Recall → 4 steps

1) Identify the Problem Statement & I/O

2) Design the Truth Table

3) Write the Expression in form of Minterm OR Maxterm, & Simplify using K-Maps

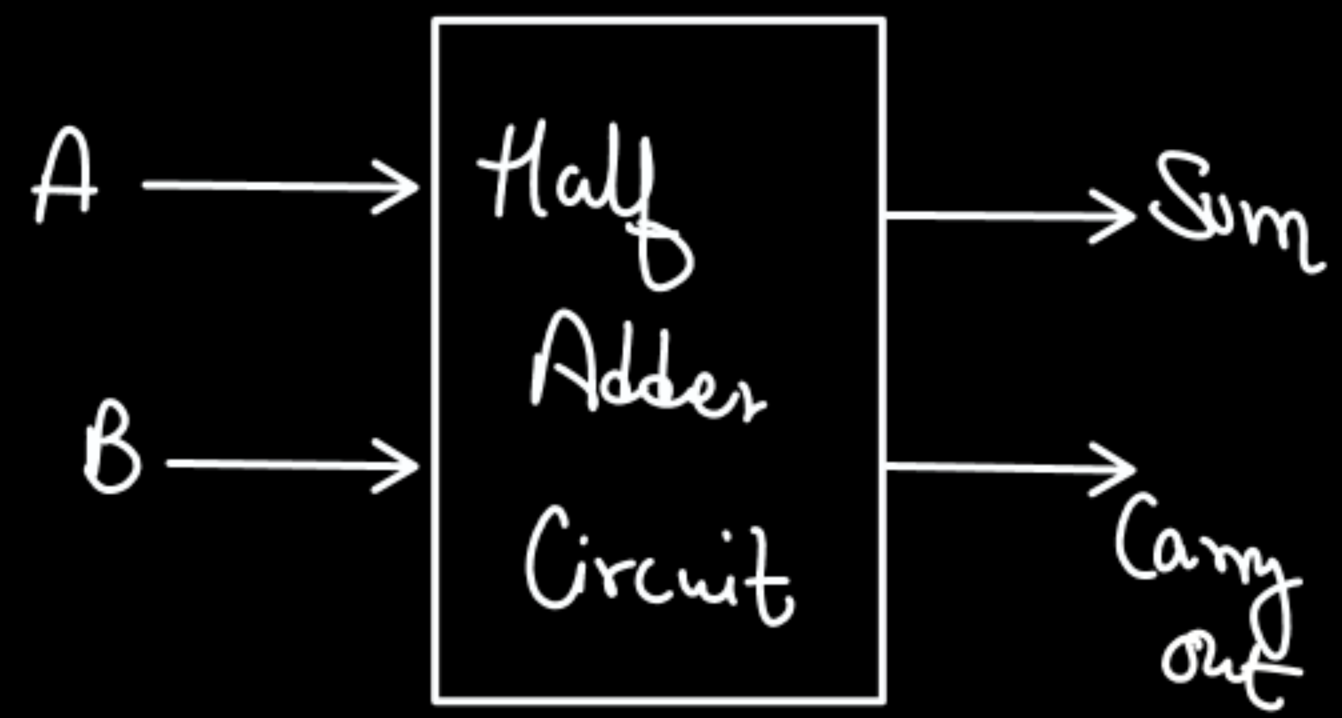
4) Design the logical Circuit.

Arithmetic Circuit

→ **Half adder**: A circuit which add two bits.

"A half adder is defined as a basic four terminals digital device which add two Binary digits".

4 Input / outputs



S.No	A	B	A+B	Decimal o/p	Binary o/p	Sum	Carry
0	0	0	0+0	0	0	0	0
1	0	1	0+1	1	1	1 $\bar{A}B$	0
2	1	0	1+0	1	1	1 $A\bar{B}$	0
3	1	1	1+1	2	10	0	1 AB

