Question : Simplify the following equation using K-Map :

$$F = xy' + x'y + x'y' \longrightarrow \text{SOP form}$$

2 variables → x, y

$$\downarrow \qquad \downarrow \qquad \downarrow$$
$$1\,0 \qquad 0\,1 \qquad 0\,0 \quad \leftarrow \text{binary}$$
$$\downarrow \qquad \downarrow \qquad \downarrow$$
$$2 \qquad 1 \qquad 0 \quad \leftarrow \text{decimal form}$$

$$\Rightarrow f = \sum m(0, 1, 2)$$

$$= \bar{x} + \bar{y}$$
$$\downarrow$$
$$\text{SOP}$$

LSB y

MSB x	$\bar{y}$ 0	y 1
$\bar{x}$ 0	0 1 $\bar{x}\bar{y}$	1 1 $\bar{x}y$
x 1	2 1 $x\bar{y}$	3 xy

$\bar{x}$

$\bar{y}$
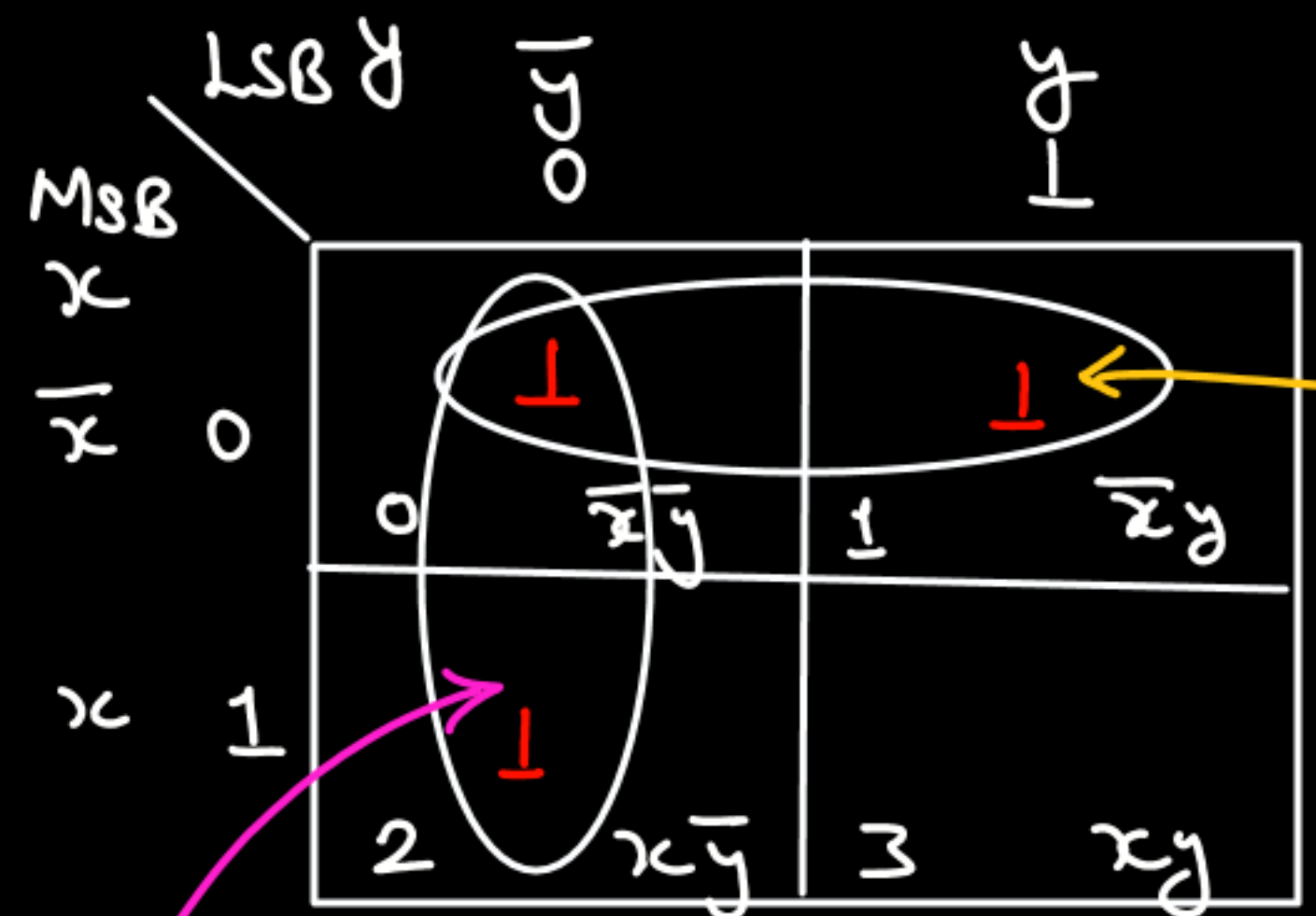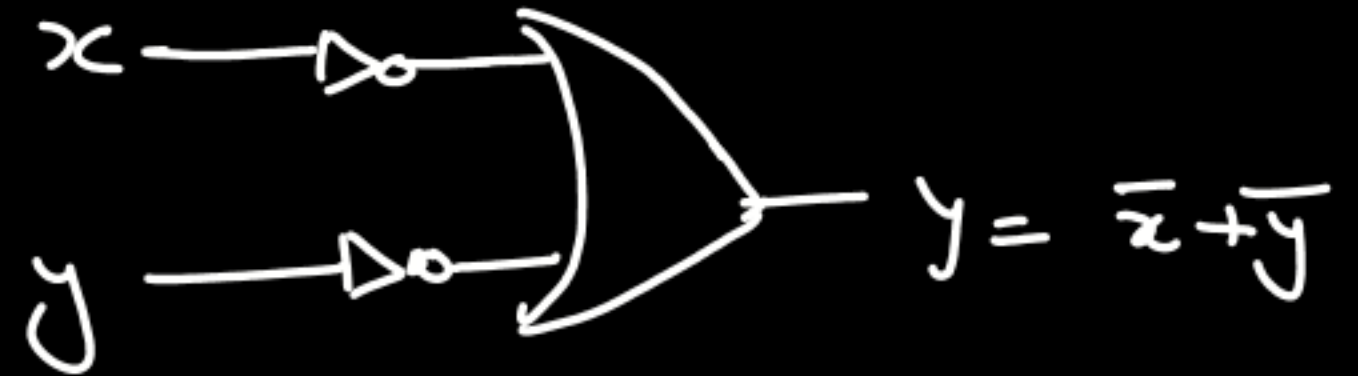
$$y = \bar{x} + \bar{y}$$

