

A distance-based ranking estimation of distribution algorithm for the permutation flowshop scheduling problem

IEEE Transactions on Evolutionary Computation
April 2013

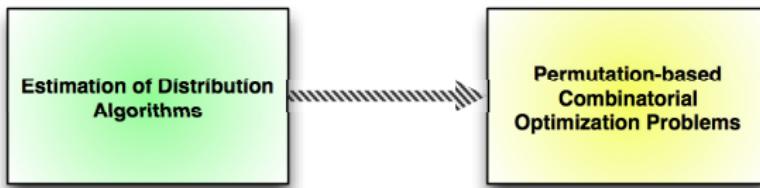
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The Background

**Estimation of Distribution
Algorithms**

The Background

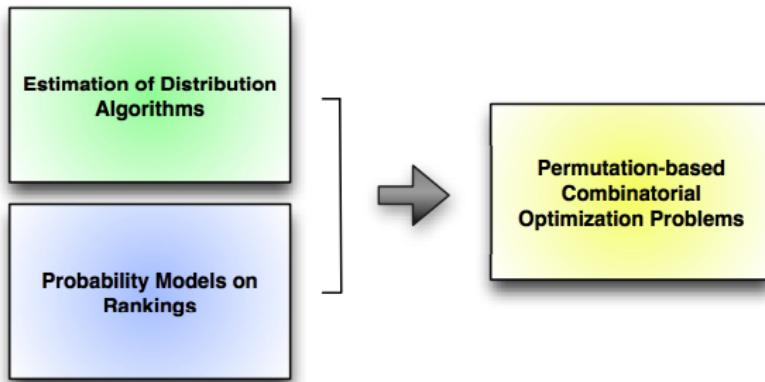


Poor performance

The Hypothesis

"Existing EDAs do not implement adequate probabilistic models for optimizing permutation domain problems."

The Contribution



Outline

- ① Estimation of Distribution Algorithms
- ② Permutation-based Optimization Problems
- ③ Distance-based Ranking Models
- ④ Our approach
- ⑤ Experiments
- ⑥ Conclusions & Future work

Outline

- 1 Estimation of Distribution Algorithms
- 2 Permutation-based Optimization Problems
- 3 Distance-based Ranking Models
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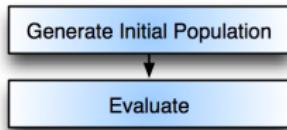
Estimation of Distribution Algorithms

