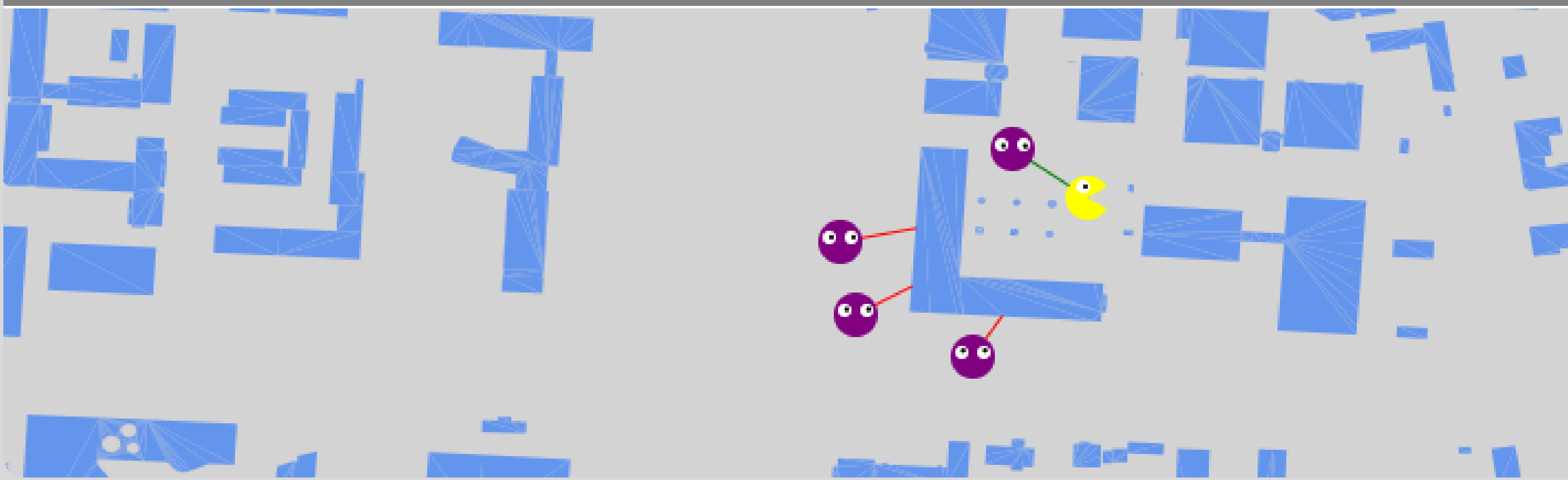
Pacman – Ghosts activate on sight

Mao L., Wang C., Sebastian G.



Problem specification

- **Given**
  - A graph $G = (V, E)$ describing a landscape with obstacles modeled through
  - simple polygons with n vertices in total
  - A straight-line drawing of the landscape $\Gamma: V \rightarrow R^2$
  - The position $P \in R^2$ (Pacman)
  - The positions $(Q_i \in R^2)_{i \in \{1,\ldots,m\}}$ of m ghosts
- **Problem**
  - Find an efficient way to determine if P is *visible* from Q_i for each $i \in \{1, \ldots, m\}$
- For positions $A, B \in R^2$, A is *visible* from B iff the segment $\overline{AB}$ has no intersection with any polygon in Γ

Our approach

- OpenStreetMap data for realistic input geometry
- Java
- Outline:
  1. Parse an OSM file for polygons of buildings
  2. Triangulate the input polygons
  3. Build a *kd*-tree on the triangle soup
  4. Perform the m visibility checks with the help of the *kd*-tree in $O(m \log n)$ time

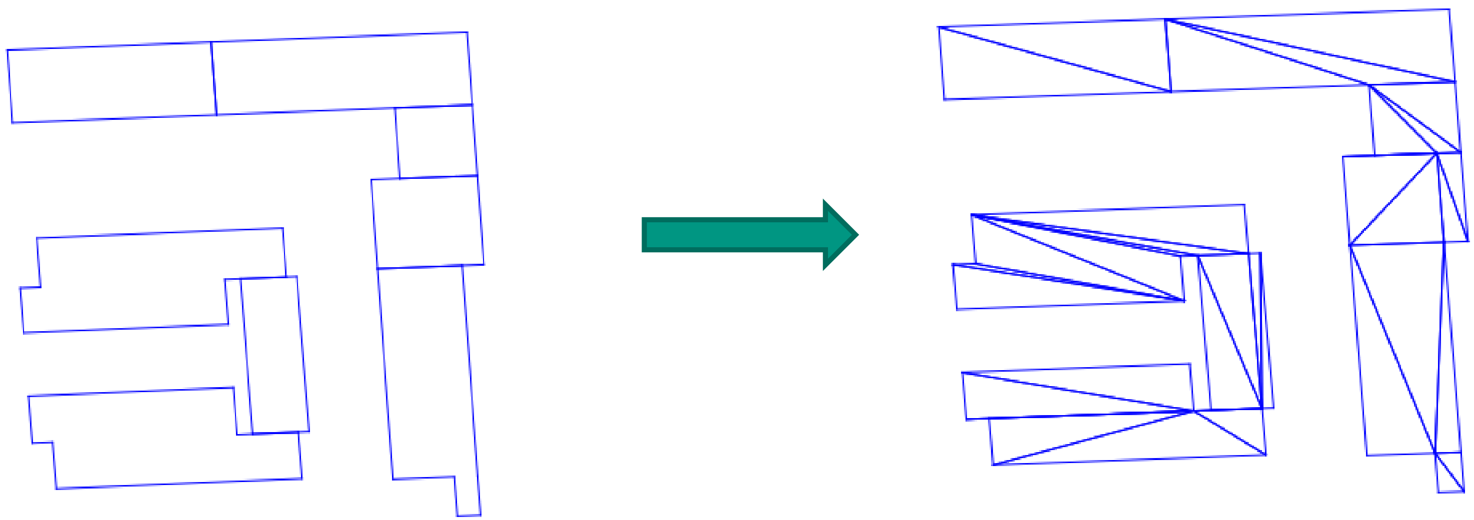
DEMO

Polygon Triangulation

EAR CLIPPING

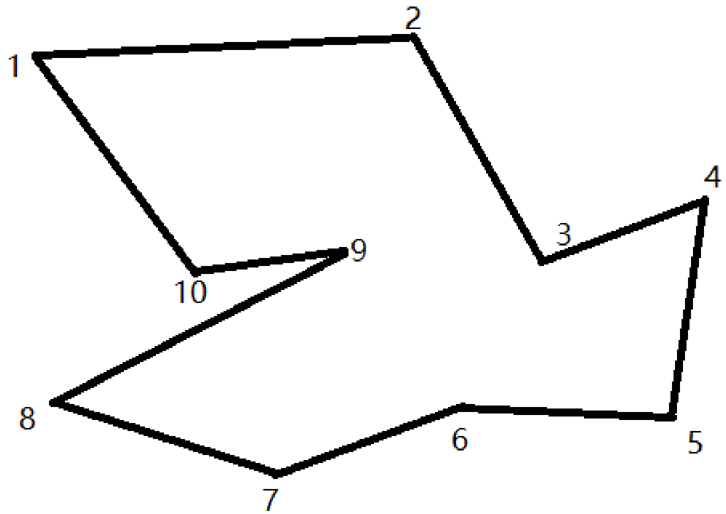
Task

- Decompose polygons into a collection of triangle

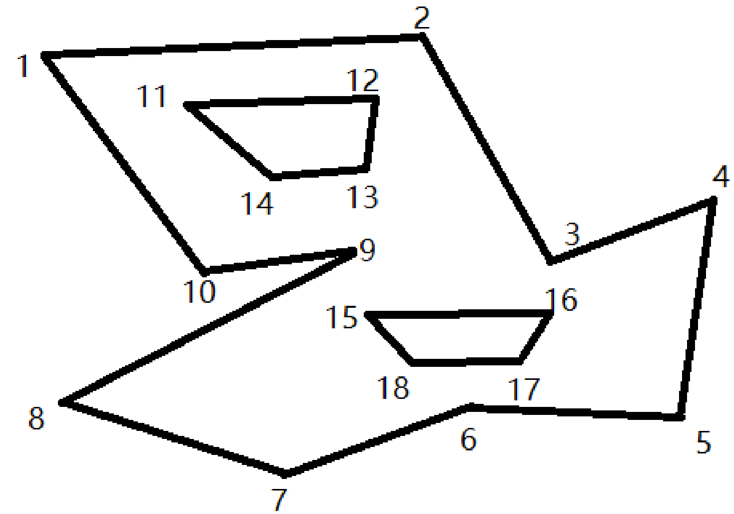


untriangulated triangulated

Types of polygons



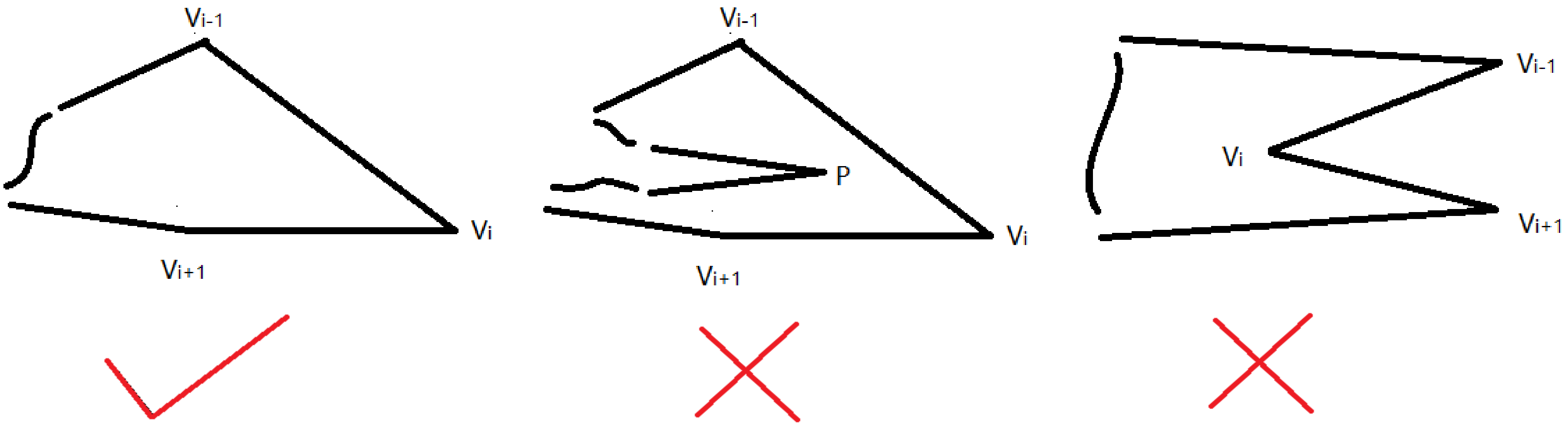
Simple polygon



Polygon with holes

Ear Clipping for the simple polygon

Is V_i an Ear in Polygon?

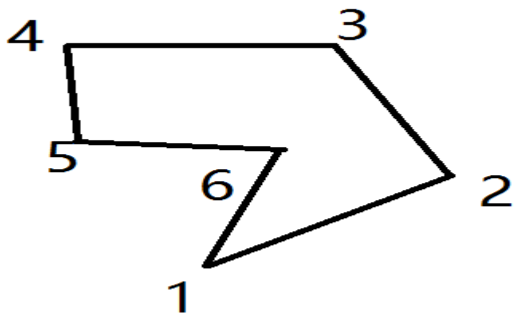


Theorem :

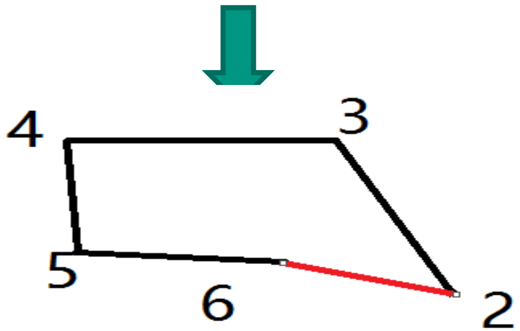
- Any triangulation of n vertices polygon has n−2 triangles
- A polygon has at least two nonoverlapping ears

