Computer Arithmetic → Binary Arithmetic

(A) Binary Addition :

2 bits = 2^2 = 4 combinations

Rule :

A + B	Sum	Carry
0 + 0	0	0
0 + 1	1	0
1 + 0	1	0
1 + 1	0	1

10

Binary		Decimal
0 0	→	0
0 1	→	1
1 0	→	2
1 1	→	3

$$\begin{array}{r} 1 \\ +\ 1 \\ \hline 2 \end{array}$$

10

$$\begin{array}{r} 1 \\ +\ 1 \\ \hline 1\ 0 \end{array}$$

cy ← 1

a)
$$\begin{array}{r} {}^{1}\,{}^{1}\,{}^{1}\quad\quad\quad \\ 0\ 1\ 1\ 1\ 0\ 1 \\ +\ 1\ 1\ 0\ 1\ 1\ 0 \\ \hline Cy=1\ \ 0\ 1\ 0\ 0\ 1\ 1 \\ \hline \end{array}$$

$1+1=10$

$$\begin{array}{r} 1 \\ +\ 1 \\ +\ 1 \\ \hline (3)_{10} \rightarrow (1\ 1)_2 \end{array}$$

$$\begin{array}{r} 1 \\ +\ 1 \\ \hline 10 \end{array}$$

b)
$$\begin{array}{r} {}^{1}\quad\quad\quad\quad\quad \\ 1\ 1\ 0\ 0\ 1\ 0 \\ +\ 0\ 1\ 1\ 0\ 0\ 1 \\ \hline Cy=1\ \ 0\ 0\ 1\ 0\ 1\ 1 \\ \hline \end{array}$$

Operands

1 bit extra

Overflow

☆ If the result is too big that requires more number of bits than operands then the Condition is called overflow.

$x + y$, $x, y \leftarrow$ operands

operator

Ex

$$\begin{array}{r} {}^{1} \\ 24 \\ +\ 18 \\ \hline 42 \\ \hline \end{array} \longrightarrow \begin{array}{r} 1\ 1\ 0\ 0\ 0 \\ +\ 1\ 0\ 0\ 1\ 0 \\ \hline cy = 1\ 0\ 1\ 0\ 1\ 0 \\ \hline \end{array}$$

decimal

Binary

$(101010)_2$

	Q	R
$42/2$	21	0
$21/2$	10	1
$10/2$	5	0
$5/2$	2	1
$2/2$	1	0
$1/2$	0	1

32	16	8	4	2	1
1	0	1	0	1	0

$32 + 8 + 2 = (42)_{10}$

(B) Binary Subtraction

Case	Sub	Borrow
0 − 0	0	0
0 − 1	$\boxed{1}$	$\boxed{1}$
1 − 0	1	0
1 − 1	0	0

Borrow

$$\begin{array}{r} 10 \leftarrow 2_{10} \\ -\ 1 \leftarrow 1_{10} \\ \hline 1 \quad\quad 1_{10} \end{array}$$

Ex

$$\begin{array}{rrrrrrr} & & 0 & 10 & 1 & & \\ 0 & 0 & \cancel{1} & \cancel{1} & 0 & 1 & 0 \\ - & 0 & 0 & 1 & 1 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ \hline \end{array}$$

$$\begin{array}{ccccccc} 16 & 8 & 4 & 2 & 1 & & \\ 1 & 1 & 0 & 1 & 0 & = & 26 \\ & 1 & 1 & 0 & 0 & = & 12 \\ \hline & & & & & & 14 \end{array}$$

$$\begin{array}{cccc} 8 & 4 & 2 & 1 \\ 1 & 1 & 1 & 0 \end{array} = 8+4+2 = 14$$

Ex :

$B = 1$

$$
\begin{array}{r}
\cancel{1}0\ \cancel{1}0\ \cancel{1}\ 1\ 0\ 1\ 0\ 1 \\
-\ 1\ 0\ 1\ 1\ 1\ 0\ 1 \\
\hline
1\ 0\ 1\ 1\ 0\ 0\ 0
\end{array}
$$

$$\begin{array}{ccccccc} \cancel{0} & 1 & 1 & 0 & 1 & 0 & 1 \\ 32 & 16 & & 8 & 4 & 2 & 1 \end{array}$$

$= 32 + 16 + 4 + 1 = (53)_{10}$

$$\begin{array}{ccccccc} 1 & 0 & 1 & 1 & 1 & 0 & 1 \\ 64 & 32 & 16 & 8 & 4 & 2 & 1 \end{array}$$

$64 + 16 + 8 + 4 + 1 = (93)_{10}$

$53 - 93$

$= (-40)$

$$\begin{array}{ccccccc} 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 64 & 32 & 16 & 8 & 4 & 2 & 1 \end{array}$$

$64 + 16 + 8 + 1 = (89)_{10}$ ← गलत

$m_1 - m_2$, $\boxed{m_1 > m_2}$

(c) Subtraction using 1's Complement

make all zeroes = 1, & all 1's = zero.

