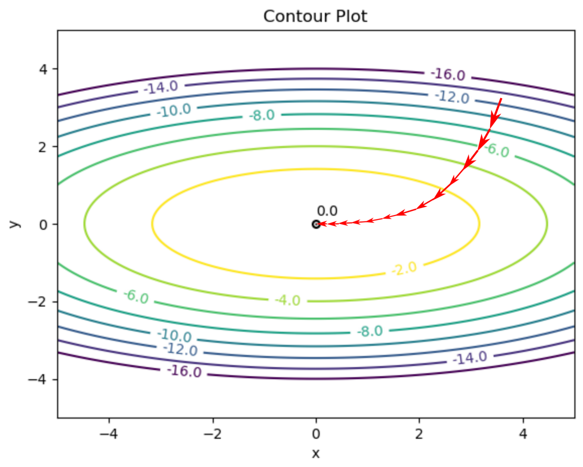
  ○ $P(o_{t+1} \mid s_{t+1})$ ○ $P(r_t \mid s_t, a_t, s_{t+1})$ ○ $P(a_{t+1} \mid o_{t+1})$
  ○ $P(o_{t+1} \mid s_{t+1})P(r_t \mid s_t, a_t, s_{t+1})$ ○ $P(o_{t+1} \mid s_{t+1})P(a_{t+1} \mid o_{t+1})$ ○ None of these

- Resample the s_{t+1} according to the normalized weighted distribution on s_{t+1}.
- **Q6.13** Finally, sample the following successor values:
  - ☐ Sample o_{t+1} from $P(O_{t+1} \mid S_{t+1})$
  - ☐ Sample a_{t+1} from $P(A_{t+1} \mid O_{t+1})$
  - ☐ Sample s_{t+1} from $P(S_{t+1} \mid S_t, A_t)$
  - ☐ Sample r_{t+1} from $P(R_{t+1} \mid S_{t+1}, A_{t+1}, S_{t+2})$

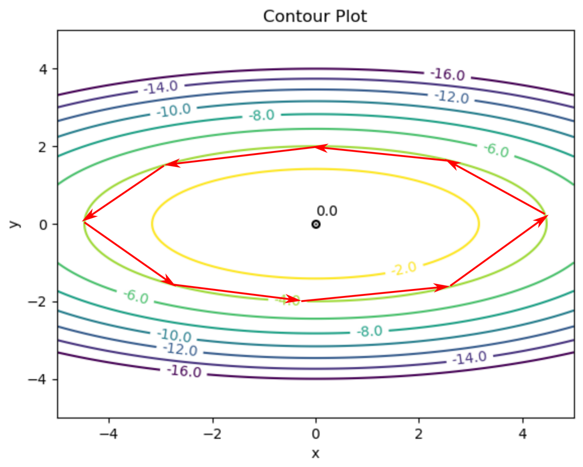
(iii) [1 pt] **Q6.14** Alternatively, we could sample the successor values (thus complete our particle) before incorporating evidence. Then, when resampling, we would resample completed particles from the weighted particle distribution. Which method would you expect to be more accurate? ○ Original method ○ Alternate method

Q7. [10 pts] Machine Learning

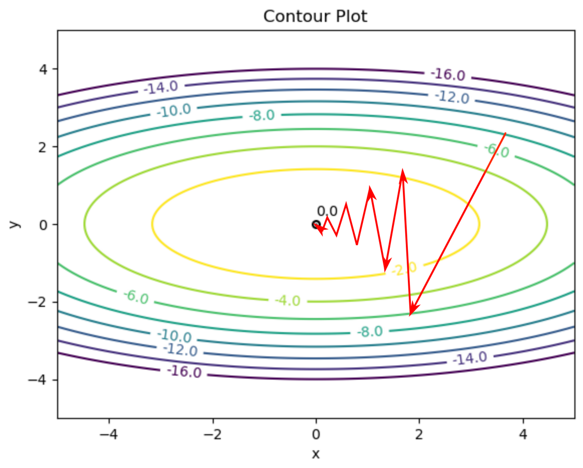
(a) **(i)** [2 pts] **Q7.1** The following four figures contain the steps of possible gradient ascent implementations each for some learning rate. Each arrow designates a step of gradient ascent. The numbers in the contour lines denote the value of the function we are maximizing on that contour. Which of the following figures, if any, contain paths that could be generated by gradient ascent?



