Anomaly Detection in Taxi Calls

Problem and Data

Anomaly Detection on Taxi Calls

Let's considere a Taxi company:

Anomaly Detection on Taxi Calls

Some context information:

- There's historical data about taxi calls in NYC (number of taxi calls over time)
- A major decision for the company is choosing the size of the car pool
- This depends on how many calls are expected
- Strong deviations from the usual patterns may also cause issues
- The company is mostly interested in detecting such "anomalies"
- Anticipating them would be a welcome addition, but it is not essential

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How can we tackle this problem?

Getting Started

A couple of good ideas:

Trying to understand the context:

- The company priorities and how their business works
- Any expectation on the data
- ...

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...And also inspecting the data

- ...So that we get a "feel" of how it works
- Formally: until we understand better its statistical distribution

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...And also inspecting the data

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- Formally: until we understand better its statistical distribution

Doing both these things early is always a good idea

A Look at the Data

Let's have a look at the available data

In [4]:
```
data, labels, windows = util.load_series(file_name, data_folder)
data.head()
```

Out[4]:

	value
timestamp	
2014-07-01 00:00:00	10844
2014-07-01 00:30:00	8127
2014-07-01 01:00:00	6210
2014-07-01 01:30:00	4656
2014-07-01 02:00:00	3820

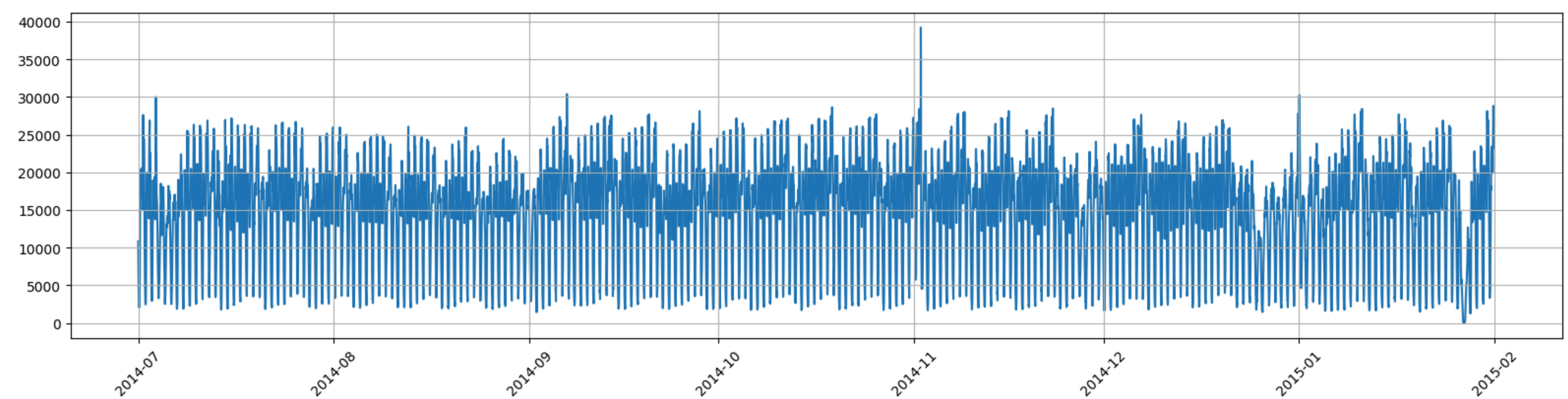
- `data` is a pandas `DataFrame` object
- It is essentially a table, in this case representing a time series
- There are well defined column names (here "value")
- There is a well defined row index (here "timestamp")

A Look at the Data

Time series are quite easy to visualize

The most direct approach is using a Cartesian plot

```
In [5]: util.plot_series(data, figsize=figsize)
```



- If are curious, all use case code is available as part of the course material

A Look at the Data

We can now move to other data structures

```
In [6]: labels.head()

Out[6]: 0   2014-11-01 19:00:00
        1   2014-11-27 15:30:00
        2   2014-12-25 15:00:00
        3   2015-01-01 01:00:00
        4   2015-01-27 00:00:00
        dtype: datetime64[ns]
```

`labels` is a pandas `Series` object

- You can think of that as a one-column table

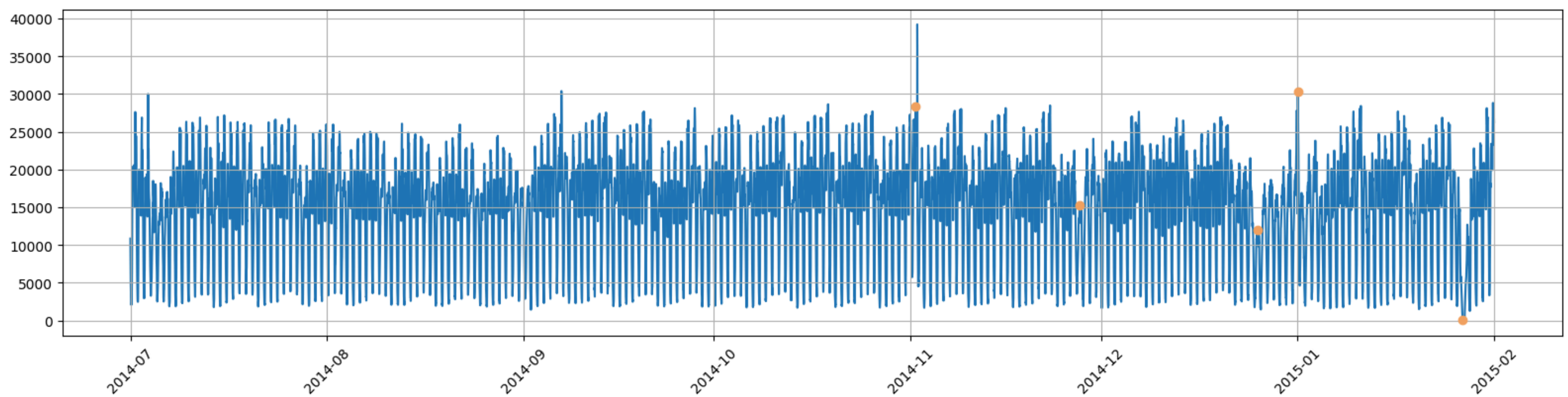
This series contains the timestamp of all known anomalies

- There are just a few of them
- ...and they are all hand-labeled

A Look at the Data

We can plot the call and anomalies together

```
In [7]: util.plot_series(data, labels, figsize=figsize)
```



- Most anomalies in the second part of the series
- ...But that's just a coincidence

A Look at the Data

Now we can check the "windows" data structure

```
In [8]: windows.head()
```

Out[8]:

	begin	end
0	2014-10-30 15:30:00	2014-11-03 22:30:00
1	2014-11-25 12:00:00	2014-11-29 19:00:00
2	2014-12-23 11:30:00	2014-12-27 18:30:00
3	2014-12-29 21:30:00	2015-01-03 04:30:00
4	2015-01-24 20:30:00	2015-01-29 03:30:00

