A new infinite family of divisible design graphs related to antipodal distance-regular graphs of diameter 3

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We consider only simple graphs. A *divisible design graph* with parameters $(v, k, \lambda_1, \lambda_2, m, n)$ is a k-regular graph on v vertices such that its vertex set can be partitioned into m classes of size n where any two distinct vertices from the same class have exactly λ_1 common neighbours and any two vertices from different classes have exactly λ_2 common neighbours. Divisible design graphs were introduced in [2] as a bridge between graph theory and the theory of group divisible designs. Since then, tens of constructions of divisible design graphs have been introduced.

In [1, Proposition 12.5.1], a construction of antipodal distance-regular graphs of diameter 3 was given. This construction uses a vector space of an even dimension equipped with a nondegenerate symplectic bilinear form. The construction admits the vector spaces over all finite fields $\mathbb{F}_q$. In our work we show that in the case when q is even it is possible to slightly modify this construction by plugging a difference set in $\mathbb{F}_q^+$ into it. The leads to a new infinite family of divisible design graphs. In the talk we discuss this modification and connection of the resulting graphs with the parabolic affine polar graphs.

The authors of [1] further discuss a few generalisations of the construction of distance-regular graphs given in [1, Proposition 12.5.1]. In our work we show that one of these generalisations, which depends on a subgroup A in $\mathbb{F}_q^+$, can be modified by plugging a difference set in the quotient group of $\mathbb{F}_q^+/A$ into it. This leads to a wider infinite family of divisible design graphs, which includes the family of divisible design graphs mentioned above. The main goal of this talk is to discuss this wider family of divisible design graphs.

References

[1] A.E. Brouwer, A. Cohen, A. Neumaier, *Distance-regular graphs*, Springer, Heidelberg, 1989.

[2] W. H. Haemers, H. Kharaghani, M. A. Meulenberg, *Divisible design graphs*, Journal of Combinatorial Theory, Series A, 118 (2011) 978–992.