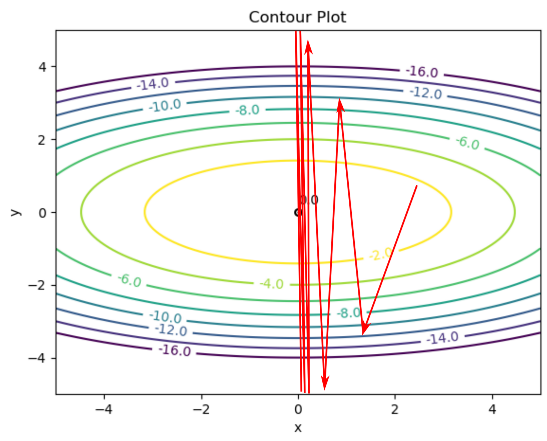
A.



B.



C.



D.

☐ A. ☐ B. ☐ C. ☐ D.

(ii) [2 pts] **Q7.2** Assume that we are trying to maximize a function that has many local maxima using gradient ascent. Which of the following variations of gradient ascent are likely to help obtain a better solution than what would be achieved with basic gradient ascent that uses a fixed step size α?

☐ Once gradient ascent has converged, store the solution, randomly perturb the solution, and run gradient ascent again until convergence. Repeat this process K times and return the best solution.

☐ Run N independent gradient ascent methods starting from N different initial solutions. Out of the ones that converge choose the best solution.

☐ For the chosen magnitude of the learning rate $|\alpha|$, at each step choose its sign with equal probability.

☐ After each iteration, increase the learning rate linearly.

(b) **(i)** [1 pt] **Q7.3** Assume that we observe some data features $x_1^{(i)} \in \mathbb{R},\ i = 1, \dots, N$ and outputs $y^{(i)} \in \mathbb{R},\ i = 1, \dots, N$. We will use a linear model of the form $\hat{y}^{(i)} = a + bx_1^{(i)}$ to predict the output y. If we use the squared euclidean norm as an objective, i.e. $loss = \frac{1}{2}\sum_{i=1}^{N}\left(\hat{y}^{(i)} - y^{(i)}\right)^2$ and a learning rate α, which of the following is the correct gradient descent rule for finding the optimal value of b?

○ $b \leftarrow b + \alpha \sum_{i=1}^{N}(a + bx_1^{(i)} - y^{(i)})x_1^{(i)}$　　○ $b \leftarrow b - \alpha \sum_{i=1}^{N}(a + bx_1^{(i)} - y^{(i)})x_1^{(i)}$

○ $b \leftarrow b + \alpha \sum_{i=1}^{N}(a + bx_1^{(i)} - y^{(i)})$　　○ $b \leftarrow b - \alpha \sum_{i=1}^{N} y^{(i)}x_1^{(i)}$

Now we observe the following dataset in which the outputs y are binary. Furthermore, we are given three features x_1, x_2, and x_3. The training dataset is as follows:

y	1	1	0	0
x_1	1	0	1	0
x_2	0	1	1	0
x_3	0	0	0	0

(ii) [1 pt] **Q7.4** We decide to use logistic regression for this classification task. What is the shape of the decision boundary we obtain from logistic regression?

○ Linear　　○ Nonlinear in general

○ Sigmoid　　○ None of the above

(iii) [2 pts] **Q7.5** We decide to train a neural network to perform classification on the whole dataset shown above. We propose using three hidden layers each of dimension 10 with a ReLu activation function except for the output layer which uses a sigmoid activation function. We will test the classifier on new unseen test data. Which of the following scenarios are likely to occur?

☐ The network will achieve zero classification error on the training set.　　☐ The network will overfit the data.

