Duality Theoram:

Any boolean function has two faces,

This theoram states that the dual of the boolean function is obtained by interchanging the logical AND with logical OR; and 1 with zero, zero with 1.

Replace : AND $\longrightarrow$ OR

OR $\longrightarrow$ AND

1 $\longrightarrow$ 0

0 $\longrightarrow$ 1

* It is also called as complement of the boolean function

* Each and every Boolean Expression has its dual.

Group 1 → Group 2 (Dual)

1. $x + 0 = x \longrightarrow x \cdot 1 = x$

2. $x + 1 = 1 \longrightarrow x \cdot 0 = 0$

3. $x + \overline{x} = 1 \longrightarrow x \cdot \overline{x} = 0$

4. $x + x = x \longrightarrow x \cdot x = x$

5. $x + y = y + x \longrightarrow x \cdot y = y \cdot x$

6. $x + (y+z) = (x+y) + z \longrightarrow x \cdot (y \cdot z) = (x \cdot y) \cdot z$

7. $x \cdot (y + z) = x \cdot y + x \cdot z \longrightarrow x + (y \cdot z) = (x+y) \cdot (x+z)$

$1 \rightarrow 0$

$0 \rightarrow 1$

AND → OR

OR → AND

Consensus Theoram/ Redundancy Theoram:

Redundancy → फालतू (Extra)

↳ Boolean trick,

Whenever a variable appears without a bar and next variable appears with a bar then other next term will be redundant.

$$\boxed{AB + \bar{A}C + BC = AB + \bar{A}C}$$

Conditions to Apply this rule:

1) Three variables must be there in an expression

2) Each variable must repeated twice.

3) One variable must Present in Complement form & uncomplemented form

Proof

$$Y = AB + \bar{A}C + BC$$

taking 1 with BC. $\Rightarrow AB + \bar{A}C + BC \cdot 1$

$$y = AB + \bar{A}C + BC(A + \bar{A})$$

$$y = \underbrace{AB + \bar{A}C + ABC}_{} + \bar{A}BC$$

$$Y = AB + ABC + \bar{A}C + \bar{A}BC$$

$$Y = AB(1+C) + \bar{A}C(1+B)$$

$$\boxed{Y = AB + \bar{A}C}$$

Redundant Literal Rule :

$$\boxed{A + \bar{A}B = A + B}$$

here $\bar{A}$ is redundant

⇓

Dual

⇓

$$\boxed{A \cdot (\bar{A} + B) = AB}$$

$$\boxed{\text{Shannon Expansion Theorem}}$$

↓

Skip

Eg let $*$ be defined as $x * y = \bar{x} + y$. let $z = x * y$. The value of $z * x$ will be ___ ?

$$x * y = \bar{x} + y$$

$$x * y = z$$

$z = \bar{x} + y$ ⟵ Implication Gate

$$\boxed{z * x \Rightarrow \bar{z} + x}$$

$$= \overline{\bar{x} + y} + x$$

$$= \underbrace{\bar{\bar{x}}}_{\text{de morgan law}} \cdot \bar{y} + x \Rightarrow x \cdot \bar{y} + x$$

$$\Rightarrow x(1 + \bar{y})$$

$$= x \quad \underline{\underline{\text{Ans}}}$$

A boolean function $x'y' + xy + x'y$ will be equivalent to ________

Sol:

$$\bar{x}\bar{y} + xy + \bar{x}y$$

$$\bar{x}\cancel{(y+\bar{y})} + xy = \bar{x} + xy \longrightarrow \text{Redundant Literal Rule}$$

$$a + \bar{a}b = a + b$$

$$= \underbrace{\bar{x} + y}$$

def $f(A,B) = \bar{A} + B$. Simplify $f(f(x+y,\, y),\, z)$

$f(a,b) = \bar{a} + b$

$$= f(\underbrace{f(x+y,\, y)},\, z)$$

$$= f(\underbrace{(\overline{x+y} + y)}_{a},\, \underset{b}{z})$$

$$= \overline{\overline{x+y} + y} + z$$

De morgan's law

$$= \overline{\bar{x} \cdot \bar{y} + y} + z$$

$b \cdot \bar{a} + a$

$= a + \bar{a} b = a + b$

$$= \overline{\bar{x} + \underbrace{y + y}} + z$$

$$\overline{\bar{x} + y} + z$$

de morgan law $\overline{A+B} = \bar{A} \cdot \bar{B}$

$$\bar{\bar{x}} \cdot \bar{y} + z$$

$$\boxed{= x \cdot \bar{y} + z}$$

Minterms:

↳ Minterms of 'n' variables is a product of the Variables so that we can get 1, by complementing the terms.

A	B	C	minterm $(A \cdot B \cdot C = 1)$	
0	0	0	$\bar{A}\,\bar{B}\,\bar{C}$	m_0
0	0	1	$\bar{A}\,\bar{B}\,C$	m_1
0	1	0	$\bar{A}\,B\,\bar{C}$	m_2
0	1	1	$\bar{A}\,B\,C$	m_3
1	0	0	$A\,\bar{B}\,\bar{C}$	m_4
1	0	1	$A\,\bar{B}\,C$	m_5
1	1	0	$A\,B\,\bar{C}$	m_6
1	1	1	$A\,B\,C$	M_7

Maxterm: (Dual of Minterm)

Maxterm = dual(minterm) : Maxterms of 'n' variable is nothing but the sum of the variables so that we can get zero by complementing the variable.

A	B	C	Maxterms (A+B+C = 0)	
0	0	0	$A+B+C$	M_0
0	0	1	$A+B+\overline{C}$	M_1
0	1	0	$A+\overline{B}+C$	M_2
0	1	1	$A+\overline{B}+\overline{C}$	M_3
1	0	0	$\overline{A}+B+C$	M_4
1	0	1	$\overline{A}+B+\overline{C}$	M_5
1	1	0	$\overline{A}+\overline{B}+C$	M_6
1	1	1	$\overline{A}+\overline{B}+\overline{C}$	M_7

$$\boxed{m_i = \overline{M_i}}$$

$F = ABC + AB\overline{C} + A\overline{B}C + \overline{A}BC$ Calculate minterms

$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$

$1 \cdot 1 \cdot 1 \quad 1 \cdot 1 \cdot 0 \quad 1\,0\,1 \quad 0\,1\,1 \leftarrow$ Natural term

$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$

$m_7 + m_6 + m_5 + m_3$

$F = \sum (m_3, m_5, m_6, m_7)$

$$\boxed{F = \sum m(3, 5, 6, 7)}$$