

# Supplementary Report: Results of the Mallows and the Generalized Mallows Estimation of Distribution Algorithms

Josu Ceberio, Ekhine Irurozki,  
Alexander Mendiburu, and Jose A. Lozano <sup>\*†</sup>

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## Abstract

In this report we introduce the raw results obtained in the experimental study carried out in the paper *A Review of Distances for the Mallows and Generalized Mallows Estimation of Distribution Algorithms* submitted to the journal *Computational Optimization and Applications*. Particularly, we summarise the results obtained with the Mallows EDAs under Kendall's- $\tau$  ( $M_{ken}$ ), Cayley ( $M_{cay}$ ) and Ulam ( $M_{ula}$ ), and the Generalized Mallows EDAs under Kendall's- $\tau$  ( $GM_{ken}$ ) and Cayley ( $GM_{cay}$ ) on a benchmark of 200 instances of the Linear Ordering Problem (LOP), the Quadratic Assignment Problem (QAP), the Permutation Flowshop Scheduling Problem (PFSP) and the Travelling Salesman Problem (TSP).

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<sup>\*</sup>J. Ceberio, E. Irurozki and J. A. Lozano are with the Intelligent Systems Group, Department of Computer Science and Artificial Intelligence, University of the Basque Country UPV/EHU, Donostia 20018, Gipuzkoa, Spain. {e-mail: josu.ceberio@ehu.es}

<sup>†</sup>A. Mendiburu is with the Intelligent Systems Group, Department of Computer Architecture and Technology, University of the Basque Country UPV/EHU, Donostia 20018, Gipuzkoa, Spain.

Table 1: Average results of 20 repetitions of  $M_{ken}$ ,  $M_{cay}$ ,  $M_{ula}$ ,  $GM_{ken}$  and  $GM_{cay}$  for the LOP instances. In addition, the best result found by any of the EDAs among the executions has been included in the table.

Instance	<i>Best</i>	$M_{ken}$	$M_{cay}$	$M_{ula}$	$GM_{ken}$	$GM_{cay}$
Cebe.lopn10.1	2384	2357	2366	2382	2367	2380
Cebe.lopn10.2	12338	12214	12164	12291	12162	12234
Cebe.lopn10.3	3210	3176	3151	3187	3165	3208
Cebe.lopn10.4	6269	6203	6177	6269	6186	6252
Cebe.lopn10.5	10543	10513	10501	10532	10536	10516
Cebe.lopn20.1	13413	13059	13136	13369	13032	13241
Cebe.lopn20.2	11554	11065	11100	11369	11124	11352
Cebe.lopn20.3	55160	53957	54172	54881	54083	54690
Cebe.lopn20.4	50661	49989	50037	50536	50039	50305