Definition

Generate Initial Population

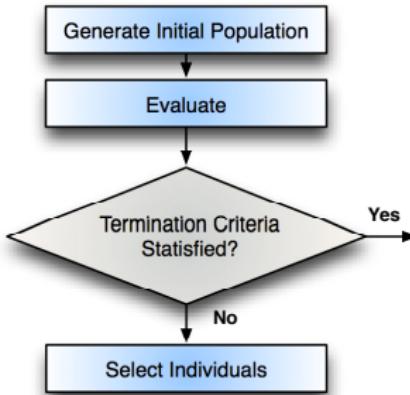
Estimation of Distribution Algorithms

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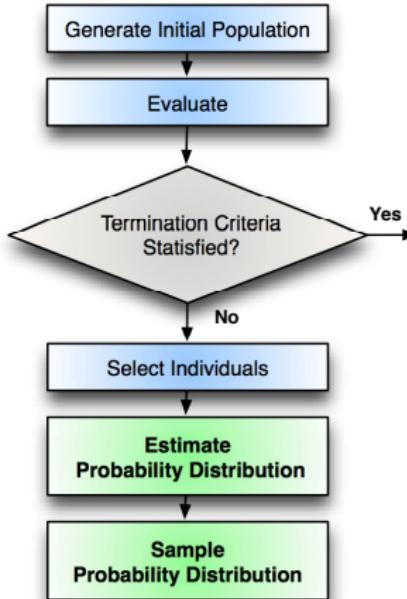
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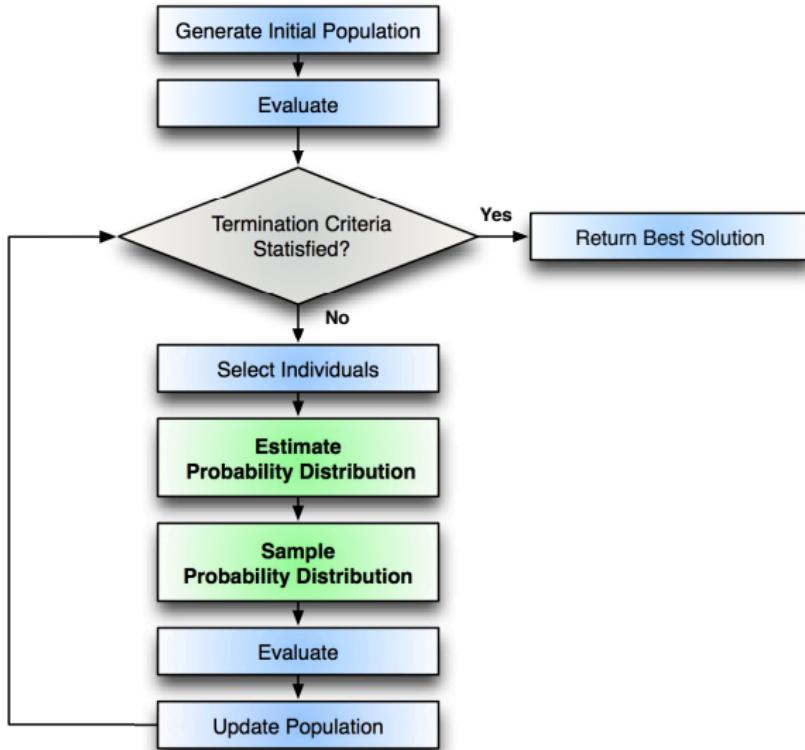
Estimation of Distribution Algorithms

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Estimation of Distribution Algorithms

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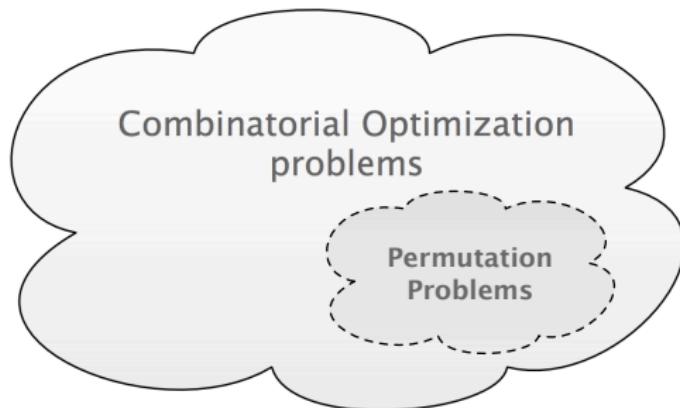


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Permutation-based Optimization Problems

Definition



Permutation-based Optimization Problems

Definition

Problems whose solution can be naturally represented as a **permutation**.

Permutation-based Optimization Problems

Examples

Travelling Salesman Problem



Which **permutation** of cities provides the shortest path?

Permutation-based Optimization Problems

Examples

Linear Ordering Problem

	1	2	3	4	5
1	0	16	11	15	7
2	21	0	14	15	9
3	26	23	0	26	12
4	22	22	11	0	13
5	30	28	25	24	0



	5	3	4	2	1
5	0	25	24	28	30
3	12	0	26	23	26
4	13	11	0	22	22
2	9	14	15	0	21
1	7	11	15	16	0

Which **permutation** of rows/columns optimizes this problem?