☐ The network will underfit the data.　　☐ The network will achieve zero classification error on the test set.

(iv) [2 pts] **Q7.6** This is a more general question independent of the aforementioned datasets. Assume that we have trained a logistic regression classifier on the last dataset obtaining the weight vector $\mathbf{w} = [w_1, w_2, w_3]$ and we observe a new feature vector $\mathbf{x}' = [x_1', x_2', x_3']$. Let $P(y = 1|\mathbf{x}; \mathbf{w}) = \frac{1}{1+e^{-\mathbf{w}\cdot\mathbf{x}}}$, with $\cdot$ denoting the inner product. Which of the following formulas describe the correct classification rule for the label y' of this data point?

☐ $y' = \underset{y'\in\{-1,1\}}{\operatorname{argmax}} P(y'|\mathbf{x}'; \mathbf{w})$　　☐ $y' = \begin{cases} 1, & \text{if } P(y' = 1|\mathbf{x}'; \mathbf{w}) \geq 0 \\ -1, & \text{otherwise} \end{cases}$

☐ $y' = \begin{cases} 1, & \text{if } P(y' = 1|\mathbf{x}'; \mathbf{w}) \geq 0.5 \\ -1, & \text{otherwise} \end{cases}$　　☐ $y' = \begin{cases} 1, & \text{if } P(y' = 1|\mathbf{x}'; \mathbf{w})(1 - P(y' = 1|\mathbf{x}'; \mathbf{w})) \geq 0 \\ -1, & \text{otherwise} \end{cases}$

Q8. [13 pts] CSPs and Bayes Nets

(a) **(i)** [1 pt] **Q8.1** *True/False*: When enforcing arc consistency in a CSP, the set of values which remain when the algorithm terminates does not depend on the order in which arcs are processed from the queue.

○ True ○ False

(ii) [1 pt] **Q8.2** In a general CSP with n variables, each taking d possible values, what is the maximum number of times a backtracking search algorithm might have to backtrack (i.e. the number of the times it generates an assignment, partial or complete, that violates the constraints) before finding a solution or concluding that none exists?

○ 0 ○ $O(1)$ ○ $O(nd^2)$ ○ $O(n^2d^3)$ ○ $O(d^n)$ ○ ∞

(iii) [1 pt] **Q8.3** What is the maximum number of times a backtracking search algorithm might have to backtrack in a general CSP, if it is running arc consistency and applying the MRV and LCV heuristics?

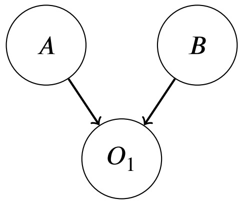
○ 0 ○ $O(1)$ ○ $O(nd^2)$ ○ $O(n^2d^3)$ ○ $O(d^n)$ ○ ∞

(b) We consider how to solve CSPs by converting them to Bayes nets.

First, assume that you are given a CSP over two binary variables, A and B, with **only one constraint:** $A = B$. Note that $A = B$ is satisfied when both are positive ($A = +a$ and $B = +b$) or both are negative ($A = -a$ and $B = -b$).

As shown below, the Bayes net is composed of nodes A and B, and an additional node for a binary random variable O_1 that represents the constraint. $O_1 = +o_1$ when A and B satisfy the constraint, and $O_1 = -o_1$ when they do not. The way you will solve for values of A and B that satisfy the CSP is by running inference for the query $P(A, B \mid +o_1)$. The setting(s) of A, B with the highest probability will satisfy the constraints.

(i) [2 pts] **Q8.4 Determine the values** in the rightmost CPT that will allow you to perform the inference query $P(A, B \mid +o_1)$ with correct results.