Ear Clipping Example

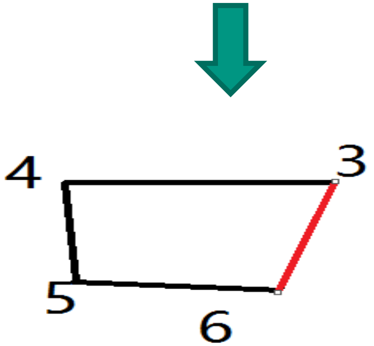
Karlsruhe Institute of Technology



The initial list of ears is E = { 1, 3, 4, 5 }

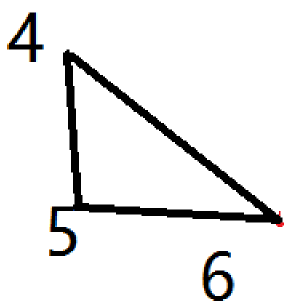


The ear at vertex 1 is removed, add vertex 2,
E = { 2, 3, 4，5 }

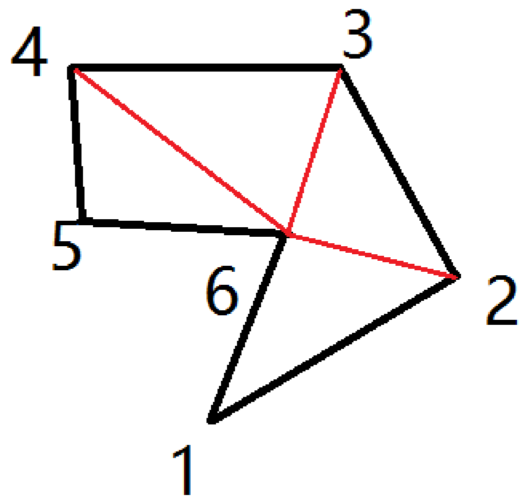


Removed vertex 2, E = { 3, 4, 5, 6 }

Ear Clipping Example



Removed vertex 3, the number of vertices less than four, then end



The full triangulation of the original polygon

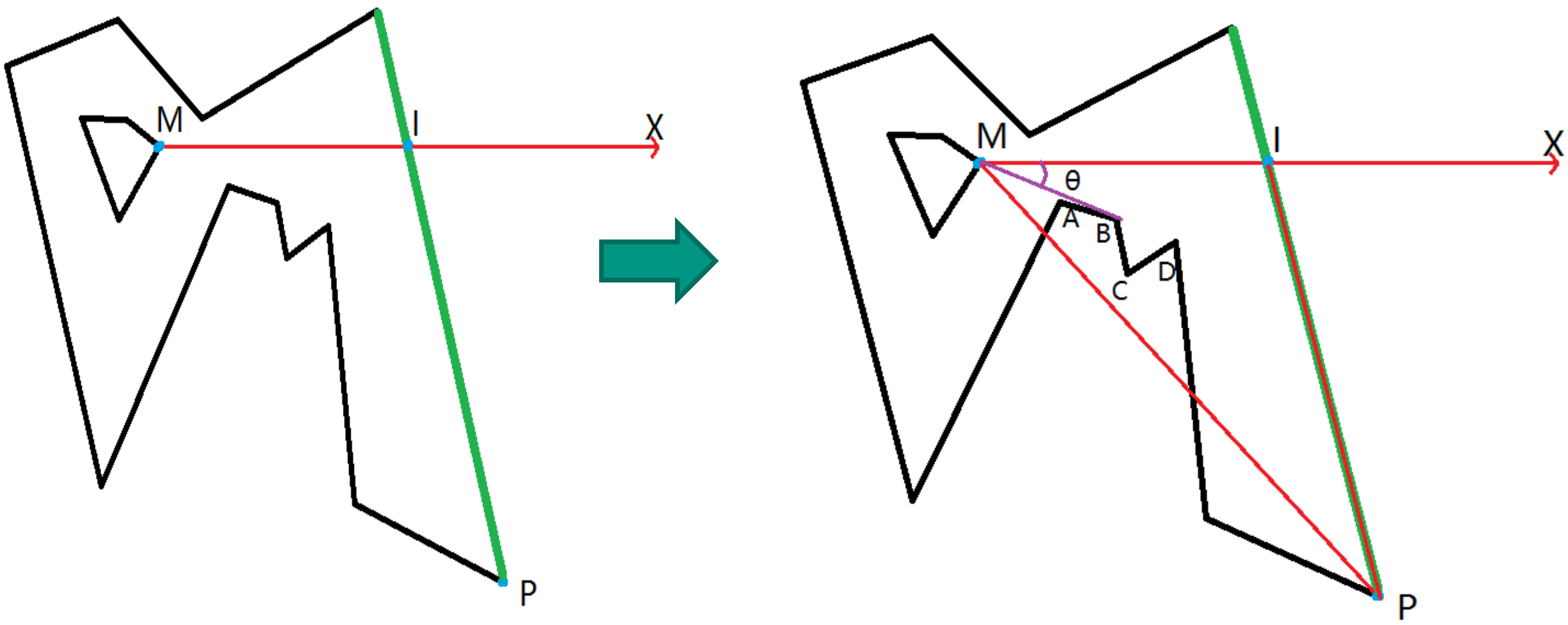
Time Complexity

- There are O(n) ears. Each update of an adjacent vertex involves an earnesstest, a process that is O(n) per update. Thus, the total removal process is O(n*n).

Polygon with holes

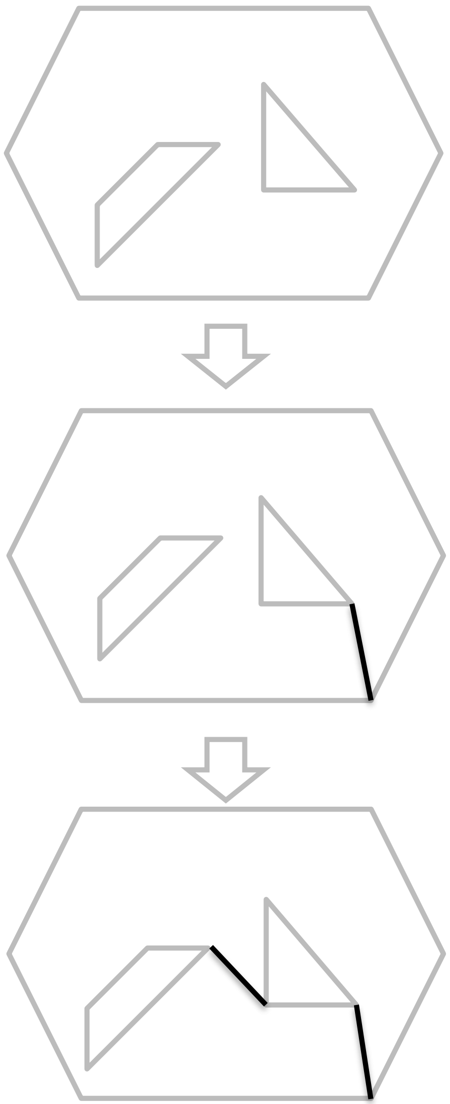
- Idea

Connect two vertex, one vertex from the outer polygon   and one vertex from the inner polygon



The Vertex M and B are what we want to connect

Polygons with Multiple Holes

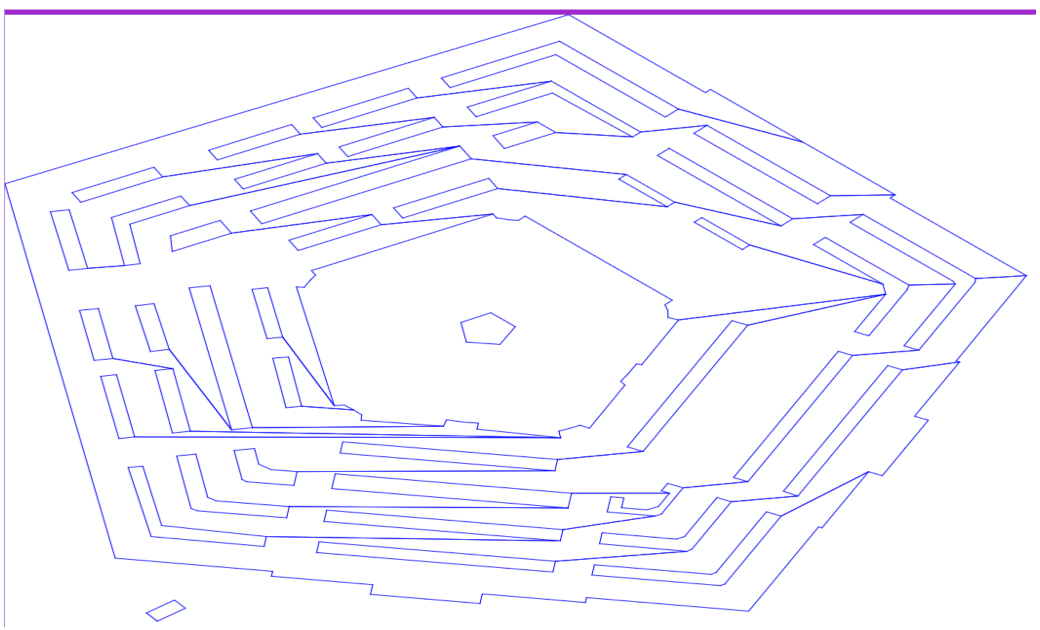


Algorithm

- Find the inner polygon with the vertex of maximum x-value
- Use the previously mentioned algorithm to combine the outer polygon and the select inner polygon
- Repeated with the new outer polygon and the remaining inner polygons

Time Complexity

- There are O(m) inner polygons. Each connection building needs O(n). Thus, the total process is O(m*n).

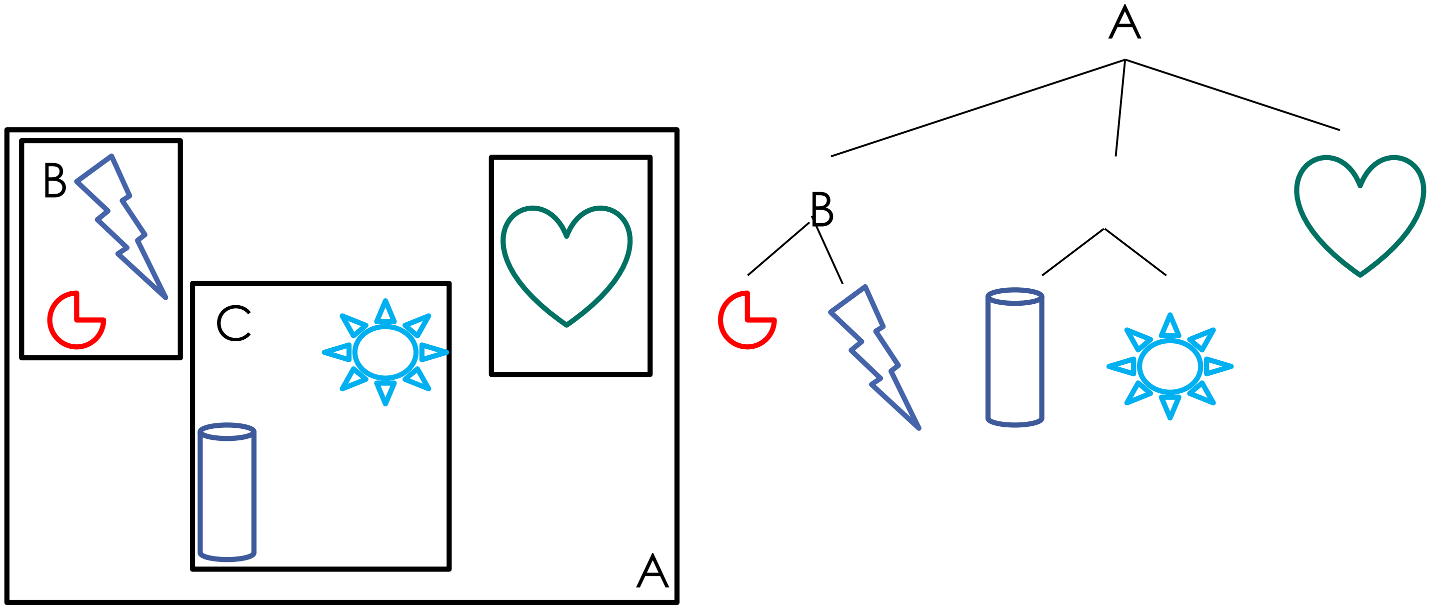


Example polygon with holes to simple polygon

Bounding Volume Hierarchy

BVH

BVH Structure



BVH Introduction

- Tree structure on a set of geometric objects

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- Leaf nodes of the tree:

  Geometric objects (wrapped in bound volumes)

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- Tree structure on a set of geometric objects
- Leaf nodes of the tree:

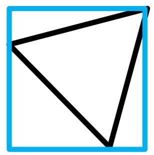
  Geometric objects (wrapped in bound volumes)
- Child Nodes → Small sets

  ➔ Father node (with larger bounding volumes)

BVH in the project

Leaves of the tree

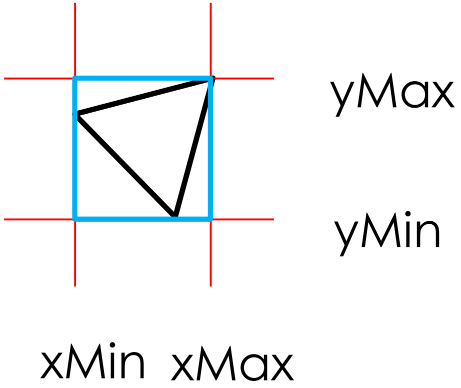
- Triangles (triangulated form the polygons)



BVH in the project

Leafs of the tree

- Triangles (triangulated form the polygons)



BVH in the project

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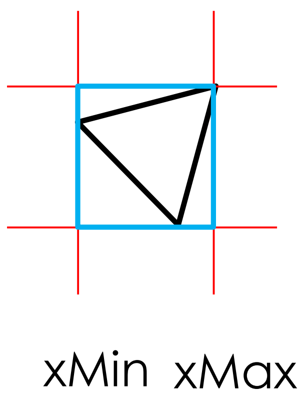
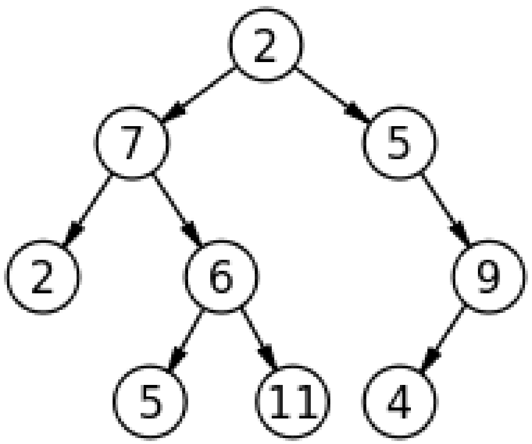


Structure of the tree

- Binary tree



BVH build up

Method :

- Find the nearest bounding volumes
- Combination → new BVH node generated
- ……
- When only one activated node exists

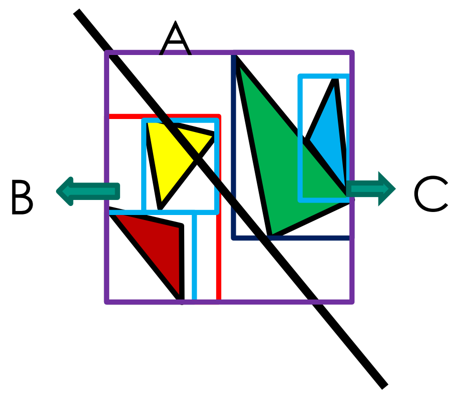
→BVH completed

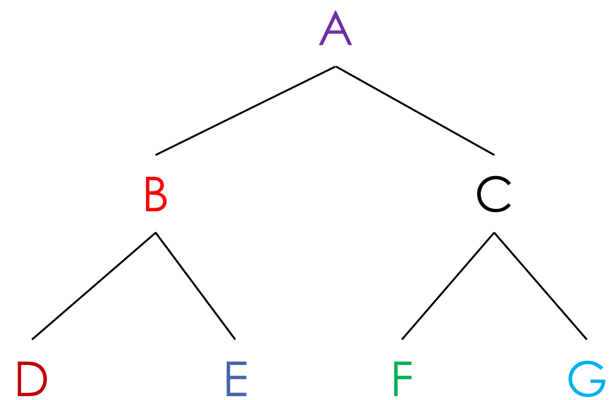
BVH search

Method :

- When node A need to be searched:
  - Segment intersects A’s bounding volume?
    - Yes
      - A is a leaf node ➔ report A
      - A is not a leaf node ➔ search A’s children

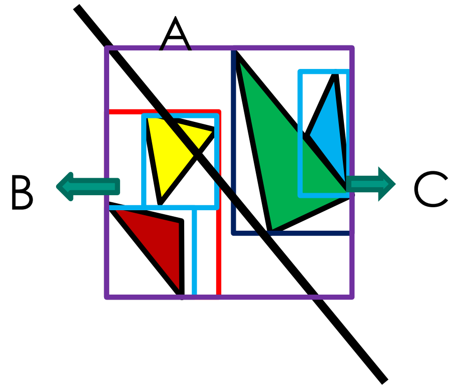
BVH search example

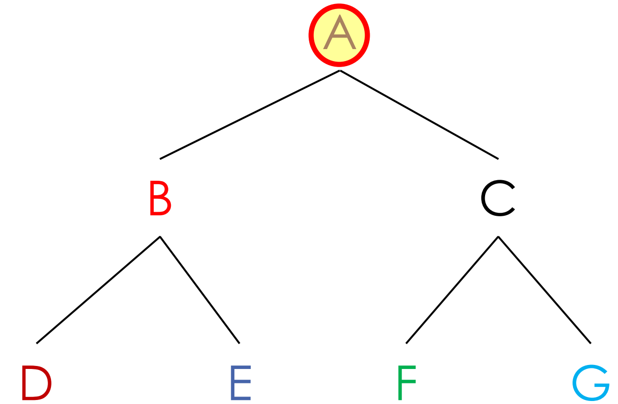




Intersect ?
Leaf node?