Permutation-based Optimization Problems

Examples

Permutation Flowshop Scheduling Problem

Permutation-based Optimization Problems

Examples

Permutation Flowshop Scheduling Problem

M1

M2

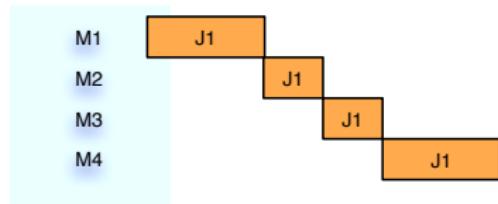
M3

M4

Permutation-based Optimization Problems

Examples

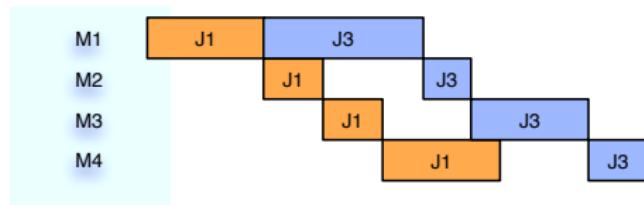
Permutation Flowshop Scheduling Problem



Permutation-based Optimization Problems

Examples

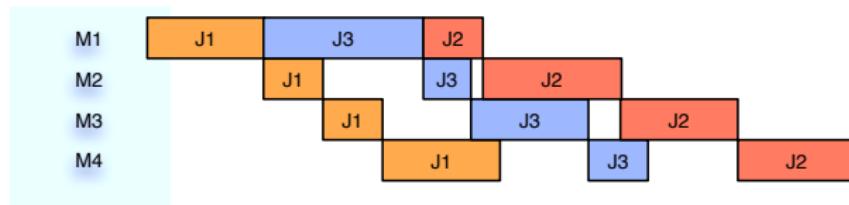
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Permutation-based Optimization Problems

Examples

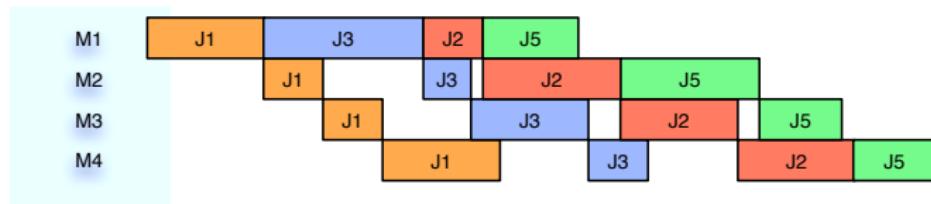
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Permutation-based Optimization Problems

Examples

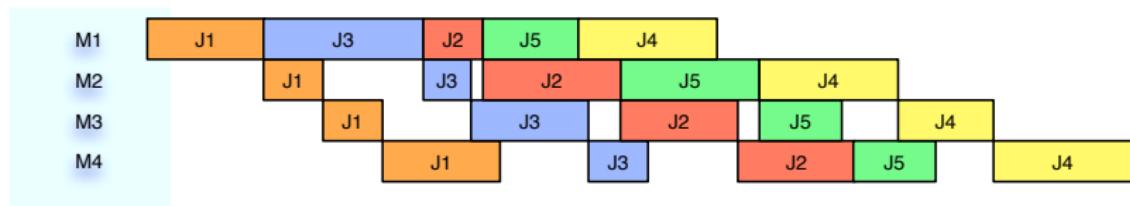
Permutation Flowshop Scheduling Problem



Permutation-based Optimization Problems

Examples

Permutation Flowshop Scheduling Problem



(1, 3, 2, 5, 4)

Estimation of Distribution Algorithms

Existing approaches

Standard representations are not effective due to the **mutual exclusivity constraints** associated with permutations.

Estimation of Distribution Algorithms

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Standard representations are not effective due to the **mutual exclusivity constraints** associated with permutations.

Integer encoding EDAs

UMDA

EBNA

MIMIC

PBIL

TREE

EHBSA

NHBSA

Learn a probability distribution over a set

$\Omega = \Omega_1 \times \Omega_2 \times \dots \times \Omega_n$ where $\Omega_i = \{1, \dots, r_i\}$ and
 $r_i \in \mathbb{N}, i = 1, \dots, n.$

Estimation of Distribution Algorithms

Existing approaches

Standard representations are not effective due to the **mutual exclusivity constraints** associated with permutations.