Cebe.lopn20.5	27186	26372	26514	26912	26261	26651
Cebe.lopn30.1	37178	36093	35618	36982	36179	36680
Cebe.lopn30.2	50965	48816	47974	50473	49252	49984
Cebe.lopn30.3	60688	58347	56992	60000	58493	59442
Cebe.lopn30.4	97551	94519	93002	96938	94908	96170
Cebe.lopn30.5	112823	108310	105827	111801	109087	110714
Cebe.lopn40.1	60966	59150	59169	60625	59540	60095
Cebe.lopn40.2	75869	72642	72951	74906	73818	74400
Cebe.lopn40.3	130752	125437	125275	130288	127597	129241
Cebe.lopn40.4	165359	158818	158607	162853	159106	161578
Cebe.lopn40.5	225132	217467	218177	223772	217279	220288
Cebe.lopn50.1	104102	99406	102279	102915	100716	102354
Cebe.lopn50.2	120484	115274	117669	119400	116421	118051
Cebe.lopn50.3	204204	192658	198651	202473	197215	200113
Cebe.lopn50.4	201359	192895	197199	199234	196187	198083
Cebe.lopn50.5	308456	294485	301283	304073	297267	301302
Cebe.lopn60.1	154096	146260	151098	153344	149459	151852
Cebe.lopn60.2	179310	168723	174251	177423	173752	175402
Cebe.lopn60.3	255747	242325	250005	253574	248437	250740
Cebe.lopn60.4	332226	313049	325276	329561	323520	325749
Cebe.lopn60.5	475090	454517	467189	472182	466726	468402
Cebe.lopn70.1	176584	164621	173171	175337	171826	173123
Cebe.lopn70.2	270183	252063	263750	268112	261804	263732
Cebe.lopn70.3	329008	308357	321106	325704	319428	321563
Cebe.lopn70.4	403908	378928	395644	401448	393040	395963
Cebe.lopn70.5	573026	536864	562381	569118	551653	560131
Cebe.lopn80.1	214487	198383	209729	212752	207966	210161
Cebe.lopn80.2	327716	302783	320064	325336	317791	319315
Cebe.lopn80.3	438083	407577	427064	433873	423868	426297
Cebe.lopn80.4	549644	509120	537736	545236	532613	537596
Cebe.lopn80.5	659128	606829	642146	652407	631929	639189
Cebe.lopn90.1	282612	261000	277845	281079	273762	276878
Cebe.lopn90.2	465947	428663	458064	463059	452665	457372
Cebe.lopn90.3	553655	501229	538048	546586	533060	539457
Cebe.lopn90.4	722522	665867	704737	715226	700304	703322
Cebe.lopn90.5	826278	760253	807944	822017	804139	808858
Cebe.lopn100.1	372605	339493	363485	370097	362147	364489
Cebe.lopn100.2	464031	418668	450868	457254	443647	449333
Cebe.lopn100.3	768924	698481	753824	764608	740274	753041
Cebe.lopn100.4	933718	852166	914252	926490	910368	913447
Cebe.lopn100.5	841592	762479	821183	832069	805133	820751