$$f(A,BC) = \Sigma m(0,3,5)$$

$\downarrow$

No Clubbing Possible

$\downarrow$

$$\overline{A}\,\overline{B}\,\overline{C} + \overline{A}BC + A\overline{B}C$$

Simplified form

LSB BC →	$\overline{B}\,\overline{C}$ 00	$\overline{B}C$ 01	BC 11	$B\overline{C}$ 10
MSB A ↓				
$\overline{A}$ 0	①		①	
	$\overline{A}\overline{B}\overline{C}$ 0	$\overline{A}\overline{B}C$ 1	$\overline{A}BC$ 3	$\overline{A}B\overline{C}$ 2
A 1		①		
	$A\overline{B}\overline{C}$ 4	$A\overline{B}C$ 5	ABC 7	$AB\overline{C}$ 6
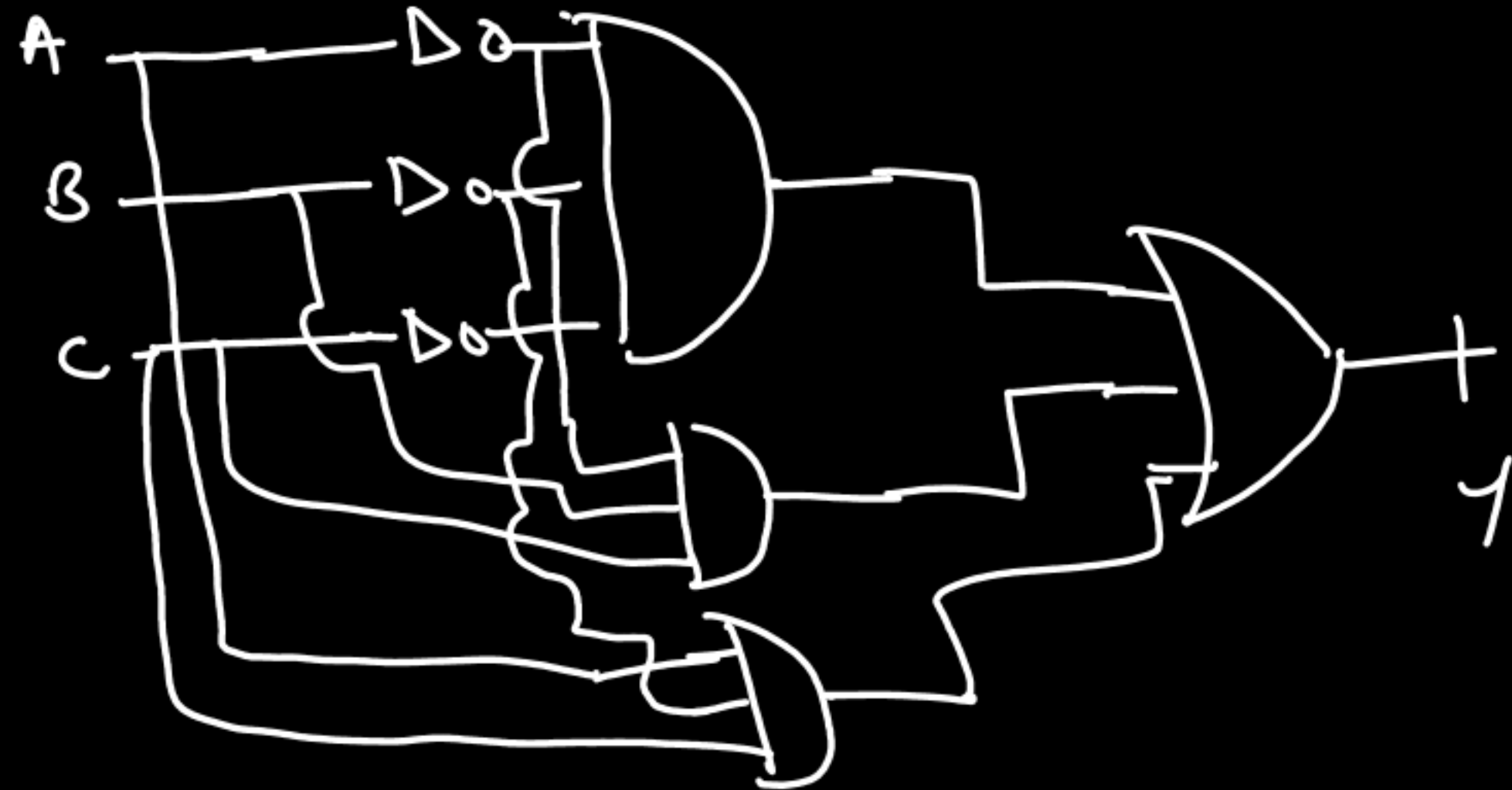
$$F = \bar{X}YZ + \bar{X}\bar{Y}Z + XY\bar{Z} + \bar{X}\bar{Y}\bar{Z} + XYZ + X\bar{Y}\bar{Z}$$

