

Solving Data-flow Problems in Syntax Trees

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September 9, 2017

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- My master's thesis¹: Call Arity vs. Demand Analysis
 - Result: Usage Analysis generalising Call Arity
 - Precision of Call Arity without co-call graphs
- Requirements led to complex analysis order
- *Specification* of data-flow problem decoupled from its *solution*

¹<https://pp.ipd.kit.edu/uploads/publikationen/graf17masterarbeit.pdf>

Strictness Analysis

- Provides lower bounds on *evaluation cardinality*
- Is this variable evaluated at least once?
 - *Strictness*: $\text{Str} ::= S \mid L$
 - Strict (Yes!)
 - Lazy (Not sure)
- Enables call-by-value, unboxing

```
main = do
  let x = ... -- S
  let y = ... -- S
  let z = ... -- L
  print (x + if odd y then y else z)
```

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GHC's Demand Analyser

- Performs strictness analysis (among other things)
- Fuels Worker/Wrapper transformation
- Backward analysis
 - Which strictness does an expression place on its free variables?
 - Which strictness does a function place on its arguments?
- *Strictness type*: $\text{StrType} = \langle \text{FVs} \rightarrow \text{Str}, \text{Str}^* \rangle$

Strictness Signatures

- Looks at the right-hand side of `const` before the `let` body!
- *Unleashes* strictness type of `const`'s RHS at call sites

```
let const a b = a -- const :: ⟨[], [S, L]⟩  
in const  
    y                -- S  
    (error "💣")      -- L
```

Call Context Matters

- Whole expression is strict in **z**
- Only digests **f** for manifest arity 1, can't look under lambda
- **f** is called with 2 arguments

```
let f x = -- f ::  $\langle [z \mapsto L], [S] \rangle$   
    if odd x  
        then \y -> y*z  
        else \y -> y+z  
in f 1 2
```

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```
let f x = -- f ::  $\langle [z \mapsto L], [S] \rangle$   
    if odd x  
        then \y -> y*z  
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in seq (f 1) 42
```


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Call Context Matters

- Solution: Analyse RHS when incoming arity is known
- Formally: Finite approximation of *strictness transformer*
 - $\text{StrTrans} = \mathbb{N} \rightarrow \text{StrType}$
- Exploit laziness to memoise results?

```
let f x = -- f1 :: ⟨ [z ↦ L], [S] ⟩
    if odd x
    then \y -> y*z
    else \y -> y+z
in f 1 2
```

Call Context Matters

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- Formally: Finite approximation of *strictness transformer*
 - $\text{StrTrans} = \mathbb{N} \rightarrow \text{StrType}$
- Exploit laziness to memoise results?

```
let f x = -- f2 :: ⟨ [z ↦ S], [S, S] ⟩  
    if odd x  
    then \y -> y*z  
    else \y -> y+z  
in f 1 2
```

Recursion

- Exploit laziness to memoise approximations?
- ✗ Recursion leads to termination problems
- Rediscovered fixed-point iteration, detached from the syntax tree
- Leads to data-flow framework, solved by worklist algorithm

```
let fac n =  
    if n == 0  
    then 1  
    else n * fac (n-1)  
in fac 12
```

Data-flow Framework for Strictness Analysis

- Allocate nodes to break recursion
 - One top-level node
 - One node per pair of (`let` binding, incoming arity)
- Initialise worklist to top-level node
- Initialise nodes with \perp

```
let f 0 = const 0
    f 1 = id
    f n = f (n 'mod' 2)
in f x y
```



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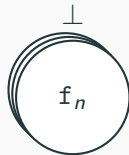
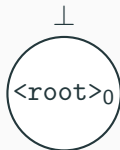


Worklist: {<root>₀}

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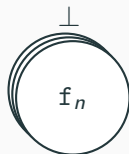
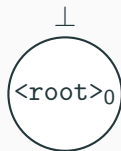
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in f x y
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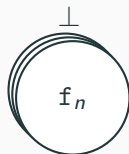
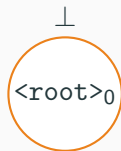
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    f 1 = id  
    f n = f (n 'mod' 2)  
in f x y
```