BVH search example





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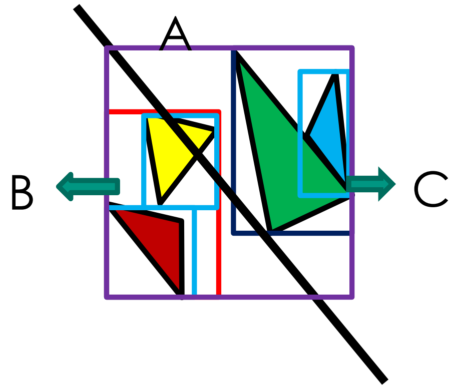

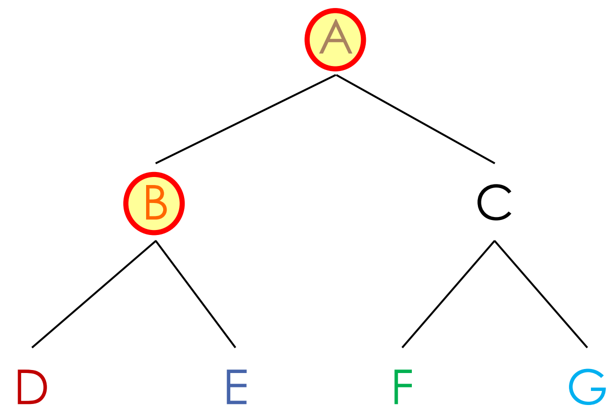

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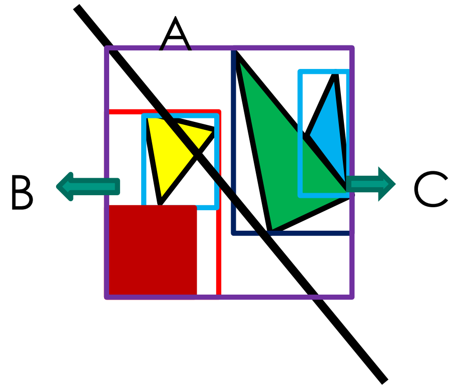

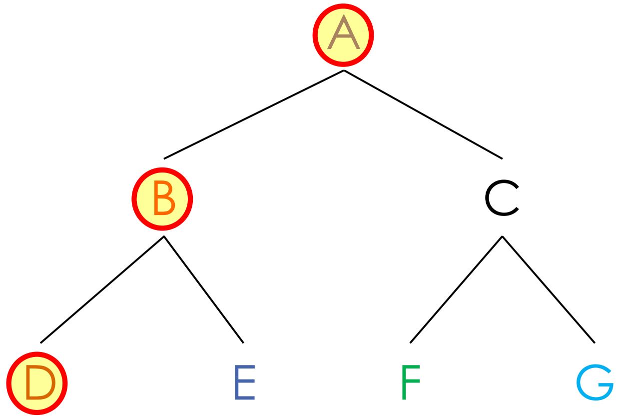

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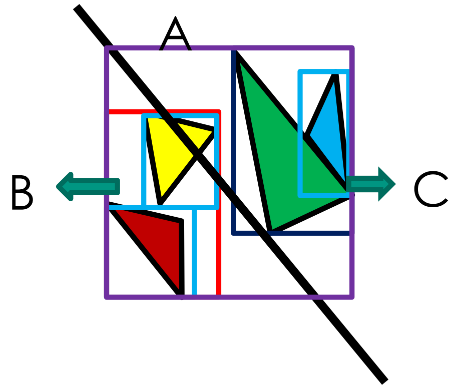

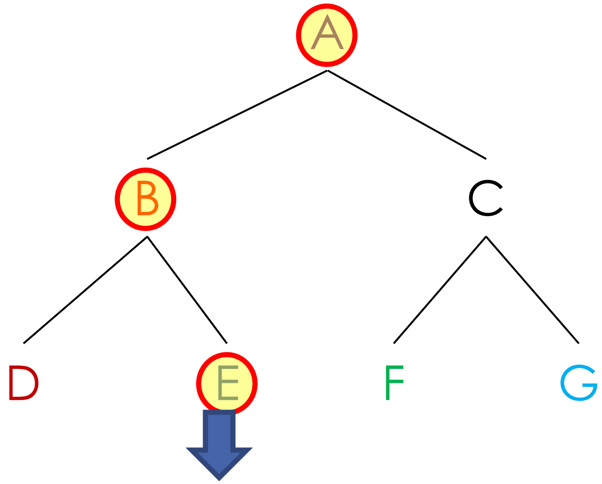

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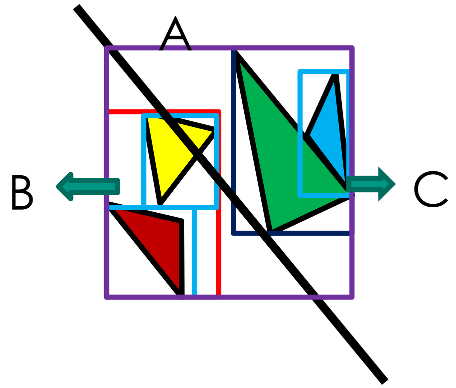

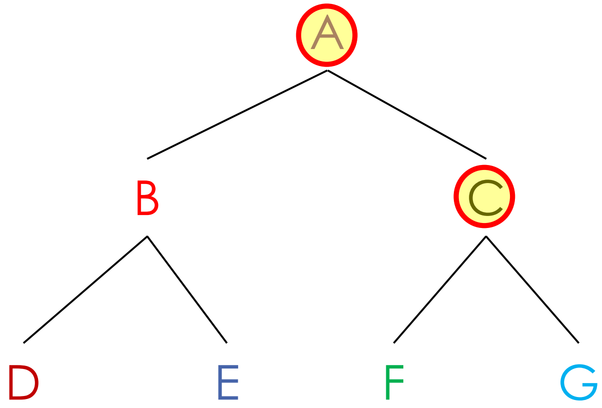

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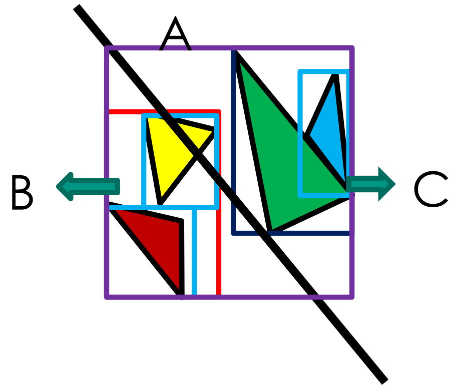

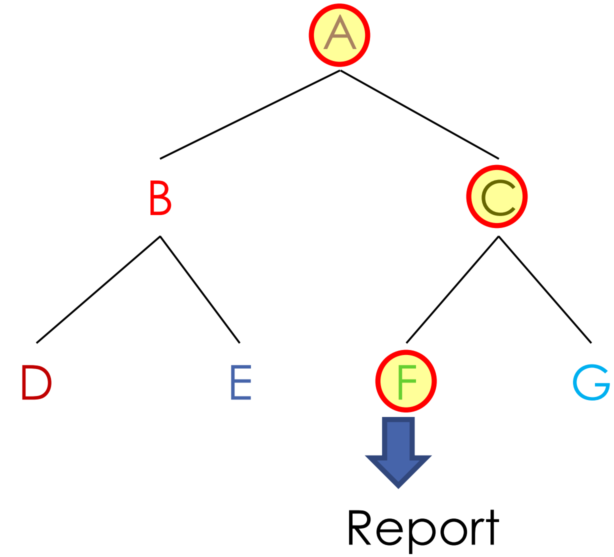

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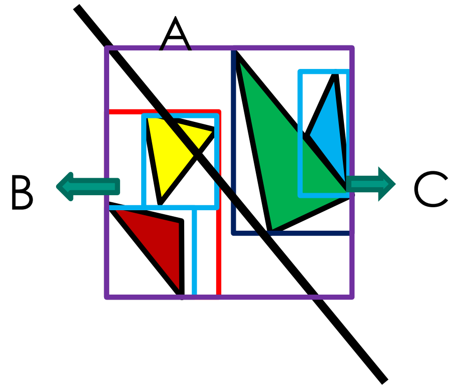

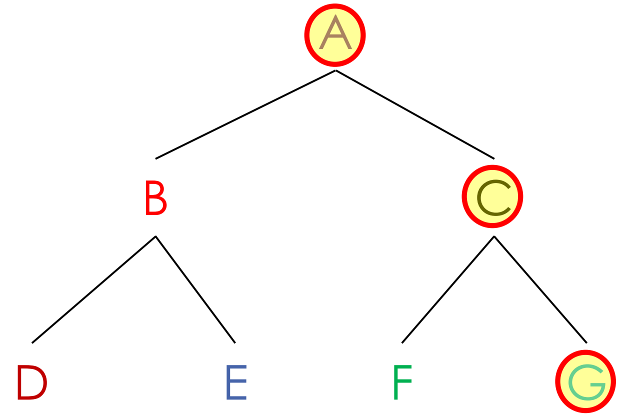

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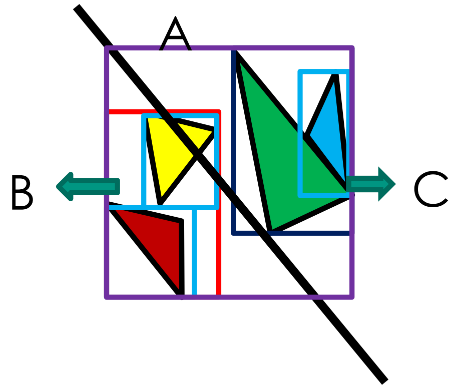
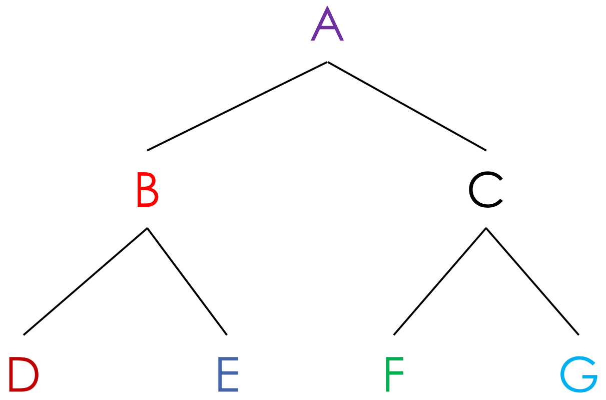




Result : E , F
reported

BVH Summary

Advantage

- Simple structure
- Allows overlapped triangles
- Fast binary search

Disadvantage

- Unexpected output (sometimes)

  <reason : segment intersects the bounding volumes but not intersects the triangles>

- Long construction time ($O(n^3)$)

  <reason : find the nearest bounding volumes costs to much time $O(n^2)$>

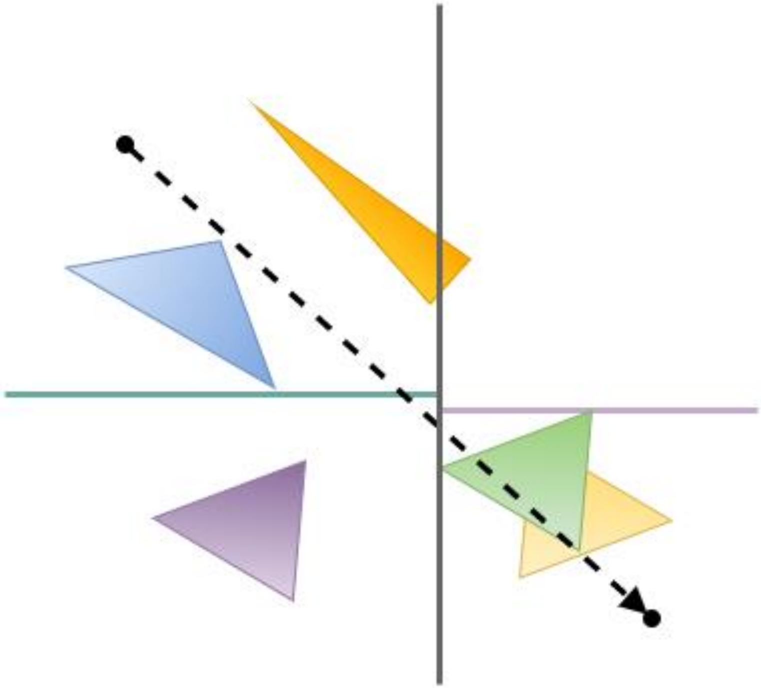
***kd*-tree**

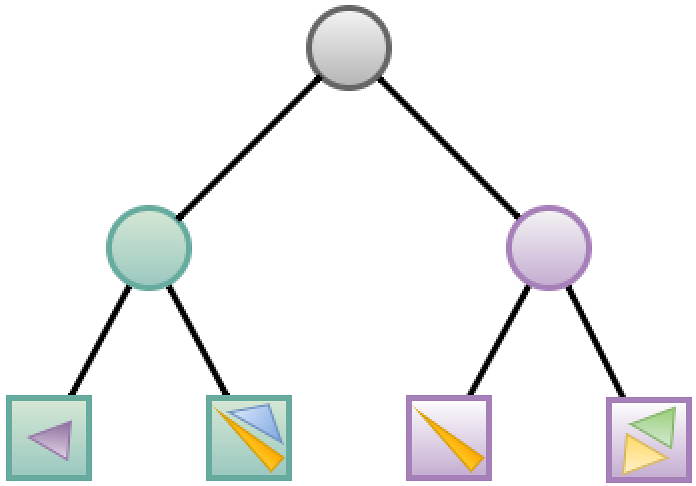
INTERSECTION TEST

Intersection test

- **Given**
  - The line-of-sight segment $\overline{AB}$ to check for intersection with the triangles
  - The currently visited node n in the *kd*-tree
- If n is not a leaf
  1. Split $\overline{AB}$ into $\overline{AC}$ and $\overline{CB}$, where C is the intersection of the line through AB with the splitting plane of n
  2. Perform the intersection test with the child of n containing A and $\overline{AC}$ as line-of-sight segment
  3. Perform the intersection test with the child of n containing B and $\overline{CB}$ as line-of-sight segment
- If n is a leaf
  1. Return the *nearest* intersection of $\overline{AB}$ with any of the associated triangles of n, if any

