Problem 1. We differentiate equation (5) in the perturbation notes with respect to u and suppress the function arguments for the sake of clarity. This gives us:

$$x_{uuu} = -\frac{F_{xxx}x_u^3 + 3F_{xxu}x_u^2 + 3F_{xx}x_{uu}x_u + 3F_{xuu}x_u + 3F_{xu}x_{uu} + F_{uuu}}{F_x}$$

Problem 2-4. Refer to my Perturbation python notebook.