

Project 02

APSC 607 Fall 2017

Seth Goodman

October 24, 2017

1 Introduction

This project explored methods for calculating the integral roots of functions. Each functions was examined in the range between zero and two, using the Composite Trapezoidal Rule, Composite Midpoint Rule, Composite Simpson's Rule, as well as an adaptive implementation of the Composite Simpson's Rule. The behavior and characteristics of these methods are reviewed by examining the effectiveness of the resulting value for the integral given a range of values for N .

All computations were performed using MATLAB using the code (Table 1) accompanying this report (in a zip file). The following Methods section will present the methods used in MATLAB to explore functions, as well as the outputs and results. The Results section of this report contains the outputs for each function and range along with related observations and discussion. All figures and tables found in this report are available in the output subdirectory of the accompanying zip file. Additionally, all code and figures found in the zip file can be accessed via GitHub¹.

¹https://github.com/sgoodm/apsc607/tree/master/project_02

2 Methods

2.1 Composite Trapezoidal Rule

TEXT

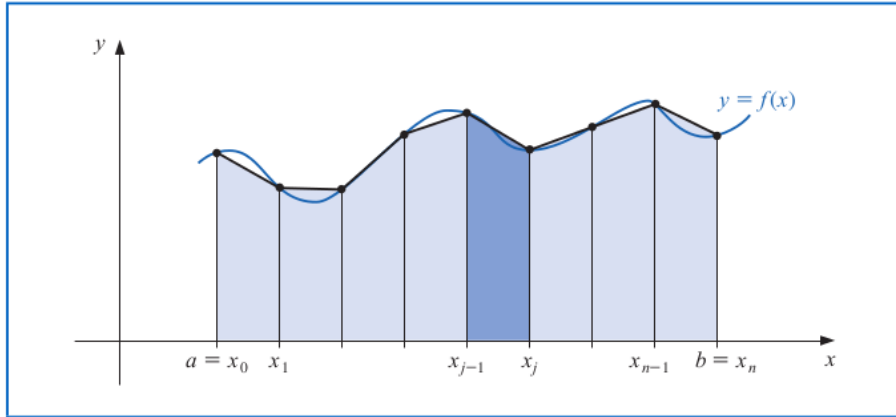


Figure 1: Trapezoidal Figure[1]

Let $f \in C^2[a, b]$, $h = (b - a)/n$, and $x_j = a + jh$, for each $j = 0, 1, \dots, n$. There exists a $\mu \in (a, b)$ for which the **Composite Trapezoidal rule** for n subintervals can be written with its error term as

$$\int_a^b f(x) dx = \frac{h}{2} \left[f(a) + 2 \sum_{j=1}^{n-1} f(x_j) + f(b) \right] - \frac{b-a}{12} h^2 f''(\mu). \quad \blacksquare$$

Figure 2: Trapezoidal Equation[1]

$$\begin{aligned} h &= (b - a)/n \\ x_j &= a + jh \\ j &= 0, 1, \dots, n \end{aligned}$$

$$\int_a^b f(x) dx = \frac{h}{2} \left[f(a) + 2 \sum_{j=1}^{n-1} f(x_j) + f(b) \right] \quad (1)$$

2.2 Composite Midpoint Rule

TEXT

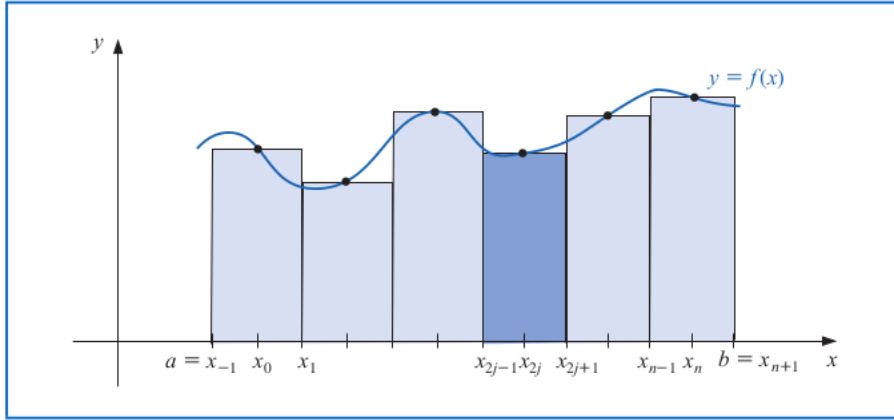


Figure 3: Midpoint Figure[1]