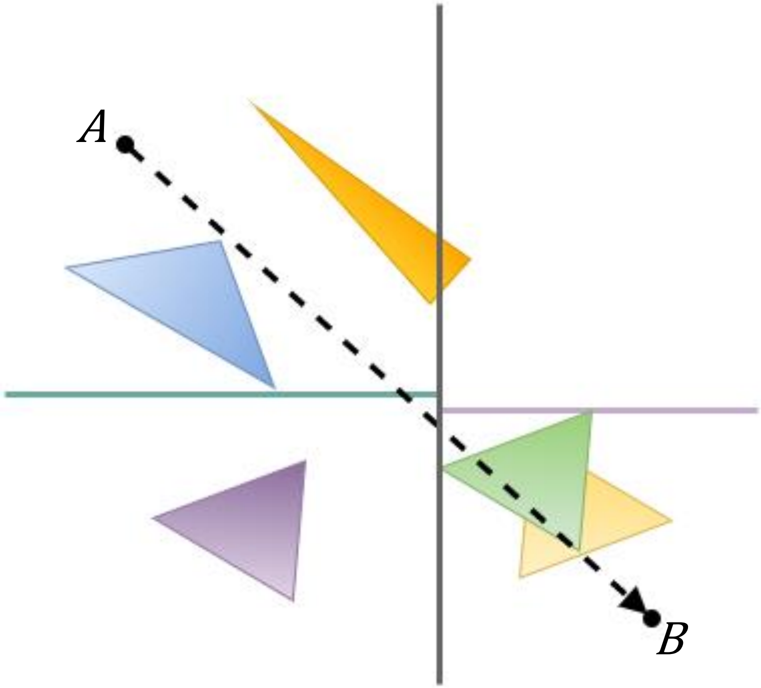
Intersection test - example

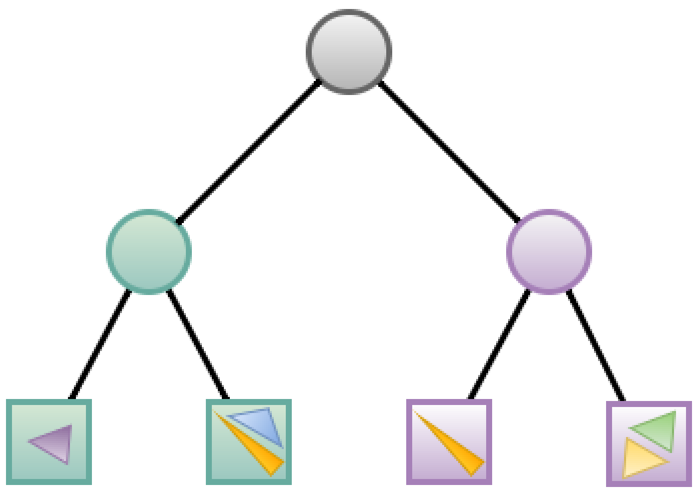




Intersection test - example

- Split $\overline{AB}$ into $\overline{AC}$ and $\overline{CB}$, where C is the intersection of the line through $\overline{AB}$ with the splitting plane of n





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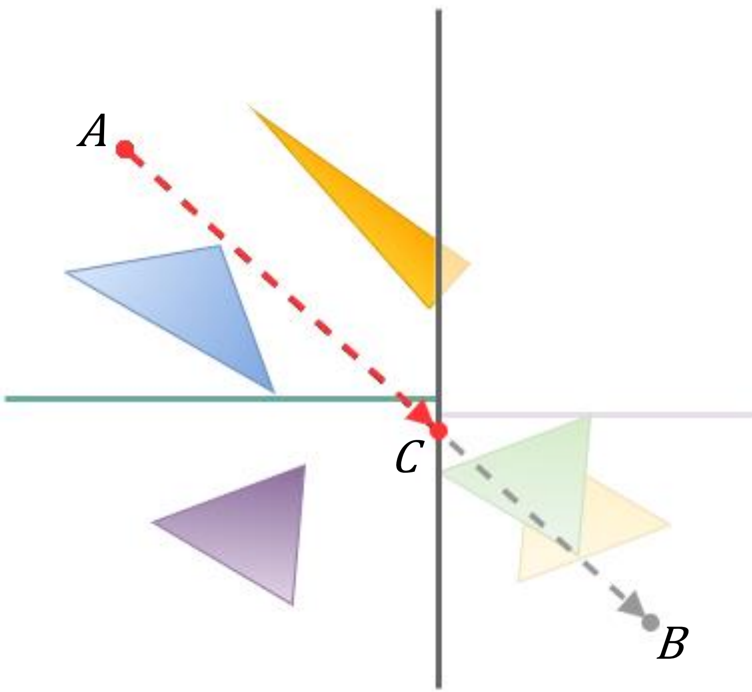
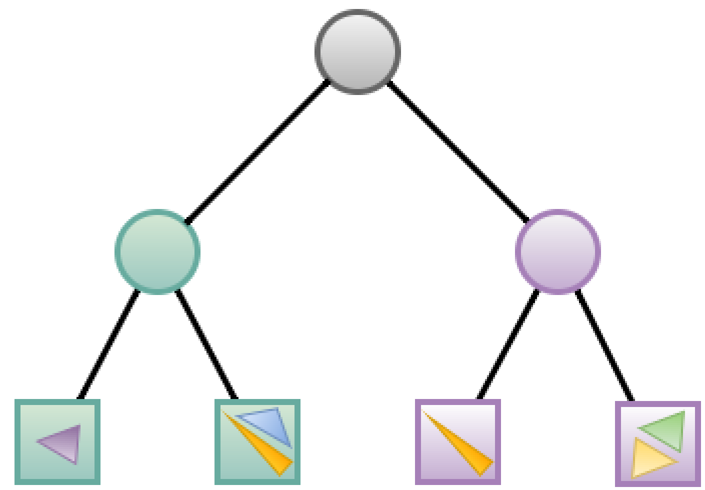
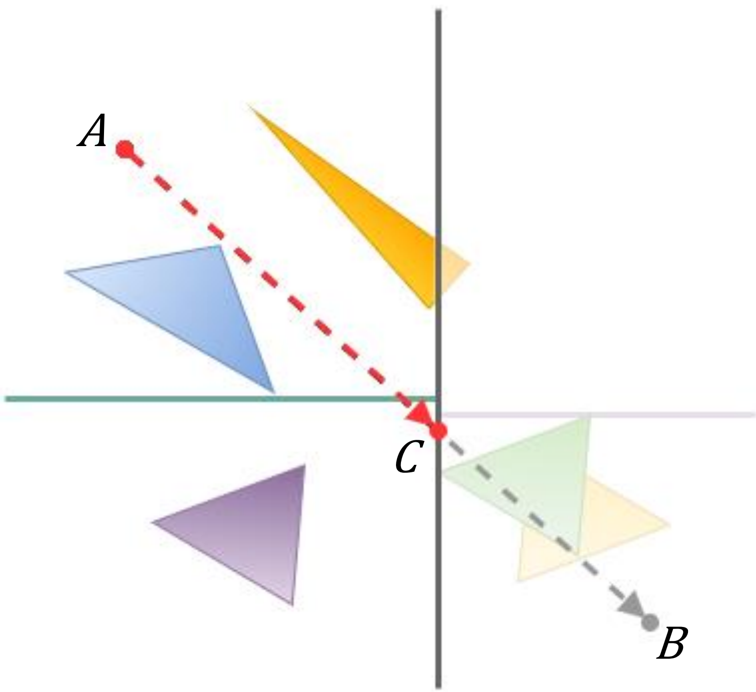
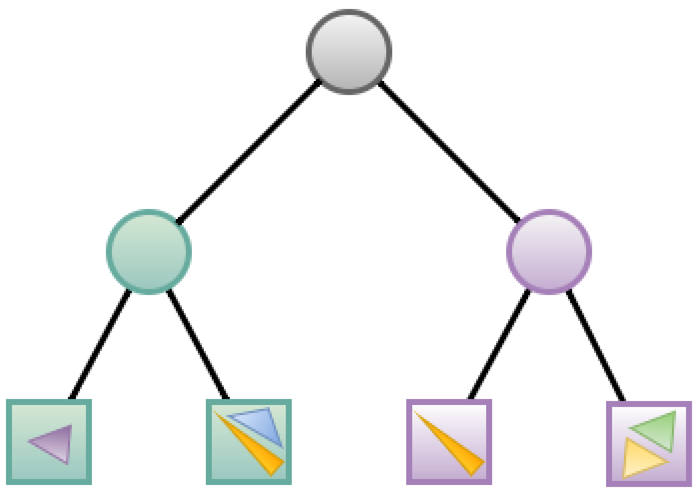




Intersection test - example

- Perform the intersection test with the child of n containing A and $\overline{AC}$ as line-of-sight segment





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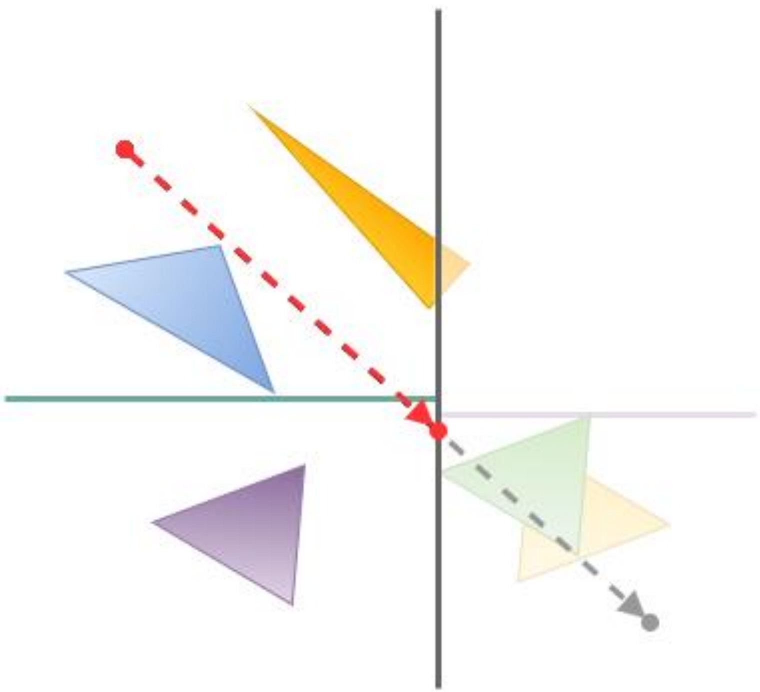

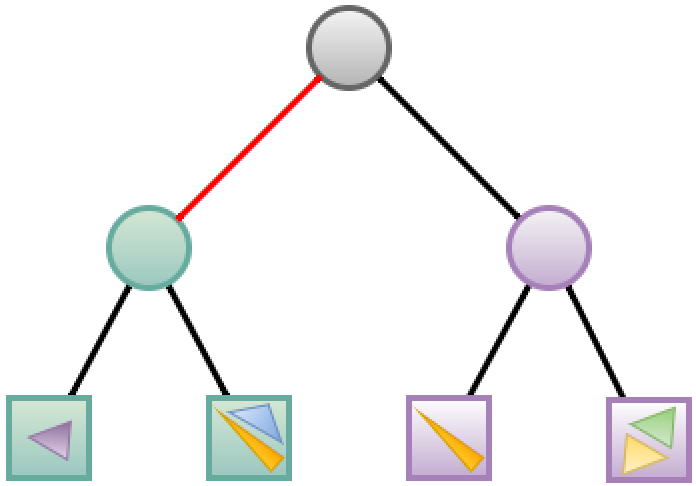

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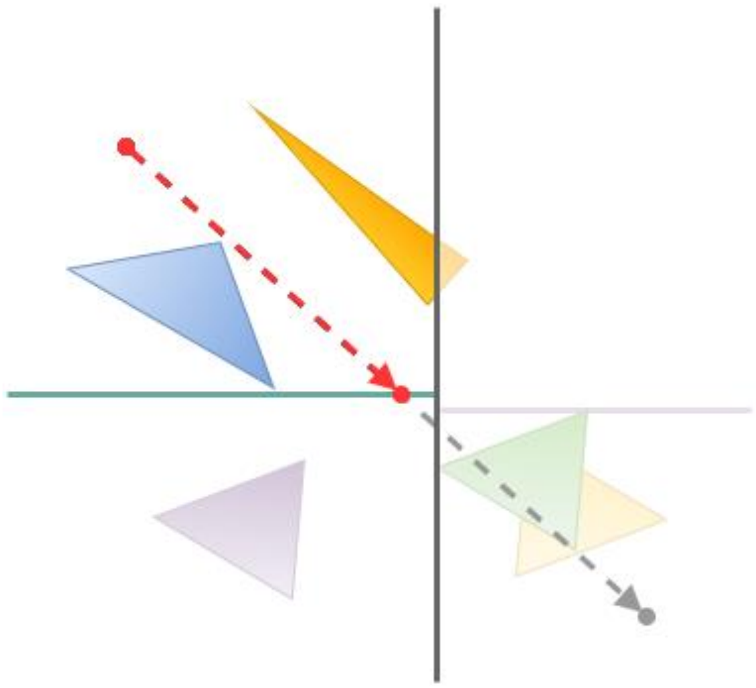
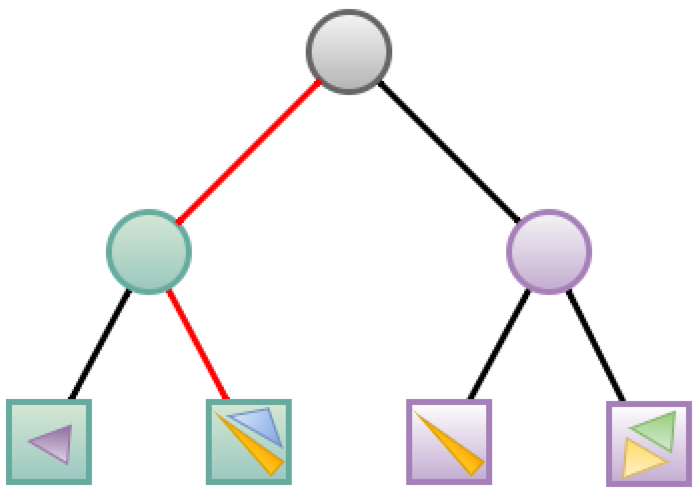
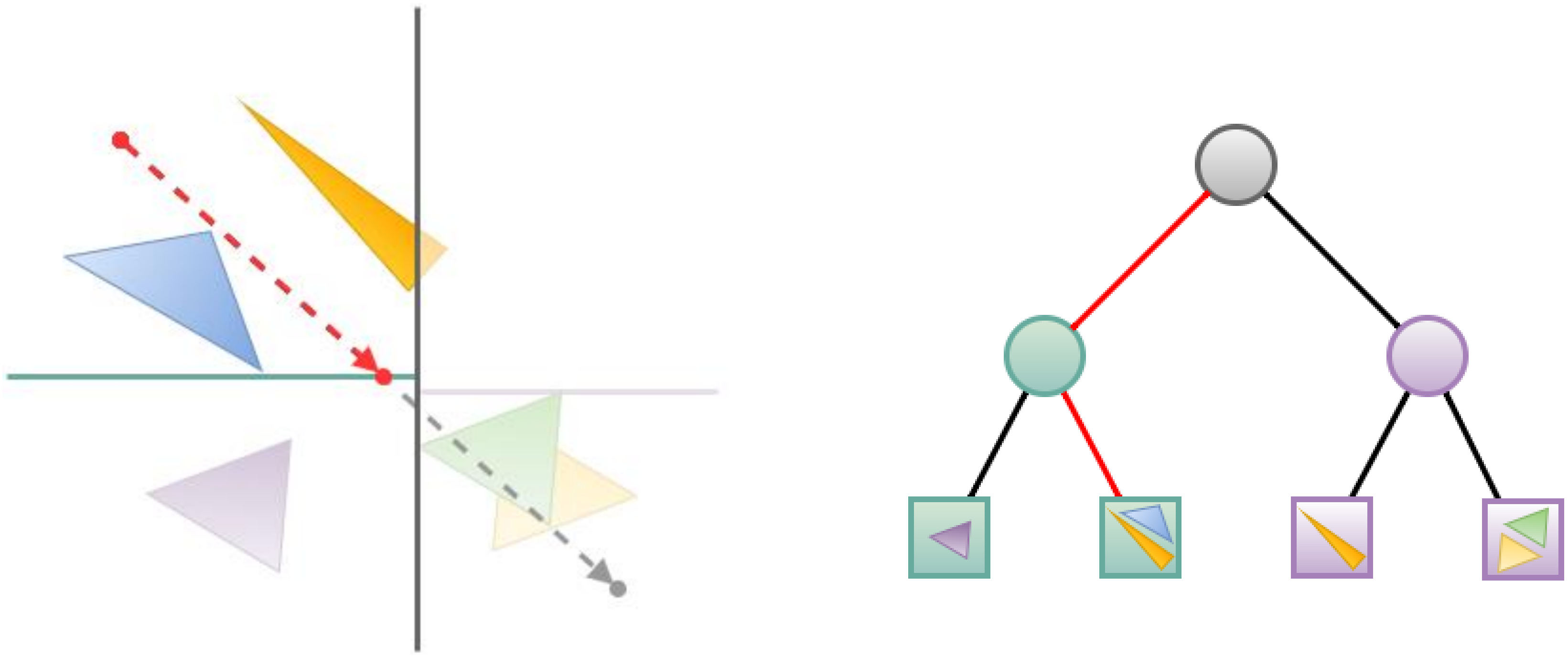




