Solving Data-flow Problems in Syntax Trees

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Introduction

- My master's thesis[1]: Call Arity vs. Demand Analysis
  - Result: Usage Analysis generalising Call Arity
  - Precision of Call Arity without co-call graphs
- Requirements led to complex analysis order
- *Specification* of data-flow problem decoupled from its *solution*

[1] https://pp.ipd.kit.edu/uploads/publikationen/graf17masterarbeit.pdf

Strictness Analysis

- Provides lower bounds on *evaluation cardinality*
- Is this variable evaluated at least once?
  - *Strictness*: Str ::= S | L
  - *S*trict (Yes!)
  - *L*azy (Not sure)
- Enables call-by-value, unboxing

```
main = do
  let  x = ... -- S
  let  y = ... -- S
  let  z = ... -- L
  print (x + if odd y then y else z)
```

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GHC's Demand Analyser

- Performs strictness analysis (among other things)
- Fuels Worker/Wrapper transformation
- Backward analysis
  - Which strictness does an expression place on its free variables?
  - Which strictness does a function place on its arguments?
- *Strictness type*: $\mathsf{StrType} = \langle \mathsf{FVs} \rightharpoonup \mathsf{Str}, \mathsf{Str}^* \rangle$

Strictness Signatures

- Looks at the right-hand side of `const` before the `let` body!
- *Unleashes* strictness type of `const`'s RHS at call sites

```
let const a b = a -- const :: ⟨[], [S, L]⟩
in const
    y             -- S
    (error "💣") -- L
```

Call Context Matters

- Whole expression is strict in `z`
- Only digests `f` for manifest arity 1, can't look under lambda
- `f` is called with 2 arguments

```
let f x = -- f :: ⟨[z ↦ L], [S]⟩
      if odd x
        then \y -> y*z
        else \y -> y+z
in f 1 2
```

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in seq (f 1) 42
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Call Context Matters

- Solution: Analyse RHS when incoming arity is known
- Formally: Finite approximation of *strictness transformer*
  - $\mathsf{StrTrans} = \mathbb{N} \to \mathsf{StrType}$
- Exploit laziness to memoise results?

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let f x = -- f₁ :: ⟨[z ↦ L], [S]⟩
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Call Context Matters

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Recursion

- Exploit laziness to memoise approximations?

✗ Recursion leads to termination problems

- Rediscovered fixed-point iteration, detached from the syntax tree
- Leads to data-flow problem, solved by worklist algorithm

```
let fac n =
      if n == 0
        then 1
        else n * fac (n-1)
in fac 12
```

Data-flow Graph for Strictness Analysis

- Allocate nodes to break recursion
  - One top-level node
  - One node per pair of (`let` binding, incoming arity)
- Initialise worklist to top-level node
- Initialise nodes with $\perp$

```
let f 0 = const 0
    f 1 = id
    f n = f (n `mod` 2)
in f x y
```

$\Longrightarrow$ <root>$_0$

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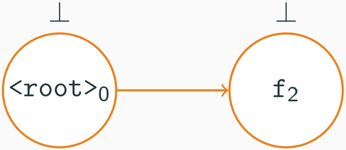

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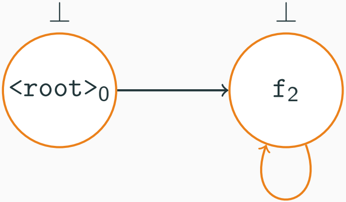

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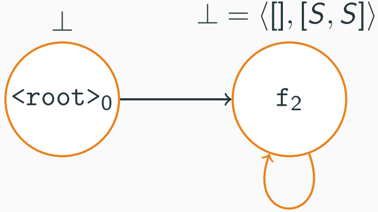

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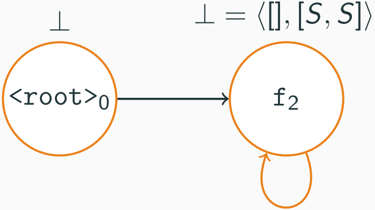

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Data-flow Graph for Strictness Analysis

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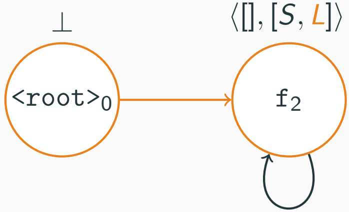

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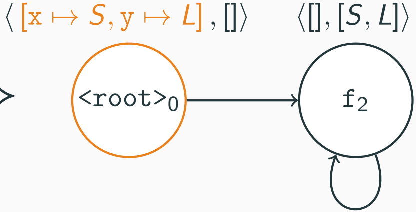

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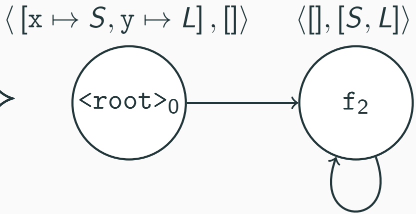

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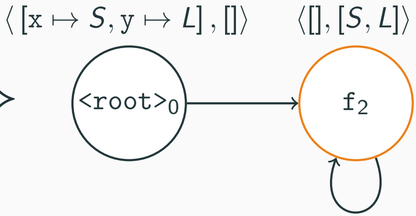

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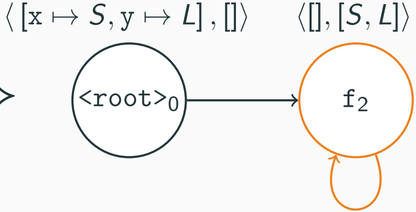