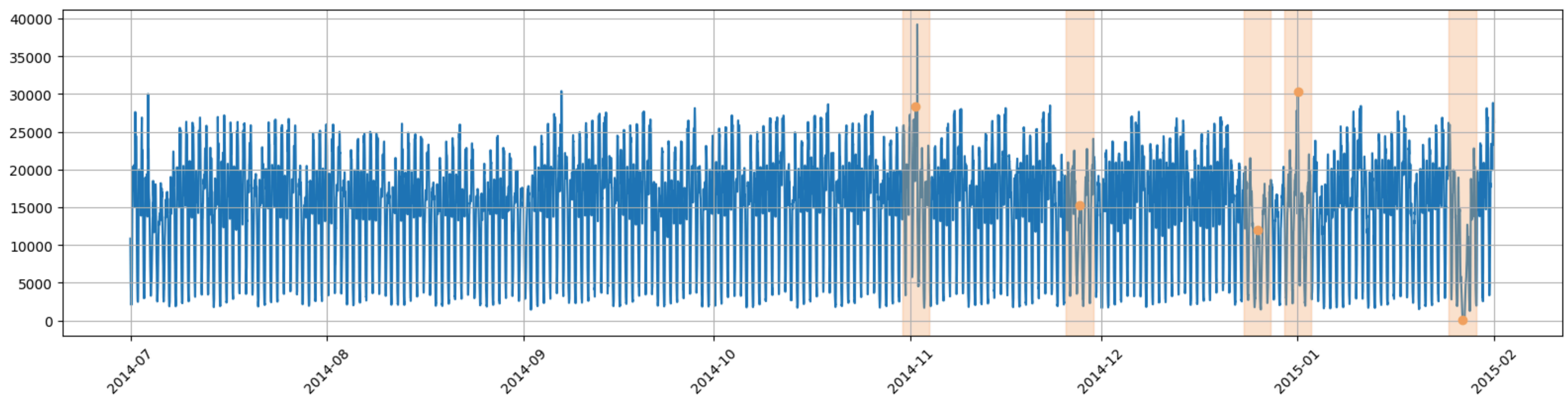
`windows` is another a pandas `DataFrame` object (a table)

- It contains the start/end of windows containing anomalies
- Detections within the window are useful and count as "hits"
- Detections outside the windows are false alarms

A Look at the Data

Let's plot all the information together

In [9]:
```
util.plot_series(data, labels, windows, figsize=figsize)
```



- Detections that occur too early/late count as misses

Anomaly Detection in Taxi Calls

Formalizing the Problem

Let's start with a question

What is our biggest difficulty right now?

On the Importance of Formalization

Right now, the problem we are tackling is too vaguely defined

This makes it much harder to think about:

- Solution approaches
- Evaluation procedures
- Key Performance Indicators

Eventually, we'll need to formally specify:

- The input and output of our solution system
- ...And a set of quality metrics

But first, we need a formal way just to reason on the system

System Formalization

Let's attempt to formalize the system, first

We can view the number of taxi call as a random variable

$$X \sim P(X)$$

- X is a source of random data
- $D(X)$ is its support, i.e. the set of possible outcomes
- $P(X)$ is its distribution, i.e. the probability of every outcome

System Formalization

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- $D(X)$ is its support, i.e. the set of possible outcomes
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Can we use this to define our car pool size, or to detect anomalies?

Anomaly Detection and Car Pool Sizing

Formally, we could size the car pool via a rule like

$$\operatorname*{argmin}_{q \in D(X)} F(q) \geq \alpha$$

- $F(x)$ tells use the probability that $X \leq x$ (Cumulative Distribution Function)
- α is the total probability of the scenario we want to cover with our pool

...And we could detect anomalies via a rule like:

$$P(x) \leq \varepsilon$$

- ε is a threshold value
- If the probability of observing x call is below θ, we say we have an anomaly

We've already made good progress!

What do we need to use this idea in practice?

Density Estimation

The main issue we have now is that we really don't know $P(X)$

...But we can learn it from our data

- Given a dataset $\{x_i\}_{i=1}^m$ containing observed numbers of taxi calls
- ...We can try to approximate $P(X)$ with a parametric function $\hat{f}(x; \theta)$

This is the gist of density estimation

In practice, $\hat{f}(x; \theta)$ is often trained for maximum likelihood estimation

- This is a very common training method based on the idea that a good model
- ...Should assign a high-probability to real data

MLE Training

Formally, MLE training consists in solving:

$$\operatorname*{argmax}_{\theta} \prod_{i=1}^{m} \hat{f}(x_i; \theta)$$

- Given our dataset $\{x_i\}_{i=1}^{m}$...
- ...We choose the model parameters θ...
- ...So that the estimated probability is as high as possible

Now we need to define two things:

- Which data we should use for training
- Which function (i.e. model) to use as an estimator

Anomaly Detection in Taxi Calls

Data and Model

Training and Testing

We will split our data in two segments

A training set, used for learning the estimator:

- This will include only data about the normal behavior
- Ideally, there should be no anomalies here (we do not want to learn them!)

A test set, use for evaluation

- We should never optimize anything on this

Training and Testing

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A test set, use for evaluation

- We should never optimize anything on this

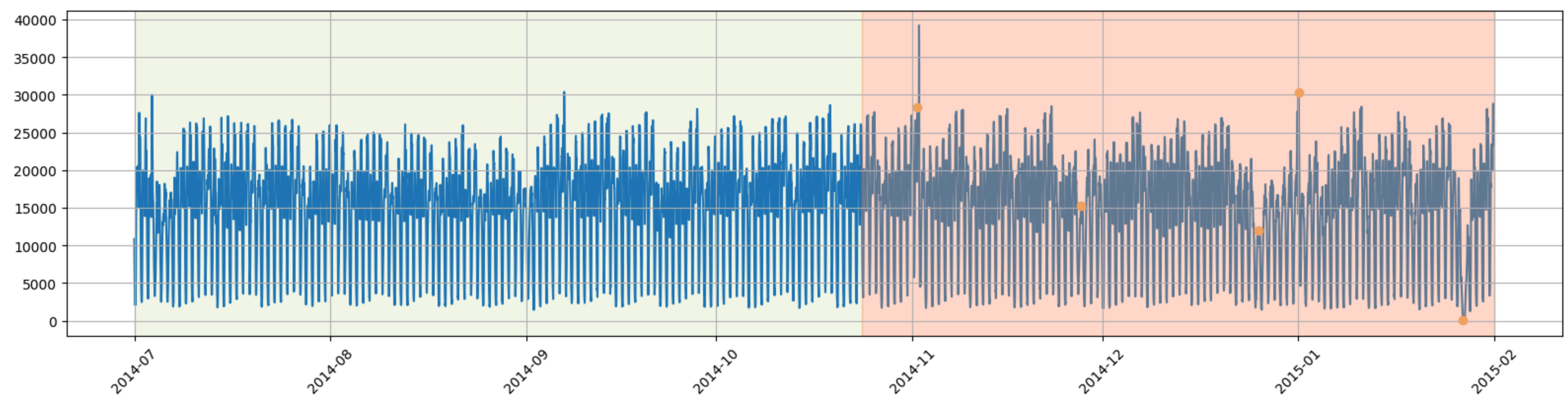
If the training set contains some anomalies

- Things are still mostly fine!
- ...As long as they are very infrequent

Training and Testing

In time series data sets are often split chronologically:

```
In [10]: train_end = pd.to_datetime('2014-10-24 00:00:00')
         util.plot_series(data, labels, test_start=train_end, figsize=figsize)
```



