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ASSIGNMENT 6

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1\. MLE

Sample (x_1, x_2, x_n) from a normal population

Population parameters $\mu = \theta_1$ & $\sigma^2 = \theta_2$

$$L(\theta_1, \theta_2 | x) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\theta_2}} \exp\left(-\frac{(x_i - \theta_1)^2}{2\theta_2}\right)$$

Maximize w.r.t θ_1 & θ_2

$$\ln(L) = \frac{-n}{n} \ln(2\pi) - \frac{n}{2} \ln(\theta_2) - \frac{1}{2\theta_2} \Sigma_{i=1}^{n} (x_i - \theta_1)^2$$

mean

$$\frac{\partial \ln(L)}{\partial \theta_1} = \frac{1}{\theta_2} \Sigma_{i=1}^{n} (x_i - \theta_1)$$

$$\frac{1}{\theta_2} \Sigma_{i=1}^{n} (x_i - \theta_1) = 0$$

$$\Sigma_{i=1}^{n} x_i - n\theta_1 = 0$$

$$\theta_1 = \frac{1}{n} \Sigma_{i=1}^{n} x_i = \bar{x}$$

variance

$$\frac{\partial \ln(L)}{\partial \theta_2} = \frac{-n}{2\theta_2} + \frac{1}{2\theta_2^2} \Sigma_{i=1}^{n} (x_i - \theta_2)^2$$

$$\frac{-n}{2\theta_1^2} \Sigma_{i=1}^{n} (x_i - \theta_1)^2 = \frac{n}{2\theta_1^2}$$

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$$\Sigma_{i=1}^{n} (x_i - \bar{x})^2 = n\theta_2$$

$$s^2 = \frac{1}{n} \Sigma_{i=1}^{n} (x_i - \bar{x})^2$$

2. $$L(\theta|x) = \theta^{\Sigma_{i=1}^{n} x_i} (1-\theta)^{n \cdot m - \Sigma_{i=1}^{n} x_i}$$

$$\ln(L) = \Sigma_{i=1}^{n} x_i \ln(\theta) + (n \cdot m - \Sigma_{i=1}^{n} x_i) \ln(1-\theta)$$

$$\frac{\partial \ln(L)}{\partial \theta} = \frac{\Sigma_{i=1}^{n} x_i}{\theta} - \frac{n \cdot m - \Sigma_{i=1}^{n} x_i}{1-\theta}$$

solving for θ

$$\frac{\Sigma_{i=1}^{n} x_i}{\theta} = \frac{n \cdot m - \Sigma_{i=1}^{n} x_i}{1-\theta}$$

$$\theta = \frac{\Sigma_{i=1}^{n} x_i}{n \cdot m}$$