Supplementary report for the paper *A Distance-based Ranking Model Estimation of Distribution Algorithm for Flowshop Scheduling Problem* submitted to the journal *IEEE Transactions on Evolutionary Computation.*

Supplementary Report: Results for the Random Benchmark instances

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Abstract

In this report we introduce the supplementary data for the random benchmark experimentation carried out in the paper *A Distance-based Ranking Model Estimation of Distribution Algorithms for Flowshop Scheduling Problem* submitted to the journal *IEEE Transactions on Evolutionary Computation.*Experimentation parameters, maximum number of evaluations and the final results of the experimentation are extensively introduced. Finally, a complete statistical analysis is introduced to confirm the results obtained.

1 Parameters

Table 1: Upper θ values for the random benchmark instances. Note the first instance of each set was selected for the experimentation. The θ that provided the best fitness average of 10 repetitions was selected.

Instance	θ range	θ_{upper}
250×10	4.0 - 7.0	5.2
250×20	4.0 - 7.0	4.4
300×10	4.0 - 7.0	4.6
300×20	4.0 - 7.0	5.2
350×10	4.0 - 7.0	6.6
350×20	4.0 - 7.0	7.0
400×10	4.0 - 7.0	5.5
400×20	4.0 - 7.0	5.0
450×10	4.0 - 7.0	4.0
450×20	4.0 - 7.0	6.7

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Table 2: Maximum number of evaluations for the random benchmark instances. Note that the first instance of each set was selected for the experimentation. The evaluation numbers reported are the average of 20 repetitions of the evaluations performed by AGA algorithm running n×m×0.4 seconds.

Instance	Evaluations
250×10	267779100
250×20	284574350
300×10	273847500
300×20	284672900
350×10	278369000
350×20	286225300
400×10	275491800
400×20	283913500
450×10	277455350
450×20	269271450

2 Results

In the Tables 3, 4 and 5 the results for the random benchmark instances of the algorithms AGA, VNS_4, GM-EDA, VNS and HGM-EDA are reported.

Table 3: Results for the random benchmark instances. Min, Max, Average and Standard deviation of the results obtained from 20 replications are introduced. Results in bold denote optimum or best known solutions of 20 repetitions.

