ApSc608 MidTerm Exam Spring 2018

1. Identify as linear, quasilinear, or nonlinear and prove it:
   a. $y^3 u_{xx} + x u_y = u$
   b. $u u_x - 2xy u_y = 0$
   c. $u_x^2 + u u_y = 1$
   d. $x^4 u_{xxxx} + 2u_{xxyy} + u_{yyyy} = 0$
   e. $u_{xx} + 2u_{xy} + u_{yy} = \sin x$
   f. $u_{xxx} + u_{yyy} + (\log x)u = 0$
2. Determine region in which given equation is hyperbolic, parabolic, or elliptic and prove it:
   a. $x u_{xx} + x u_{xy} + y u_{yy} = 0$
   b. $x u_{xx} + y u_{xy} + K = 0$
   c. $x^2 y u_{xx} + xy u_{xy} - y^2 u_{yy} = 0$
   d. $x^2 u_{xx} - 2xy u_{xy} + y^2 u_{yy} = e^x$
   e. $(\sin^2 x)u_{xx} + (\sin 2x)u_{xy} + (\cos^2 x)u_{yy} = x$
3. Find the Fourier series for the following functions (values in parentheses are the ranges) and plot both the actual function and your Fourier series approximation:
   a. $f(x) = 0$ $(-\pi < x < 0)$; x^2 $(0 \le x \le \pi)$
   b. $f(x) = 1$ $(-5 < x < 0)$; $1+x$ $(0 \le x < 5)$
   c. $f(x) = -1$ $(-\pi < x < 0)$; 2 $(0 \le x < \pi)$
4. Coleman Text: Exercises 1.6 #6, 10, 19
5. Solve the wave equation

   $u_{tt} = a^2 u_{xx}$ with $0 < x < L$ and $t > 0$ with following conditions

   $u(0,t) = 0$, $u(L,t) = 0$, $u(x,0) = (1/4)(x(L-x))$, and $u_t(x,0) = 0$

6. Coleman Text: Exercises 4.2 #3a, b
7. Coleman Text: Exercises 6.1 #1a, b
8. Coleman Text: Exercises 6.3 #1b, e
9. Coleman Text: Exercises 11.2 #1a, b, c
10. Demonstrate the use of the FFT on a multi-sine signal with at least three frequencies. Then demonstrate the concept of Aliasing. Your presentation and discussion of results will be very important.

END OF TEST