### **Nested CPR**

How to Keep a MR Intern Busy

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## CPR – Motivation

```
fac :: Int -> Int
fac 1 = 1
fac n = n * fac (n-1)
```

### **CPR** – Motivation

```
fac :: Int -> Int
fac (I# n) = case n of
   1# -> I# 1#
   _ -> case fac (I# (n -# 1#)) of
   I# m -> I# (n *# m)
```

### **CPR** – Motivation

```
fac (I# n) = case n of
  1# -> I# 1#
  _ -> case fac (I# (n -# 1#)) of
     I# m -> I# (n *# m)
```

```
fac n = I# (facw n)
facw (I# n) = case (case n of
    1# -> I# 1#
    _ -> case fac (I# (n -# 1#)) of
    I# m -> I# (n *# m)) of
    I# res -> res
```

```
fac n = I# (facw n)
facw (I# n) = case n of
  1# -> 1#
     -> n *# facw (I# (n -# 1#))
```

```
fac (I# n) = I# (facw n)
facw n = case n of
   1# -> 1#
        -> n *# facw (n -# 1#)
```

### When is this beneficial?

```
data E = Add E E | Lit Int
eval (Add l r) = eval l + eval r
eval (Lit n) = n
```

### When is this beneficial?

```
data E = Add E E | Lit Int
eval e = Int# (evalw e)
evalw (Add l r) = evalw l +# evalw r
evalw (Lit n) = case n of I# n' -> n'
```

### When is this beneficial?

```
data E = Add E E | Lit Int
eval e = Int# (evalw e)
evalw (Add l r) = evalw l +# evalw r
evalw (Lit n) = case n of I# n' -> n'
sum $ map (eval . Lit) [1..1000000]
```

#### Nested CPR - Motivation

```
facI0 :: Int -> IO Int
facI0 1 = pure 1
facI0 n = do
    m <- facI0 (n-1)
    pure (n * m)</pre>
```

#### Nested CPR - Motivation

```
facIO :: Int -> World -> (World, Int)
facIO 1 s = (s, 1)
facIO n s =
  let (s', m) = facIO (n-1) s in
  (s', n * m)
```

#### Nested CPR – Motivation

```
facIO :: Int -> World -> (World, Int)
facIO (I# n) s = case facIOw n s of
   (# s, n #) -> (s, n)
facIOw :: Int# -> World -> (# World, Int #)
facIOw 1# s = (# s, 1 #)
facIOw n s = case facIOw (n -# 1#) s of
   (# s', m #) -> (# s', I# n * m #)
```

#### Nested CPR – Motivation

```
facIO :: Int -> World -> (World, Int)
facIO (I# n) s = case facIOw n s of
   (# s, n #) -> (s, n)
facIOw :: Int# -> World -> (# World, Int #)
facIOw 1# s = (# s, I# 1 #)
facIOw n s = case facIOw (n -# 1#) s of
   (# s', I# m #) -> (# s', I# (n *# m) #)
```

#### Nested CPR – Motivation

```
facIO :: Int -> World -> (World, Int)
facIO (I# n) s = case facIOww n s of
   (# s, n #) -> (s, I# n)
facIOww :: Int# -> World -> (# World, Int# #)
facIOww 1# s = (# s, 1# #)
facIOww n s = case facIOww (n -# 1#) s of
   (# s', m #) -> (# s', n *# m #)
```

```
f :: Int -> (Int, Int)
f x = (x-1, x+1)
```

#### Careful!

• Divergence

### Careful!

Divergence

```
f :: Int -> (Int, Int)
f x = (x-1, x+1)

f' :: Int -> (Int, Int)
f' x = case f'w x of
  (# a, b #) -> (I# a, I# b)
f'w (I# x) = (# x -# 1#, x +# 1# #)
```

### Careful!

Divergence

```
f :: Int -> (Int, Int)
f x = (x-1, x+1)
f' :: Int -> (Int, Int)
f' x = case f'w x of
  (\# a, b \#) \rightarrow (I\# a, I\# b)
f'w(I\# x) = (\# x - \# 1\#, x + \# 1\# \#)
> f (error "") `seq` ()
()
> f' (error "") `seq` ()
```

```
f :: Int -> (Int, Int)
f x = x `seq` (x-1, g x)
```

#### Careful!

- Divergence
- Speculation

$$g :: Int \rightarrow Int$$

$$g n = 0$$

$$g n = n + g (n-1)$$

#### Careful!

- Divergence
- Speculation

```
f :: Int -> (Int, Int)
f x = case fw x of
   (# a, b #) -> (I# a, I# b)
fw x = (# x -# 1#, gw x #)

gw :: Int# -> Int#
gw n = 0#
```

gw n = n + # g (n - # 1#)

#### Careful!

- Divergence
- Speculation

```
f :: Int -> (Int, Int)
f x = case fw x of
  (\# a, b \#) \rightarrow (I\# a, I\# b)
fw x = (# x - # 1#, gw x #)
gw :: Int# -> Int#
gw n = 0#
gw n = n + \# g (n - \# 1\#)
> sum $ map (fst . f) [0..1000000]
\Sigma
```

```
data E = Add E E | Lit Int
eval (Add l r) = eval l + eval r
eval (Lit n) = n
```

```
data E = Add E E | Lit Int
eval e = Int# (evalw e)
evalw (Add l r) = evalw l +# evalw r
evalw (Lit n) = case n of I# n' -> n'
```

```
data E = Add E E | Lit Int
eval e = Int# (evalw e)
evalw (Add l r) = evalw l +# evalw r
evalw (Lit n) = case n of I# n' -> n'
sum $ map (eval . Lit) [1..1000000]
```

```
data E = Add E E | Lit Int
eval (Add l r) = eval l + eval r
eval (Lit n) = n
```

```
data E = Add E E | Lit Int
eval (Add l r) = I# (eval_ub l #+ eval_ub r)
eval (Lit n) = n
eval_ub (Add l r) = eval_ub l #+ eval_ub r
eval_ub (Lit n) = case n of I# n' -> n'
sum $ map (eval . Lit) [1..1000000]

\[
\iff "Return-pattern specialisation"
```

# Return Pattern Specialisation vs. Nested CPR

```
facIO :: Int -> IO Int
facIO 1 = pure 1
facIO n = do
    m <- facIO (n-1)
    pure (n * m)

main = facIO 1000 >>= print
```

## Return Pattern Specialisation vs. Nested CPR

```
facIO :: Int -> World -> (World, Int)
facIO 1 s = (s, 1)
facIO n s = case facIO (n-1) s of
   (s', m) -> (s', n * m)

main s = case facIO 1000 s of
   (s', I# n) -> printw n s'
```

## Return Pattern Specialisation vs. Nested CPR

```
facIO :: Int -> World -> (World, Int)
facIO 1 s = (s, 1)
facIO n s = case facIO (n-1) s of
  (s', m) \rightarrow (s', n * m)
facIO_ub :: Int# -> World -> (# World, Int# #)
facIO ub 1# s = (# s, 1# #)
facIO_ub n s = case facIO_ub (n -# 1#) s of
  (# s', m #) -> (# s', n *# m #)
main s = case facIO_ub 1000 s of
  (# s', n #) -> printw n s'
```

## Other Possible Topics

- Reliable Constructor Specialisation for functions/lambdas
  - Zero-overhead concat in Stream Fusion!
- Pattern-match Checking
- Backend work?
- Frontend work?