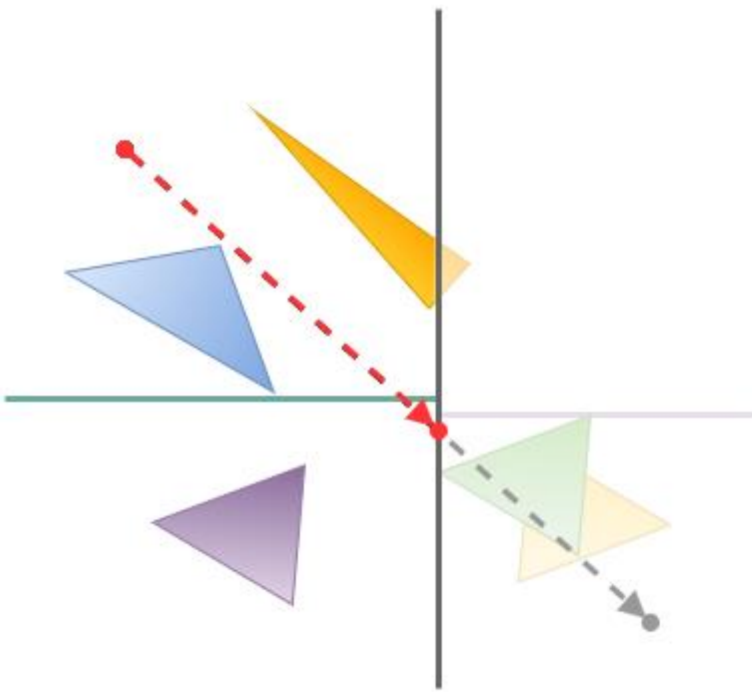
Intersection test - example

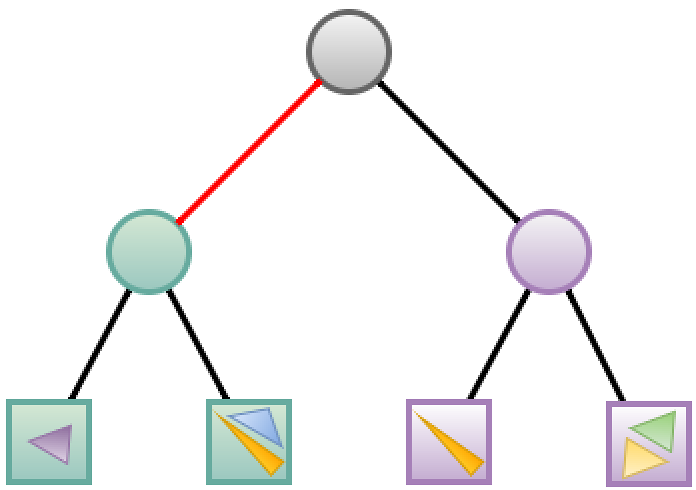
- Return the *nearest* intersection of $\overline{AB}$ with any of the associated triangles of n, if any



Intersection test - example

- Perform the intersection test with the child of n containing B and $\overline{CB}$ as line-of-sight segment





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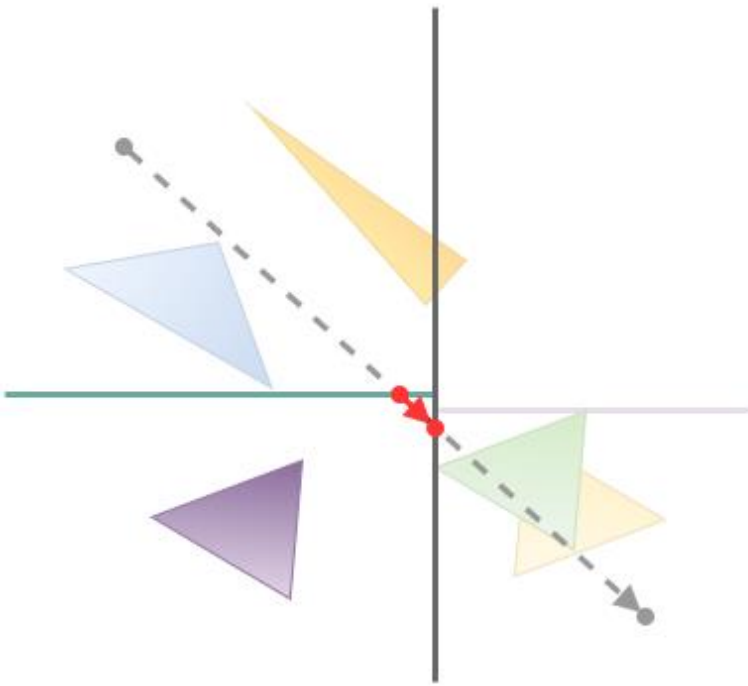

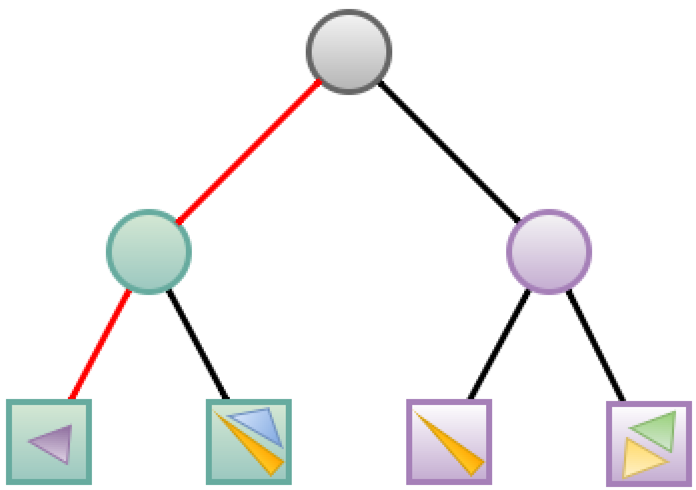

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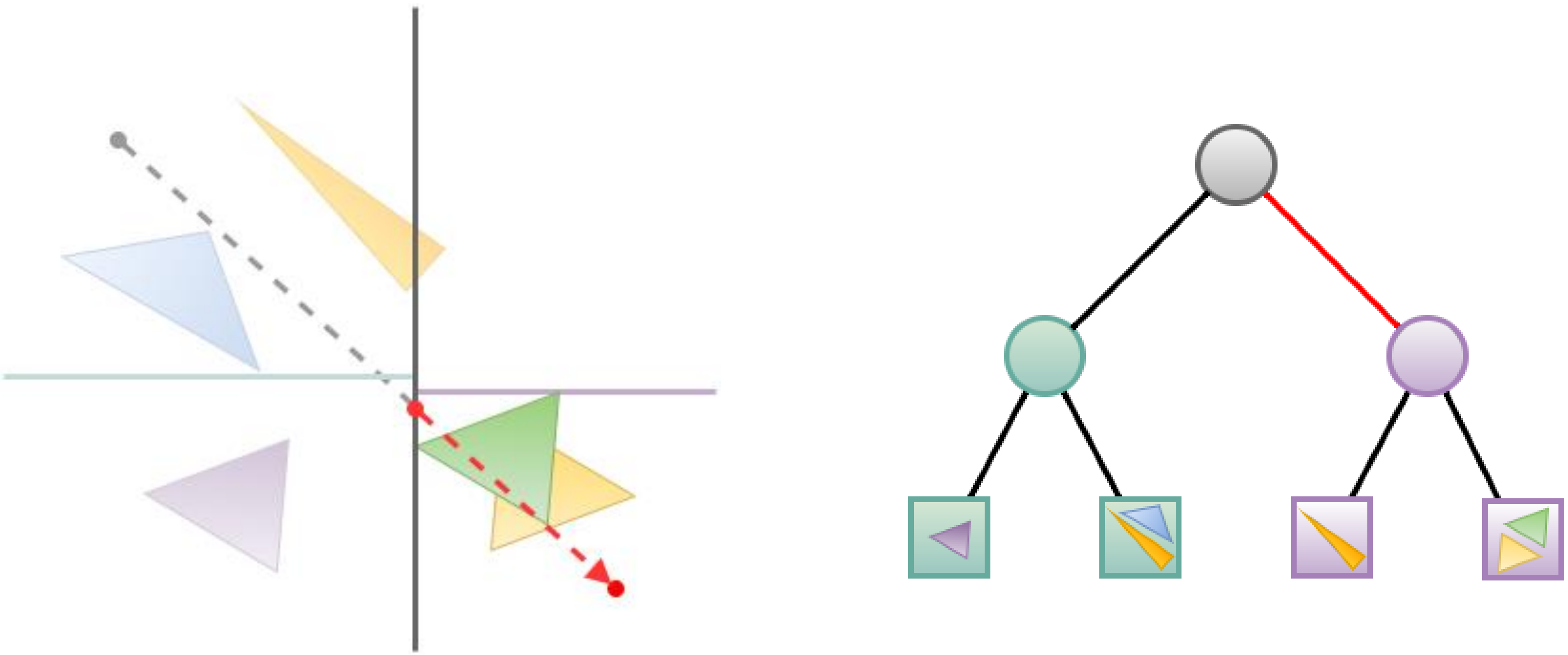

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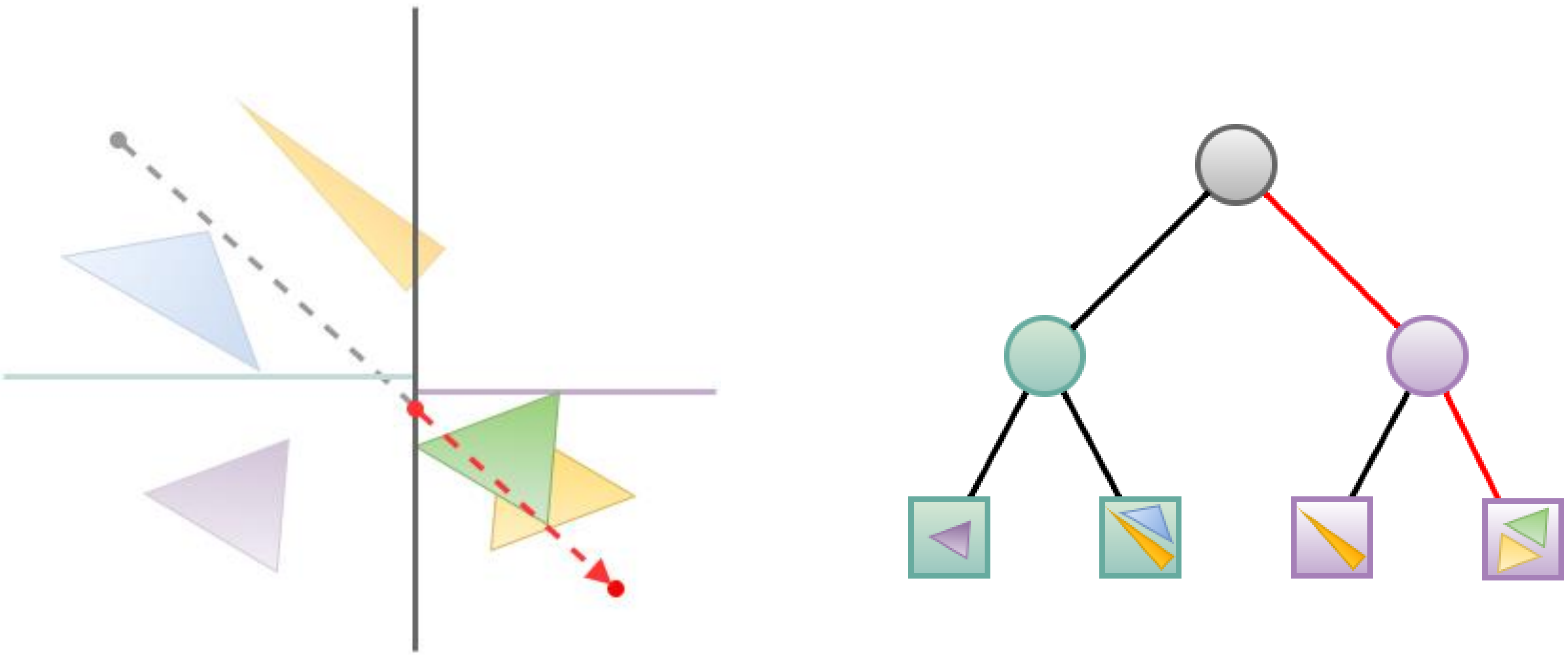

