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MOND or Dark Matter?

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ABSTRACT

This paper presents a statistical analysis of the rotation curves of eight spiral galaxies from the SPARC dataset, with the aim of evaluating the relative merits of Modified Newtonian Dynamics (MOND) and Dark Matter as models for galaxy dynamics. The analysis relies on several statistical techniques, including least squares parameter fitting, Markov Chain Monte Carlo (MCMC) sampling of posterior distributions, and leave-p-out cross-validation.

1. INTRODUCTION

The dynamics of stars in galaxies have long puzzled astrophysicists, who have proposed two main models to explain their anomalous behavior. The first model posits the existence of Dark Matter (DM), a form of matter that does not interact with electromagnetic radiation but has significant gravitational effects. The second model, Modified Newtonian Dynamics (MOND), introduces corrections to Newtonian dynamics to explain the observed rotation curve data. The DM model suggests that there is a vast[1] halo of dark matter surrounding the visible matter in the galaxy. For our purposes, I will use a simplified model of a spiral galaxy. I assume the spiral galaxy has two components: a disk and a DM halo. For the disk, I use the rotational velocity model derived from Freeman (1970) exponential disk model for density.

$$V_{disk}(R) = \sqrt{G\frac{M_{disk}}{R_{disk}}}\frac{1}{\sqrt{2}}(\frac{R}{R_{disk}})\sqrt{B[\frac{R}{R_{disk}}]} \tag{1}$$

where $B[x] \equiv I_0(x)K_0(x) - I_1(x)K_1(x)$. For the halo, I use the rotation velocity model derived from the NFW Profile, introduced by Navarro et al. (1996).

$$V_{halo}(R) = \sqrt{G\frac{M_{halo}(R)}{R}} \tag{2}$$

In total, this model has 4 parameters for fitting:M_{disk}, R_{disk}, M_{halo}, R_{halo}. Alternatively, MOND suggests that a correction to the gravitational constant could explain the anomalous rotation curve data. We can include a simplified MOND galaxy model that includes the exponential disk model and an additional correction term. Given this, the MOND model has 3 parameters for fitting: M_{disk}, R_{disk}, and the correction term a_{crit}. To be a valid model, we expect a_{crit} to be consistent across all galaxies. Using rotation curve data for 8 spiral galaxies from the SPARC data catalog (Lelli et al. (2016)), I employed several statistical techniques to discern which model is more favorable. The following sections detail my methodology and results.

2. LEAST SQUARES PARAMETER FITTING

I used the SciPy (Virtanen et al. (2020)) curve_fit function to find the best fitting parameters for each model on each galaxy. The curve_fit function uses non-linear least squares for fitting, which is good for our non-linear models. The resulting chi-squared values for the DM and MOND models are compared in Table 1 for each of the eight spiral galaxies from the SPARC data catalog. The DM model outperformed the MOND model in most of the galaxies, except for NGC 3198 and NGC 300. It should be noted that the chi-squared values for the DM and MOND models alone do not provide conclusive evidence for either model, as a good fit can be achieved with various parameter combinations. Therefore, additional statistical techniques were used to further investigate the validity of the models.

[1] The halo is vast relative to the disk

Chi-Squared Values								
Model	NGC 3198	NGC 2403	NGC 0300	NGC 0024	NGC 1003	NGC 2903	NGC 2998	NGC 0100
DM	39.29	12.98	9.33	10.15	172.10	120.53	275.78	9.51
MOND	12.80	76.21	9.11	12.34	284.68	86.11	16.19	9.94

Table 1.

3. MCMC

To test the validity of the claim that a_{crit} is consistent across all galaxies, I ran an MCMC algorithm to sample from all of the a_{crit} posterior distributions for comparison. The a_{crit} posterior distribution was consistently skewed, so I sampled from the $log(a_{crit})$ distribution instead. I used an uninformative log-uniform prior for all parameters, since all are physically required to be greater than 0. Further, the size (the radius and mass) of the galaxies vary greatly, so it is beneficial to use a prior that makes no assumptions about scale. The bounds, $0.5 < R_{disk} < 50$, $0.1 < M_{disk} < 3000$, $0.1 < a_{crit} < 20$, were set based on the best fit parameters.