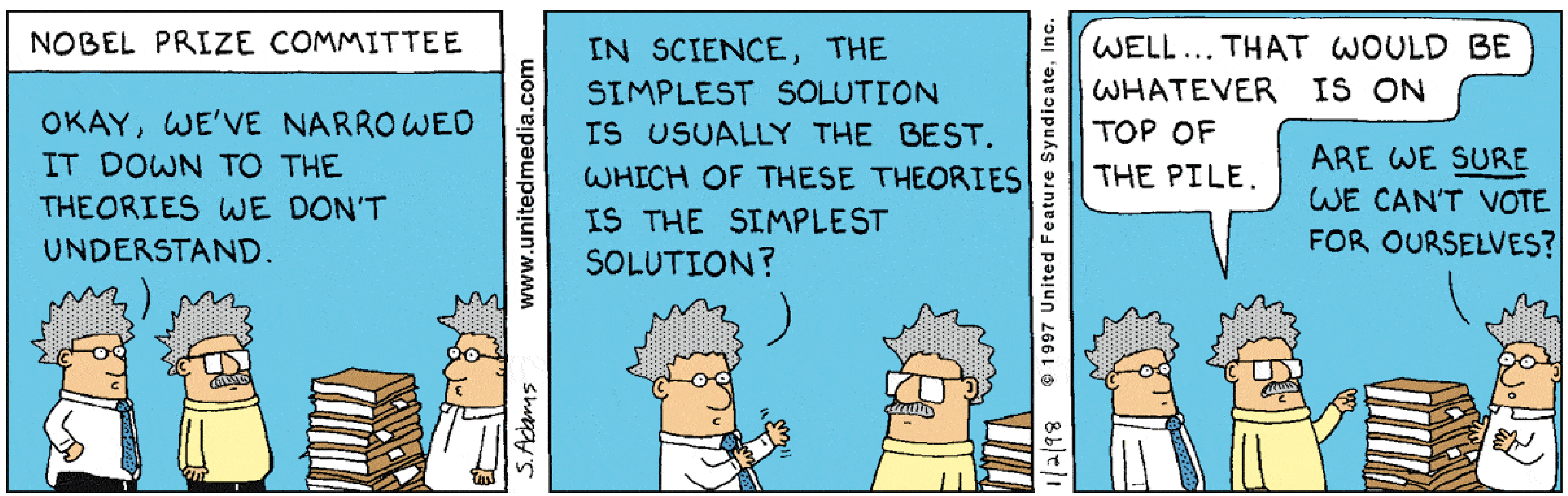
- Green: training set, orange: test set

Choosing an Estimator

Which estimation model should we use?

- Lacking any strong reason for doing otherwise
- Using Occam's razor is usually a good idea

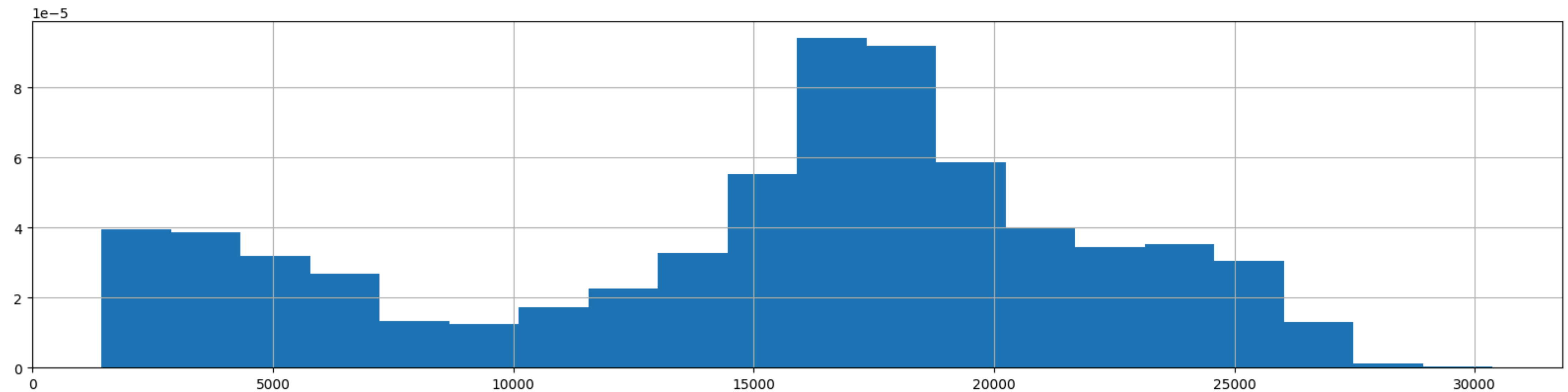
So, we'll go for a simple approach



Histograms as Density Estimators

A histogram is a (very) simple density estimator

In [11]:
```
data_tr = data[data.index < train_end]
util.plot_histogram(data_tr, bins=20, figsize=figsize)
```



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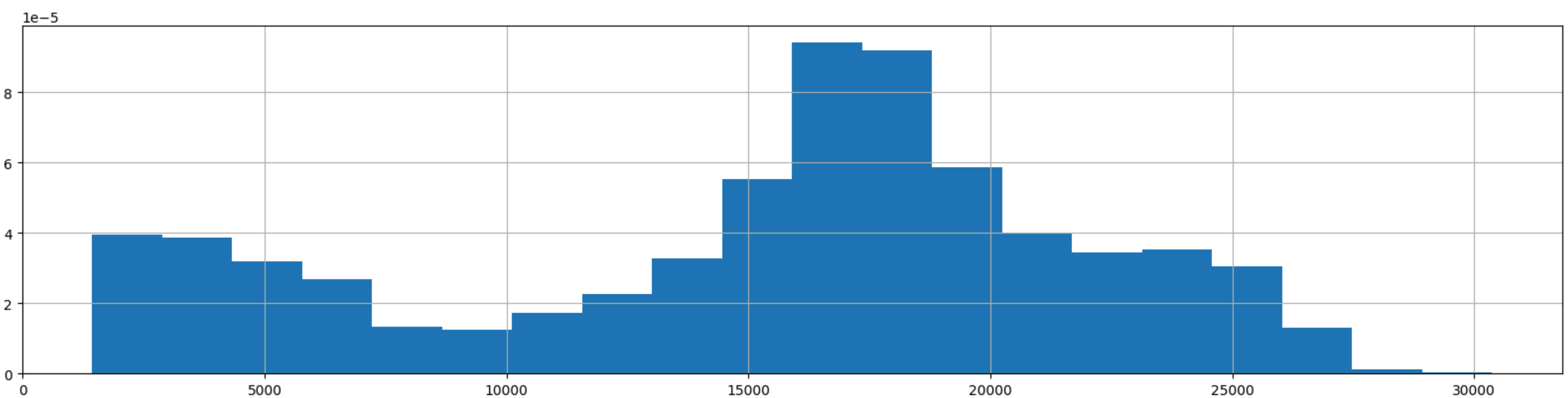
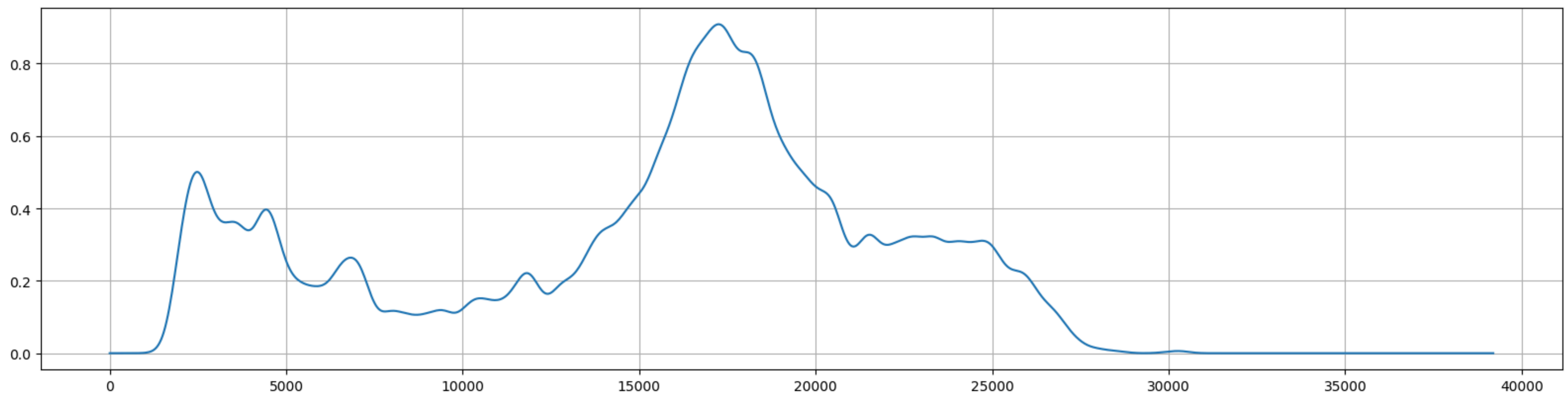