Instance	ID	AGA				VNS_4				HGM-EDA			
		Min	Max	Avg.	Std.	Min	Max	Avg.	Std.	Min	Max	Avg.	Std.
250×10	1	1574153	1581347	1576793	1910	1586380	1594457	1590532	2384	**1566623**	1571195	1568969	1309
	2	1597409	1609723	1603218	3336	1610891	1617265	1613911	1908	**1589117**	1595517	1593026	1673
	3	1641678	1648066	1644198	2214	1653769	1661267	1657944	2114	**1633818**	1641297	1637359	1757
	4	1613745	1627833	1620304	4010	1625247	1629952	1628924	1388	**1603824**	1610632	1607455	1889
	5	1659509	1665551	1662724	2001	1671810	1674417	1673685	950	**1652808**	1658454	1655890	1378
	6	1573535	1576645	1574782	813	1579130	1579130	1579130	0	**1568866**	1574151	1571112	1664
	7	1601934	1612036	1606517	2975	1612754	1619272	1615579	1770	**1592812**	1597167	1595378	1186
	8	1559891	1567166	1563174	1735	1573504	1577360	1575642	1046	**1552918**	1559740	1555627	1537
	9	1601827	1610973	1607141	2824	1610429	1619915	1615760	2089	**1594956**	1599865	1597051	1514
	10	1602815	1614489	1607090	3070	1612719	1615403	1615040	769	**1592563**	1599247	1595545	1562
250×20	1	1850851	1864788	1860515	3868	1869785	1877499	1873254	2078	**1847471**	1855536	1851442	2415
	2	1903678	1920300	1911599	4865	1911928	1921935	1918633	2323	**1892483**	1901469	1895743	2360
	3	1886670	1898773	1891349	3295	1897111	1904093	1901045	2098	**1879796**	1886797	1883014	2093
	4	1862538	1880019	1868715	4619	1870860	1881674	1876540	2584	**1852134**	1859560	1855407	2222
	5	1841336	1858636	1850767	5178	1856684	1867562	1862001	2624	**1836747**	1847349	1841798	3078
	6	1869022	1883615	1875509	4188	1880239	1885729	1883180	1311	**1859562**	1867674	1863531	2390
	7	1888192	1901625	1894832	4285	1899923	1904322	1902067	1506	**1876186**	1884239	1880572	2334
	8	1860016	1875647	1867022	4067	1873160	1881313	1878047	2181	**1850963**	1861320	1855947	2531
	9	1869203	1884333	1878257	4881	1880612	1889633	1886133	2067	**1859994**	1867028	1863290	1505
	10	1845996	1864233	1854047	5377	1853385	1865047	1860643	2868	**1837633**	1846244	1841679	2333
300×10	1	2251273	2264908	2257521	4388	2265860	2274217	2270819	2160	**2241165**	2245153	2243244	1186
	2	2235429	2242415	2239233	2169	2250027	2257033	2253968	2031	**2221209**	2227987	2224911	1934
	3	2254615	2262236	2257063	2358	2269477	2269477	2269477	0	**2239369**	2249918	2242945	2511
	4	2226053	2231089	2228788	1509	2239258	2239258	2239258	0	**2216046**	2222773	2220188	1972
	5	2265952	2277455	2272071	3051	2276761	2284768	2281545	2025	**2255074**	2262582	2258006	1592
	6	2271241	2282930	2277022	2783	2284501	2290214	2289542	1350	**2259913**	2265242	2263223	1472
	7	2255633	2266368	2261652	3646	2271746	2278769	2275025	1901	**2244410**	2251265	2248515	1921
	8	2215727	2225787	2221583	2259	2227617	2228859	2228773	293	**2203072**	2211296	2207658	2128
	9	2258816	2269780	2264020	2848	2272774	2272774	2272774	0	**2245325**	2254840	2249943	1901
	10	2249814	2255433	2252759	1765	2263764	2267799	2266514	1140	**2239058**	2245999	2240830	1981
300×20	1	2606191	2621755	2612288	4619	2616255	2629221	2622216	3524	**2586158**	2599528	2593160	3393
	2	2585492	2599796	2593007	4656	2592489	2608024	2600676	3885	**2569577**	2586106	2577047	4640
	3	2607218	2626567	2613544	4613	2617677	2629928	2624656	3109	**2588574**	2603421	2594564	3521
	4	2598254	2614032	2603574	4342	2613856	2622132	2616541	2270	**2584806**	2594420	2588579	2945
	5	2614286	2636119	2624804	5245	2628377	2640414	2634406	3875	**2601077**	2614917	2606162	3282
	6	2626320	2645825	2633105	4122	2639338	2648286	2643736	2934	**2605560**	2624648	2613938	4506
	7	2607273	2625869	2616274	5827	2614325	2629589	2622098	3398	**2593450**	2605485	2598782	3275
	8	2575862	2588341	2582165	3880	2589180	2603068	2596387	3549	**2556756**	2574146	2565667	4155
	9	2604157	2627195	2612483	5486	2611147	2620995	2617420	2549	**2583311**	2604376	2590413	4196
	10	2637143	2651563	2645603	4030	2642776	2661786	2652874	4754	**2625824**	2638126	2632085	3711

Table 4: Results for the random benchmark instances. Min, Max, Average and Standard deviation of the results obtained from 20 replications are introduced. Results in bold denote optimum or best known solutions of 20 repetitions.

