1.1. $W = 2\times 2$; $x = 3\times 3$;

$$\begin{bmatrix} w_{00} & w_{01} \\ w_{10} & w_{11} \end{bmatrix} .* \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & x_{00} & x_{01} & x_{02} & 0 \\ 0 & x_{10} & x_{11} & x_{12} & 0 \\ 0 & x_{20} & x_{21} & x_{22} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} w_{11}x_{00} & w_{10}x_{02} \\ w_{01}x_{20} & x_{22}w_{00} \end{bmatrix}$$

Flattening X gives a 9×1 array. $\Rightarrow$ A should be a 4×9 array.

$$A = \begin{bmatrix} w_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & w_{10} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & w_{01} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & w_{00} \end{bmatrix} \begin{bmatrix} x_{00} \\ x_{01} \\ x_{02} \\ x_{10} \\ x_{11} \\ x_{12} \\ x_{20} \\ x_{21} \\ x_{22} \end{bmatrix}$$

$$= \begin{bmatrix} w_{11}\,x_{00} \\ w_{10}\,x_{02} \\ w_{01}\,x_{20} \\ w_{00}\,x_{22} \end{bmatrix} \leftarrow \text{flattened } Y.$$

1.2. $w = \begin{bmatrix} w_{00} & w_{01} \\ w_{10} & w_{11} \end{bmatrix}$ $\quad X = \begin{bmatrix} x_{00} & x_{01} \\ x_{10} & x_{11} \end{bmatrix}$.

transpose conv with 2 stride is $\begin{bmatrix} w_{00}\,X & w_{01}\,X \\ w_{10}\,X & w_{11}\,X \end{bmatrix}$

each x is 2x2.

So, we get a 4x4 matrix.

Flattened $x = [x_{00} \;\; x_{01} \;\; x_{10} \;\; x_{11}]^T$.

Flattened Y = 16x1 array.

$\therefore$ A = 4x16 array.

$$A = \begin{bmatrix} w_{00} & 0 & 0 & 0 \\ w_{01} & 0 & 0 & 0 \\ w_{10} & 0 & 0 & 0 \\ w_{11} & 0 & 0 & 0 \\ 0 & w_{00} & 0 & 0 \\ 0 & w_{10} & 0 & 0 \\ 0 & w_{01} & 0 & 0 \\ 0 & w_{11} & 0 & 0 \\ 0 & 0 & w_{00} & 0 \\ 0 & 0 & w_{01} & 0 \\ 0 & 0 & w_{10} & 0 \\ 0 & 0 & w_{11} & 0 \\ 0 & 0 & 0 & w_{00} \\ 0 & 0 & 0 & w_{01} \\ 0 & 0 & 0 & w_{10} \\ 0 & 0 & 0 & w_{11} \end{bmatrix}$$

1.3. Given $X = \begin{bmatrix} x_{00} & x_{01} \\ x_{10} & x_{11} \end{bmatrix}$ Consider conv filter

$= [w_0]\ [w_1]\ [w_2]\ [w_3]$

$\rightarrow 4\times1\times1\times1.$

Output of conv (stride = 1, pad = 0)

$$\text{layer } i = \begin{bmatrix} w_i x_{00} & w_i x_{01} \\ w_i x_{10} & w_i x_{11} \end{bmatrix} = w_i X.$$

Let transpose conv filter $= \begin{bmatrix} w_0 & w_1 \\ w_2 & w_3 \end{bmatrix}$.

transpose output (stride = 2, pad = 0)

$$= \begin{bmatrix} w_0 X & w_1 X \\ w_2 X & w_3 X \end{bmatrix},$$

each $X = 2\times2$ mat

$\Rightarrow$ output $= 4\times4$ mat.

See that 4 conv layers can be placed next to each other to become output of transpose conv.

$$= \begin{bmatrix} \text{conv-out}[0] & \text{conv-out}[1] \\ \text{conv-out}[1] & \text{conv-out}[2] \end{bmatrix}$$

Hence we can prove the property.

