Conversion of Bases in Different Number System

Revise

$(x)_{10}$ ←

1) Decimal → Base 10

↳ each digit only consist 10 possible Numbers

→ Range → 0 to 9

$(x_n \dots x_1 x_0)_r$, $r > x_i$, $r > 0$

2) Binary → Base 2 ← $(x)_2$

↳ digits $\in \{0, 1\}$

$(x_n \dots x_1 x_0)_r$, $r > x_i$, $r > 0$

3) Octal $\longrightarrow$ Base - 8 $\longleftarrow$ range of 8 possible numbers in each digit

↳ Range $\rightarrow$ 0 to 7

$(x_n \ldots x_1\, x_0)_b\,, \quad x_i < b\,, \quad b > 0$

4) Hexadecimal $\longrightarrow$ Base - 16 $\longleftarrow$ 16 possible numbers in each digit

↳ Range $\rightarrow$ 0 to F $\longrightarrow$ 0 to 9

10 — A

11 — B

12 — C

13 — D

14 — E

15 — F

Ex $(21F)_{16}$

Conversion of Radix

→ Each Number System is dependent on Radix

$$(14)_{10} \neq (14)_{8} \neq (14)_{16}$$

Number
- Without decimal Point
- With decimal Point

a) Decimal to Other Base System

↳ Any

ex $(54)_{10} \longrightarrow (x)_2$

Algorithm : Repeated division Algorithm

1) Divide the decimal Number to the radix of other Number

$\frac{54}{2}$, then we'll get Quotient & Remainder

$Q = 27$
$R = 0$

$$\begin{array}{r|l} 2 & 54 \;\; 27 \\ & \underline{4} \\ & 14 \\ & \underline{14} \\ & R=0 \end{array}$$

2) Store the quotient & remainder, Put the remainder on Top and divide the quotient from same base.

$\frac{27}{2}$, Store the Quotient & Remainder

$R = 1$
$Q = 13$

0
1

→ Repeat the Process until Quotient become '0'.

3) Write the remainders from bottom to top. then the final number is converted value.

$(54)_{10} \rightarrow (x)_{2}$

division	Quotient	Remainder
$\frac{54}{2}$	27	0
$\frac{27}{2}$	13	1
$\frac{13}{2}$	6	1
$\frac{6}{2}$	3	0
$\frac{3}{2}$	1	1
$\frac{1}{2}$	$q = 0$	1

$(54)_{10}$

$\downarrow$

$(110110)_2$

Eg $(619)_{10} \longrightarrow (x)_8$ { Decimal $\longrightarrow$ Octal }

division	Quotient	Remainder
$\frac{619}{8}$	77	3
$\frac{77}{8}$	9	5
$\frac{9}{8}$	1	1
$\frac{1}{8}$	q=0	1

$(1153)_8 \rightleftharpoons (619)_{10}$

Eg $(514)_{10} \longrightarrow (x)_{16}$

division	Quotient	Remainder
$\frac{514}{16}$	32	2
$\frac{32}{16}$	2	0
$\frac{2}{16}$	q=0	2

$(514)_{10} \longleftrightarrow (202)_{16}$

Ex $(9677)_{10} \longrightarrow (x)_{16}$

A = 10, B = 11, C = 12, D = 13, E = 14, F = 15

division	quotient	Remainder
$\frac{9677}{16}$	604	13 (D)
$\frac{604}{16}$	37	12 (C)
$\frac{37}{16}$	2	5
$\frac{2}{16}$	q = 0	2

$(9677)_{10} \longleftrightarrow (25CD)_{16}$

Eg $(177)_{10} \longrightarrow (x)_2$

division	Quotient	Remainder
$\frac{177}{2}$	88	1
$88/2$	44	0
$\frac{44}{2}$	22	0
$22/2$	11	0
$11/2$	5	1
$5/2$	2	1
$2/2$	1	0
$1/2$	q=0	1

$(177)_{10}$

$\downarrow$

$(10110001)_2$

Place Value System

↳ Each digit has its own weightage

$$(1234)_{10}$$

$$1000 + 200 + 30 + 4$$

$$= (1 \times 1000) + (2 \times 100) + (3 \times 10) + (4 \times 1)$$

$$= 1 \times 10^3 + 2 \times 10^2 + 3 \times 10^1 + 4 \times 10^0$$

Most Significant Number

least Significance Number

Radix → weightage ⇒ 3 (highest)

Radix ← weightage → low

b) Other Base System to decimal

1) Determine the position of each digit of the number.

$$(512)_8 \longrightarrow \underset{\text{MSN}}{5} \quad 1 \quad \underset{\text{LSN}}{2}$$

2) Multiply the digits of the number on the Basis of their weightage. & calculate the sum of Product.

$$5 \times 8^2 + 1 \times 8^1 + \underset{\text{LSN}}{2} \times 8^0$$

$$= 5 \times 64 + 8 + 2$$

$$= (330)_{10}$$

$$(512)_8 \longleftrightarrow (330)_{10}$$

Eg $(1153)_8 \longrightarrow (x)_{10}$

LSN

MSN $\longrightarrow 1\times 8^3 + 1\times 8^2 + 5\times 8^1 + 3\times 8^0$

$= 512 + 64 + 40 + 3$

$= (619)_{10}$

Eg $(1011001)_2 \longrightarrow (x)_{10}$

$1\times 2^6 + \cancel{0\times 2^5} + 1\times 2^4 + 1\times 2^3 + \cancel{0\times 2^2} + \cancel{0\times 2^1} + 1\times 2^0$