Instance	ID	AGA				VNS_4				HGM-EDA			
		Min	Max	Avg.	Std.	Min	Max	Avg.	Std.	Min	Max	Avg.	Std.
350×10	1	3065912	3076051	3072182	2505	3084744	3093112	3090163	2528	**3051090**	3062857	3055806	2945
	2	3010979	3015930	3013093	1820	3021883	3021883	3021883	0	**2996234**	3004933	2999430	2779
	3	3011165	3017258	3015093	1425	3016891	3017479	3017431	153	**2977597**	2990792	2984841	3098
	4	3042887	3061112	3052818	5848	3064140	3070473	3067863	1876	**3026374**	3044715	3031778	4507
	5	3052190	3066334	3056989	3820	3072390	3080591	3076491	2550	**3031433**	3045762	3036396	3396
	6	3039389	3052171	3045145	4518	3058774	3068271	3063253	2831	**3023478**	3037643	3028553	3779
	7	3054327	3062327	3058004	2450	3069719	3076137	3073861	1774	**3037375**	3046678	3041731	2072
	8	3095955	3103257	3100344	2414	3105729	3105729	3105729	0	**3078709**	3090803	3083100	3030
	9	3049066	3062688	3054702	4220	3062955	3074067	3070845	2591	**3029894**	3046914	3034932	3876
	10	3029292	3041719	3035785	3215	3048587	3057479	3052237	2400	**3014352**	3026134	3020810	2858
350×20	1	3483233	3504896	3498255	4799	3491694	3509228	3500998	4850	**3462591**	3483172	3470716	4627
	2	3488903	3507941	3497138	4767	3501417	3510806	3506411	2991	**3472888**	3500079	3487248	8872
	3	3465021	3485781	3475952	5388	3477174	3490673	3482641	3553	**3448696**	3464552	3455661	4260
	4	3488964	3510910	3499478	5699	3510746	3524062	3517190	3798	**3487674**	3510219	3499036	5548
	5	3441716	3460868	3451913	4917	3449548	3466074	3460903	5082	**3422898**	3456290	3435055	9054
	6	3461065	3492595	3476020	7246	3483332	3498394	3489514	4004	**3451601**	3488460	3465020	8285
	7	3490261	3508580	3500593	4326	3502329	3523039	3514279	4903	**3482315**	3502424	3492962	5822
	8	3460497	3492602	3481046	7229	3481989	3505227	3492165	5207	**3448931**	3482175	3462981	7736
	9	3494192	3516318	3507493	6112	3498925	3521232	3510165	4697	**3471376**	3494609	3483249	5837
	10	3454235	3474468	3467141	5695	3471309	3486437	3477256	4197	**3432934**	3460623	3444867	6774
400×10	1	**3933307**	3946831	3940408	3486	3953376	3953376	3953376	0	3947105	3978898	3965250	9291
	2	3913626	3924694	3917274	3064	3937745	3937745	3937745	0	**3910048**	3955026	3924313	11985
	3	**3934557**	3942910	3938381	2444	3954822	3954822	3954822	0	3936392	3975825	3955525	10872
	4	3891700	3907083	3899321	4952	3916944	3928950	3923015	3545	**3875265**	3903755	3889359	7973
	5	**3929183**	3946391	3937956	4324	3945640	3945640	3945640	0	3932228	3969255	3949245	11094
	6	**3913491**	3926900	3920493	3088	3930115	3930115	3930115	0	3931953	3972628	3951860	10979
	7	3869753	3877726	3873155	2529	3880045	3884523	3884000	1321	**3865710**	3921303	3884016	11942
	8	3925389	3937590	3932163	3068	3946990	3947456	3947433	104	**3918728**	3985113	3949931	16749
	9	3910625	3923279	3917789	3673	3921963	3930638	3929987	1979	**3903109**	3959599	3927476	14645
	10	3936169	3944038	3939571	1856	3952256	3952256	3952256	0	**3934891**	3979551	3956974	12355
400×20	1	**4465942**	4493539	4480917	6372	4473212	4494718	4487498	4758	4488339	4545991	4515697	13822
	2	**4442588**	4464397	4454015	5871	4455085	4475599	4468002	5669	4471430	4528310	4503037	14324
	3	**4419467**	4437383	4430321	4457	4438085	4441469	4441059	1023	4445568	4482757	4462419	11434
	4	**4450298**	4481322	4469057	8577	4473051	4488169	4480069	4589	4481842	4531124	4509070	13364
	5	**4380600**	4408163	4394538	6893	4401272	4423334	4409630	5934	4404712	4457527	4437232	12753
	6	**4460906**	4480574	4471233	5368	4467146	4490854	4483005	6660	4483966	4533733	4506287	11377
	7	**4472922**	4505260	4495309	7933	4487424	4520797	4512639	7650	4494560	4545423	4526379	13295
	8	**4438057**	4449718	4444973	3445	4450479	4469245	4459819	5036	4466578	4513838	4491073	13587
	9	**4419300**	4434078	4426108	4213	4441031	4442341	4442238	330	4450333	4498461	4472159	13188
	10	**4402417**	4418523	4410963	4719	4415587	4430118	4423594	4029	4426702	4473361	4447537	11145

Table 5: Results for the random benchmark instances. Min, Max, Average and Standard deviation of the results obtained from 20 replications are introduced. Results in bold denote optimum or best known solutions of 20 repetitions.