$$\text{Sum} = \bar{A}B + A\bar{B} = A \oplus B$$

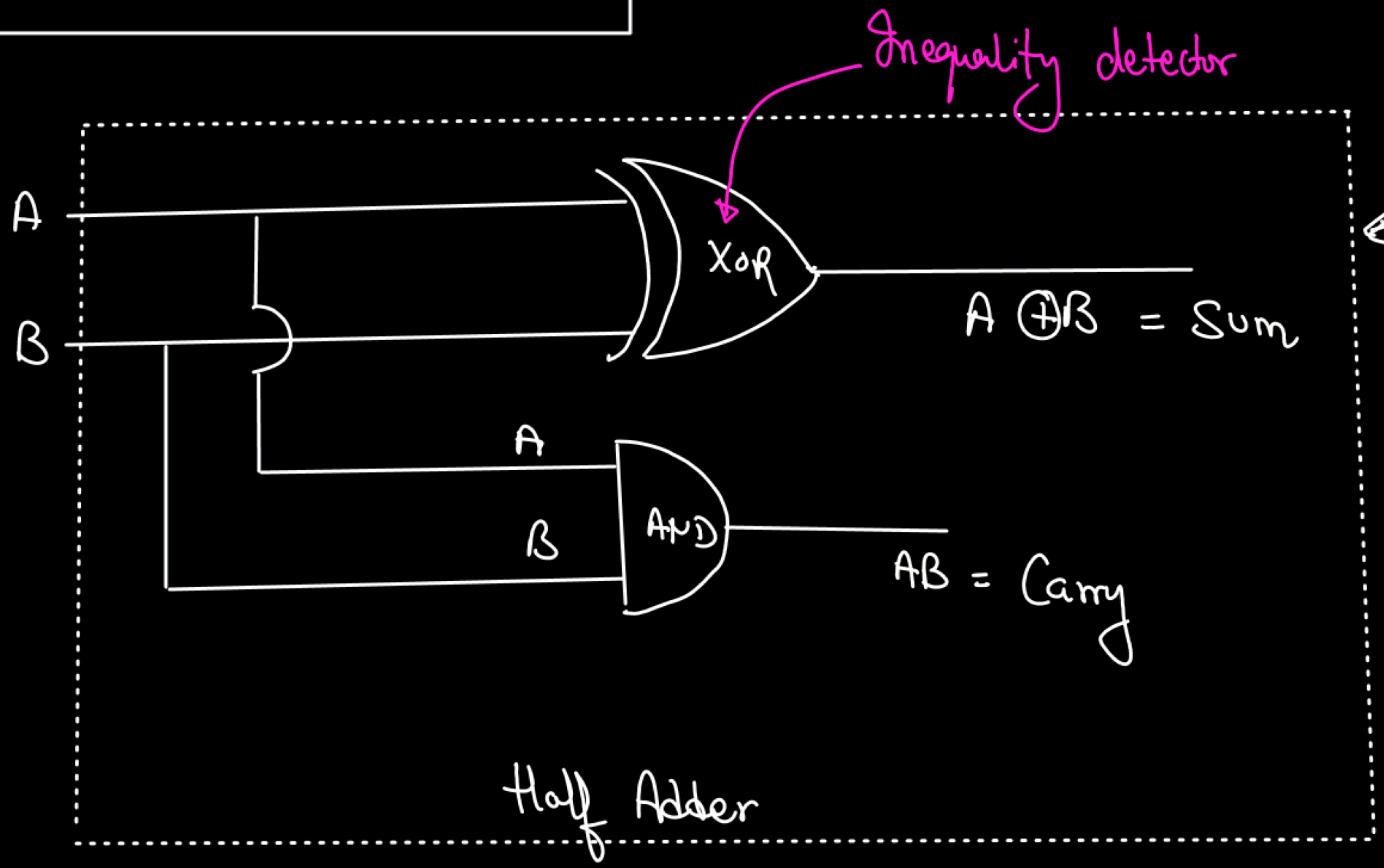
$$\text{Carry}_{out} = AB$$

AND

$$\text{Sum} = \bar{A}B + A\bar{B} = A \oplus B$$

$$\text{Carry} = AB$$

Task :→ Make Half Adder using NAND & NOR gate



two outputs

* Half adder requires 5 NAND OR 5 NOR gates

Half subtractor:

→ Is a Circuit that subtracts two bits

→ No. of I/P = 2 & O/P = 2 (4 terminals)