- It gives us a probability for every value
- The model parameters θ are in this case the bins

Kernel Density Estimation

Another simple approach is Kernel Density Estimation

In [12]:
```
kde = util.train_kde(data_tr, bandwidth_range=np.linspace(0.01, 0.1, 10))
util.plot_density_estimator_1D(kde, np.linspace(0, data['value'].max(), 1000), figsize=figs
```

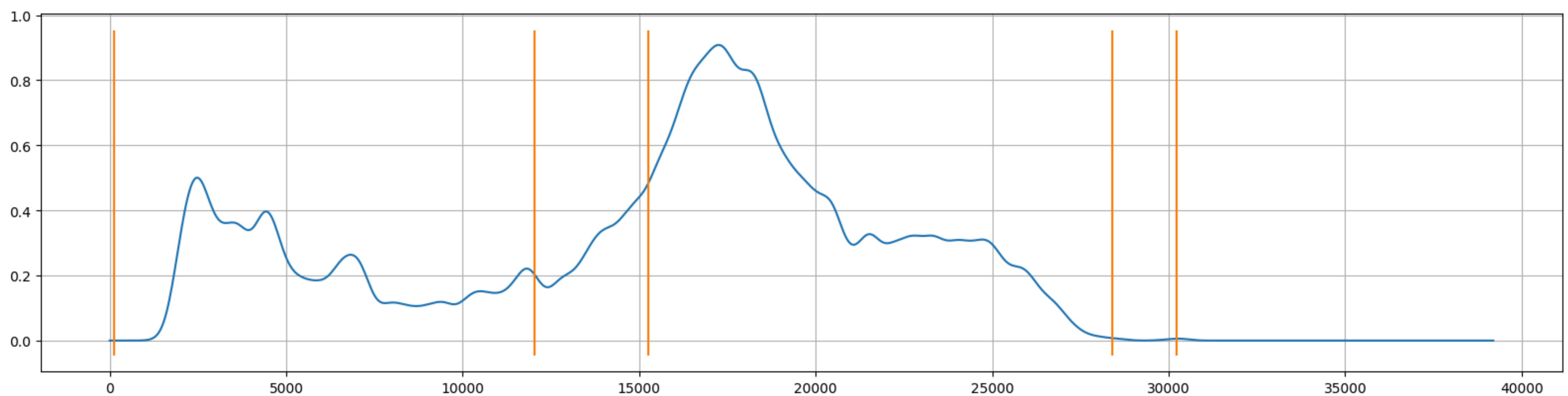


- KDE plances one small kernel (e.g. a Gaussian) on every training point
- A distribution is the obtained by averaging

Density Estimation for Anomaly Detection

We can test our idea by checking the probability of anomalous points

In [13]:
```
util.plot_density_estimator_1D(kde, np.linspace(0, data['value'].max(), 1000), figsize=figs
```



- Several of the anomalous points have very low estimated probabilities

Alarm Signal

In anomaly detection, it is actually customary to work with alarm signals

- Rather than checking for low probabilities
- ...We check for a high "alarm"

We can obtain an alarm signal from our estimator as:

$$-\log \hat{f}(x; \theta) \geq \varepsilon$$

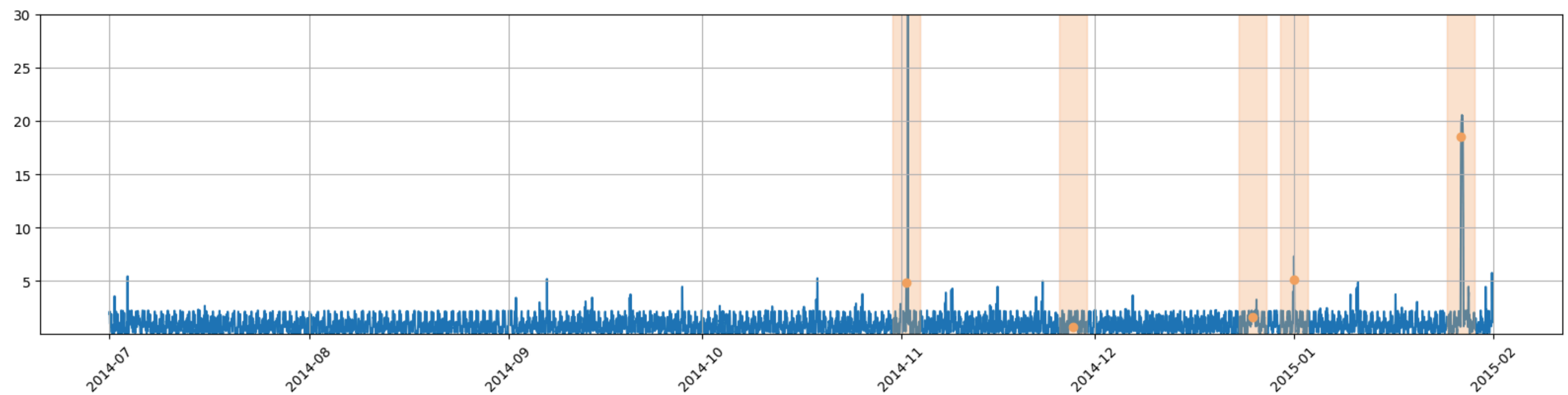
- We use log probabilities (to reduce a bit the scale)
- ...And we change the sign to interpret them as an "alarm"

It is still equivalent to the previous formulation

Alarm Signal

We can now obtain (and plot) our alarm signal:

In [14]:
```
ldens = kde.score_samples(data.values) # Obtain log probabilities
signal = pd.Series(index=data.index, data=-ldens) # Build series with neg. prob.
util.plot_series(signal, labels=labels, windows=windows, figsize=figsize, y_cap=30) # Plot
```