Ex n = $(1010)_2$ → one's complement

max 4-digit number =

$$
\begin{array}{rcccc}
 & 1 & 1 & 1 & 1 \\
 & 1 & 0 & 1 & 0 \\
\hline
x = & 0 & 1 & 0 & 1 \\
\hline
\end{array}
$$

3cd1

In Binary, $1 \rightarrow 0$
$0 \rightarrow 1$

number

1's Complement

* n digit number

→ subract the number from max.

36 → 9's Complement

$$
\begin{array}{r}
99 \\
36 \\
\hline
63 \\
\hline
\end{array}
$$

Suppose $z = x - y$ ← subtrahend (s)

x ← minuend (m)

1) Take 1's Complement of Subtrahend (s) = $\bar{y}$

2) Add with minuend $\bar{y} + x$

↳ if the result has Carry bit = 1, then add to the LSB of the result

→ If the result has No Carry then take 1's Complement of the result which will be negative.

$z = x - y$

$\boxed{\bar{y} + x}$

↓

$r = \bar{y} + x$

↳ $C_y = 1$,

↳ Add to LSB

↳ $C_y = 0$

↳ $\bar{r}$ (-ve)

Ex Case 1: $10101 - \underbrace{00101}$

$z = x - y$ ← Subtrahend

minuend

$\bar{y} = \overline{00101} \longrightarrow 11010$

$$
\begin{array}{rrrrrrl}
r = & 1 & 0 & 1 & 0 & 1 & (x) \\
+ & 1 & 1 & 0 & 1 & 0 & (\bar{y}) \\
\hline
Cy=1 & 0 & 1 & 1 & 1 & 1 & \leftarrow \text{LSB}
\end{array}
$$

$$
\begin{array}{rrrrrrl}
 & 1 & 1 & 1 & 1 & & \\
\Rightarrow & 0 & 1 & 1 & 1 & 1 & \\
 & & & + & & 1 & \\
\hline
 & 1 & 0 & 0 & 0 & 0 & (+ve) \\
\hline
\end{array}
$$

$z = x - y$

$$\boxed{\bar{y} + x}$$

$r = \bar{y} + x$

- $Cy = 1$, → Add to LSB
- $Cy = 0$ → $\bar{r}$ (−ve)

Ex: $Z = 0\,1\,1\,1\,0\,1 - 1\,1\,0\,0\,1\,0$

$$Z = \underset{\text{(minuend)}}{x} - \underset{\text{(sub)}}{y}$$

$$\bar{y} = \overline{110010} = 001101$$

$$r = \bar{y} + x = \begin{array}{r} \scriptstyle 1\;1\;1\;\;\;1\;\; \\ 0\,0\,1\,1\,0\,1 \\ +\;0\,1\,1\,1\,0\,1 \\ \hline 1\,0\,1\,0\,1\,0 \end{array}$$

no carry $\Rightarrow$ $Cy = 0$

$$\bar{r} \Rightarrow \overline{101010} = -(010101)$$

$Z = x - y$

$\boxed{\bar{y} + x}$

$\downarrow$

$r = \bar{y} + x$

→ $Cy = 1$,
↳ Add to LSB

→ $Cy = 0$
↳ $-\bar{r}$ (−ve)

Subtraction Using 2's Complement

↳ 1's Complement + 1

$z = x - y$

x → minuend

y ← subtrahend

a) Calculate the 2's Complement of subtrahend

b) Add with minuend

↳ If we get the Carry bit then discard the Carry, & the resultant number is +ve

→ else take 2's Complement of the result.

a) $\bar{y} + 1$

b) $x + (\bar{y} + 1) = r$

- $cy = 1$ } discard Carry
  $r = +ve$
- $cy = 0$
  ↳ $(\bar{r} + 1)$
  ↓
  (−ve number)

Ex $\overset{x}{10101} - \overset{y}{00111}$

2's Complement of y = 1's comp + 1

$$\begin{array}{r} 11000 \\ +\quad 1 \\ \hline \bar{y}+1 = 11001 \\ \hline \end{array}$$

$r = x + (\bar{y}+1)$

$$\begin{array}{r} 10101 \\ 11001 \\ \hline \end{array}$$

$(Cy = 1)$ 01110

discard

result (+ve)

a) $\bar{y} + 1$

b) $x + (\bar{y}+1) = r$

- $Cy = 1$ } discard Carry
  r = +ve
- $Cy = 0$
  - $(\bar{r} + 1)$
    - (−ve number)

Ex $\overset{x}{10101} - \overset{y}{10111}$

$z = x - y$

2's Complement of $y \Rightarrow \bar{y} + 1$

$$\begin{array}{r} 01000 \\ + \quad 1 \\ \hline 01001 \end{array} = \bar{y} + 1$$

$r = x + (\bar{y} + 1)$

$$\begin{array}{r} 10101 \\ + 01001 \\ \hline 11110 \end{array}$$

Cy=0

2's Complement $(\bar{r} + 1)$

$$\begin{array}{r} 00001 \\ + \quad 1 \\ \hline 00010 \end{array}$$

← result (−ve number)

a) $\bar{y} + 1$

b) $x + (\bar{y} + 1) = x$

- Cy = 1 } discard Carry
  - r = +ve
- Cy = 0
  - $(\bar{r} + 1)$
  - (−ve number)