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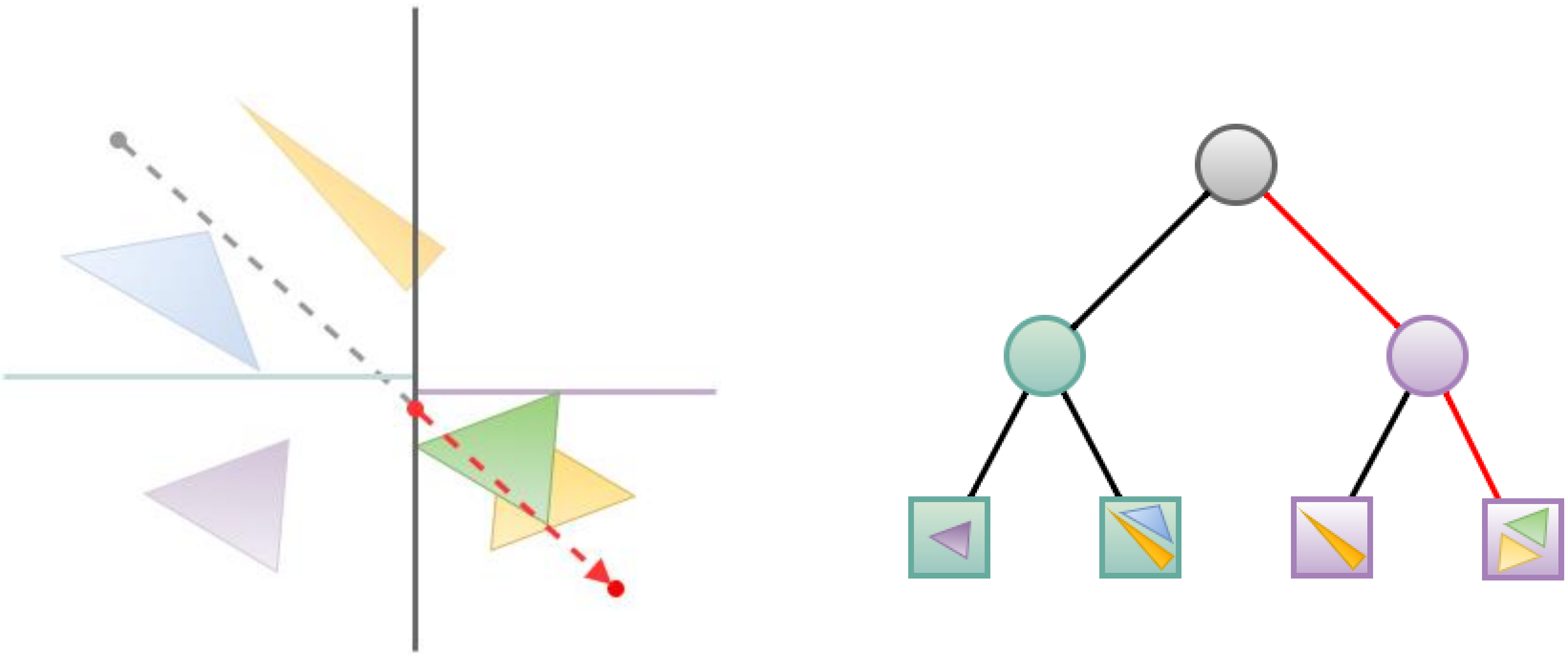

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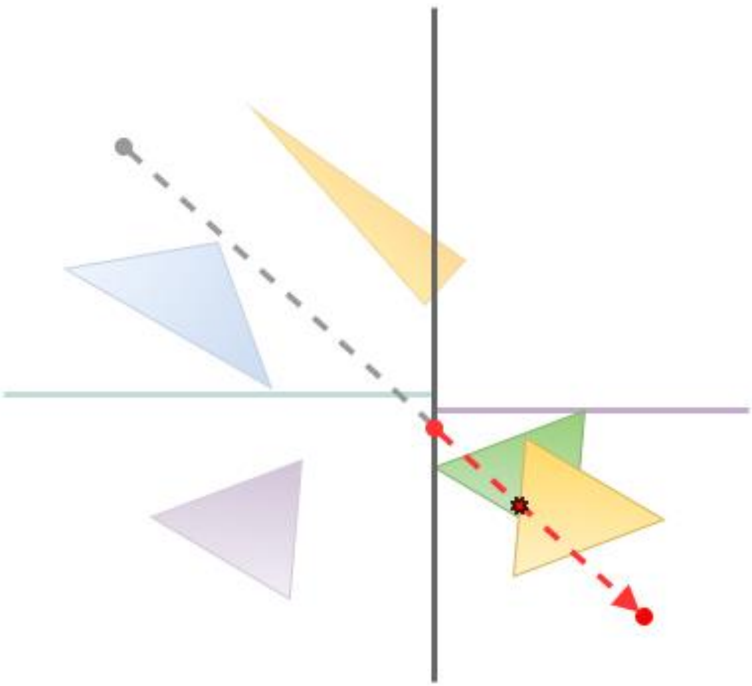

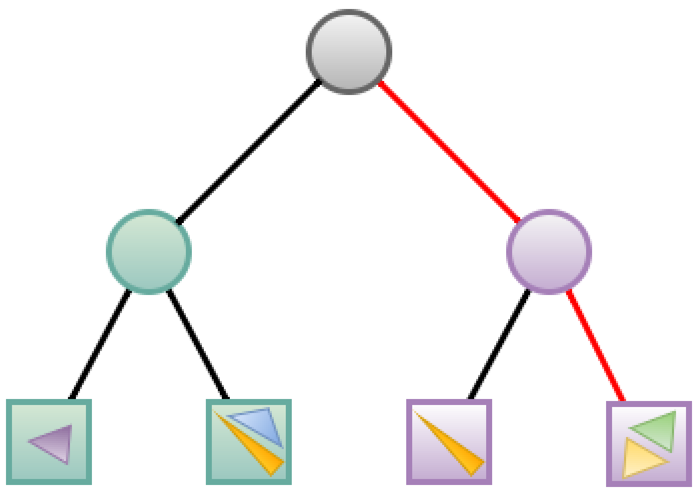

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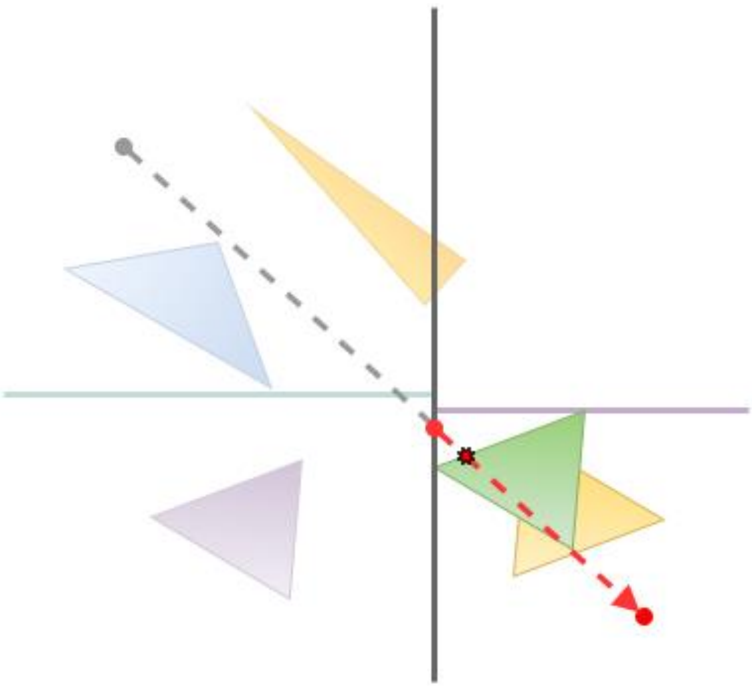
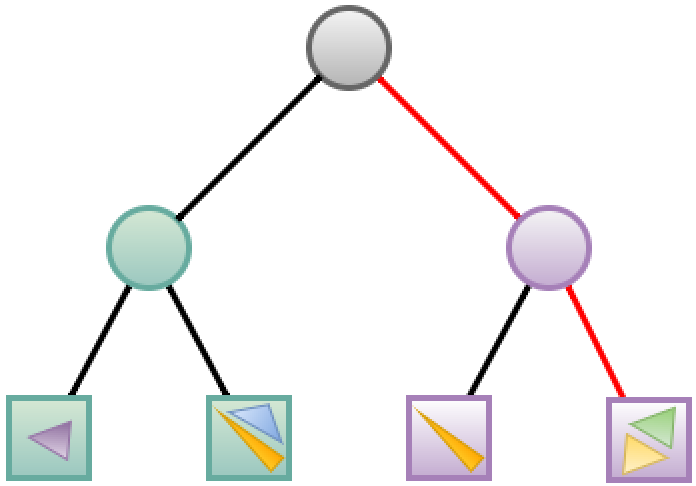


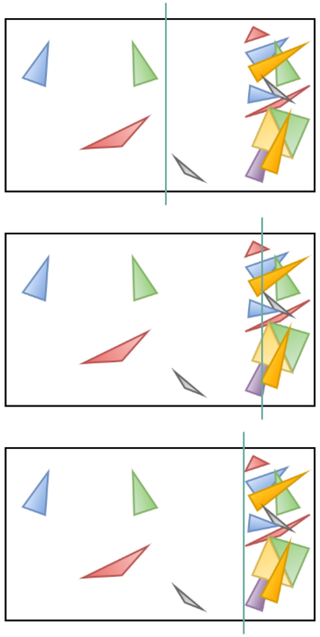


kd-tree

CONSTRUCTION

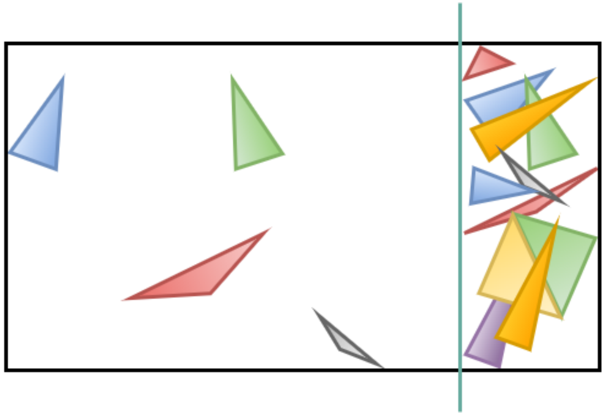
Construction

- Given a list of polygons, build a *kd*-tree such that average query time is low
- Different heuristics
  - Spatial median
  - Object median
  - Cost function (Surface Area Heuristic)
- Implemented *kd*-tree construction based on Surface Area Heuristic
- Based on the $O(n \log^2 n)$ approach in [1]
  - Paper also describes $O(n \log n)$ algorithm



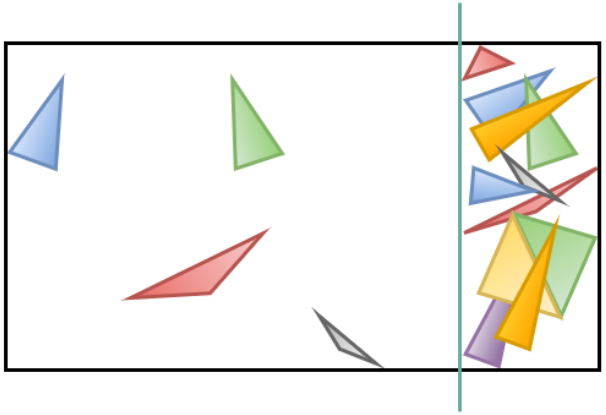
Surface Area Heuristic

- Assigns costs to splits
- Lowest scoring splits (*perfect* splits) are optimal under the following assumptions:
  - Assuming uniformly distributed rays penetrating the bounding box
  - kd-tree traversal costs and triangle intersection costs are known



Surface Area Heuristic

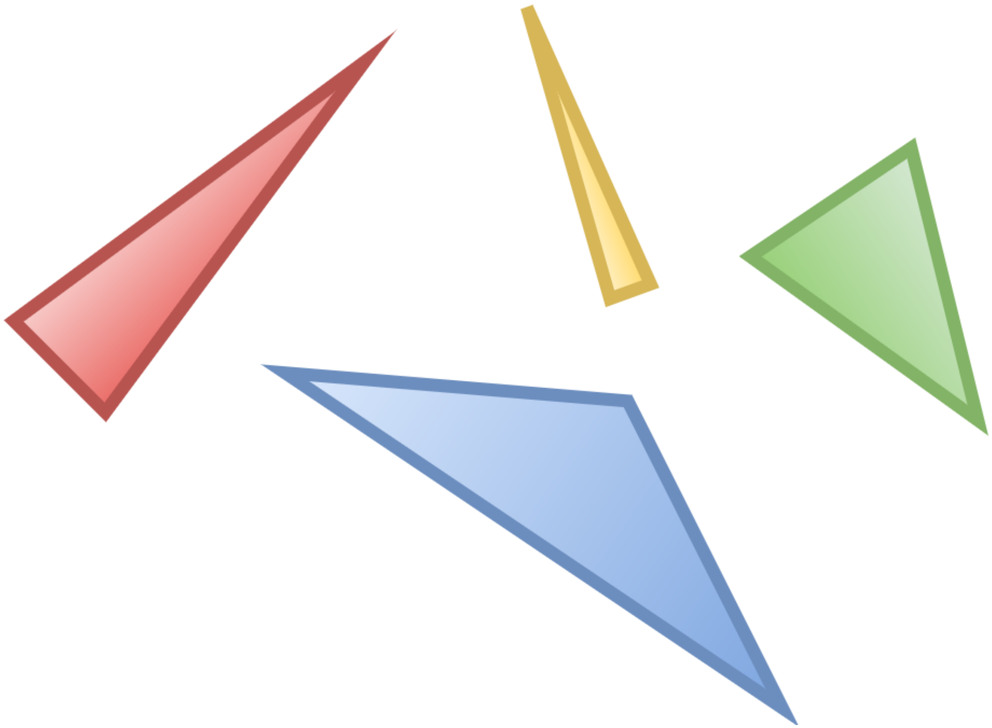
- The cost function depends on
  - The surface area of the current node‘s bounding box
  - The surface areas of the split bounding boxes
  - The number of triangles intersecting the right or left split bounding box