Instance	ID	AGA				VNS_4				HGM-EDA			
		Min	Max	Avg.	Std.	Min	Max	Avg.	Std.	Min	Max	Avg.	Std.
450×10	1	**4870929**	4889070	4883626	5293	4904632	4904933	4904918	67	4935781	4980292	4952295	12697
	2	**4884041**	4897775	4892084	4260	4907294	4907294	4907294	0	4945213	4996317	4969349	12025
	3	**4930649**	4945845	4940080	3508	4952505	4965100	4959690	3382	4963724	4997216	4979093	11280
	4	**4933807**	4936130	4934735	677	4936947	4936947	4936947	0	4979497	5034532	5006915	11814
	5	**4934094**	4942892	4940075	2301	4947333	4947333	4947333	0	4998720	5028228	5015788	8649
	6	**4922342**	4931113	4926956	2850	4942281	4950666	4947278	2697	4971752	5013337	4989858	10189
	7	**4931140**	4941073	4935709	2901	4950775	4956910	4955556	1859	4976883	5035827	5001841	13303
	8	**4933524**	4946098	4938891	3557	4957049	4960695	4960448	809	4985428	5014363	5000896	7985
	9	**4906973**	4919973	4913383	4213	4934814	4942637	4939385	2292	4960811	4994291	4975019	9210
	10	**4976526**	4989451	4981896	3014	5003081	5005074	5004837	597	5039960	5077268	5057628	9996
450×20	1	**5532585**	5563136	5549313	8637	5542765	5562406	5554299	5754	5608227	5654304	5631164	14428
	2	**5545906**	5580165	5564135	9327	5569431	5592450	5580214	5468	5622073	5687086	5643452	14835
	3	**5584602**	5617291	5600264	8083	5597985	5616850	5607715	5043	5633716	5701506	5660685	19114
	4	5588641	5610737	5599575	7745	**5587972**	5612248	5600785	6465	5653949	5710515	5678634	14685
	5	**5556343**	5578183	5570557	6890	5573864	5592928	5585438	6195	5627182	5678570	5652449	15105
	6	**5507299**	5543428	5521705	9903	5532708	5557544	5547247	7347	5583783	5650496	5618789	16503
	7	**5534081**	5572680	5556858	8935	5555781	5574585	5567192	4674	5611787	5659727	5636271	14464
	8	**5580460**	5614739	5591816	9442	5594845	5616896	5604431	6569	5644147	5697429	5669628	16092
	9	**5558982**	5594373	5579344	8296	5561410	5585250	5575892	7182	5607597	5671253	5641866	15478
	10	5559057	5574801	5567613	4295	**5549109**	5572643	5562585	7477	5592803	5656208	5630423	16764

3 Statistical Analysis

In order to state whether there exist statistical differences among the observed results, we consider applying a non-parametric Friedman's test to the average results of the 5 algorithms separately for each job-machine configuration. A level $\alpha = 0.05$ of significance was set. Results reported significant differences between the algorithms. Therefore a post-hoc method is used to carry out all pairwise comparisons and determined which algorithms stand out from the rest of the approaches. Particularly, Shaffer's static procedure is used. Again, the significance level has been fixed to $\alpha = 0.05$.

3.1 Instances of 250 jobs: 250×10 and 250×20.

3.1.1 Non-parametric test: Friedman Test

Table 6: Average ranking of the algorithms for the instances 250×10 and 250×20.

Algorithm	250×10	250×20
AGA	1.9	1.9
VNS_4	0.9	0.9
HGM-EDA	2.9	2.9

Table 7: Adjusted p-values for the algorithms for the configuration 250×10. The hypothesis that have a p-value higher than ≤ 0.05 are rejected.

Hypothesis	unadjusted p	p_{Shaf}
VNS_4 vs. HGM-EDA	7.74×10^{-6}	2.32×10^{-5}
AGA vs. VNS_4	0.025	0.025
AGA vs. HGM-EDA	0.025	0.025

Table 8: Adjusted p-values for the algorithms for the configuration 250×20. The hypothesis that have a p-value higher than ≤ 0.05 are rejected.

Hypothesis	unadjusted p	p_{Shaf}
VNS_4 vs. HGM-EDA	7.74×10^{-6}	2.32×10^{-5}
AGA vs. VNS_4	0.025	0.025
AGA vs. HGM-EDA	0.025	0.025

3.2 Instances of 300 jobs: 300×10 and 300×20.

3.2.1 Non-parametric test: Friedman Test

Table 9: Average ranking of the algorithms for the instances 300×10 and 300×20.

Algorithm	300×10	300×20
AGA	1.9	1.9
VNS_4	0.9	0.9
HGM-EDA	2.9	2.9

3.2.2 Pairwise comparison: Shaffer's Static Procedure

Table 10: Adjusted p-values for the algorithms for the configuration 300×10. The hypothesis that have a p-value higher than ≤ 0.05 are rejected.