Worklist: {<root>_0}

Data-flow Framework for Strictness Analysis

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    f 1 = id  
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in f x y
```



Worklist: {}

Data-flow Framework for Strictness Analysis

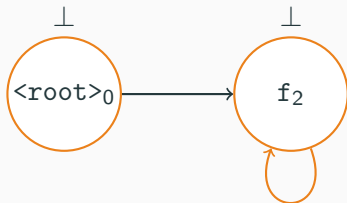
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Worklist: $\{\}$

Data-flow Framework for Strictness Analysis

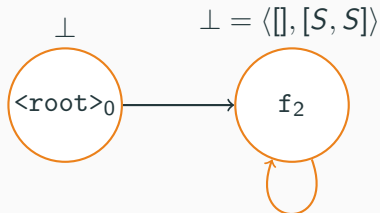
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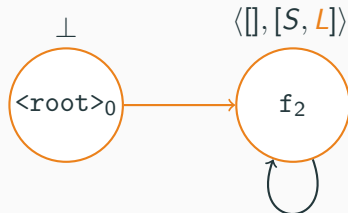
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Worklist: $\{\}$

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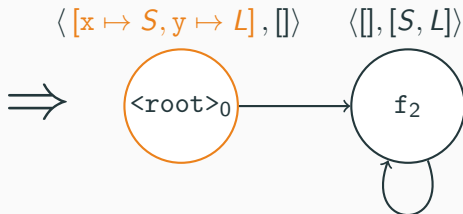
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    f 1 = id  
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```



Worklist: $\{f_2, \langle \text{root} \rangle_0\}$

Data-flow Framework for Strictness Analysis

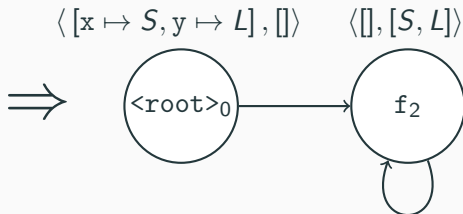
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Worklist: $\{f_2\}$

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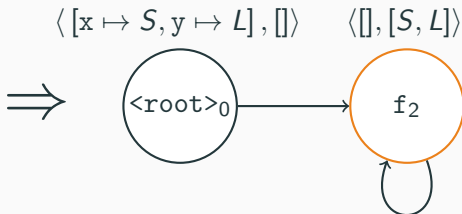
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in f x y
```



Worklist: $\{f_2\}$

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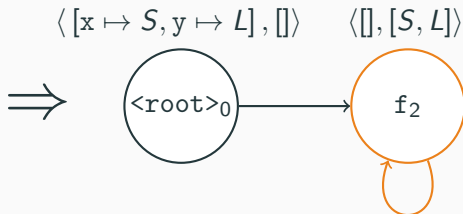
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in f x y
```



Worklist: $\{\}$

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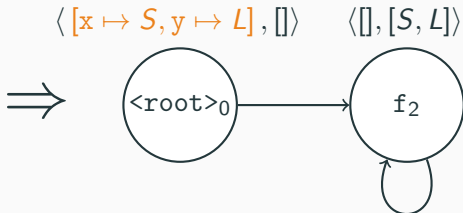
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    f 1 = id  
    f n = f (n 'mod' 2)  
in f x y
```



Worklist: $\{\}$

Data-flow Framework for Strictness Analysis

```
let f 0 = const 0  
    f 1 = id  
    f n = f (n 'mod' 2)  
in f x y
```



Worklist: $\{\}$

Implementation

- Hide iteration strategy behind `TransferFunction` monad
- Data-flow nodes `k`, denoting lattice `v`
- Single 'impure' primitive `dependOn`

```
data TransferFunction k v a
instance Monad (TransferFunction k v)
```

```
dependOn
  :: Ord k
  => k
  -> TransferFunction k v (Maybe v)
```