Integer encoding EDAs

Sample

4	5	3	1	2
1	2	4	5	3
3	1	2	4	5
1	3	2	4	5

Estimation of Distribution Algorithms

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Integer encoding EDAs

Univariate model

Job	X1	X2	X3	X4	X5
1	0.5	0.25	-	0.25	-
2	-	0.25	0.5	-	0.25
3	0.25	0.25	0.25	-	0.25
4	0.25	-	0.25	0.5	-
5	-	0.25	-	0.25	0.5

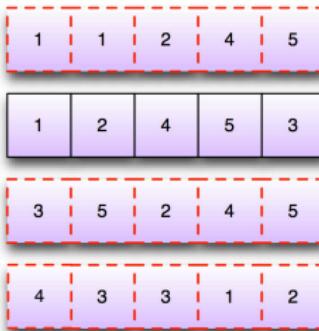
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Integer encoding EDAs

New Sample



Estimation of Distribution Algorithms

Existing approaches

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Integer encoding EDAs

New Sample

1	1	2	4	5
1	2	4	5	3
3	5	2	4	5
4	3	3	1	2

The sampling does not always provide an individual in \mathbb{S}_n , but an individual in Ω .

Estimation of Distribution Algorithms

Existing approaches

Standard representations are not effective due to the **mutual exclusivity constraints** associated with permutations.

Real-value encoding EDAs

UMDA_c

MIMIC_c

EGNA

IDEA

...

The sampled real vectors are transformed into permutations with the **Random Keys** strategy.

$$\begin{array}{c} (0.30, 0.10, 0.40, 0.20) \\ (0.25, 0.14, 0.35, 0.16) \\ (0.60, 0.20, 0.80, 0.40) \end{array} \rightarrow (3, 1, 4, 2)$$

High redundancy in the codification.

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Distance-based Ranking Models

The Mallows model

A distance-based unimodal exponential model.

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A distance-based unimodal exponential model.

Two parameters:

- ▶ Consensus ranking σ_0
- ▶ Spread parameter θ

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Probability distribution:

$$P(\sigma) = \frac{1}{\psi(\theta)} e^{-\theta D_\tau(\sigma, \sigma_0)}$$

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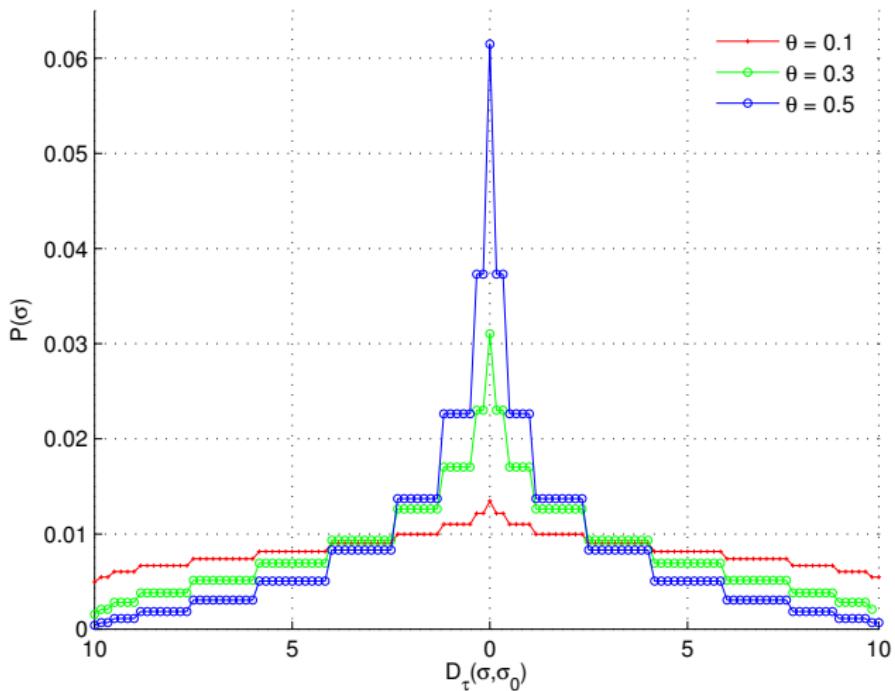
Probability distribution:

$$P(\sigma) = \frac{1}{\psi(\theta)} e^{-\theta D_\tau(\sigma, \sigma_0)}$$

Analogous over permutations to the Gaussian distribution.

