Cyclic difference families, perfect difference families and their applications

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Let v be a positive odd integer. A (v, k, λ)-perfect difference family (PDF) is a collection $\mathcal{F}$ of k-subsets of $\{0, 1, \ldots, v-1\}$ such that the multiset $\bigcup_{F\in\mathcal{F}} \{x - y : x, y \in F, x > y\}$ covers each element of $\{1, 2, \ldots, (v-1)/2\}$ exactly λ times. Perfect difference families are a special class of perfect systems of difference sets. This talk shows that a $(v, 4, \lambda)$-PDF exists if and only if $\lambda(v-1) \equiv 0 \pmod{12}$, $v \geq 13$, and $(v, \lambda) \notin \{(25, 1), (37, 1)\}$. This result resolves a nearly 50-year-old conjecture posed by Bermond. Perfect difference families find applications in radio astronomy, optical orthogonal codes for optical code-division multiple access systems, geometric orthogonal codes for DNA origami, difference triangle sets, additive sequences of permutations, and graceful graph labelings. Perfect difference families can be also seen as a special kind of cyclic difference families. This talk will also give a survey on the recent progress on constructions for cyclic difference families.

References

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