Implementation

- `DataFlowFramework` assigns `TransferFunction` and `ChangeDetector` to nodes

```
type ChangeDetector k v  
  = k -> v -> v -> Bool
```

```
data DataFlowFramework k v  
  = DFF  
  { transfer      :: k -> TransferFunction k v v  
  , detectChanges :: k -> ChangeDetector k v  
  }
```

Implementation

- `runFramework` solves data-flow problems
- Specification as `DataFlowFramework`
- Implements iteration strategy
 - Can use worklist algorithm, starting from a specified root set

```
runFramework
  :: Ord k
  => DataFlowFramework k v
  -> Set k
  -> Map k v
```

Applied to Strictness Analysis

- Denote expressions by their strictness transformer
- Model points of strictness transformer separately
- Instantiate as
`DataFlowFramework (ExprNode, Arity) StrType`
- `ExprNode`: Totally ordered, allocated as needed
 - Dictates priority in worklist
 - Performance depends on suitable priorities

Comparison to hoopl

- hoopl (Ramsey et al. 2010) works on CFGs
 - Our data-flow graph is implicit in functions/closures
- Designed for imperative languages
- ‘Operational’ rather than ‘denotational’
 - Control-flow rather than data-flow
- Makes (join semi-)lattice explicit
 - We should, too!
- Also includes a solution for transformations

Conclusion

- ✓ Goal: Decouple analysis logic from iteration logic by a graph-based approach
- ✗ Coupling not as painful as it would be in imperative programs
- ✓ Still obscures intent, even obstructs ideas
- ✓ Caching of analysis results as in Peyton Jones et al. (2006) between iterations for free
- ✗ Unclear how performance is affected
- ✗ Can only shine if shared concerns are actually extracted

References



Peyton Jones, Simon, Peter Sestoft, and John Hughes (2006).
Demand Analysis. URL: <https://www.microsoft.com/en-us/research/publication/demand-analysis/>.



Ramsey, Norman, João Dias, and Simon Peyton Jones (2010).
“Hoopl: A Modular, Reusable Library for Dataflow Analysis and Transformation”. In: *Proceedings of the Third ACM Haskell Symposium on Haskell*. Haskell '10. Baltimore, Maryland, USA: ACM, pp. 121–134. ISBN: 978-1-4503-0252-4. DOI: 10.1145/1863523.1863539. URL: <http://doi.acm.org/10.1145/1863523.1863539>.

Backup

Example

```
let f 0 = const 0  
    f 1 = id  
    f n = f (n 'mod' 2)  
in f x y
```



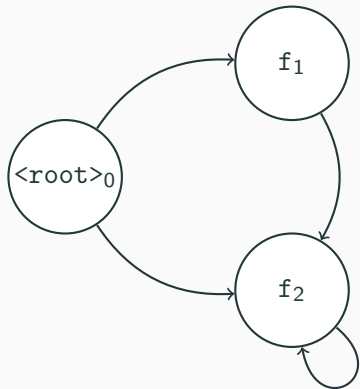
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```
let f 0 = const 0  
    f 1 = id  
    f n = f (n 'mod' 2)  
in seq (f x) (f x y)
```



Example

```
let f 0 = const 0  
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in seq (f x) (f x y)
```



Implementation: Behind the Curtain

- `TransferFunction` is a `State` monad around `WorklistState`

```
data TransferFunction node lattice a
  = TFM (State (WorklistState node lattice ) a)
  deriving (Functor, Applicative, Monad)
```

Threading annotated expressions

- Annotated **CoreExprs** are the reason why we do this!
- Thread it through all nodes:
DataFlowFramework (**ExprNode**, **Arity**) (**StrType**, **CoreExpr**)
- Complicates change detection
 - Expressions follow AST structure
 - Possibly change when strictness type did not
 - **ChangeDetector** has to check set of changed dependencies
- $\text{Str} ::= S \mid L$ not enough for annotating functions
 - $\text{Str} ::= S^n \mid L$ with arity $n \in \mathbb{N}$
 - 'f was called at least once, with at least n arguments'
- ... Or do it as the Demand Analyser does: Assume manifest arity for annotation
 - Be careful not to inline unsaturated wrappers!