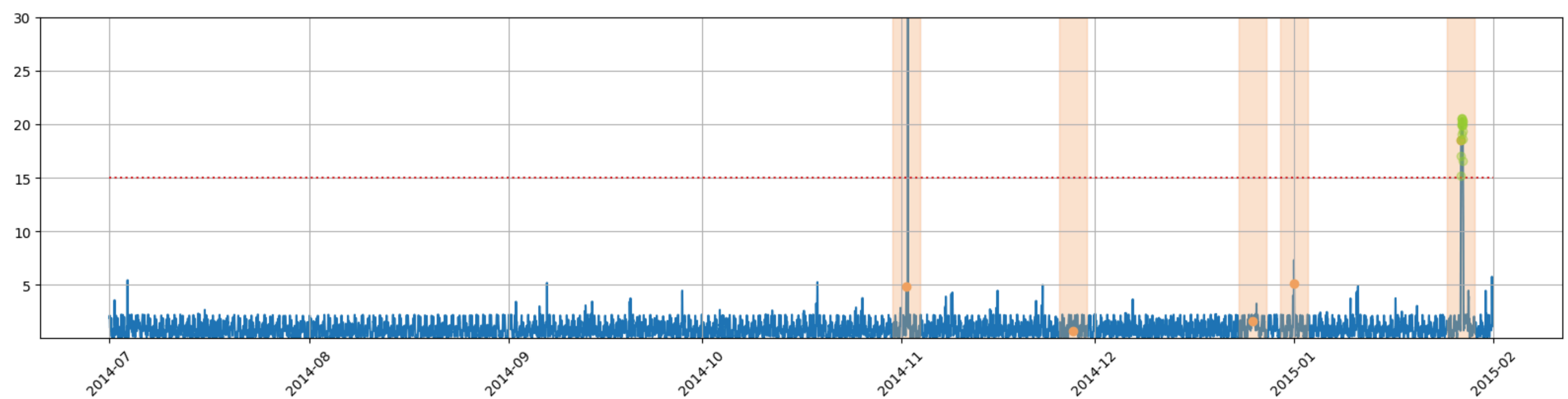


- Again, some anomalies stand out

Detecting Anomalies

By picking a threshold, we can simulate the operation of our anomaly detector

```
In [15]: eps = 15
         pred = pd.Series(signal.index[signal >= eps]) # the threshold il 12
         util.plot_series(signal, labels=labels, windows=windows, predictions=pred, threshold=eps, fi
```



- Not very good, but the threshold is chosen almost at random now
- There are a many false positives, which are very common in anomaly detection

Anomaly Detection in Taxi Calls

Metrics and Threshold Choice

For choosing a threshold, we need to determine its quality

...But how do we evaluate a system like this?

Metrics for Anomaly Detection

Evaluating the quality of an Anomaly Detection system can be tricky

- Usually, we do not need to match the anomalies exactly
- Sometimes we wish to anticipate anomalies
- ...But sometimes we just want to detect them in past data

There is no "catch-all" metric, like accuracy in classification

It is much better to devise a cost model

- We evaluate the cost and benefits of our predictions:
- By doing this, we focus on the value for our customer

This is important for all industrial problems!

A Simple Cost Model

We will use a simple cost model

Remember that our goals are:

- Analyzing anomalies
- Anticipating anomalies

We will use a simple model based on:

- True Positives as windows for which we detect at least one anomaly
- False Positives as detected anomalies that do not fall in any window
- False negatives as anomalies that go undetected
- Late detections as windows where a detection was correct, but late

A Simple Cost Model

In our example, we'll assign a somewhat arbitrary cost to every error

In [16]:
```
c_alrm = 1 # Cost of investigating a false alarm
c_missed = 10 # Cost of missing an anomaly
c_late = 5 # Cost for late detection
cmodel = util.ADSimpleCostModel(c_alrm, c_missed, c_late)
cost = cmodel.cost(signal, labels, windows, thr=3.5)
print(f'The cost with the current predictions is: {cost}')
```

```
The cost with the current predictions is: 56
```

This is just an example, but the idea of focusing on acutal cost is important

In general, our goal is to find some kind of cost function $c(\{x_i\}_{i=1}^m, \theta, \varepsilon)$ depending on:

- An evaluation dataset $\{x_{i=1}\}^m$
- The estimator parameters θ
- The threshold ε

Choosing the Threshold

Ideally, we wish to choose the best threshold

For that, we need a dataset to evaluate $c(\{x_i\}_{i=1}^{m}, \theta, \varepsilon)$

- ...But we cannot use the test data!
- ...Since that would lead to overfitting

Choosing the Threshold

Ideally, we wish to choose the best threshold

For that, we need a dataset to evaluate $c(\{x_i\}_{i=1}^{m}, \theta, \varepsilon)$

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- ...Since that would lead to overfitting

Most data-driven AI approaches have both parameters and hyper-parameters

- In our case, θ represents the parameter
- ...And ε is a hyper-parameter

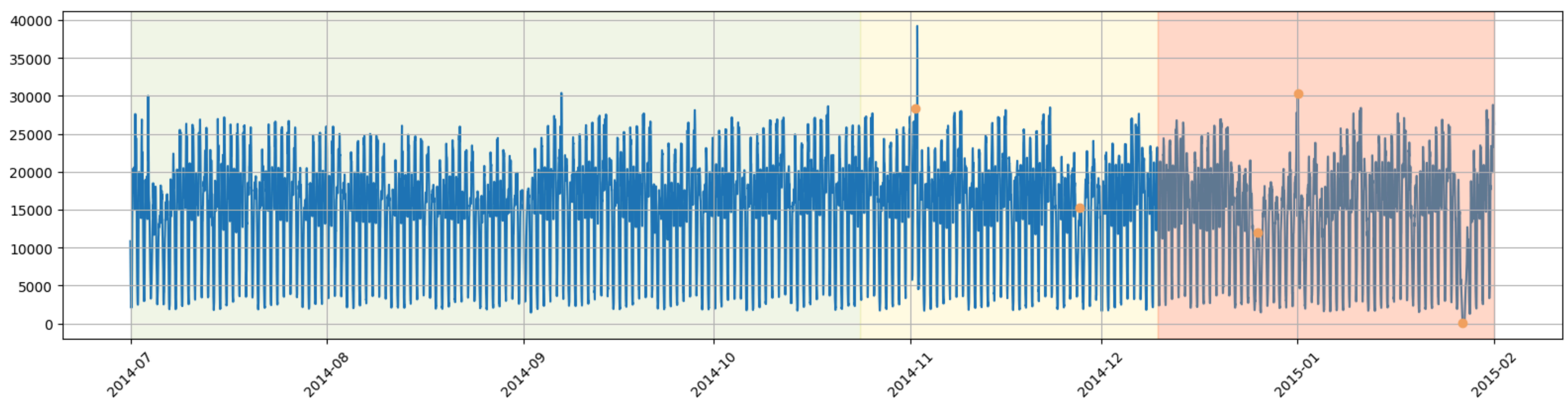
Neither should be optimized on the test data

Define a Validation Set

We can however define a separate validation set

We need a fraction of the data containing anomalies

In [17]:
```
val_end = pd.to_datetime('2014-12-10 00:00:00')
util.plot_series(data, labels, val_start=train_end, test_start=val_end, figsize=figsize)
```

