

Divisible design graphs with selfloops

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A k -regular graph Γ on v vertices is called a divisible design graph (DDG for short) with parameters $(v, k, \lambda_1, \lambda_2, m, n)$ if the vertex set V of Γ can be partitioned into m classes each of size n such that any two distinct vertices of the same class have precisely λ_1 common neighbours and any two distinct vertices of different classes have precisely λ_2 common neighbours. DDG's were introduced in [1] as a bridge between (group) divisible designs and graphs, and the graphs under consideration were undirected and without loops. In this work, we introduce a natural extension of DDGs to include possible selfloops (LDDG's for short) and develop a basic theory for the same. We describe two infinite families of such graphs, some members of which are (undirected) DDG's. We state some theoretical results including the spectrum of such LDDG's. We also classify all examples satisfying parameter restrictions. Finally, we discuss a procedure called dual Seidel switching, which constructs new LDDGs from others. This is based on joint work with Bart De Bruyn and Sergey Goryainov [2].

References

- [1] W. H. Haemers, H. Kharaghani, M. A. Meulenber, *Divisible design graphs*, Journal of Combinatorial Theory Series A, Volume 118, 2011, p. 978–992.
- [2] A. Bhowmik, B. De Bruyn, S. Goryainov, *Divisible design graphs with selfloops*, Discrete Mathematics, Volume 349, Issue 3, 2026, p. 114824.