Scanned with CamScanner

2.1. $w_{And} = [1, 1]$. $b_{And} = -1.5$

3.1 Given $x=1$.

$$\max(0, w_1 x + b_1) = w_1 x + b_1 = h_1$$

$$\max(0, w_2 h_1 + b_2) = w_2 h_1 + b_2 = h_2$$

$$\max(0, w_3 h_2 + b_3) = w_3 h_2 + b_3 = h(x).$$

Calculating, we get.

$$h(x) = 2x+3.$$

$$\boxed{\frac{\partial h}{\partial x} = 2.}$$

3.2. Given $x = -1$.

$$\max(0, w_1 x + b_1) = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}(-1) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -0.5 \\ 0.5 \end{bmatrix}$$

$$\max\left(0, \begin{bmatrix} -0.5 \\ 0.5 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix}.$$

So, w_1 can be rewritten as $\begin{bmatrix} 0 \\ 0.5 \end{bmatrix}$

~~Howeve~~ However, rest of the weights won't change as all the outputs are > 0.

Finally $h(x) = x + 3$.

$$\boxed{\frac{\partial h}{\partial x} = 1.}$$

Scanned with CamScanner

3.3. Given $x = -0.5$

$$W_1 x + b_1 = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} (-0.5) + \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -0.25 \\ 0.75 \end{bmatrix}.$$

$$\max_0 (W_1 x + b_1) = \begin{bmatrix} 0 \\ 0.75 \end{bmatrix}.$$

$\therefore W_1$ can be rewritten as $\begin{bmatrix} 0 \\ 0.5 \end{bmatrix}$.

Rest of W_2, W_3 & b's won't be affected as all outputs are > 0.

Solving, we get $h(x) = x + 3$.

$$\boxed{\frac{\partial h}{\partial x} = 1.}$$

4.1 $f_1(x) = |w' x + b|$

$$= \begin{bmatrix} |2x_1 - 1| \\ |2x_2 - 1| \\ \vdots \\ |2x_d - 1| \end{bmatrix} \rightarrow d \text{ dim vector.}$$

Take a particular coordinate, say 1.

$|2x_1 - 1|$ can map to $[0,1]$ when $x \in [0, 1/2]$ & $[1/2, 1]$

$\therefore$ Each coordinate has 2 ways.

d coordinates have $\underbrace{2 \times 2 \ldots \times 2}_{d}$ ways.

$\therefore$ $\boxed{2^d \text{ input regions are identified onto } 0}$

4.2. Given $f(g(x))$. let $g(x) = h$.

$f(h) \in (0,1)^d \Rightarrow h$ can be any of n_f regions

$h = g(x)$. Each h can be any of n_g regions

$\Rightarrow$ $\boxed{f(g(x)) \text{ identifies } n_f \times n_g \text{ regions.}}$

4.3. Using 4.1 & 4.2,

$h_L = |w_L h_{L-1} + b_L|$ $\Rightarrow$ h_{L-1} can be any of 2^d regions.

For each, $h_{L-1} = |w_{L-1} h_{L-2} + b_{L-1}|$ $\Rightarrow$ h_{L-2} can be any of 2^d regions.

$\Rightarrow$ total regions $\Rightarrow$ $2^d \times 2^d$.

When we go till $h_1 = |w_1 x + b_1|$,

we similarly get any of $\underbrace{2^d \times 2^d \cdots \times 2^d}_{L} = 2^{Ld}$ region of x can be mapped to each region of $f(x) = h_L$.

2.2 $w_{OR} = [1, 1]$ $b_{or} = -0.5.$

2.3. let $w = [w_1, w_2]$ $b = b$.

As per eqns. $0 \cdot w_1 + 0 \cdot w_2 + b < 0$ — (1)

$0 \cdot w_1 + 1 \cdot w_2 + b \geq 0$ — (2)

$1 \cdot w_1 + 0 \cdot w_2 + b \geq 0$ — (3)

$1 \cdot w_1 + 1 \cdot w_2 + b < 0$ — (4)

$1 + 4 \Rightarrow$ $1 \cdot w_1 + 1 \cdot w_2 + 2b < 0$

$2 + 3 \Rightarrow$ $1 \cdot w_1 + 1 \cdot w_2 + 2b \geq 0$

These 2 eqns are contradictory. Hence no such set of W, b exists.