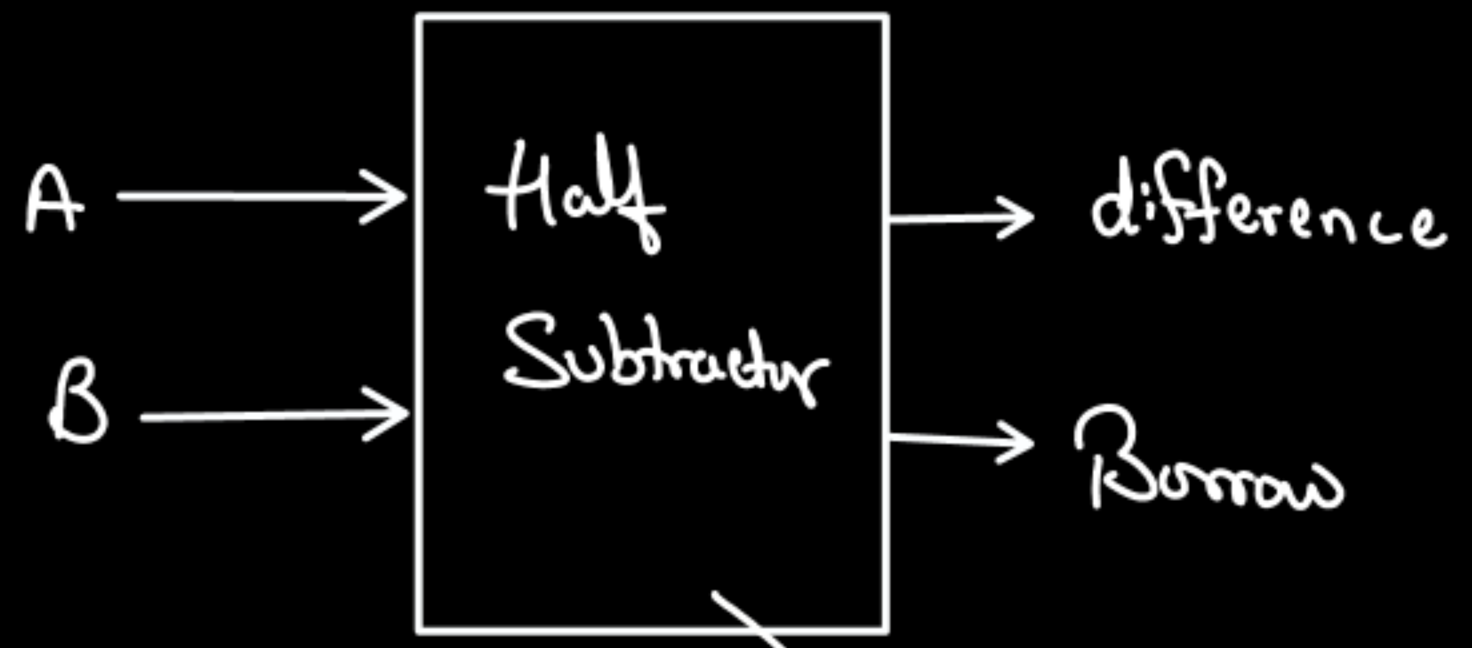
S.No	A	B	difference	Borrow
0	0	0	0	0
1	0	1	1 $\bar{A}B$	1 $\bar{A}B$
2	1	0	1 $A\bar{B}$	0
3	1	1	0	0