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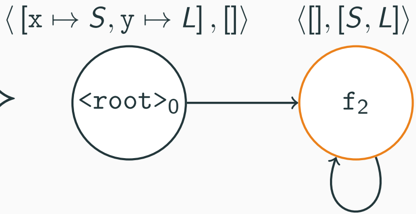

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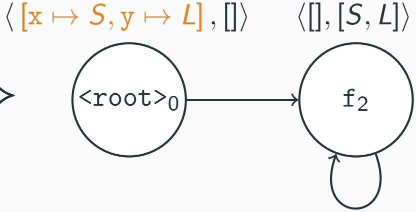

Worklist: $\{\}$

Implementation

- Hide iteration strategy behind `TransferFunction` monad
- Data-flow nodes `k`, denoting lattice `v`
- Single 'impure' primitive `dependOn`

```haskell
data TransferFunction k v a
instance Monad (TransferFunction k v)

dependOn
  :: Ord k
  => k
  -> TransferFunction k v (Maybe v)
```

Implementation

- DataFlowFramework assigns TransferFunction and ChangeDetector to nodes

```
type ChangeDetector k v
  = k -> v -> v -> Bool

data DataFlowFramework k v
  = DFF
  { transfer      :: k -> TransferFunction k v v
  , detectChanges :: k -> ChangeDetector k v
  }
```

Implementation

- `runFramework` solves data-flow problems
- Specification as `DataFlowFramework`
- Implements iteration strategy
  - Can use worklist algorithm, starting from a specified root set

```
runFramework
  :: Ord k
  => DataFlowFramework k v
  -> Set k
  -> Map k v
```

Applied to Strictness Analysis

- Denote expressions by their strictness transformer
- Model points of strictness transformer separately
- Instantiate as
  `DataFlowFramework (ExprNode, Arity) StrType`
- `ExprNode`: Totally ordered, allocated as needed
  - Dictates priority in worklist
  - Performance depends on suitable priorities

Comparison to `hoopl`

- `hoopl` (Ramsey et al. 2010) works on CFGs
    - Our data-flow graph is much less restrictive
    - Edges implicit in DSL
- Designed for imperative languages
- ‘Operational’ rather than ‘denotational’
    - Analysis along control-flow rather than data-flow
- Makes (join-semi)lattice explicit
    - TODO
- Also includes a solution for transformations

Discussion

- ✓ Decouple analysis logic from iteration logic by a graph-based approach
- ✗ Coupling not as painful as it would be in imperative programs
- ✓ Still obscures intent, even obstructs ideas
- ✓ ‘Hacks’ such as caching of analysis results as in Peyton Jones et al. (2006, §9.2) between iterations for free
- ✗ Unclear how performance is affected
- ✗ Can only shine if shared concerns are actually extracted from a number of analyses

Conclusion

- Pitched an interesting idea that came out of my thesis
- Separate *specification* of the data-flow problem from computing its *solution*
- Graph-based abstraction, single solver
- Future Work:
  1. Monotone maps with partially-ordered keys[2]
  2. Polish API, make a package
  3. Testdrive and measure it in GHC

[2] `https://github.com/sgraf812/pomaps/`

Bibliography

Peyton Jones, Simon, Peter Sestoft, and John Hughes (2006). *Demand Analysis*. URL: https://www.microsoft.com/en-us/research/publication/demand-analysis/.

Ramsey, Norman, João Dias, and Simon Peyton Jones (2010). "Hoopl: A Modular, Reusable Library for Dataflow Analysis and Transformation". In: *Proceedings of the Third ACM Haskell Symposium on Haskell*. Haskell '10. Baltimore, Maryland, USA: ACM, pp. 121–134. ISBN: 978-1-4503-0252-4. DOI: 10.1145/1863523.1863539. URL: http://doi.acm.org/10.1145/1863523.1863539.

Backup

Example

```
let f 0 = const 0
    f 1 = id
    f n = f (n `mod` 2)
in f x y
```

$\Longrightarrow$

Example

```
let f 0 = const 0
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in seq (f x) (f x y)
```

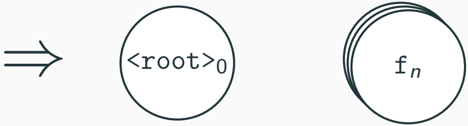

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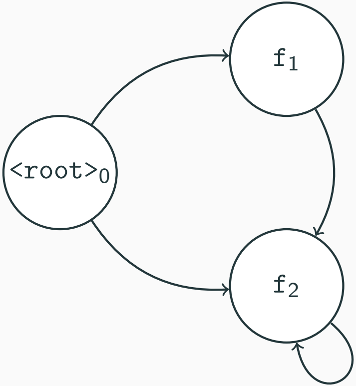

Implementation: Behind the Curtain

- `TransferFunction` is a `State` monad around `WorklistState`

```
data TransferFunction node lattice a
  = TFM (State (WorklistState node lattice ) a)
  deriving (Functor, Applicative, Monad)
```

Threading annotated expressions

- Annotated `CoreExpr`s are the reason why we do this!
- Thread it through all nodes:
  `DataFlowFramework (ExprNode, Arity) (StrType, CoreExpr)`
- Complicates change detection
  - Expressions follow AST structure
  - Possibly change when strictness type did not
  - `ChangeDetector` has to check set of changed dependencies
- Str $::= S \mid L$ not enough for annotating functions
  - Str $::= S^n \mid L$ with arity $n \in \mathbb{N}$
  - '`f` was called at least once, with at least n arguments'
- … Or do it as the Demand Analyser does: Assume manifest arity for annotation
  - Be careful not to inline unsaturated wrappers!