I used the emcee python package (Foreman-Mackey et al. (2013)) to run the algorithm with 50 walkers and 10000 accepted steps. Figure 1 shows the mixing and corner plot for NGC 2903. The plots for the rest of the galaxies can be found by running the following notebook: https://github.com/gabrielpfaffman/Astrostatistics-Final-Project.

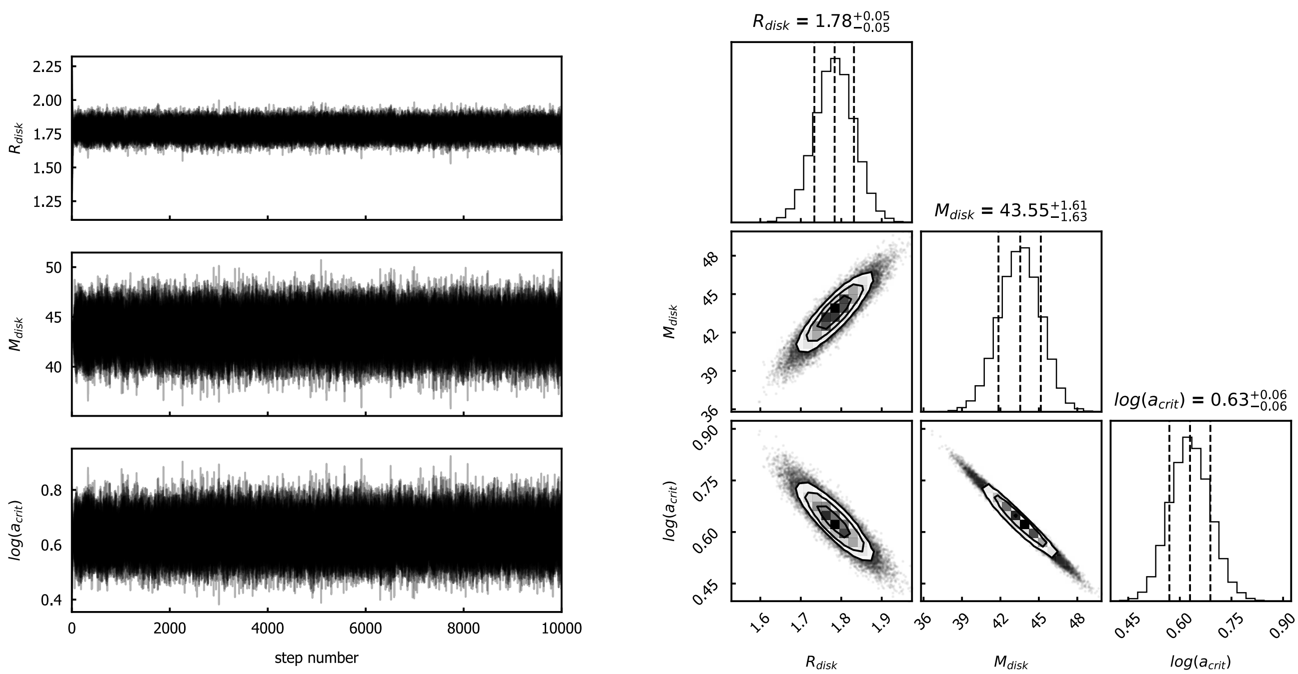

Figure 1. Mixing and corner plot for 50 walker, 10000 iteration MCMC algorithm on NGC 2903.

As expected, there is a strong relationship between the mass and radius distributions. Interestingly, there is also a strong relationship between the these physical parameters parameter and the $log(a_{crit})$ distribution. The box plots in Figure 2 show that the posterior distributions of a_{crit} vary widely, contradicting a key tenant of MOND. However, it is important to note that the distributions are broad.