Table 2: Average results of 20 repetitions of  $M_{ken}$ ,  $M_{cay}$ ,  $M_{ula}$ ,  $GM_{ken}$  and  $GM_{cay}$  for the QAP instances. In addition, the best result found by any of the EDAs among the executions has been included in the table.

Instance	<i>Best</i>	$M_{ken}$	$M_{cay}$	$M_{ula}$	$GM_{ken}$	$GM_{cay}$
Cebe.qap.n10.1	3971134	4716584	4617052	5316593	5237562	4214033
Cebe.qap.n10.2	191405	196282	194592	198022	197688	192914
Cebe.qap.n10.3	6588668	7697975	7061705	7554308	7741003	6896134
Cebe.qap.n10.4	149411	156168	154366	157443	157073	153517
Cebe.qap.n10.5	7833127	9387983	8909872	9440965	9545409	8570250
Cebe.qap.n20.1	17655023	32106413	31129246	34695317	31046916	22553382
Cebe.qap.n20.2	817479	857572	847538	866199	855495	828244
Cebe.qap.n20.3	23561217	37696119	33476323	39771008	35912672	27517760
Cebe.qap.n20.4	802798	840678	836848	850180	840033	811414
Cebe.qap.n20.5	28198973	39669972	37904415	41607148	37544194	31515376
Cebe.qap.n30.1	105682995	161345222	167377768	177829787	158607014	137326398
Cebe.qap.n30.2	1791398	1894352	1908560	1920073	1892211	1827807
Cebe.qap.n30.3	67061805	104758236	107817677	115749920	103015019	78380156
Cebe.qap.n30.4	1905605	2012215	2019920	2034039	2003342	1948472
Cebe.qap.n30.5	57519526	91515591	91743148	99086735	87199201	74097675
Cebe.qap.n40.1	156949623	251577760	207457107	283611879	254214733	177204519
Cebe.qap.n40.2	3222598	3393759	3301905	3441444	3381341	3256783
Cebe.qap.n40.3	145735702	214200618	176963528	233070512	208860669	155627475
Cebe.qap.n40.4	3358581	3534590	3437173	3581022	3522784	3392303
Cebe.qap.n40.5	133934433	195587349	182326784	220394334	193779776	145197076
Cebe.qap.n50.1	260406098	409559258	304933311	464773301	412191613	284288425
Cebe.qap.n50.2	5369433	5609736	5423341	5694258	5611483	5397904
Cebe.qap.n50.3	262145145	375271981	287331071	408446258	371726185	274243506
Cebe.qap.n50.4	5393073	5671278	5466982	5751719	5650873	5434852
Cebe.qap.n50.5	213713867	310508802	239458328	347030490	306677582	228367185
Cebe.qap.n60.1	457918667	674446906	504542328	744657246	668473955	491747669
Cebe.qap.n60.2	7769698	8127779	7855752	8243453	8131262	7825235
Cebe.qap.n60.3	344818390	497651383	377159788	537733399	493288378	364746837
Cebe.qap.n60.4	7877537	8246539	7952356	8354904	8244617	7916373
Cebe.qap.n60.5	321208245	464419692	345596933	503970232	460893600	335051252
Cebe.qap.n70.1	676724432	990333344	745992579	1017631873	979366637	722564122
Cebe.qap.n70.2	10660152	11136312	10752714	11166903	11126099	10714370
Cebe.qap.n70.3	528926962	717851674	561884419	731180567	712225224	543877786
Cebe.qap.n70.4	10414034	10909395	10526230	10927617	10908153	10492359
Cebe.qap.n70.5	467101324	662721565	500523364	672072025	651825463	492538256
Cebe.qap.n80.1	881065952	1287145679	951493796	1278736222	1266037517	939080214
Cebe.qap.n80.2	14032237	14701285	14190521	14683124	14700827	14125414
Cebe.qap.n80.3	697721421	951475721	738211406	946096376	941335293	728586428
Cebe.qap.n80.4	14132988	14776105	14258793	14785487	14768489	14223037
Cebe.qap.n80.5	639756786	879243973	676293319	873947849	879351243	661207664
Cebe.qap.n90.1	1177175998	1683776890	1273910210	1673190293	1661407371	1249288181
Cebe.qap.n90.2	17392931	18210354	17559798	18151483	18153415	17509326
Cebe.qap.n90.3	989650560	1330335973	1055085352	1316675232	1318025188	1031720067
Cebe.qap.n90.4	17481164	18231044	17582522	18187948	18219214	17543124
Cebe.qap.n90.5	753545529	1068077779	814996438	1052403746	1054674053	793711531
Cebe.qap.n100.1	1495614972	2109090425	1582146450	2044123370	2105710491	1539653068
Cebe.qap.n100.2	22184316	23051256	22297295	22934837	23032708	22238052
Cebe.qap.n100.3	1158421359	1532141520	1207911505	1495760226	1522402624	1191053300
Cebe.qap.n100.4	21734674	22680911	21891772	22567325	22636622	21840145
Cebe.qap.n100.5	993190021	1387845179	1044687158	1329255505	1373518413	1023579103

Table 3: Average results of 20 repetitions of  $M_{ken}$ ,  $M_{cay}$ ,  $M_{ula}$ ,  $GM_{ken}$  and  $GM_{cay}$  for the PFSP instances. In addition, the best result found by any of the EDAs among the executions has been included in the table.

