

**New infinite families of divisible design graphs, which are covers of strongly regular polar graphs**

Weihao Yan

*Hebei Normal University*

weihao.yan.hebnu@outlook.com

We consider only simple graphs. A *divisible design graph* with parameters  $(v, k, \lambda_1, \lambda_2, m, n)$  is a  $k$ -regular graph on  $v$  vertices such that its vertex set can be partitioned into  $m$  classes of size  $n$  where any two distinct vertices from the same class have exactly  $\lambda_1$  common neighbours and any two vertices from different classes have exactly  $\lambda_2$  common neighbours. Divisible design graphs were introduced in [3] as a bridge between graph theory and the theory of group divisible designs. Since then, tens of constructions of divisible design graphs have been introduced.

In [1, Proposition 12.5.3], a construction of antipodal distance-regular graphs of diameter 3 was given. This construction uses a vector space of dimension 2 equipped with a nondegenerate symplectic bilinear form. Note that the construction admits the vector spaces over all finite fields  $\mathbb{F}_q$ . Another ingredient of the construction above is a subgroup  $N$  of index  $r \geq 2$  in  $\mathbb{F}_q^*$ . In our work we show that it is possible to slightly modify this construction by plugging a difference set in the (cyclic) quotient group  $\mathbb{F}_q^+/N$  into it and letting the dimension of the vector space be an arbitrary positive even integer. This leads to a new infinite family of divisible design graphs that are  $r$ -covers of the corresponding symplectic strongly regular polar graphs [2, Section 2.5].

Further, we mimic the construction of divisible design graphs above by replacing the symplectic bilinear form with a bilinear (resp. sequilinear) form obtained by the polarisation of hyperbolic, elliptic and parabolic quadratic forms (resp. by the polarisation of the Hermitian form). This gives a few more infinite families of  $r$ -covers of strongly regular polar graphs (see [2, Section 2.6, Section 2.7]). These infinite families of  $r$ -covers contain infinite subfamilies of non-trivial divisible design graphs in the elliptic, parabolic and Hermitian cases.

This is joint work with Bart De Bruyn and Sergey Goryainov.

## References

- [1] A.E. Brouwer, A. Cohen, A. Neumaier, *Distance-regular graphs*, Springer, Heidelberg, 1989.
- [2] A. E. Brouwer, H. Van Maldeghem, *Strongly Regular Graphs*, Cambridge University Press, Cambridge (2022).
- [3] W. H. Haemers, H. Kharaghani, M. A. Meulenberg, *Divisible design graphs*, Journal of Combinatorial Theory, Series A, 118 (2011) 978–992.