

Creating divisible design graphs using affine designs

Vladislav Kabanov

School of Mathematical Sciences, Hebei Key Laboratory of Computational Mathematics and Applications, Hebei Normal University

vvkabanov@icloud.com

A k -regular graph with v vertices is called a divisible design graph if its vertex set can be partitioned into m classes of size n , and any two distinct vertices in the same class have λ_1 common neighbours, and any two vertices from different classes have λ_2 common neighbours [5], [1]. Recently, we presented in [6] and [7] several new constructions that generate infinite sequences of divisible design graphs. These constructions develop ideas of W.D. Wallis [10], D.G. Fon-Der-Flaass [2], and M. Muzychuk [9]. Some of the divisible design graphs from [6] were used in [8] to construct the new strongly regular graphs with the parameters of the complement symplectic graphs. Possible divisible design graphs suitable for constructing strongly regular graphs were studied in [3] and [4]. In our talk we discuss old and introduce a new prolific constructions of infinite families of divisible design graphs with new parameters.

We use a symmetric 2-design with parameters (m, κ, λ) and m affine designs $AR(q, r)$. Let q be a power of 2 and let $\kappa = (q^2r - 1)/(q - 1)$ be the number of parallel classes of blocks in affine designs. then there are divisible design graphs with parameters

$$v = q^2rm, \quad k = qr(q^2r - 1)/(q - 1), \quad \lambda_1 = qr(qr - 1)/(q - 1), \quad \lambda_2 = \lambda r, \quad m, \quad n = q^2r,$$

and

$$v = q^3r(qr - 1)/(q - 1), \quad k = q^2r(qr - 1)/(q - 1), \quad \lambda_1 = qr(qr - q)/(q - 1), \quad \lambda_2 = qr(qr - 1)/(q - 1), \\ m = q^2(qr - 1)/(q - 1), \quad n = qr.$$

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