$2^6 + 2^4 + 2^3 + 1$

$= 64 + 16 + 8 + 1 = (89)_{10}$

Ex $(13A)_{16} \rightarrow (x)_{10}$

$$\begin{array}{ccccc} 1 & & 3 & & A \\ \times & + & \times & + & \times \\ 16^2 & & 16^1 & & 16^0 \end{array}$$

$$= 1\times16^2 + 3\times16 + \overset{10}{A}\times16^0$$

$$= 256 + 48 + 10$$

$$= (314)_{10}$$

Ex $(1210)_3 \rightarrow (x)_{10}$

$$1\times3^3 + 2\times3^2 + 1\times3^1 + \cancel{0\times3^0}$$

$$27 + 18 + 3$$

$$= (48)_{10}$$

Ex $(1760)_8 \rightarrow (x)_{10}$

$$1\times8^3 + 7\times8^2 + 6\times8^1 + \underbrace{0\times8^0}$$

$$8^3 + 7\times64 + 6\times8$$

$$= 512 + 448 + 48 = (1008)_{10}$$

Ex $\underbrace{0x}\,\underbrace{1A62} \rightarrow (x)_{10}$

Hexadecimal Notation

Number

$$1\times16^3 + \overset{}{A}\times16^2 + 6\times16^1 + 2\times\cancel{16^0}$$

$$4096 + 10\times256 + 96 + 2$$

$$\begin{array}{r} = 4096 \\ + 2560 \\ 96 \\ 2 \\ \hline \end{array} \Rightarrow (6754)_{10}$$

Prefix → 0_0 → Octal

Prefix → $0x$ → Hex

Decimal to Other Base System

↳ Repeated Division Method

Other Base System to Decimal

↳ Weightage Sum of Product Method

$$(N)_r = [x_n \ldots\ldots x_2 \ x_1 \ x_0]_r$$

Decimal Representation:

$$[x_n r^n \ldots x_2 r^2 + x_1 r^1 + x_0 r^0]$$

$$(N)_{10} = x_n r^n + \ldots\ldots x_2 r^2 + x_1 r^1 + x_0 r^0, \ \forall r \in \mathbb{N}, \ x < r$$

Ex: $(1001)_2 \longrightarrow (x)_3$

$(1001)_2$ ← Binary

$(x)_3$ ← trinary

Binary → Decimal → trinary

$\underline{1} \times 2^3 + \underline{0} \times 2^2 + \underline{0} \times 2^1 + \underline{1} \times 2^0$

$2^3 + 1 = 8 + 1 = (9)_{10} \longrightarrow (x)_3$

Decimal → other

$\Rightarrow (9)_{10} \longrightarrow (x)_3$

division	Quotient	Remainder
9/3	3	0
3/3	1	0
1/3	q = 0	1

$(9)_{10} \longrightarrow (100)_3$

$\Rightarrow (1001)_2 \longrightarrow (9)_{10} \longrightarrow (100)_3$

Special Cases

1 bit < 0, 1

A) Binary to Octal :

↳ Octal number system → 3 bits
↳ 8 possible numbers

→ To Convert Binary to octal, we will make the pair of three bits from the least Significant Bit (LSB)

MSB, LSB

Ex: $(0\,0\,1\,1\,0\,0\,1\,0\,1)_2 \longrightarrow (x)_8$ { Bin → Oct }

001 100 101
1 4 5

$\Rightarrow (145)_8$

Octal	Binary
0	0
1	1
2	1 0
3	1 1
4	1 0 0
5	1 0 1
6	1 1 0
7	1 1 1

MSB → 1 1 1 ← LSB

3 binary digits

Ex: $(0100\ 1000\ 111\ 00\ 11\ 01\ 0001)_2 \rightarrow (x)_8$

MSB, LSB

$$\underbrace{010}_{2}\ \underbrace{010}_{2}\ \underbrace{001}_{1}\ \underbrace{110}_{6}\ \underbrace{011}_{3}\ \underbrace{010}_{2}\ \underbrace{001}_{1}$$

$= (2216321)_8$

Ex: $(0101011101)_2 \rightarrow (x)_8$

LSB

$$\underbrace{010}_{2}\ \underbrace{101}_{5}\ \underbrace{101}_{5} \Rightarrow (255)_8$$

Octal	Binary
0	0
1	1
2	1 0
3	1 1
4	1 0 0
5	1 0 1
6	1 1 0
7	1 1 1

MSB, LSB

3 binary digits

B) Octal to Binary :- 3+2+1

↳ Represent each digit of the octal number in 3 bits

Ex: $(246357)_8 \rightarrow (x)_2$

$$\begin{array}{cccccc} 2 & 4 & 6 & 3 & 5 & 7 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 010 & 100 & 110 & 011 & 101 & 111 \end{array}$$

$= (010\ 100\ 110\ 011\ 101\ 111)_2$

Ex: $(3240)_8 \rightarrow (x)_2$

$$\begin{array}{cccc} 3 & 2 & 4 & 0 \\ \downarrow & \downarrow & \downarrow & \downarrow \\ 011 & 010 & 100 & 000 \end{array} \Rightarrow (011010100000)_2$$

Octal	Binary
0	0
1	1
2	10
3	11
4	100
5	101
6	110
7	MSB → 111 ← LSB

3 binary digits