011   001   110   000   111   100

3   1   6   0   7   4 $\Rightarrow F = \sum m(0,1,3,4,6,7)$

K-map:

MSB \ YZ (LSB)	$\bar{Y}\bar{Z}$ 00	$\bar{Y}Z$ 01	YZ 11	$Y\bar{Z}$ 10
$\bar{X}$ 0	1 (0)	1 (1)	1 (3)	1 (2)
X 1	1 (4)	(5)	1 (7)	1 (6)

$\bar{Y}\bar{Z}$ is Common

* Larger group, & values must in power of 2

$\bar{X}\bar{Y}\bar{Z} \quad \bar{X}\bar{Y}Z \quad \bar{X}YZ \quad \bar{X}Y\bar{Z}$
$\bar{X}$ is Common

$XYZ \quad \bar{X}YZ \quad XY\bar{Z} \quad \bar{X}Y\bar{Z}$
$= Y$ is Common

$$f = Y + \bar{X} + \bar{Y}\bar{Z} \quad \leftarrow \text{simplified}$$

$Ex:$ $f(A,B,c) = \Pi M(1, 2, 4, 6, 7) \leftarrow POS\,(Product\,of\,Sum) \rightarrow$ Answer 'zero'

	$\overrightarrow{BC}$ $B+c$ 00	$B+\bar{c}$ 01	$\bar{B}+\bar{c}$ 11	$\bar{B}+c$ 10
A 0		0		0 ⊗
	0	1	3	2
$\bar{A}$ 1	0		0	0
	4	5	7	6

$\bar{B}+c$

$\bar{A}+B+c$
$\bar{A}+\bar{B}+c$
$= \bar{A}+c$

$A+B+\bar{c}$

$\bar{A}+\bar{B}$

POS

$f(A,B,c) = (\bar{B}+c)\cdot(\bar{A}+\bar{B})\cdot(\bar{A}+c)\cdot(A+B+\bar{c})$

$\Downarrow$ Dual

$f(A,B,c) = B\cdot\bar{c} + AB + A\bar{c} + \bar{A}\bar{B}c$

SOP

4 Variable k Map:

A	B	C	D	SN	Minterm
0	0	0	0	0	$\bar{A}\bar{B}\bar{C}\bar{D}$
0	0	0	1	1	$\bar{A}\bar{B}\bar{C}D$
0	0	1	0	2	$\bar{A}\bar{B}C\bar{D}$
0	0	1	1	3	$\bar{A}\bar{B}CD$
0	1	0	0	4	$\bar{A}B\bar{C}\bar{D}$
0	1	0	1	5	$\bar{A}B\bar{C}D$
0	1	0	0	6	$\bar{A}B\bar{C}\bar{D}$
0	1	1	1	7	$\bar{A}BCD$
1	0	0	0	8	$A\bar{B}\bar{C}\bar{D}$
1	0	0	1	9	$A\bar{B}\bar{C}D$
1	0	1	0	10	$A\bar{B}C\bar{D}$
1	0	1	1	11	$A\bar{B}CD$
1	1	0	0	12	$AB\bar{C}\bar{D}$
1	1	0	1	13	$AB\bar{C}D$
1	1	1	0	14	$ABC\bar{D}$
1	1	1	1	15	$ABCD$

