On signed graphs with fixed smallest eigenvalue

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Let G be a graph with smallest eigenvalue $\lambda_{\min}(G)$. In 1973, Hoffman [1] showed that: (i) for any real number $\lambda \leq -1$, if $\lambda_{\min}(G) \geq \lambda$, then there exists a positive integer $t = t_\lambda$, such that G is $\{K_{1,t}, \widetilde{K_{2t}}\}$-free; (ii) for any integer t, if G is $\{K_{1,t}, \widetilde{K_{2t}}\}$-free, then there exists a positive integer $\lambda = \lambda_t$, such that $\lambda_{\min}(G) \geq \lambda$. In 2016, Kim, Koolen and Yang [2] gave a structure theory for graphs with fixed smallest eigenvalue.

In this talk, I will present a generalization of these results to signed graphs. Let (G, σ) be a signed graph with smallest eigenvalue $\lambda_{\min}((G, \sigma))$. We showed that: (i) for any real number $\lambda \leq -1$, if $\lambda_{\min}((G, \sigma)) \geq \lambda$, then there exists a positive integer $t = t_\lambda$, such that (G, σ) is $\{(K_{1,t}, +), (K_t, -)\ (\widehat{K_{2t}}, +), \widehat{K_{2t}}\}$-switching-free; (ii) for any integer t, if (G, σ) is $\{(K_{1,t}, +), (K_t, -), (\widetilde{K_{2t}}, +),\ \widehat{K_{2t}}\}$-switching-free, then there exists a positive integer $\lambda = \lambda_t$, such that $\lambda_{\min}((G, \sigma)) \geq \lambda$. Moreover, we gave a structure theory for signed graphs with fixed smallest eigenvalue. In the end, I will introduce an application of our method on signed graphs with smallest eigenvalue greater than $-1 - \sqrt{2}$.

This is based on a joint work with Prof. Koolen and Mr. Liu Jing-Yuan.

References

[1] A.J. Hoffman, *On spectrally bounded graphs*, A Survey of Combinatorial Theory, 1973, 277-283.

[2] H.K. Kim, J.H. Koolen, and J.Y. Yang. *A structure theory for graphs with fixed smallest eigenvalue*, Linear Algebra Appl., 504, 2016, 1-13.