A	$P(A)$
$-a$	0.5
$+a$	0.5

B	$P(B)$
$-b$	0.5
$+b$	0.5

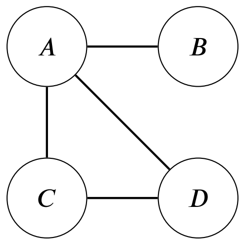
A	B	O_1	$P(O_1 \mid A, B)$
$-a$	$-b$	$-o_1$	**(1)**
$-a$	$-b$	$+o_1$	**(2)**
$-a$	$+b$	$-o_1$	**(3)**
$-a$	$+b$	$+o_1$	**(4)**
$+a$	$-b$	$-o_1$	**(5)**
$+a$	$-b$	$+o_1$	**(6)**
$+a$	$+b$	$-o_1$	**(7)**
$+a$	$+b$	$+o_1$	**(8)**

(ii) [2 pts] **Q8.5 Compute the Posterior Distribution:** Now you can find the solution(s) using probabilistic inference. Fill in the values below.

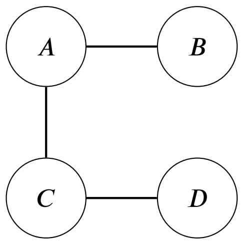
O_1	A	B	$P(A, B \mid +o_1)$
$+o_1$	$-a$	$-b$	**(1)**
$+o_1$	$-a$	$+b$	**(2)**
$+o_1$	$+a$	$-b$	**(3)**
$+o_1$	$+a$	$+b$	**(4)**

(c) Consider the following CSP:
Variables: $\{A, B, C, D\}$
Domains: $\{1, 2, 3, 4\}$ for each variable
Constraints:
$C_1 : A < 3$
$C_2 : B > 2$
$C_3 : C < D$
$C_4 : D < B$
C_5: all variables take on different values

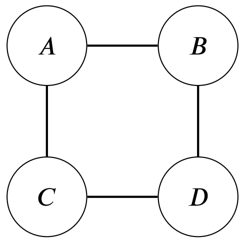
(i) [1 pt] **Q8.6** Choose which of the following indicates the constraint graph of this CSP:



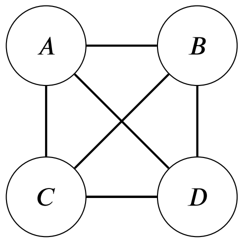
(1)



(2)



(3)



(4)

○ **(1)**
○ **(2)**
○ **(3)**
○ **(4)**
○ None of the above

(ii) [1 pt] Enforce all unary constraints. For parts 8.7 to 8.10, mark all the values that **are pruned** from the domains of each node.

Variable	Domain
A	1 2 3 4
B	1 2 3 4
C	1 2 3 4
D	1 2 3 4

Q8.7 A: ☐ 1 ☐ 2 ☐ 3 ☐ 4 ○ Nothing is pruned when enforcing unary constraints
Q8.8 B: ☐ 1 ☐ 2 ☐ 3 ☐ 4 ○ Nothing is pruned when enforcing unary constraints
Q8.9 C: ☐ 1 ☐ 2 ☐ 3 ☐ 4 ○ Nothing is pruned when enforcing unary constraints
Q8.10 D: ☐ 1 ☐ 2 ☐ 3 ☐ 4 ○ Nothing is pruned when enforcing unary constraints

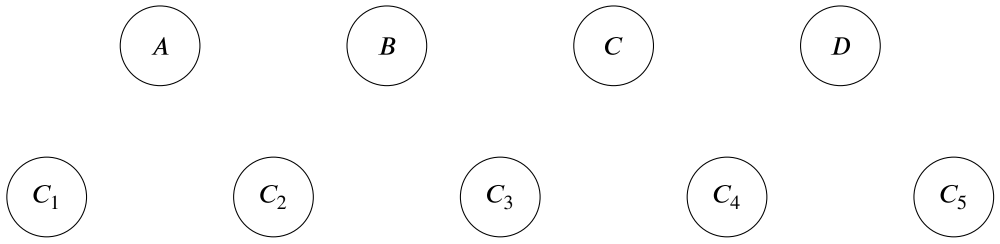
(iii) [2 pts] Run arc consistency. For parts 8.11 to 8.14, mark all the additional values that are pruned from each domain. **(Don't mark values that were already pruned from unary constraints).**i