Instance	<i>Best</i>	$M_{ken}$	$M_{cay}$	$M_{ula}$	$GM_{ken}$	$GM_{cay}$
Cebe.pfsp.n10.m10.1	7741	7781	7791	7768	7796	7774
Cebe.pfsp.n10.m10.2	8212	8227	8268	8227	8274	8212
Cebe.pfsp.n10.m10.3	7740	7854	7867	7819	7881	7815
Cebe.pfsp.n10.m10.4	7638	7678	7660	7654	7668	7648
Cebe.pfsp.n10.m10.5	8088	8144	8169	8110	8191	8107
Cebe.pfsp.n20.m10.1	20091	20666	20685	20329	20461	20438
Cebe.pfsp.n20.m10.2	19285	19992	19885	19594	19894	19712
Cebe.pfsp.n20.m10.3	19701	20316	20298	19957	20136	20133
Cebe.pfsp.n20.m10.4	19658	20152	20058	19817	19987	19854
Cebe.pfsp.n20.m10.5	21836	22516	22302	22077	22328	22141
Cebe.pfsp.n30.m10.1	38258	39390	40571	38763	38951	39047
Cebe.pfsp.n30.m10.2	38125	39225	40015	38449	38643	38755
Cebe.pfsp.n30.m10.3	38239	39471	40814	38781	38795	38767
Cebe.pfsp.n30.m10.4	37720	38976	40008	38359	38635	38376
Cebe.pfsp.n30.m10.5	40105	41177	42170	40705	40766	40928
Cebe.pfsp.n40.m10.1	60844	62943	63553	62321	61525	61824
Cebe.pfsp.n40.m10.2	63659	65640	66070	65400	64352	64682
Cebe.pfsp.n40.m10.3	60099	62508	62449	61530	60815	61048
Cebe.pfsp.n40.m10.4	66125	68252	68671	67366	66845	66937
Cebe.pfsp.n40.m10.5	63541	65612	65392	64643	64202	64410
Cebe.pfsp.n50.m10.1	90680	94155	92881	93050	91464	92084
Cebe.pfsp.n50.m10.2	90831	95082	93344	93408	91795	92558
Cebe.pfsp.n50.m10.3	88816	92935	91433	91436	90050	91160
Cebe.pfsp.n50.m10.4	89317	93271	91125	92176	89991	90783
Cebe.pfsp.n50.m10.5	92763	96240	94698	95665	93604	93799
Cebe.pfsp.n60.m10.1	120014	127033	123125	122908	121803	122554
Cebe.pfsp.n60.m10.2	121648	127780	124515	124567	122628	123464
Cebe.pfsp.n60.m10.3	121471	127713	124352	123425	122759	123461
Cebe.pfsp.n60.m10.4	122981	129016	125880	125795	124256	125091
Cebe.pfsp.n60.m10.5	128953	135160	131522	131533	129809	130742
Cebe.pfsp.n70.m10.1	159178	168324	163649	162311	161752	163229
Cebe.pfsp.n70.m10.2	163214	172380	167553	166773	165391	166919
Cebe.pfsp.n70.m10.3	157560	166688	160417	160041	159471	160090
Cebe.pfsp.n70.m10.4	164948	173122	168169	167594	166203	167624
Cebe.pfsp.n70.m10.5	165475	173655	168068	167964	166771	167477
Cebe.pfsp.n80.m10.1	200142	213702	203486	203459	202092	202856
Cebe.pfsp.n80.m10.2	195254	206669	198606	197454	197619	197832
Cebe.pfsp.n80.m10.3	199263	213188	204313	203299	202738	203582
Cebe.pfsp.n80.m10.4	193008	206564	197779	196161	196734	197080
Cebe.pfsp.n80.m10.5	208180	219996	211314	210879	210455	211099
Cebe.pfsp.n90.m10.1	244772	261911	249869	248693	248726	248999
Cebe.pfsp.n90.m10.2	246614	263319	250949	250075	250984	250190
Cebe.pfsp.n90.m10.3	247882	264612	253509	252067	251330	251744
Cebe.pfsp.n90.m10.4	249724	265429	254699	253103	253130	253034
Cebe.pfsp.n90.m10.5	253354	269268	258555	256995	257865	256742
Cebe.pfsp.n100.m10.1	301487	322925	306790	305816	309823	305178
Cebe.pfsp.n100.m10.2	301631	323109	306734	305589	309350	306245
Cebe.pfsp.n100.m10.3	299787	321191	304856	302991	304532	304072
Cebe.pfsp.n100.m10.4	308706	328927	311914	313020	314793	312642
Cebe.pfsp.n100.m10.5	296978	316385	300583	300054	304066	299974

Table 4: Average results of 20 repetitions of  $M_{ken}$ ,  $M_{cay}$ ,  $M_{ula}$ ,  $GM_{ken}$  and  $GM_{cay}$  for the TSP instances. In addition, the best result found by any of the EDAs among the executions has been included in the table.