Let $f \in C^2[a, b]$, n be even, $h = (b - a)/(n + 2)$, and $x_j = a + (j + 1)h$ for each $j = -1, 0, \dots, n + 1$. There exists a $\mu \in (a, b)$ for which the **Composite Midpoint rule** for $n + 2$ subintervals can be written with its error term as

$$\int_a^b f(x) dx = 2h \sum_{j=0}^{n/2} f(x_{2j}) + \frac{b-a}{6} h^2 f''(\mu).$$

■

Figure 4: Midpoint Equation[1]

2.3 Composite Simpson's Rule

TEXT

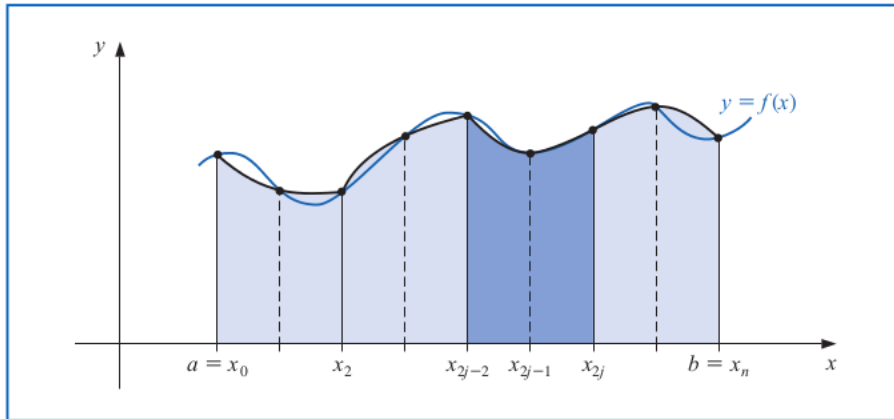


Figure 5: Simpson's Figure[1]

With $h = (b - a)/n$ and $x_j = a + jh$, for each $j = 0, 1, \dots, n$, we have

$$\begin{aligned}\int_a^b f(x) \, dx &= \sum_{j=1}^{n/2} \int_{x_{2j-2}}^{x_{2j}} f(x) \, dx \\ &= \sum_{j=1}^{n/2} \left\{ \frac{h}{3} [f(x_{2j-2}) + 4f(x_{2j-1}) + f(x_{2j})] - \frac{h^5}{90} f^{(4)}(\xi_j) \right\},\end{aligned}$$

Figure 6: Simpson's Equation[1]

2.4 Adaptive Simpson's Rule

TEXT

3 Results

TEXT

- 1. Like this,
- 2. and like this.
- Like this,
- and like this.

Word Definition

Concept Explanation

Idea Text

...using dots ...

$$f(x) = e^{2x} * sin(3x)$$
 (2)

$$f(x) = \frac{1}{x + 4}$$
 (3)

before figure 1a

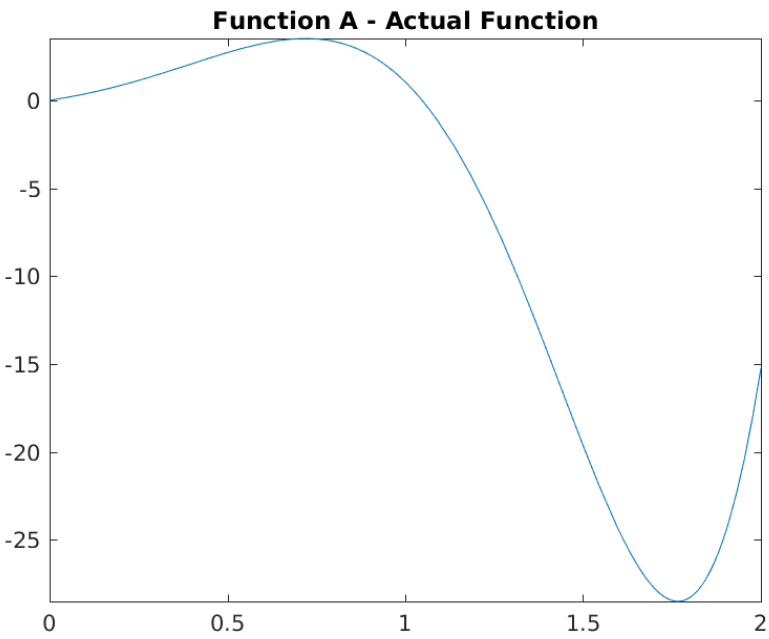


Figure 7: caption text a

after figure 1a

Item	Quantity
Widgets	42
Gadgets	13

Table 1: An example table.

Something about minimum error in footnote².

²See following link on MATLAB precision limitations (general limitations of floating point representations apply) <https://www.mathworks.com/help/fixedpoint/ug/limitations-on-precision.html>

References

- [1] Burden, R., Faires, J., Numerical Analysis 9th Edition. 2010