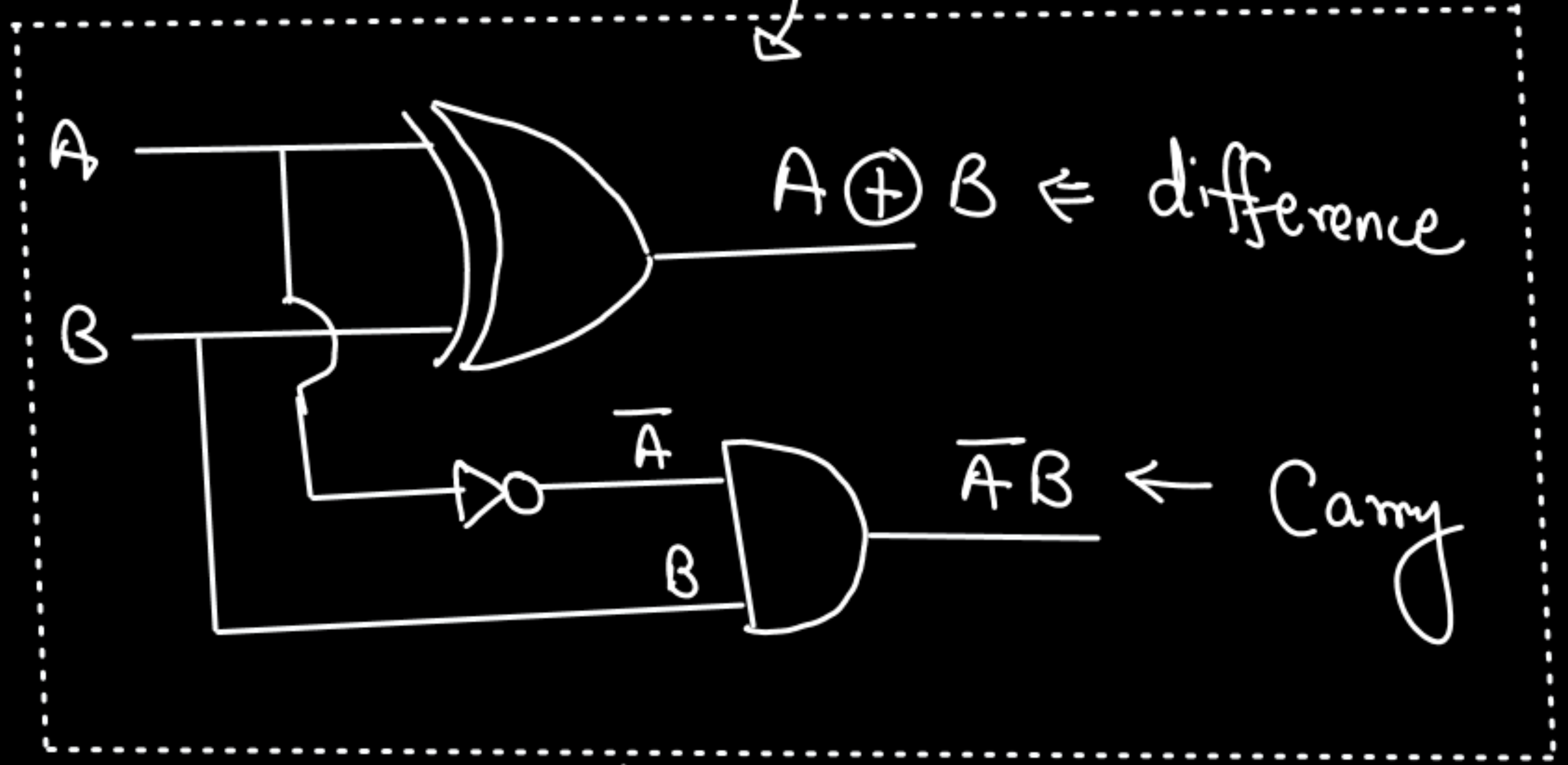
$$\boxed{\begin{aligned} \text{difference} &= A\bar{B} + \bar{A}B \\ &= A \oplus B \end{aligned}}$$