Distance-based Ranking Models

The Mallows model

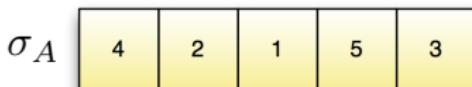


Distance-based Ranking Models

The Mallows model

Kendall's- τ distance

Counts the number of pairwise disagreements.

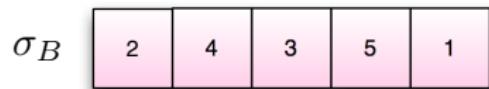
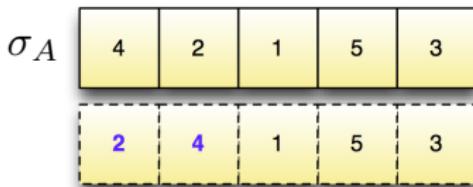


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Distance-based Ranking Models

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Counts the number of pairwise disagreements.

σ_A	4	2	1	5	3
	2	4	1	5	3
	2	4	5	1	3

σ_B	2	4	3	5	1
------------	---	---	---	---	---

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σ_B	2	4	3	5	1
------------	---	---	---	---	---

$$D_\tau(\sigma_A, \sigma_B) = 4$$

Distance-based Ranking Models

The Generalized Mallows model

If the D can be decomposed as... $D_\tau(\sigma, \sigma_0) = \sum_{j=1}^{n-1} V_j(\sigma, \sigma_0)$

$$P(\sigma) = \frac{1}{\psi(\theta)} e^{-\theta D_\tau(\sigma, \sigma_0)}$$

Mallows

Distance-based Ranking Models

The Generalized Mallows model

If the D can be decomposed as... $D_\tau(\sigma, \sigma_0) = \sum_{j=1}^{n-1} V_j(\sigma, \sigma_0)$

$$P(\sigma) = \frac{1}{\psi(\theta)} e^{-\theta D_\tau(\sigma, \sigma_0)} \quad P(\sigma) = \frac{1}{\psi(\theta)} e^{-\sum_{j=1}^{n-1} \theta_j V_j(\sigma, \sigma_0)}$$

