(2)

C2.2 # 10

A) $-\Delta E = \underbrace{i(x,t)}^{i} \cdot R \cdot \Delta x + L \underbrace{\frac{di}{dt}(x,t)}^{i_t} \cdot \Delta x$

$$\frac{\Delta E}{\Delta x} + \frac{R \cdot i \cdot \cancel{\Delta x}}{\cancel{\Delta x}} + \frac{L \cdot i_t \cdot \cancel{\Delta x}}{\cancel{\Delta x}} = 0 \qquad \frac{\Delta E}{\Delta x} = E_x$$

$$E_x + Ri + Li_t = 0$$

$$-\Delta i = G \cdot E \cdot \Delta x + C \cdot \overbrace{\frac{dE}{dt}(x,t)}^{E_t} \cdot \Delta x$$

$$\frac{\Delta i}{\Delta x} + \frac{G \cdot E \cdot \cancel{\Delta x}}{\cancel{\Delta x}} + \frac{C \cdot E_t \cdot \cancel{\Delta x}}{\cancel{\Delta x}} = 0 \qquad \frac{\Delta i}{\Delta x} = i_x$$

$$i_x + GE + CE_t = 0$$

B) $E_{xx} + Ri_x + Li_{tx} = 0$

$$i_{xt} + GE_t + CE_{tt} = 0$$

$$E_{xx} + R(-GE - CE_t) + L(-GE_t - CE_{tt}) = 0$$

$$E_{xx} = RGE + RCE_t + LGE_t + LCE_{tt}$$

$$E_{xx} = LCE_{tt} + (RC + LG)E_t + RGE$$

(2)

c) $\mathcal{E}_{xt} + Ri_t + Li_{tt} = 0$

$i_{xx} + G\mathcal{E}_x + C\mathcal{E}_{tx} = 0$

$i_{xx} + G(-Ri - Li_t) + C(-Ri_t - Li_{tt}) = 0$

$i_{xx} = GRi + GLi_t + CRi_t + CLi_{tt}$

$$i_{xx} = LCi_{tt} + (RC + LG)i_t + RGi$$

D) Results from B + C both satisfy telegraph eq.

$$u_{xx} = LCu_{tt} + (RC + LG)u_t + RGu$$

If L and G can be neglected $(L = 0,\ G = 0)$

$$u_{xx} = RCu_t$$

$$\mathcal{E}_{xx} = \cancel{LC\mathcal{E}_{tt}} + (RC + \cancel{LG})\mathcal{E}_t + \cancel{RG\mathcal{E}} = RC\mathcal{E}_t$$

$$i_{xx} = \cancel{LCi_{tt}} + (RC + \cancel{LG})i_t + \cancel{RGi} = RCi_t$$

Scanned by CamScanner

(3)

c3.3 #10

$$f(x) = \begin{cases} 0 & \text{if } -\pi \le x < -\pi/2 \\ 1 & \text{if } -\pi/2 < x < \pi/2 \\ 0 & \text{if } \pi/2 \le x \le \pi \end{cases}$$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx = \cancel{\frac{1}{\pi}\int_{-\pi}^{-\pi/2} f(x)\,dx}^{\,0} + \frac{1}{\pi}\int_{-\pi/2}^{\pi/2} f(x)\,dx + \cancel{\frac{1}{\pi}\int_{\pi/2}^{\pi} f(x)\,dx}^{\,0}$$

$$a_0 = \frac{1}{\pi}\int_{-\pi/2}^{\pi/2} 1\,dx = \frac{1}{\pi}\left(\pi/2 - (-\pi/2)\right) = \frac{1}{\pi}\cdot\pi = 1$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos\left(\frac{n\pi x}{\pi}\right)dx$$

$$a_n = \cancel{\frac{1}{\pi}\int_{-\pi}^{-\pi/2} 0\cdot\cos(nx)\,dx}^{\,0} + \frac{1}{\pi}\int_{-\pi/2}^{\pi/2} 1\cdot\cos(nx)\,dx + \cancel{\frac{1}{\pi}\int_{\pi/2}^{\pi} 0\cdot\cos(nx)\,dx}^{\,0}$$

$$a_n = \frac{1}{\pi}\left[\frac{\sin(nx)}{n}\Big|_{-\pi/2}^{\pi/2}\right] = \frac{1}{\pi}\cdot\frac{2}{n}\sin\left(\frac{\pi n}{2}\right)$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin\left(\frac{n\cancel{\pi}x}{\cancel{\pi}}\right)dx = \frac{1}{\pi}\int_{-\pi/2}^{\pi/2} 1\cdot\sin(nx)\,dx = \frac{1}{\pi}\left[-\frac{\cos(nx)}{n}\Big|_{-\pi/2}^{\pi/2}\right]$$

$$b_n = \frac{1}{\pi}\cdot 0 = 0$$

$$F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n\cdot\cos\left(\frac{n\pi x}{L}\right) + \cancel{b_n\cdot\sin\left(\frac{n\pi x}{L}\right)}^{\,0}\right)$$

$$F(x) = \frac{1}{2} + \frac{2}{\pi}\sum_{n=1}^{\infty}\left[\frac{\cancel{2}}{\cancel{\pi} n}\cdot\sin\left(\frac{\pi n}{2}\right)\cdot\cos\left(\frac{n\cancel{\pi}x}{\cancel{\pi}}\right)\right] = \frac{1}{2} + \frac{2}{\pi}\sum_{n=1}^{\infty}\left[\frac{1}{n}\sin\left(\frac{\pi n}{2}\right)\cos(nx)\right]$$