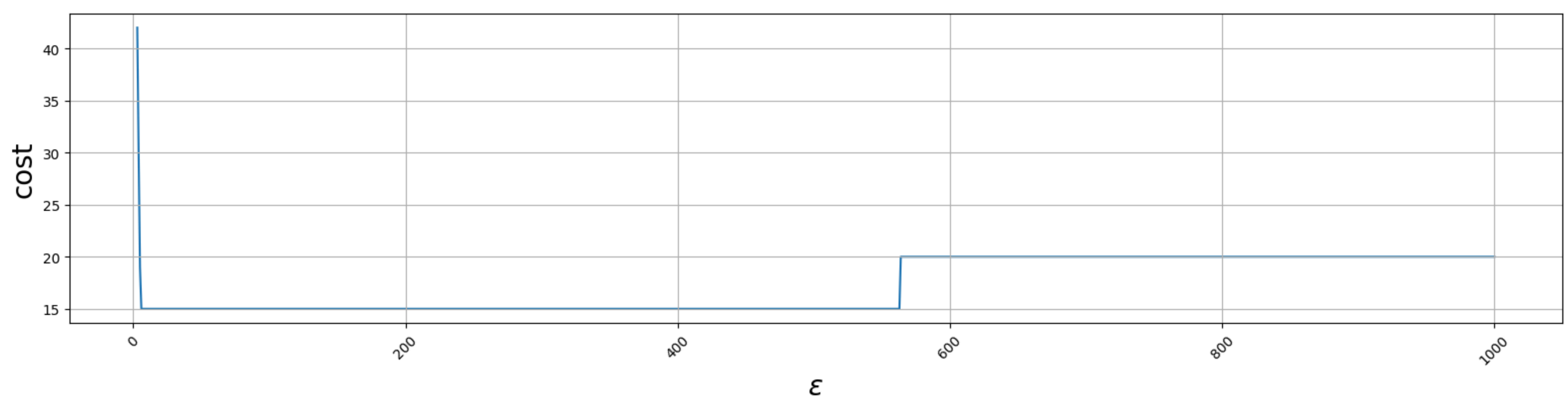


Effect of Changing the Threshold

We can visualize the cost associated to different thresholds on the validation set

In [18]:

```
signal_opt = signal[signal.index < val_end]
labels_opt = labels[labels < val_end]
windows_opt = windows[windows['end'] < val_end]
thr_range = np.linspace(3, 1000, 1000)
cost_range = pd.Series(index=thr_range, data=[cmodel.cost(signal_opt, labels_opt, windows_o
util.plot_series(cost_range, figsize=figsize, xlabel=r'$\varepsilon$', ylabel='cost')
```



Threshold Optimization

We can now define our threshold ε by optimizing over the validation set:

$$\mathop{\mathrm{argmin}}_{\varepsilon} c(\{x_i\}_{i=1}^m, \theta, \varepsilon)$$

```
In [19]: best_thr, best_cost = util.opt_thr(signal_opt, labels_opt, windows_opt,  cmodel, thr_range)
         print(f'Best threshold: {best_thr:.3f}, corresponding cost: {best_cost:.3f}')
```

```
Best threshold: 5.994, corresponding cost: 15.000
```

Then we can check how our detector performed on the test data:

```
In [20]: signal_test = signal[signal.index >= val_end]
         labels_test = labels[labels >= val_end]
         windows_test = windows[windows['begin'] >= val_end]
         ctst = cmodel.cost(signal_test, labels_test, windows_test, best_thr)
         print(f'Cost on the test data {ctst}')
```

```
Cost on the test data 10
```

Anomaly Detection in Taxi Calls

Improving the Results

Reassess and Plan

Let's recap our current situation

- We have a formalization for our anomaly detector
- ...And one for threshold optimization

Which means that we have a full problem formalization

We also have a simple prototype

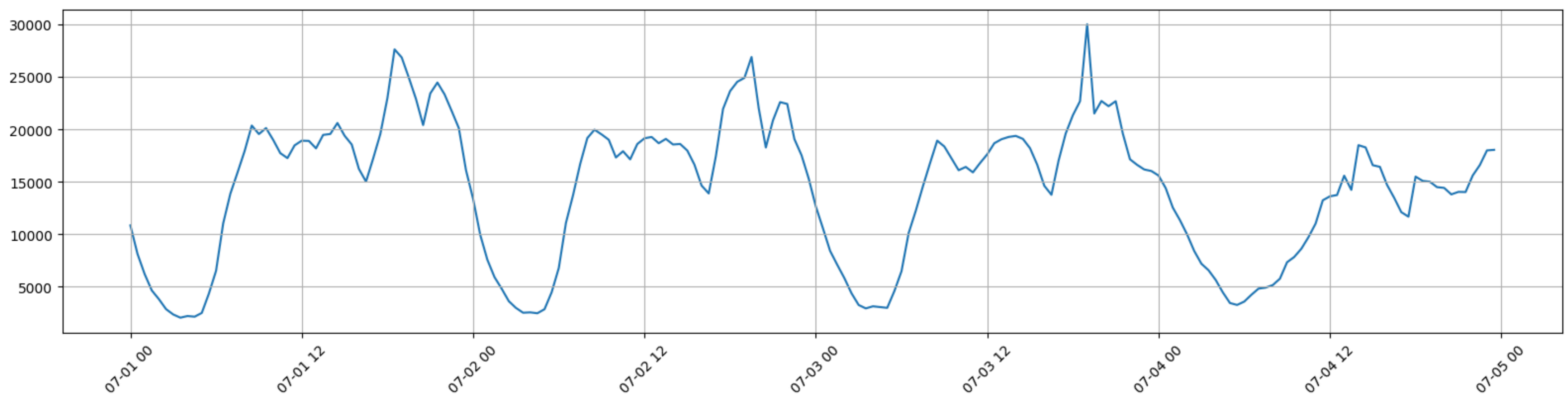
- KDE is used for density estimation
- Grid search for threshold optimization

Can we do better?