→

Mallows

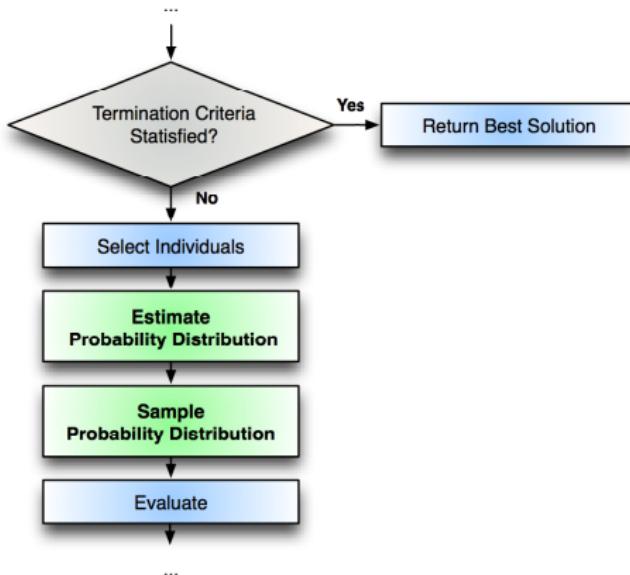
Generalized Mallows

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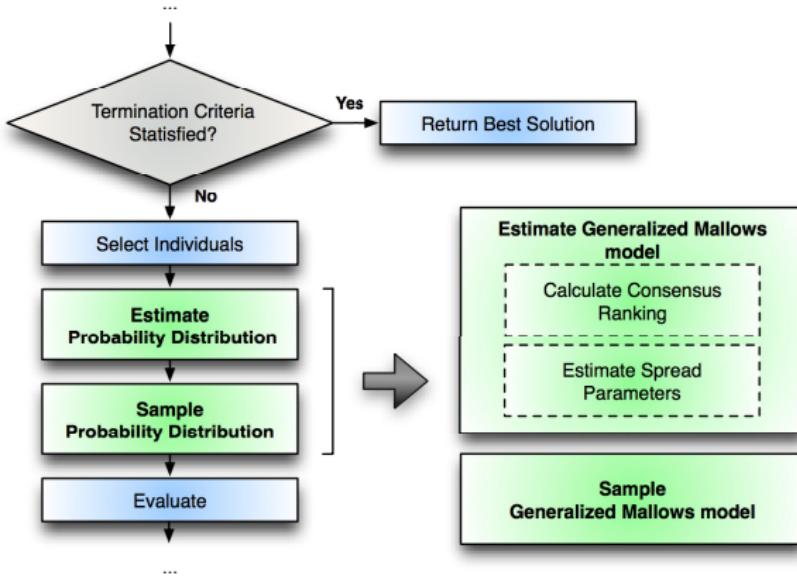
Generalized Mallows EDA

Scheme



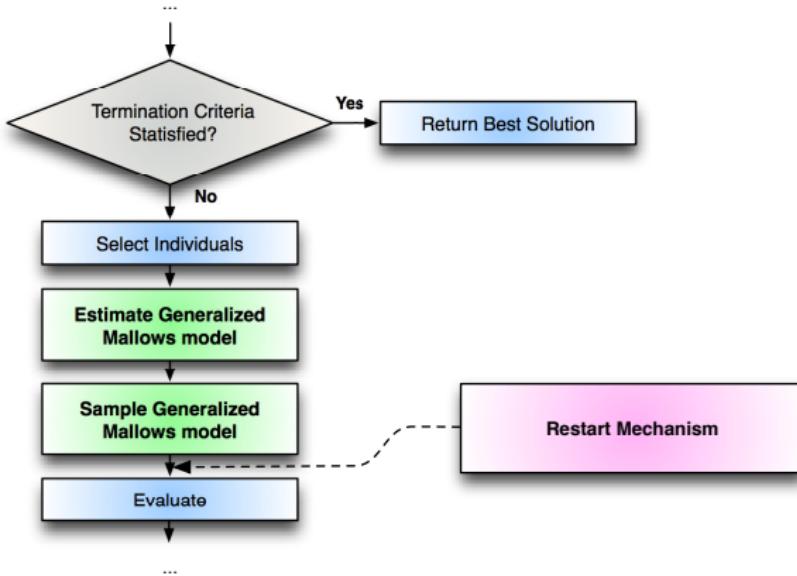
Generalized Mallows EDA

Scheme



Generalized Mallows EDA

Scheme



Generalized Mallows EDA

Preliminary Experiments - Comparing to NHBSA and EHBSA

Algorithms

- ▶ Edge Histogram-based Sampling Algorithm (EHBSA)
- ▶ Node Histogram-based Sampling Algorithm (NHBSA).
- ▶ Generalized Mallows EDA (GMEDA).

Instances

- ▶ Taillard's Benchmark: 120 instances.

Other

- ▶ Best solution of 20 repetitions.
- ▶ Max. number of evaluations.

Generalized Mallows EDA

Preliminary Experiments - Comparing to NHBSA and EHBSA

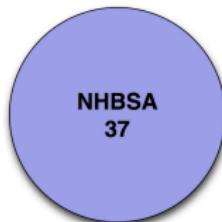
Results

120 instances

Generalized Mallows EDA

Preliminary Experiments - Comparing to NHBSA and EHBSA

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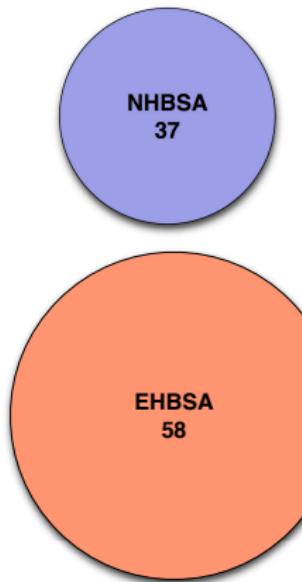