$$\text{Carry} = \bar{A}B$$

↓

AND



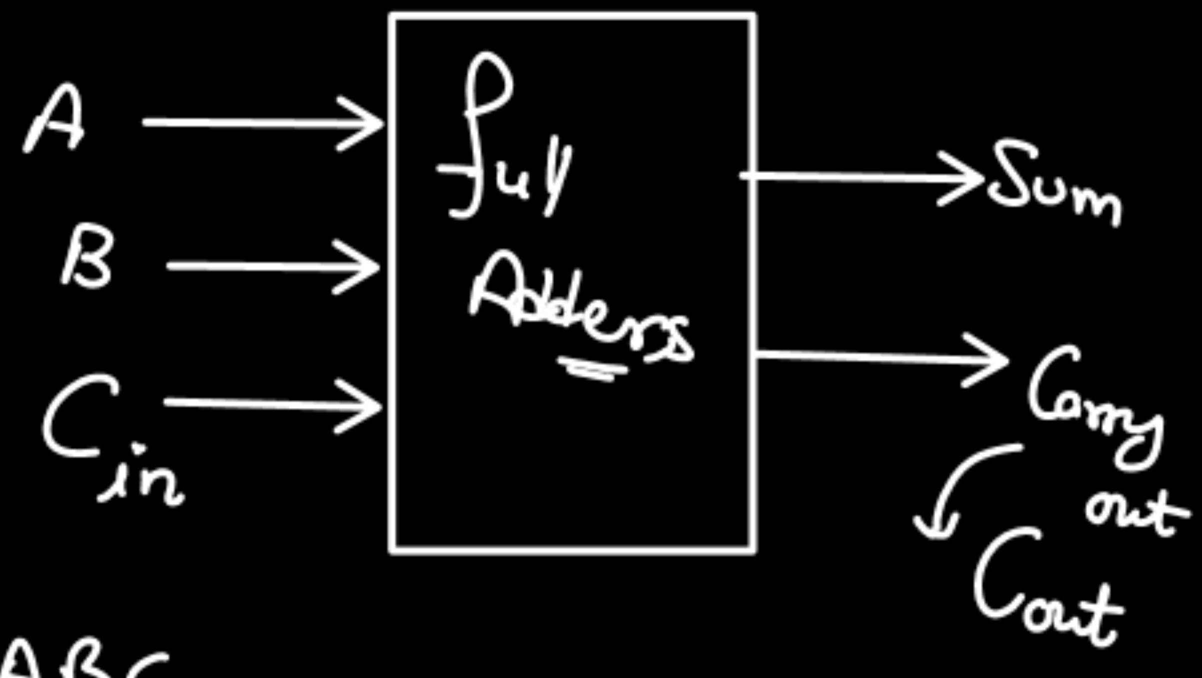


Half Subtractor

* Difference of half subtractor is same as sum in a half adder. but in Borrow, we has to take the complement of Minuend.

Full Adder Concept:

* It has three inputs & produce two outputs. first two I/Ps are A, B but the third input represented as C_{in}.



	A	B	C_{in}	Sum	C_{out}
0	0	0	0	0	0
1	0	0	1	1 $\bar{A}\bar{B}C$	0
2	0	1	0	1 $\bar{A}B\bar{C}$	0
3	0	1	1	0	1 → $\bar{A}BC$
4	1	0	0	1 $A\bar{B}\bar{C}$	0
5	1	0	1	0	1 → $A\bar{B}C$
6	1	1	0	0	1 → $AB\bar{C}$
7	1	1	1	1 ABC	1 → ABC

For Sum:

$$f = \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + ABC$$

$$\boxed{f(A,B,C_{in}) = \sum m(1,2,4,7)}$$

For Carry: $f = \bar{A}BC + A\bar{B}C + AB\bar{C} + ABC$

$$\boxed{f(A,B,C_{in}) = \sum m(3,5,6,7)}$$

for Sum :

$$f(A,B,C) = \sum m(1,2,4,7)$$

$A \backslash BC$	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
$\bar{A}$ 0	0	1 (1)	3	1 (2)
A 1	1 (4)	5	1 (7)	6

$\Rightarrow$ No grouping possible

→ XOR & XNOR

→ In this case, all minterms has odd no. of 1's as input by using, XOR property for multiple inputs,

$$\boxed{\begin{aligned} f(A,B,C) &= \sum m(1,2,4,7) \\ &= A \oplus B \oplus C_{in} \end{aligned}}$$

Pattern

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for Carry Out :

$$f(A,B,C) = \sum m(3,5,6,7)$$

$A \backslash BC$	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
$\bar{A}$ 0	0	1	1 (3)	2
A 1	4	1 (5)	1 (7)	1 (6)

AC BC AB

$$C_{out} = f = AB + BC + AC \quad \text{(Simplified) AND-OR}$$

$$f = \underbrace{\bar{A}BC + A\bar{B}C} + AB\bar{C} + ABC \quad \text{(Complex) (SoP)}$$

$$C(\bar{A}B + A\bar{B}) + AB\cancel{(C+\bar{C})}$$

$$\boxed{\Rightarrow AB + (A \oplus B)\,C_{in}}$$

Revision

Let $y = A \odot B \odot C$

↳ 3 input XNOR Gate is same as 3 input XOR operation

* for multiple inputs, XOR searches for odd number of 1's → $A \oplus B \oplus C$ if no of 1's are odd the o/p = 1

But XNOR gate searches for even number of 0's

* for odd number of Inputs

↳ odd number of 1's will be same as even no. of zeroes

$$\boxed{XNOR = XOR}$$

* for even number of Input,

$$\boxed{XNOR = \overline{XOR}}$$

for Sum

$$f(A,B,C) = \sum m(1,2,4,7)$$
$$= A \oplus B \oplus C_{in}$$

for Carry

$$\Rightarrow AB + (A \oplus B)\, C_{in}$$