Variable	Domain
A	1 2 3 4
B	1 2 3 4
C	1 2 3 4
D	1 2 3 4

Q8.11 A: ☐ 1 ☐ 2 ☐ 3 ☐ 4 ○ Nothing additional is pruned
Q8.12 B: ☐ 1 ☐ 2 ☐ 3 ☐ 4 ○ Nothing additional is pruned
Q8.13 C: ☐ 1 ☐ 2 ☐ 3 ☐ 4 ○ Nothing additional is pruned
Q8.14 D: ☐ 1 ☐ 2 ☐ 3 ☐ 4 ○ Nothing additional is pruned

(d) [2 pts] Now, in order to formulate the CSP from part (c) as a Bayes net we create 5 nodes for each of the constraints $(C_1, ..., C_5)$.

We wish to construct the Bayes net below such that, when queried for the posterior $P(A, B, C, D \mid +c_1, +c_2, +c_3, +c_4, +c_5)$, will assign the highest probability to the setting(s) of values that satisfy all constraints. In the following graph, determine all the edges that need to be added to the Bayes Net.



For each subpart, select all nodes that should be a parent of the given constraint node:

Q8.15 C_1:	☐ A	☐ B	☐ C	☐ D	☐ C_1	☐ C_2	☐ C_3	☐ C_4	☐ C_5
Q8.16 C_2:	☐ A	☐ B	☐ C	☐ D	☐ C_1	☐ C_2	☐ C_3	☐ C_4	☐ C_5
Q8.17 C_3:	☐ A	☐ B	☐ C	☐ D	☐ C_1	☐ C_2	☐ C_3	☐ C_4	☐ C_5
Q8.18 C_4:	☐ A	☐ B	☐ C	☐ D	☐ C_1	☐ C_2	☐ C_3	☐ C_4	☐ C_5
Q8.19 C_5:	☐ A	☐ B	☐ C	☐ D	☐ C_1	☐ C_2	☐ C_3	☐ C_4	☐ C_5

Q9. [12 pts] Potpourri

(a) **(i)** [2 pts] **Q9.1** Consider two heuristics h_1 and h_2. We are given that one of them is always equal to the true distance h^* and the other one is consistent, but not which one. Select all of the following heuristics h which are guaranteed to be consistent:

- ☐ $h = h_1 + h_2$
- ☐ $h = \max(h_1, h_2)$
- ☐ $h = \min(h_1, h_2)$
- ☐ $h = p \cdot h_1 + (1 - p) \cdot h_2 \quad \forall\, p$ where $0 < p < 1$
- ☐ At each state $s \in S$, we randomly choose $h(s)$ to be either $h_1(s)$ or $h_2(s)$ with equal probability.
- ○ None of the above.

(ii) [2 pts] **Q9.2** Consider 4 new heuristics h_1, h_2, h_3, and h_4 which are all **nonnegative**. We are given that $h_3 = h_1 + h_2$ and h_3 is admissible. Additionally, we know that h_4 is consistent. Select all of the following statements which are guaranteed to be true given the above information:
Recall that a heuristic h_i dominates h_j if the estimated goal distance for h_i is greater than or equal to the estimated goal distance for h_j at every node in the state space graph.

- ☐ Both h_1 and h_2 are guaranteed to be admissible
- ☐ $\frac{1}{2}\left(h_1 + h_2\right)$ is admissible
- ☐ h_4 dominates h_3
- ☐ $\max(h_3, h_4)$ is consistent
- ☐ $\min(h_3, h_4)$ is consistent
- ☐ $\frac{1}{2}\left(h_3 + h_4\right)$ is consistent
- ○ None of the above.