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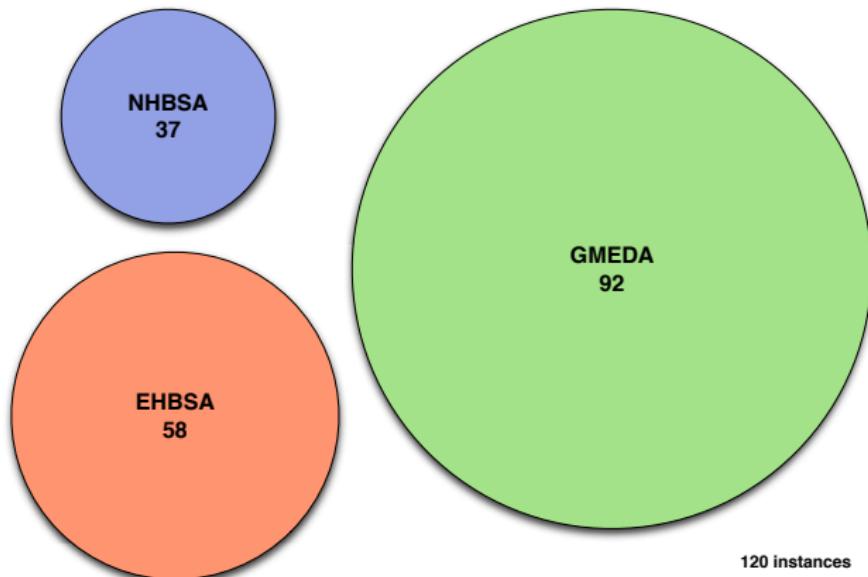


120 instances

Generalized Mallows EDA

Preliminary Experiments - Comparing to NHBSA and EHBSA

Results



Generalized Mallows EDA

Preliminary Experiments - Comparing to NHBSA and EHBSA

The challenge is to outperform the state-of-the-art algorithms of the PFSP.

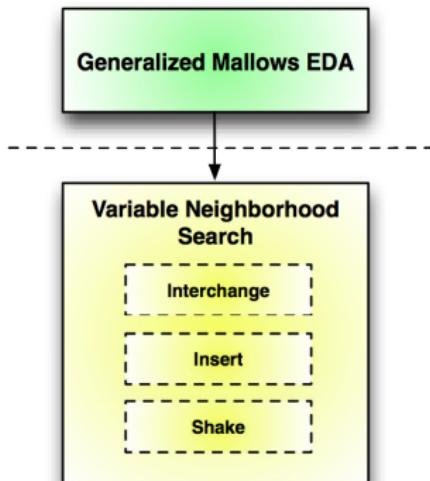
Hybrid Generalized Mallows EDA

The challenge: State-of-the-art algorithms

Generalized Mallows EDA

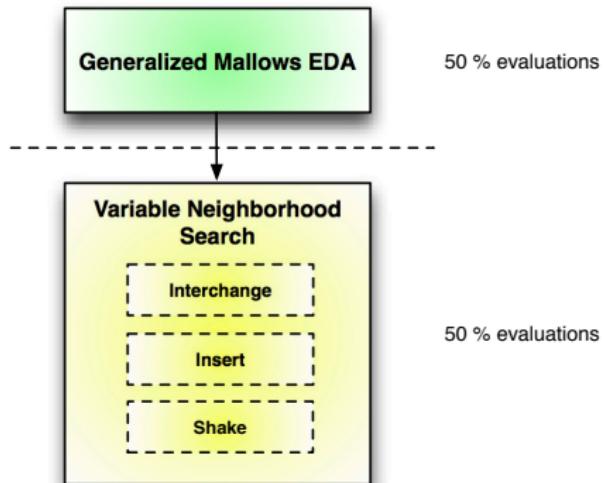
Hybrid Generalized Mallows EDA

The challenge: State-of-the-art algorithms



Hybrid Generalized Mallows EDA

The challenge: State-of-the-art algorithms



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Experiments

Settings

Algorithms

- ▶ Asynchronous Genetic Algorithm (AGA).
- ▶ Variable Neighborhood Search 4 (VNS_4).
- ▶ Generalized Mallows EDA (GMEDA).
- ▶ Hybrid Generalized Mallows EDA (HGMEDA).
- ▶ Variable Neighborhood Search (VSN) - own design.

