Canonical SOP & POS notations:

⇒ A truth table consists of a set of all inputs and outputs.

⇒ If there are 'n' inputs, then there will be 2^n of possible combinations of zeros & ones

↳ So we can express each output in following ways:

Canonical SOP form → Sum of Product → Sum of minterm

Canonical POS form :→ Product of Sum → Product of Maxterms

Canonical SOP form:

→ In this, each product term Consists all literals. These products are nothing but the minterms.

$$f = \underbrace{\bar{p}qr}_{m3} + \underbrace{p\bar{q}r}_{m5} + \underbrace{pq\bar{r}}_{m6} + \underbrace{pqr}_{m7} + \cancel{\bar{p}qr} + \cancel{p\bar{q}r} + \cancel{pq\bar{r}} + \cancel{pqr}$$

	p	q	r	minterms	
0	0	0	0	$\bar{p}\bar{q}\bar{r}$	
1	0	0	1	$\bar{p}\bar{q}r$	
2	0	1	0	$\bar{p}q\bar{r}$	
3	0	1	1	$\bar{p}qr$	→ m3
4	1	0	0	$p\bar{q}\bar{r}$	
5	1	0	1	$p\bar{q}r$	→ m5
6	1	1	0	$pq\bar{r}$	— m6
7	1	1	1	pqr	→ m7

$$f = m3 + m5 + m6 + m7$$

$$f = \sum m(3,5,6,7)$$

↓

Canonical SOP form

Canonical POS form :

↳ Product of Maxterms

	p	q	r	maxterms	
1	0	0	0	$p+q+r$	
2	0	0	1	$p+q+\bar{r}$	
3	0	1	0	$p+\bar{q}+r$	— M3
4	0	1	1	$p+\bar{q}+\bar{r}$	— M4
5	1	0	0	$\bar{p}+\bar{q}+\bar{r}$	
6	1	0	1	$\bar{p}+q+\bar{r}$	— M6
7	1	1	0	$\bar{p}+\bar{q}+r$	— M7
8	1	1	1	$\bar{p}+\bar{q}+\bar{r}$	

$$f = (\bar{p}+q+\bar{r})\cdot(p+\bar{q}+r)\cdot(\bar{p}+\bar{q}+r)\cdot(p+\bar{q}+\bar{r})$$

$$\downarrow \qquad \downarrow \qquad \downarrow \qquad \downarrow$$

$$M6 \cdot M3 \cdot M7 \cdot M4$$

$$f = M3 \cdot M4 \cdot M6 \cdot M7$$

$$f = \prod M(3,4,6,7) \leftarrow \text{(आसान)}$$

↓

Canonical POS form

Standard SOP form:

↳ The simplified form of Canonical SOP is noted to the Standard SOP form.

Example: $f = \bar{p}qr + p\bar{q}r + pq\bar{r} + pqr$

$$= \bar{p}qr + p\bar{q}r + pq\underbrace{\cancel{(r+\bar{r})}}_{1}$$

$$= \bar{p}qr + p\bar{q}r + pq$$

$$= q(\bar{p}r + p) + p\bar{q}r$$

Redundant literal Rule → $A + \bar{A}B = A + B$

$$= q(p + \bar{p}r) + p\bar{q}r$$

$$= q(p + r) + p\bar{q}r$$

$$= pq + qr + p\bar{q}r$$

$$p(q + \bar{q}r) + qr$$

RLR

$$p(q + r) + qr$$

$\boxed{pq + pr + qr}$ ← Standard SOP form

Standard POS form:

↳ Minimised / simplified form of Canonical POS form.