A Closer Look at Our Data

Let's have a closer look at our series

In [21]:
```
util.plot_series(data.iloc[:4*48], figsize=figsize)
```



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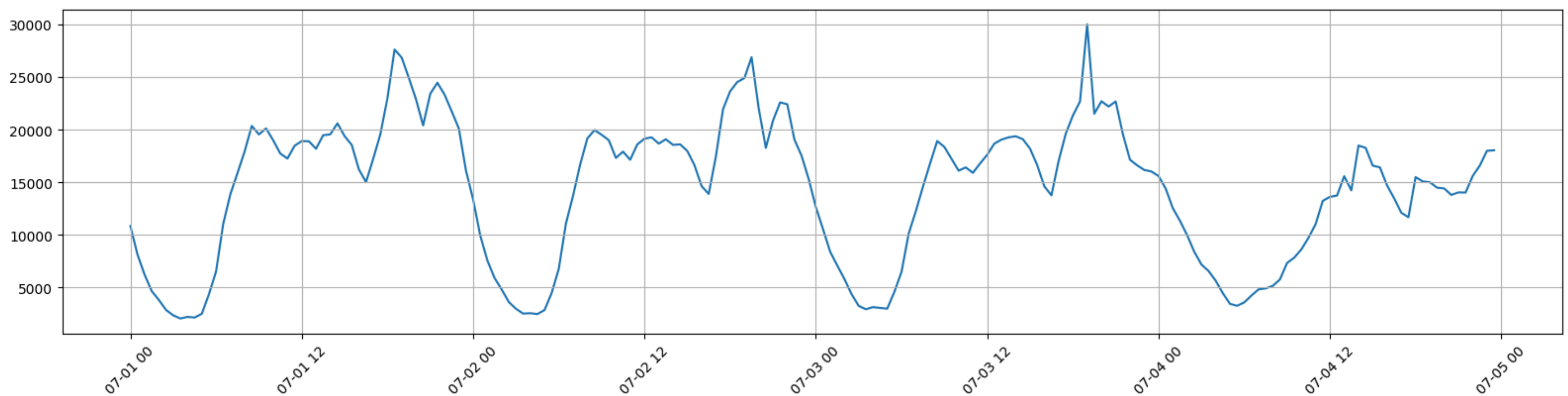
```

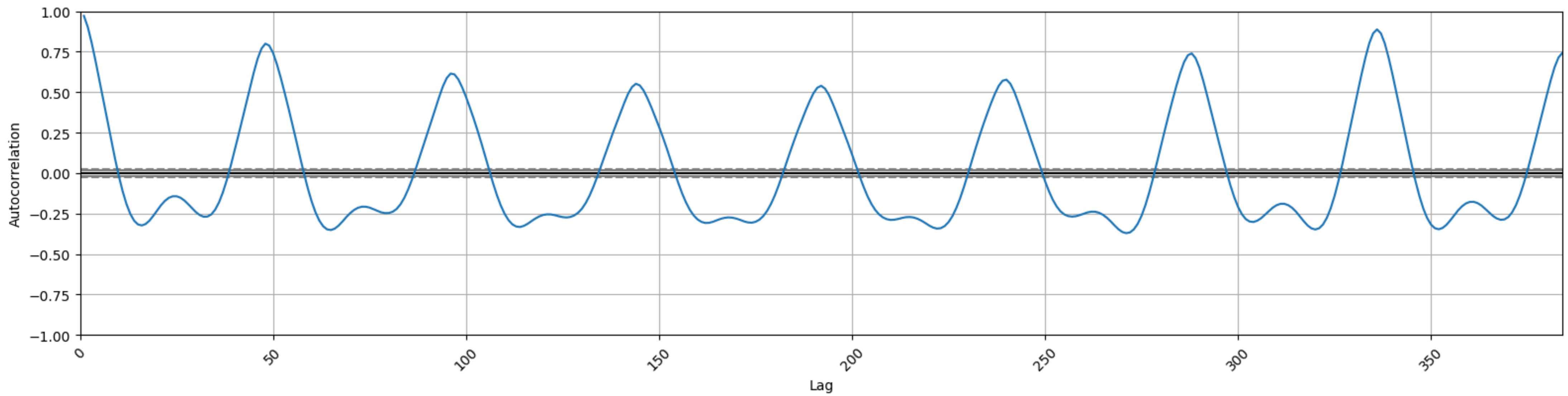


- The number of calls seems to be roughly following a period
- Which is quite normal, given that it's a local, human, activity

Determine the Period

This is even clearer if we draw an autocorrelation plot

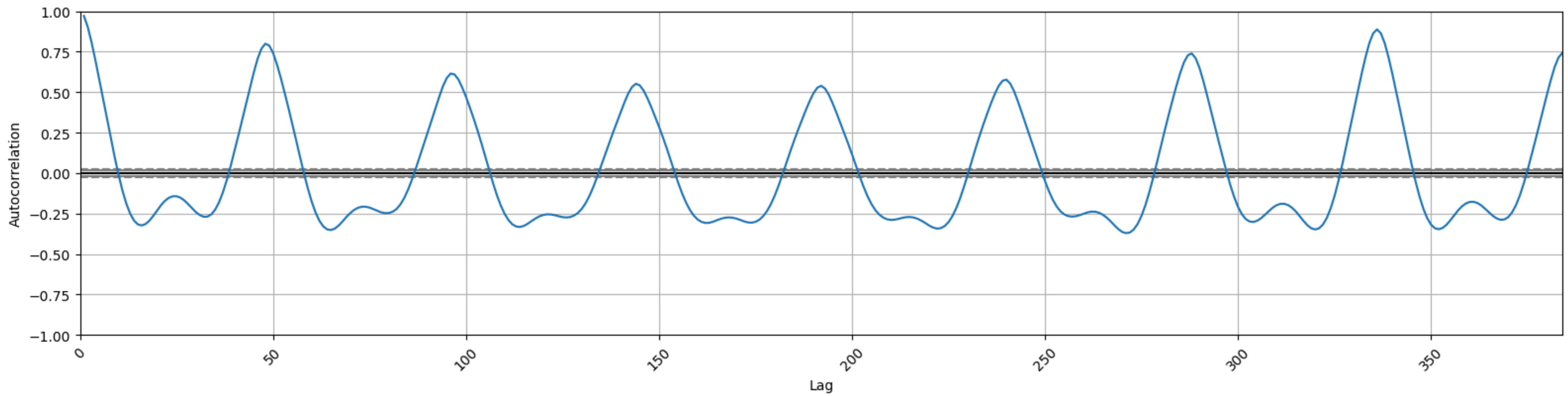
In [22]:
```
util.plot_autocorrelation(data, max_lag=8*48, figsize=figsize)
```



Determine the Period

This is even clearer if we draw an autocorrelation plot

```
In [22]: util.plot_autocorrelation(data, max_lag=8*48, figsize=figsize)
```



- There are peaks every 48 time steps (a time step is 30 minutes)
- And the peak at 7×48 steps (one week) is particularly tall

Time as an Additional Input

One way to look at that

...Is that the distribution depends on the position within the period

- Therefore, we should consider the number of taxi calls x
- ...And the time of the week t together

Let us extract (from the index) the time information information:

```
In [23]: hour_of_week = (24 * data.index.weekday + data.index.hour + data.index.minute / 60)
```

We can then add it as a separate column to the data:

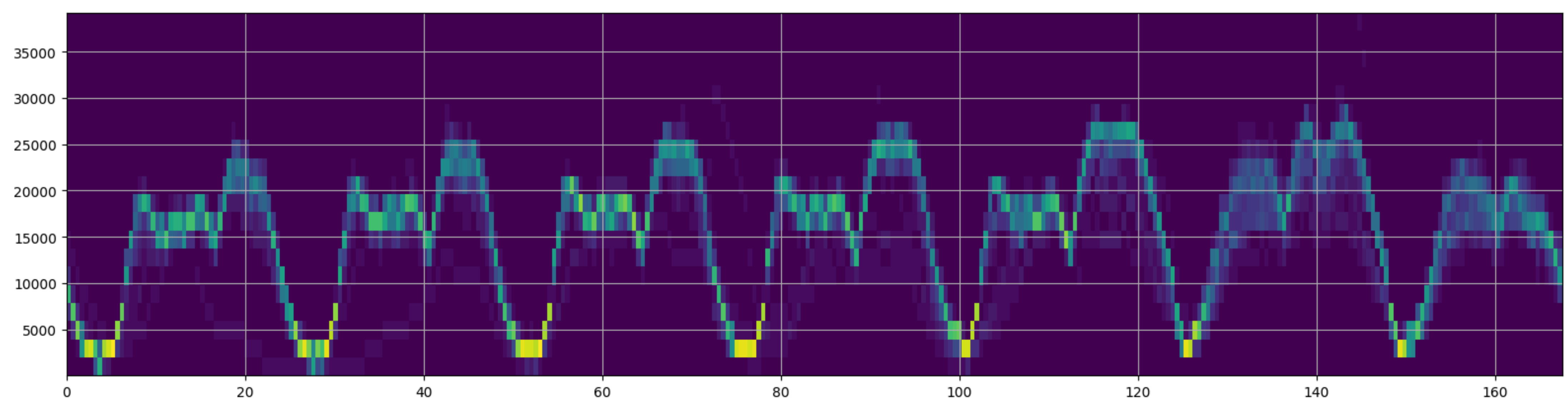
```
In [24]: data2 = data.copy()
         data2['hour_of_week'] = hour_of_week
```

Multivariate Distribution

Let us examine the resulting multivariate distribution

We can use a 2D histogram:

```
In [25]: util.plot_histogram2d(data2['hour_of_week'], data2['value'], bins=(7*48, 20), figsize=figsi
```



- x = time, y = value, color = frequency of occurrence

Training a Density Estimator

We can train a KDE model for this new dataset, too

```
In [26]: data2_tr = data2[data2.index < train_end]
         kde2 = util.train_kde(data2_tr, bandwidth_range=np.linspace(0.01, 0.1, 10))
```

The model will now estimate a joint distribution (calls & time):

$$\hat{f}(X, T) \simeq P(X, T)$$

We can use this model for anomaly detection just like in the previous case

$$\hat{f}(X, T) \leq \varepsilon$$

- In truth, things are bit more complicated, but we'll skip the details

Alarm Signal

We can obtain an alarm signal like in the previous case

In [27]:
```
ldens2 = kde2.score_samples(data2.values) # Obtain log probabilities
signal2 = pd.Series(index=data2.index, data=-ldens2) # Build series with neg. prob.
util.plot_series(signal2, labels=labels, windows=windows, figsize=figsize, y_cap=30) # Plot
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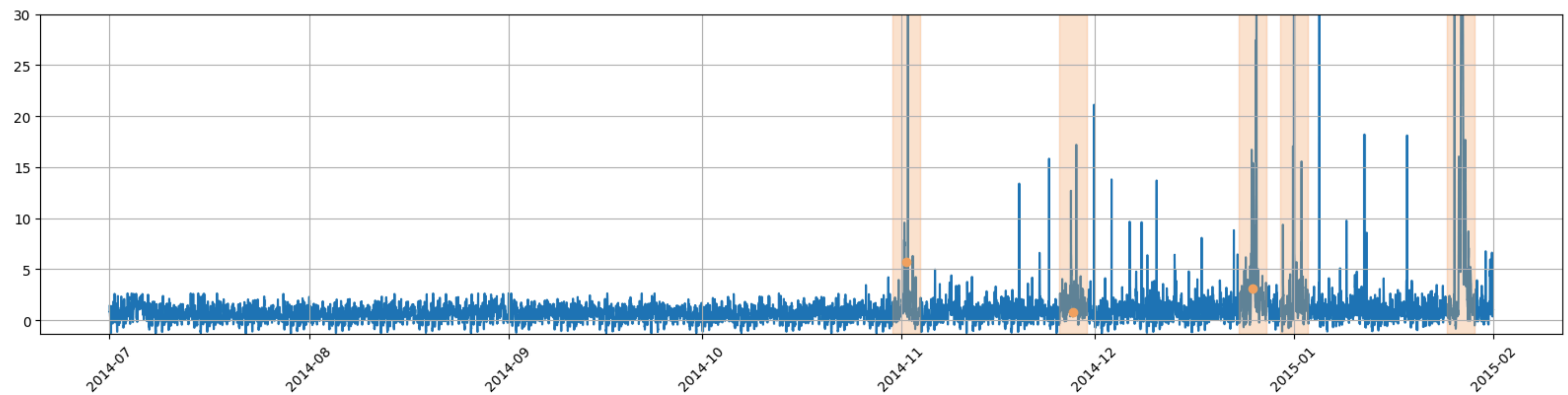
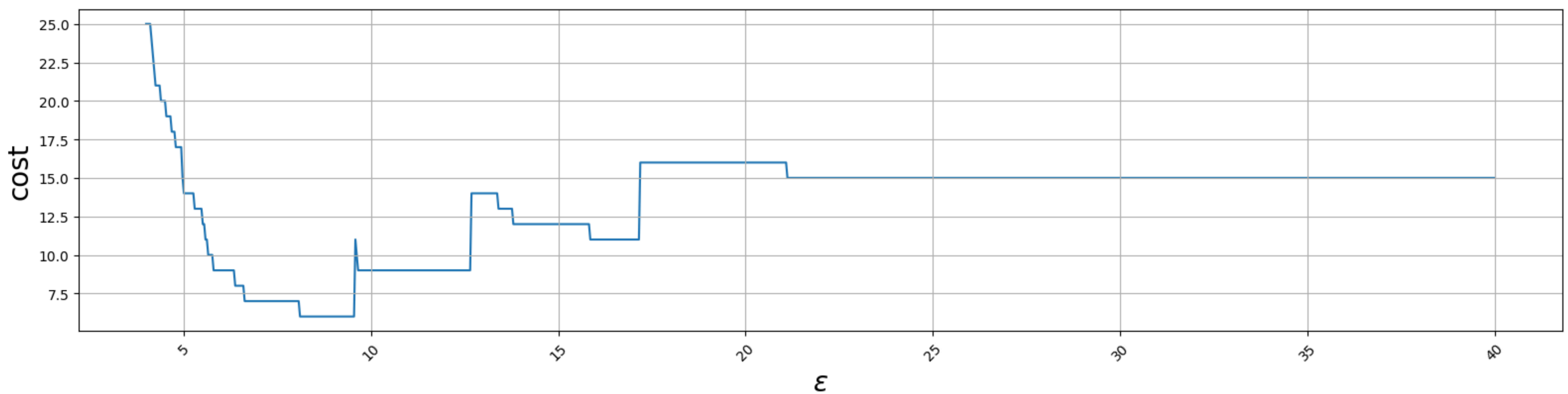


- There are not several peaks around some of the previously missed anomalies

Threshold Selection

The cost surface we get for ε is also more varied

In [28]:
```
signal_opt2 = signal2[signal2.index < val_end]
thr_range2 = np.linspace(4, 40, 1000)
cost_range2 = pd.Series(index=thr_range2, data=[cmodel.cost(signal_opt2, labels_opt, window
util.plot_series(cost_range2, figsize=figsize, xlabel=r'$\varepsilon$', ylabel='cost')
```



Evaluation

Let's see which kind of costs we get for the new model

We'll start from the training and validation data

- This is the data for which ε is directly optimized
- So, improvement can be taken almost for granted here

```
In [29]: best_thr2, best_cost2 = util.opt_thr(signal_opt2, labels_opt, windows_opt,  cmodel, thr_rang
         print(f'Best threshold: {best_thr:.3f}, corresponding cost: {best_cost2:.3f}')
```

```
Best threshold: 5.994, corresponding cost: 6.000
```

But we also get better results on the test data!

```
In [30]: signal_test2 = signal2[signal2.index >= val_end]
         ctst = cmodel.cost(signal_test2, labels_test, windows_test, best_thr2)
         print(f'Cost on the test data {ctst}')
```

```
Cost on the test data 9
```