Instance	<i>Best</i>	$M_{ken}$	$M_{cay}$	$M_{ula}$	$GM_{ken}$	$GM_{cay}$
Cebe.tsp.n10.1	143	150	149	147	154	145
Cebe.tsp.n10.2	17745	19689	19453	19798	19860	17821
Cebe.tsp.n10.3	23339	24626	24523	25634	25287	23886
Cebe.tsp.n10.4	27260	28377	27842	28188	29467	27260
Cebe.tsp.n10.5	433	453	446	457	464	441
Cebe.tsp.n20.1	250	307	329	280	290	275
Cebe.tsp.n20.2	23114	33961	39805	29180	31834	28054
Cebe.tsp.n20.3	36047	48787	56484	43928	45911	41370
Cebe.tsp.n20.4	26130	40450	50943	37493	37438	33751
Cebe.tsp.n20.5	473	661	723	568	606	548
Cebe.tsp.n30.1	267	437	517	392	373	419
Cebe.tsp.n30.2	41414	59875	72130	55760	54690	59730
Cebe.tsp.n30.3	46671	75072	87847	66621	64010	62735
Cebe.tsp.n30.4	46973	82459	98491	76323	71269	67243
Cebe.tsp.n30.5	731	1086	1334	943	964	961
Cebe.tsp.n40.1	399	633	580	706	603	448
Cebe.tsp.n40.2	43460	81237	73229	89440	71672	53099
Cebe.tsp.n40.3	57580	109353	93812	122042	98429	70667
Cebe.tsp.n40.4	64369	103842	89258	126838	93835	72344
Cebe.tsp.n40.5	1053	1633	1619	1755	1454	1184
Cebe.tsp.n50.1	487	856	568	648	750	556
Cebe.tsp.n50.2	54971	104572	67057	79963	90164	61754
Cebe.tsp.n50.3	69139	130021	83096	91114	118334	77648
Cebe.tsp.n50.4	87677	150839	99992	119021	137651	97563
Cebe.tsp.n50.5	1125	2033	1357	1597	1870	1238
Cebe.tsp.n60.1	524	1039	604	625	934	583
Cebe.tsp.n60.2	66151	133412	78811	78386	124286	75386
Cebe.tsp.n60.3	77986	169103	94538	92983	149465	89184
Cebe.tsp.n60.4	87918	180536	102522	106486	157381	98988
Cebe.tsp.n60.5	1387	2415	1599	1582	2341	1534
Cebe.tsp.n70.1	621	1249	703	701	1053	683
Cebe.tsp.n70.2	78017	157630	92138	91523	149988	87575
Cebe.tsp.n70.3	78950	195206	100563	92443	166202	94401
Cebe.tsp.n70.4	102442	220776	120543	120986	193732	117783
Cebe.tsp.n70.5	1586	2931	1793	1744	2736	1695
Cebe.tsp.n80.1	682	1476	826	788	1312	806
Cebe.tsp.n80.2	88301	183586	100288	98498	169203	96156
Cebe.tsp.n80.3	91450	226978	110783	106258	196413	105251
Cebe.tsp.n80.4	117773	253512	138001	131668	225972	132414
Cebe.tsp.n80.5	1692	3534	2023	1929	3288	1895
Cebe.tsp.n90.1	769	1633	894	870	1524	860
Cebe.tsp.n90.2	91236	210501	105183	103715	188011	103352
Cebe.tsp.n90.3	98573	254910	119731	113256	225962	112426
Cebe.tsp.n90.4	128819	300260	149639	145085	256557	141518
Cebe.tsp.n90.5	1927	4097	2147	2105	3589	2114
Cebe.tsp.n100.1	778	1815	940	834	1632	926
Cebe.tsp.n100.2	88685	237335	114425	99708	212770	112037
Cebe.tsp.n100.3	101363	285378	129400	116040	255315	125432
Cebe.tsp.n100.4	128048	327336	155722	142366	302223	149257
Cebe.tsp.n100.5	1775	4435	2154	1996	3944	2127