Instances

- ▶ Taillard's Benchmark: 120 instances.
- ▶ Random Benchmark: 100 instances.

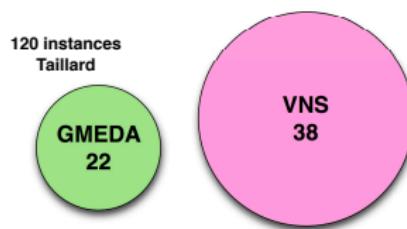
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Experiments

Results

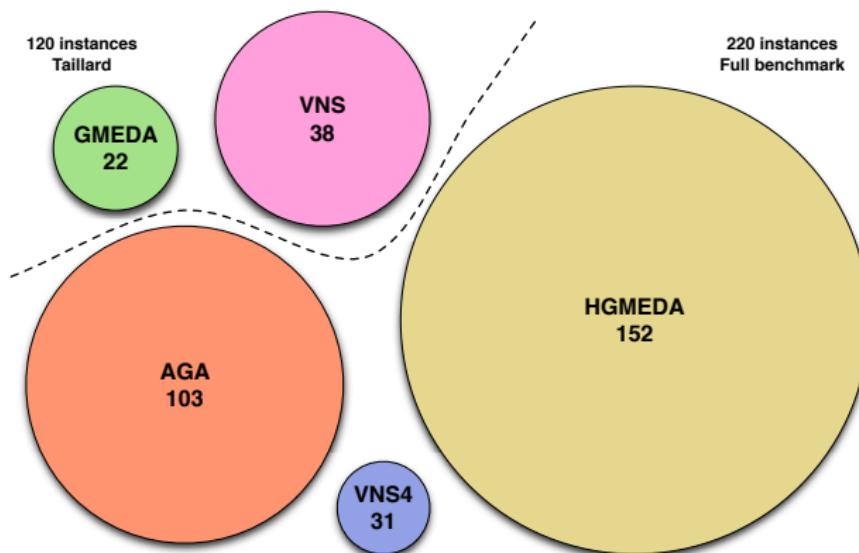
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Results



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Conclusions

1

2

3

4

Conclusions

1

We propose
a novel algorithm for
permutation problems:
Generalized Mallows EDA

2

3

4

Conclusions

1

We propose
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It introduces an explicit
probability model for
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Conclusions

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We propose
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Generalized Mallows EDA
outperforms existing
EDAs for the PFSP

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Conclusions

1

We propose
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Generalized Mallows EDA

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It introduces an explicit
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Generalized Mallows EDA
outperforms existing
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4

The hybrid GMEDA is the
new state-of-the-art
algorithm for the PFSP

Open Questions

About the model

- ▶ Accurate consensus ranking σ_0
- ▶ Self-adaptive spread parameters θ

Open Questions

About the EDA

- ▶ Initialization methods.
- ▶ Tuning the parameters of the EDA.
- ▶ Restart techniques.
- ▶ Advanced hybridizations.

Open Questions

Future Work

- ▶ Mallows + [Cayley](#) distance
- ▶ Mallows + [Ulam](#) distance
- ▶ Plackett-Luce
- ▶ Bradley-Terry
- ▶ Travelling Salesman Problem
- ▶ Linear Ordering Problem
- ▶ Quadratic Assignment Problem

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Parameters Settings

Population size	$10n$
Selection size	n
Offspring size	$10n - 1$
Max. Evaluations	$1000n^2$

n denotes the size of the problem.