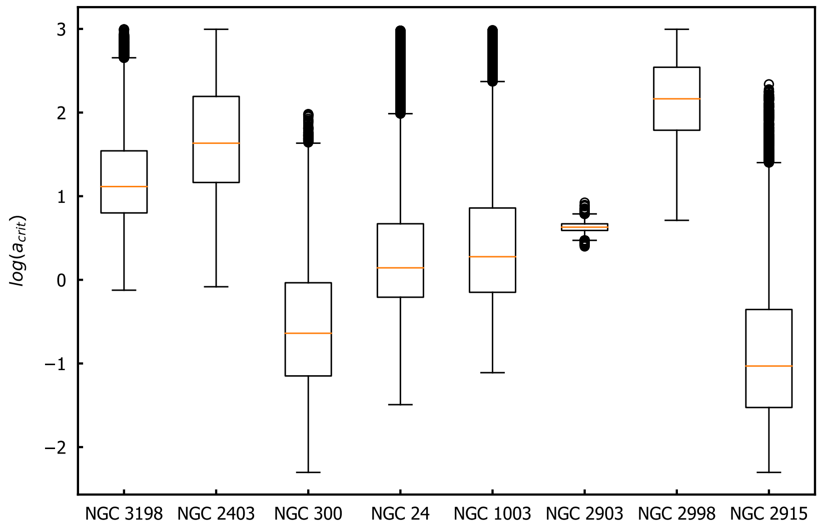

Figure 2. Box plots of $log(a_{crit})$ posterior distributions, sampled using an MCMC algorithm.

4. MODEL SELECTION : CROSS-VALIDATION

To directly compare the performance of the DM and MOND models, I used the leave-p-out cross-validation technique. This method is particularly suited for non-linear models like DM and MOND, whereas other model comparison metrics such as the Bayesian Information Criterion (BIC) and Akaike Information Criterion (AIC) may not be as effective.

In this approach, I split the data into training and validation sets, with p set to 20% of the data. I used the SciPy curve_fit function to fit the model parameters to the training set, and then calculated the chi-squared value of the fits against the validation set. For each galaxy, I randomly selected 1000 different training and validation sets.

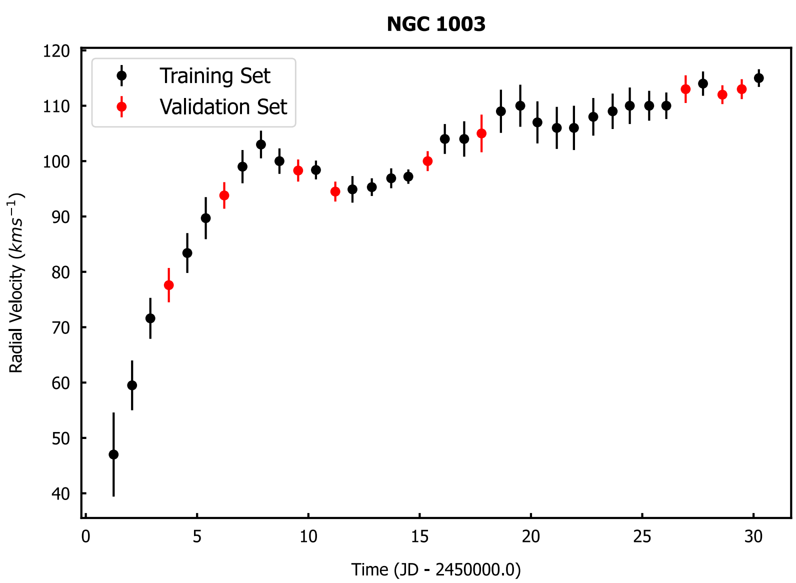

Figure 3. The randomly chosen training and validation sets for the 50th iteration of cross-validation on NGC 1003.

Figure 3 shows an example of the randomly chosen training and validation sets for the 50th iteration of cross-validation on NGC 1003. For each iteration, the model with the lower chi-squared value received one point. I tallied up the scores across all models and galaxies, resulting in a final score of DM: 4336 and MOND: 3664. From this, we can estimate an odds ratio of 1.183 in favor of the DM model.

5. CONCLUSIONS

The lack of consistency of the a_{crit} posterior distribution appears like a major problem for the MOND model. Adding further concern for MOND, it was outperformed by the DM model in cross-validation. However, we still do not have enough information to draw any confident conclusions. The posterior distributions for a_{crit} are too broad to make any claims about the legitimacy of MOND. Further, the odds ratio between DM and MOND is not great enough to confidently claim that DM is the preferred model. One option to increase the precision of our results is to use data with smaller errors. Another option is to consider more rotation curves in order to make the posterior a_{crit} distributions narrower and refine the odds ratio. It is possible that there is already enough rotation curve data available in the SPARC catalog to refine the results to a statistically significant regime. However, the computational expense of the method employed in this study requires further optimization or the use of high-performance computing.

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