Eg $f = (p+q+r)\cdot(p+q+\bar{r})\cdot(p+\bar{q}+r)\cdot(\bar{p}+q+r)$

$= \underbrace{(p+q+r)\cdot(p+q+r)}_{(p+q+r)}\cdot\underbrace{\underbrace{(p+q+r)}_{m}\underbrace{(p+q+\bar{r})}_{n}}_{(p+q)}\cdot\underbrace{\underbrace{(p+q+r)}_{m}\cdot\underbrace{(p+\bar{q}+r)}_{m}}_{(p+r)}\cdot\underbrace{\underbrace{(p+q+r)}_{m}\cdot\underbrace{(\bar{p}+q+r)}_{m}}_{(q+r)}$

$= \boxed{(p+q+r)\cdot(p+q)\cdot(p+r)\cdot(q+r)}$

$(m+r)(m+\bar{r})$

$\cancel{mm} + m\bar{r} + rm + \cancel{r\bar{r}}$

$m + \underbrace{m\bar{r} + rm} + 0$

$m + m\,\cancel{(\bar{r}+r)}\,1$

$= m$ — $p+q$

Q Convert Standard POS form to Canonical POS form

WRONG

$f = (\bar{A}+B+C)\cdot(\bar{B}+C+D)\cdot(A+\bar{B}+\bar{C}+D)$ → Assuming that this is simplified form

$(\bar{A}+B+C+D\bar{D})\cdot(A\bar{A}+\bar{B}+C+D)\cdot(A+\bar{B}+\bar{C}+D)$

$1+0+0+0 \quad\cdot\quad 0+1+0+0 \quad\cdot\quad (0+1+1+0)$

$1000 \qquad 0100 \qquad 0110$

$8 \qquad 4 \qquad 6$

$f = \prod M(8,4,6)$

Q Convert non Standard SOP function in Minterms.

$f = xy + yz + zx$, 3 variables

$xy(z+\bar{z}) + yz(x+\bar{x}) + zx(y+\bar{y})$

$xyz + xy\bar{z} + \cancel{xyz} + \bar{x}yz + \cancel{xyz} + x\bar{y}z$

$$\begin{array}{cccc} xyz & xy\bar{z} & \bar{x}yz & x\bar{y}z \\ \downarrow & \downarrow & \downarrow & \downarrow \\ 111 & 110 & 011 & 101 \\ \downarrow & \downarrow & \downarrow & \downarrow \\ 7 & 6 & 3 & 5 \end{array}$$

* Order of variable matters

$f = m_7 + m_6 + m_3 + m_5$

$f = \sum m(3,5,6,7)$

Q $f(w,x,y,z) = \Sigma(1,3,4,6,9,11,12,14)$ ← minterm default

$$\begin{array}{cccccccc}
1 & 3 & 4 & 6 & 9 & 11 & 12 & 14 \\
\downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\
0001 & 0011 & 0100 & 0110 & 1001 & 1011 & 1100 & 1001
\end{array}$$

$$f = \bar{w}\bar{x}\bar{y}z + \bar{w}\bar{x}yz + \bar{w}x\bar{y}\bar{z} + \bar{w}xy\bar{z} + w\bar{x}\bar{y}z + w\bar{x}yz + wx\bar{y}\bar{z} + w\bar{x}\bar{y}z$$

→ $\bar{w}\bar{x}z$ Common

$$\bar{w}\bar{x}z\cancel{(\bar{y}+y)} + \bar{w}x\bar{z}\cancel{(\bar{y}+y)} + w\bar{x}z\cancel{(\bar{y}+y)} + wx\bar{y}\bar{z} + w\bar{x}\bar{y}z$$

$$\bar{w}\bar{x}z + \bar{w}x\bar{z} + w\bar{x}z + wx\bar{y}z + w\bar{x}\bar{y}z$$

$$\bar{x}\bar{z}\cancel{(\bar{w}+w)} + \bar{w}x\bar{z} + wx\bar{y}z + w\bar{x}\bar{y}z$$

$$\bar{x}\bar{z} + \bar{w}x\bar{z} + wx\bar{y}z + w\bar{x}\bar{y}z$$

$$\bar{x}\bar{z} + \bar{w}x\bar{z} + wz\bar{y}\cancel{(x+\bar{x})}$$

$$\bar{x}\bar{z} + \bar{w}x\bar{z} + wz\bar{y}$$

$$\bar{z}(\bar{x} + x\bar{w}) + wz\bar{y}$$

$$\bar{z}(x + \bar{w}) + wz\bar{y}$$

$$f = \underline{\bar{z}x + \bar{z}\bar{w} + wz\bar{y}}$$