(4)

c4.2 # 4d

$U_{tt} = a^2 U_{xx}$ (wave eq)

IC: $u(x,0) = f(x)$ BC: $u_x(0,t) = 0$ $\Big\{$ Type 10: $\lambda_n = \left(\frac{\pi(2n-1)}{2L}\right)^2$

$u_t(x,0) = g(x)$ $u(L,t) = 0$ $\Big\}$

$X_n = \cos(\lambda_n^{1/2} x)$

$n = 1, 2, 3, \ldots$

$u(x,t) = X(x)T(t)$ ... $X'' + \lambda X = 0$

$U_{xx} = X''T$ $T'' + c^2 \lambda T = 0$

$U_{tt} = XT''$ $\uparrow$

wave eq: $\frac{XT'' = a^2 X'' T}{a^2 X T} \longrightarrow \frac{T''}{a^2 T} = \frac{X''}{X} = -\lambda$

$u_x(0,t) = 0 \rightarrow X'(0)T(t) = 0 \rightarrow X'(0) = 0$ or $T(t) = 0$

$u(L,t) = 0 \rightarrow X(L)T(t) = 0 \rightarrow X(L) = 0$ or $T(t) = 0$

$T'' + c^2 \left(\frac{(2n-1)^2 \pi^2}{4L^2}\right) T = 0$ ...

$T_n = A_n \cos(\sqrt{a} \cdot \sqrt{\lambda} \cdot t) + B_A \sin(\sqrt{a} \cdot \sqrt{\lambda} \cdot t)$

$T_n = A_n \cos\left(\frac{(2n-1)}{2L} \cdot \pi a t\right) + B_A \sin\left(\frac{(2n-1)}{2L} \cdot \pi a t\right)$

$u(x,t) = X_n T_n \Rightarrow \sum_{n=1}^{\infty} \cos\left(\frac{(2n-1)\pi}{2L}\right) \left[A_n \cos\left(\frac{(2n-1)}{2L} \cdot \pi a t\right) + B_n \sin\left(\frac{(2n-1)}{2L} \cdot \pi a t\right)\right]$

$\longrightarrow$

(4)

$$u(x,0) = \sum_{n=1}^{\infty} \cos\left(\frac{(2n-1)\pi}{2L}\right) \cdot \left[A_n \underbrace{\cancel{\cos(0)}}_{1} + B_n \underbrace{\cancel{\sin(0)}}_{0}\right]$$

$$u(x,0) = \sum_{n=1}^{\infty} A_n \cdot \cos\left(\frac{(2n-1)\pi}{2L}\right) = f(x)$$

$$A_n = \frac{2}{L}\int_0^L f(x)\cos\left(\frac{(2n-1)\pi}{2L}\right)dx$$

$$u_t(x,t) = \sum_{n=1}^{\infty} \cos\left(\frac{(2n-1)\pi}{2L}\right) \cdot \left[-A_n\left(\frac{(2n-1)\pi a}{2L}\right)\sin\left(\frac{(2n-1)\pi a t}{2L}\right) + B_n\left(\frac{(2n-1)\pi a}{2L}\right)\cos\left(\frac{(2n-1)\pi a t}{2L}\right)\right]$$

(A_n)

$$u_t(x,0) = \sum_{n=1}^{\infty} \cos\left(\frac{(2n-1)\pi}{2L}\right) \cdot B_n\left(\frac{(2n-1)\pi a}{2L}\right) = g(x)$$

$$\hookrightarrow B_n \cdot \frac{(2n-1)\pi a}{2L} = A_n = \frac{2}{L}\int_0^L g(x)\cos\left(\frac{(2n-1)\pi}{2L}\right)dx$$

$$B_n = \frac{4}{(2n-1)\pi a}\int_0^L g(x)\cos\left(\frac{(2n-1)\pi}{2L}\right)dx$$

$$u(x,t) = \sum_{n=1}^{\infty} \cos\left(\frac{(2n-1)\pi}{2L}\right) \cdot \left[A_n \cos\left(\frac{(2n-1)}{2L}\cdot \pi a t\right) + B_n \sin\left(\frac{(2n-1)}{2L}\cdot \pi a t\right)\right]$$

⑤

c 6.3 #1e

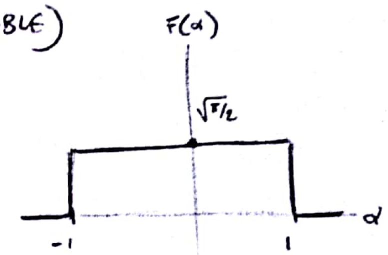
$$f(x) = \frac{\sin(x)}{x}$$

$$F(\alpha) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-i\alpha x}\, dx = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{\sin(x)}{x} e^{-i\alpha x}\, dx$$