Construction

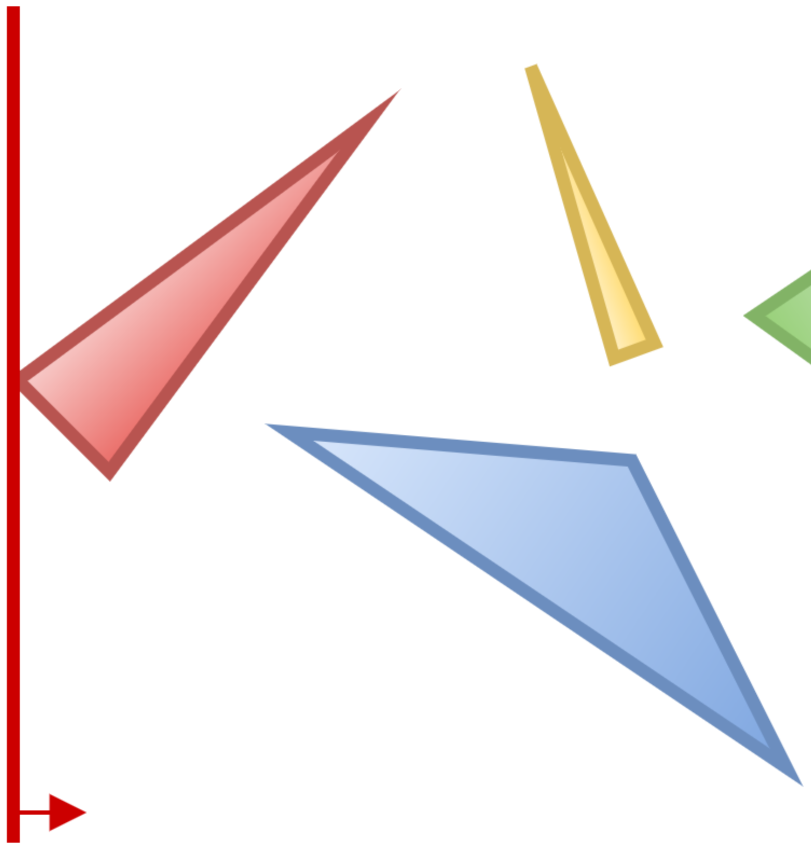
- SAH needs to evaluate the cost for each possible split
  - Perfect split will be at the begin or end of a triangle
  - $O(n)$ split candidates to evaluate per recursion step
    - Naive: Compute number of triangles in right and left child in $O(n)$, resulting in $O(n^2)$ overall
- Sweep-line based algorithm for $O(n \log^2 n)$
  - Sweep along dimension d
  - Regard begin and end of polygons as events ($O(n \log n)$ for sorting)
  - Compute the numbers of triangles and the associated minimum SAH cost incrementelly in $O(n)$

Sweep-line-based Perfect Split



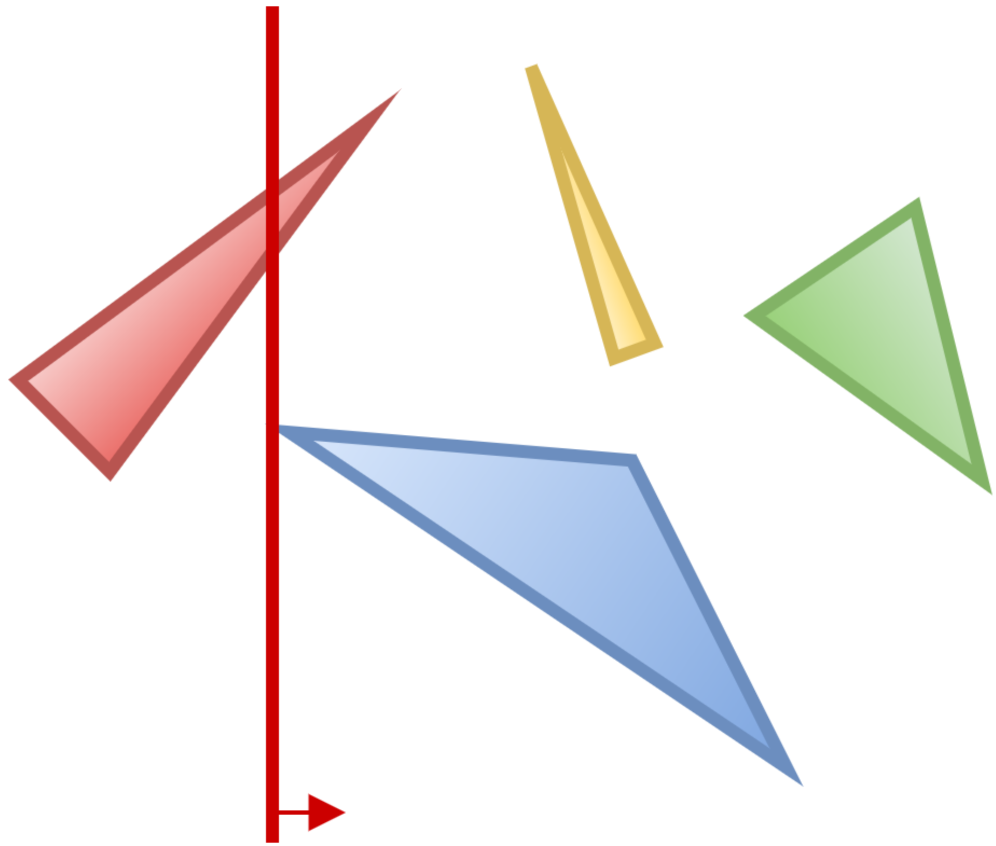
Triangles left of sweep line: 0
Triangles right of sweep line: 4

Sweep-line-based Perfect Split



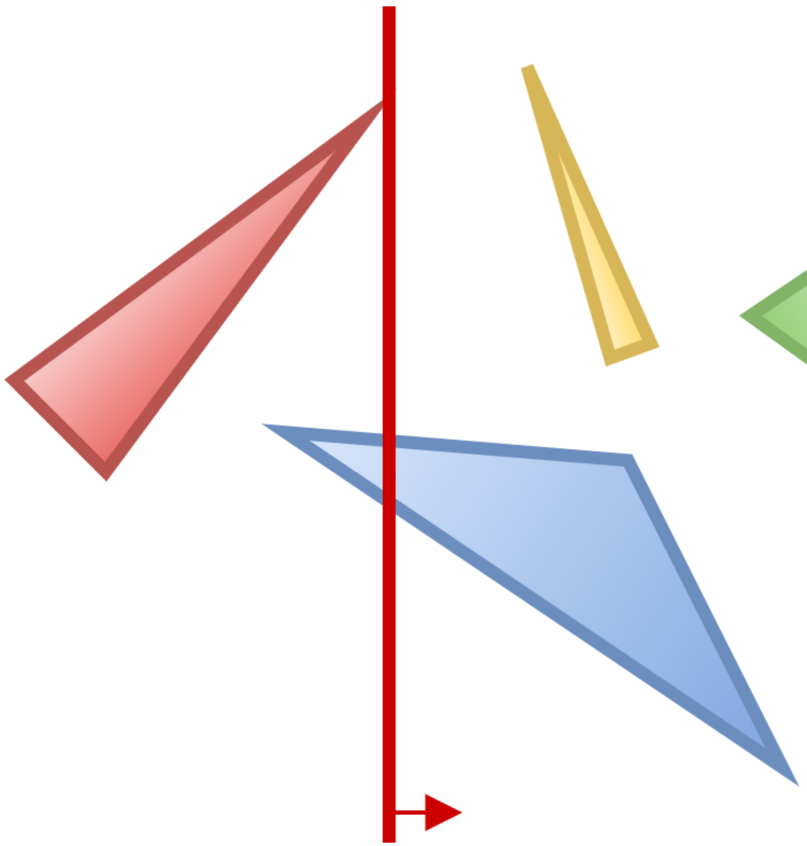
Triangles left of sweep line: 1
Triangles right of sweep line: 4

Sweep-line-based Perfect Split



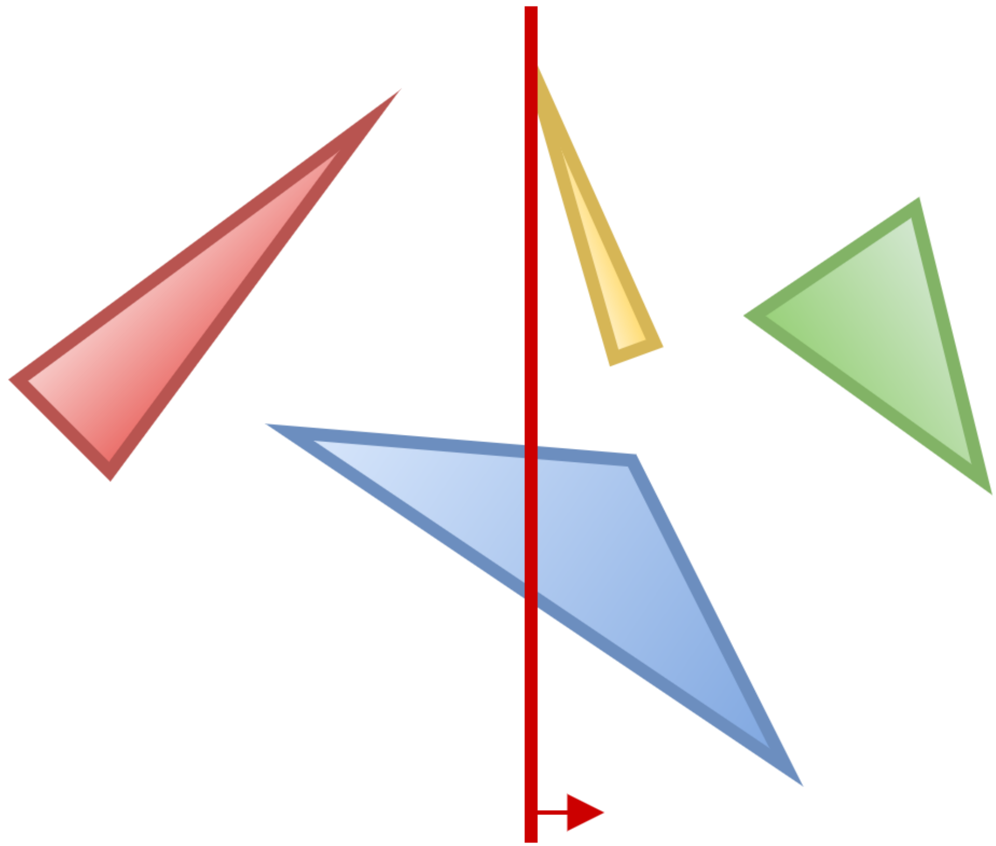
Triangles left of sweep line: 2
Triangles right of sweep line: 4

Sweep-line-based Perfect Split



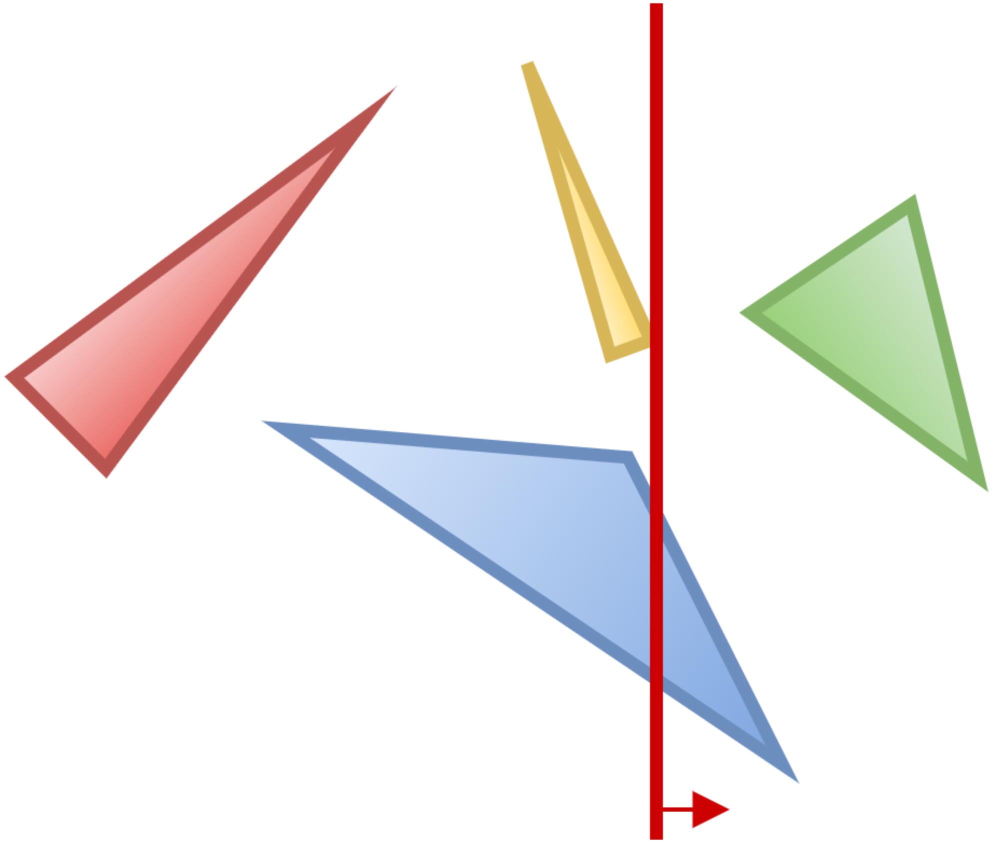
Triangles left of sweep line: 2
Triangles right of sweep line: 3

Sweep-line-based Perfect Split



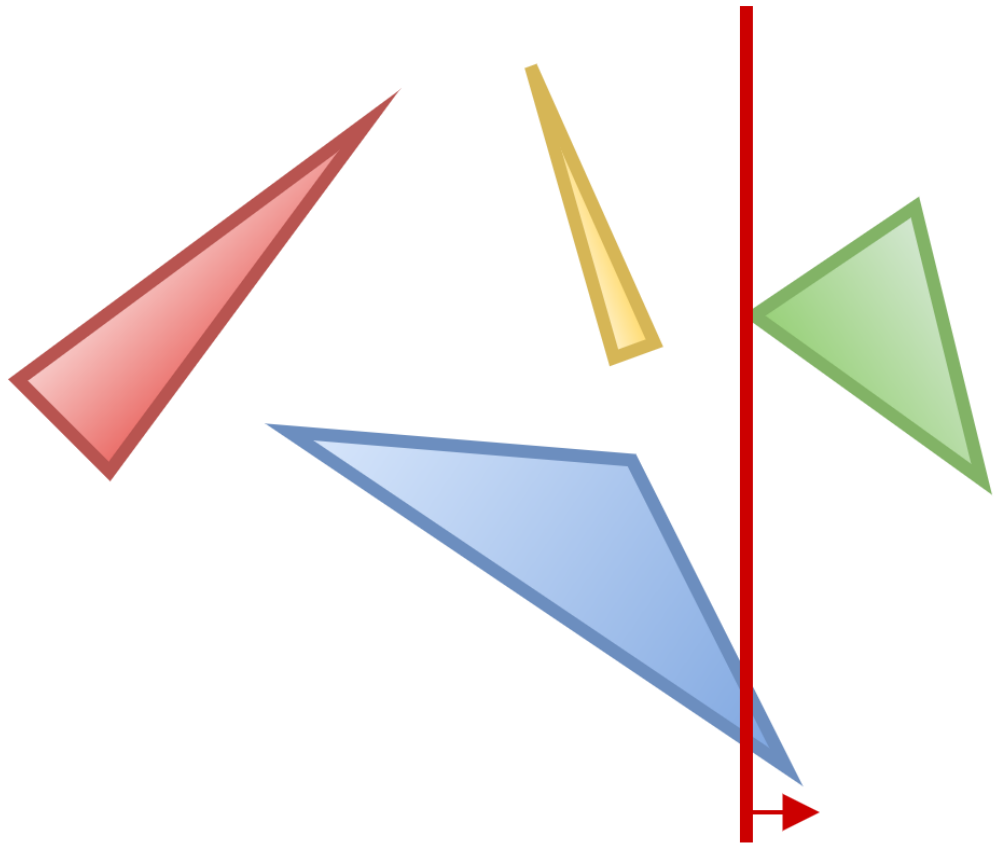
Triangles left of sweep line: 3
Triangles right of sweep line: 3