LSB CD→

MSB
AB↓

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	$\bar{A}\bar{B}\bar{C}\bar{D}$ 0	$\bar{A}\bar{B}\bar{C}D$ 1	$\bar{A}\bar{B}CD$ 3	$\bar{A}\bar{B}C\bar{D}$ 2
$\bar{A}B$ 01	$\bar{A}B\bar{C}\bar{D}$ 4	$\bar{A}B\bar{C}D$ 5	$\bar{A}BCD$ 7	$\bar{A}BC\bar{D}$ 6
AB 11	$AB\bar{C}\bar{D}$ 12	$AB\bar{C}D$ 13	$ABCD$ 15	$ABC\bar{D}$ 14
$A\bar{B}$ 10	$A\bar{B}\bar{C}\bar{D}$ 8	$A\bar{B}\bar{C}D$ 9	$A\bar{B}CD$ 11	$A\bar{B}C\bar{D}$ 10

Minterms (SOP)

POS form

A	B	C	D	SN	Maxterm
0	0	0	0	0	$A+B+C+D$
0	0	0	1	1	$A+B+C+\bar{D}$
0	0	1	0	2	$A+B+\bar{C}+D$
0	0	1	1	3	$A+B+\bar{C}+\bar{D}$
0	1	0	0	4	$A+\bar{B}+C+D$
0	1	0	1	5	$A+\bar{B}+C+\bar{D}$
0	1	1	0	6	$A+\bar{B}+C+D$
0	1	1	1	7	$A+\bar{B}+\bar{C}+\bar{D}$
1	0	0	0	8	$\bar{A}+B+C+D$
1	0	0	1	9	
1	0	1	0	10	
1	0	1	1	11	
1	1	0	0	12	
1	1	0	1	13	
1	1	1	0	14	
1	1	1	1	15	

LSB $CD \rightarrow$

MSB

AB	$C+D$ $0+0$	$C+\bar{D}$ $0+1$	$\bar{C}+\bar{D}$ $1+1$	$\bar{C}+D$ $1+0$
$A+B$ $0+0$	$A+B+C+D$ 0	$A+B+C+\bar{D}$ 1	$A+B+\bar{C}+\bar{D}$ 3	$A+B+\bar{C}+D$ 2
$A+\bar{B}$ $0+1$	$A+\bar{B}+C+D$ 4	$A+\bar{B}+C+\bar{D}$ 5	$A+\bar{B}+\bar{C}+\bar{D}$ 7	$A+\bar{B}+\bar{C}+D$ 6
$\bar{A}+\bar{B}$ $1+1$	$\bar{A}+\bar{B}+C+D$ 12	$\bar{A}+\bar{B}+C+\bar{D}$ 13	$\bar{A}+\bar{B}+\bar{C}+\bar{D}$ 15	$\bar{A}+\bar{B}+\bar{C}+D$ 14
$\bar{A}+B$ $1+0$	$\bar{A}+B+C+D$ 8	$\bar{A}+B+C+\bar{D}$ 9	$\bar{A}+B+\bar{C}+\bar{D}$ 11	$\bar{A}+B+\bar{C}+D$ 10

Maxterms

① $f = \sum m (0, 1, 3, 5, 7, 10, 11, 13, 15)$ — SOP $\quad f(A, B, C, D)$

$\overline{A}\,\overline{B}\,\overline{C}$

quad CD

$A\,\overline{B}\,C$

BD

$$f = \overline{A}\,\overline{B}\,C + A\,\overline{B}\,C + CD + BD$$

	$\overline{C}\,\overline{D}$ 00	$\overline{C}D$ 01	CD 11	$C\overline{D}$ 10
$\overline{A}\,\overline{B}$ 00	1 ___0	1 ___1	1 ___3	___2
$\overline{A}B$ 01	___4	1 ___5	1 ___7	___6
AB 11	___12	1 ___13	1 ___15	___14
$A\overline{B}$ 10	___8	___9	1 ___11	1 ___10

Questions $\longrightarrow$ 4 vac K-Maps
                                $\downarrow$
                    (don't care condition)
                                    $\downarrow$
                                Simple