Hypothesis	unadjusted p	p_{Shaf}
VNS_4 vs. HGM-EDA	7.74×10^{-6}	2.32×10^{-5}
AGA vs. VNS_4	0.025	0.025
AGA vs. HGM-EDA	0.025	0.025

Table 11: Adjusted p-values for the algorithms for the configuration 300×20. The hypothesis that have a p-value higher than ≤ 0.05 are rejected.

Hypothesis	unadjusted p	p_{Shaf}
VNS_4 vs. HGM-EDA	7.74×10^{-6}	2.32×10^{-5}
AGA vs. VNS_4	0.025	0.025
AGA vs. HGM-EDA	0.025	0.025

3.3 Instances of 350 jobs: 350×10 and 350×20.

3.3.1 Non-parametric test: Friedman Test

Table 12: Average ranking of the algorithms for the instances 350×10 and 350×20.

Algorithm	350×10	350×20
AGA	1.9	1.9
VNS_4	0.9	0.9
HGM-EDA	2.9	2.9

3.3.2 Pairwise comparison: Shaffer's Static Procedure

Table 13: Adjusted p-values for the algorithms for the configuration 350×10. The hypothesis that have a p-value higher than ≤ 0.05 are rejected.

Hypothesis	unadjusted p	p_{Shaf}
VNS_4 vs. HGM-EDA	7.74×10^{-6}	2.32×10^{-5}
AGA vs. VNS_4	0.025	0.025
AGA vs. HGM-EDA	0.025	0.025

Table 14: Adjusted p-values for the algorithms for the configuration 350×20. The hypothesis that have a p-value higher than ≤ 0.05 are rejected.

Hypothesis	unadjusted p	p_{Shaf}
VNS_4 vs. HGM-EDA	7.74×10^{-6}	2.32×10^{-5}
AGA vs. VNS_4	0.025	0.025
AGA vs. HGM-EDA	0.025	0.025

3.4 Instances of 400 jobs: 400×10 and 400×20.

3.4.1 Non-parametric test: Friedman Test

Table 15: Average ranking of the algorithms for the instances 400×10 and 400×20.

Algorithm	400×10	400×20
AGA	2.4	2.9
VNS_4	1.0	1.9
HGM-EDA	2.5	0.9

3.4.2 Pairwise comparison: Shaffer's Static Procedure

Table 16: Adjusted p-values for the algorithms for the configuration 400×10. The hypothesis that have a p-value higher than ≤ 0.05 are rejected.

Hypothesis	unadjusted p	p_{Shaf}
VNS_4 vs. HGM-EDA	0.001	0.005
AGA vs. VNS_4	0.003	0.025
AGA vs. HGM-EDA	0.8	0.025

Table 17: Adjusted p-values for the algorithms for the configuration 400×20. The hypothesis that have a p-value higher than ≤ 0.05 are rejected.

Hypothesis	unadjusted p	p_{Shaf}
AGA vs. HGM-EDA	7.74×10^{-6}	2.32×10^{-5}
VNS_4 vs. HGM-EDA	0.025	0.025
AGA vs. VNS_4	0.025	0.025

3.5 Instances of 450 jobs: 450×10 and 450×20.

3.5.1 Non-parametric test: Friedman Test

Table 18: Average ranking of the algorithms for the instances 450×10 and 450×20.

Algorithm	450×10	450×20
AGA	2.9	2.8
VNS_4	1.9	2.1
HGM-EDA	0.9	0.9

3.5.2 Pairwise comparison: Shaffer's Static Procedure

Table 19: Adjusted p-values for the algorithms for the configuration 450×10. The hypothesis that have a p-value higher than ≤ 0.05 are rejected.

Hypothesis	unadjusted p	p_{Shaf}
AGA vs. HGM-EDA	7.74×10^{-6}	2.32×10^{-5}
VNS_4 vs. HGM-EDA	0.025	0.025
AGA vs. VNS_4	0.025	0.025

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Table 20: Adjusted p-values for the algorithms for the configuration 450×20. The hypothesis that have a p-value higher than ≤ 0.05 are rejected.

Hypothesis	unadjusted p	p_{Shaf}
AGA vs. HGM-EDA	5.69×10^{-5}	1.70×10^{-4}
VNS_4 vs. HGM-EDA	0.007	0.007
AGA vs. VNS_4	0.17	0.17