(MATCHES TRANSFORM TABLE)

If $|\alpha| > 1$, $F(\alpha) = 0$

If $|\alpha| \leq 1$, $F(\alpha) = \frac{\pi}{\sqrt{2\pi}} = = \sqrt{\frac{\pi^2}{2\pi}} = \sqrt{\pi/2}$



$$g(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{i\alpha x} \int_{-\infty}^{\infty} f(z) e^{-i\alpha x}\, dz\, d\alpha$$

$$g(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(\alpha) e^{i\alpha x}\, d\alpha = \mathcal{F}^{-1}[F(\alpha)]$$ ... i.e., the inverse

$$g(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty} \sqrt{\pi/2}\, e^{i\alpha x}\, d\alpha$$

If $x = 0 \rightarrow g(x) = 1$

If $x \neq 0 \longrightarrow \frac{\sin \alpha}{\alpha} = g(\cancel{\alpha}) = f(x)$

⑤

C6.3 # 11c

$$f(x) = \frac{2}{3+(x-1)^2} \left(= \frac{2}{x^2-2x+4} \right)$$

$$F(\alpha) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-i\alpha x} dx = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{2}{3+(x-1)^2} e^{-i\alpha x} dx$$

$$F(\alpha) = \frac{1}{\sqrt{2\pi}} \cdot \left(-\frac{\pi}{3} \cdot \left[(i\sqrt{3}+3)\sin(\alpha) + (3i-\sqrt{3})\cos(\alpha) \right] \cdot e^{-\alpha\sqrt{3}} \right)$$

$$F(\alpha) = -\frac{\sqrt{\pi}}{3\sqrt{2}} \cdot e^{-\alpha\sqrt{3}} \cdot \left[(i\sqrt{3}+3)\sin(\alpha) + (3i-\sqrt{3})\cos(\alpha) \right]$$

(7)

c11.2 #6

$u_t + u_{xx} = 0 \quad x > 0, \; t > 0$

Compare w/ Exact $\Rightarrow u = \dfrac{3x}{1+3t}$

$u(x,0) = 3x$

$u(0,t) = 0$

Use: $0 \leq x \leq 1$

Compute at: $t = 0.2$

$\Delta x = 0.2$

$\Delta t = 0.1$

$(i,j \approx x,t)$

$$u_{i,j+1} = u_{i,j} - \frac{c\,\Delta t}{2\,\Delta x}\left(u_{i+1,j} - u_{i-1,j}\right) + \frac{1}{2}\left(\frac{c\,\Delta t}{\Delta x}\right)^2\left(u_{i+1,j} - 2u_{i,j} + u_{i-1,j}\right)$$

$$u_{i,j+1} = (1-\epsilon^2)\,u_{i,j} + \frac{\epsilon}{2}(1+\epsilon)\,u_{i-1,j} + \frac{\epsilon}{2}(\epsilon-1)\,u_{i+1,j}$$

$$\epsilon = \frac{c\,\Delta t}{\Delta x} = \frac{1 \cdot 0.1}{0.2} = 0.5$$

$$u_{i,j+1} = 0.75 \cdot u_{i,j} + 0.375 \cdot u_{i-1,j} - 0.125 \cdot u_{i+1,j}$$

RESULTS FROM MATLAB IN ATTACHED CSV (CODE ATTACHED TOO)

	t: 0	0.1	0.2	Exact (0.2)
x: 0	0	0	0	0
.2	0.6			0.375
.4	1.2			0.75
.6	1.8			1.125
.8	2.4			1.5
1	3			1.875

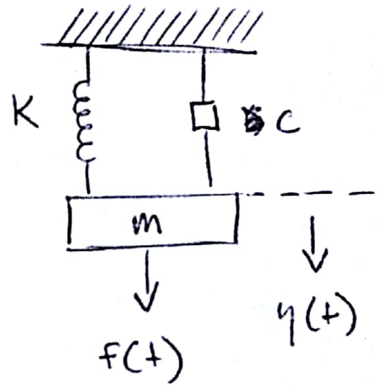
(9)

$M = 0.2\ kg$

$K = 2\ N/m$

$C = 1.2$ Damping, Ns/m

$f(t) = 5 \cdot \cos(4t)$



$x_1(t) = y(t)$

$x_2(t) = M \frac{dy}{dt}$

$$\dot{x}_1 = \left(\frac{1}{m}\right) x_2$$

$$\dot{x}_2 = -K x_1 - \left(\frac{C}{m}\right) x_2 + f(t)$$

$$\dot{\bar{x}} = \underbrace{\begin{bmatrix} 0 & 1/m \\ -k/m & -C/m \end{bmatrix}}_{A} \bar{x} + \underbrace{\begin{bmatrix} 0 \\ 1/m \end{bmatrix}}_{B} f(t)$$

$$\bar{y} = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{C} \bar{x} + \underbrace{\begin{bmatrix} 0 \end{bmatrix}}_{D} f(t)$$

$\left[\text{SEE MATLAB + RESULTS ATTACHED}\right]$