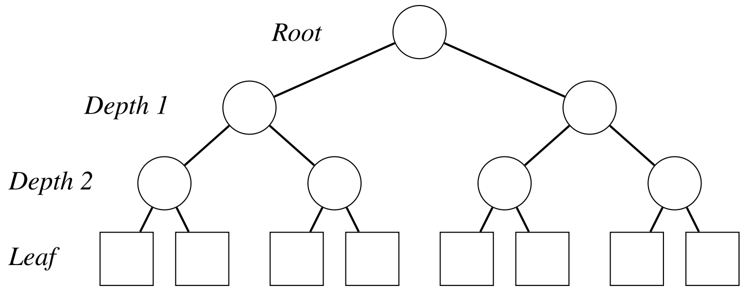
(b) [2 pts] **Q9.3** Select all of the following statements about MDP and RL that are true.

- ☐ Let π^* be the optimal policy. Then value iteration starting from random values will converge to $V^{\pi^*}(s)$ for all states.
- ☐ Approximate Q-learning is guaranteed to return the optimal policy upon convergence.
- ☐ In environments with deterministic transitions, no exploration is required for Q-learning to converge to the optimal policy.
- ☐ A large discount factor γ (approaching 1) on an MDP means that the agent emphasizes long-term rewards.
- ☐ Unlike ϵ-Greedy which requires randomness, using an exploration function provides a deterministic method to explore new states.

(c) [2 pts] **Q9.4** Select all of the following statements about Variable Elimination that are true.

- ☐ To find an optimal elimination ordering, we always want to choose the variable involved in the least amount of edge connections.
- ☐ In variable elimination, we continue eliminating until every single node in the bayes net has been eliminated.
- ☐ Variable elimination is at least as efficient as inference by enumeration.
- ☐ Variable elimination runs in polynomial time
- ○ None of the above

(d) [2 pts] **Q9.5** Consider the following game tree which contains only expectation nodes and **leaf values which are strictly positive**.



For the following options, select the option if there exists a set of leaf values in the positive domain that would allow you to prune. Consider each option independently.

- ☐ It is possible to prune the tree in its current form.
- ☐ Replace the root node with a maximizer.
- ☐ Replace both nodes at depth 1 with maximizers.
- ☐ Replace all 4 nodes at depth 2 with maximizers.
- ☐ Replace the root node with a minimizer.
- ☐ Replace both nodes at depth 1 with minimizers.
- ☐ Replace all 4 nodes at depth 2 with minimizers.
- ○ None of the above

(e) [2 pts] **Q9.6** In class we saw a formulation for approximate Q-learning that uses a linear function on the features. Consider using the sigmoid function $s(x) = \frac{1}{1+e^{-x}}$ as our approximation function, which we could potentially do if we wanted to bound the values of the Q functions. Let α denote the learning rate and $w_i, f_i, i = 1. \dots, n$ denote the weights and features respectively. If we use $Q(s,a) = \frac{1}{1+e^{-\mathbf{w}\cdot\mathbf{f}}}$ as our Q-function approximation, where $\mathbf{w}\cdot\mathbf{f} = \sum_{i=1}^{n} w_i f_i(s,a)$, and use the squared norm metric for the approximation, which of the following is the correct update for parameter w_j?
Hint: The derivative of the sigmoid function is $s'(x) = s(x)(1 - s(x))$.

- ○ $w'_j = w_j + \alpha\left(r + \gamma \max_{a'} Q_w(s', a') - Q_w(s,a)\right) \frac{1}{1+e^{-\mathbf{w}\cdot\mathbf{f}}}\left(1 - \frac{1}{1+e^{-\mathbf{w}\cdot\mathbf{f}}}\right) f_j$
- ○ $w'_j = w_j + \alpha\left(r + \gamma \max_{a'} Q_w(s', a') - Q_w(s,a)\right) f_j$
- ○ $w'_j = w_j - \alpha\left(r + \gamma \max_{a'} Q_w(s', a') - Q_w(s,a)\right) \frac{1}{1+e^{-\mathbf{w}\cdot\mathbf{f}}}\left(1 - \frac{1}{1+e^{-\mathbf{w}\cdot\mathbf{f}}}\right) f_j$
- ○ $w'_j = w_j - \alpha\left(r + \gamma \max_{a'} Q_w(s', a') - Q_w(s,a)\right) \frac{1}{1+e^{-\mathbf{w}\cdot\mathbf{f}}} f_j$