Sweep-line-based Perfect Split



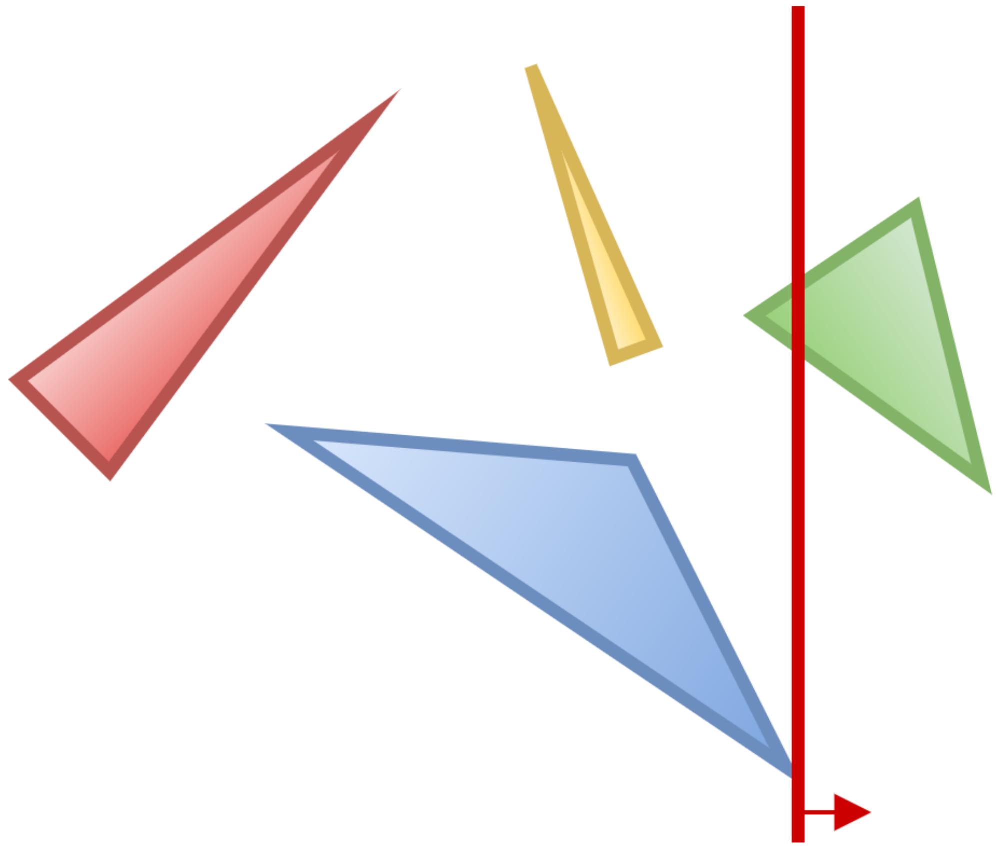
Triangles left of sweep line: 3
Triangles right of sweep line: 2

Sweep-line-based Perfect Split



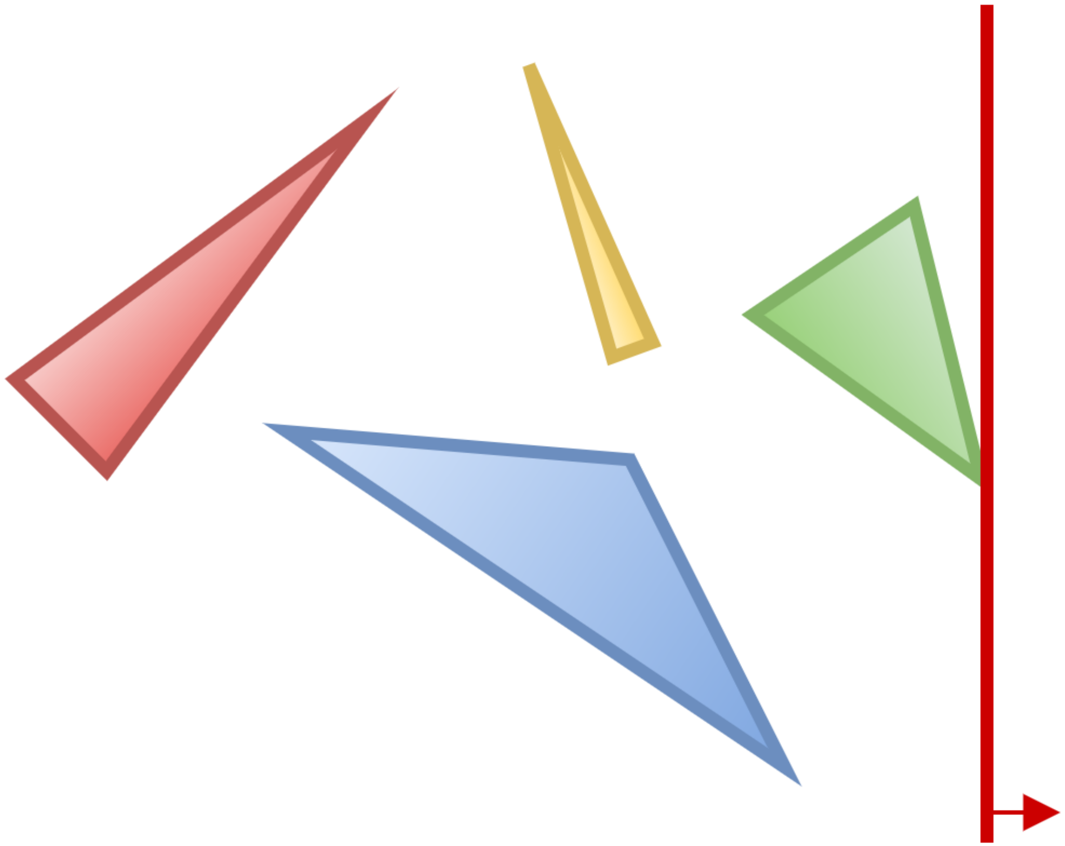
Triangles left of sweep line: 4
Triangles right of sweep line: 2

Sweep-line-based Perfect Split



Triangles left of sweep line: 4
Triangles right of sweep line: 1

Sweep-line-based Perfect Split



Triangles left of sweep line: 4
Triangles right of sweep line: 0

Construction

- Branch or leaf it?
- Branch if and only if the perfect split costs less than intersecting every triangle
- Since tree depth is in $O(\log n)$, we construct the *kd*-tree in $O(n \log^2 n)$
- By sorting only once and maintaining the sort order, construction could happen in $O(n \log n)$ [1]

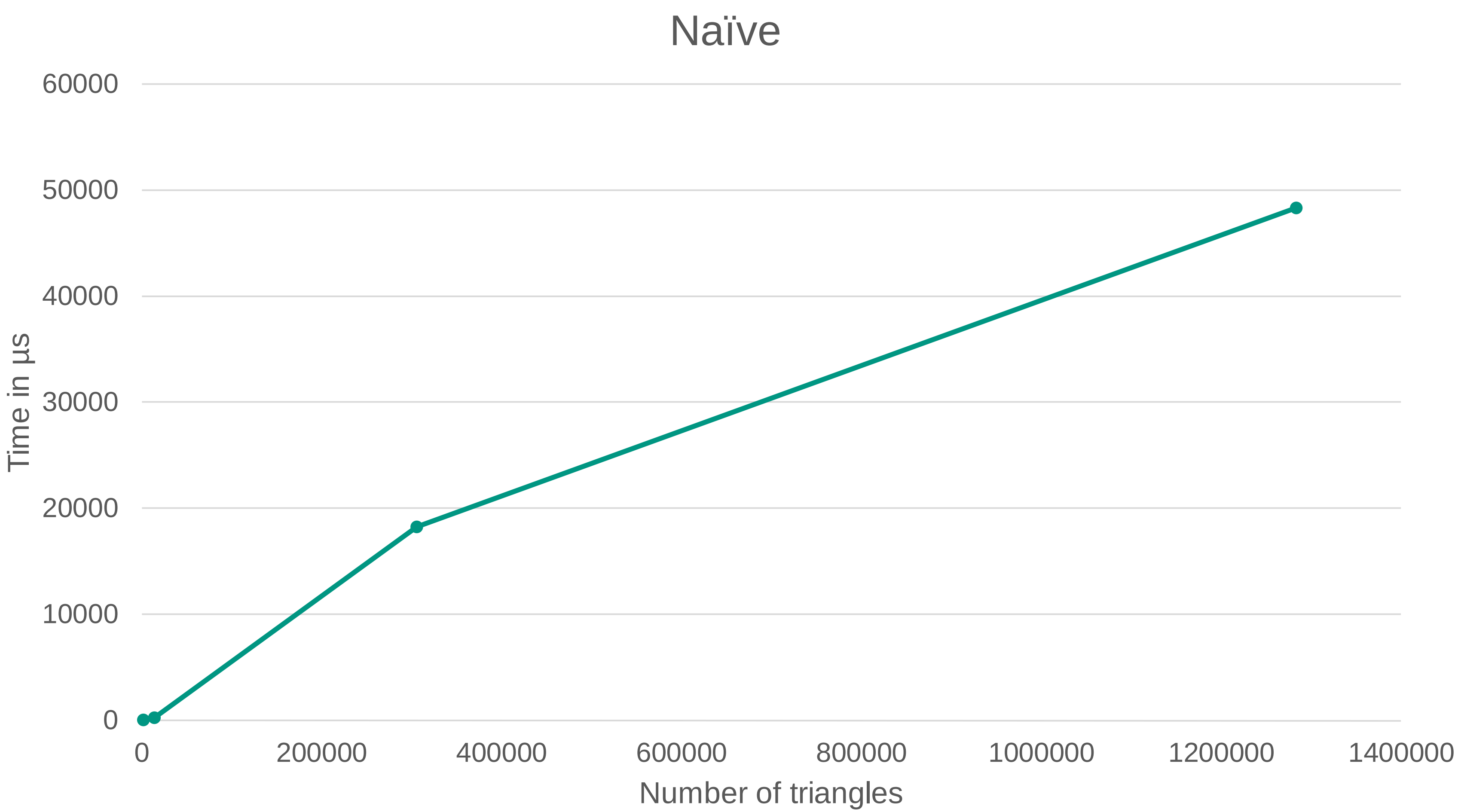
Performance

Performance

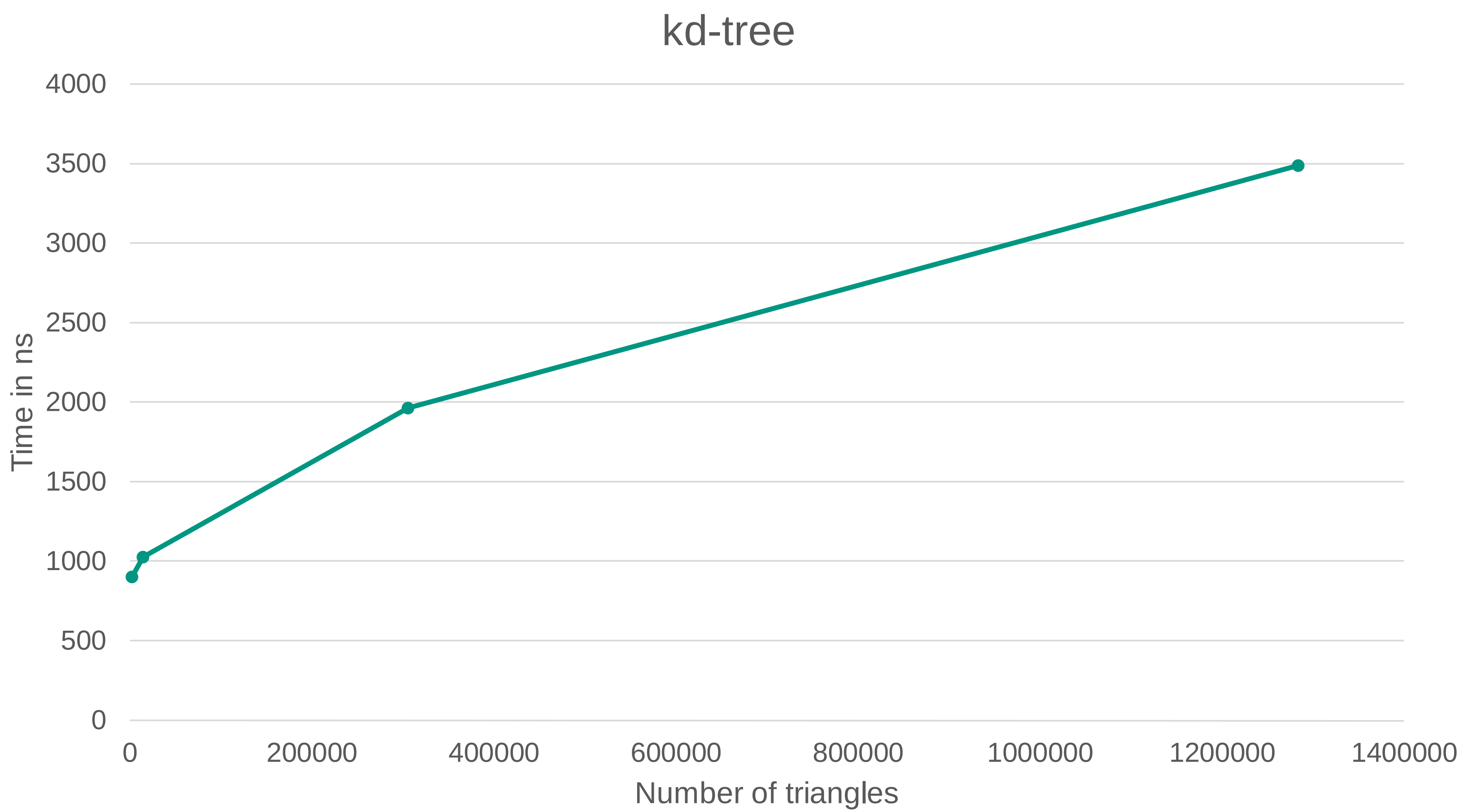
- Benchmark system:
  - Intel Xeon E3-1231v3 4x 3.40GHz
  - Windows
  - Java HotSpot 64-Bit Server VM (build 25.66-b18)
  - Max. JVM heap size: 4 GB

number of triangles	**naïve** (*µs* per random query)	**kd-tree** (*ns* per random query)
2,117	29.9	902.2
14,485	242.4	1025.0
305,662	18244.5	1961.5
1,283,858	48345.6	3489.4

Performance



Performance



References

- [1] On building fast kd-Trees for Ray Tracing, and on doing that in O(N log N) (2006), Ingo Wald, Vlastimil Havran